COMP108
Data Structures and Algorithms

Data structures - Arrays (Part I Searching)

Professor Prudence Wong

pwong@liverpool.ac.uk

2022-23

Outline

Data structures - arrays

- What are data structures?
- What are arrays?
- Using arrays to look for a number in a sequence of numbers
  - Sequential/Linear search
  - Binary search
  - Finding maximum/minimum

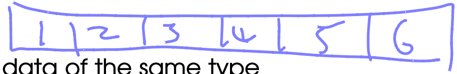
Learning outcome:

- Be able to describe the principles of and apply arrays and their associated algorithms

What are data structures?

- Dynamic data
  - data may grow, shrink, change over time
- Operations on dynamic data
  - search, insert, delete, minimum, maximum, successor, predecessor
  - some operations actually change the data: insert, delete
- Data structures
  - systematic way of organising and accessing data
- Examples
  - Arrays
  - Queues and Stacks
  - Linked lists
  - Trees and Graphs
  - Hash tables

Arrays



- Array: numbered collection of data of the same type
- Each cell has an index, containing an element
- Array A of size n with indices 1 to n: $A[1..n]$
  - note: in certain programming languages, arrays are indexed as $A[0..(n-1)]$
  - in the lecture notes, we use $1..n$
- Out of bounds: any index not in the range $1..n$

Arrays

- Array: numbered collection of data of the same type
- Each cell has an index, containing an element
- Array A of size n with indices 1 to n: $A[1..n]$
    - note: in certain programming languages, arrays are indexed as $A[0..(n-1)]$
    - in the lecture notes, we use $1..n$
- Out of bounds: any index not in the range $1..n$
- Use a loop to access the elements of the array

// summing all elements of an array
$sum \leftarrow 0$, $i \leftarrow 1$
while $i \leq n$ do
begin
    $sum \leftarrow sum + A[i]$
    $i \leftarrow i + 1$
end
output sum

1	2	3	4	5
10	20	5	3	6

sum
0 + A[1]
10 + A[2]
30 + A[3]
35
38
44

Sequential Search . . .

Sequential/Linear search

- Input: n numbers stored in an array $A[1..n]$, and a target number *key*
- Output: determine if *key* is in the array or not
- Algorithm (Sequential / Linear search)
  1. From $i \leftarrow 1$, compare *key* with $A[i]$ one by one as long as $i \leq n$.
  2. Stop and report ``Found!'' when *key* is the same as $A[i]$.
  3. Repeat and report ``Not Found!'' when $i > n$.

Sequential search - Example - To find 7

12	34	2	9	7	5	$\leftarrow$ 6 numbers
7						$\leftarrow$ key

Sequential search - Example - To find 7

12	34	2	9	7	5	$\leftarrow$ 6 numbers
7						$\leftarrow$ key
12	**34**	2	9	7	5	
	7					

Sequential search - Example - To find 7

12	34	2	9	7	5	← 6 numbers
7						← key
12	**34**	2	9	7	5	
	7					
12	34	**2**	9	7	5	
		7				

Sequential search - Example - To find 7

12	34	2	9	7	5	← 6 numbers
7						← key
12	**34**	2	9	7	5	
	7					
12	34	**2**	9	7	5	
		7				
12	34	2	**9**	7	5	
			7			

Sequential search - Example - To find 7

12	34	2	9	7	5	← 6 numbers
7						← key
12	**34**	2	9	7	5	
	7					
12	34	**2**	9	7	5	
		7				
12	34	2	**9**	7	5	
			7			
12	34	2	9	**7**	5	
				7		**FOUND!**

Sequential search - Example - To find 10

12	34	2	9	7	5	$\leftarrow$ 6 numbers
10						$\leftarrow$ key

Sequential search - Example - To find 10

12	34	2	9	7	5	← 6 numbers
10						← key
12	**34**	2	9	7	5	
	10					

Sequential search - Example - To find 10

12	34	2	9	7	5	← 6 numbers
10						← key
12	**34**	2	9	7	5	
	10					
12	34	**2**	9	7	5	
		10				

Sequential search - Example - To find 10

12	34	2	9	7	5	← 6 numbers
10						← key
12	**34**	2	9	7	5	
	10					
12	34	**2**	9	7	5	
		10				
12	34	2	**9**	7	5	
			10			

Sequential search - Example - To find 10

12	34	2	9	7	5	← 6 numbers
10						← key
12	**34**	2	9	7	5	
	10					
12	34	**2**	9	7	5	
		10				
12	34	2	**9**	7	5	
			10			
12	34	2	9	**7**	5	
				10		

Sequential search - Example - To find 10

12	34	2	9	7	5	← 6 numbers
10						← key
12	**34**	2	9	7	5	
	10					
12	34	**2**	9	7	5	
		10				
12	34	2	**9**	7	5	
			10			
12	34	2	9	**7**	5	
				10		
12	34	2	9	7	**5**	
					10	**NOT FOUND!**

Sequential search: Pseudo code - Ideas

variable *i* to step through the array
boolean *found* to indicate whether *key* is found

Sequential search: Pseudo code - Ideas

variable *i* to step through the array
boolean *found* to indicate whether *key* is found

$i \leftarrow ??$
$found \leftarrow ??$

if $found == true$ then
    output ``Found!''
else
    output ``Not found!''

Sequential search: Pseudo code - Ideas

variable *i* to step through the array
boolean *found* to indicate whether *key* is found

false

$i \leftarrow ??$ 1
found $\leftarrow ??$ n
while $i \leq ??$ AND *found* $== ??$ do false
begin

end
if *found* $==$ *true* then
    output ``Found!''
else
    output ``Not found!''

Sequential search: Pseudo code - Ideas

variable i to step through the array

boolean *found* to indicate whether *key* is found

$i \leftarrow ??$

$found \leftarrow ??$

while $i \leq ??$ AND $found == ??$ do

begin

/* check whether the i-th element

of the array equals key and if so

set *found* accordingly */

end

if $found ==$ *true* then

output ``Found!''

else

output ``Not found!''

Sequential search: Pseudo code - Ideas

variable i to step through the array
boolean *found* to indicate whether *key* is found

$i \leftarrow ??$
found $\leftarrow ??$
while $i \leq ??$ AND *found* $== ??$ do
begin
    /* check whether the i-th element
        of the array equals key and if so
        set *found* accordingly */
    $i \leftarrow i + 1$
end
if *found* $==$ *true* then
    output ``Found!''
else
    output ``Not found!''

Sequential search: Pseudo code

variable i to step through the array
boolean *found* to indicate whether *key* is found

$i \leftarrow 1$
found $\leftarrow$ *false*
while $i \leq$ AND *found* $==$ do
begin

$i \leftarrow i + 1$
end
if *found* $==$ *true* then
output ``Found!''
else
output ``Not found!''

Sequential search: Pseudo code

variable i to step through the array
boolean *found* to indicate whether *key* is found

$i \leftarrow 1$
found $\leftarrow$ *false*
while $i \leq n$ AND *found* $==$ *false* do
begin

    $i \leftarrow i + 1$
end
if *found* $==$ *true* then
    output ``Found!''
else
    output ``Not found!''

Sequential search: Pseudo code

variable i to step through the array

boolean *found* to indicate whether *key* is found

$i \leftarrow 1$

$found \leftarrow false$

while $i \leq n$ AND $found == false$ do

begin

    if $key == A[i]$ then

        $found \leftarrow true$

        $i \leftarrow i + 1$

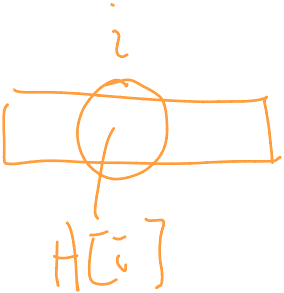
end

if $found == true$ then

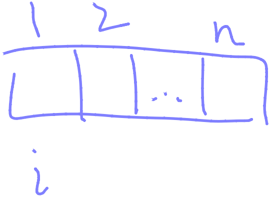
    output ``Found!''

else

    output ``Not found!''



Sequential search: Pseudo code



variable *i* to step through the array
boolean *found* to indicate whether *key* is found

$i \leftarrow 1$
$found \leftarrow false$
while $i \leq n$ AND $found == false$ do
begin
    if $key == A[i]$ then
        $found \leftarrow true$
    else
        $i \leftarrow i + 1$
end
if $found == true$ then
    output ``Found!''
else
    output ``Not found!''

when loop terminates,
i stores position if found; i stores n+1 otherwise

i?

Sequential search: Pseudo code (see `SampleSeqSearch.java` on Canvas)

variable i to step through the array
boolean *found* to indicate whether *key* is found

$i \leftarrow 1$
found $\leftarrow$ *false*
while $i \leq n$ AND *found* $==$ *false* do
begin
    if *key* $== A[i]$ then
        found $\leftarrow$ *true*
    else
        $i \leftarrow i + 1$
end
if *found* $==$ *true* then
    output ``Found!''
else
    output ``Not found!''

Sequential search: Time complexity

$i \leftarrow 1$
$found \leftarrow false$
while $i \leq n$ AND $found == false$ do
begin
    if $key == A[i]$ then
        $found \leftarrow true$
    else
        $i \leftarrow i + 1$
end

How many comparisons this algorithm takes?

Best case:

Worst case:

Sequential search: Time complexity

$i \leftarrow 1$
$found \leftarrow false$
while $i \leq n$ AND $found == false$ do
begin
    if $key == A[i]$ then
        $found \leftarrow true$
    else
        $i \leftarrow i + 1$
end

How many comparisons this algorithm takes?

Best case: *key* is first number
$\Rightarrow$ 1 comparison
$\Rightarrow O(1)$

Worst case:

Sequential search: Time complexity

$i \leftarrow 1$
found $\leftarrow$ *false*
while $i \leq n$ ~~AND *found* == *false*~~ do
begin
  if *key* $==$ $A[i]$ then
    found $\leftarrow$ *true*
  else found <- false
    $i \leftarrow i + 1$
end

How many comparisons this algorithm takes?

Best case: *key* is first number
$\Rightarrow$ 1 comparison
$\Rightarrow O(1)$

Worst case: *key* is last number OR *key* is not found
$\Rightarrow$ n comparison
$\Rightarrow O(n)$

30	20	12	5	6	10

key is 12

Sequential search: Time complexity

$i \leftarrow 1$
$found \leftarrow false$
while $i \leq n$ AND $found == false$ do
begin
    if $key == A[i]$ then
        $found \leftarrow true$
    else
        $i \leftarrow i + 1$
end

How many comparisons this algorithm takes?

Best case: *key* is first number
$\Rightarrow$ 1 comparison
$\Rightarrow O(1)$

Worst case: *key* is last number OR *key* is not found
$\Rightarrow n$ comparison
$\Rightarrow O(n)$

If the data is sorted in ascending/descending order, can we improve the time complexity?

Binary Search . . .

Binary search

- more efficient way of searching when the sequence of numbers is pre-sorted
- Input: a sequence of n sorted numbers $A[1], A[2], \cdots, A[n]$ in ascending order and a target number *key*
- Idea of algorithm:
  1. compare *key* with number in the middle
  2. then focus on only the first half or the second half (depend on whether *key* is smaller or greater than the middle number)
  3. reduce the amount of numbers to be searched by half

Binary search - Example - To find 24

3	7	11	12	**15**	19	24	33	41	55	$\leftarrow$ 10 numbers
				24						$\leftarrow$ key

Binary search - Example - To find 24

3	7	11	12	**15**	19	24	33	41	55	← 10 numbers
				24						← key
					19	24	**33**	41	55	← 5 numbers left
							24			← key

Binary search - Example - To find 24

3	7	11	12	**15**	19	24	33	41	55	← 10 numbers
				24						← key
					19	24	**33**	41	55	← 5 numbers left
							24			← key
					19	24				← 2 numbers left
					24					← key

Binary search - Example - To find 24

3	7	11	12	**15**	19	24	33	41	55	← 10 numbers
				24						← key
					19	24	**33**	41	55	← 5 numbers left
							24			← key
					19	24				← 2 numbers left
					24					← key
						24				← 1 number left
						24				**FOUND!**

Binary search - Example 2 - To find 30

3	7	11	12	**15**	19	24	33	41	55	← 10 numbers
				30						← key

Binary search - Example 2 - To find 30

3	7	11	12	**15**	19	24	33	41	55	← 10 numbers
				30						← key
					19	24	**33**	41	55	← 5 numbers left
							30			← key

Binary search - Example 2 - To find 30

3	7	11	12	**15**	19	24	33	41	55	← 10 numbers
				30						← key
					19	24	**33**	41	55	← 5 numbers left
							30			← key
					19	24				← 2 numbers left
					30					← key

Binary search - Example 2 - To find 30

3	7	11	12	**15**	19	24	33	41	55	← 10 numbers
				30						← key
					19	24	**33**	41	55	← 5 numbers left
							30			← key
					19	24				← 2 numbers left
					30					← key
						24				← 1 number left
						30				**NOT FOUND!**

Binary search - Pseudo code

```
first ← 1
last ← n
found ← false
while first ≤ last AND found == false do
begin

    // check with number in the middle

end
if found == true then
    output ``Found!''
else
    output ``Not found!''
```

$A[first], \cdots, A[mid], \cdots, A[last]$

Binary search - Pseudo code

$first \leftarrow 1$
$last \leftarrow n$
$found \leftarrow false$
while $first \leq last$ AND $found == false$ do
begin

    // check with number in the middle

end
if $found == true$ then
    output ``Found!''
else
    output ``Not found!''

$$A[first], \cdots, A[mid], \cdots, A[last]$$

$$key = A[mid]$$

Binary search - Pseudo code

$first \leftarrow 1$
$last \leftarrow n$
$found \leftarrow false$
while $first \leq last$ AND $found == false$ do
begin

    // check with number in the middle

end
if $found == true$ then
    output ``Found!''
else
    output ``Not found!''

$$A[first], \cdots, A[mid], \cdots, A[last]$$

$$key = A[mid]$$

$$\longleftarrow \quad key < A[mid]$$

Binary search - Pseudo code

$first \leftarrow 1$
$last \leftarrow n$
$found \leftarrow false$
while $first \leq last$ AND $found == false$ do
begin

// check with number in the middle

end
if $found == true$ then
output ``Found!''
else
output ``Not found!''

$$A[first], \cdots, A[mid], \cdots, A[last]$$

$$key = A[mid]$$

$$\longleftarrow \quad key < A[mid]$$

$$key > A[mid] \quad \longrightarrow$$

Binary search - Pseudo code

$first \leftarrow 1$
$last \leftarrow n$
$found \leftarrow false$
while $first \leq last$ AND $found == false$ do
begin

    // check with number in the middle

end
if $found == true$ then
    output ``Found!''
else
    output ``Not found!''

$\lfloor\ \rfloor$ is the floor function
truncates the decimal part

$$mid \leftarrow \left\lfloor \frac{first + last}{2} \right\rfloor$$

Binary search - Pseudo code

$first \leftarrow 1$
$last \leftarrow n$
$found \leftarrow false$
while $first \leq last$ AND $found == false$ do
begin

    // check with number in the middle

end
if $found == true$ then
    output ``Found!''
else
    output ``Not found!''

$\lfloor\ \rfloor$ is the floor function
truncates the decimal part

$mid \leftarrow \lfloor \frac{first + last}{2} \rfloor$
if $key == A[mid]$ then
    $found \leftarrow true$

Binary search - Pseudo code

first $\leftarrow 1$
last $\leftarrow n$
found $\leftarrow$ *false*
while *first* $\leq$ *last* AND *found* $==$ *false* do
begin

    // check with number in the middle

end
if *found* $==$ *true* then
    output ``Found!''
else
    output ``Not found!''

$\lfloor\ \rfloor$ is the floor function
truncates the decimal part

$mid \leftarrow \lfloor \frac{first + last}{2} \rfloor$
if $key == A[mid]$ then
    found $\leftarrow$ *true*
else
    if $key < A[mid]$ then
        $last \leftarrow mid - 1$

Binary search - Pseudo code

$first \leftarrow 1$
$last \leftarrow n$
$found \leftarrow false$
while $first \leq last$ AND $found == false$ do
begin

    // check with number in the middle

end
if $found == true$ then
    output ``Found!''
else
    output ``Not found!''

$\lfloor \; \rfloor$ is the floor function
truncates the decimal part

$mid \leftarrow \lfloor \frac{first + last}{2} \rfloor$
if $key == A[mid]$ then
    $found \leftarrow true$
else
    if $key < A[mid]$ then
        $last \leftarrow mid - 1$
    else
        $first \leftarrow mid + 1$

Binary search - Pseudo code

$first \leftarrow 1$, $last \leftarrow n$, $found \leftarrow false$
while $first \leq last$ AND $found == false$ do
begin
    $mid \leftarrow \lfloor \frac{first+last}{2} \rfloor$
    if $key == A[mid]$ then
        $found \leftarrow true$
    else if $key < A[mid]$ then
            $last \leftarrow mid - 1$
        else $first \leftarrow mid + 1$
end
if $found == true$ then
        output ``Found!''
else
        output ``Not found!''

Binary search - Pseudo code

$first \leftarrow 1$, $last \leftarrow n$, $found \leftarrow false$
while $first \leq last$ AND $found == false$ do
begin
  $mid \leftarrow \lfloor \frac{first+last}{2} \rfloor$
  if $key == A[mid]$ then
    $found \leftarrow true$
  else if $key < A[mid]$ then
      $last \leftarrow mid - 1$
    else $first \leftarrow mid + 1$
end
if $found == true$ then
    output ``Found!''
else
    output ``Not found!''

Why $first \leq last$?

Binary search - Pseudo code

$first \leftarrow 1$, $last \leftarrow n$, $found \leftarrow false$
while $first \leq last$ AND $found == false$ do
begin
  $mid \leftarrow \lfloor \frac{first+last}{2} \rfloor$
  if $key == A[mid]$ then
    $found \leftarrow true$
  else if $key < A[mid]$ then
      $last \leftarrow mid - 1$
    else $first \leftarrow mid + 1$
end
if $found == true$ then
    output ``Found!''
else
    output ``Not found!''

Why $first \leq last$?

- When there is one number left, both *first* and *last* (and *mid*) point to the same location

Binary search - Pseudo code

```
first ← 1, last ← n, found ← false
while first ≤ last AND found == false do
begin
   mid ← ⌊(first+last)/2⌋
   if key == A[mid] then
      found ← true
   else if key < A[mid] then
         last ← mid − 1
      else first ← mid + 1
end
if found == true then
      output ``Found!''
else
      output ``Not found!''
```

Why $first \leq last$?

- When there is one number left, both *first* and *last* (and *mid*) point to the same location
- If this number isn't the key, then either *first* becomes $mid + 1$ or *last* becomes $mid - 1$.

Binary search - Pseudo code

first $\leftarrow 1$, *last* $\leftarrow n$, *found* $\leftarrow$ *false*
while *first* $\leq$ *last* AND *found* $==$ *false* do
begin
  mid $\leftarrow \lfloor \frac{first+last}{2} \rfloor$
  if *key* $==$ $A[mid]$ then
    found $\leftarrow$ *true*
  else if *key* $<$ $A[mid]$ then
      last $\leftarrow$ *mid* $- 1$
    else *first* $\leftarrow$ *mid* $+ 1$
end
if *found* $==$ *true* then
    output ``Found!''
else
    output ``Not found!''

Why *first* $\leq$ *last*?

- When there is one number left, both *first* and *last* (and *mid*) point to the same location
- If this number isn't the key, then either *first* becomes *mid* $+ 1$ or *last* becomes *mid* $- 1$.
- In both cases, *first* becomes larger than *last* and the while condition becomes false; hence the loop terminates.

Binary search - Time complexity

Best case:

Worst case:

$first \leftarrow 1$, $last \leftarrow n$, $found \leftarrow false$
while $first \leq last$ AND $found == false$ do
begin
  $mid \leftarrow \lfloor \frac{first+last}{2} \rfloor$
  if $key == A[mid]$ then
    $found \leftarrow true$
  else if $key < A[mid]$ then
      $last \leftarrow mid - 1$
    else $first \leftarrow mid + 1$
end

Binary search - Time complexity

Best case:

- *key* is the number in the middle

$\Rightarrow$ 1 comparison

$\Rightarrow$ $O(1)$

Worst case:

$first \leftarrow 1$, $last \leftarrow n$, $found \leftarrow false$
while $first \leq last$ AND $found == false$ do
begin
  $mid \leftarrow \lfloor \frac{first+last}{2} \rfloor$
  if $key == A[mid]$ then
    $found \leftarrow true$
  else if $key < A[mid]$ then
      $last \leftarrow mid - 1$
    else $first \leftarrow mid + 1$
end

Binary search - Time complexity

Best case:

- *key* is the number in the middle

$\Rightarrow$ 1 comparison

$\Rightarrow$ $O(1)$

Worst case:

- at most $\lceil \log_2 n \rceil + 1$ comparisons

$\Rightarrow$ $O(\log n)$

$first \leftarrow 1$, $last \leftarrow n$, $found \leftarrow false$
while $first \leq last$ AND $found == false$ do
begin
  $mid \leftarrow \lfloor \frac{first+last}{2} \rfloor$
  if $key == A[mid]$ then
    $found \leftarrow true$
  else if $key < A[mid]$ then
      $last \leftarrow mid - 1$
    else $first \leftarrow mid + 1$
end

Binary search - Time complexity

Best case:

- *key* is the number in the middle

$\Rightarrow$ 1 comparison

$\Rightarrow$ $O(1)$

Worst case:

- at most $\lceil \log_2 n \rceil + 1$ comparisons

$\Rightarrow$ $O(\log n)$

Why?

Every comparison reduces the amount of numbers by at least half

E.g., $16 \Rightarrow 8 \Rightarrow 4 \Rightarrow 2 \Rightarrow 1$

```
first ← 1, last ← n, found ← false
while first ≤ last AND found == false do
begin
   mid ← ⌊(first+last)/2⌋
   if key == A[mid] then
      found ← true
   else if key < A[mid] then
         last ← mid − 1
      else first ← mid + 1
end
```

Summary: Arrays - linear search and binary search

Next: Arrays - finding maximum/minimum

For note taking