Review of First Part

Numbers, Polynomials, Vectors

Recap – Key Points

- In the first parts of the module we reviewed:

The concept of *number* and its *different types*

The notion of *polynomial*: *form*, *attributes*, *properties*

Basic models of *vector* and *operations* on these

Simple *Matrix* structures

Linear Transformations and their role in *Graphics*

Number

- You should be able to:

1. Recall the *different classes* of number (*Natural*, *Whole*, *Integer*, *Rational*, *Real*) and *recognize* which is the *most suitable* in *different contexts*.
2. Be *aware* of how the class of *Real* number *originates* and the idea of that *subset of Reals* obtained as *roots of polynomial* expressions.

Typical Example Questions

- What is the *most suitable* class of numbers to use in modelling the following situations?

1. The *number* of *tenors* who have recorded the role of *Tristan* in Wagner's *Tristan und Isolde*.
2. The *points* total of a *football team* at the *end of the season*.
3. The *average mark* obtained by a student in *Year 1*.
4. The *points* the UK entry in the *Eurovision song contest* gets.
5. The *exact time* it takes a car to reach a *speed of 80 Km/hour*.

Polynomial Forms

- You should be aware of:

1. The *formal* definition of *polynomial of degree* k in x.
2. The notions of *coefficient* and the set $H_k[X]$ for numbers H.
3. Basic *operations* (*addition*, *scalar multiplication*).
4. The concept of the *roots of a polynomial* and *properties*.

- You do ***NOT*** need to be aware of:

1. Root finding *algorithms* or *methods*.
2. Operations: *multiplying*, *factoring*, *dividing* polynomials.

Typical Questions

1. What is the *coefficient* of x^2 after expanding
$$(x-1)(x-2)(x+3)$$
2. An company estimates its *annual* tax liability by using a scheme in which x^2 pounds are paid for *each manager*, x^3 for each worker and -500 pounds for *charitable causes*. If the company employs 40 *managers* and 3 *workers* what is the *polynomial* describing its *annual tax liability*?
3. What is the *degree* of the polynomial formed from
$$(x-7)(x^3+6)(x^2-2)$$

Vectors and Matrices I

- You should be aware of:

1. The *form* and *attributes* of n-vectors: *direction*, *size*. *Component* properties (*ordered*, *number types* used).
2. Basic *vector operations*: *addition*, *scalar multiple*, *size*.
3. The *properties* and *requirements* of *vector spaces*.
4. The form taken by *matrices* and *simple operations*.
5. The process of *multiplying* a *matrix* by a *vector*.
6. The notions of *vector space dimension*, *linear combination*, *linear independence* and *basis* of a *vector space*.

Vectors and Matrices II

- You should be able to form the *product* of "*small*" *matrices* ($2 \times 2, 3 \times 3, 4 \times 4$ etc) you *do not need* the general $p \times q$ by $q \times r$ rules.
- You should also know the process of multiplying a matrix by a vector and its outcome.

Typical Example Questions

1. What is the size of the *3-vector* $\langle -3,12,5\rangle$?
2. A *robot* records its *position* and *current time* (in *seconds*) using a *4-vector* $\langle x, y, z, t\rangle$. If this shows $\langle 10,0,25,6\rangle$ and it is instructed to move 5 *places left* (x) and 10 *places higher* (z) after 4 *more seconds*, what is its *new coordinate 4-vector*?
3. Are the following set of vectors in $\mathbb{R}^3$ *linearly independent*?
$$\{\langle 1,3,2\rangle\langle 2,1,3\rangle\langle 3,2,1\rangle\langle 1,2,3\rangle\}$$
4. If these are *not* express *one* as a *linear combination* of the other *three*.

Linear Transformations and Graphics

- You should be aware of:

1. How *Linear Transformations* and *Matrix-Vector* are related.
2. What's needed for a *mapping* to be a *Linear Transformation*.
3. The structure of 2×2 and 3×3 *scaling matrices*.
4. The concept of *Homogenous coordinates* and *matrices*.
5. The way in which *matrix-products* are *combined* to realise different *graphic effects*.
6. The *consequences* of effects not being *commutative*.

More Example Questions

- What is the effect of the following 2×2 matrices when applied to a point $\langle p, q \rangle$ in 2-dimensional space?

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} ; \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} ; \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} ; \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

- A robot controller wishes to implement a command that instructs a robot to move from $\langle x, y, z, t \rangle$ to a *new position* after *19 seconds*. The new position will be $\langle 2x, 2y, 2z, t + 19 \rangle$. What are the *dimensions* of the matrix *needed* to realise this effect as a matrix-vector product?
- The instruction is *modified* so that the move should take place after a further t seconds. *Comment on the matrix needed*.

Review of Second Part

Calculus and Complex Numbers

Calculus Recap – Key Points

- In the second section we looked at:

The background to *Differential Calculus*

Its use as a method for studying *function behaviour*

The *relationship* between *Calculus* & *Optimization*

Developments for *Several Variables*

Brief overview of *Integral Calculus*

Complex Numbers Recap – Key Points

- We continued by looking at:

The *origins* of *Complex Numbers*

The different *representations* for these

Operations on Complex Numbers

The extension of *Calculus* for *Complex Numbers*

Important *applications* in *CS*

Differential Calculus

- You should be aware of:

The concept of *line functions* and their *attributes*

The interpretation of “*first derivative*” as “*gradient at a point*”

How *first derivatives* are used to discover *critical points*

How *The Second Derivative Test* is used to *classify* these

The notion of “*partial derivative*”

The background to *optimization* of *multivariable* cases

Typical Example Questions

- A vehicle moves from a *starting* position of $(6,3)$ to a *final* position of $(42,327)$. What is the *gradient* of the incline?
- When the vehicle has reached a *height* $y = 200$ what is its corresponding *horizontal* (x) position?
- An *effective utility value* is given by a *function*

$$E(x) = 30(\log x)^3 - 15(\log x)^4$$

Find an *expression* for the *critical* point(s) of $E(x)$.

Integral Calculus

- You should be aware of:

  The origins and use of *Integration* in *Area Measurement*

  The relationship via "*anti-derivatives*" with *Differentiation*

  The distinction between *definite* and *indefinite* integral

- You ***DO NOT*** need to be aware of:

  The developments to *Line Length* and *Volume Computation*

  "*Creative*" techniques such as *Integration by Parts*

Typical Example Question

- Consider the *indefinite integral*:

$$\int \frac{(x+6)dx}{3x^2+2x-1}$$

- show how this may be *rewritten* in the form

$$\int \frac{A(x)dx}{(x+1)} + \int \frac{B(x)dx}{(3x-1)}$$

- for appropriate functions $A(x)$ and $B(x)$.
- Use this *rewritten form* to *compute the integral*.

Complex Numbers

- You should be aware of:

The *attributes* of *Complex Numbers*

(*Real* and *Imaginary parts*, *phase*, *size*)

The *different methods* for *representing* Complex Numbers

The *basic operations* on Complex Numbers

(*addition*, *modulus*, *conjugate*, *multiplication*, *division*)

The notion of “*Primitive Root of Unity*”

Some sense of how *Calculus* is extended.

Important *applications* in *Computer Science*

Complex Number Forms & Operations

- Principle *attributes*: *Real* and *Imaginary parts*, *size* and *phase*.
- Forms: $x + iy$; *Matrix*; *Argand*; *polar form*; *Euler Form*.
- *Addition* and *Scalar multiplication*: analogue of *2-vector* ops.
- *Multiplication*: expansion of $(u + iv)(p + iq)$ using $i^2 = -1$.
- *Division*: expresses z^{-1} using properties of $\overline{z}$ and $|z|$.
- *Primitive* (k*'th*) *Root* (*of Unity*): $w \in \mathbb{C} : \ w^k = 1$.

More Example Questions

- Give the *Polar* and *Euler Forms* for $3 - 4i, 4 - 3i, 5 + 12i$.
- Compute the *product* $(1 + 10i)(2 - 3i)$.
- *Simplify* the expressions

$$\frac{2+i}{2-i}\ ;\ \frac{2-i}{2+i}\ ;\ \frac{5+12i}{3+4i}$$

- If $z_{n+1} = (z_n)^2 + 1 \ - 0.5i$ with $z_0 = 1$ *find the value* of $|z_2|$.

Calculus in the Complex Plane

- You should be able to apply the *Cauchy-Riemann conditions* to determine if a Complex Valued function is *differentiable*.
- You should be aware of the *background* affecting *Integration of Complex Valued functions* and the notions such as *Contour* and *Line Integral*.
- You should be able to *formulate* the functions relevant to a *parameterized curve* between two distinct Complex numbers.
- You are ***NOT*** expected to be able to *apply Cauchy's Integral Theorem* but you should be *aware* of its *significance in CS*.

Example Questions

- Which of the following functions satisfy the *Cauchy-Riemann conditions*?

$$f(z) = |z|^2; f(z) = \sqrt{z}\ ; f(z) = z^{-1}; f(z) = (\overline{z})^2$$

- Find

$$\int_\gamma z^3 dz$$

- between $z_0 = 1 - i$ and $z_1 = 2 + i$ with γ a *straight line* connecting these two points.

Applications in Computer Science

- It is *important* to be aware of *significant uses* of *Complex Numbers* and the supporting theory in *Computer Science*:

  Quaternion Algebra (in Advanced Graphics)

  Algorithm Analysis (especially average-case studies)

  The Fourier Transform

  Computer Creativity – Fractal art, Music

- Some of these will be met next year.
- Others have provided the basis for *Final Year Project* work.

Review of Third Part

Experiment and Matrices
Spectral Methods

Computing as Experiment – Recap

- In the third part of the module we looked at:

The treatment of problems in CS by *experiment*

The basic *statistical* tools and ideas involved

Notions of *population*, *sampling* and *distribution*

Measures such as *Expectation*, *Variance*, *Standard Deviation*

The idea of *Random Variable*

Some simple *Data Analysis* techniques – *Regression*.

Matrix and Spectral Methods – Recap

- We then considered the topic of *Spectral Methods* in CS:

Matrix attributes of *determinant* and *inverse*

The notions of *singular* and *non-singular matrix*

The *Eigenvalue Problem* and its analysis

The *Perron-Frobenius Theorem*

Conditions for *Real Spectra*

Computational Techniques

Applications in CS – *Graph analysis*, *Page Rank*, *Compression*

Statistical Elements

- You should be aware of (although the exam will not require complicated calculations):

1. The concepts of *Population*, *Sampling* and *Distribution*.
2. The distinction between *biased* and *unbiased* sampling.
3. The measures *Expectation*, *Variance*, *Standard Deviation*.
4. The form of *random variables* and their importance.
5. How standard deviation is *estimated* in "*real contexts*".
6. The basic structure of *experiments by sampling*.

Typical Example Questions

- Given the following collection of marks:

$$(10,10,5,25,30,75,80,52,90,60,52)$$

1. What is the *median mark*?
2. Using an *unbiased distribution*, what is the *expected mark*?
3. For a distribution, *D*, in which

$$\mathrm{P}[D = X] = \begin{cases} 0 \; if \; X \leq 30 \\ {}^{1}/_{8} \; if \; 30 < X < 60 \\ {}^{3}/_{16} \; if \; X \geq 60 \end{cases}$$

what is the *expected mark* using *D*?

Data Analysis and Regression

- You should be aware of:

1. The *framework* and *background* to *regression methods*.
2. The formulation of "*best fit*" as an *optimization problem*
3. Methods for *casting functions* as *linear functions*
4. How to *apply* techniques to "*real data*"

- You ***DO NOT*** need to
- *Memorize formulae* for "*best-fit*" *line functions*

Typical Example Question

- Find the gradient of the best-fit line for the data:

X	**5**	**7**	**9**	**11**	**17**	**20**	**38**	**40**	**51**
Y	**10**	**15**	**16**	**24**	**33**	**41**	**80**	**79**	**98**

Matrix Methods

- You should be aware of:

1. Matrix *operations*: *addition*, *product*, *transpose*.
2. The concept of *matrix determinant* and *inverse*.
3. The distinction between *singular* and *non-singular matrices*.
4. Basic conditions guaranteeing *singularity*.

Spectral Methods

- You should be aware of:

1. The concept of eigenvalue and eigenvector.
2. Conditions ensuring dominant and all real spectra.
3. Basic computational techniques (Power method, etc).
4. The background to important applications.

Typical Examples

- Which of the following matrices is/are *singular*?

$$\begin{pmatrix} 1 & 2 & 3 & 0 \\ 2 & 3 & 1 & 4 \\ 3 & 2 & 1 & 4 \\ 3 & 6 & 9 & 0 \end{pmatrix};\begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{pmatrix};\begin{pmatrix} 1 & 2 & 1 & 4 \\ 2 & 2 & 5 & 4 \\ 3 & 2 & 7 & 4 \\ 4 & 4 & 9 & 8 \end{pmatrix}$$

- Which of the following statements is *always* true?

The result of *multiplying* non-singular matrices is *non-singular*

The result of *adding* non-singular matrices is *non-singular*

The *transpose* of a non-singular matrix is *non-singular*

The *product* of an $n \times k$ and $k \times n$ matrix is *singular*

Example Questions - Spectral Methods

- If the value 0 is an eigenvalue of the matrix, P, what property can be deduced of P?
- Find an eigenvector for the dominant eigenvalue of the matrix below by using the Power Method.

$$\begin{pmatrix} 0 & 0.25 & 0 & 0.25 & 0.15 \\ 0.5 & 0.25 & 0 & 0 & 0.15 \\ 0 & 0 & 0.75 & 0.5 & 0.25 \\ 0 & 0.25 & 0 & 0.25 & 0.25 \\ 0.5 & 0.25 & 0.25 & 0 & 0.2 \end{pmatrix}$$

Conclusion

And in your head do you feel
What you're not supposed to feel?
And you take what you want
But you don't get it for free
You need more time
'Cause your thoughts and words won't last forever more
And I'm not sure if it'll ever work out right
But it's ok
It's all right
Sunday Morning Call
(from Standing on the Shoulders of Giants by Oasis)

Overview

- A reprise of the main parts of the module.
- How the final exam is structured.
- Some hints on *what to do*
- And what ***NOT*** to do.

Main Topics

- *Number Types* and Properties.
- *Polynomial Structures*.
- *Vectors* and *Vector Attributes*.
- *Simple Matrix effects* for Graphics
- *Calculus* and *Optimization*.
- *Complex Numbers* and their *use in CS*.
- *Computing as Experiment*
- *Spectral methods* and applications.

Exam Format

- The final exam contributes 60%.
- It will use *30 questions* (all MCQ) with *5 options* per question.

Exam Arrangements

- The final exam will be hosted ON CAMPUS
- Calculators are NOT PERMITTED.
- Notice that as a result you will NOT see certain types of questions involving complicated arithmetic calculations.

Final words

- If you have coped with the 3 class tests the exam itself should not prove too taxing.
- If you attempted but had a few slip-ups be sure to revise those areas causing problems.
- Remember you have already completed 40%

Good luck to everyone!

"La commedia è finita!"

Ruggero Leoncavallo
Pagliacci (closing line)