Problem set 1
Mathematical Preliminaries

Excercise 1

Given $\overline{X} = (1, 2, 3)^T$ and $\overline{Y} = (3, 2, 1)^T$ find

1. $\overline{X} + \overline{Y}$
2. $\overline{X}^T \overline{Y}$
3. $\overline{Y}\,\overline{X}^T$

Excercise 2

Given two matrices $\overline{A} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ and $\overline{B} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & 3 \\ -1 & 0 & 1 \end{pmatrix}$

1. Compute $\overline{A} + \overline{B}$
2. Compute $\overline{B} + \overline{A}$. Is it equal to $\overline{A} + \overline{B}$? Is it always the case?
3. Compute $\overline{A} \cdot \overline{B}$
4. Compute $\overline{B} \cdot \overline{A}$. Is it equal to $\overline{A} \cdot \overline{B}$?

Excercise 3

Compute the inverse of the following matrix $\overline{A} = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$, if one exsits. Verify that the matrix product of $\overline{A}$ and its inverse is the 2x2 identity matrix.

Excercise 4

Show that the vectors $\overline{A} = (1, 2, -3, 4)^T$, $\overline{B} = (1, 1, 0, 2)^T$, and $\overline{C} = (-1, -2, 1, 1)^T$ are linearly independent.

Excercise 5

Find the ranks of the following matrices $\overline{A} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ and $\overline{B} = \begin{pmatrix} 1 & 2 & 1 \\ -2 & -3 & 1 \\ 3 & 5 & 0 \end{pmatrix}$.

Excercise 6

Find the eigenvalues and the corresponding eigenvectors of $\overline{A} = \begin{pmatrix} 4 & 2 \\ 1 & 3 \end{pmatrix}$

Excercise 7

Given $f(x) = \log(x)$ (where log denotes the natural logarithm) and $g(x) = 2x + 1$, compute

1. $f'(x)$
2. $g'(x)$
3. $(f(x) + g(x))'$
4. $(f(x)g(x))'$
5. $\left(\frac{f(x)}{g(x)}\right)'$
6. $(g(f(x)))'$

Excercise 8

Given $f(x, y) = (x + 2y^3)^2$ compute

1. $\frac{\partial f}{\partial x}$
2. $\frac{\partial f}{\partial y}$
3. $\nabla_{(x,y)} f$