Rupture Dynamics and Memory Persistence in Discontinuous Systems

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Abstract

We present a novel mathematical framework for analyzing systems with discrete rupture points, where information and structure persist through discontinuities. By introducing memory trace subspaces and resonance functions, we demonstrate how global coherence emerges from local discontinuities. Applications to dynamical systems, quantum mechanics, and cognitive science are explored, illustrating the universality of this approach.

1 Introduction

2 Fundamental Properties

Definition 1 (Rupture System)**.** A rupture system consists of a triple $(V, M, \{t_1, \ldots, t_k\})$ satisfying three key conditions:

1. **Local Invertibility:** $\forall t \notin \{t_1, \ldots, t_k\}, \exists \varepsilon > 0$ such that $M(t)$ is invertible in $(t - \varepsilon, t + \varepsilon)$.

2. **Memory Trace:** $\exists S \subseteq V$ with $\dim(S) > 0$ such that information in S persists across rupture points.

3. **Resonance:** For every pair of rupture points $t_i < t_j$, there exists a resonance function f mapping between states.

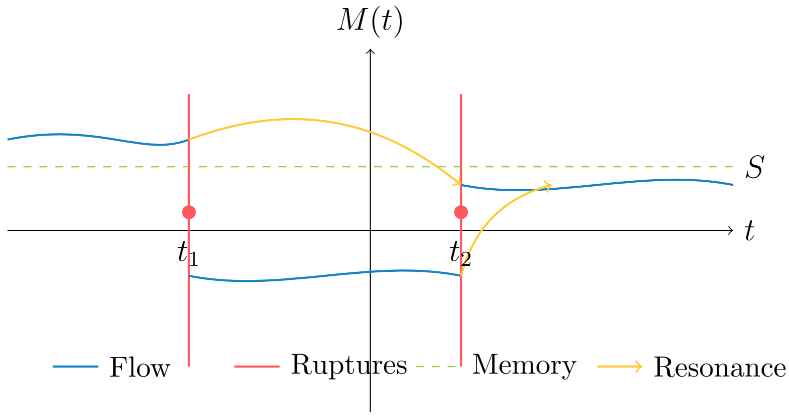

Figure 1: A rupture system showing flow dynamics (blue), rupture points (red), memory trace (green), and resonance connections (gold).

Theorem 2 (Memory Persistence)**.** *For any rupture system satisfying the above conditions, there exists a non-trivial subspace $W \subset V$ such that information encoded in W persists across all rupture points.*

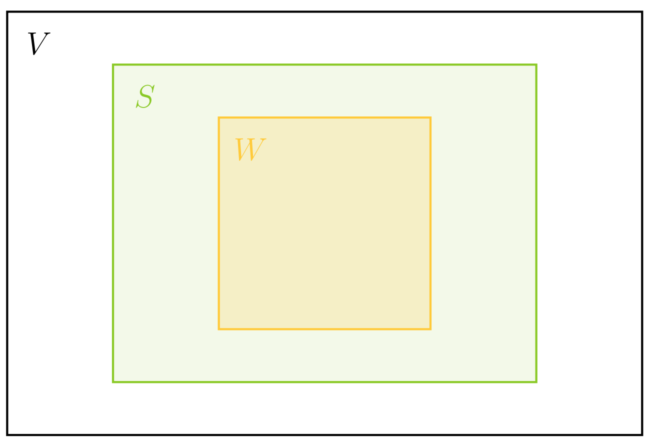

Figure 2: The persistent subspace W as intersection of kernels within memory trace subspace S.

3 Applications

3.1 Quantum Mechanics

In quantum measurement theory, rupture points correspond to state collapse events:

$$M(t) = U(t)PU^{\dagger}(t).$$

3.2 Cognitive Science

Neural networks exhibit rupture points during learning events.

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