Beyond integers: Rationals and Reals

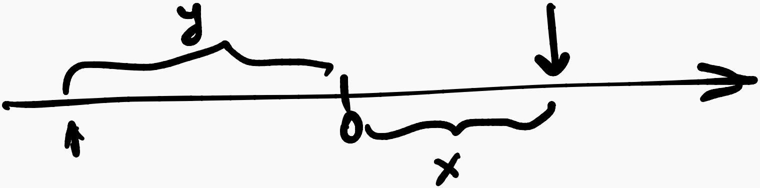
The Rational Numbers all numbers that can be written as $\dfrac{m}{n}$ where m and n are integers and n is not 0.

?

$0.3 = \frac{3}{10}$

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \dots$

The Real Numbers all (decimal) numbers — distances to points on a number line.



$$\frac{1}{3} = \underbrace{0.3333\ldots}$$

Mathematical proof

Solving and computing

Mathematics underpins STEM subjects. In many cases, we are concerned with solving and computing

The quadratic equation $2x^2 + 6x + 7 = 0$ has roots α and β.

Write down the value of $\alpha + \beta$ and the value of $\alpha\beta$.

Complete the table of values for $y = 3 - x^2$

x	-3	-2	-1	0	1	2	3
y		-1	2		2		-6

Work out $\frac{1}{3} \times \frac{1}{5}$

Find the general solution, in degrees, of the equation

$$2\sin(3x + 45°) = 1$$

5 miles = 8 kilometres

Which is longer, 26 miles or 45 km?

Statements

Which of the following are true?

- "26 miles is longer than 45 km."
- An integer doubled is larger than the integer.
- The sum of any two odd numbers is even.

The moral of the story

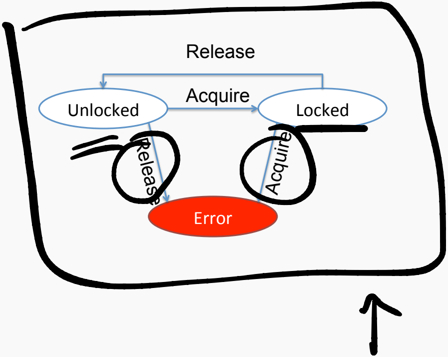
- We can't believe a statement just because it appears to be true.

We need a proof that the statement is true or a proof that it is false.

Example: Drivers behaviour[1]

```
do {
    KeAcquireSpinLock();
    nPacketsOld = nPackets;
    if (request) {
        request = request->Next;
        KeReleaseSpinLock();
        nPackets++;
    }
} while (nPackets != nPacketsOld);
KeReleaseSpinLock();
```

Does this code obey the locking rules?



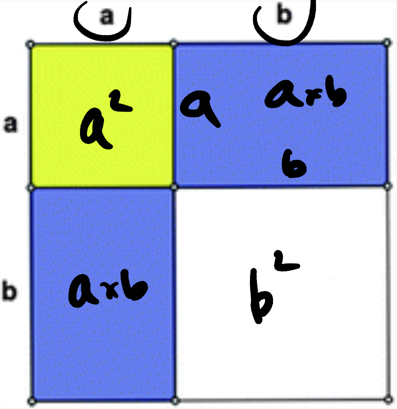
You don't need to understand the actual code!

[1] from Microsoft presentations on Static Driver Verifier (part of Visual Studio)

Historical detour: Visual proofs

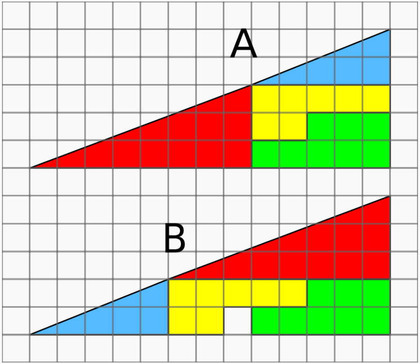
a+b

a+b



Visual proof of

$(a+b)^2 = a^2 + 2ab + b^2$



Visual "proof" of

32.5 = 31.5

Proofs

- A mathematical proof is as a **carefully reasoned argument** to convince a sceptical listener (often yourself) that a given statement is true.
- Both discovery and proof are integral parts of problem solving. When you think you have discovered that a certain statement is true, try to figure out why it is true.
- If you succeed, you will know that your discovery is genuine. Even if you fail, the process of trying will give you insight into the nature of the problem and may lead to the discovery that the statement is false.

Example: Properties of odd and even numbers

1. Is 0 even? ✓
2. Is -301 odd? ✓
3. Is the sum of any two odd numbers even? ???
4. Is every number either even or odd?

x 0 1 2 -44

Definition 1 An integer is even if it can be divided by 2

What about $1/2$?

Definition 2 An integer is even if the result of dividing this integer by 2 is an integer

What about odd numbers

<u>Definition</u> an <u>integer</u> is odd if divided by 2 it's not an integer

3

Odd and even numbers

Definition
An integer n is **even** if, and only if, n equals twice some integer.

An integer n is **odd** if, and only if, n equals twice some integer plus 1.

Symbolically, if n is an integer, then

n is even $\Leftrightarrow \exists$ an integer k such that $n = 2k$.

n is odd $\Leftrightarrow \exists$ an integer k such that $n = 2k + 1$.

Notice the use of $\boxed{\Leftrightarrow \quad \exists \quad \forall.}$

Exists

All

$\exists$

$\boxed{\text{Compute two plus five}}$

$2+5$

Def2 Even

Def Odd

?

Def Even

Def Odd

0, 1, 2, 3, 4, 5 . . .

COMP109

~~0~~ 1, 2, 3, 4

COMP116