

Foundations of Computer Science

Comp109

University of Liverpool

Boris Konev

konev@liverpool.ac.uk

Part 1. Number Systems and Proof Techniques

Comp109 Foundations of Computer Science

- S. Epp. *Discrete Mathematics with Applications*
Chapter 4, Sections 5.2 and 5.3.
- E. Bloch. *Proofs and Fundamentals*
Chapter 2, Section 6.3.
- K. Rosen. *Discrete Mathematics and Its Applications*
Section 5.1.

Contents

- The most basic datatypes
 - Natural Numbers
 - Integers
 - Rationals
 - Real Numbers
 - Prime Numbers
- Proof Techniques
 - Direct proof and disproof
 - Proof of existence
 - Disproof by counterexample
 - Generalising from the generic particular
 - Indirect Proof
 - Proof by contradiction
 - Proof by contrapositive
 - Proof by mathematical induction

$$\textcircled{11} = 2$$

M

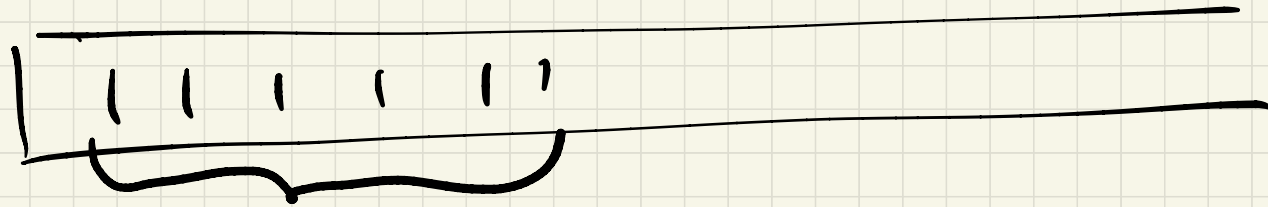
W

C

L

111

1111



$x + 11'$

$$12305_{\text{q}} = 5 + \underbrace{10 \cdot 0} + \underbrace{10^2 \cdot 3} + 10^3 \cdot 2 + \underbrace{10^4 \cdot 1}$$

The natural numbers

0, 1, 2, 3, ...

Key property: Any natural number can be obtained from 0 by applying the operation $S(n) = n + 1$ some number times.

Examples: $S(0) = 1.$

$$S(S(0)) = 2.$$

$$S(S(S(0))) = 3.$$



Beyond naturals: Integers

The Integers $\dots, -2, -1, 0, 1, 2, \dots$

God made the integers, all else is the work of man

(Leopold Kronecker)

