Lab 4: Fuzzy Foundations & Visualisation

Module: Artificial Intelligence
Topic: Fuzzy Logic – Part 1

Learning Objectives

By the end of this lab, you will be able to:

1. Define and plot common membership functions (triangular, trapezoidal, Gaussian, sigmoid)
2. Compute membership degrees for crisp inputs
3. Implement fuzzy hedges from scratch and visualise their effect on membership functions
4. Apply fuzzy set operations: AND (min), OR (max), NOT (complement), and probabilistic OR
5. Fuzzify crisp inputs for the *dapping* example

Setup

Run the cell below to import the required libraries. We use `scikit-fuzzy` for its convenient membership function generators and `matplotlib` for plotting.

In [ ]:
```
import numpy as np
import skfuzzy as fuzz
import matplotlib.pyplot as plt

# Configure matplotlib for better display
plt.rcParams['figure.figsize'] = [10, 5]
plt.rcParams['font.size'] = 12
plt.rcParams['axes.grid'] = True
plt.rcParams['grid.alpha'] = 0.3
```

---

1. Crisp Sets vs. Fuzzy Sets

In classical (Boolean) set theory, an element either belongs to a set or it doesn't. The **characteristic function** of a crisp set A maps elements from a universe of discourse X to $\{0, 1\}$:

$$f_A(x) : X \to \{0, 1\}, \quad f_A(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{if } x \notin A \end{cases}$$

A **fuzzy set** generalises this by allowing partial membership. The membership function $\mu_A(x)$ maps elements to the continuous range $[0, 1]$:

$$\mu_A(x) : X \to [0, 1]$$

The **universe of discourse** is the range of all possible values for a chosen variable (e.g., human height in cm).

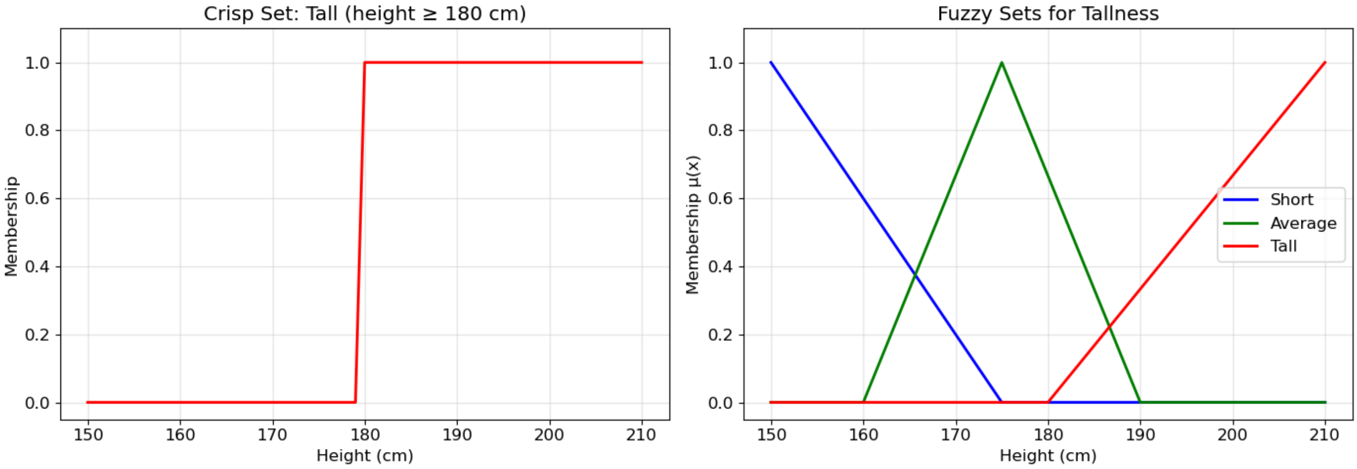
Exercise 1.1: Crisp vs. Fuzzy Tallness

The slides show crisp and fuzzy sets for *tallness*. The universe of discourse for height ranges from 150 cm to 210 cm.

Task: Plot a crisp set where "tall" means height ≥ 180 cm (Boolean boundary), and then plot a fuzzy version where tallness is a gradual transition. Use the following triangular fuzzy sets:

- **Short**: Triangle(150, 150, 175)
- **Average**: Triangle(160, 175, 190)
- **Tall**: Triangle(180, 200, 210)

In [ ]:
```
# Universe of discourse: height in cm
x_height = np.arange(150, 211, 1)

# --- Crisp set: tall if height >= 180 ---
crisp_tall = np.where(x_height >= 180, 1.0, 0.0)

# --- Fuzzy sets ---
short   = fuzz.trimf(x_height, [150, 150, 175])
# TODO: create here the "average" and "tall" sets


fig, (ax1, ax2) = plt.subplots(1, 2, figsize=(14, 5))

# Crisp
ax1.plot(x_height, crisp_tall, 'r-', linewidth=2)
ax1.set_title('Crisp Set: Tall (height ≥ 180 cm)')
ax1.set_xlabel('Height (cm)')
ax1.set_ylabel('Membership')
ax1.set_ylim(-0.05, 1.1)

# Fuzzy
ax2.plot(x_height, short,   'b-',  linewidth=2, label='Short')
ax2.plot(x_height, average, 'g-',  linewidth=2, label='Average')
ax2.plot(x_height, tall,    'r-',  linewidth=2, label='Tall')
ax2.set_title('Fuzzy Sets for Tallness')
ax2.set_xlabel('Height (cm)')
ax2.set_ylabel('Membership μ(x)')
ax2.set_ylim(-0.05, 1.1)
ax2.legend()

plt.tight_layout()
plt.show()

# Demonstrate partial membership
height = 184
mu_avg = fuzz.interp_membership(x_height, average, height)
# TODO: compute here the membership value for "tall"


print(f"A person who is {height} cm tall:")
print(f"  μ_average({height}) = {mu_avg:.2f}")
print(f"  μ_tall({height})    = {mu_tall:.2f}")
print(f"  → Partial membership in MULTIPLE sets simultaneously.")
```




```
A person who is 184 cm tall:
  μ_average(184) = 0.40
  μ_tall(184)    = 0.13
  → Partial membership in MULTIPLE sets simultaneously.
```

---

2. Common Membership Functions

The shape of a membership function determines how elements map to degrees of membership. The most common types are:

Type	Parameters	scikit-fuzzy function
Triangular	a (left foot), b (peak), c (right foot)	`fuzz.trimf(x, [a, b, c])`
Trapezoidal	a, b (left shoulder), c, d (right shoulder)	`fuzz.trapmf(x, [a, b, c, d])`
Gaussian	μ (mean), σ (std. dev.)	`fuzz.gaussmf(x, mean, sigma)`
Sigmoid	c (centre), a (slope)	`fuzz.sigmf(x, c, a)`

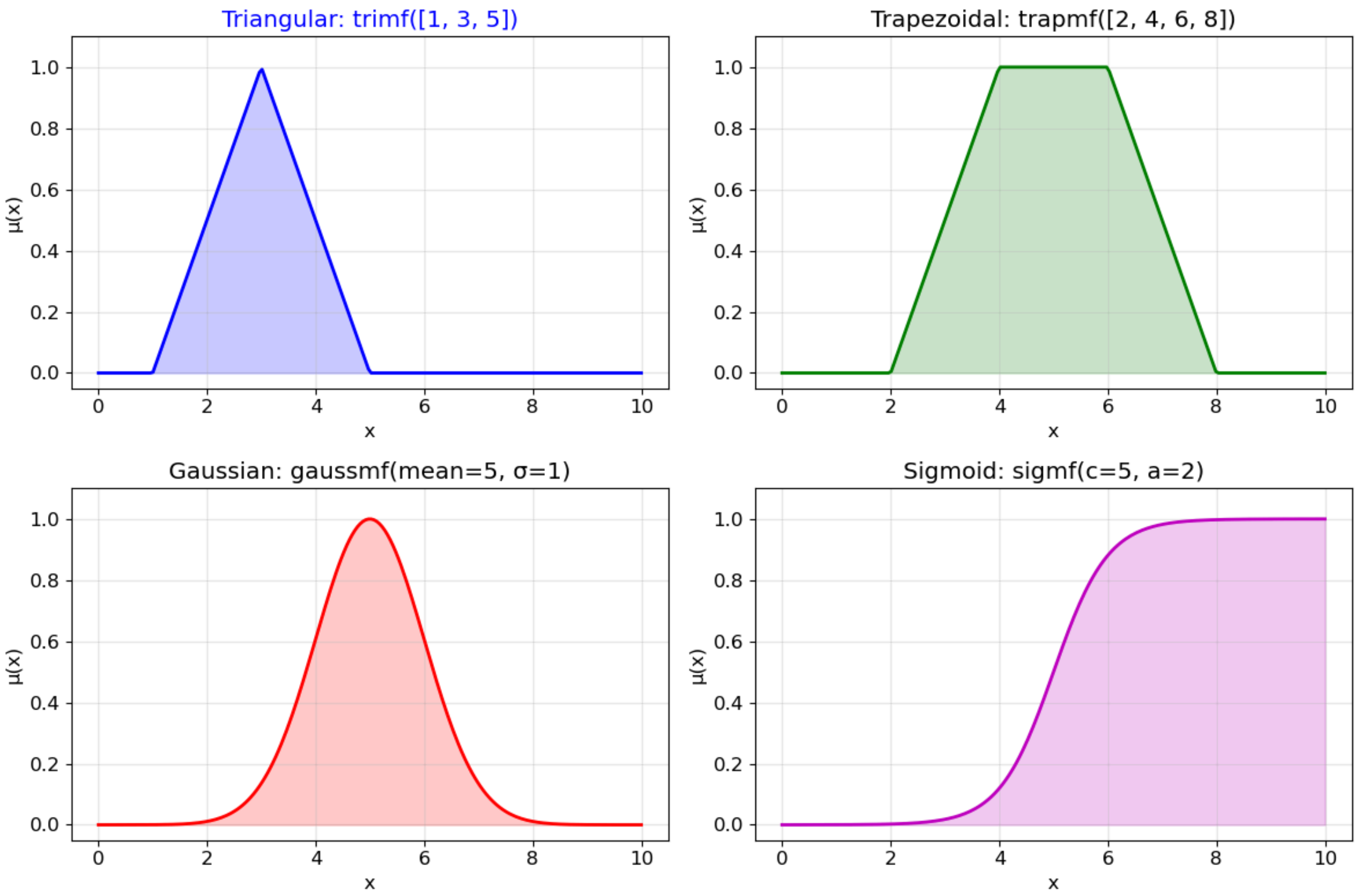
Exercise 2.1: Plot All Four Membership Function Types

Task: Using a universe of discourse from 0 to 10, create and plot one example of each membership function type. Choose parameters that make the differences visually clear.

In [ ]:

```python
x = np.linspace(0, 10, 200)

# Triangular: peak at 3
# TODO: create here a triangular membership function


# Trapezoidal: plateau from 4 to 6
# TODO: create here a trapezoidal membership function


# Gaussian: centred at 5, sigma=1
# TODO: create here a gaussian membership function


# Sigmoid: inflection at 5, slope=2 (positive = rising)
# TODO: create here a sigmoidal membership function


fig, axes = plt.subplots(2, 2, figsize=(12, 8))

axes[0, 0].plot(x, mf_tri, 'b-', linewidth=2)
```

```python
axes[0, 0].set_title('Triangular: trimf([1, 3, 5])', color='b')
axes[0, 0].fill_between(x, mf_tri, alpha=0.2, color='b')

# TODO: code here to visualise the other 3 membership functions


for ax in axes.flat:
    ax.set_xlabel('x')
    ax.set_ylabel('μ(x)')
    ax.set_ylim(-0.05, 1.1)

plt.tight_layout()
plt.show()
```



Exercise 2.2: The Dapping Example — Defining Linguistic Variables

Dapping is a traditional form of fly fishing common in the West of Ireland on the great western lakes (Lough Corrib, Lough Mask, Lough Conn, etc.). It involves impaling a live insect on a hook and letting the wind carry it across the water to tempt a trout. The success depends on **wind** and **temperature**: too calm and nothing happens, too stormy and you lose control.

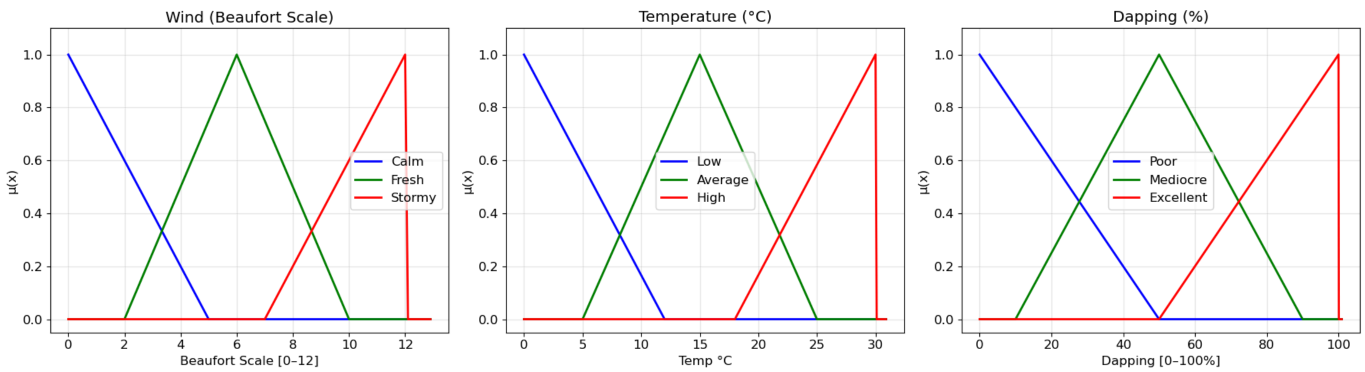
The following linguistic variables are used here:

Variable	Universe of Discourse	Fuzzy Sets
Wind	0–12 (Beaufort scale)	Calm(0, 0, 5), Fresh(2, 6, 10), Stormy(7, 12, 12)
Temperature	0–30 (°C)	Low(0, 0, 12), Average(5, 15, 25), High(18, 30, 30)
Dapping	0–100 (%)	Poor(0, 0, 50), Mediocre(10, 50, 90), Excellent(50, 100, 100)

Task: Define and plot all three linguistic variables with their fuzzy sets.

In [ ]:
```python
# --- Define universes of discourse ---
x_wind = np.arange(0, 13, 0.1)      # Beaufort scale 0-12
# TODO: code here the universes of discurse for
```

```python
#       temperature and dapping effectiveness


# --- Wind membership functions ---
wind_calm   = fuzz.trimf(x_wind, [0, 0, 5])
wind_fresh  = fuzz.trimf(x_wind, [2, 6, 10])
wind_stormy = fuzz.trimf(x_wind, [7, 12, 12])

# --- Temperature membership functions ---
# TODO: code the temperature membership functions here


# --- Dapping membership functions ---
# TODO: code the dapping membership functions here


# --- Plot ---
fig, (ax1, ax2, ax3) = plt.subplots(1, 3, figsize=(18, 5))

# Wind
ax1.plot(x_wind, wind_calm,   'b-', linewidth=2, label='Calm')
ax1.plot(x_wind, wind_fresh,  'g-', linewidth=2, label='Fresh')
ax1.plot(x_wind, wind_stormy, 'r-', linewidth=2, label='Stormy')
ax1.set_title('Wind (Beaufort Scale)')
ax1.set_xlabel('Beaufort Scale [0-12]')
ax1.set_ylabel('μ(x)')
ax1.legend()

# Temperature
# TODO: code here the figure for temperature


# Dapping
# TODO: code here the figure for dapping


for ax in [ax1, ax2, ax3]:
    ax.set_ylim(-0.05, 1.1)

plt.tight_layout()
plt.show()
```



Exercise 2.3: Computing Membership Degrees

Given crisp inputs, **fuzzification** determines the degree of membership in each fuzzy set. For a triangular MF with parameters (a, b, c):

$$\mu(x) = \begin{cases} 0 & \text{if } x \leq a \text{ or } x \geq c \\ \frac{x-a}{b-a} & \text{if } a < x \leq b \\ \frac{c-x}{c-b} & \text{if } b < x < c \end{cases}$$

Task: For the inputs **wind = 8** (Beaufort) and **temperature = 10°C**, compute the membership degree in every fuzzy set.

Note: The exact membership values depend on the precise triangle coordinates, which are read from the handout diagrams. The triangles we defined above are a close but imperfect approximation of those diagrams.

In [ ]:
```python
# Crisp inputs
wind_val = 8
temp_val = 10

# Fuzzify wind
mu_calm   = fuzz.interp_membership(x_wind, wind_calm,   wind_val)
mu_fresh  = fuzz.interp_membership(x_wind, wind_fresh,  wind_val)
mu_stormy = fuzz.interp_membership(x_wind, wind_stormy, wind_val)

# Fuzzify temperature
# TODO: code the temperature fuzzification here


print(f"Fuzzification for wind = {wind_val}, temperature = {temp_val}")
print(f"{'─' * 45}")
print(f"  μ_calm({wind_val})    = {mu_calm:.4f}")
print(f"  μ_fresh({wind_val})   = {mu_fresh:.4f}")
print(f"  μ_stormy({wind_val})  = {mu_stormy:.4f}")
print()
print(f"  μ_low({temp_val})     = {mu_low:.4f}")
print(f"  μ_average({temp_val}) = {mu_average:.4f}")
print(f"  μ_high({temp_val})    = {mu_high:.4f}")
```

```
Fuzzification for wind = 8, temperature = 10
─────────────────────────────────────────────
  μ_calm(8)    = 0.0000
  μ_fresh(8)   = 0.5000
  μ_stormy(8)  = 0.2000

  μ_low(10)     = 0.1667
  μ_average(10) = 0.5000
  μ_high(10)    = 0.0000
```

---

3. Hedges — Fuzzy Set Modifiers

Hedges are linguistic modifiers that alter the shape of a membership function. They either **concentrate** (narrow) or **dilate** (widen) a fuzzy set.

Hedge	Formula	Type	Exponent
Very	$\mu_A^{very}(x) = [\mu_A(x)]^2$	Concentration	2
Extremely	$\mu_A^{extremely}(x) = [\mu_A(x)]^3$	Concentration	3
Very Very		Concentration	4

Hedge	Formula	Type	Exponent
	$\mu_A^{very\ very}(x) = [\mu_A(x)]^4$		
Slightly	$\mu_A^{slightly}(x) = [\mu_A(x)]^{1.7}$	Concentration	1.7
A Little	$\mu_A^{a\ little}(x) = [\mu_A(x)]^{1.3}$	Concentration	1.3
More or Less	$\mu_A^{mol}(x) = \sqrt{\mu_A(x)}$	Dilation	0.5
Somewhat	$\mu_A^{somewhat}(x) = \sqrt[3]{\mu_A(x)}$	Dilation	1/3
Indeed	$2[\mu_A(x)]^2$ if $\mu \leq 0.5$; $1 - 2[1 - \mu_A(x)]^2$ if $\mu > 0.5$	Mixed	—
Not	$\mu_A^{not}(x) = 1 - \mu_A(x)$	—	—

Concentration (exponent > 1): narrows the set, reduces membership degrees.
Dilation (exponent < 1): widens the set, increases membership degrees.

Exercise 3.1: Implement Hedges from Scratch

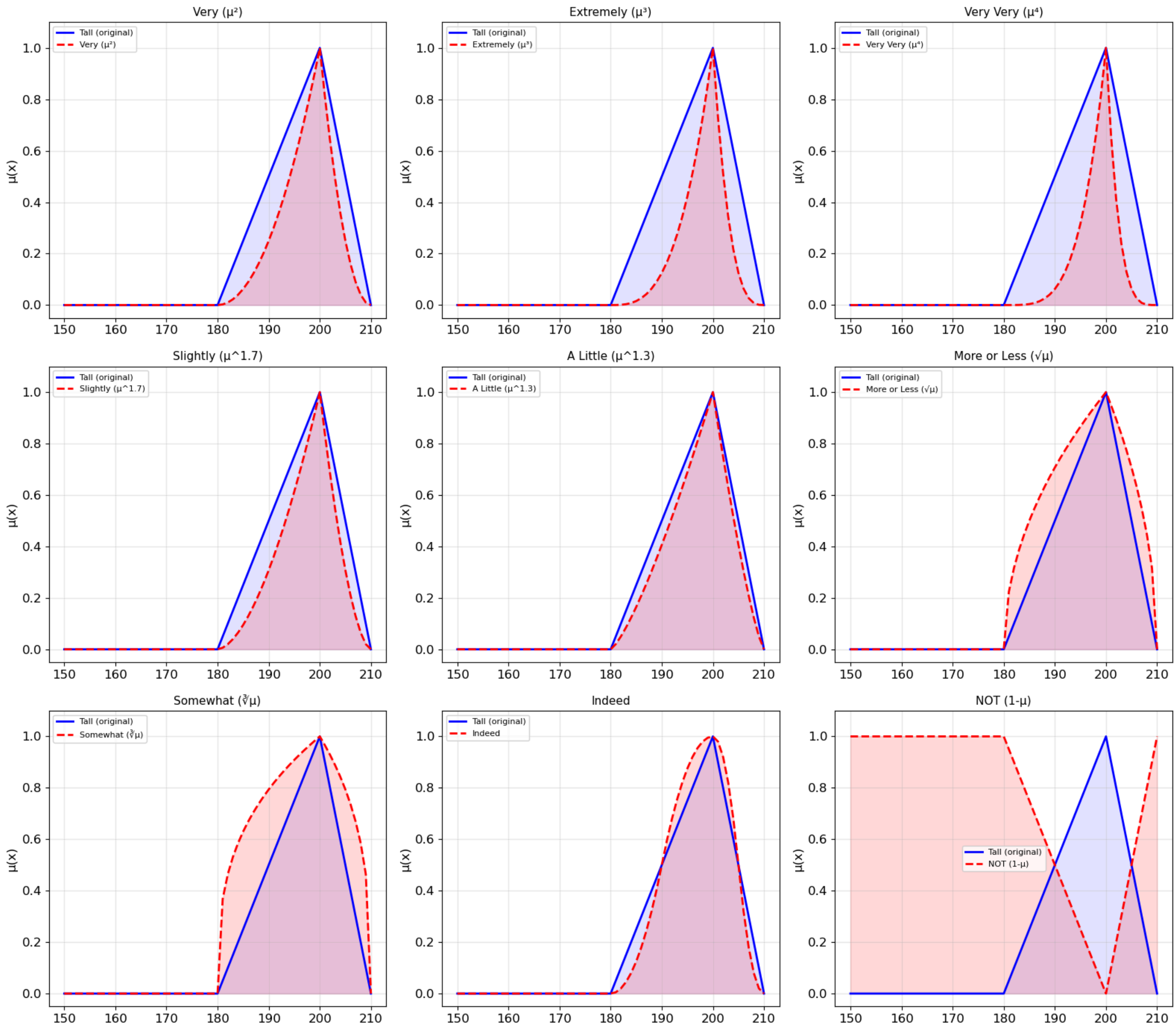
Task: Implement all the hedges from the table above as Python functions. Each function takes an array of membership values and returns the modified values. Do **not** use any fuzzy library for this — implement the formulas directly.

In [ ]:
```
def hedge_very(mu):
    """Very: μ^2 (concentration)"""
    return mu ** 2

# TODO: code all the remaining hedge functions here
## Tip 1: use NumPy's "where" function to code
##       the "hedge_indeed" function
## Tip 2: see the Exercise 3.2 code cell for the
##         names of the functions
```

Exercise 3.2: Visualise Hedge Effects

Task: Apply each hedge to the **Tall** fuzzy set and plot the original alongside the modified version. This shows exactly how concentration narrows a set and dilation widens it.

In [ ]:
```
x_height = np.arange(150, 211, 1)
tall = fuzz.trimf(x_height, [180, 200, 210])

hedges = {
    'Very (μ²)':          hedge_very,
    'Extremely (μ³)':     hedge_extremely,
    'Very Very (μ⁴)':     hedge_very_very,
    'Slightly (μ^1.7)':   hedge_slightly,
    'A Little (μ^1.3)':   hedge_a_little,
    'More or Less (√μ)':  hedge_more_or_less,
    'Somewhat (∛μ)':      hedge_somewhat,
    'Indeed':             hedge_indeed,
    'NOT (1-μ)':          hedge_not,
}

fig, axes = plt.subplots(3, 3, figsize=(16, 14))

for ax, (name, func) in zip(axes.flat, hedges.items()):
    hedged = func(tall)
    ax.plot(x_height, tall, 'b-', linewidth=2, label='Tall (original)')
    ax.plot(x_height, hedged, 'r--', linewidth=2, label=name)
    ax.fill_between(x_height, tall, alpha=0.1, color='blue')
    ax.fill_between(x_height, hedged, alpha=0.15, color='red')
    ax.set_title(name, fontsize=11)
    ax.set_ylabel('μ(x)')
    ax.set_ylim(-0.05, 1.1)
    ax.legend(fontsize=8)

plt.tight_layout()
plt.show()
```



Exercise 3.3: Hedge Computations for Practice

The slides give these examples:

- A membership of 0.86 in *tall* becomes **0.7396** in *very tall* ($0.86^2 = 0.7396$)
- A membership of 0.86 in *tall* becomes **0.6361** in *extremely tall* ($0.86^3 = 0.6361$)
- A membership of 0.86 in *tall* becomes **0.5470** in *very very tall* ($0.86^4 = 0.5470$)
- A membership of 0.86 in *tall* becomes **0.9274** in *more or less tall* ($\sqrt{0.86} = 0.9274$)

Task: Verify these computations; then for the the dapping example, calculate the hedged membership values for $\mu = 0.5$ ($\mu_{stormy}(8)$), $\mu = 0.3$ ($\mu_{low}(10)$), and $\mu = 0.7$ ($\mu_{average}(10)$).

In [ ]:

```
mu = 0.86
print(f"Slide verification (μ = {mu}):")
print(f"  Very:         {hedge_very(mu):.4f}         (expected: 0.7396)")
print(f"  Extremely:    {hedge_extremely(mu):.4f}    (expected: 0.6361)")
print(f"  Very Very:    {hedge_very_very(mu):.4f}    (expected: 0.5470)")
print(f"  More or Less: {hedge_more_or_less(mu):.4f} (expected: 0.9274)")
print()

mu = 0.5
# TODO: code here to compute the hedged membership values for the
#       dapping example for μ_stormy(8)


mu = 0.3
print(f"Dapping example (μ_low(10) = {mu}):")
```

```
print(f"  Very low:          {hedge_very(mu):.4f}         → μ^2 = 0.09")
print()

mu = 0.7
print(f"Dapping example (μ_average(10) = {mu}):")
print(f"  More or less avg: {hedge_more_or_less(mu):.4f} → √0.7 ≈ 0.8367")
```

```
Slide verification (μ = 0.86):
  Very:          0.7396          (expected: 0.7396)
  Extremely:     0.6361     (expected: 0.6361)
  Very Very:     0.5470     (expected: 0.5470)
  More or Less: 0.9274 (expected: 0.9274)

Dapping example (μ_stormy(8) = 0.5):
  Extremely stormy: 0.1250    → μ^3 = 0.125
  Slightly stormy:  0.3078     → μ^1.7 ≈ 0.307
  Very stormy:      0.2500         → μ^2 = 0.25

Dapping example (μ_low(10) = 0.3):
  Very low:         0.0900         → μ^2 = 0.09

Dapping example (μ_average(10) = 0.7):
  More or less avg: 0.8367 → √0.7 ≈ 0.8367
```

---

4. Fuzzy Set Operations

Fuzzy logic extends the standard Boolean operations to work with continuous membership values:

Operation	Crisp Logic	Fuzzy Logic	Formula
AND (∩)	Intersection	Minimum	$\mu_{A \cap B}(x) = \min[\mu_A(x), \mu_B(x)]$
OR (∪)	Union	Maximum	$\mu_{A \cup B}(x) = \max[\mu_A(x), \mu_B(x)]$
NOT	Complement	$1 - \mu$	$\mu_{\overline{A}}(x) = 1 - \mu_A(x)$
Probabilistic OR	—	Algebraic sum	$\text{probOR}(a, b) = a + b - ab$

These functions preserve the standard Boolean truth tables when inputs are restricted to $\{0, 1\}$, but also extend naturally to all values in $[0, 1]$.

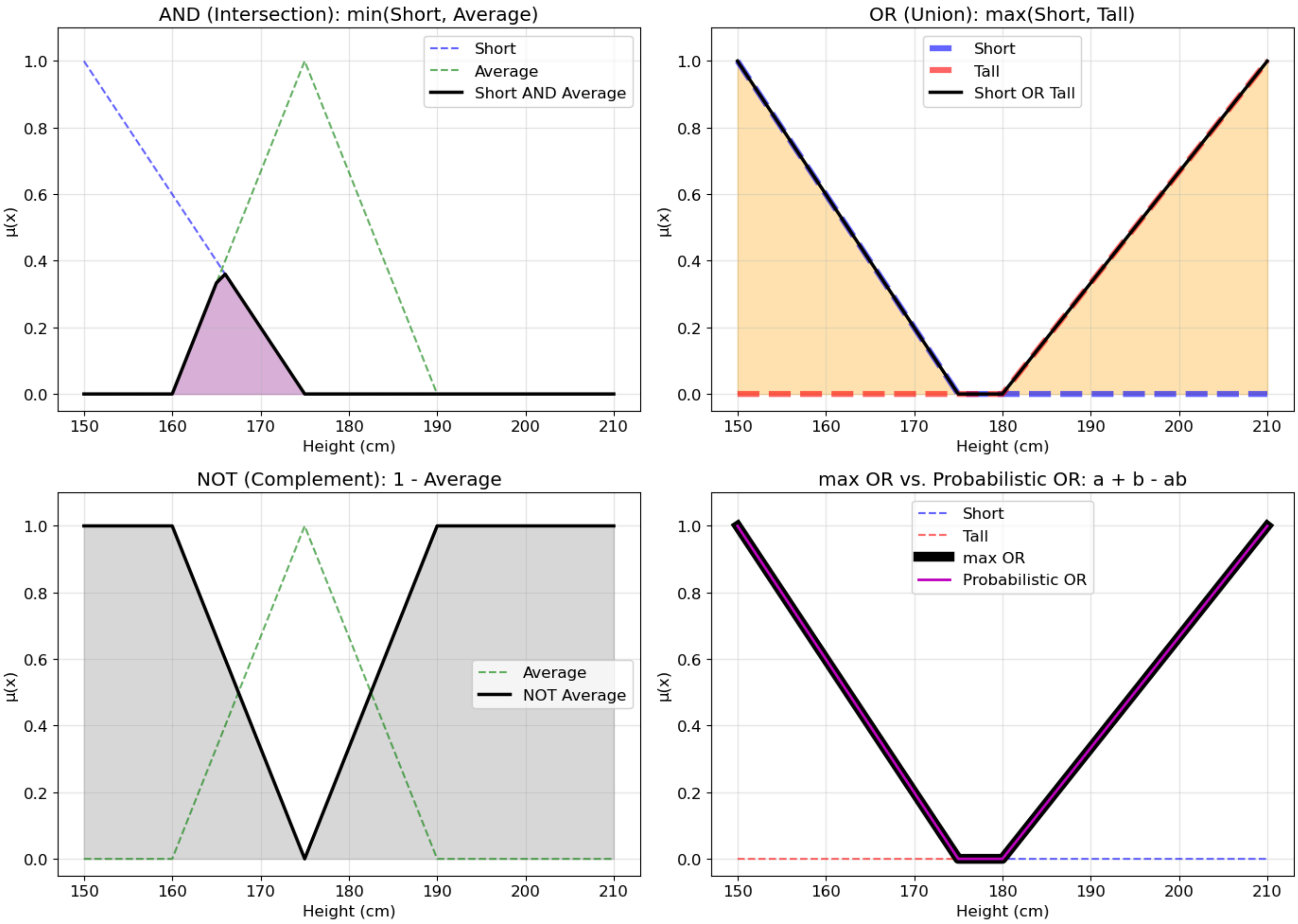
Exercise 4.1: Implement and Visualise Fuzzy Operations

Task: Using the *tallness* fuzzy sets (Short, Average, Tall), compute and plot:

1. **Short AND Average** (min)
2. **Short OR Tall** (max)
3. **NOT Average** (complement)
4. **Short OR Tall** using probabilistic OR

In [ ]:
```
x_height = np.arange(150, 211, 1)

# TODO: create here the "short", "average", and "tall" sets
#       as per Exercise 1.1



# Fuzzy operations
and_short_avg    = np.minimum(short, average)        # AND = min
or_short_tall    = np.maximum(short, tall)           # OR  = max
not_average      = 1 - average                       # NOT = complement
probor_short_tall = short + tall - short * tall      # Probabilistic OR

fig, axes = plt.subplots(2, 2, figsize=(14, 10))

# AND
axes[0, 0].plot(x_height, short, 'b--', linewidth=1.5, alpha=0.6, label='Short')
axes[0, 0].plot(x_height, average, 'g--', linewidth=1.5, alpha=0.6, label='Average')
axes[0, 0].plot(x_height, and_short_avg, 'k-', linewidth=2.5, label='Short AND Average')
axes[0, 0].fill_between(x_height, and_short_avg, alpha=0.3, color='purple')
axes[0, 0].set_title('AND (Intersection): min(Short, Average)')
axes[0, 0].legend()

# OR
axes[0, 1].plot(x_height, short, 'b--', linewidth=4.5, alpha=0.6, label='Short')
axes[0, 1].plot(x_height, tall, 'r--', linewidth=4.5, alpha=0.6, label='Tall')
# TODO: plot and fill the OR (max) operation as per the AND (min) above


# NOT
axes[1, 0].plot(x_height, average, 'g--', linewidth=1.5, alpha=0.6, label='Average')
axes[1, 0].plot(x_height, not_average, 'k-', linewidth=2.5, label='NOT Average')
axes[1, 0].fill_between(x_height, not_average, alpha=0.3, color='gray')
axes[1, 0].set_title('NOT (Complement): 1 - Average')
axes[1, 0].legend()

# Probabilistic OR vs Max OR
axes[1, 1].plot(x_height, short, 'b--', linewidth=1.5, alpha=0.6, label='Short')
axes[1, 1].plot(x_height, tall, 'r--', linewidth=1.5, alpha=0.6, label='Tall')
axes[1, 1].plot(x_height, or_short_tall, 'k-', linewidth=7.5, label='max OR')
axes[1, 1].plot(x_height, probor_short_tall, 'm-', linewidth=2, label='Probabilistic OR')
axes[1, 1].set_title('max OR vs. Probabilistic OR: a + b - ab')
axes[1, 1].legend()

for ax in axes.flat:
    ax.set_xlabel('Height (cm)')
    ax.set_ylabel('μ(x)')
    ax.set_ylim(-0.05, 1.1)

plt.tight_layout()
plt.show()

# Numerical comparison
print("Probabilistic OR vs. max OR at height = 170 cm:")
h = 170
s = fuzz.interp_membership(x_height, short, h)
t = fuzz.interp_membership(x_height, tall, h)
print(f"  Short({h}) = {s:.3f}, Tall({h}) = {t:.3f}")
```

```
print(f"  max OR    = {max(s, t):.3f}")
print(f"  prob OR   = {s + t - s*t:.3f}")
print("  → When one value is 0, both methods give the same result.")
print()
# TODO: code here the same comparison as above (h = 170) but now
#       for h = 182
```




```
Probabilistic OR vs. max OR at height = 170 cm:
  Short(170) = 0.200, Tall(170) = 0.000
  max OR    = 0.200
  prob OR   = 0.200
  → When one value is 0, both methods give the same result.

  Short(182) = 0.000, Tall(182) = 0.067
  max OR    = 0.067
  prob OR   = 0.067
  → When both are non-zero, prob OR is always ≥ max OR.
```

---

5. Putting It All Together: Manual Fuzzification with Hedges

This exercise combines everything from this lab into a single, worked example.

Exercise 5.1: Dapping Rules with Hedges

Consider the following three rules for the **Mamdani** inference method (wind = 8, temperature = 10):

1. **IF** wind IS **extremely** stormy **OR** temperature IS **very** low **THEN** dapping IS **not very** poor
2. **IF** wind IS fresh **AND** temperature IS **more or less** average **THEN** dapping IS mediocre
3. **IF** wind IS **slightly** stormy **AND** temperature IS **NOT** low **THEN** dapping IS **a little** excellent

Important: The exact membership values depend on the triangle parameters read from the lecture handout diagrams. Adopt the fuzzified values wind = 8 and temp = 10:

Input	Set	μ
wind = 8	Stormy	0.5
wind = 8	Fresh	0.38
temp = 10	Low	0.3
temp = 10	Average	0.7
temp = 10	High	0.0

In practice, you will read these from the provided figures. Here, we hard-code them to match the handout and work through the inference steps, which helps practice for the assessment.

Task: Compute, step by step, the fuzzified and hedged antecedent values for each rule. Show all intermediate calculations.

In [ ]:

```
# Step 1: Raw ("visual"/"manual") fuzzification (wind=8, temp=10)
# This approach helps practice for the assessment
# These values come from reading the lecture handout diagrams
mu_stormy  = 0.5
mu_fresh   = 0.38
mu_low     = 0.3
mu_average = 0.7
mu_high    = 0.0

print("Step 1: Raw Fuzzification (from lecture handout figures)")
print(f"  μ_stormy(8)   = {mu_stormy}")
print(f"  μ_fresh(8)    = {mu_fresh}")
print(f"  μ_low(10)     = {mu_low}")
print(f"  μ_average(10)= {mu_average}")
print(f"  μ_high(10)    = {mu_high}")

# === Rule 1 ===

print()
print("=" * 60)
print("RULE 1: IF wind IS extremely stormy OR temp IS very low")
print("        THEN dapping IS not very poor")
print("=" * 60)

# Antecedent 1: extremely stormy
extremely_stormy = hedge_extremely(mu_stormy)
print(f"  μ_stormy(8) = {mu_stormy}")
print(f"  extremely(μ) = μ³ = {mu_stormy}³ = {extremely_stormy:.4f}")

# Antecedent 2: very low
very_low = hedge_very(mu_low)
print(f"  μ_low(10) = {mu_low}")
print(f"  very(μ) = μ² = {mu_low}² = {very_low:.4f}")

# OR = max
rule1_antecedent = max(extremely_stormy, very_low)
print(f"  OR (max) = max({extremely_stormy:.4f}, {very_low:.4f}) = {rule1_antecedent:.4f}")

# Consequent: not very poor
very_rule1 = hedge_very(rule1_antecedent)
not_very_rule1 = hedge_not(very_rule1)
print(f"  Consequent: very(μ) = {rule1_antecedent:.4f}² = {very_rule1:.4f}")
print(f"              NOT = 1 - {very_rule1:.4f} = {not_very_rule1:.4f}")
```

```python
print(f"  → Rule 1 fires: clip 'poor' at {not_very_rule1:.4f}")

# === Rule 2 ===

print()
print("=" * 60)
print("RULE 2: IF wind IS fresh AND temp IS more or less average")
print("        THEN dapping IS mediocre")
print("=" * 60)

# Antecedent 1: fresh (no hedge)
print(f"  μ_fresh(8) = {mu_fresh}")

# Antecedent 2: more or less average
# TODO: compute the "more or less average" value

print(f"  μ_average(10) = {mu_average}")
print(f"  more_or_less(μ) = √μ = √{mu_average} = {mol_average:.4f}")

# AND = min
# TODO: compute the rule antecedent (i.e. AND (min))

print(f"  AND (min) = min({mu_fresh}, {mol_average:.4f}) = {rule2_antecedent:.4f}")
print(f"  → Rule 2 fires: clip 'mediocre' at {rule2_antecedent:.4f}")

# === Rule 3 ===

print()
print("=" * 60)
print("RULE 3: IF wind IS slightly stormy AND temp IS NOT low")
print("        THEN dapping IS a little excellent")
print("=" * 60)

# Antecedent 1: slightly stormy
# TODO: compute the "slightly stormy" value

print(f"  μ_stormy(8) = {mu_stormy}")
print(f"  slightly(μ) = μ^1.7 = {mu_stormy}^1.7 = {slightly_stormy:.4f}")

# Antecedent 2: NOT low
not_low = hedge_not(mu_low)
print(f"  μ_low(10) = {mu_low}")
print(f"  NOT = 1 - {mu_low} = {not_low:.4f}")

# AND = min
# TODO: compute the rule antecedent (i.e. AND (min))

print(f"  AND (min) = min({slightly_stormy:.4f}, {not_low:.4f}) = {rule3_antecedent:.4f}")

# Consequent: a little excellent
# TODO: compute the "a little" value for the antecedent

print(f"  Consequent: a_little(μ) = {rule3_antecedent:.4f}^1.3 = {a_little_rule3:.4f}")
print(f"  → Rule 3 fires: clip 'excellent' at {a_little_rule3:.4f}")

print()
print("=" * 60)
print("SUMMARY: These are the values used in the Mamdani handout")
print("=" * 60)
print(f"  Rule 1 clips 'poor' at:      {not_very_rule1:.4f}  (handout: ~1.0)")
print(f"  Rule 2 clips 'mediocre' at:  {rule2_antecedent:.4f}  (handout: 0.38)")
print(f"  Rule 3 clips 'excellent' at: {a_little_rule3:.4f}  (handout: 0.215)")
```

```
Step 1: Raw Fuzzification (from lecture handout figures)
  μ_stormy(8)  = 0.5
  μ_fresh(8)   = 0.38
  μ_low(10)    = 0.3
  μ_average(10)= 0.7
  μ_high(10)   = 0.0

============================================================
RULE 1: IF wind IS extremely stormy OR temp IS very low
        THEN dapping IS not very poor
============================================================
  μ_stormy(8) = 0.5
  extremely(μ) = μ³ = 0.5³ = 0.1250
  μ_low(10) = 0.3
  very(μ) = μ² = 0.3² = 0.0900
  OR (max) = max(0.1250, 0.0900) = 0.1250
  Consequent: very(μ) = 0.1250² = 0.0156
              NOT = 1 - 0.0156 = 0.9844
  → Rule 1 fires: clip 'poor' at 0.9844

============================================================
RULE 2: IF wind IS fresh AND temp IS more or less average
        THEN dapping IS mediocre
============================================================
  μ_fresh(8) = 0.38
  μ_average(10) = 0.7
  more_or_less(μ) = √μ = √0.7 = 0.8367
  AND (min) = min(0.38, 0.8367) = 0.3800
  → Rule 2 fires: clip 'mediocre' at 0.3800

============================================================
RULE 3: IF wind IS slightly stormy AND temp IS NOT low
        THEN dapping IS a little excellent
============================================================
  μ_stormy(8) = 0.5
  slightly(μ) = μ^1.7 = 0.5^1.7 = 0.3078
  μ_low(10) = 0.3
  NOT = 1 - 0.3 = 0.7000
  AND (min) = min(0.3078, 0.7000) = 0.3078
  Consequent: a_little(μ) = 0.3078^1.3 = 0.2161
  → Rule 3 fires: clip 'excellent' at 0.2161

============================================================
SUMMARY: These are the values used in the Mamdani handout
============================================================
  Rule 1 clips 'poor' at:      0.9844  (handout: ~1.0)
  Rule 2 clips 'mediocre' at:  0.3800  (handout: 0.38)
  Rule 3 clips 'excellent' at: 0.2161  (handout: 0.215)
```

Summary

In this lab you have:

1. **Distinguished** crisp sets from fuzzy sets and understood the concept of partial membership
2. **Defined and plotted** the four main membership function types: triangular, trapezoidal, Gaussian, and sigmoid
3. **Computed membership degrees** by fuzzifying crisp inputs against defined fuzzy sets
4. **Implemented all hedges from scratch**: Very, Extremely, Very Very, Slightly, A Little, More or Less, Somewhat, Indeed, and NOT
5. **Applied fuzzy set operations**: AND (min), OR (max), NOT (complement), and probabilistic OR

6. **Worked through the dapping example** with hedges, which helps practice for the assessment

Key Equations Revisited

Triangular MF:

$\mu(x; a, b, c) = \max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right)$

Hedge (power-based):

$\mu_A^{hedged}(x) = [\mu_A(x)]^p$ where $p > 1$ = concentration, $p < 1$ = dilation

Fuzzy operations:

AND: $\min(\mu_A, \mu_B)$ OR: $\max(\mu_A, \mu_B)$ NOT: $1 - \mu_A$ probOR: $\mu_A + \mu_B - \mu_A \cdot \mu_B$

Preparation for Lab 2

In the next lab, we will:

- Build a complete **Mamdani fuzzy inference system** from scratch (fuzzification → rule evaluation → aggregation → defuzzification)
- Implement **COG, MOM, SOM, and LOM** defuzzification methods
- Use the **pyfuzzylite** library to build and compare **Mamdani, Sugeno, and Tsukamoto** inference systems
- Apply everything to the **project staffing** and **dapping** examples from lectures