Prove for any integers m, n that if $m \cdot n$ is even then one of them is even.

Proof

Suppose for a proof by contradiction that for some m, n odd we have $m \cdot n$ is even.

$m = 2k+1$ for some int. k

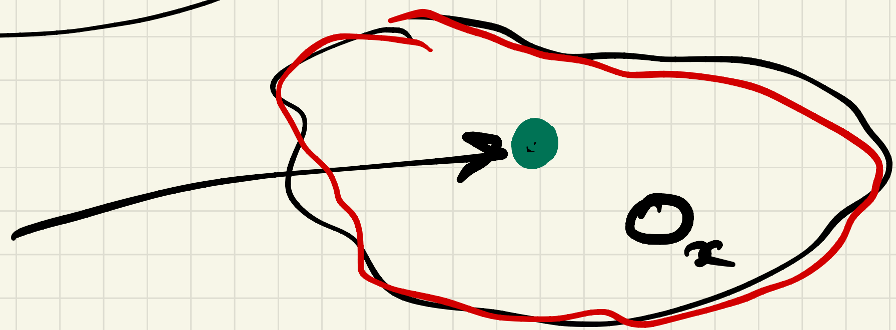
$n = 2l+1$ for some int l

$$\Rightarrow m \cdot n = (2k+1)(2l+1) = 4kl + 2k + 2l + 1$$

So $m \cdot n$ is odd, a contradiction

Universal Statement

$\forall x$ if $\underline{\qquad}$ then $\underline{\qquad}$



Negate

$\exists x$ $\underline{\qquad}$ and not $\underline{\qquad}$

Prove that

$\forall\, m, n$ if m, n are integers s.t $m+n$ is even then either m, n are odd or m, n are even

$8 = 4+4 = 3+5$

Proof Suppose for a proof by contradiction that $\exists\, m, n$ integers $m+n$ is even and one of them is odd and another is even.

Case 1

m is odd, n is even
By def. of odd m = . _ ,
by def. of even n = - -
Then $m+n = 2(\_ \ \_)+1$ which is <u>odd</u>

Case 2

m is even, n is odd
By def. of even m = . - .
By def of odd n = _ -
Then $m+n = 2(\_ \ \_)+1$ which is <u>odd</u>

proof by contradiction

conjecture
(univ. statement)

¬ conjecture
(existential statement)

case 1

case 2

When to use indirect proof

- Many theorems can be proved either way. Usually, however, when both types of proof are possible, indirect proof is clumsier than direct proof.
- In the absence of obvious clues suggesting indirect argument, try first to prove a statement directly. Then, if that does not succeed, look for a counterexample.
- If the search for a counterexample is unsuccessful, look for a proof by contradiction

Mathematical induction

Mathematical induction

- Mathematical induction is one of the more *recently* developed techniques of proof in the history of mathematics.
- It is used to check conjectures about the outcomes of processes that occur repeatedly and according to definite patterns.
- In general, mathematical induction is a method for proving that a property defined for integers n is true for all values of n that are greater than or equal to some initial integer

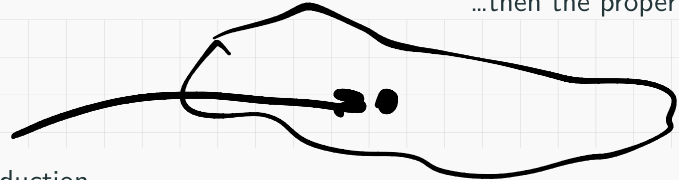
Generic particular vs induction for universal statements

- Generalisation from the generic particular:

  "Suppose that x is a particular but arbitrarily chosen ..."

  ..."property holds for this x"...

  "...then the property holds for all x"



- Induction

  Some kind of a process that goes over the elements of a set