Comp305

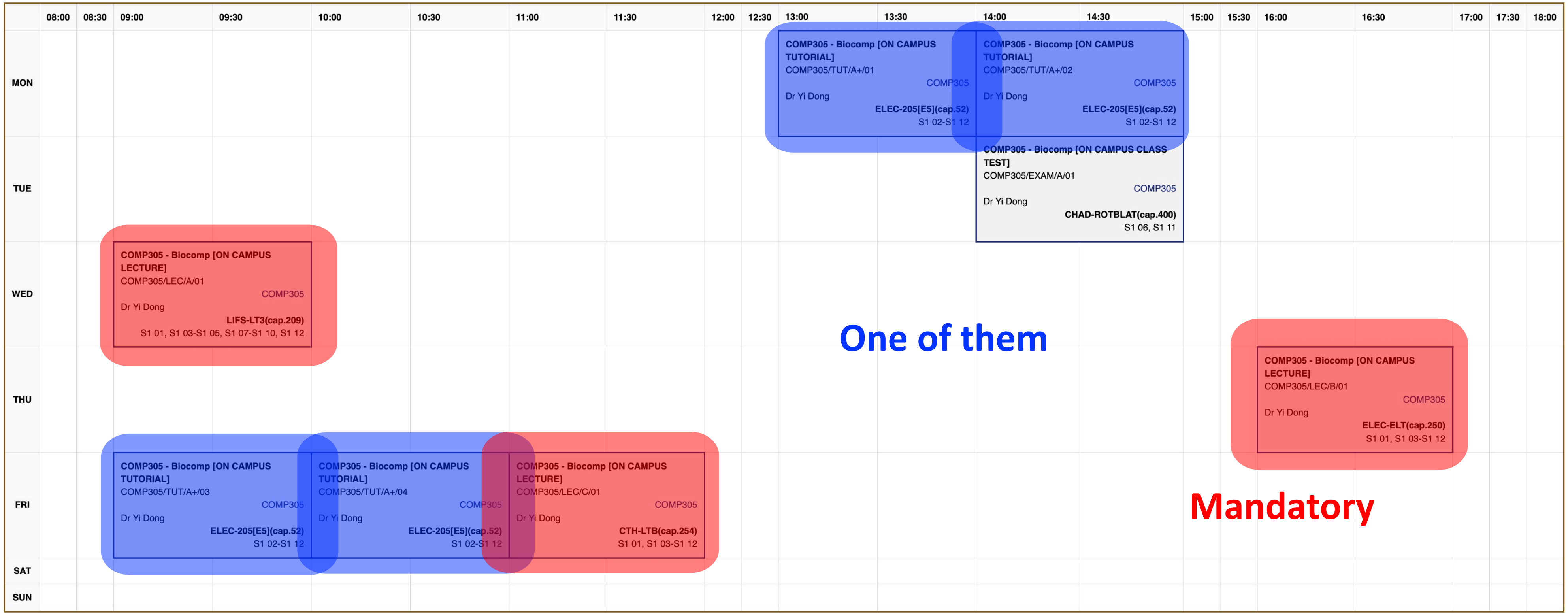
Biocomputation

Lecturer: Yi Dong

Comp305 Module Timetable

UNIVERSITY OF LIVERPOOL

Semester 1 View - My Timetable:



There will be **26-30** lectures, thee per week. The lecture slides will appear on Canvas. Please use Canvas to access the lecture information. There will be **9** tutorials, one per week.

Lecture/Tutorial Rules

Questions are welcome as soon as they arise, because

1. Questions give feedback to the lecturer;

2. Questions help your understanding;

3. Your questions help your classmates, who might experience difficulties with formulating the same problems/doubts in the form of a question.

Comp305 Part I.

Artificial Neural Networks

Topic 2.

The McCulloch-Pitts Neuron (1943)

Topic of Today's Lecture

What can we do with a McCulloch-Pits Neuron?

Some examples.

The McCulloch-Pitts Neuron (1943)

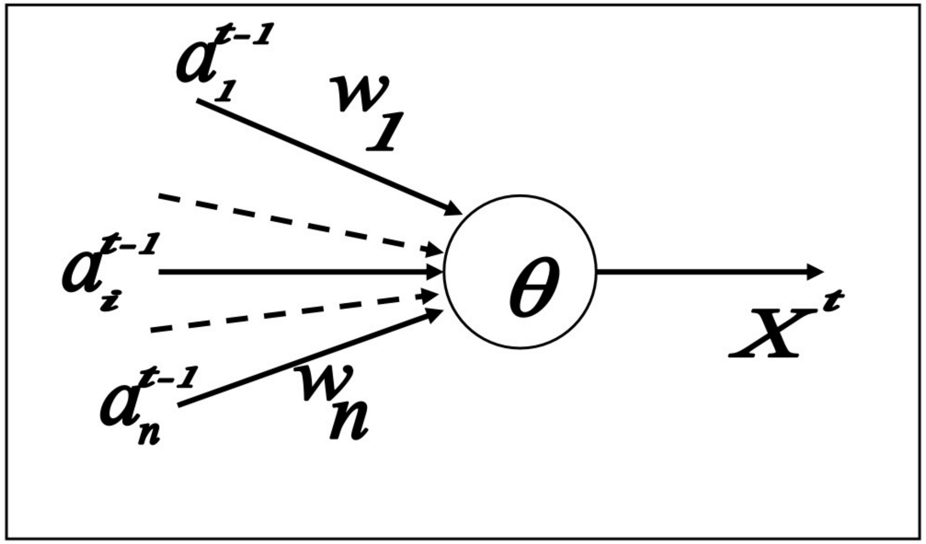
McCulloch and Pitts demonstrated that

"…because of the all-or-none character of nervous activity, neural events and the relations among them can be treated by means of the propositional logic".

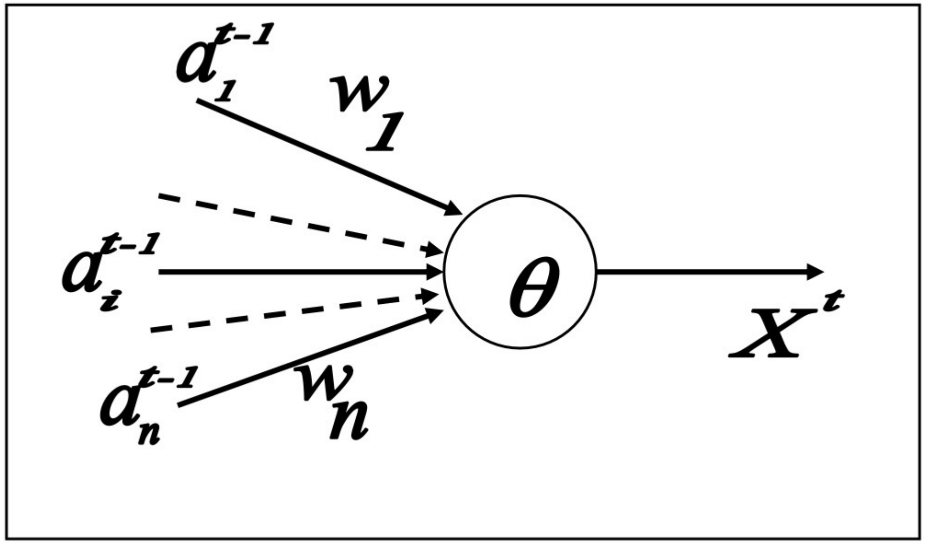
The McCulloch-Pitts Neuron (1943)

The authors modelled the neuron as

- a ***binary,*** ***discrete-time*** input
- with ***excitatory*** ***and*** ***inhibitory*** ***connections and an*** ***excitation threshold***.



MP Neuron: Computation



Output X^t of the neuron at the following instant t is defined according to the rule:

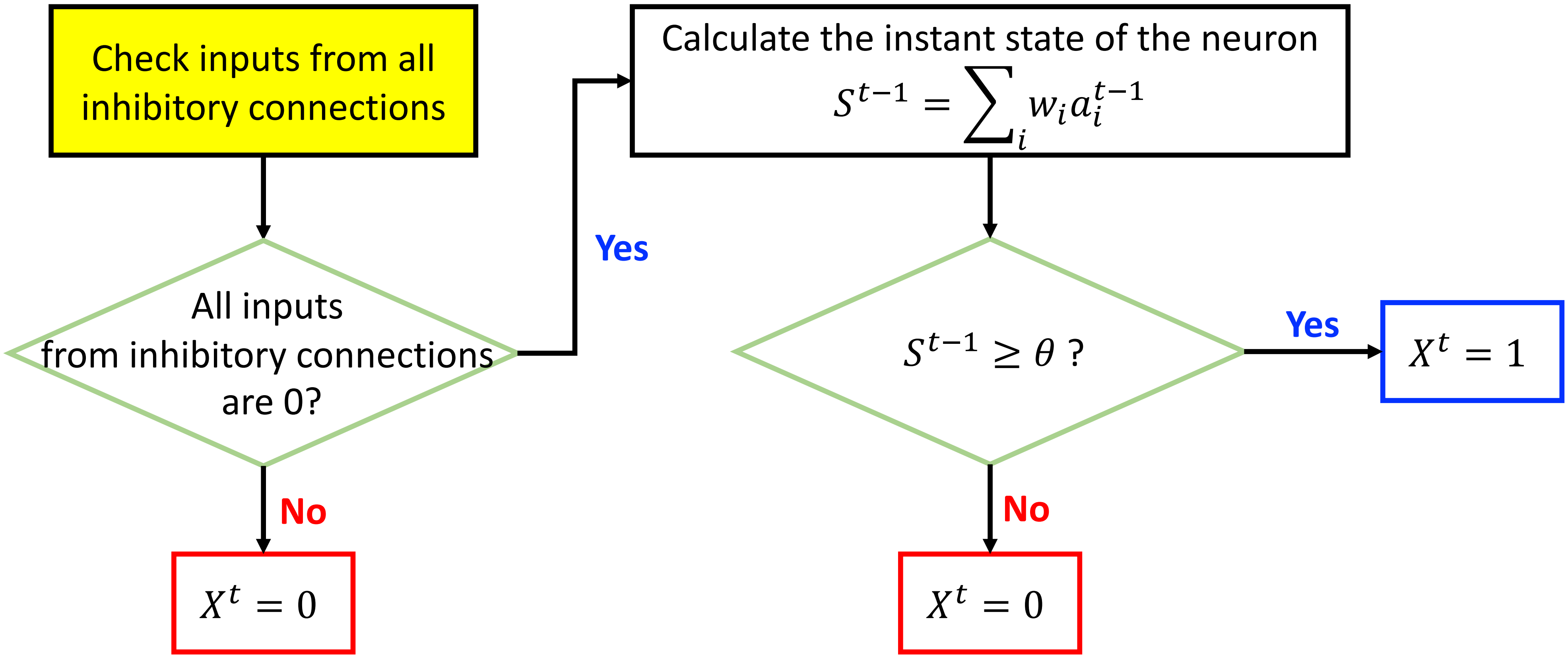
$X^t = 1$ if and only if $S^{t-1} = \sum_i w_i a_i^{t-1} \geq \theta$, and $w_i > 0, \forall a_i^{t-1} > 0$.

Comparison with Different Models

	Biological neuron	Basic abstract model	**McCulloch-Pitts Neuron**
Input	Dendrites: From huge number of neurons, sometimes very distant ones	Multiple Inputs	Multiple ***binary, discrete-time*** inputs
Excitation	From a repetition of impulses in time at the same synapse (***temporal summation***) or from the simultaneous arrival of impulses at a sufficient number of adjacent synapses (***spatial summation***) to make the "density" of signal high enough at some region of the neuron to overcome ***the excitation threshold***.	The abstract neuron is excited (output is equal to 1) when weighted sum is above the threshold 0.	The output is obtained by computing a threshold **activation function**, if there is no inhibitory inputs. Single Output.
Output	Axon: To huge number of neurons, sometimes very distance ones	Single Output.	

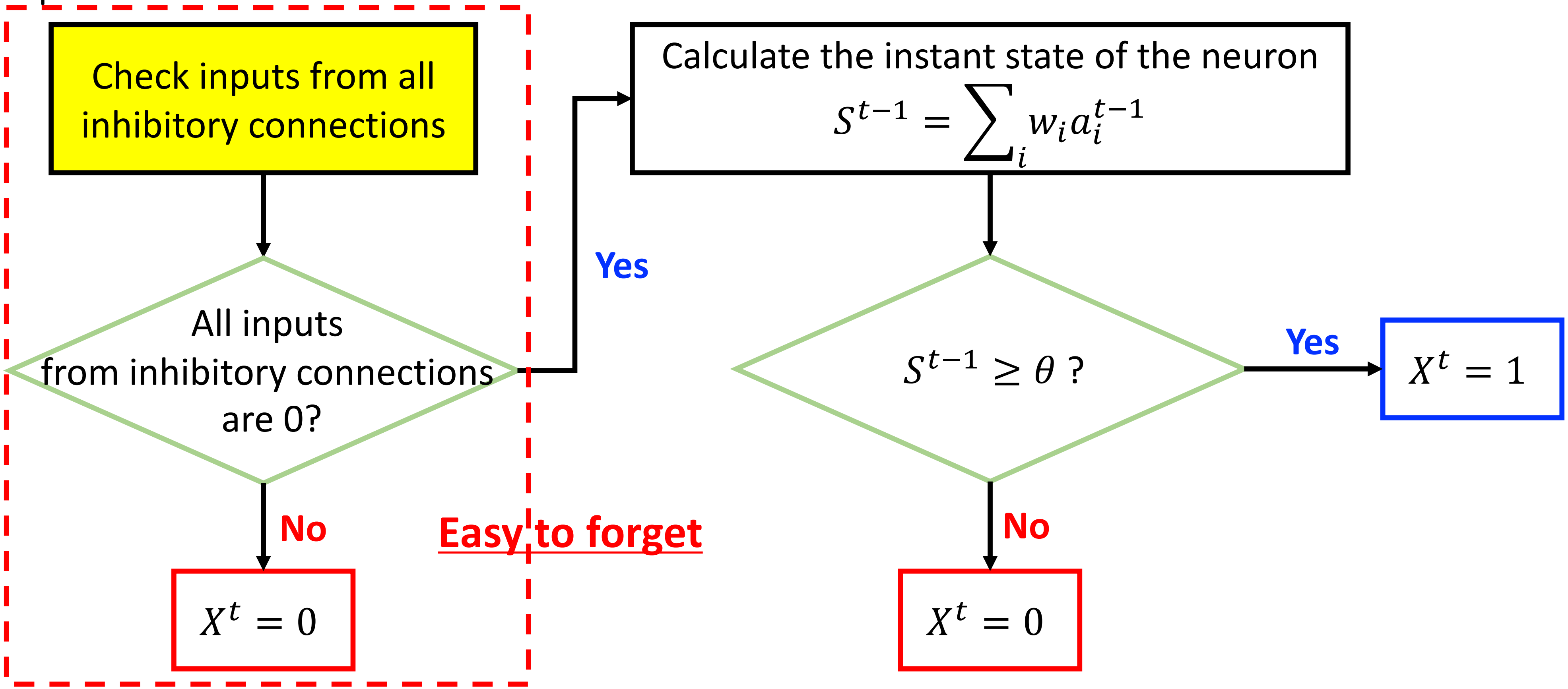
MP Neuron Computation Algorithm

Given a fixed MP neuron, that is, all weights of connections and neuron threshold are set up in advance.

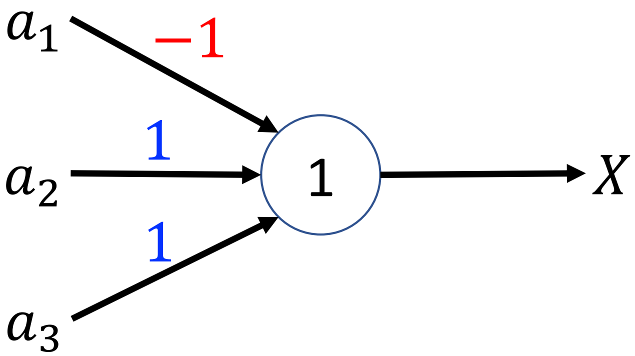


MP Neuron Computation Algorithm

Given a fixed MP neuron, that is, all weights of connections and neuron threshold are set up in advance.



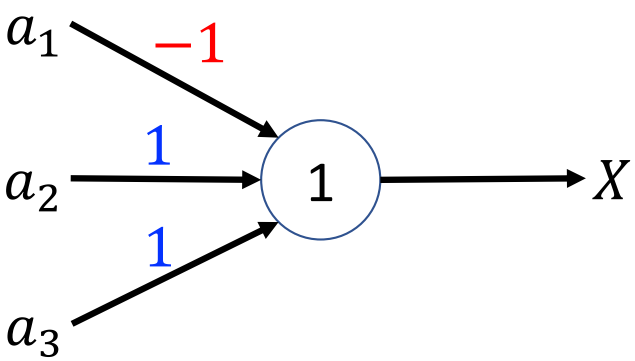
An Example



$$\theta = 1$$
$$w_1 = -1$$
$$w_2 = 1$$
$$w_3 = 1$$

$$X = ?$$

An Example

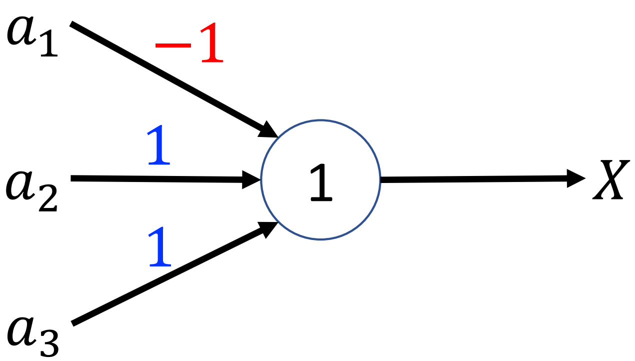


$$\theta = 1$$
$$w_1 = -1$$
$$w_2 = 1$$
$$w_3 = 1$$

$$X = ?$$

1) Input $a_1 = 0$, $a_2 = 1$, $a_3 = 1$

An Example

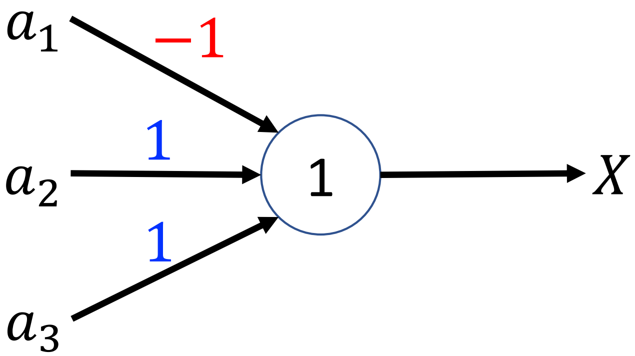


$$\theta = 1$$
$$w_1 = -1$$
$$w_2 = 1$$
$$w_3 = 1$$

$X = ?$

1) Input $a_1 = 0$, $a_2 = 1$, $a_3 = 1$
2) All inhibitory connections are silent
3) Instant state $S = 0 \times (-1) + 1 \times 1 + 1 \times 1 = 2 > \theta$
4) Check activation function $S > \theta \Longrightarrow X = 1$

An Example

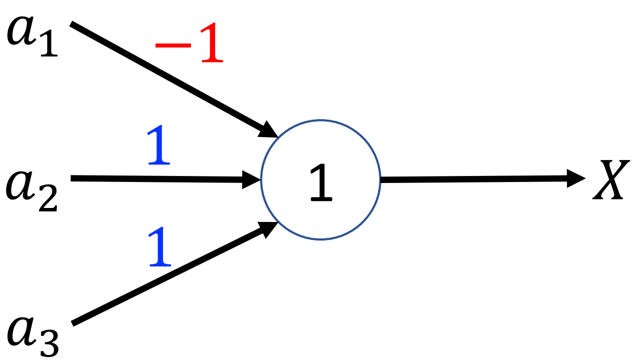


$\theta = 1$
$w_1 = -1$
$w_2 = 1$
$w_3 = 1$

$X = ?$

1) Input $a_1 = 1$, $a_2 = 1$, $a_3 = 1$

An Example



$\theta = 1$
$w_1 = -1$
$w_2 = 1$
$w_3 = 1$

$X = ?$

1) Input $a_1 = 1$, $a_2 = 1$, $a_3 = 1$
2) There is an inhibitory connection activated (a_1)
3) We have $X = 0$

MP-Neuron as a Binary Unit

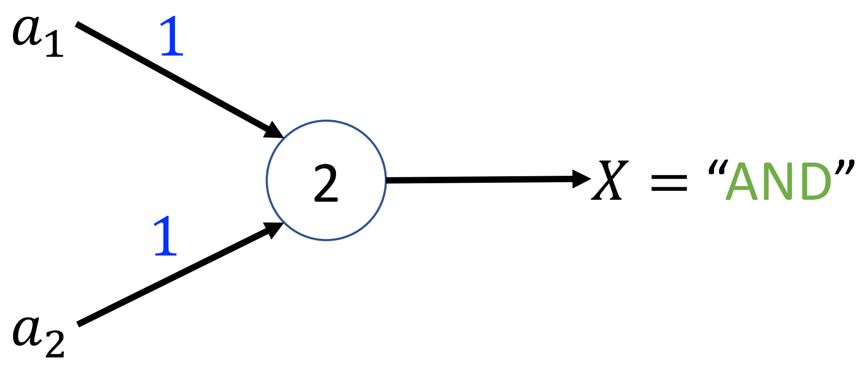
- ***Simple logical functions*** can be implemented directly with a single McCulloch-Pitts unit.

- The output value **1** can be associated with the logical value ***true*** and **0** with the logical value ***false***.

- Now, let us demonstrate how weights and thresholds can be set to yield neurons which realise the logical functions **AND**, **OR** and **NOT**.

MP-Neuron Logic: Two Inputs

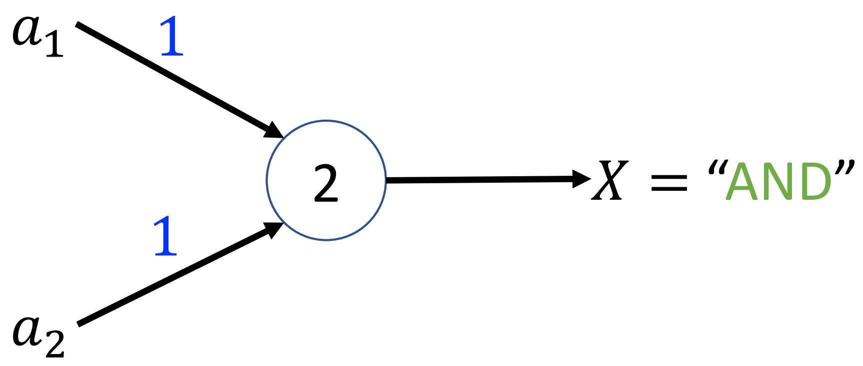
$$a_1 \wedge a_2$$



"AND" – the output fires if a_1 and a_2 <u>both</u> fire.

MP-Neuron Logic: Two Inputs

$a_1 \land a_2$



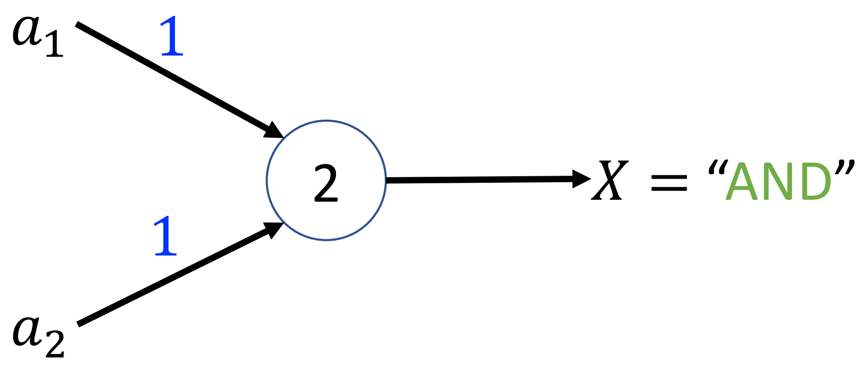
"AND" – the output fires if a_1 and a_2 <u>both</u> fire.

Q: How to prove?

MP-Neuron Logic: Two Inputs

a_1	a_2	"AND"
1	1	1
0	1	0
1	0	0
0	0	0

$$a_1 \wedge a_2$$

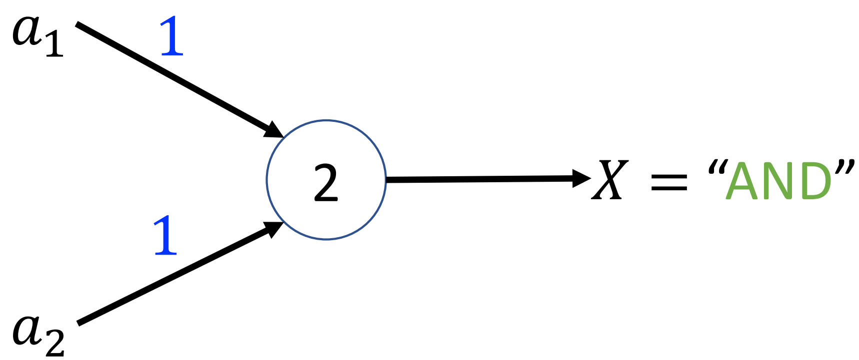


"AND" – the output fires if a_1 and a_2 both fire.

MP-Neuron Logic: Two Inputs

$\boldsymbol{a_1}$	$\boldsymbol{a_2}$	**"AND"**
1	1	1
0	1	0
1	0	0
0	0	0

$$a_1 \wedge a_2$$



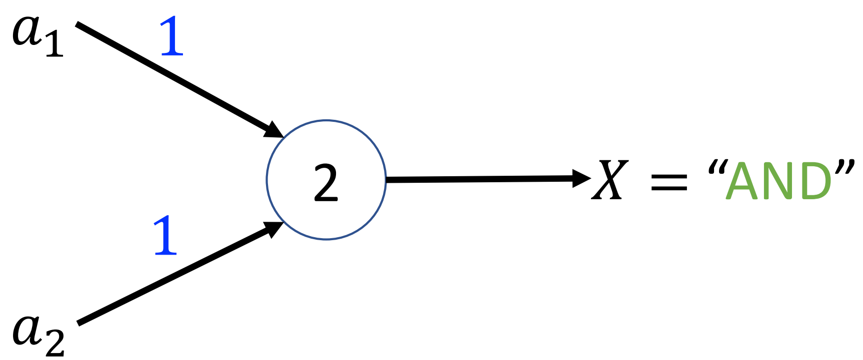
No inhibitory connections!

1) Input $a_1 = 1, a_2 = 1.$ $S = 1\times1 + 1\times1 = 2 \geq \theta.$ $X = 1.$
2) Input $a_1 = 0, a_2 = 1.$ $S = 0\times1 + 1\times1 = 1 < \theta.$ $X = 0.$
3) Input $a_1 = 1, a_2 = 0.$ $S = 1\times1 + 0\times1 = 1 < \theta.$ $X = 0.$
4) Input $a_1 = 0, a_2 = 0.$ $S = 0\times1 + 0\times1 = 0 < \theta.$ $X = 0.$

MP-Neuron Logic: Two Inputs

a_1	a_2	"AND"
1	1	1
0	1	0
1	0	0
0	0	0

$$a_1 \wedge a_2$$

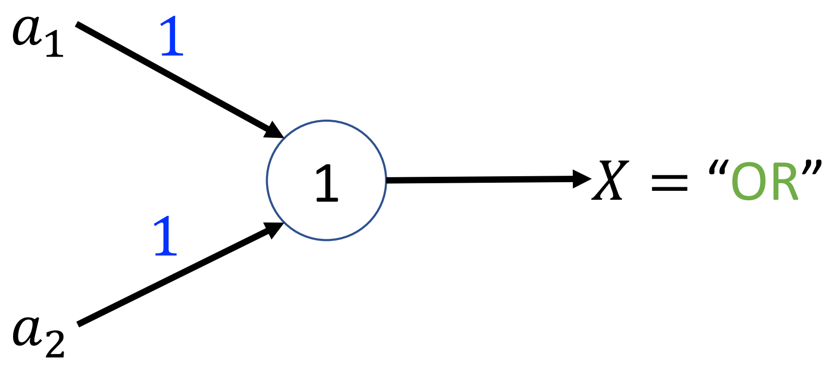


1) Input $a_1 = 1, a_2 = 1.$ $S = 1\times1 + 1\times1 = 2 \geq \theta.$ $X = 1.$
2) Input $a_1 = 0, a_2 = 1.$ $S = 0\times1 + 1\times1 = 1 < \theta.$ $X = 0.$
3) Input $a_1 = 1, a_2 = 0.$ $S = 1\times1 + 0\times1 = 1 < \theta.$ $X = 0.$
4) Input $a_1 = 0, a_2 = 0.$ $S = 0\times1 + 0\times1 = 0 < \theta.$ $X = 0.$

MP-Neuron Logic: Two Inputs

a_1	a_2	"OR"
1	1	1
0	1	1
1	0	1
0	0	0

$$a_1 \vee a_2$$

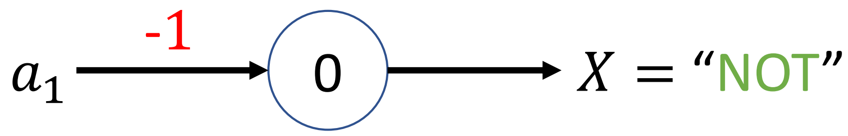


"OR" – the output fires if a_1 fires or a_2 fires or both fire.

MP-Neuron Logic: Two Inputs

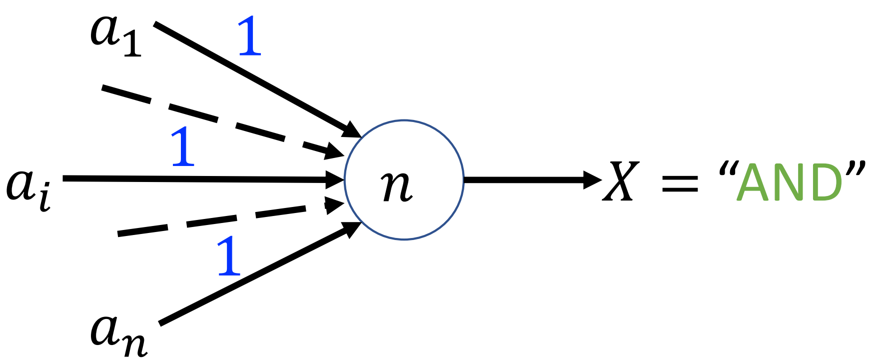
$\boldsymbol{a_1}$	**“NOT”**
1	0
0	1

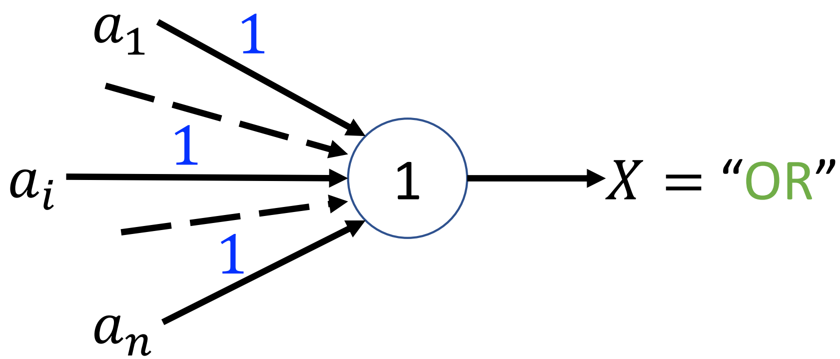
$\neg a_1$



“NOT” – the output fires if a_1 does **<u>NOT</u>** fire and vice versa.

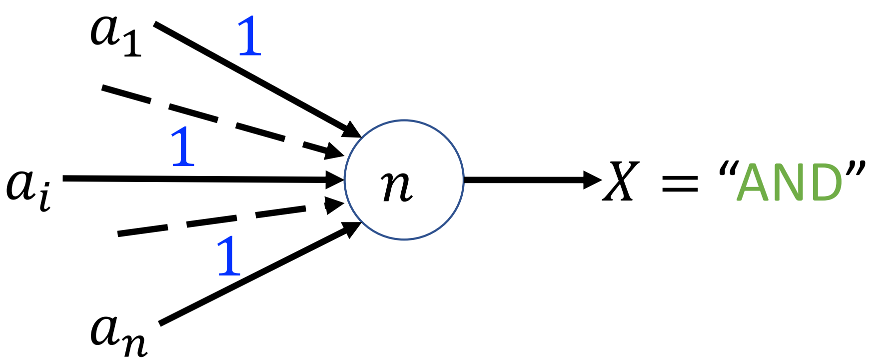
MP-Neuron Logic: Multiple Inputs

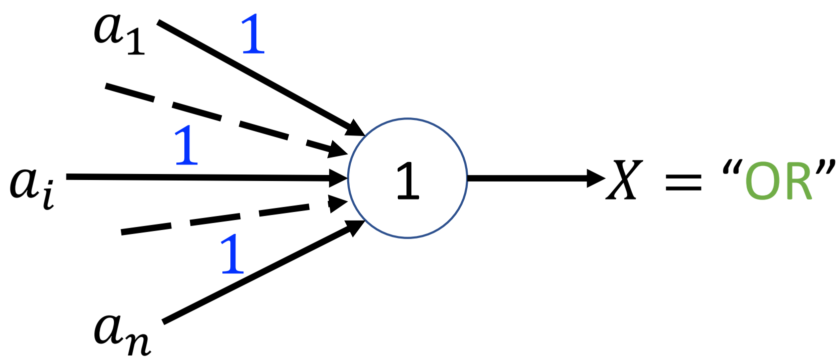




A single MP neuron can compute the conjunction or disjunction of n arguments, as it is shown, while two conventional logic units are needed to perform the conjunction of just three inputs.

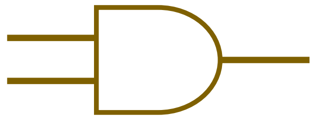
MP-Neuron Logic: Multiple Inputs





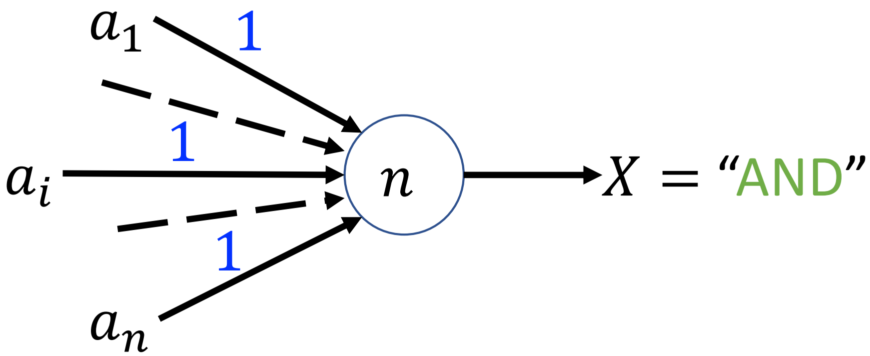
A single MP neuron can compute the conjunction or disjunction of n arguments, as it is shown, while two conventional logic units are needed to perform the conjunction of just three inputs.

Logic gate: AND

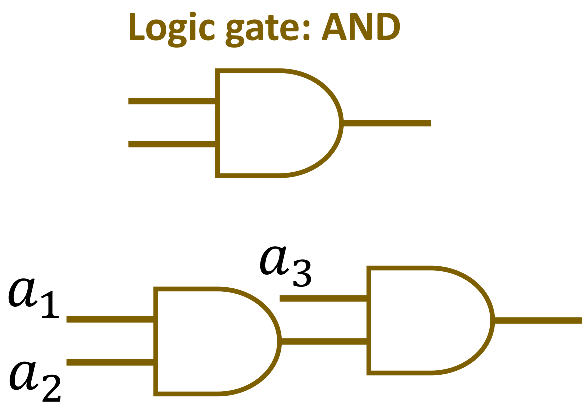


Q: How to use this logic gate to perform the conjunction of three inputs?

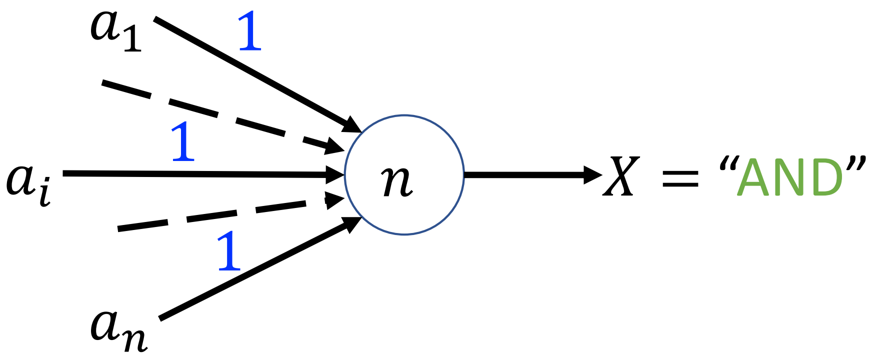
MP-Neuron Logic: Multiple Inputs

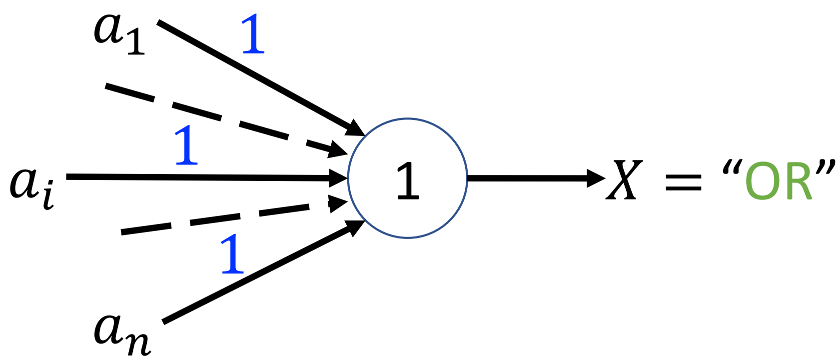


A single MP neuron can compute the conjunction or disjunction of n arguments, as it is shown, while two conventional logic units are needed to perform the conjunction of just three inputs.



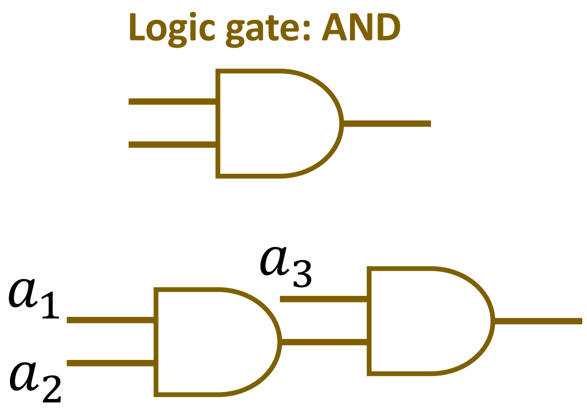
MP-Neuron Logic: Multiple Inputs



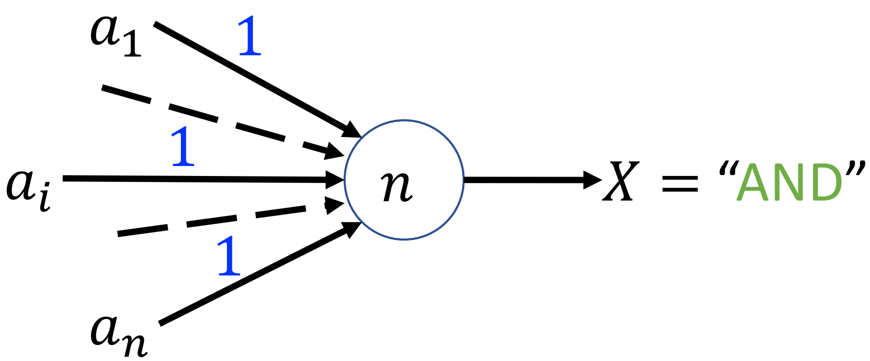


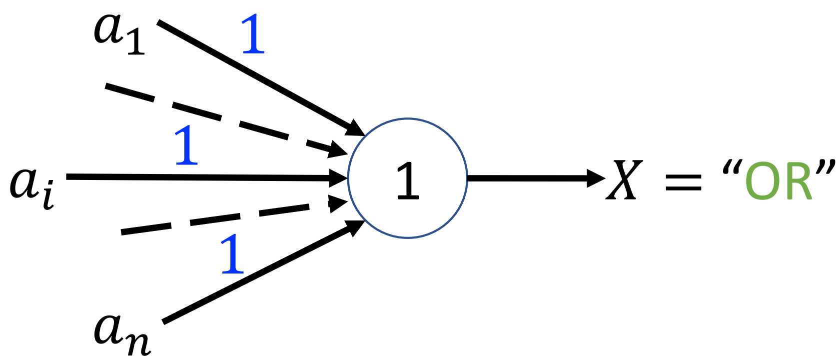
A single MP neuron can compute the conjunction or disjunction of n arguments, as it is shown, while two conventional logic units are needed to perform the conjunction of just three inputs.

Q: How to prove?



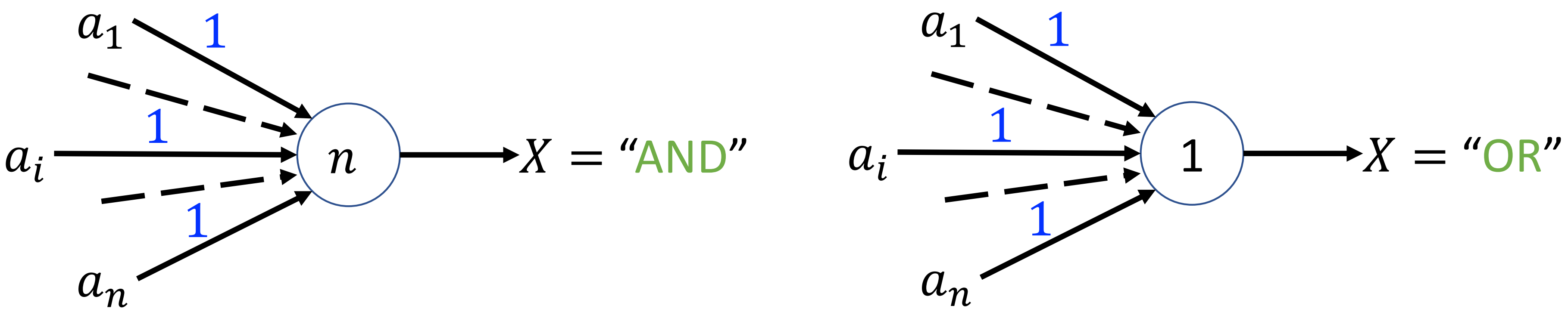
MP-Neuron Logic: Multiple Inputs





In general, the same kind of computation requires several conventional logic gates with two inputs.

MP-Neuron Logic: Multiple Inputs



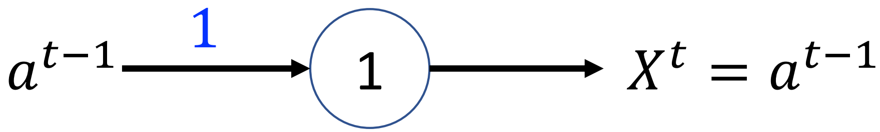
The threshold logic elements reduce the complexity of the circuit used to implement a given logical function.

More Interesting with Time

If we consider the time, there are more interesting applications.

MP-Neuron as a Register Cell

a_1	"Reg"
1	1
0	0

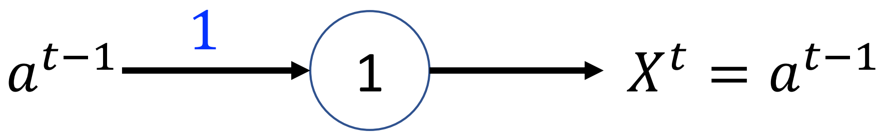


A single neuron with a single input a and with the weight and threshold values both of unity, computes the output

$$X^t = a^{t-1}$$

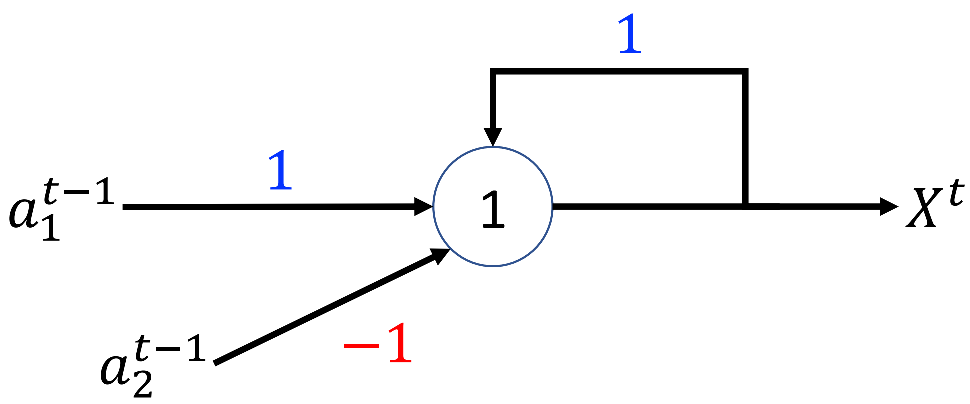
MP-Neuron as a Register Cell

a_1	"Reg"
1	1
0	0



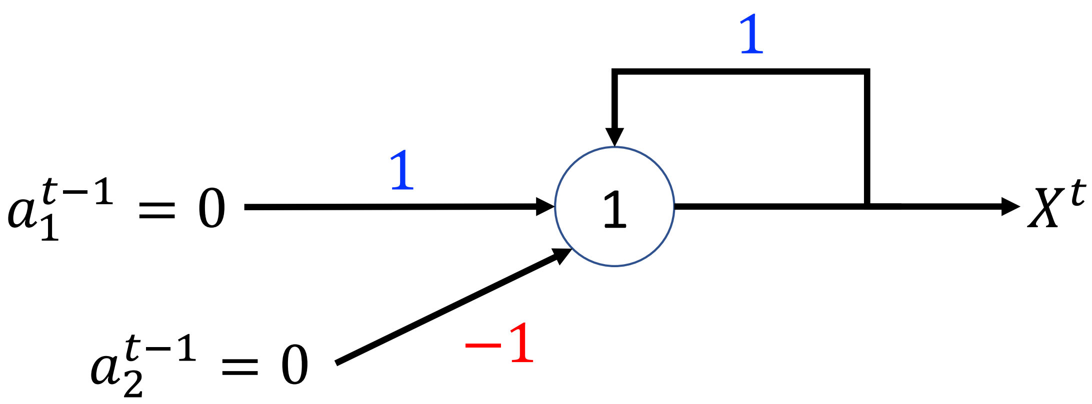
Such a single neuron thus behaves as a single register cell able to retain the input for one period elapsing between two instant.

(Extended) MP-Neuron as a Memory Cell



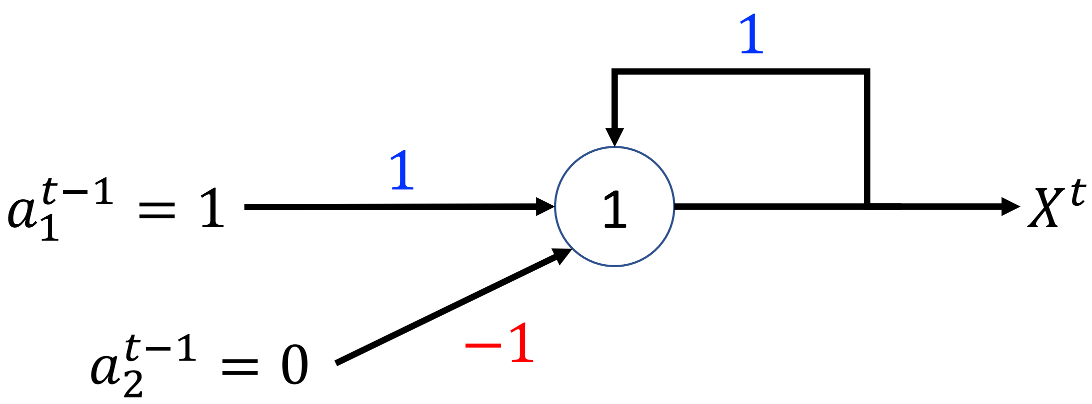
With a feedback loop closed around the neuron, as it is shown above, we obtain a memory cell. Note that it is not a single MP-neuron with the classical definition.

(Extended) MP-Neuron as a Memory Cell



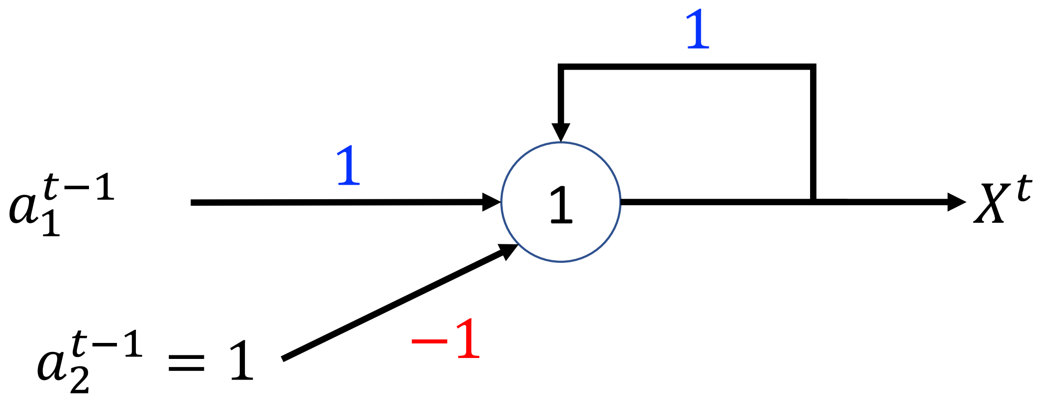
In the absence of inputs, the output value is sustained indefinitely. This is because of the output of 0 feeds back to the input does not cause firing at the next instant, while the output of 1 does.

(Extended) MP-Neuron as a Memory Cell



An excitatory input of 1, via connection with weight $w_1 = 1$, initializes the firing in this memory cell.

(Extended) MP-Neuron as a Memory Cell



An inhibitory input of 1, via connection with weight $w_2 = -1$, initializes a nonfiring state in this memory cell.

Representation Power (Without time)

What kind of propositions can be represented by a single MP neuron (without time)?

Representation Power (Without time)

What kind of propositions can be represented by a single MP neuron (without time)?

In the next lecture, we try to address this question.