Russel’s paradox

$$\{x \in S \mid x \ldots\}$$

$$\{x \in \mathbb{Z} \mid x \text{ is even}\}$$

$$\{x \mid x \text{ is even}\}$$

$$2^A = \{ B \subseteq A \}$$

$\{ x \text{ is a set} \}$

Why is this set theory “naive”

It suffers from paradoxes.

A leading example:

> *A barber is the man who shaves all those, and only those, men who do not shave themselves.*

- Who shaves the barber?

Russell's Paradox

Russell's paradox shows that the 'object' $\{x \mid P(x)\}$ is not always meaningful.

Set $A = \{B \mid B \notin B\}$

Problem: do we have $A \in A$?

Abbreviate, for any set C, by $P(C)$ the statement $C \notin C$. Then $A = \{B \mid P(B)\}$.

- If $A \in A$, then (from the definition of P), not $P(A)$. Therefore $A \notin A$.
- If $A \notin A$, then (from the definition of P), $P(A)$. Therefore $A \in A$.