Inflation dynamics over the net-zero transition: Carbon price shocks in a rationally inattentive world *

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Abstract

In recent years, a growing theoretical literature has made great strides in relaxing restrictive assumptions of full information and full rationality amongst agents, and embedding more realistic alternatives within DSGE models. However, one field that has remained largely immune to these developments is that of climate macroeconomics. And yet the settings that such environmental, or 'E', -DSGE models seek to model are ones in which agents' expectations are *especially* important, given the long-run horizon of physical risks and net-zero transition policies. We show that relaxing full information rational expectation (FIRE) assumptions completely reverses the macroeconomic implications of certain net-zero transition policies. In particular, under FIRE assumptions, the macroeconomic consequences of such policies, such as carbon taxes, are predicted to be relatively small, and potentially even *deflationary*. However, we show that, if instead agents are rationally inattentive (and do not fully internalise the outlook for the economy), permanent increases in carbon prices will be *inflationary* and *persistently* so. This could result in potentially significantly different monetary policy implications of the net-zero transition.

Keywords: Environmental DSGE, Net-Zero Transition, Carbon Taxes, Information Frictions, Rational Inattention

JEL codes: D83, Q50, E32, E58, E71

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1 Introduction

Standard New Keynesian (NK) models assume that agents have fully informed (FI) and perfectly rational expectations (RE). The implication is that 'FIRE' agents are perfectly able to internalise shocks that will happen long into the future, and accordingly and optimally amend their behaviour today. Acknowledgment that this is unrealistic is not novel. Empirical evidence has long shown that macroeconomic variables react far less sensitively to news about the future than predicted by standard NK models. This is known as the 'Forward Guidance puzzle' Del Negro et al. (2023). Broad literatures across various economic fields have sought to develop intuitive ways of solving this puzzle by relaxing either RE[1] or FI.[2] Yet, despite this work, the tendency to maintain FIRE assumptions persists in a range of literatures.

One such literature is climate macroeconomics. Yet, expectations are *especially* important within a climate setting, where both physical risks – such as adverse weather events – and transition policies – such as carbon taxes – will impact investment, saving, consumption, and production decisions over long horizons. Take the latter. In order to meet net-zero targets, the OECD estimates that the price of carbon will have to increase by 80% in the UK and Euro Area from 2021 to 2030. Insofar as the substitution of energy inputs from carbon-intensive to green resources is not immediate or perfect, this will entail an increase in production costs for firms, with macroeconomic consequences. Yet, both empirical studies (McKibbin et al. 2021, Konradt & di Mauro 1999, Moessner 2022) – based on high-frequency identification – and E-DSGE modelling to date (Diluiso et al. 2021, Annicchiarico & Di Dio 2017) – estimate / predict a relatively small impact of carbon price increases on macroeconomic variables.[3]

However, each strand faces shortcomings. Predictions from empirical work are extrapolated from estimates that are based on extremely small increases in carbon prices relative to those that will be needed to meet Net-Zero targets. Meanwhile, the majority of the modelling conducted to date has assumed that transition policies are credible, individuals are able to fully internalise the likely impact on the macroeconomy, and alter their behaviour today as a result. That is, they assume that agents are FIRE.

In this paper, we relax this unrealistic assumption. This can be done in a number of ways. We follow Sims (2003) and Mackowiak & Wiederholt (2015) in assuming that agents are rationally inattentive. We estimate attention based on UK household survey data from the Inflation Attitudes Survey, and build on Pfaeuti (2021) in demonstrating how to apply it tractably to a benchmark environmental , or 'E', -DSGE model a la Annicchiarico & Di Dio (2015).

We show that relaxing FIRE assumptions has very different implications for the impact of carbon price shocks on the macroeconomy and potentially for monetary policy. In particular, we show that, contrary to the predictions of models with FIRE agents, if agents are rationally inattentive, carbon price shocks will be *inflationary*, rather than deflationary, and *persistently* so. These predictions have potentially significantly different monetary policy implications for the net-zero transition.

[1]For example by supposing that agents: behave 'habitually' in their consumption decisions (Fuhrer 2000); are boundedly rational, or 'myopic', (Gabaix 2020); form expectations adaptively (based on previous period's observations) (Galí & Gertler 1999); learn adaptively from forecast errors (Evans & Honkapohja 2003); or have 'Level-K' thinking (Crawford & Iriberri 2007)

[2]For example by supposing that information is: sticky (Mankiw & Reis 2007); dispersed unevenly across individuals (Melosi 2014); or a scarce resource such that agents are 'rationally inattentive' (Sims 2003)

[3]There are exceptions to this. For instance, Coenen et al. (2022) estimate a 0.9pp impact on inflation in a credible carbon price scenario in the EU.

2 Rational Inattention Framework

The core concept behind the rational inattention framework is that information is a scarce resource and costly to acquire (Sims 2003). Agents pay attention to different sources of information based on the relative costs and benefits of doing so. The optimal degree of attention, γ, that an individual may pay to the state of the economy, x, is given by (with full derivation in Appendix A):

$$\gamma^* = \max\left(0, 1 - \frac{\lambda}{\sigma_x^2}\right) \tag{1}$$

where $\lambda = \frac{c}{b}$ is the relative cost and benefit the individual faces from paying attention, and σ_x^2 is the degree of uncertainty around a macroeconomic variable, x. Thus, the optimal level of attention is, of course, increasing with b and decreasing with c, but more interestingly, increasing in the degree of uncertainty σ_x^2. That is, it is state-dependent.

2.1 Tractable application to a dynamic setup

We can apply a tractable reduced-form version of the rational inattention framework to a dynamic setup. Suppose in period $t-1$ individual i has some belief $\pi^{e,RI}_{i,t|t-1}$ about what they expect inflation to be in the next period. They then receive a signal $s_{i,t}$ in period t with information about what inflation is expected to be in period $t+1$ (e.g. communicated by the central bank), given by:

$$s_{i,t} = E_t[\pi_{t+1}] + \epsilon_{i,t} \tag{2}$$

where $\epsilon \sim N(0, \sigma_\epsilon^2)$ reflects noise in the signal. $E_t[\pi_{t+1}]$ denotes the fully informed rational expectation in period t for π_{t+1}. Individual i can then decide to what degree they wish to pay attention to this signal, and to update their one-period ahead expectation. Their Bayesian updated one-period ahead expectation is given by:

$$\pi^{e,RI}_{i,t+1|t} \equiv E[\pi_{t+1}|s_{i,t}] = (1-\gamma)\pi^{e,RI}_{i,t|t-1} + \gamma s_{i,t} \tag{3}$$

Where $\pi^{e,RI}_{i,t+1|t}$ denotes agent i's rationally inattentive one-period ahead expectation. At one extreme, an individual choosing to pay full attention to the signal ($\gamma = 1$), would form a 'rationally inattentive' expectation that is equal to the fully informed expectation, such that $\pi^{e,RI}_{i,t+1|t} = E_t[\pi_{t+1}]$.[4] At the other extreme, an individual paying zero attention ($\gamma = 0$), would not update their one-period ahead inflation expectation at all, and simply carry forward that from the previous period: $\pi^{e,RI}_{i,t+1|t} = \pi^{e,RI}_{i,t|t-1}$. Somewhere in between these extreme cases, $0 < \gamma < 1$, agents' updating of beliefs has a component that is fully informed and rational, and another that is autoregressive. This results in *persistence* of expectations. We compare the predictions with the Gabaix (2020) bounded rationality framework in Appendix B.

2.2 Estimating attention based on UK household data

We can estimate the attention parameter, γ, based on UK household expectations data from the Inflation Attitudes Survey (IAS).[5] We follow a similar method to Pfaeuti (2021) in doing so

[4]We show in Appendix A that $\sigma_\epsilon^2 = 0$ when $\gamma = 0$, so we can drop it here.

[5]Future work should also compute attention estimates based on UK firm and financial market expectations data.

by rearranging equation (3) to get:[6]

$$\pi^{e,RI}_{i,t+1|t} = \pi^{e,RI}_{i,t|t-1} + \gamma \left[E_t[\pi_{t+1}] - \pi^{e,RI}_{i,t|t-1} \right] \tag{4}$$

And estimating the following equation:

$$\pi^{e,RI}_{i,t+1|t} = \beta_0 + \beta_1 \pi^{e,RI}_{i,t|t-1} + \beta_2 \left[E_t[\pi_{t+1}] - \pi^{e,RI}_{i,t|t-1} \right] \tag{5}$$

We then calculate attention $\hat{\gamma}$ scaled to 1, such that: $\hat{\gamma} = \frac{\hat{\beta}_2}{\hat{\beta}_1 + \hat{\beta}_2}$, using Newey-West standard errors with 4 lags. Table 1 reports a summary of the regression results, with the full regression table reported in Appendix C.

Table 1: Estimating Attention

	UK Household Data (IAS)		
	Full Sample	High Income	Low & Medium Income
	(1)	(2)	(3)
$\hat{\gamma}_{full}$	0.22***	0.28***	0.22***
$\hat{\gamma}_{low\pi}$	0.20***	0.28***	0.19***
$\hat{\gamma}_{high\pi}$	0.27***	0.29***	0.27***
Sample Period	Full	Low inflation	High inflation
Observations	68	68	68
Note:			*p<0.1; **p<0.05; ***p<0.01

Row (1), column (1) reports the estimated attention to one-period inflation, across the full sample and time period of available data (Q3 2006 - Q2 2023): $\hat{\gamma}_{full} = 0.22$.[7] In rows (2) and (3), we distinguish between different time periods in the sample. Row (2) reports the estimated coefficients ($\hat{\gamma}_{low\pi}$) during the period of low inflation after the GFC (Q1 2010 - Q1 2021), while row (3) reports the estimates ($\hat{\gamma}_{high\pi}$) during the recent period of high inflation. Consistent with the theoretical predictions of the rational inattention model, we find evidence consistent with attention being greater during times of greater uncertainty ($\hat{\gamma}_{high\pi} > \hat{\gamma}_{low\pi}$).[8]

Columns (2) and (3) report the estimated coefficients distinguishing by income. From row (1), we see that high income households typically pay more attention than low income households. Interestingly, however, we see that the degree of attention paid by high income households does not change very much between the low and high inflation periods. Instead, that of low income households does (relatively) considerably; driving the average effect observed in column (1).

[6]We drop $\epsilon_{i,t}$ in equation (7) for simplicity, as noise is entirely reflected and captured by γ, as per footnote 3.
[7]This is consistent with similar empirical estimates in the US by Pfaeuti (2021)
[8]This is also consistent with findings from survey data (e.g. Weber et al. (2023)).

3 Application to a climate DSGE model

3.1 Model

We take a climate DSGE model a la Annicchiarico & Di Dio (2015) (henceforth, ADD15) and relax the FIRE assumptions therein. The set-up is similar to a standard NK model: infinitely-lived households maximise utility through their choice of consumption and labour; final goods-producing firms operate under perfect competition; intermediate goods-producing firms operate under monopolistic competition using labour and capital inputs; there are nominal rigidities in the form of sticky prices a la Calvo (1983); and a simple Taylor rule closes the model. The key difference with standard NK models is that (i) intermediate-good producing firms release pollutant emissions as a by-product of production, and (ii) the stock of pollutant affects firms' productivity (as in Heutel (2012)) such that the production function is given by:

$$Y_{j,t} = (1 - \Gamma(M_t))\, A_t K_{j,t}^{\alpha} L_{j,t}^{1-\alpha}, \alpha \in (0,1), \tag{6}$$

where $Y_{j,t}$ is the output of a firm producing intermediate good j at time t, M_t is the stock of pollutant at time t, Γ is an increasing and convex function, α is the elasticity of output w.r.t. capital, and A_t is total factor productivity (TFP).

In order to reduce emissions, $Z_{j,t}$, which at the firm-level are proportional to output, $Y_{j,t}$, firms can exert 'abatement effort', $U_{j,t}$, such that:

$$Z_{j,t} = (1 - U_{j,t})\psi Y_{j,t}, \tag{7}$$

where ψ reflects the emissions per unit output absent any abatement. However, emissions abatement is costly to firms, and is a function of abatement effort and output:

$$C_A(U_{j,t}, Y_{j,t}) = \phi_1 U_{j,t}^{\phi_2} Y_{j,t}, \phi_1 > 0, \phi_2 > 1, \tag{8}$$

where ϕ_1 and ϕ_2 are exogenous technological parameters. Firms' marginal cost of an additional unit of output, $Y_{j,t}$, is given by:

$$MC_t = \Psi_t + \phi_1 U_{j,t}^{\phi_2} + \frac{P_{z,t}}{P_t}(1 - U_t)\psi \tag{9}$$

This function has three components. First, Ψ_t, reflects the standard marginal cost associated with extra capital and labour inputs, which depends on the production technology and input prices. Second, $\phi_1 U_{j,t}^{\phi_2}$ is the cost associated with abatement. Third, $\frac{P_{z,t}}{P_t}(1 - U_t)\psi$ is the cost associated with emitting pollutant, where P_z is the exogenous 'price' of emission and can be thought of as a carbon tax imposed by the government. The rest of the model setup is standard, and we have an optimal price-setting equation given by:

$$\frac{P_t^*}{P_t} = \frac{\theta - 1}{\theta} \frac{E_t \Sigma_{i=0}^{\infty} \xi^i Q_{t,t+i}^R MC_{t+i} (\frac{P_{t+i}}{P_t})^{\theta} Y_{t+i}}{E_t \Sigma_{i=0}^{\infty} \xi^i Q_{t,t+i}^R (\frac{P_{t+i}}{P_t})^{\theta - 1} Y_{t+i}} \tag{10}$$

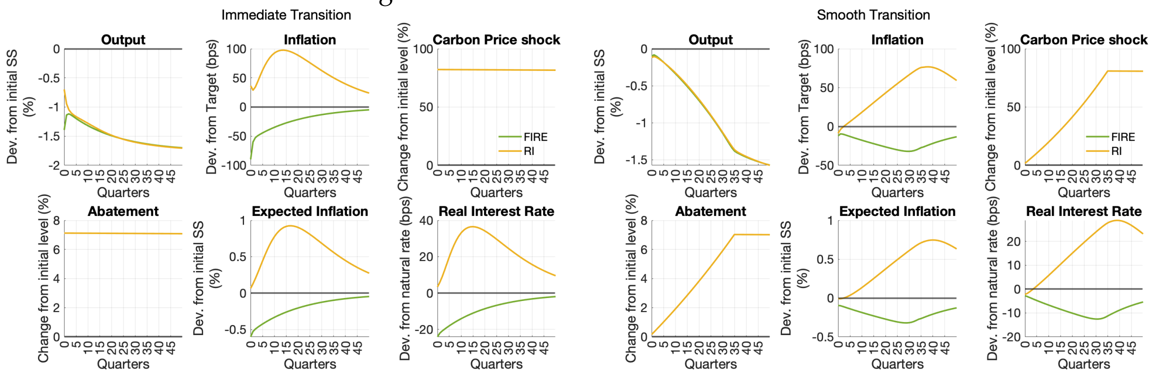
where prices today depend both on expected future prices, as well as expected future marginal costs and output.

With these key ingredients, we can compare the baseline ADD15 model with FIRE agents with an augmented model in which the FIRE expectations operator on forward-looking variables, $E_t[x_{t+1}]$, is replaced with the RI counterpart, $x^{e,RI}_{i,t+1|t}$. Attention is calibrated with $\gamma = 0.22$, consistent with UK empirical evidence on inflation expectations presented in Section 2.2.[9][10]

3.2 Results

Figure 1 reports the impulse responses to permanent carbon price shocks across the standard ADD15 model with FIRE agents (in green), and my augmented model with RI agents (in yellow).[11] On the LHS, we model an 'immediate' transition where the carbon price unexpectedly jumps in one single go, while on the RHS, we model a 'smooth' transition with carbon price increasing gradually each period, and this transition is announced in period $t = 1$. In each case, carbon prices increase by 80% (in the latter case, in 36 quarters), consistent with OECD estimates of the required 2021 - 2030 increase to hit net-zero targets.

Figure 1: Carbon Price Shocks[12]



Focusing first on the LHS panel – the one-time permanent increase in carbon price – there are a few points to make. First, under FIRE assumptions the impact of a permanent increase in carbon price is deflationary. The intuition is that, in this model, there are two competing effects on inflation. On the one hand, firms pass on the increase in carbon prices to consumers by increasing prices. This 'cost-push' component of the shock is inflationary. However, on the other hand, output falls significantly, over a long-horizon.[13] FIRE agents fully internalise this expected future lower output and smooth consumption by reducing demand today, thereby pushing down on inflation. In the FIRE model, this latter deflationary effect outweighs the

[9]This calibration is used also for other forward-looking variables: output, consumption, and investment; extrapolating from the data on inflation. Future work should estimate these values separately for each respective forward-looking variable.

[10]For now, attention is modelled as constant. Future work should allow γ to vary both across individuals and over time, consistent with empirical evidence presented in Section 2.3.

[11]Carbon price shocks are modelled as an exogenous increase in P_z. The 'permanent' nature of the increase is modelled through a shock persistence of (very nearly) 1. At this stage, we have not modelled this shock as a change to the steady-state, which more developed work will seek to do.

[12]NB: the bottom right figure in each panel is mis-labelled as 'Real Interest Rate'. This is in fact the nominal interest rate.

[13]To note, We have not modelled the permanently higher carbon price as a change in the steady-state. Future work could develop this further.

inflationary cost-push effect. Monetary policy responds by cutting interest rates and inflation returns smoothly towards target.

In contrast, when agents are rationally inattentive, the impact on inflation is positive and persistent. Intuitively, the deflationary affect coming from expected future lower output is dampened, and becomes outweighed by the inflationary cost-push component.[14] Additionally, expectations are sluggish. They do not fall as quickly to target, resulting in persistently higher inflation.

Indeed, this persistence of inflation under RI is even more acute in the case of a 'smooth' transition on the RHS panel. Agents do not fully pay attention to news received in period $t = 1$ that the carbon price will subsequently increase. Instead, each time the carbon price does increase, this represents an unexpected shock to some proportion of agents, pushing up on their inflation expectations. In turn, these expectations are slow to return to target once the transition has finished, further delaying the return of inflation to target.

In each case, the macroeconomic and potential monetary policy implications are significantly different to those predicted by standard FIRE models.

4 Conclusions

Expectations are particularly important in a climate setting given the long-run horizon of transition policies. Yet, modelling in this field has focused on the unrealistic assumption that agents are fully informed and rational. We show that the macroeconomic effects of the net-zero transition could be significantly different in a world in which agents are rationally inattentive, compared to one in which they are FIRE. Benchmark 'E-DSGE' models predict that the macroeconomic effects of carbon price increases are likely to be small, and potentially even deflationary. We show that these predictions are sensitive to the assumption that agents are FIRE. If instead agents are rationally inattentive and do not fully interanlise the outlook for the economy, carbon price increases could be *inflationary* and *persistently*, with potentially significantly different implications for monetary policy.

Future work could extend the simple application of RI within a benchmark E-DSGE model in a number of ways. For instance, extensions could endogenise state-dependent attention, with particular relevance given potential recent hihgh inflation episode, and potential for 'scarring' effects therein (Malmendier & Nagel 2016). Extensions could also apply this framework to a broader range of transition policies, to compare both their emissions abatement effectiveness and impact on the macroeconomy in this setting.

[14]This intuition is qualitatively consistent with findings from Annicchiarico et al. (2022) in a behavioural model.

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Appendix A

Appendix A.1 Micro-foundation: static setup

The core concept behind the rational inattention framework is that information is a scarce resource and costly to acquire (Sims 2003). Agents pay attention to different sources of information based on the relative costs and benefits of doing so. Mathematically, following Mackowiak & Wiederholt (2009), an individual may receive a noisy signal, s, containing information about inflation, π, given by:

$$s = \pi + \epsilon$$

where $\epsilon \sim N(0, \sigma_\epsilon^2)$ reflects noise in the signal. Based on this signal, an individual will update whatever prior belief they had about inflation, $\bar{\pi}$, to form a Bayesian posterior belief, $\tilde{\pi}$:

$$\tilde{\pi} \equiv E[\pi|s] = (1-\gamma)\bar{\pi} + \gamma s \tag{A-1}$$

where $\gamma \equiv 1 - \frac{\sigma^2_{\pi|s}}{\sigma^2_\pi} = [0,1]$ denotes the degree of attention an individual chooses to pay to the signal, s. Higher values of gamma reflect greater attention paid, which in turn reduces uncertainty, $\sigma^2_{\pi|s}$.[15]

Individuals derive utility from being well informed, $u(\pi, \tilde{\pi})$, such that $\frac{du}{d(|\pi - \tilde{\pi}|)} < 0$, but incur some cost, $c(\gamma)$, from paying attention, such that $\frac{dc}{d\gamma} > 0$. Thus, their optimal choice of attention problem is described by:

$$\max_\gamma E[u(x, \tilde{x})] - c(\gamma) \tag{A-2}$$

which (after some maths) yields optimal attention:

$$\gamma^* = \max\left(0, 1 - \frac{\lambda}{\sigma^2_\pi}\right) \tag{A-3}$$

$\lambda = \frac{c}{b}$ is the relative cost and benefit the individual faces from paying attention. Thus, the optimal level of attention is, of course, increasing with b and decreasing with c, but more interestingly increasing in the degree of uncertainty around inflation, σ^2_π. That is, it is state-dependent.

Individuals then update their belief as per equation (A-1), and form a posterior belief about inflation, $\tilde{\pi}$ that deviates from actual inflation, π, by:

$$\pi - \tilde{\pi} = \frac{\lambda}{\sigma^2_\pi}\pi + \eta \tag{A-4}$$

where $\eta \equiv \gamma\epsilon \sim N(0, \sigma^2_\eta)$ can be interpreted as resulting noise in action. Note that $\lambda > 0$ implies under-reaction of beliefs to shocks to π, with individuals choosing $\gamma < 1$ and paying less attention to inflation than in a perfect information setting.

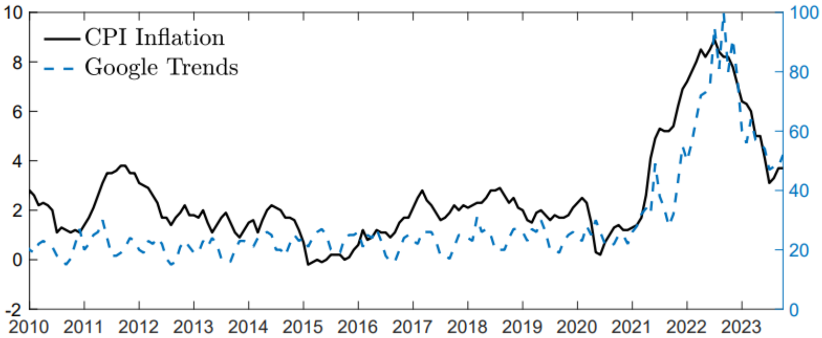
[15]In the extreme case where an individual pays full attention (i.e. where $\gamma = 1$ and $\sigma^2_{\pi|s}$), an individual forms posterior belief $\tilde{\pi} = \pi + \epsilon$. By Gaussian properties and Bayes' Rule, it can be shown that $\sigma^2_{\pi|s} = \frac{\sigma^2_\pi \sigma^2_\epsilon}{\sigma^2_\pi + \sigma^2_\epsilon}$, implying that $\tilde{\pi} = \pi$ and we are in the full information world.

Appendix B

This Appendix provides some additional empirical results.

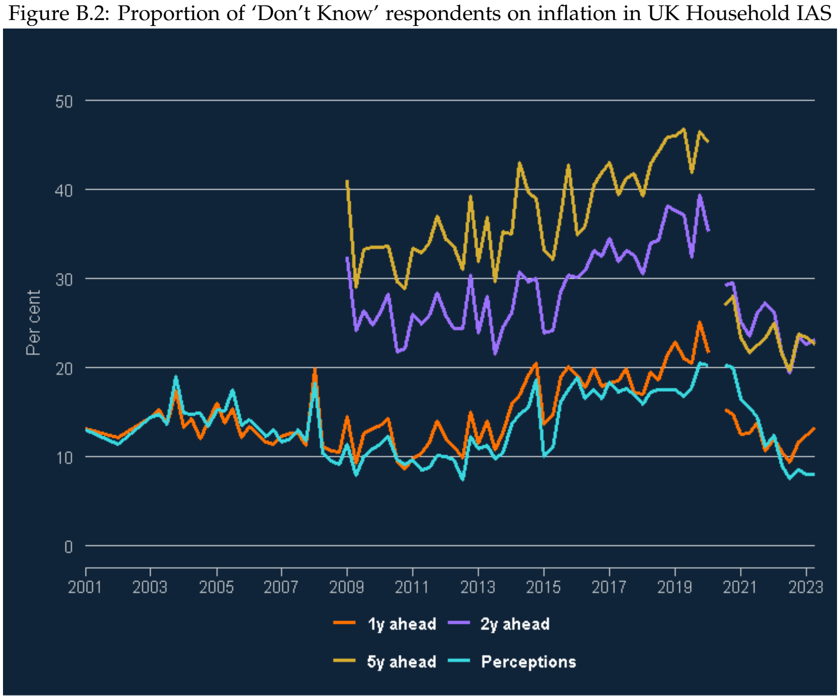
Empirical Observations

Figure B.1: Google searches for inflation



Source: Pfaeuti (2021) The black-solid line shows monthly year-on-year US CPI inflation rate (left axis), and the blue-dashed line the number of Google searches of inflation in the US (right axis, normalised to have a maximum of 100).

Figure B.2: Proportion of 'Don't Know' respondents on inflation in UK Household IAS



Notes: This figure plots the quarterly evolution of participants who respond 'Don't Know' in relation to questions about (i) current perceived inflation (aqua), (ii) 1y ahead (orange), (iii) 2y ahead (purple), and (iv) 5y ahead inflation expectations (yellow).

Regression Results

Table Appendix B.A-1: Estimating Attention: Full Regression Results

	UK Household data (IAS)								
	Full Sample			High Income			Low Income		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Lagged Exp ($\hat{\beta}_1$)	0.906***	1.008***	0.850***	0.845***	0.927***	0.770***	0.921***	1.020***	0.872***
	(0.047)	(0.047)	(0.086)	(0.053)	(0.048)	(0.145)	(0.055)	(0.049)	(0.085)
Updated Exp ($\hat{\beta}_2$)	0.263***	0.250***	0.319***	0.325***	0.364***	0.313**	0.257***	0.240***	0.319***
	(0.041)	(0.045)	(0.099)	(0.049)	(0.049)	(0.130)	(0.042)	(0.039)	(0.095)
$\hat{\gamma} = \frac{\hat{\beta}_2}{\hat{\beta}_1+\hat{\beta}_2}$	0.22	0.20	0.27	0.28	0.28	0.29	0.22	0.19	0.27
Constant	0.657***	0.345**	0.827***	0.850**	0.698***	1.144***	0.616*	0.298*	0.764***
	(0.171)	(0.169)	(0.219)	(0.391)	(0.175)	(0.411)	(0.353)	(0.175)	(0.216)
Sample Period	Full	Low inflation	High inflation	Full	Low inflation	High inflation	Full	Low inflation	High inflation
Observations	68	45	9	68	45	9	68	45	9
R^2	0.860	0.789	0.901	0.785	0.606	0.859	0.868	0.804	0.899

Note: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$