1. A biased die produces numbers with the following probability distribution:

1	2	3	4	5	6
0.15	0.1	0.2	0.25	0.15	0.15

   Calculate

   (a) The expected value of a single roll of the die.

   (b) The expected value if the die is rolled twice and added together.

   (c) The variance of a single roll of the die.

2. The mean of a set of n data points is

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

   (a) Calculate an expression for the change in the mean if a new data point x_{n+1} is measured.

   (b) Calculate (i) the mean and (ii) the median of the data set { 1.9, 2.3, 3.1, 3.6, 3.7, 4.0, 4.1, 4.5 }.

   (c) Calculate the change in (i) the mean and (ii) the median if the data points { 198.2, 206.6 } are added to the data set.

   (d) Comment on these changes. Do you think either average is still a useful measure of the location? How would you use summary statistics to understand the changes in the data set?

3. The following data represents the daily energy output for a small solar power generator.

Date	**Energy generated/kJ**
2023-06-01	101.29
2023-06-02	112.41
2023-06-03	-1
2023-06-04	122.54
2023-06-05	1.12
2023-06-06	75.56
2023-06-07	87.10
2023-06-08	69.49
2023-06-09	96.43
2023-06-10	121.11
2023-06-11	0.00
2023-06-12	59.35

   (a) What type of data is this?

(b) Which of the data points are unusual? What is unusual about each one, and can you make a suggestion for what might have caused the anomaly?

(c) What external data source do you think would be important to help you analyse this data series?

4. A car racing team are testing their new car on a straight track. They have a camera set up alongside the track, radars to measure the velocity and an accelerometer inside the car which records the acceleration in the directions along (x) and across (y) the track. They would like to use a Kalman filter to model the position and velocity of the car.

   (a) What assumptions are they making about the noise that affects the car's movement and the accelerometer measurements?

   (b) At the start of a step, the car's position (in metres) and velocity (in metres per second) are estimated to be

   $$\hat{\mathbf{x}}_{t-1|t-1} = \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} 123.0 \\ 10.0 \\ 90.0 \\ 1.2 \end{bmatrix}$$

   The accelerations were measured as $(a_x, a_y) = (1.4, -0.2)$. Use the dynamical equations

   $$s = vt + \frac{1}{2}at^2$$
   $$v = at$$

   to calculate the dynamical estimate of the car's position and velocity after a step of 1 second.

   (c) At the new step, the camera measures the car's position in the x direction with an error of 0.12 m, and the radars measure the x and y velocities with an error of 0.04 m/s; the velocity errors are correlated with correlation 0.5. What is the covariance matrix used for the measurement errors?

   (d) How would you expect the covariance of the state to change when a measurement is made? Does this suggest a problem with the testing set-up?