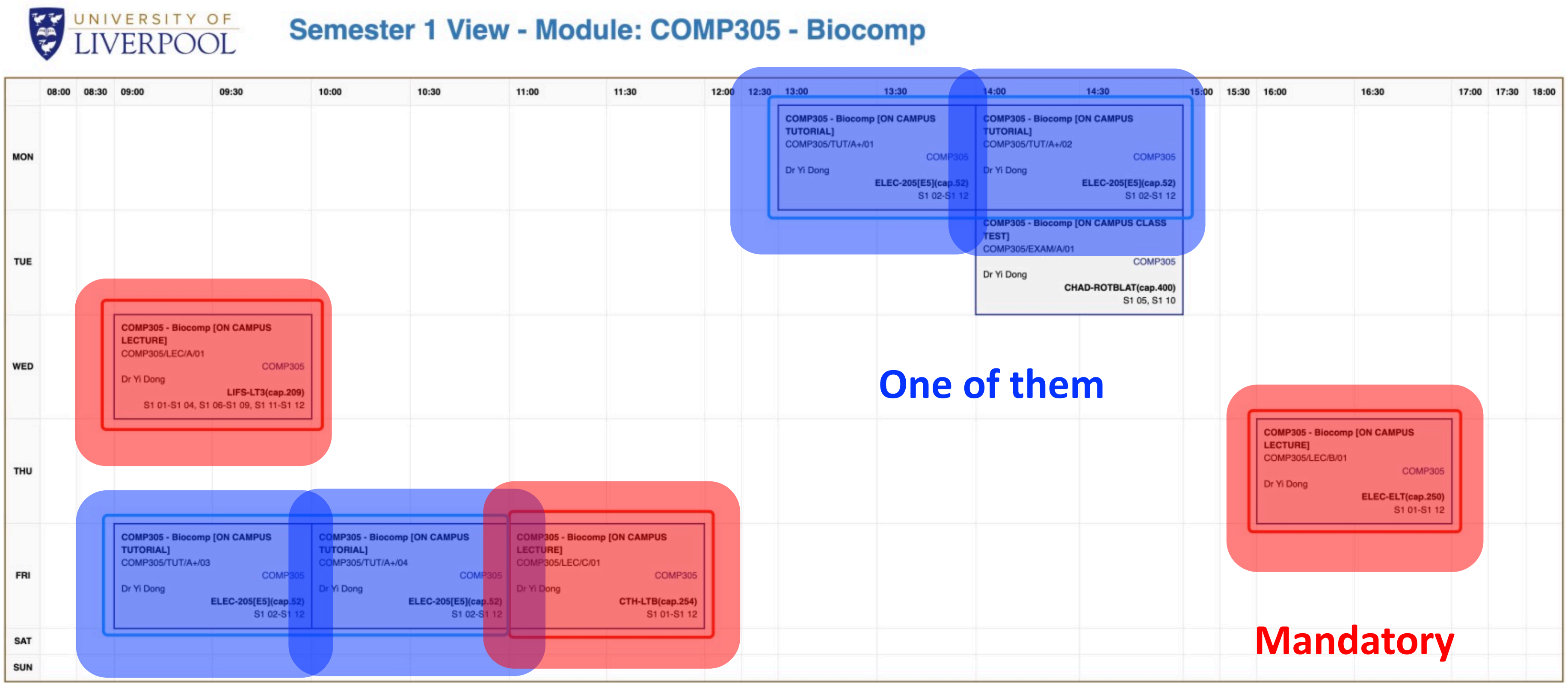
Comp305

Biocomputation

Lecturer: Yi Dong

Comp305 Module Timetable



There will be **26-30** lectures, thee per week. The lecture slides will appear on Canvas. Please use Canvas to access the lecture information. There will be **9** tutorials, one per week.

Lecture/Tutorial Rules

Questions are welcome as soon as they arise, because

1. Questions give feedback to the lecturer;

2. Questions help your understanding;

3. Your questions help your classmates, who might experience difficulties with formulating the same problems/doubts in the form of a question.

Class Test 1

- 9 am, Thursday, Week 6. 31/Oct/2024. CHAD-ROTBLAT: Chadwick Building, Rotblat Lecture Theatre, Room G/171.
- Closed book written exam: Biological background, MP network, unsupervised learning (Hebb's rule, Oja's rule, Kohonen's rule.).
- 50 minutes. Calculators are allowed.
- 3 questions in total. Each has 50 marks. Only need to answer **2** of them. Contribute 15% of the overall module mark.

Comp305 Part I.

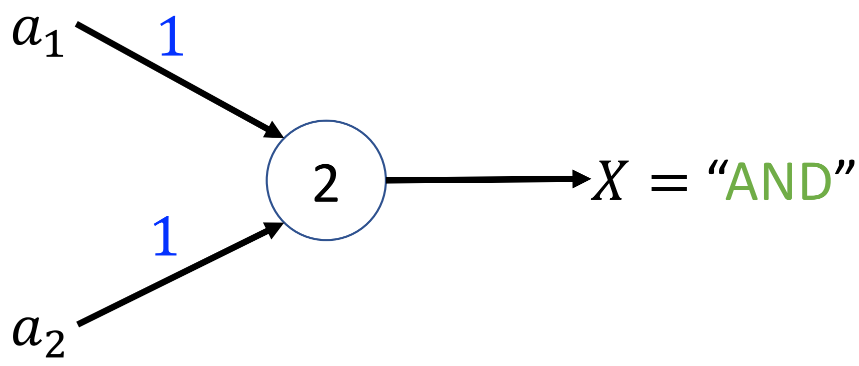
Artificial Neural Networks

Topic 2.

The McCulloch-Pitts Neuron (1943)

MP-Neuron Logic: Two Inputs

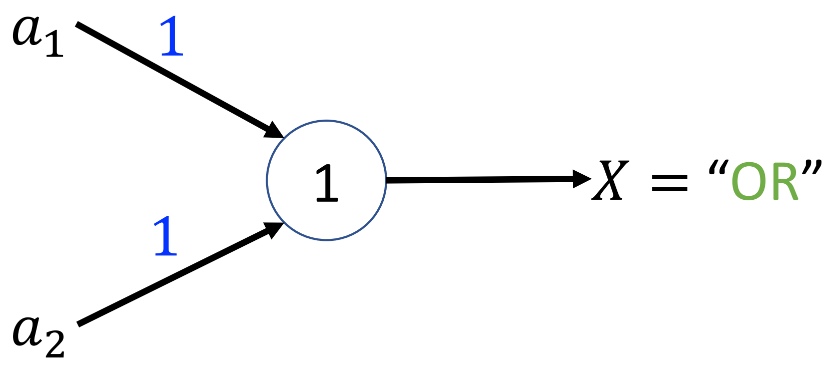
a_1	a_2	"AND"
1	1	1
0	1	0
1	0	0
0	0	0



"AND" – the output fires if a_1 and a_2 both fire.

MP-Neuron Logic: Two Inputs

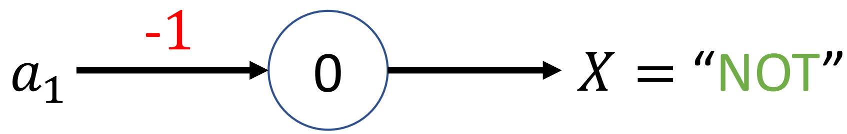
a_1	a_2	"OR"
1	1	1
0	1	1
1	0	1
0	0	0



"OR" – the output fires if a_1 fires <u>or</u> a_2 fires <u>or</u> both fire.

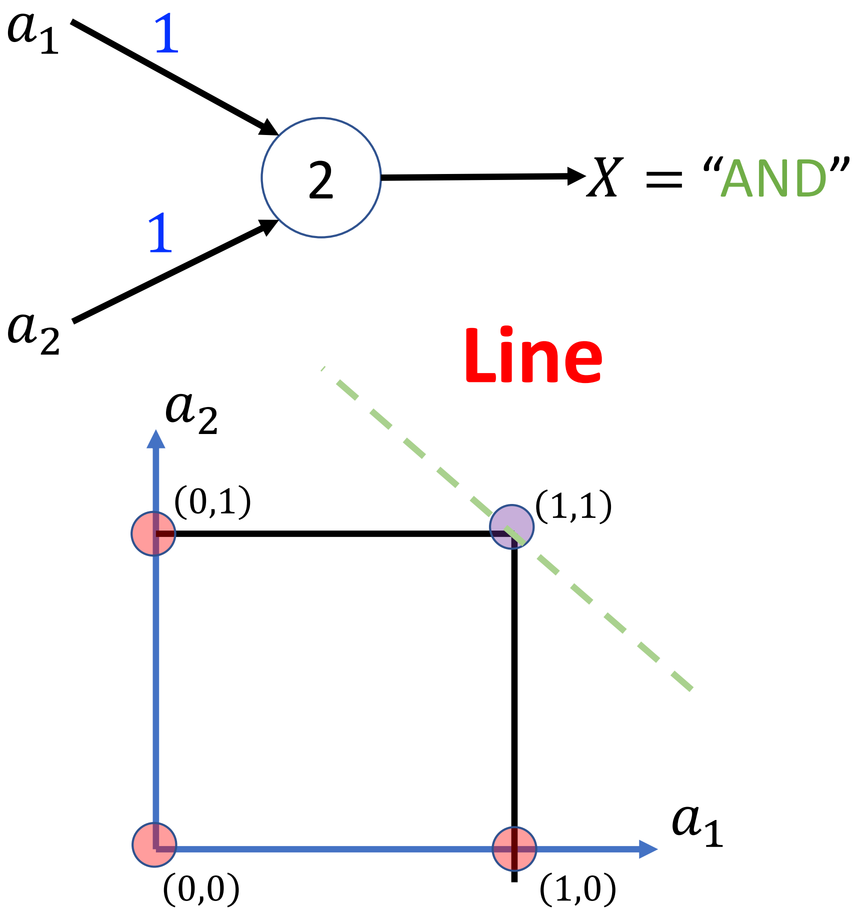
MP-Neuron Logic: Input Inputs

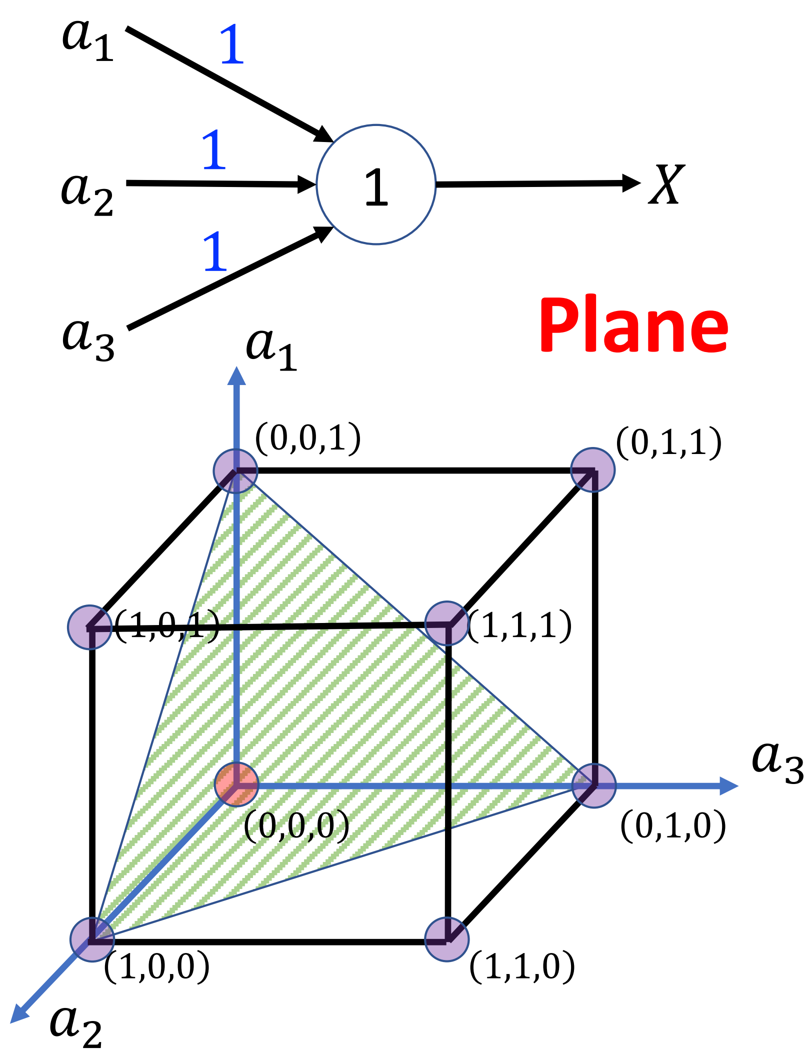
a_1	"NOT"
1	0
0	1



"NOT" – the output fires if a_1 does **NOT** fire and vice versa.

Geometric Interpretation





Revisit definition

Recall the definition of MP neuron.

$$X^t = 1 \text{ if and only if } S^{t-1} = \sum_{i=1}^{n} w_i a_i^{t-1} \geq \theta, \text{ and } w_i > 0, \forall a_i^{t-1} > 0.$$

Assume that $w_1 = -1$,

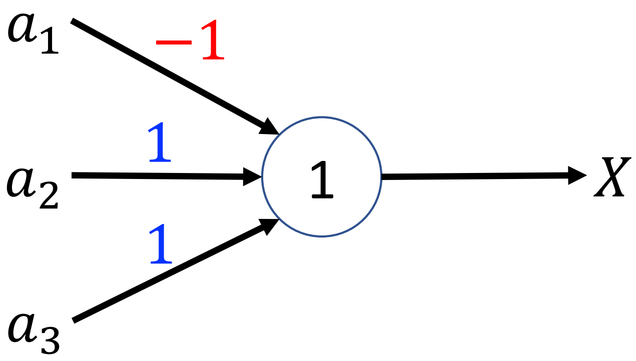
$$S^{t-1} = \begin{cases} -1, & \boxed{a_1 = 1} \\ \sum_{i=1}^{n} a_i^{t-1}, & \boxed{a_1 = 0} \end{cases}$$

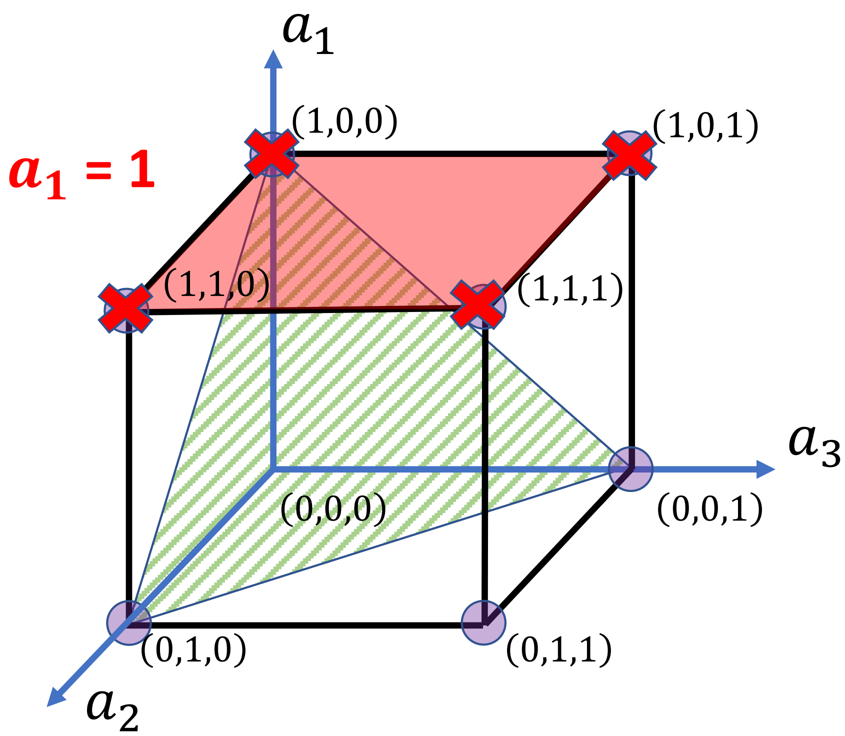
The face of $a_1 = 1$.

For a plane $\sum_{i=1}^{n} a_i^{t-1} = \theta$, remove the face of $a_1 = 1$ from the positive side.

Iteratively do the above, until all the inhibitory connections are considered.

3-Input Case





- The inputs that fire the neuron are (0,1,1), (0,0,1), (0,1,0).

Representation Power of a single MP Neuron

- A single MP neuron can be used to represent some Boolean functions which are linearly separable.

- Linear separability (for Boolean functions): There exists a line (plane) such that all inputs which produce a 1 for the function lie on one side of the line (plane) and all inputs which produce a 0 lie on other side of the line (plane).

- Completeness: Can each linear separable function be represented by a single MP neuron? **No**!

- A single MP neuron describes a specific linear boundary that is only determined by the threshold in the hyper-cube state space, removing the faces corresponding to inhibitory connections.

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- A single MP neuron describes a spe determined by the threshold in the faces corresponding to inhibitory connections.

Difficult to utilize the whole representation power. Instead, its capacity to represent AND, OR, NOT is used.

Propositional Logic

- It deals with propositions (which can be true or false) and relations between propositions, including the construction of arguments based on them. Compound propositions are formed by connecting propositions by logical connectives. Propositions that contain no logical connectives are called atomic propositions. Unlike first-order logic, propositional logic does NOT deal with non-logical objects, predicates about them, or quantifiers.

- Operations: AND, OR, NOT, Implies (→)

Presentation Power of a MP neural network

- Although the McCulloch-Pitts neuron model was very simplistic, it can ***perform*** the ***basic logic operations*** **AND, OR** and **NOT**

- An MP neural network ***can implement*** any ***multivariable propositional logic function***, with the thresholds and weights being appropriately selected.

An Example

Construct an MP network for the following propositional logic formula.

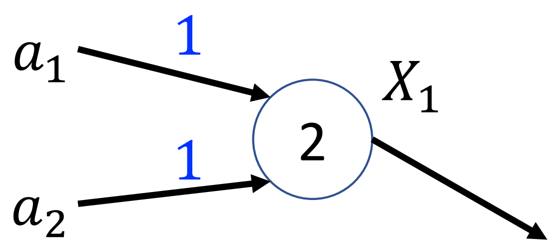
$$X = a_1 \wedge a_2 \vee \neg a_3$$

An Example

Construct an MP network for the following propositional logic formula.

$$X = a_1 \land a_2 \lor \neg a_3$$

Step 1: $X_1 = a_1 \land a_2$



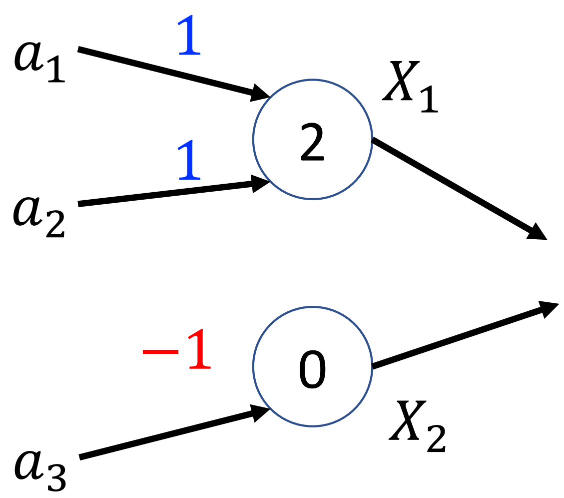
An Example

Construct an MP network for the following propositional logic formula.

$$X = a_1 \wedge a_2 \vee \neg a_3$$

Step 1: $X_1 = a_1 \wedge a_2$

Step 2: $X_2 = \neg a_3$



An Example

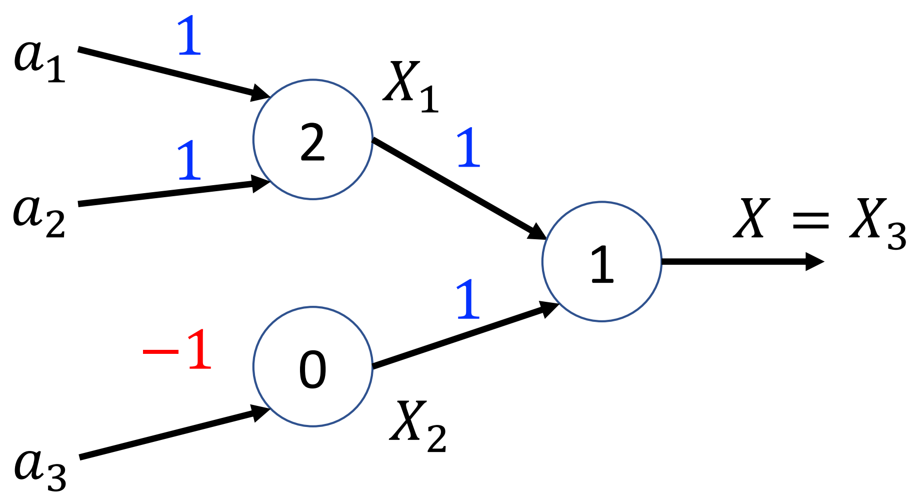
Construct an MP network for the following propositional logic formula.

$$X = a_1 \wedge a_2 \vee \neg a_3$$

Step 1: $X_1 = a_1 \wedge a_2$

Step 2: $X_2 = \neg a_3$

Step 3: $X_3 = X_1 \vee X_2$



Practice

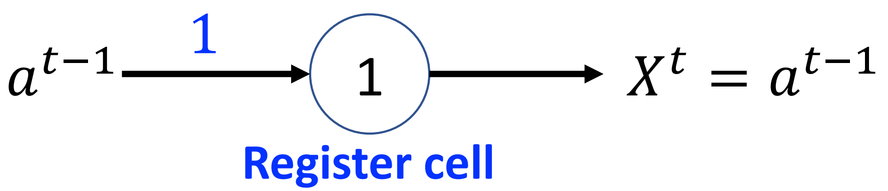
Construct an MP network for the following propositional logic formula.

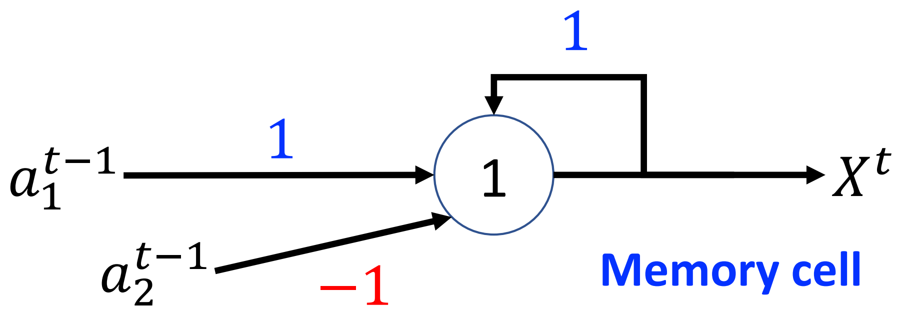
$$X = a_1 \wedge (a_2 \vee \neg a_3)$$

Presentation Power of a MP neural network

- Although the McCulloch-Pitts neuron model was very simplistic, it can ***perform*** the ***basic logic operations*** **AND, OR** and **NOT**

- A MP neural network ***can implement*** any ***multivariable propositional logic function***, with the thresholds and weights being appropriately selected.
- Furthermore, the discrete time, *or* unity delay property of the model makes it even ***possible to build*** ***a sequential digital circuitry***.





MP Neuron Conclusion

- **The McCulloch and Pitts 1943 paper** had a huge influence on the thoughts and studies that **led to modern digital computer design.**

- **MP-neuron outlined the first formal model of an elementary computing unit.**

MP Neuron Conclusion

- The McCulloch and Pitts neuron model included all necessary elements to perform logic operations, and thus
- **MP-neuron** could function as an

arithmetic-logic computing element to compute any computable function.

MP Neuron Conclusion

- The McCulloch-Pitts neuron was a very significant result and with it, it is generally agreed that the disciplines of

neural networks and ***artificial intelligence***

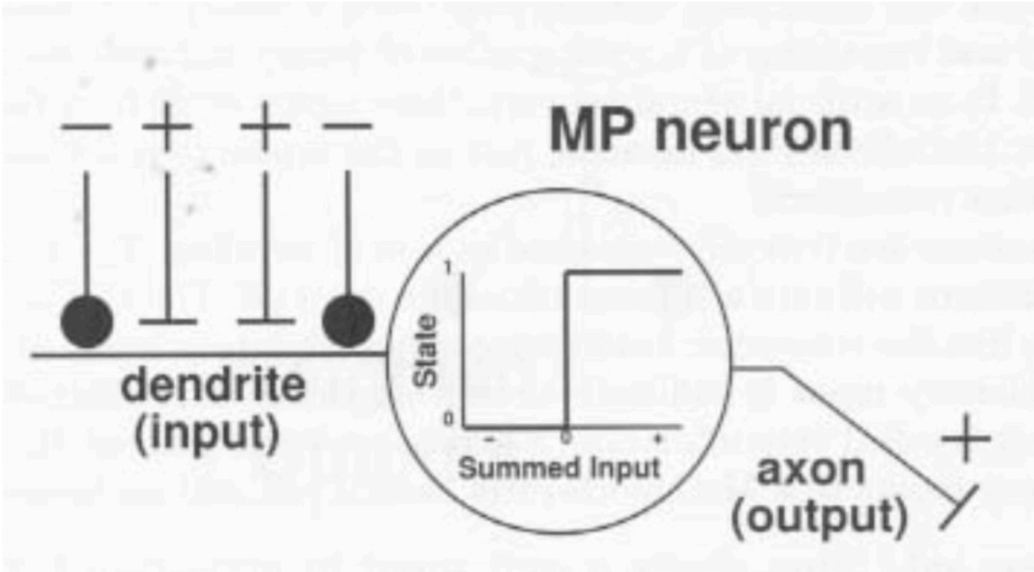
were born.

MP Neuron Conclusion

- Though the threshold elements are, from the combinatorial point of view, more versatile than conventional logic gates, there was a problem with assumed ***unlimited fan-in*** (number of inputs of a logic gate):

the implementation of the compact electronic model was not feasible in the days of bulky vacuum tubes.

MP Neuron Conclusion



The formal neuron model was not widely adopted for the vacuum tube hardware description, and the model never became technically significant.

MP Neuron Conclusion

- **Now-days a possible way** of overcoming the hardware difficulties could be the use of

  optical computing elements

  capable of providing *unlimited* fan-in by changing the angle of lens.

- But still not quite clear yet…

MP Neuron Conclusion

- The main "ideological" problems of the McCulloch- Pitts model were that.
  - The network must be **completely specified before its using**
  - There were **no free parameters** to suit different problems.

- ***Learning can only be implemented by modifying the connection pattern of the network***, which is necessarily more complex than just adjusting numerical parameters.