Time-Evolving Paradoxical Spaces: A Dynamic Graph Framework with Surreal Distances

Author Name

December 16, 2024

Abstract

We present a novel mathematical framework for modeling time-evolving graphs with surreal ordinal distances and recursive paradoxical spaces. Building on surreal number theory, dynamic graph systems, and memory-like structures from sheaf theory, we analyze the evolution of spaces over time. This approach incorporates infinitesimal and transfinite changes, resonance phenomena, and singularities. Applications include non-Euclidean geometries, dynamic systems, and semiotic paradoxes inspired by recursive spaces such as the *House of Leaves*.

Contents

1 Introduction

Spaces that evolve recursively over time, exhibit paradoxical growth, or involve transfinite measures require new mathematical tools. Traditional graph theory is insufficient for representing:

- Flows through spaces with infinitesimal and infinite distances,
- Dynamic transformations of nodes and edges over time,
- Singularities and resonance points where spatial continuity breaks.

Our framework builds on three primary tools:

1. **Surreal Numbers:** Recursive structures that model infinite and infinitesimal changes,
2. **Dynamic Graphs:** Graphs with evolving nodes, edges, and surreal weights,
3. **Monodromy and Memory:** Structures inspired by sheaf theory to capture time evolution and resonance.

This paper introduces the mathematical definitions, properties, and examples for analyzing such spaces.

2 Surreal Numbers and Ordinals

2.1 Surreal Numbers

Definition 2.1 (Surreal Numbers)**.** *A surreal number $\mathcal{S}$ is defined recursively as:*

$$\mathcal{S} = \{L \mid R\}, \quad \text{where } L, R \subseteq \mathcal{S} \text{ and } L < R.$$

Surreal numbers extend real numbers to include infinitesimal (ϵ) and transfinite (ω) values.

Example 2.1 (Basic Surreal Numbers)**.**

$$0 = \{\mid\}, \quad 1 = \{0 \mid\}, \quad -1 = \{\mid 0\}, \quad \frac{1}{2} = \{0 \mid 1\}.$$

Infinitesimals and infinities are given as:

$$\epsilon = \{0 \mid\}, \quad \omega = \{\mid 0\}.$$

2.2 Ordinal Arithmetic

Surreal numbers naturally generalize ordinals, allowing operations such as addition, multiplication, and exponentiation.

Definition 2.2 (Ordinal)**.** *An ordinal $\mathcal{O}$ is expressed as:*

$$\mathcal{O} = \omega^w + k, \quad w, k \in \mathbb{N}.$$

Remark 2.1. *Ordinal arithmetic includes:*

$$\omega + 1 > \omega, \quad \omega \cdot 2 = \omega, \quad \omega^\omega > \omega^2.$$

3 Dynamic Graphs with Surreal Distances

3.1 Graph Framework

We define a dynamic graph $\mathcal{H} = (V, E, W)$:

- V: Nodes (e.g., spaces),
- E: Edges connecting nodes,
- W: Weights as surreal distances.

Definition 3.1 (Surreal Distance)**.** *The weight $w_{uv} \in W$ of an edge (u, v) is a surreal ordinal:*

$$w_{uv} = \omega^w + k \textit{ or an infinitesimal } \epsilon.$$

3.2 Edge Types and Recursions

Edges can take the following forms:

1. **Single Edge:** A standard edge with a surreal weight,
2. **Infinite Edge:** A recursive edge repeating transfinite times,
3. **Fractional Edge:** A fractional path with rational weight,
4. **Paradoxical Edge:** An edge with undefined behavior.

Example 3.1 (The Five-Minute Hallway)**.** *This hallway grows infinitely when entered:*

$$\textit{Nodes: Entrance} \rightarrow \textit{Hallway} \rightarrow \textit{Abyss.}$$

Edges are weighted as:

$$\textit{Entrance} \xrightarrow{\omega} \textit{Hallway}, \quad \textit{Hallway} \xrightarrow{\omega^2} \textit{Abyss.}$$

4 Time Evolution and Monodromy

4.1 Transformation Rules

A time-evolving graph $\mathcal{H}(t)$ evolves based on transformations:

$$\text{Transformation:} \quad T : E \rightarrow E', \quad w' = w + \epsilon.$$

Transformations may include:

- Local perturbations,
- Discontinuous jumps,
- Resonance interactions.

Definition 4.1 (Monodromy)**.** *The* monodromy *of a time-evolving graph captures how nodes and edges transform over time, including:*

$$\textit{Singularities: Special times where continuity breaks.}$$

4.2 Conditions for Stability

Theorem 4.1 (Stability Condition)**.** *A dynamic graph $\mathcal{H}(t)$ is stable if:*

$$\forall u, v \in V : \quad \textit{flow through } E \textit{ remains bounded as } t \to \infty.$$

5 Paradoxical Spaces and Sheaf Memory

Example 5.1 (Mirror Room)**.** *The mirror room generates infinite reflections at infinitesimal steps:*

$$\textit{Origin} \xrightarrow{\epsilon} \textit{First Reflection} \xrightarrow{\epsilon} \textit{Second Reflection} \to \dots.$$

The sequence collapses to an abyss as $n \to \infty$.

Remark 5.1. *The* sheaf-like memory *structure ensures that information about nodes persists through singularities and transformations.*

6 Conclusion

We have developed a framework for time-evolving graphs with surreal distances, transformations, and paradoxical structures. Applications span recursive geometries, dynamic systems, and semiotic spaces.

Future directions include:

- Extending monodromy to higher-dimensional spaces,
- Applying the framework to physical systems with non-standard flows,
- Investigating connections to game theory and logic.

References

[1] Conway, J. H. (1976). *On Numbers and Games.*

[2] Harary, F. (1969). *Graph Theory.*

[3] Danielewski, M. Z. (2000). *House of Leaves.*