Set operations

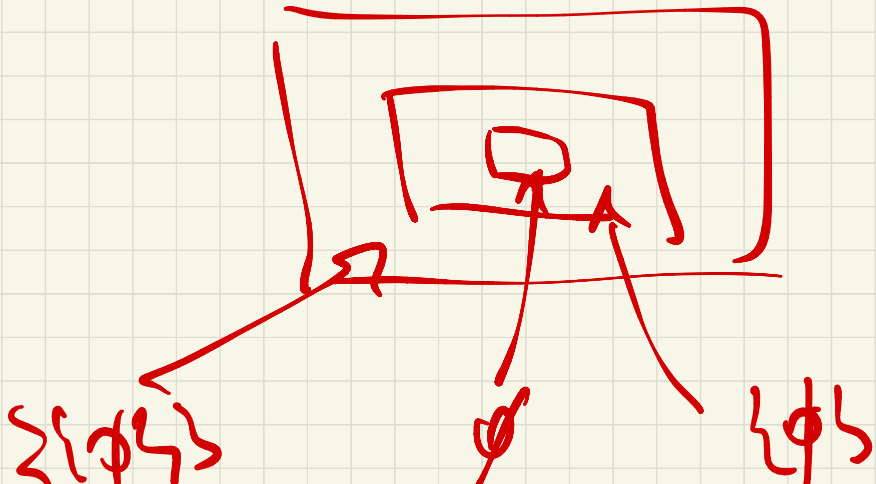
$A = \{1, 2, 3\}$

$1 \in A$ $\qquad$ $4 \notin A$

$1 \notin \emptyset$

$\emptyset \in \{\emptyset\}$

$\{\{\emptyset\}\}$



$A \subseteq B$ $\quad\quad$ $\{1,2,3\} \not\subseteq \{1,2\}$

$$\boxed{\{1,2\} \subseteq \{1,2,3\}}$$

$\leqslant$

$\{a,b,c\} \supseteq \{a,c\}$

$\geqslant$

$\{1,2,3\} \not\subseteq \{a,c\}$

$\not\supseteq$

$\subseteq$

$1 \leq 1$

$\subset$

$A \subseteq A$

$\subsetneq$

$\{1\} \subsetneq \{1, 2\}$

$\{1\} \subseteq \{1, 2\}$

$1 < 2$

$1 \leq 2$

$1 \leq 2$

The union of two sets

Definition The union of two sets A and B is the set

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}.$$

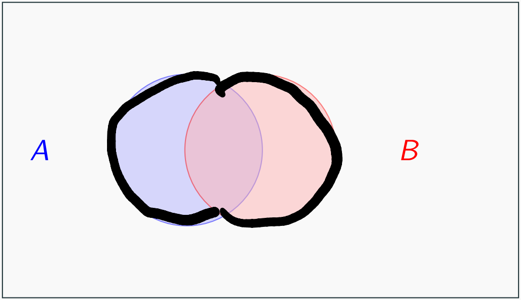


Figure 2: Venn diagram of $A \cup B$.

Example

Suppose

$$A = \{4, 7, 8\}$$

and

$$B = \{4, 9, 10\}$$

Then

$$A \cup B = \{4, 7, 8, 9, 10\}.$$

Detour: Set union in Python

```
def union(A, B):
    result = set()
    for x in A:
        result.add(x)
    for x in B:
        result.add(x)
    return result
```

Testing the method:

```
print union(m, n)
```

But then there is a built-in operation:

```
print m.union(n)
```

Union of sets represented by bit vectors

Let $S = \langle 1, 2, 3, 4, 5 \rangle$, $A = \{1, 3, 5\}$ and $B = \{3, 4\}$.

- Compute $A \cup B$.

$$\chi_A = [1, 0, 1, 0, 1]$$
$$\chi_B = [0, 0, 1, 1, 0]$$
$$\chi_{A \cup B} = [1, 0, 1, 1, 1]$$

- Compute the union of the set C, represented by $[1, 0, 0, 0, 1]$, and the set D, represented by $[1, 1, 0, 0, 1]$.

Union of sets represented by bit vectors

Let $S = \langle 1, 2, 3, 4, 5 \rangle$, $A = \{1, 3, 5\}$ and $B = \{3, 4\}$.

- Compute $A \cup B$.

- Compute the union of the set C, represented by $[1, 0, 0, 0, 1]$, and the set D, represented by $[1, 1, 0, 0, 1]$.

$$S = \{a, b, c, x, y, z\}$$

$$\chi_A = [0, 1, 1, 0, 1, 1] \qquad A = \{b, c, y, z\}$$

$$\chi_B = [1, 0, 1, 0, 1, 0] \qquad B = \{a, c, y\}$$

$$\chi_{A \cup B} = [1, 1, 1, 0, 1, 1]$$

$A = \{ \ldots \}$ $\quad$ $V = [0, 1 \ldots]$

$A \dashrightarrow V$

χ_A

The intersection of two sets

Definition The intersection of two sets A and B is the set

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$

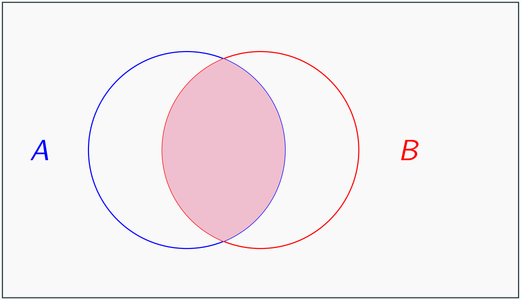


Figure 3: Venn diagram of $A \cap B$.

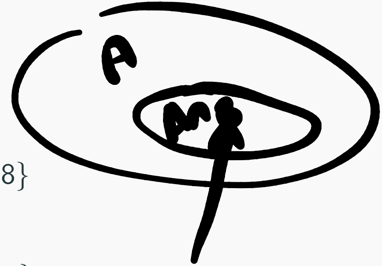
Example

Suppose

$$A = \{4, 7, 8\}$$

and

$$B = \{4, 9, 10\}.$$



Then

$$A \cap B = \{4\}$$

Detour: Set intersection in Python

```python
def intersection(A, B):
    result = set()
    for x in A:
        if x in B:
            result.add(x)
    return result
```

Testing the method:

```python
print intersection(m, n)
print intersection(n, {1})
```

But then there is a built-in operation:

```python
print n.intersection({1})
```

Intersection of sets represented by bit vectors

Let $S = \langle 1, 2, 3, 4, 5 \rangle$, $A = \{1, 3, 5\}$ and $B = \{3, 4\}$.

- Compute $A \cap B$.

$$\chi_A = [1, 0, 1, 0, 1]$$
$$\chi_B = [0, 0, 1, 1, 0]$$
$$\chi_{A \cap B} = [0, 0, 1, 0, 0]$$

- Compute the intersection of the set C, represented by $[1, 0, 0, 0, 1]$, and the set D, represented by $[1, 1, 0, 0, 1]$.

The relative complement

Definition The relative complement of a set B relative to a set A is the set

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}.$$

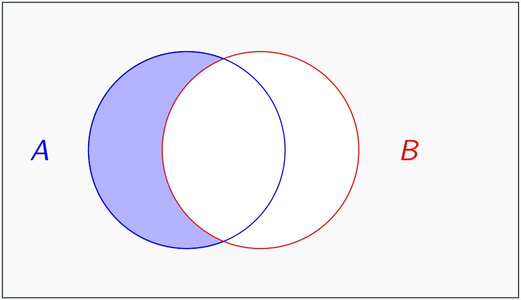


Figure 4: Venn diagram of $A - B$.

Example

Suppose

$$A = \{4, 7, 8\}$$

and

$$B = \{4, 9, 10\}.$$

Then

$$A - B = \{7, 8\}$$

$$\{1, 2\} - \{1, 2, 3\} = \emptyset$$

$$\{1, 2\} - \emptyset = \{1, 2\}$$

$$A - A = \emptyset$$

Detour: Set complement in Python

```
def complement(A, B):
    result = set()
    for x in A:
        if x not in B:
            result.add(x)
    return result
```

Testing the method:

```
print complement(m, {'a'})
```

But then there is a built-in operation:

```
print m-{'a'}
```

Relative complement and bit vectors

Let $S = \langle 1, 2, 3, 4, 5 \rangle$, $A = \{1, 3, 5\}$ and $B = \{3, 4\}$.

- Compute $A - B$.

$$\chi_A = [1, 0, 1, 0, 1]$$
$$\chi_A = [0, 0, 1, 1, 0]$$
$$\chi_{A-B} = [1, 0, 0, 0, 1]$$

- Compute the relative complement of the set C, represented by $[1, 0, 0, 0, 1]$, related to the set D, represented by $[1, 1, 0, 0, 1]$.

The complement

When we are dealing with subsets of some large set U, then we call U the *universal set* for the problem in question.

Definition The complement of a set A is the set

$$\sim A = \{x \mid x \notin A\} = U - A.$$

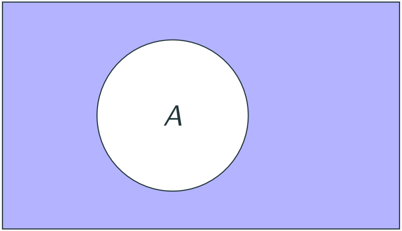


Figure 5: Venn diagram of $\sim A$. (The rectangle is U)

$U = \mathbb{Z}$

$A = \{x \in \mathbb{Z} \mid x \text{ is odd}\}$

$\sim A$ - even int

Complement and bit vectors

Let $S = \langle 1, 2, 3, 4, 5 \rangle$, $A = \{1, 3, 5\}$ and $B = \{3, 4\}$.

- Compute $\sim A$.

$\chi_A = [1, 0, 1, 0, 1]$

$\boxed{\chi_{\sim A} = [0, 1, 0, 1, 0]}$

- Compute $\sim B$.

$\chi_B = \{0, 0, 1, 1, 0\}$

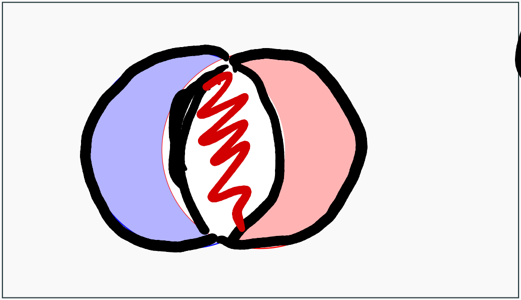
$\chi_{\sim B} = \{1, 1, 0, 0, 1\}$

- Compute the complement of the set C, represented by $[1, 0, 0, 0, 1]$.

The symmetric difference

Definition The symmetric difference of two sets A and B is the set

$$A \Delta B = \{x \mid (x \in A \text{ and } x \notin B) \text{ or } (x \notin A \text{ and } x \in B)\}.$$



$(A \cup B) - (A \cap B)$

Figure 6: Venn diagram of $A \Delta B$.

Example

Suppose

$$A = \{4, \underline{7, 8}\}$$

and

$$B = \{4, \underline{9, 10}\}.$$

Then

$$A \Delta B = \{\underline{7, 8}, \underline{9, 10}\}$$

$$\underbrace{\{1,2,3\}}_{A} \triangle \underbrace{\{a,b,c\}}_{B} = \{1,2,3,a,b,c\}$$

$$\underline{\{1,2,3\} \cup \{a,b,c\}} = \{1,2,3,a,b,c\}$$

$$A \cap B = \emptyset$$