More examples of direct proof

Prove that every integer is rational

$\forall n$ if n is an integer then n is rational

$$n = \frac{n}{1} = \frac{2n}{2} = \frac{3n}{3}$$

Proof: Suppose that n is a particular but arbitrarily chosen integer.

As $n = \frac{n}{1}$ and $1 \neq 0$, n is rational

Prove that the sum of any two rational numbers is rational

$\forall\ r, q$ if r, q are rational then

$r+q$ is rational

Proof Suppose that r and q are particular but arbitrarily chosen rational numbers

Then $r = \frac{k}{l}$ where k, l are some integers, $l \neq 0$

$q = \frac{m}{n}$ where m, n are int., $n \neq 0$

$$r+q = \frac{k}{l} + \frac{m}{n} = \frac{kn+ml}{ln}$$

$kn+ml$, ln are integers

Since l, n are not 0,

$l \cdot n$ is non-zero

So. $r+q$ is rational

Prove that the product of any two rational numbers is rational

$\forall r, q$ st . . . . .

⋮

$r = \frac{k}{l}$ . . .

⋮

$q = \frac{m}{n}$ . .

$r \cdot q = \frac{k \cdot m}{n \cdot l}$

Prove that the double of a rational number is rational

$\forall r$ if r is rational then $2r$ is rational

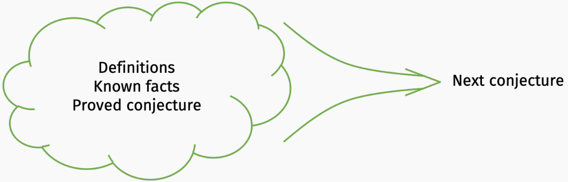
Proof

Suppose that r is a particular but arbitrarily chosen rational number

As 2 is an integer, 2 is rational

So $\underline{r \cdot 2}$ is rational

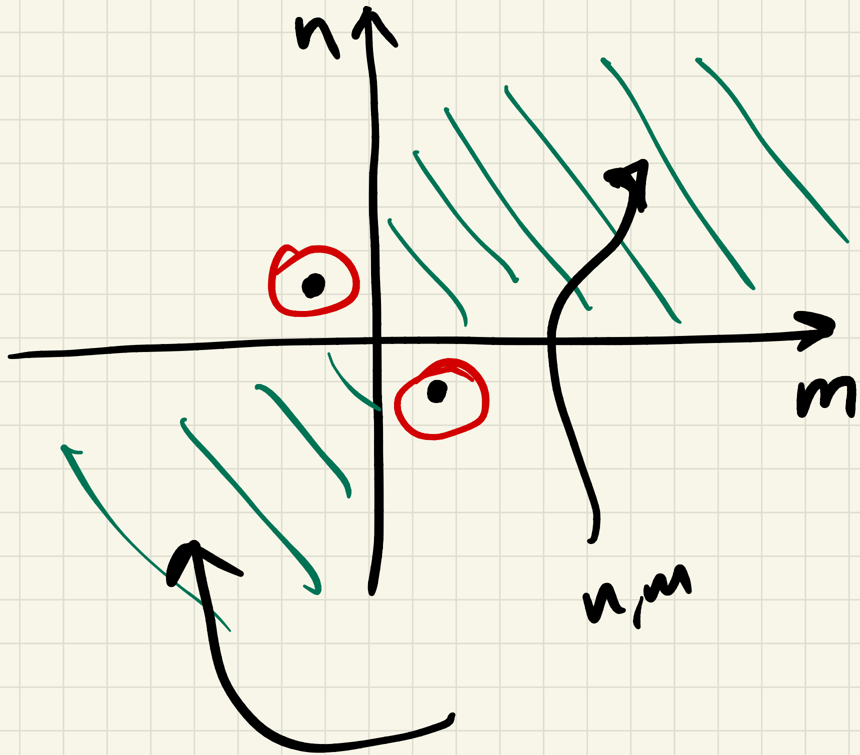
Mathematical discovery



Proof by cases

$\forall\, m, n$ if $m^2 = n^2$ then $m = n$

Not true e.g for $m = 1,\ n = -1$




n
m

Prove by cases: Combine generic particulars and proof by exhaustion

Statement: For all integers n, $n^2 + n$ is even

Case 1: n is even

Suppose that n is a particular but arbitrarily chosen even number

$n = 2k$, where k is an int. $n^2+n = (2k)^2 + 2k$

$= 4k^2 + 2k = 2(2k^2+k)$

Case 2: n is odd

Suppose that n is a particular but arb. chosen odd number

$n = 2k+1$ ...

$n^2+n = (2k+1)^2 + (2k+1) = 4k^2+4k+1+2k+1$

$= 4k^2+6k+2 = 2(2k^2+3k+1)$

Prove that the product of any two consecutive integers is even

The square of any integer is of the form $3k$ or $3k+1$

Case 1 n is even

Suppose that n is a particular but arbitrarily chosen even integer

Then $n = 2k$, where k is an int.

$$n^2 = (2k)^2 = 4k^2$$

0	1	2

$$n = 3 \cdot k + \ldots \begin{matrix} 0 \\ 1 \\ 2 \end{matrix}$$

$$7 = 3 \cdot 2 + 1$$
$$8 = 3 \cdot 2 + 2$$
$$9 = 3 \cdot 2 + 0$$
$$\vdots$$

Case 1 $\quad n = 3k$

Case 2 $\quad n = 3k+1$

Case 3 $\quad n = 3k+2$