Using the definition to justify an answer

$\exists$ an integer l s.t $n = 2l$

Definition

n is even $\Leftrightarrow$ $\exists$ an integer k such that $n = 2k$.

n is odd $\Leftrightarrow$ $\exists$ an integer k such that $n = 2k + 1$.

1. Is 0 even? $k = 0$

2. Is -301 odd? $k = -151$

$2 \cdot k = -302$

$2k + 1 = -301$

Show that **both** 20 and 30 are even integers

$20 = 2 \cdot 10$ $\quad k = 10$

$30 = 2 \cdot 15$ $\quad l = 15$

$20 = 2k$

$30 = 2l$

k, l are integers

More examples

Definition

n is even $\Leftrightarrow \exists$ an integer k such that $n = 2k$.

n is odd $\Leftrightarrow \exists$ an integer k such that $n = 2k + 1$.

3. If a and b are integers, is $6a^2b$ even?

$$2(\underbrace{3a^2b})$$

4. If a and b are integers, is $\underbrace{10a + 8b +} 1$ odd?

$$2(5a+4b)+1$$

5. Is every integer either even or odd?

Proving existential statements

Knowledge $\Rightarrow$

- conjecture

Existential statements

Statements of the **form** $\exists x\ Q(x)$

- The easiest way to prove
$$\exists x\ Q(x)$$
is to find an x that makes $Q(x)$ true.

Examples of constructive proof

1. Prove the following: $\exists$ an even integer n that can be written in two ways as a sum of two prime numbers.

   $10 = 5+5 = 7+3$

   $20 = 17+3 = 13+7$

2. Suppose that r and s are integers. Prove the following: $\exists$ an integer k such that $22r + 18s = 2k$.

More than one variable

$\exists x\ Q(x)$

- there $\exists$ integers m and n such that $m > 1$, $n > 1$ and $\frac{1}{m} + \frac{1}{n}$ is an integer

$m = n = 2$

$\frac{1}{2} + \frac{1}{2} = 1$

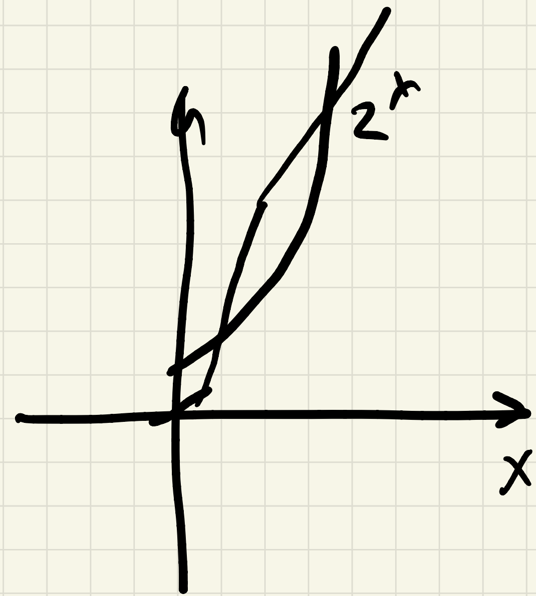
$\exists\ a, b:\quad \sqrt{a+b} = \sqrt{a} + \sqrt{b}$

$$\underbrace{\sqrt{1+0}}_{1} = \underbrace{\sqrt{1}}_{1} + \underbrace{\sqrt{0}}_{0}$$

$$\sqrt{a+0} = \sqrt{a} + \sqrt{0}$$

$\exists x > 1: \quad 2^x > x^{10}$

x			
1	2^1	$>$	1^{10}
2	2^2	$<$	2^{10}
3	2^3	$<$	3^{10}
4	2^4	$<$	4^{10}
5	2^5		
6	2^6		
$\vdots$			



$$\boxed{2^{10} = 1024}$$

$$\exists x > 1 \ : \ 2^x > x^{10}$$

$$x = 2^{10}$$

$$2^{2^{10}} \quad (2^{10})^{10}$$