Problem set 8
Social Network Analysis

Excercise 1

For the graph shown in Figure 1, compute the highest

- degree centrality, - closeness centrality, and - betweenness centrality.

The nodes that take on these highest values are already marked in the figure.

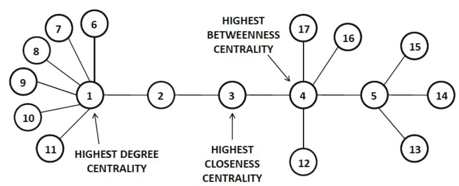

Figure 1: Caption

Solution Using the formula for the degree centality of a node i

$$C_D(i) = \frac{\deg(i)}{n-1}$$

we find out that $C_D(1) = \frac{\deg(1)}{17-1} = \frac{7}{16}$.

2. We use the formula for the closeness centrality of a node i

$$C_C(i) = \frac{1}{\text{AvDist}(i)},$$

where $\text{AvDist}(i) = \frac{\sum_{j=1}^{n} \text{dist}(i,j)}{n-1}$ is the average distance of shortest paths starting from i.

We have $\text{AvDist}(3) = \frac{\text{dist}(3,1)+\text{dist}(3,2)+\ldots+\text{dist}(3,17)}{16} = \frac{2+1+0+1+2+3+3+3+3+3+3+2+3+3+3+2+2}{16} = \frac{39}{16}$ and

$$C_C(3) = \frac{16}{39}$$

We use the formula for the betweenness centraily of a node i

$$C_B(i) = \frac{\sum_{j<k} f_{jk}(i)}{\binom{n}{2}},$$

where $f_{jk}(i) = \frac{q_{jk}(i)}{q_{jk}}$, and q_{jk} is the number of shortest paths between nodes j and k, and $q_{jk}(i)$ is the number of shortest paths between nodes j and k that pass through node i.

The shortest path from 4 to any other node will always go through 4. There are 16 such paths.

Any shortest path from $\{6, 7, 8, 9, 3, 10, 11, 1, 2\}$ to $\{17, 16, 12, 15, 14, 5, 13\}$ will go through 4. There are $9 * 7 = 63$ such paths.

Similarly any shortest path between $\{17, 16, 12\}$ will also go through 4. There are 3 such paths. Any shortest path between $\{5, 15, 14, 13\}$ and $\{17, 16, 12\}$ will also go through 4. There are $4 * 3 = 12$ such paths.

Hence the total number of shortest paths going through 4 is $16+63+3+12 = 94$. Betweenness centrality=94/136

Excercise 2

For every vertex of the graph shown in Figure 2, compute its degree prestige.

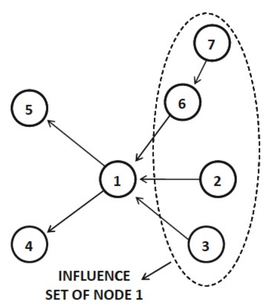

Figure 2: Graph 1

Solution Using the formula $P_D(i) = \frac{\deg_+(i)}{n-1}$ we find

i	$P_D(i)$
1	3/6
2	0
3	0
4	1/6
5	1/6
6	1/6
7	0

Excercise 3

For every vertex of the graph shown in Figure 2, compute its proximity prestige.

Solution

$$\text{AvDist}(i) = \frac{\sum_{j \in \text{Influence}(i)} \text{dist}(j, i)}{|\text{Influence}(i)|}.$$

i	Influence(i)	AvDist(i)	InfluenceFraction(i)	$P_P(i)$
1	4	5/4	2/3	8/15
2	0	0	0	0
3	0	0	0	0
4	5	2	5/6	5/12
5	5	2	5/6	5/12
6	1	1	1/6	1/6
7	0	0	0	0