Professor: Peter Rupert
TA: Matthew Fitzgerald
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Midterm Exam
Winter 2020

Problem 1

Consider a representative agent model in which agents choose consumption c_t and labor l_t each period to maximize expected lifetime utility.

$$\mathbb{E}_0\Big[\sum_{t=0}^{\infty}\beta^t u(c_t, l_t)\Big]$$

Output y_t is produced according to the following production function:

$$y_t = z_t k_t^{\alpha} l_t^{1-\alpha}$$

where z_t is an AR(1) technology shock and k_t is capital in period t. Assume the depreciation rate of capital is 100%.

(a) Write down the Bellman Equation for the social planner's problem.

(b) Find the Euler equation for consumption.

(c) Assume the representative agent has the following utility function:

$$u(c,h) = ln(c) - \frac{1}{2}l^2$$

Guess that the policy function for capital takes the form $k' = (1-\gamma)y$, and solve for γ. Write out the policy functions for l and c.

(d) Assume the representative agent has the following utility function:

$$u(c,h) = ln\Big(c - \frac{1}{2}l^2\Big)$$

Guess that the policy function for capital takes the form $k' = (1-\eta)y$, and solve for η. Write out the policy functions for l and c.

(e) Looking back at your answers to (c) and (d), which of the utility functions resulted in a constant labor supply? Why does this occur?

Problem 2

Consider an infinite horizon cake eating problem where the cake eating agent is given a cake of size k_0 and she has to determine how to optimally consume it over her infinite lifespan. Assume the agent has instantaneous utility $u(c)$ and has discount factor $0 < \beta < 1$.

Part I: Consider the case when the individual has multiplicative taste shocks. In other words, the individual's instantaneous utility becomes $zu(c)$ where z is stochastic. Further, assume the shocks take on one of two values, z_{L} or z_{H} (where $z_{\mathrm{L}} < z_{\mathrm{H}}$), and that the shocks are a first order Markov process with the following transition matrix:

$$\pi = \begin{bmatrix} \pi_{LL} & \pi_{LH} \\ \pi_{HL} & \pi_{HH} \end{bmatrix}$$

(Recall: $\pi_{ij} = p(z' = z_j | z = z_i)$ for $i \in \{L, H\}$, $j \in \{L, H\}$)

(a) Write down the cake eater's Bellman equation. What is (are) the choice variable(s) and what is (are) the state variable(s)?

(b) Derive the Euler equation when the cake eater receives shock z_i where $i \in \{L, H\}$. Next write out the Euler equation when the cake eater receives a shock of z_H using the appropriate transition probabilities from the transition matrix given above. Finally, write out the Euler equation when the cake eater receives a shock of z_L using the appropriate transition probabilities from the transition matrix given above.

(c) Imagine that the cake eater received a low shock (z_L) this period. How does her optimal cake eating behavior differ when her current shock is z_L and π_{LL} is near 1 versus when her current shock is z_L and π_{LL} is near 0? In other words, given that she gets a shock of z_L today, compare the cake eater's optimal consumption policy when π_{LL} is near 1 and when π_{LL} is near 0. Explain the intuition.

Part II: Consider the same setup as in Part I, only now the cake eater doesn't know the current period shock when she makes her consumption decision.

(a) Write down the cake eater's Bellman equation. What is (are) the choice variable(s) and what is (are) the state variable(s)?

(b) Derive the Euler equation.

(c) Will the optimal policy depend on this period's shock? Briefly explain why or why not.

Part III: Consider the same setup as in Part I (the cake eater once again sees the current period shock before making her consumption decision), only now the cake eater cannot eat a fraction of the cake but instead each period must decide to either consume the entire cake or save the entire cake. Assume that if the cake is not consumed in the current period, then the size of the cake is θk in the next period. If $\theta < 1$, then the cake depreciates each period it is not eaten (this would be the case if a portion of the cake goes bad each period or if you have a sibling that eats a fraction of the cake each night while you are asleep. If $\theta > 1$, then the cake grows each period that it is kept. Finally, if $\theta = 1$, then the cake does not change in size from one period to the next.

(a) What is V_s, the value for the cake eater if she saves the cake? What is V_e, the value for the cake eater if she consumers the cake? Use V_s and V_e to write the value function for the cake eater. What is (are) the choice variable(s) and what is (are) the state variable(s)?

(b) Write the value function for the cake eater when the current period shock is z_H, using the appropriate transition probabilities from the transition matrix given above. Write the value function for the cake eater when the current period shock is z_L, using the appropriate transition probabilities from the transition matrix given above.

(c) Assume $\theta = 1$, under what conditions does the cake eater consume the entire cake this period? Briefly explain the intuition.

(d) If the current shock is z_L, how large does π_{LH} need to be in order to make the cake eater wait to consume the cake (i.e. to not consume the cake in this period)?

Part IV: Consider the same setup as in Part 1 except now the cake eater does not have any preference shocks and her instantaneous utility is once again $u(c)$. Instead, the cake size receives an additive shock each period. The cake magically grows each period by z_t **where $\boldsymbol{z_t}$ is i.i.d.** and can take two values z_H and z_L (where $z_H > z_L$). Therefore, in each period t, you observe the additive shock to the cake size z_t and then make your consumption decision.

(a) Write down the cake eater's Bellman equation. What is (are) the state variable(s) and what is (are) the control variable(s)?