Week 9

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Today

- Talk about Euler Equations
- Talk about midterm
  - We'll go over problem 1 from the midterm
  - Talk about general strategies for taking the final
- When do you want your final to be?

Setup

- Consider the following problem:

$$\max_{\{c_t k_{t+1}\}_{t=0}^{T}} \sum_{t=0}^{T} \beta^t u(c_t)$$

$$\text{s.t. } c_t + k_{t+1} \leq f(k_t), \forall t$$

$$c_t,\ k_{t+1} \geq 0$$

$$k_0 \text{ given}$$

- $u()$ increasing $\implies$ resource constraint binding: $c_t + k_{t+1} = f(k_t)$
- Inada Conditions $\implies$ $c_t,\ k_{t+1} > 0$

Euler Equation

$$\underbrace{u'(f(k_t) - k_{t+1})}_{\substack{\text{Cost in utility of}\\\text{saving additional}\\\text{unit of capital}\\\text{for } t+1}} = \underbrace{\beta u'(f(k_{t+1}) - k_{t+2})}_{\substack{\text{Discounted additional}\\\text{utility from one more}\\\text{unit of consumption}}} \quad \underbrace{f'(k_{t+1})}_{\substack{\text{Additional production}\\\text{possible with one more}\\\text{unit of capital in } t+1}} \qquad \text{for } t = 0, \cdots, T-1$$

- **Idea:** Social planner deciding whether to save one more unit of capital for tomorrow
- Saving one more unit reduces consumption by one unit at utility cost of $u'(f(k_t - k_{t+1})$
- One more unit of capital tomorrow allows additional production of $f(k_{t+1})$
- Each additional unit of production (when used for consumption) is worth $u'(f(k_{t+1}) - k_{t+2})$ in utility tomorrow, and thus $\beta u'(f(k_{t+1}) - k_{t+2})$
- Note: T equations with $T + 1$ unknowns ($\{k_{t+1}\}_{t=0}^{T}$)

Now let's look at the midterm