Week 7

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February 21, 2020

Today

- First, how was the exam?
- Today we will talk about worker turnover (i.e. you can get fired)
- We will also talk about on the job search

Worker Turnover

Last time we assumed that the employment state was absorbing. Now, let's generalize the model slightly to allow employed workers to be exogenously separated from their job at a rate s. The expression for the flow value of unemployment remains unchanged:

$$rU = b + \alpha \int_{\underline{w}}^{\overline{w}} \max\left\{V(w) - U, 0\right\} dF(w).$$

Let's derive the flow value of employment (at a wage w). A good habit to have is to start with discrete time and take a limit as $dt \to 0$.

$$V(w) = wdt + \frac{1}{1 + rdt}\Big[(1 - sdt)V(w) + sdtU\Big]$$

$$(1 + rdt)V(w) = (1 + rdt)wdt + (1 - sdt)V(w) + sdtU$$

$$rdtV(w) = (1 + rdt)wdt + sdt\Big[U - V(w)\Big]$$

$$V(w) = \frac{(1 + rdt)w}{r} + \frac{s}{r}\Big[U - V(w)\Big]$$

Take the limit as $dt \to 0$.

$$V(w) = \frac{w}{r} + \frac{s}{r}\Big[U - V(w)\Big]$$

$$rV(w) = w + s\Big[U - V(w)\Big] \qquad \Longrightarrow \qquad V(w) = \frac{w + sU}{r + s}$$

First let's look at, and rewrite, rU.

$$
\begin{aligned}
rU &= b + \alpha \int_{\underline{w}}^{\overline{w}} \max\left\{V(w) - U, 0\right\} dF(w) \\
&= b + \alpha \int_{\underline{w}}^{\overline{w}} \max\left\{ \underbrace{\frac{w + sU}{r + s}}_{V(w)} - \underbrace{\frac{w_R + sU}{r + s}}_{U}, 0 \right\} dF(w) \\
&= b + \frac{\alpha}{r + s} \int_{w_R}^{\overline{w}} (w - w_R) dF(w) \\
&= b + \frac{\alpha}{r + s} \int_{w_R}^{\overline{w}} [1 - F(w)] dw \qquad \text{(int. by parts)}
\end{aligned}
$$

Now we have all we need to determine the reservation wage. Recall this is defined by the wage that solves $rV(w_R) = rU$.

$$r\frac{w_R + sU}{r+s} = rU \quad \Longrightarrow \quad w_R + sU = rU + sU \quad \Longrightarrow \quad w_R = rU$$

Thus

$$w_R = b + \frac{\alpha}{r+s}\int_{w_R}^{\overline{w}} [1 - F(w)]dw.$$

From here we can do the same comparative static exercises we talked about last time.

Equilibrium Unemployment

- If employment is absorbing, all agents will eventually become employed
  - no real notion of an equilibrium unemployment / employment rate
- Now that we have added features allowing workers to re-enter unemployment, we can begin discussing equilibrium unemployment
- Here, let's think about equating the flows of workers into and out of unemployment
- Remembering that we typically normalize the mass of agents to be equal to 1, we have

$$\underbrace{\alpha[1 - F(w_R)]u}_{\text{mass leaving UE}} = \underbrace{s(1-u)}_{\text{mass entering UE}} \quad \implies \quad u = \frac{s}{\alpha[1 - F(w_R)] + s}$$

On-the-Job Search

- Something that is in the data, that so far we cannot account for, are job-to-job transitions
- Many individuals will change jobs without ever entering into unemployment
- Here, we will allow *already employed* workers to continue searching for jobs
- Suppose that unemployed workers receive job offers at a rate α_0, while employed workers receive job offers at a rate α_1

As before, the flow value of unemployment will look similar.

$$rU = b + \alpha_0 \int_{w_R}^{\overline{w}} [V(w) - U] dF(w)$$

Now for the flow value of employment.

$$\begin{aligned} V(w) = wdt + \frac{1}{1 + rdt} \Bigg[& \alpha_1 dt(1 - sdt) \int_{\underline{w}}^{\overline{w}} \max\left\{ V(\widetilde{w}), V(w) \right\} dF(\widetilde{w}) \\ & + (1 - \alpha_1 dt)(1 - sdt) V(w) \\ & + \alpha_1 sdt^2 \int_{\underline{w}}^{\overline{w}} \max\left\{ V(\widetilde{w}), U \right\} dF(\widetilde{w}) \\ & + (1 - \alpha_1 dt) sdtU \Bigg] \end{aligned}$$

Now let's simplify. Pay careful attention to combining like-terms and cancellations.

$$
\begin{aligned}
(1 + rdt)V(w) = &(1 + rdt)wdt \\
&+ (\alpha_1 dt - \alpha_1 sdt^2) \int_{\underline{w}}^{\overline{w}} \max\{V(\widetilde{w}), V(w)\}\, dF(\widetilde{w}) \\
&+ (1 - sdt - \alpha_1 dt + \alpha_1 sdt^2) V(w) \\
&+ \alpha_1 sdt^2 \int_{\underline{w}}^{\overline{w}} \max\{V(\widetilde{w}), U\}\, dF(\widetilde{w}) \\
&+ (sdt - \alpha_1 sdt^2) U
\end{aligned}
$$

$$
\begin{aligned}
rdtV(w) =& (1 + rdt)wdt \\
&+ (\alpha_1 dt - \alpha_1 sdt^2) \int_{\underline{w}}^{\overline{w}} \max\left\{V(\widetilde{w}) - V(w), 0\right\} dF(\widetilde{w}) \\
&+ sdt\Big[U - V(w)\Big] \\
&+ \alpha_1 sdt^2 \int_{\underline{w}}^{\overline{w}} \max\left\{V(\widetilde{w}) - U, 0\right\} dF(\widetilde{w})
\end{aligned}
$$

Now divide both sides by rdt.

$$
\begin{aligned}
V(w) = \frac{(1 + rdt)w}{r} &+ \left(\frac{\alpha_1 - \alpha_1 sdt}{r}\right) \int_{\underline{w}}^{\overline{w}} \max\left\{V(\widetilde{w}) - V(w), 0\right\} dF(\widetilde{w}) \\
&+ \frac{s}{r}\Big[U - V(w)\Big] \\
&+ \frac{\alpha_1 sdt}{r} \int_{\underline{w}}^{\overline{w}} \max\left\{V(\widetilde{w}) - U, 0\right\} dF(\widetilde{w})
\end{aligned}
$$

Take the limit as $dt \to 0$.

$$rV(w) = w + \alpha_1 \int_{\underline{w}}^{\overline{w}} \max\left\{V(\widetilde{w}) - V(w), 0\right\} dF(\widetilde{w}) + s\Big[U - V(w)\Big]$$

- That is, the flow value of employment is given by the instantaneous payoff w plus the expected increase in payoff of a higher wage opportunity plus the expected decrease in payoff associated with losing one's job

- Now, what about the reservation wage?
  - for already employed workers, they will accept a job if $\widetilde{w} > w$ (so the "reservation wage" for an employed worker is her current wage)
  - for unemployed workers, we want to look for the w_R such that $rV(w_R) = rU$

Reservation Wage

Here, unlike with the very basic model, the reservation wage derivation is *slightly* more involved. I recommend following these steps.

1. find $rV(w_R)$
2. construct $rV(w_R) = rU$ and then solve for w_R
3. determine $V'(w)$ and then plug in

1. find $rV(w_R)$

$$rV(w_R) = w_R + \alpha_1 \int_{\underline{w}}^{\overline{w}} \max\{V(\widetilde{w}) - V(w_R), 0\}\, dF(\widetilde{w}) + s\Big[\underbrace{U - V(w_R)}_{=0}\Big]$$

$$= w_R + \alpha_1 \int_{w_R}^{\overline{w}} [V(\widetilde{w}) - V(w_R)] dF(\widetilde{w})$$

Now, we can apply integration by parts (similar to how we've done it before), as follows.

$$\int_{w_R}^{\overline{w}} [V(\widetilde{w}) - V(w_R)] dF(\widetilde{w}) \qquad \Longrightarrow \qquad \begin{array}{ll} u = V(\widetilde{w}) - V(w_R) & v = F(\widetilde{w}) \\ du = V'(\widetilde{w}) d\widetilde{w} & dv = dF(\widetilde{w}) \end{array}$$

$$
\begin{aligned}
\int_{w_R}^{\overline{w}} [V(\widetilde{w}) - V(w_R)] dF(\widetilde{w}) &= [V(\widetilde{w}) - V(w_R)] F(\widetilde{w}) \Big|_{w_R}^{\overline{w}} - \int_{w_R}^{\overline{w}} V'(\widetilde{w}) F(\widetilde{w}) d\widetilde{w} \\
&= V(\overline{w}) - V(w_R) - \int_{w_R}^{\overline{w}} V'(\widetilde{w}) F(\widetilde{w}) d\widetilde{w} \\
&= \int_{w_R}^{\overline{w}} V'(\widetilde{w}) d\widetilde{w} - \int_{w_R}^{\overline{w}} V'(\widetilde{w}) F(\widetilde{w}) d\widetilde{w} \\
&= \int_{w_R}^{\overline{w}} V'(\widetilde{w}) [1 - F(\widetilde{w})] d\widetilde{w}
\end{aligned}
$$

We can plug this into the expression for $rV(w_R)$ that we had before...

$$rV(w_R) = w_R + \alpha_1 \int_{w_R}^{\overline{w}} V'(\widetilde{w})[1 - F(\widetilde{w})]d\widetilde{w}$$

2. construct $rV(w_R) = rU$ and then solve for w_R

Before beginning, notice that we can transform the integral in the expression for rU just like we did above noting that $V(w_R) = U$.

$$
\begin{aligned}
w_R + \alpha_1 \int_{w_R}^{\overline{w}} V'(\widetilde{w})[1 - F(\widetilde{w})]d\widetilde{w} &= b + \alpha_0 \int_{w_R}^{\overline{w}} [V(\widetilde{w}) - U]dF(\widetilde{w}) \\
&= b + \alpha_0 \int_{w_R}^{\overline{w}} V'(\widetilde{w})[1 - F(\widetilde{w})]d\widetilde{w}
\end{aligned}
$$

$$
w_R = b + (\alpha_0 - \alpha_1) \int_{w_R}^{\overline{w}} V'(\widetilde{w})[1 - F(\widetilde{w})]d\widetilde{w}
$$

3. determine $V'(w)$ and then plug in

Recall: $$rV(w) = w + \alpha_1 \int_{w}^{\overline{w}} [V(\widetilde{w}) - V(w)] dF(\widetilde{w}) + s\Big[U - V(w)\Big]$$

Define: $$\varphi(w) \equiv \int_{w}^{\overline{w}} [V(\widetilde{w}) - V(w)] dF(\widetilde{w}) \qquad \text{(the surplus function)}$$

$$rV'(w) = 1 + \alpha_1 \varphi'(w) - sV'(w)$$

Now let's work on the surplus function...

$$\varphi(w) = \int_{w}^{\overline{w}} [V(\widetilde{w}) - V(w)] dF(\widetilde{w})$$

$$= \int_{w}^{\overline{w}} V'(\widetilde{w})[1 - F(\widetilde{w})] d\widetilde{w} \qquad \text{(int. by parts)}$$

$$\varphi'(w) = -V'(w)[1 - F(w)]$$

Plugging this result into the prior expression, we can determine that

$$rV'(w) = 1 - \alpha_1 V'(w)[1 - F(w)] - sV'(w)$$

$$V'(w) = \frac{1}{r + s + \alpha_1[1 - F(w)]}$$

Plug this in and we've found the reservation wage.

$$w_R = b + (\alpha_0 - \alpha_1) \int_{w_R}^{\overline{w}} \frac{[1 - F(w)]}{r + s + \alpha_1[1 - F(w)]} dw$$

As we did with the first model today, we can very easily determine the equilibrium unemployment rate.

$$\underbrace{\alpha_0[1 - F(w_R)]u}_{\text{mass leaving UE}} = \underbrace{s(1 - u)}_{\text{mass entering UE}} \implies u = \frac{s}{\alpha_0[1 - F(w_R)] + s}$$