PHYS 265 Lab 3

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I Introduction

The standard model of particle physics is one of the most important modern scientific theories, providing an description of three out of the four fundamental forces of the universe. Each of the three forces described by the standard model is meditated by a characteristic particle; the electromagnetic force for example, is mediated by the photon. The neutral force mediating particle for the weak nuclear force is known as the Z_0 particle and unlike the photon has a nonzero mass. Since the mass of a particle is important to understanding its interactions, the ***A*** ***T***oroidal ***L***HC ***A***pparatu***S*** (ATLAS) detector has been tasked with collecting data to accurately determine its mass

In order to do this, we will take advantage of the fact that the Z_0 particle is unstable and decays into a lepton particle-antiparticle pair, known as the $Z_0 \to \ell\bar{\ell}$ interaction. Since energy and momentum is conserved, if we can accurately measure the energies and momentum of each decay pair, we know what the energy and momentum of the Z_0 particle must be and from there can calculate what its mass should be. Since the energy a Z_0 particle can have before decaying can vary, its equivalent mass can also vary due to the famous $E = mc^2$ relation so to find the mass of the Z_0 particle, we are really looking for a peak in the mass distribution of these Z_0 decay interactions.

II Invariant Mass distribution

Each particle detection by ATLAS contains the following information: the magnitude of the particle's transverse momentum (p_T), the azimuthal angle of the particle relative to the beam (ϕ), and the particle's pseudorapidity (η). These values can then be used to find the momentum of the detected particle using the following equations:

$$p_x = p_T \cos(\phi) \qquad p_y = p_T \sin(\phi) \qquad p_z = p_T \sinh(\eta) \tag{1}$$

To calculate the invariant mass of a particle, we can use the following equation:

$$M = \sqrt{E^2 - (p_x^2 + p_y^2 + p_z^2)} \tag{2}$$

Since we are trying to calculate the invariant mass of the Z_0 particle which is the parent particle of each pair of particle detections, we have to sum both the momenta and energy of each pair of detections. These summed values are then used in the calculation of the invariant mass. The reason we can reasonably add a quantity with units of energy and a quantity with units of momentum is because we are assuming we are using a unit system where the speed of light, $c = 1$.

With the invariant mass for each Z_0 decay calculated, the invariant mass was then plotted in a histogram with 41 bins between 87 GeV and 93 GeV. We then assumed that these counts follow Poisson statistics so we can calculate the uncertainty in the number of counts as being the square root of the recorded number of counts. The true invariant mass of the Z_0 particle will be located at the peak of this histogram. To find the location of this peak, we then fit this histogram to the Breit-Wigner distribution, described by the following equation where m is the measured invariant mass, m_0 is the true rest mass, and Γ is a parameter related to the width of the peak.

$$D(m, m_0, \Gamma) = \frac{1}{\pi} \frac{(\Gamma/2)}{(m - m_0)^2 + (\Gamma/2)^2} \tag{3}$$

One property of this distribution is that $\int_{-\infty}^{\infty} D(m, m_0, \Gamma) dm = 1$ which means that this distribution has to be scaled by a certain quantity to match the expected peak value of the binned data. The normalization value used in this investigation was 2500, which is equal to half the number of detected events.

The function fitting was performed using the `curve_fit` function in the `scipy.optimize` package which uses the Levenberg-Marquardt algorithm as implemented in the fortran library `MINPACK`. The x-coordinates of the data was shifted to the right by half a bin so that the data points were located in the centers of the bins instead of on the left edges. To ensure the uncertainties in the fitting function were used correctly without any additional scaling, we made sure to set the `absolute_sigma` argument equal to true. In addition to returning the parameters of best fit, `curve_fit` also returns a covariance matrix which can be used to find the uncertainty in the fitting parameters by taking the square root of the diagonal.

To determine the fit parameters, only the data points between 87 GeV and 93 GeV were used, as these were the bins with the highest signal-to-noise ratio (SNR) due to being around the peak of the distribution. These bins contain approximately half (47.6%) of the particle detections in the data set.

The calculated fit parameters for this central data range was $m_0 = 90.3 \pm 0.1$ GeV and $\Gamma = 6.4 \pm 0.2$ GeV. Figure 1 reports the units of these values as being GeV/c^2 as Einstein's mass-energy equivalence equation should demand the c^2 be there, but since we are working in units where c is normalized to be 1, we can drop that unit for brevity. We also know that Γ and m_0 must have the same units as they are added in the distribution equation.

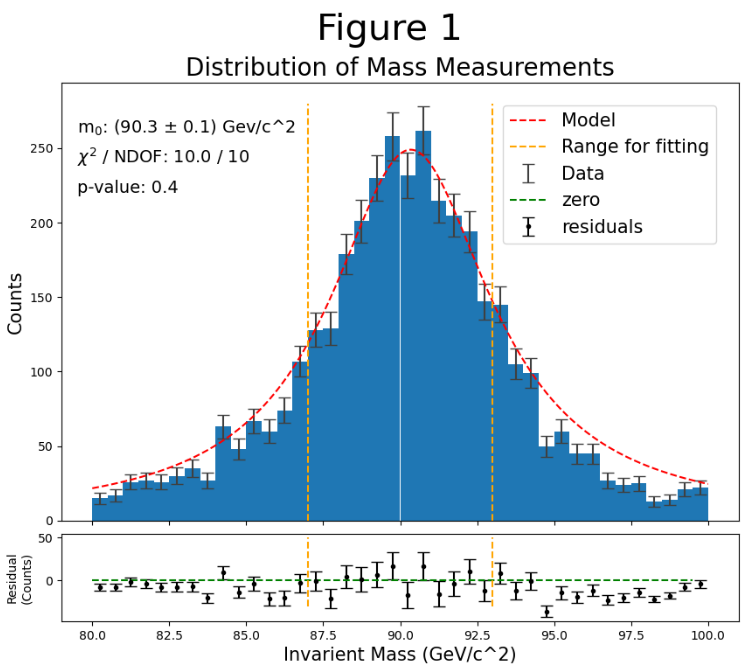

Figure 1: Histogram of decay counts with model fit to 12 highest SNR data bins

As can be seen in Figure 1, a model distribution found by fitting the this central data range qualitatively appears to follow the shape of the rest of the curve where the SNR is lower. The difference between the data and the model is known as the residual and is plotted below the main histogram plot. Perfect agreement between the data and the model would result in all the residuals being zero and falling on the dashed green line, but we don't expect the data points to perfectly agree with the model. On average, we instead expect each data point to fall one standard deviation from the perfect agreement point. If we divide each residual by its uncertainty and then square the result, we are left with a value known as χ^2. Summing the χ^2 values for the fitting region gives the χ^2 of the fit. Since a model with n fitting parameters can perfectly pass through n data points, we subtract the number of fitting parameters from the number of fitted data points to find the expectation value of χ^2, also known as the degrees of freedom. It is particularly useful in analysis to know the p-value of a fit, or the probability that you would get a χ^2 greater than or equal to a given χ^2 with a certain number of degrees of freedom.

For this function, there are two fitting parameters, m_0 and Γ and 12 data bins used in fitting, so the number of degrees of freedom for this fit is 10. The χ^2 evaluated for this fit was 10.0, which corresponds to a p-value of 0.4. A fit perfectly matching the expectation values would have a p-value of 0.5 as half the data points would be above the expectation value and half would be below. Since our p-value of 0.4 is less than the expectation value of 0.5, our data doesn't agree perfectly with our expectations, but there is still a 40%

chance this χ^2 value is produced purely by chance showing that there is no significant tension between our fit and our data. In order for there to be tension at the 2σ level of significance, our p-value would have to be less than 0.05.

III Parameter Scan

The uncertainties provided by the `curve_fit` function are helpful for getting an initial estimate of the confidence of a fit, but it is often beneficial to visualize how the two parameters relate to each other as fitting parameters are not necessarily orthogonal. To make this visualization, a χ^2 value was calculated for the fitting region for a wide range of parameter values ($89 \leq m_0 \leq 91$, $5 \leq \Gamma \leq 8$) and shifted down by the minimum χ^2 value in the parameter space. These $\Delta\chi^2$ values were plotted as a heat map in the parameter space. Solid and dashed contours were placed in the parameter space at the 1σ and 3σ levels respectively.

Since our fitting function has two free parameters, the $\Delta\chi^2$ values corresponding to 1σ and 3σ is 2.30 and 9.21 respectively. A red marker was also placed at the location of the parameters the optimizer found. As expected, this location is found at the minimum of the $\Delta\chi^2$ map.

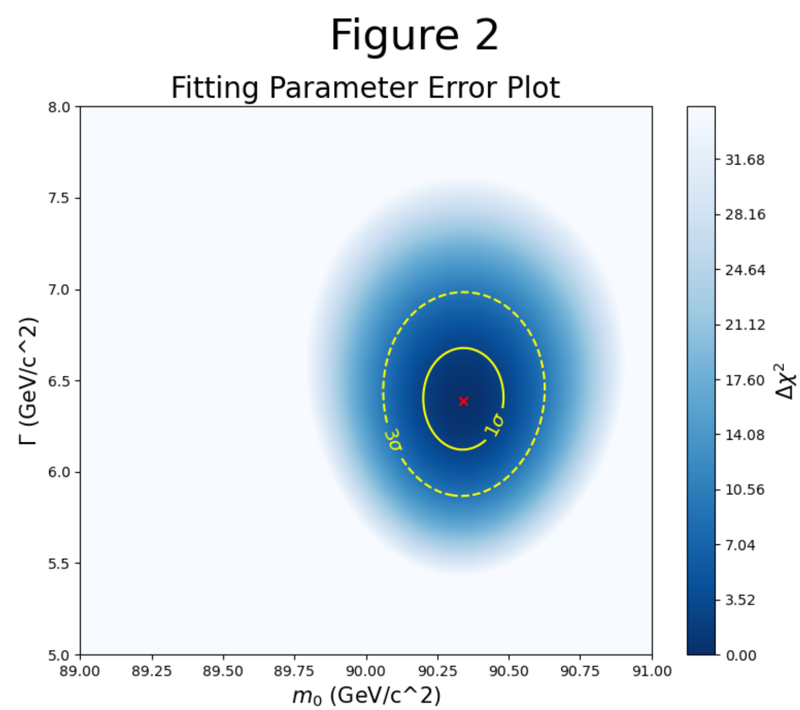

Figure 2: $\Delta\chi^2$ plot showing the contours of fitting parameter uncertainty. $\Delta\chi^2$ clipped to 32 for nicer viewing.

IV Discussion and Future work

In this lab, the rest mass of the Z_0 boson was determined to be 90.3 ± 0.1 GeV which is notably different from the accepted value of 91.1876 ± 0.0021 GeV as found by R.L. Workman et al. in the Particle Data Group (PDG). Since the difference in the two values is approximately 9 times larger than the uncertainty in the difference, there is a significant disagreement between the two values. Our investigation assumed that there were no systematic errors contained within the ATLAS data which could very well be far from the truth, leading a calculated value over 5σ away from the accepted value. Additionally, we assumed that 41 bins was the optimal number for this detector, but without any information on the energy resolution of the detector, we can not be sure that estimation is correct.

Future steps that can be taken to improve the accuracy of investigations using this detector include collecting data for far more pairs of Z_0 particle decays, as that would continue to increase the SNR of our dataset, utilizing the energy resolution of the detector to ensure we are using the optimal bin size, and using calibration data for the detector to ensure our starting data is accurate.