

A Surreal Treasure Hunt: Extending John Conway's Legacy

Inspired by the Spirit of John Horton Conway

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Abstract

Once upon a time, on a vast, uncharted island, a group of adventurers embarked on a treasure hunt. Guided by incomplete maps, random markers, and a shared goal, they uncovered treasures that none could have found alone. In this tale, we weave together the magic of surreal numbers, ordinal arithmetic, and hidden proposals to tackle the age-old problem of group decision-making. Inspired by John Conway's playful genius, this story explores how mathematical tools can transform negotiation and collaboration.

Chapter 1: The Call to Adventure

Our tale begins on the shores of an island unlike any other. This was no ordinary land; it was the realm of Surrealia, where every number—be it real, imaginary, infinite, or infinitesimal—had a home. Surrealia was ruled by the ancient laws of the binary tree, where each number was born from the choices of its ancestors.

A group of adventurers arrived, each with their own map of treasures scattered across the island. But the maps were incomplete and secret. Sharing them outright would invite chaos: distrust, competition, and the risk of losing treasures to others. Yet, working alone meant missing out on treasures that only collaboration could reveal.

Their challenge was clear: How could they combine their maps, uncover treasures, and ensure fairness for all?

Chapter 2: The Neutral Guide and the Hidden Maps

To solve their dilemma, the adventurers turned to a neutral guide. This guide had a clever idea. Instead of revealing everyone's maps outright, the adventurers would submit their maps secretly. These hidden maps represented each adventurer's preferences—locations they believed held treasures.

But how could the guide ensure fairness? The guide introduced randomness into the mix, placing random markers across the island. These markers would shake things up, encouraging exploration of areas beyond the adventurers' initial preferences.

The guide announced: "We will only reveal the treasures where maps overlap—where your preferences and the random markers intersect. That way, no single adventurer's map will dominate, and we'll uncover treasures together."

Chapter 3: Measuring the Paths to Treasure

The island of Surrealia was vast, its treasures scattered across infinite landscapes. To navigate this complexity, the adventurers needed a way to measure the distance between their maps. The guide explained:

”In Surrealia, every treasure has a path—a lineage through the great binary tree. To measure how close two treasures are, we calculate the path distance between them: the steps from each treasure to their first common ancestor in the tree.”

This measure, called $\tau(x, y)$, helped the adventurers decide which treasures were compatible. Treasures with shorter path distances were more likely to represent shared interests.

Example: The Tale of the Fractions

Two treasures, $x = \frac{1}{2}$ and $y = -\frac{1}{2}$, were located on opposite sides of the island. Their common ancestor was 0, the great origin point of all numbers. Each treasure was two steps away from 0, making the total path distance:

$$\tau\left(\frac{1}{2}, -\frac{1}{2}\right) = 2 + 2 = 4.$$

Example: The Meeting of Infinity and Infinitesimal

Another adventurer sought treasures at ϵ (an infinitesimal treasure) and ω (an infinite treasure). Though seemingly worlds apart, both treasures traced their lineage back to 0. The path distance was:

$$\tau(\epsilon, \omega) = (\omega + 1) + \omega = \omega + 1.$$

Chapter 4: The Final Protocol

The guide explained the steps to uncover treasures:

1. **Step 1: Submit Hidden Maps.** Each adventurer secretly submits their map of preferred treasures.
2. **Step 2: Add Random Markers.** The guide places random markers across the island, expanding the search space.
3. **Step 3: Reveal Overlaps.** The guide reveals only the treasures where maps overlap, ensuring fairness and collaboration:

$$P_{\text{final}} = \bigcup_{i,j} (S_i \cap S_j) \cup \bigcup_i (S_i \cap R).$$

Chapter 5: The Wisdom of the Island

The adventurers marveled at how their hidden maps and random markers had led to treasures they could not have discovered alone. The guide reflected:

”Surrealia is a land of infinite possibilities, where collaboration thrives when we honor both the known and the unknown. Your maps represented your preferences; the random

markers, the potential for surprise. Together, they uncovered treasures beyond imagination.”

Conclusion

This tale of Surrealia illustrates a profound truth: By combining hidden preferences, randomness, and careful measurement, we can navigate the infinite landscapes of decision-making. Inspired by the visionary work of John Horton Conway, this framework offers a playful yet rigorous approach to negotiation and collaboration.