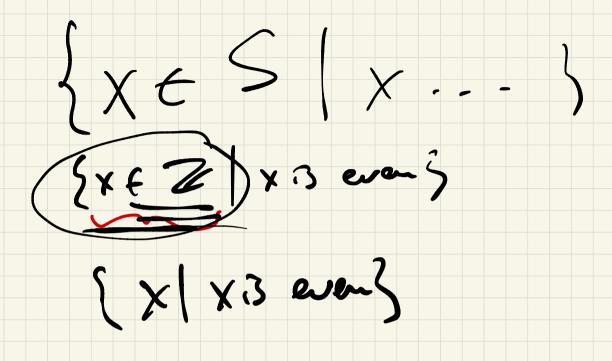
Russel's paradox



2° = { B < A ?

1 x 3 a set 3

Why is this set theory "naive"

It suffers from paradoxes.

A leading example:

A barber is the man who shaves all those, and only those, men who do not shave themselves.

■ Who shaves the barber?

Russell's Paradox

Russell's paradox shows that the 'object' $\{x \mid P(x)\}$ is not always meaningful.

Set
$$A = \{B \mid B \notin B\}$$

Problem: do we have $A \in A$?

Abbreviate, for any set C, by P(C) the statement $C \notin C$. Then $A = \{B \mid P(B)\}$.

- If $A \in A$, then (from the definition of P), not P(A). Therefore $A \notin A$.
- If $A \notin A$, then (from the definition of P), P(A). Therefore $A \in A$.