Matthew Fitzgerald
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# Problem Set # 3

## Problem 1: Be careful what you study

You just graduated from UCSB and got a job as a two year postdoc in environmental economics at the University of Antarctica (that's what you get for doing your job market paper on emperor penguins). The University pays your room and board, and in addition you receive your entire two year salary of $x_0$ on your first day [1]. The university is located in a remote town called Bellingshausentown. There is only one bank, Bank of the South Pole, which offers an interest rate of $r$ each month on the savings you deposit.

One day you're having lunch with the other postdocs and they start discussing their compensation packages. It turns out they all have the same payment scheme as you, except each postdoc received a different salary and has been hired by the university for a different number of months. Since you are the only economist in the group, they ask you to tell them the optimal amount to save each month given their compensation packages. Assume each individual discounts the future at a rate of $\beta < 1$ and that none of the postdocs plan on having any savings when they move on to their next job (hopefully somewhere warmer).

**(a)** Write down the postdoc's saving problem recursively (hint: Let the contract for a given postdoc last $T$ months, and note that you do not receive any interest on your savings in the first period).

**(b)** Define period $T+1$ as the month after the postdoc's contract is over. Solve for $V_{T+1}$ (hint: at what points can the maximum occur for a linear function?).

**(c)** Using your answer from the previous part, write out the postdoc's problem in period $T$. Solve for $V_T$.

**(d)** Does it ever make sense to consume part of the current period income? Explain the intuition (note that since the University is covering your room and board, zero consumption is fine here if it is optimal).

**(e)** Under what conditions are you indifferent between saving and consuming? Carefully explain your answer.

**(f)** Assume $\beta = 1$. How does this change your consumption behavior? Explain the intuition.

## Problem 2: The Fountain of Youth

Tired of the cold, you decide to go to the Bahamas for spring break. After flying in to the South Bimini airport, you order a Lyft to take you to your hotel. During the ride you start up a conversation with your driver Ponce, a Spanish expat who's lived on the island for many years. Ponce tells you about a beautiful remote area on the island and after dropping your things at the hotel, you decide to check it out. On your way you run out of water

[1] Not to add insult to injury, but had you listened to Chris Costello in Bren and worked on fisheries management for your job market paper, the University of the Bahamas also had an open postdoc position this year focusing on the study of optimal management of the Caribbean Spiny Lobster fishery which offered a larger compensation plan. Perhaps now you regret telling Chris during your meeting that you were "more of a bird person".

and notice a small spring in your path. You take a sip of water from the spring and instantly feel rejuvenated. It turns out the spring is actually the Fountain of Youth and now you will live forever[2]! That night after dinner you buy a cake to celebrate, but the economist in you instantly realizes your mistake. You now have to decide how to optimally eat the cake over an infinite lifespan.

**(a)** Write down your maximization problem recursively, taking into account your recently acquired immortality.

**(b)** Derive the Euler equation.

**(c)** Assume you have log utility. Then guess that the value function takes the following form:

$$V(k) = A + Bln(K)$$

Verify that this guess is consistent with optimization, and solve for $A$ and $B$.

**(d)** Now guess that the policy function takes the following form:

$$k' = \theta k$$

Verify that this guess is consistent with optimization, and solve for $\theta$.

**Problem 3: Fill in the blank value function iteration**

Open the file "ValueFunctionIterationIncomplete.jl".

**(a)** This file has a simple version of value function iteration, but it is incomplete. Fill in the code where necessary and make sure it runs properly.

**(b)** Try the function "val_fun_iter" using the production function you created in the script, 500 grid points, an $\alpha$ of 0.3, and a $\delta$ of 0.5.

**(c)** Write additional code in the script that plots the value function and the policy function against $k$ in separate graphs (Turn this script in, and rename it "ValueFunctionIterationYOURNAME.jl").

**(d)** Now try increasing the number of grid points to 2000. Why is *each iteration* slower when the number of grid points increases?

[2]The interested reader can find a history of the legend of the Fountain of Youth here: https://www.history.com/news/the-myth-of-ponce-de-leon-and-the-fountain-of-youth