Burger's Equation in 2D

Matthew Portman

Computational Science Research Center
San Diego State University

December 19, 2017

Outline

What is Burger's Equation?

What is Burger's Equation?

Burger's Equation and you

- Fluid dynamics – the (brief) physics.
- Used to model: Traffic Flow, Nonlinear Acoustics, Gas Dynamics.
- Burger's equation and Navier-Stokes.

Johannes (Jan) Martinus Burger By Source (WP:NFCC4), Fair use,

On Wikipedia

Burger's Equation(s)

- Inviscid Burger's Equation in 1D

$$\frac{\partial u(x,t)}{\partial t} + \frac{\partial f(u)}{\partial x} = 0$$

Burger's Equation(s)

- Inviscid Burger's Equation in 1D

$$\frac{\partial u(x,t)}{\partial t} + \frac{\partial f(u)}{\partial x} = 0$$

- Inviscid Burger's Equation in 2D

$$\frac{\partial F(x,y,t)}{\partial t} + u(x,y)\frac{\partial F}{\partial x} + v(x,y)\frac{\partial F}{\partial y} = 0$$

Numerical Schemes

Numerical Schemes

FTCS and Mimetic

- FTCS

$$F_{i,j}^{n+1} = F_{i,j}^{n} - \frac{u\Delta t}{\Delta x^2}(F_{i-1,j}^{n} - F_{i+1,j}^{n}) - \frac{v\Delta t}{\Delta y^2}(F_{i-1,j}^{n} - F_{i+1,j}^{n})$$

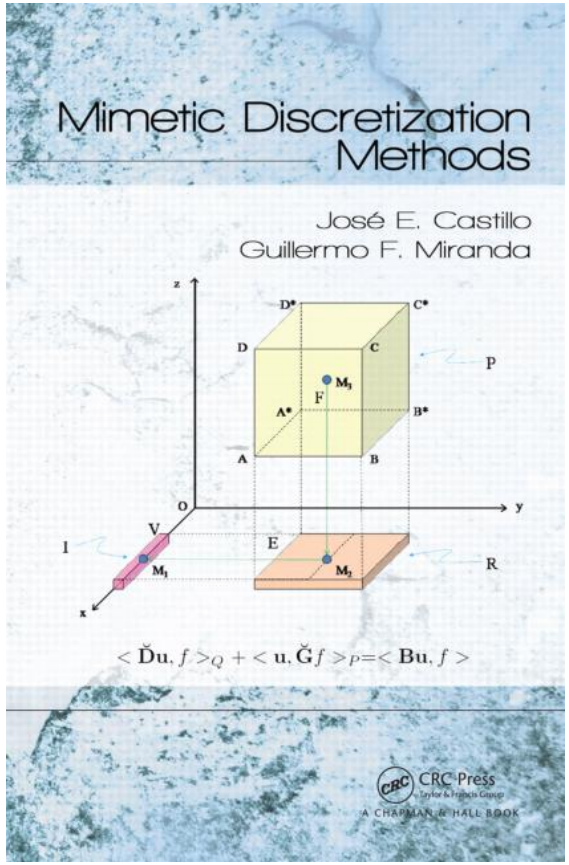
- Lax-Wendroff

$$\begin{aligned} F(x,y,t) &= F + \Delta t\ F_t + \frac{1}{2}\Delta t^2 F_{tt} + \ldots \\ &= F - \Delta t(uF_x + vF_y) \\ &\quad + \frac{1}{2}\Delta t^2(u(u_x F_x + uF_x x) + u(v_x F_y + uF_{yx}) \\ &\quad + v(u_y F_x + uF_{xy}) + v(v_y F_y + uF_y y)) + \ldots \end{aligned}$$

Mimetic

Mimetic

- Divergence Operator: $\nabla \cdot F$
- Mimetic Interpolator



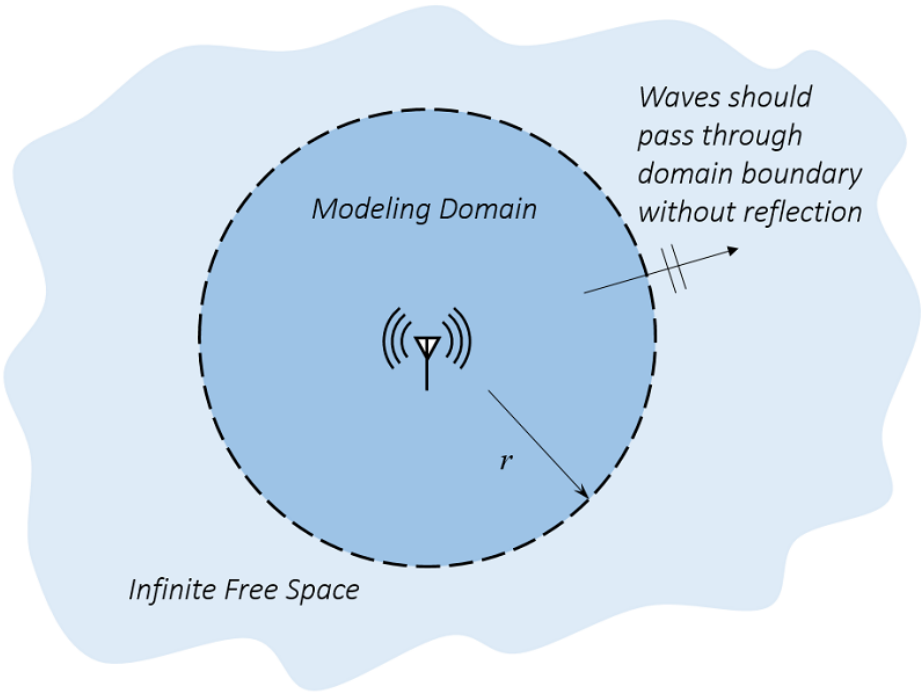
CRC Press

Numerical Analysis and Results

Numerical Analysis and Results

Conservation of mass, Boundary and Initial Conditions

- Conservation of Flux/Mass
- Initial Condition: Gaussian Wave at x,y $= 0$.
- Boundary Conditions: "Open"
- x $=$ y, $\Delta x = \Delta y$



Credit: Walter Frei

Results

- The Mimetic method was the only fully conservative method.

Results

- The Mimetic method was the only fully conservative method.
- The three methods evolved similarly.

Results

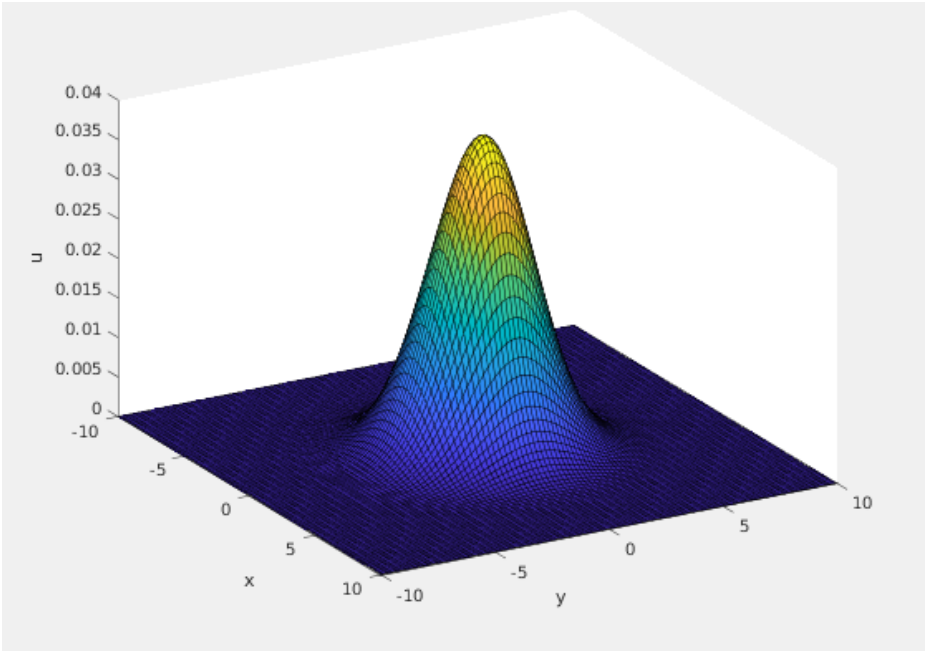
- The Mimetic method was the only fully conservative method.
- The three methods evolved similarly.
- Adjusting Δx and/or Δt did not affect the conservation numbers of FTCS and Lax-Wendroff significantly until they approached 0.5+.

Results

- The Mimetic method was the only fully conservative method.
- The three methods evolved similarly.
- Adjusting Δx and/or Δt did not affect the conservation numbers of FTCS and Lax-Wendroff significantly until they approached 0.5+.
- The schemes were all stable to a large range of Δx and Δt.

Plots

- FTCS
- Lax-Wendroff
- Mimetic



From Plots

Thanks to:

- Angel Boada
- Dr. Jose Castillo and CSRC