Recursive Games, Surreal Payoffs, and Impossible Spaces: A Unified Framework for Dynamic Graph Systems

Your Name

December 16, 2024

Abstract

We propose a novel mathematical framework that unifies recursive game theory, dynamic graph systems, and surreal-valued payoffs. By integrating infinitesimal and transfinite numbers into game dynamics, we model decision-making across evolving graphs where perturbations, singularities, and resonance points govern equilibria. Monodromy, inspired by algebraic topology, describes the behavior of games under recursion, while impossible spaces provide a conceptual metaphor for decision systems that collapse or stabilize across infinite paths. This work bridges game theory, graph dynamics, and recursive systems, with applications in AI training, climate policy, and strategic negotiations.

1 Introduction

Classical game theory models static equilibria, but many real-world systems involve recursion, evolution, and infinite decision layers. Negotiations, climate agreements, and AI training systems exhibit delays, adjustments, and perturbations that can only be captured by a recursive framework.

We introduce a new framework for recursive games defined on dynamic graphs:

- Nodes represent decision states that evolve under recursive strategies.
- Edges carry surreal-valued weights, including infinitesimal penalties (ϵ) and transfinite rewards (ω).
- Perturbations dynamically adjust nodes, edges, and payoffs, leading to emergent equilibria at resonance points.

Inspired by *impossible spaces* like those in *House of Leaves* [2], we model recursive decision-making in systems where strategies collapse, stabilize, or oscillate under infinite recursion. Monodromy maps track strategy evolution across singularities, providing tools to analyze game equilibria under perturbations.

2 Recursive Games on Dynamic Graphs

Let $G_t = (N_t, E_t)$ be a graph at time t, where:

- N_t is the set of nodes (decision states),
- E_t is the set of edges with weights $w_t \in \mathbb{S}$ (surreal numbers).

2.1 Surreal-Valued Payoffs

Surreal numbers generalize payoffs to include:

- **Infinitesimal penalties**: ϵ, where $\epsilon > 0$ but $\epsilon < r \, \forall r > 0$,
- **Transfinite rewards**: ω, representing infinite magnitudes,
- **Collapsed costs**: Negative infinities ($-\omega$) when recursion breaks down.

The payoff for a recursive strategy is:

$$u(t) = \sum_{i=0}^{t} w_i - k\epsilon, \quad \text{where } k \to \infty \text{ collapses the strategy.}$$

2.2 Game Evolution Rules

The graph G_t evolves according to:

$$G_{t+1} = f(G_t, \Delta_t),$$

where Δ_t introduces perturbations:

- Adding new nodes: Expanding the strategy space,
- Adjusting edge weights: Rebalancing payoffs under recursion,
- Removing edges: Eliminating dominated strategies.

2.3 Example: Recursive Negotiation

Consider two agents negotiating recursively over time:

- Cooperation yields payoff 1 at time t,
- Delay imposes an infinitesimal penalty ϵ,
- Infinite delays collapse the negotiation into $-\omega$.

The recursive payoff is:

$$u(t) = \begin{cases} 1 - k\epsilon & \text{for finite delays,} \\ -\omega & \text{for infinite delays.} \end{cases}$$

Equilibrium occurs at **resonance points** where:

$$\text{Cost(Cooperate)} = \text{Cost(Delay)}.$$

3 Monodromy and Resonance Analysis

Monodromy tracks changes in graph states under recursion:

$$\Phi : G_0 \to G_t,$$

where Φ describes how nodes, edges, and weights transform as recursion progresses.

3.1 Resonance Points

Resonance points stabilize recursive dynamics. For a node n, resonance occurs when:

$$\sum_{t=0}^{\infty} \Delta u(t) = 0,$$

where $\Delta u(t)$ captures recursive changes in payoffs.

3.2 Singularities

Singularities arise when strategies collapse:

$$u(t) \to -\omega.$$

These points correspond to boundaries where recursion destabilizes the system.

4 Impossible Spaces: Conceptual Metaphors

Inspired by *House of Leaves* [2], we interpret recursive graphs as impossible spaces:

- Nodes represent decision "rooms" that evolve recursively,
- Edges describe surreal-valued paths that connect states,
- Singularities correspond to spatial boundaries dissolving under infinite recursion.

Impossible spaces model phenomena such as:

- Infinite delays collapsing strategies ($-\omega$),
- Emerging pathways under perturbations,
- Recursive oscillations stabilizing at resonance points.

5 Applications

5.1 AI Training Systems

Recursive games provide tools for AI agents balancing:

- Immediate exploration (finite payoffs),
- Long-term exploitation (transfinite gains),
- Infinitesimal penalties for delays.

5.2 Strategic Negotiations

Recursive delays in negotiations impose increasing costs:

$$u(t) = 1 - k\epsilon, \quad \text{where infinite delays collapse to } -\omega.$$

5.3 Climate Policy Models

Infinite recursion models procrastination in climate agreements. Delays increase costs until catastrophic collapse:

$$u(t) \to -\omega \quad \text{as } t \to \infty.$$

6 Future Work

This framework opens avenues for:

- Stochastic extensions for recursive games,
- Empirical validation in multi-agent AI systems,
- Topological analysis of dynamic graphs under perturbations.

7 Conclusion

We developed a unified framework for recursive games, surreal-valued payoffs, and dynamic graph systems. Monodromy and resonance describe equilibria under infinite recursion, while impossible spaces serve as conceptual tools for understanding boundary-breaking dynamics. Applications include AI, negotiations, and climate policy, with future work focusing on stochastic and empirical extensions.

References

[1] J. H. Conway, *On Numbers and Games*, Academic Press, 1976.

[2] M. Z. Danielewski, *House of Leaves*, Pantheon Books, 2000.

[3] J. von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*, Princeton University Press, 2007.