Compact Stars

via Smoothed Particle Hydrodynamics

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Outline

Compact Object Physics and Rotation

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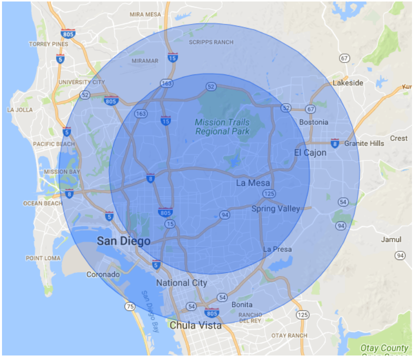
Compact Objects: A Primer

- Classical Mechanics and General Relativity.
- White Dwarfs, Black Holes

Compact Objects: A Primer

- Classical Mechanics and General Relativity.
- White Dwarfs, Black Holes
- ...And Neutron Stars.



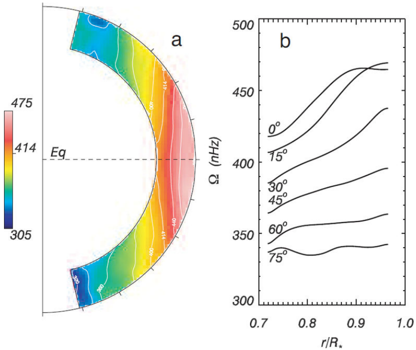
A neutron star the size of San Diego
Thanks to: Google Maps and obeattie.github.io

(Differential) Rotation

- Rotation in Compact Objects
- Differential Rotation AKA Rotating w.r.t. Rotation

$$j(\Omega) = A^2(\Omega_c - \Omega)$$
$$\frac{\Omega}{\Omega_c} = \frac{A^2}{A^2 + r^2 sin^2\theta}$$
$$A = aR_{\text{star}}$$

Muller and Eriguchi (1985)



Differential Rotation of the Sun (Brun et. al. 2004)

Smoothed Particle Hydrodynamics

SPH: What is it? How does it work? Let's find out!

Smoothed Particle Hydrodynamics (SPH)

- The N-Body Problem and the Mean-Field approximation... solving the Advection Equation.

Smoothed Particle Hydrodynamics (SPH)

- The N-Body Problem and the Mean-Field approximation... solving the Advection Equation.
- SPH Mathematics: the Basics

$$\mathcal{L}_{sph} = \sum_{i=1}^{N} m_i [\frac{1}{2} v_i^2 - u(\rho_i)]$$

$$A(r) = \int A(r') W(r - r', h) dr' \rightarrow A(r) = \sum_b m_b \frac{A_b}{\rho_b} W(r - r_b, h)$$

The Smoothing Kernel W

$$W = \frac{1}{\pi h^2} e^{-\frac{|r|^2}{2h^2}}; \quad W_{ij} = W(|r_i - r_j|; h)$$

Smoothed Particle Hydrodynamics (SPH) - More Physics

- After some Euler-Lagranging... (Monaghan 1992)

$$\rho(r) = \sum_j m_j W(|r - r_j|; h)$$

$$\frac{dv_i}{dt} = -\sum_j m_j (\frac{P_j}{\rho_j^2} + \frac{P_i}{\rho_i^2}) \nabla_i W_{ij}$$

$$\frac{du_i}{dt} = \frac{1}{2} \sum_j m_j (\frac{P_j}{\rho_j^2} + \frac{P_i}{\rho_i^2}) v_{ij} \cdot \nabla_i W_{ij}$$

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- Other considerations include:

$$\Pi_{ij},\ h \to h(i),\ \text{and damping}$$

Implementation

Implementation

or: How I learned to Stop Doing Everything at Once

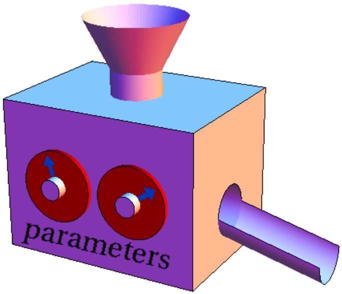
Initial Conditions and Parameters

- Spatial Discretization?
  Grid/Mesh? Boundaries?
- Initial Conditions: x, v, u, ρ
- Equation of State - Polytropic (simple)
- Parameters: k, ν, λ, a

$$P = k\rho^2$$

$$a_i = -nuv_i - \lambda x_i - a_{i,v1}$$

Monaghan and Price (2004)

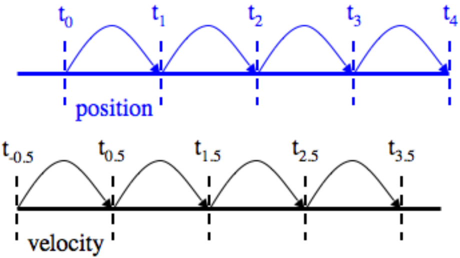


Nykamp DQ, "Function machine parameters." From Math Insight.

Leapfrogging

- Leapfrogging Through Time

$$v_{n+\frac{1}{2}} = v_{n-\frac{1}{2}} + a_n \Delta t$$
$$x_{n+1} = \Delta t v_{n+\frac{1}{2}} + x_n$$



Leapfrogging for position and velocity

Parallel Approach

Parallel Approach feat. CUDA

Parallelization

- Solving the Advection Equation: Forward Differencing $\rightarrow$ Leap-Frog

for $t = 1$ **to** $MaxTime$ **do**
    Update $\vec{x}$;
    Update $\vec{v}$;
    Initialize $\vec{a}$;
    for $i = 1$ **to** N **do**
        for $j = i + 1$ **to** N **do**
            $\vec{a}_i \mathrel{+}= ...\nabla W_{ij}$;
            $\vec{a}_j \mathrel{-}= ...\nabla W_{ij}$;
            $ut_i \mathrel{+}= ... \cdot \nabla W_{ij}$;
            $ut_j \mathrel{+}= ... \cdot \nabla W_{ij}$;
        end
    end
end

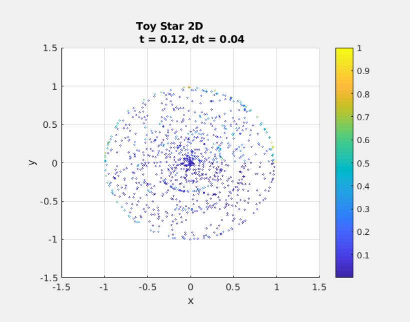
Implementation

```
! PARALLELIZING -------------------------------

  tblock = dim3(32,32,1)
  grid = dim3(ceiling(real(maxn)/tblock%x),ceiling(real(maxn)/tblock%y),1)

  pos_d(:,:) = pos(:,:)

  CALL gweighf<<<grid, tblock>>>(pos_d, hsm, grad_d)

  grad(:,:) = grad_d(:,:)

  do i = 1,maxn

    ratioi = P(i)/(rho(i)*rho(i))

    do j = i+1, maxn

      ratioj = P(j)/(rho(j)*rho(j))

! Monaghan & Price (2004)
! Acceleration due to pressure

      temp = m(j)*(ratioi + ratioj)

      accel(:,i) = accel(:,i) - temp*grad(1,j)
      accel(:,j) = accel(:,j) + temp*grad(2,j)

! Thermal Energy per unit mass/dt.

      temp = temp*abs(dot_product(vel(:,i)-vel(:,j),grad(:,j)))

      ut(i) = ut(i) + temp
      ut(j) = ut(j) + temp
! Assume no thermal conduction between adjacent particles. Just another term that would have to be added.
    enddo

  enddo
```

Results

Normalized x, y, Energy/time:

- Without rotation (of any kind)
- Without differential rotation
- With both kinds of rotation



Conclusions and Extensions

- Energy and (Angular) Momentum Conservation.
- Boundary Conditions and the extent of usefulness of the 'toy' model.
- Better approximation to self-gravity and relativistic approaches.
- Extending to 3D.

Conclusions and Extensions for Parallelization

- Expected Speed-up
- Parameterization of the model
- Extending to 3D in problem space and CUDA

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Further references available by request.