COMP108
Data Structures and Algorithms

Graphs (Part II Paths, Circuits, BFS/DFS)

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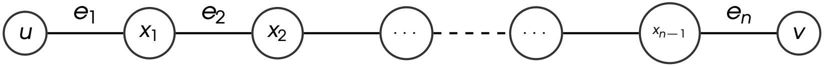
Undirected graph - Paths and circuits

- In an undirected graph, a **path** from a vertex u to a vertex v is a sequence of edges $e_1 = \{u, x_1\}, e_2 = \{x_1, x_2\}, \cdots, e_n = \{x_{n-1}, v\}$, where $n \geq 1$.
- The **length** of this path is n.
- Note that a path from u to v implies a path from v to u in an undirected graph.
- If $u \equiv v$, this path is called a **circuit (cycle)**.



Euler Circuit

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  (NB. A vertex can be repeated.)
- Does every graph has an Euler circuit ?

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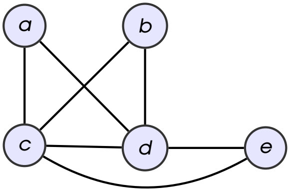

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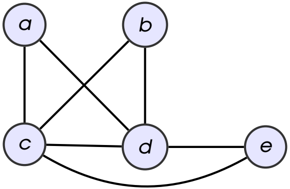

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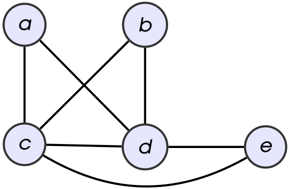

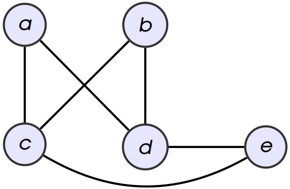

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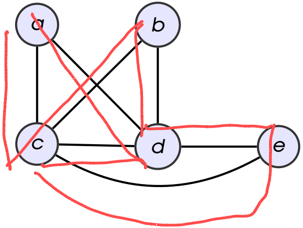

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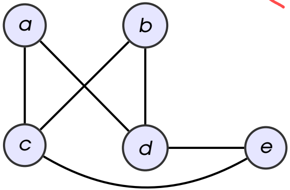
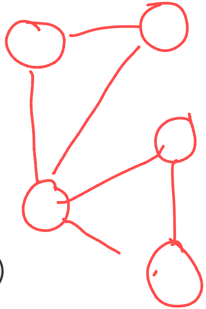


no Euler circuit



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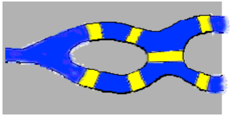

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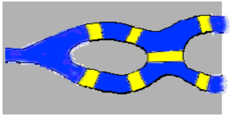

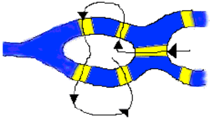

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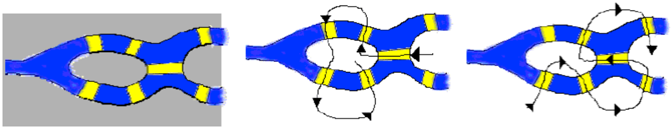

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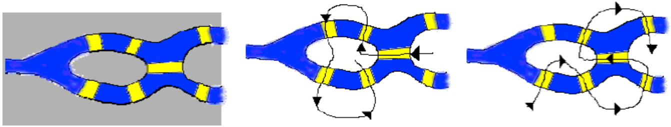

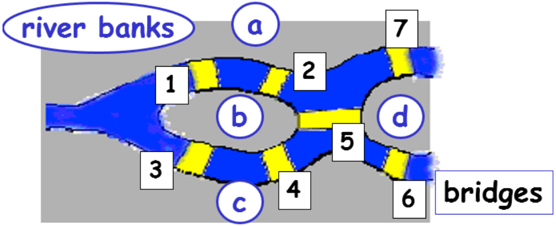

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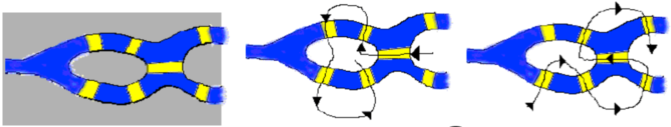

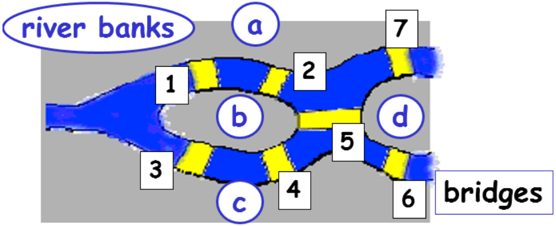

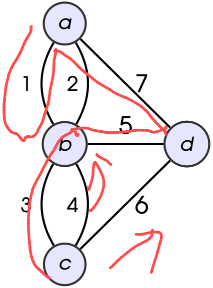

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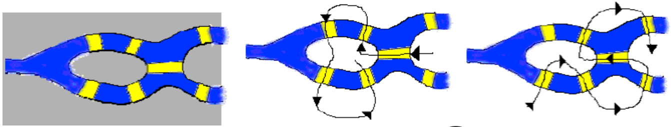
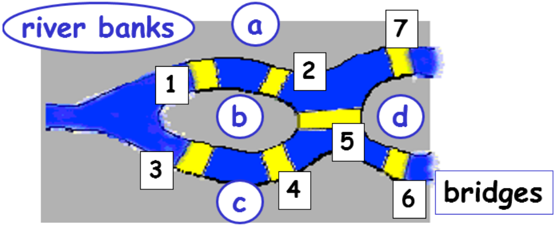
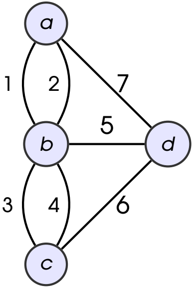


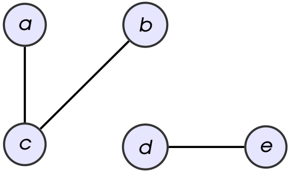




How to determine if there is Euler circuit?

A trivial condition for Euler circuit

- An undirected graph G is said to be **connected** if there is a path between every pair of vertices.
- If G is not connected, there is no single circuit to visit all edges or vertices.
- Being connected is a necessary condition but not sufficient.



Necessary and Sufficient Condition

Let G be a connected graph.

Lemma

G contains an Euler circuit if and only if the degree of every vertex is even.

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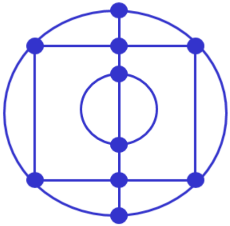

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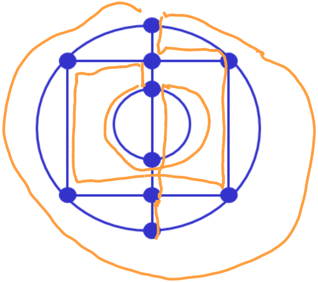

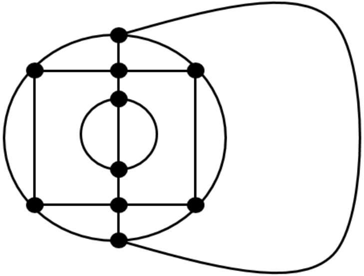

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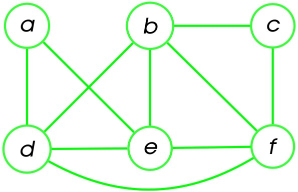

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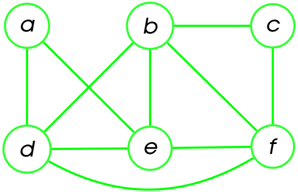

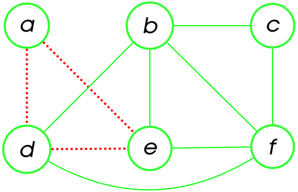

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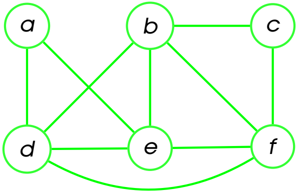

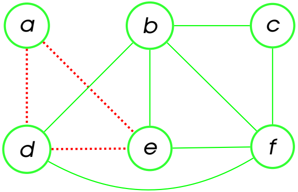

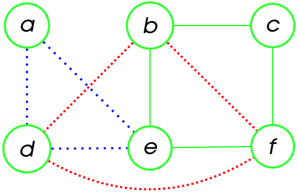

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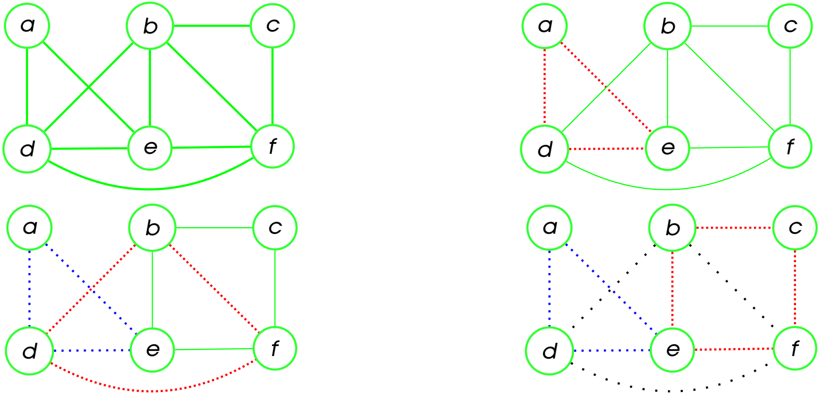

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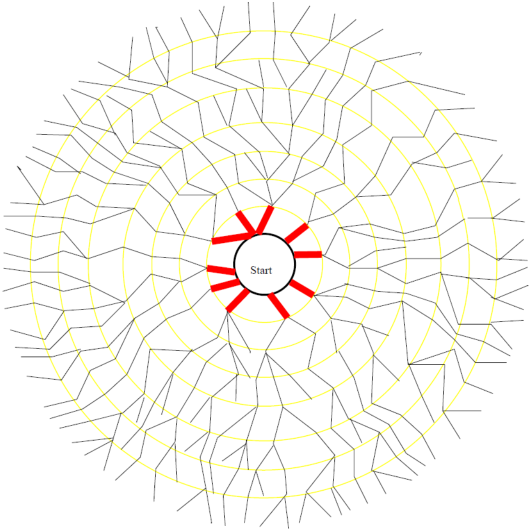
Hamiltonian circuit

- Let G be an undirected graph.
- A **Hamiltonian circuit** is a circuit containing every vertex of G exactly once.
- Note that a Hamiltonian circuit may NOT visit all edges.
- Unlike the case of Euler circuits, determining whether a graph contains a Hamiltonian circuit is a very difficult problem. (NP-hard)

Breadth First Search (BFS) - figure adopted from COMP111



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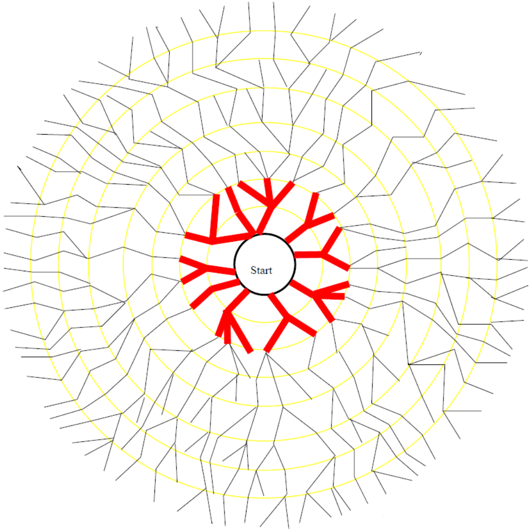
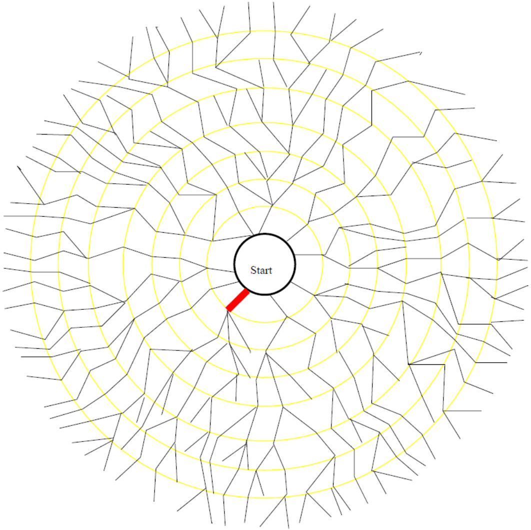


Depth First Search (DFS) - figure adopted from COMP111



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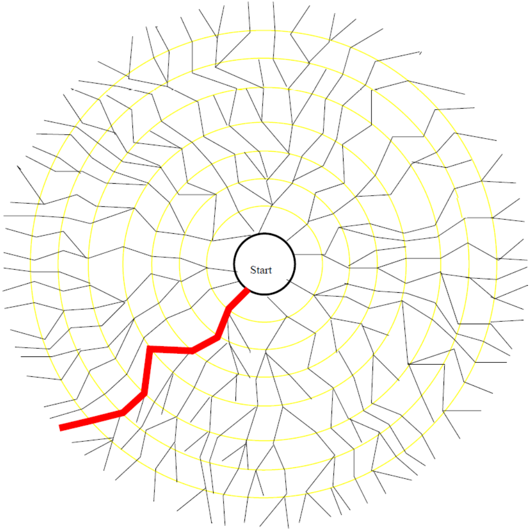
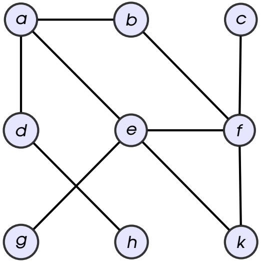


BFS...

BFS - with queue / linked list

BFS:

Suppose a is the starting vertex.



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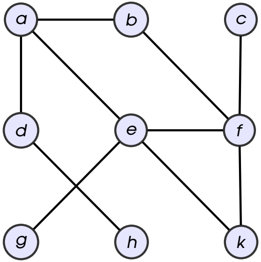
head $\longrightarrow$ | a | / | $\longleftarrow$ *tail*



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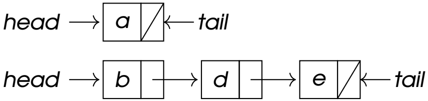

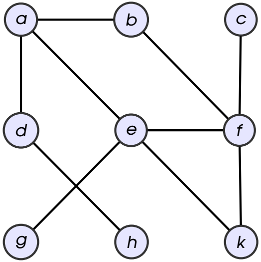

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BFS: *a*, *b*, *d*,

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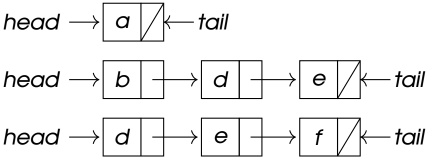

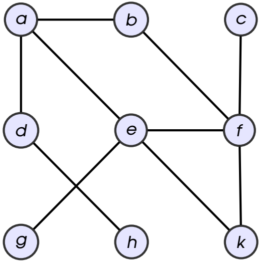

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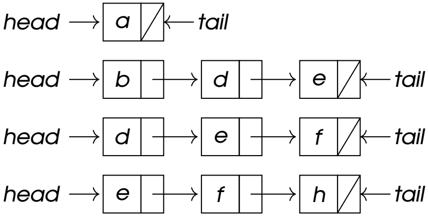

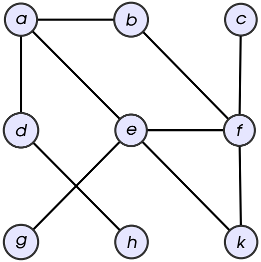

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BFS: a, b, d, e, f,

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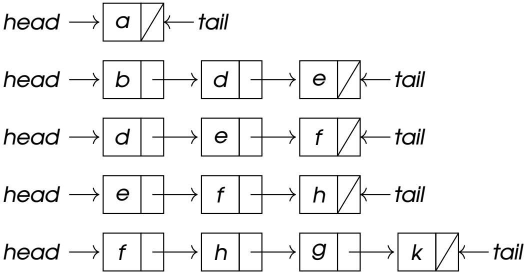

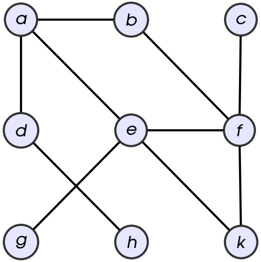

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BFS: *a*, *b*, *d*, *e*, *f*, *h*,

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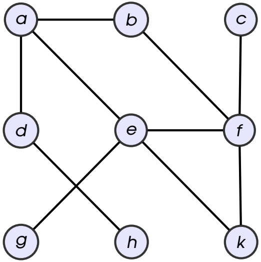

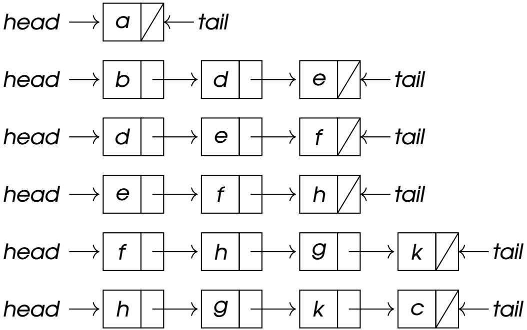

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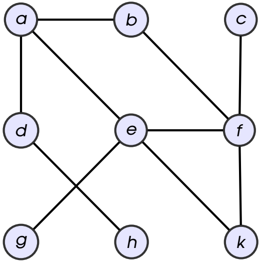

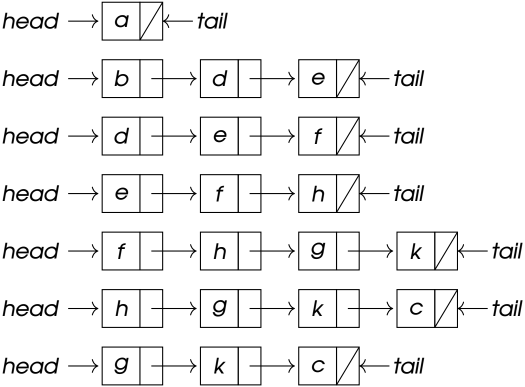

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BFS: *a, b, d, e, f, h, g, k,*

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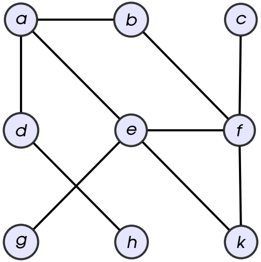

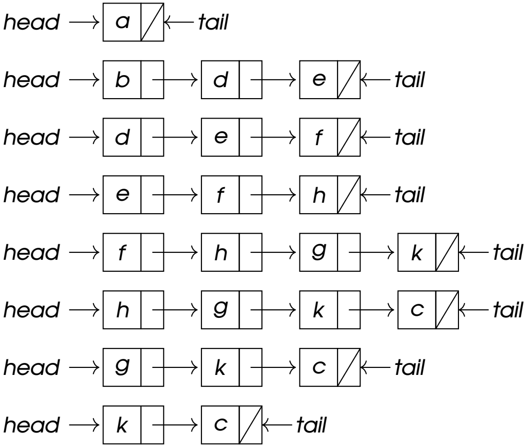

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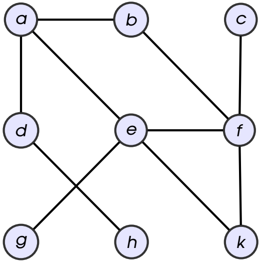

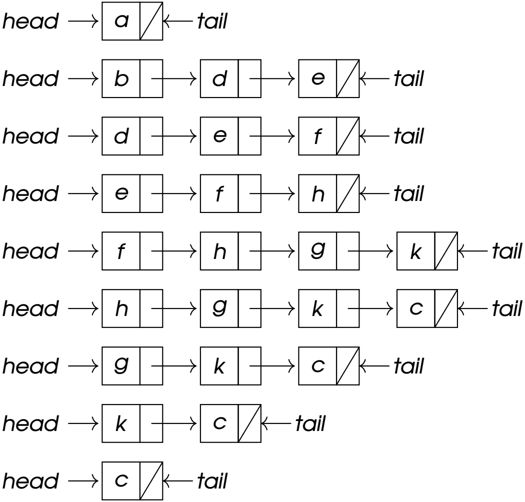

BFS - pseudo code - with linked list / queue

unmark all vertices

choose some starting vertex s

mark s and insert s into tail of list L

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mark s and insert s into tail of list L
while L is nonempty do
begin

end

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    remove a vertex v from **head of L**

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while L is nonempty do
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choose some starting vertex *s*
mark *s* and insert *s* into tail of list *L*
while *L* is nonempty do
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    visit *v* // e.g., print its data
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end

BFS - pseudo code - with linked list / queue

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while L is nonempty do
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    remove a vertex v from head of L
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DFS...

DFS - with stack

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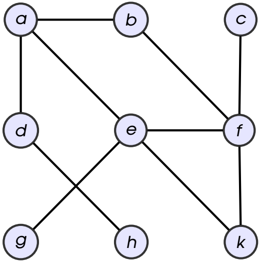

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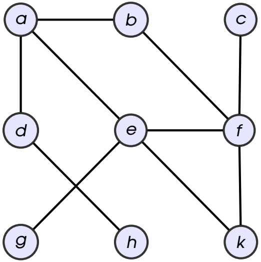

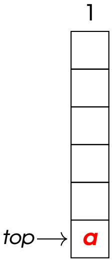

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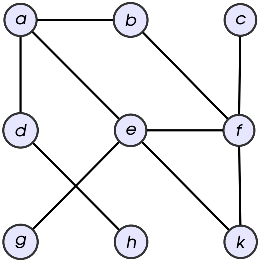
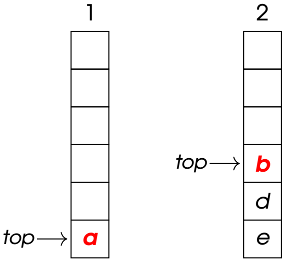
Suppose a is the starting vertex.

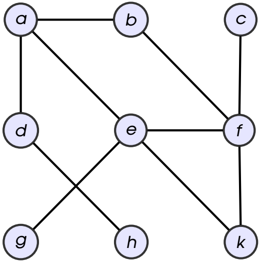


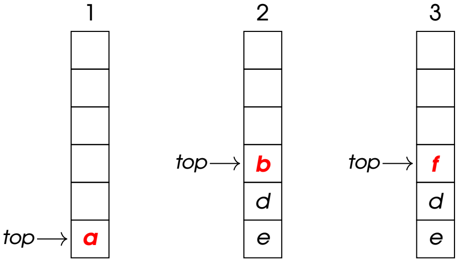


DFS - with stack

DFS: a, b, f,

Suppose a is the starting vertex.

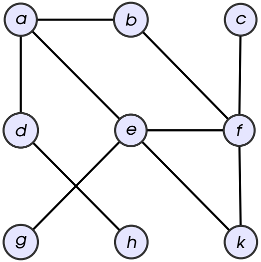


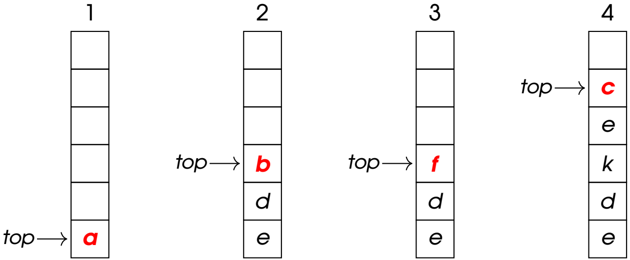


DFS - with stack

DFS: a, b, f, c,

Suppose a is the starting vertex.

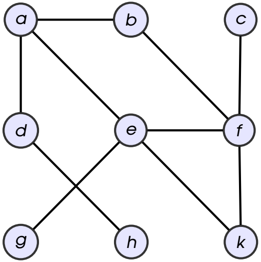


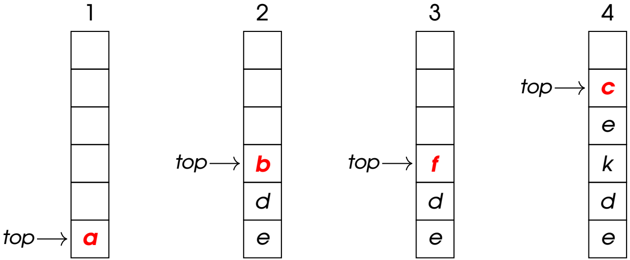


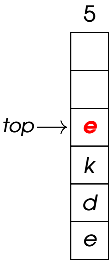
DFS - with stack

DFS: a, b, f, c, e,

Suppose a is the starting vertex.



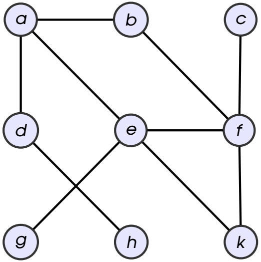


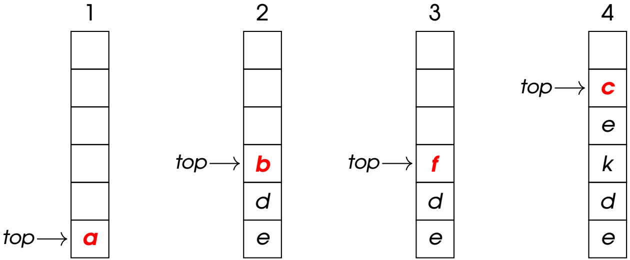


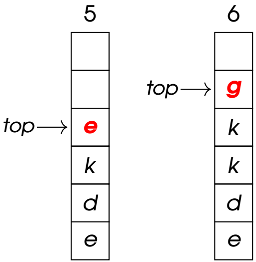
DFS - with stack

DFS: a, b, f, c, e, g,

Suppose a is the starting vertex.



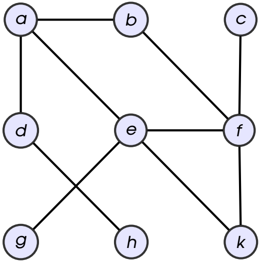


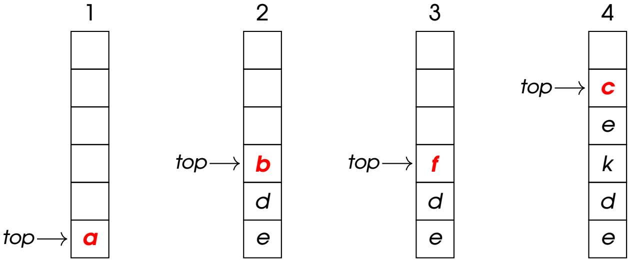


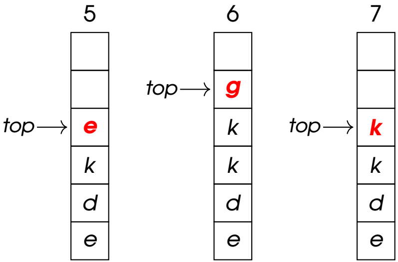
DFS - with stack

DFS: a, b, f, c, e, g, k,

Suppose a is the starting vertex.



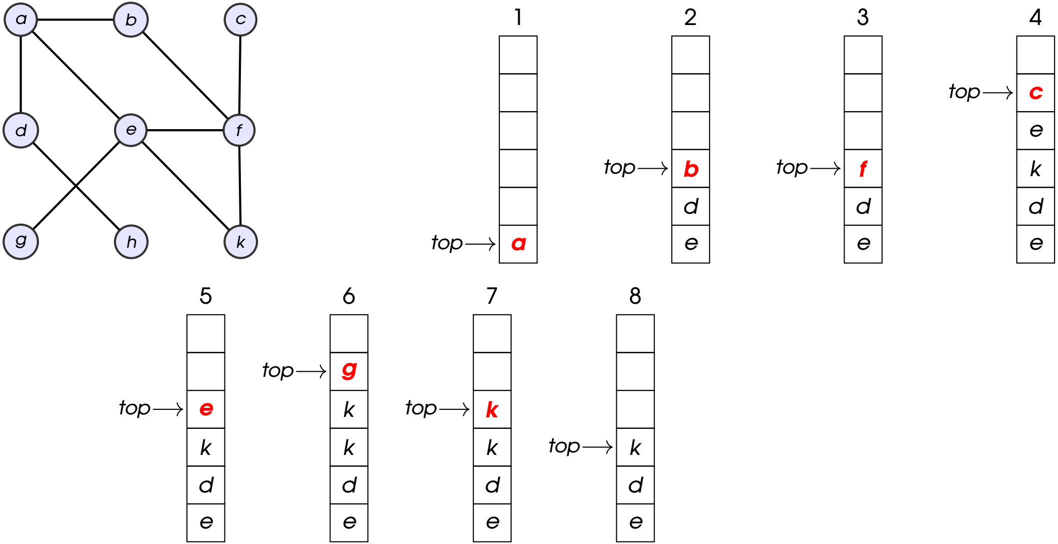




DFS - with stack

DFS: a, b, f, c, e, g, k,

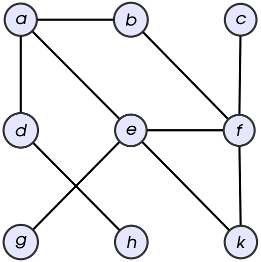
Suppose a is the starting vertex.

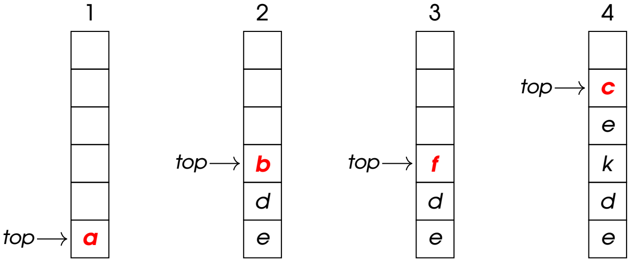


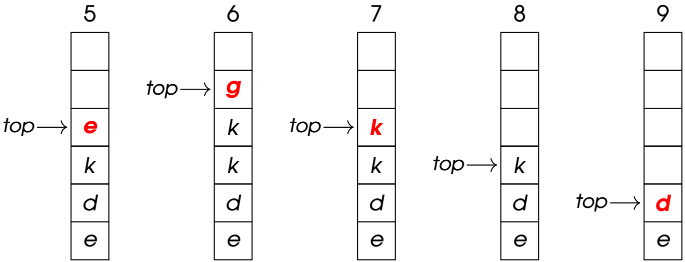
DFS - with stack

DFS: *a*, *b*, *f*, *c*, *e*, *g*, *k*, *d*,

Suppose *a* is the starting vertex.



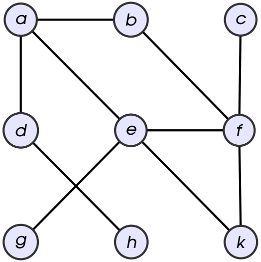


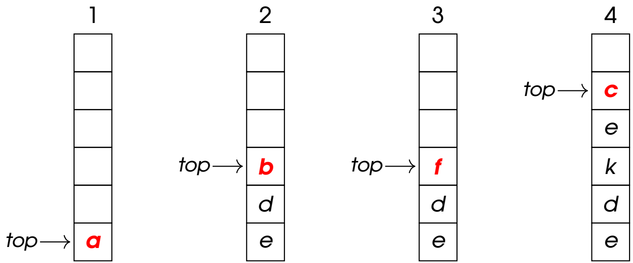


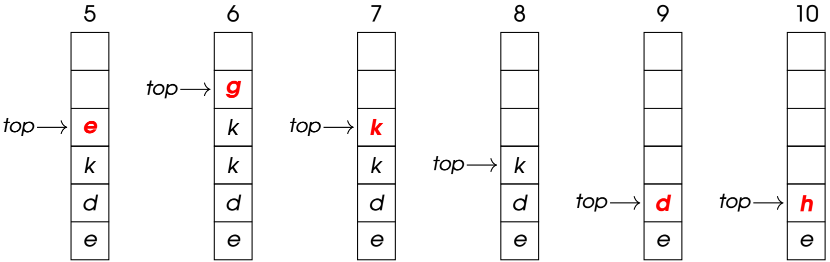
DFS - with stack

DFS: *a*, *b*, *f*, *c*, *e*, *g*, *k*, *d*, *h*

Suppose *a* is the starting vertex.



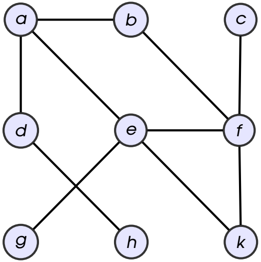


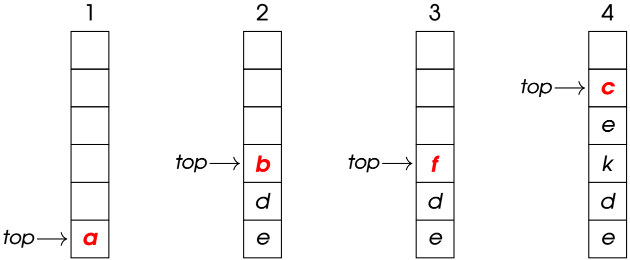


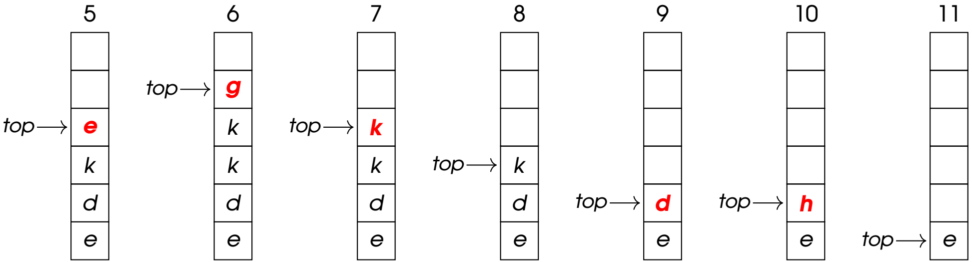
DFS - with stack

DFS: *a, b, f, c, e, g, k, d, h*

Suppose *a* is the starting vertex.







DFS - pseudo code - with stack

unmark all vertices

DFS - pseudo code - with stack

unmark all vertices

push starting vertex u onto **top of stack** S

DFS - pseudo code - with stack

```
unmark all vertices
push starting vertex u onto top of stack S
while S is nonempty do
begin



end
```

DFS - pseudo code - with stack

unmark all vertices

push starting vertex ***u*** onto **top of stack *S***

while S is nonempty do

begin

  pop a vertex ***v*** from **top of *S***

end

DFS - pseudo code - with stack

```
unmark all vertices
push starting vertex u onto top of stack S
while S is nonempty do
begin
    pop a vertex v from top of S
    if v is unmarked then
    begin



    end
end
```

DFS - pseudo code - with stack

```
unmark all vertices
push starting vertex u onto top of stack S
while S is nonempty do
begin
    pop a vertex v from top of S
    if v is unmarked then
    begin
        visit and mark v


    end
end
```

DFS - pseudo code - with stack

```
unmark all vertices
push starting vertex u onto top of stack S
while S is nonempty do
begin
    pop a vertex v from top of S
    if v is unmarked then
    begin
        visit and mark v
        for each unmarked neighbor w of v do

    end
end
```

DFS - pseudo code - with stack

```
unmark all vertices
push starting vertex u onto top of stack S
while S is nonempty do
begin
    pop a vertex v from top of S
    if v is unmarked then
    begin
        visit and mark v
        for each unmarked neighbor w of v do
            push w onto top of S
    end
end
```

Summary

Summary: Traversals

Next: Greedy Algorithms

For note taking