

Burger's Equation in 2D

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Outline

- 1 What is Burger's Equation?
- 2 Numerical Schemes
- 3 Numerical Analysis and Results

What is Burger's Equation?

What is Burger's Equation?

Burger's Equation and you

- Fluid dynamics – the (brief) physics.
- Used to model: Traffic Flow, Nonlinear Acoustics, Gas Dynamics.
- Burger's equation and Navier-Stokes.



Johannes (Jan) Martinus Burger By Source (WP:NFCC4), Fair use,

On Wikipedia

Burger's Equation(s)

- Inviscid Burger's Equation in 1D

$$\frac{\partial u(x, t)}{\partial t} + \frac{\partial f(u)}{\partial x} = 0$$

Burger's Equation(s)

- Inviscid Burger's Equation in 1D

$$\frac{\partial u(x, t)}{\partial t} + \frac{\partial f(u)}{\partial x} = 0$$

- Inviscid Burger's Equation in 2D

$$\frac{\partial F(x, y, t)}{\partial t} + u(x, y) \frac{\partial F}{\partial x} + v(x, y) \frac{\partial F}{\partial y} = 0$$

Numerical Schemes

Numerical Schemes

FTCS and Mimetic

- FTCS

$$F_{i,j}^{n+1} = F_{i,j}^n - \frac{u\Delta t}{\Delta x^2}(F_{i-1,j}^n - F_{i+1,j}^n) - \frac{v\Delta t}{\Delta y^2}(F_{i,j-1}^n - F_{i,j+1}^n)$$

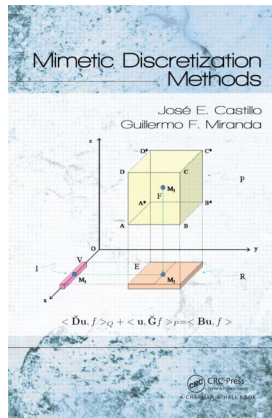
- Lax-Wendroff

$$\begin{aligned} F(x, y, t) &= F + \Delta t F_t + \frac{1}{2}\Delta t^2 F_{tt} + \dots \\ &= F - \Delta t(uF_x + vF_y) \\ &\quad + \frac{1}{2}\Delta t^2(u(u_x F_x + uF_{xx}) + u(v_x F_y + uF_{yx}) \\ &\quad + v(u_y F_x + uF_{xy}) + v(v_y F_y + uF_{yy})) + \dots \end{aligned}$$

Mimetic

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- Divergence Operator: $\nabla \cdot F$
- Mimetic Interpolator



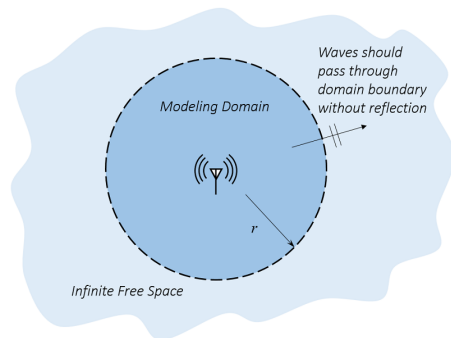
CRC Press

Numerical Analysis and Results

Numerical Analysis and Results

Conservation of mass, Boundary and Initial Conditions

- Conservation of Flux/Mass
- Initial Condition: Gaussian Wave at $x, y = 0$.
- Boundary Conditions: "Open"
- $x = y, \Delta x = \Delta y$



Credit: Walter Frei

Results

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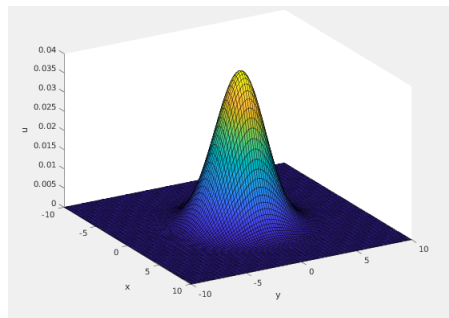
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- Adjusting Δx and/or Δt did not affect the conservation numbers of FTCS and Lax-Wendroff significantly until they approached 0.5+.

Results

- The Mimetic method was the only fully conservative method.
- The three methods evolved similarly.
- Adjusting Δx and/or Δt did not affect the conservation numbers of FTCS and Lax-Wendroff significantly until they approached 0.5+.
- The schemes were all stable to a large range of Δx and Δt .

Plots

- FTCS
- Lax-Wendroff
- Mimetic



From Plots

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