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Econ 204B
February 20, 2020

Midterm Exam
Winter 2020

Problem 1

Consider a representative agent model in which agents choose consumption c_t and labor l_t each period to maximize expected lifetime utility.

$$\mathbb{E}_0\Big[\sum_{t=0}^{\infty}\beta^t u(c_t, l_t)\Big]$$

Output y_t is produced according to the following production function:

$$y_t = z_t k_t^{\alpha} l_t^{1-\alpha}$$

where z_t is an AR(1) technology shock and k_t is capital in period t. Assume the depreciation rate of capital is 100%.

(a) [10] Write down the Bellman Equation for the social planner's problem.

$$\begin{aligned} V(k,z) = \max_{c,l,k'} & \Big\{u(c,l) + \beta\mathbb{E}[V(k',z')|z]\Big\} \\ \text{s.t. } & c + k' \leq zk^{\alpha}l^{1-\alpha} \\ & c, l, k' \geq 0 \end{aligned}$$

(b) [10] Find the Euler equation for consumption.

Assuming an interior solution (i.e. a utility function that will satisfy the Inada conditions), we have

$$\mathcal{L} = u(c,l) + \beta\mathbb{E}[V(k',z')|z] + \lambda(zk^{\alpha}l^{1-\alpha} - c - k')$$

Then the FOC's are:

$$\frac{\partial \mathcal{L}}{\partial c} = 0 \implies u_c(c,l) - \lambda = 0 \tag{$\spadesuit$}$$
$$\frac{\partial \mathcal{L}}{\partial l} = 0 \implies u_l(c,l) + (1-\alpha)\lambda(zk^{\alpha}l^{-\alpha}) = 0 \tag{$\blacklozenge$}$$
$$\frac{\partial \mathcal{L}}{\partial k'} = 0 \implies \beta\mathbb{E}[V_{k'}(k',z')|z] - \lambda = 0 \tag{$\clubsuit$}$$

Then combining ($\spadesuit$) and ($\blacklozenge$), along with ($\spadesuit$) with ($\clubsuit$), we have

$$u_l(c,l) + (1-\alpha)u_c(c,l)(zk^{\alpha}l^{-\alpha}) = 0$$
$$u_c(c,l) = \beta\mathbb{E}[V_{k'}(k',z')|z] \tag{$\heartsuit$}$$

Now we need to determine $V_{k'}(k',z')$. Using the Benveniste-Scheinkman Theorem we have:

$$V_k(k,z) = \lambda\alpha z k^{\alpha-1}l^{1-\alpha} \tag{$\boxtimes$}$$
$$V_{k'}(k',z') = \lambda'\alpha z' k'^{\alpha-1}l'^{1-\alpha} \tag{Pushing forward}$$
$$V_{k'}(k',z') = u_{c'}(c',l')\alpha z' k'^{\alpha-1}l'^{1-\alpha} \tag{Plugging in $\spadesuit$}$$

Plugging this in to ($\heartsuit$) we have our Euler equation for consumption

$$u_c(c,l) = \beta\mathbb{E}[u_{c'}(c',l')\alpha z' k'^{\alpha-1}l'^{1-\alpha}|z]$$

(c) [10] Assume the representative agent has the following utility function:

$$u(c,l) = ln(c) - \frac{1}{2}l^2$$

Guess that the policy function for capital takes the form $k' = (1-\gamma)y$, and solve for γ. Write out the policy functions for l and c.

Note that since $k' = (1-\gamma)y$, we know $c = \gamma y$ from the budget constraint. Let's first rewrite our first order conditions from part (b):

$$u_l(c,l) + (1-\alpha)u_c(c,l)(zk^{\alpha}l^{-\alpha}) = 0 \tag{1}$$
$$u_c(c,l) = \beta\mathbb{E}[u_{c'}(c',l')\alpha z' k'^{\alpha-1}l'^{1-\alpha}|z] \tag{2}$$

Now we can plug in the derivatives of the utility function. Let's start with (2):

$$
\begin{aligned}
\frac{1}{c} &= \beta\mathbb{E}[\frac{1}{c'}\alpha z' k'^{\alpha-1} l'^{1-\alpha}|z] \\
\frac{1}{c} &= \alpha\beta\mathbb{E}[\frac{1}{c'} z' k'^{\alpha-1} l'^{1-\alpha}|z] \\
\frac{1}{\gamma y} &= \alpha\beta\mathbb{E}[\frac{1}{\gamma y'}\frac{y'}{k'}|z] && \text{(Using } c = \gamma y\text{)} \\
\frac{1}{\gamma y} &= \alpha\beta\mathbb{E}[\frac{1}{\gamma k'}|z] \\
\frac{1}{\gamma y} &= \alpha\beta\mathbb{E}[\frac{1}{\gamma(1-\gamma)y}|z] && \text{(Using } k' = (1-\gamma)y\text{)} \\
\frac{1}{\gamma y} &= \alpha\beta\frac{1}{\gamma(1-\gamma)y} && (*) \\
\gamma &= 1 - \alpha\beta && (♫)
\end{aligned}
$$

Note that in $(*)$ we were able to drop the expectation because γ is a constant and all values in y have already been realized. Now let's look at (2):

$$
\begin{aligned}
-l + (1-\alpha)\frac{1}{c}(zk^{\alpha}l^{-\alpha}) &= 0 \\
l &= (1-\alpha)\frac{1}{c}(zk^{\alpha}l^{-\alpha}) \\
l &= (1-\alpha)\frac{1}{c}(\frac{y}{l}) \\
l &= (1-\alpha)\frac{1}{\gamma y}(\frac{y}{l}) && \text{(Using } c = \gamma y\text{)} \\
l^2 &= (1-\alpha)\frac{1}{\gamma} \\
l^2 &= (1-\alpha)\frac{1}{1-\alpha\beta} && \text{(Plugging in ♫)} \\
l &= \sqrt{\frac{1-\alpha}{1-\alpha\beta}}
\end{aligned}
$$

Thus we have

$$
\begin{aligned}
l &= \sqrt{\frac{1-\alpha}{1-\alpha\beta}} \\
k' &= \alpha\beta\Big(\frac{1-\alpha}{1-\alpha\beta}\Big)^{\frac{1-\alpha}{2}} zk^{\alpha} \\
c &= (1-\alpha\beta)\Big(\frac{1-\alpha}{1-\alpha\beta}\Big)^{\frac{1-\alpha}{2}} zk^{\alpha}
\end{aligned}
$$

(d) [10] Assume the representative agent has the following utility function:

$$u(c,l) = ln\Big(c - \frac{1}{2}l^2\Big)$$

Guess that the policy function for capital takes the form $k' = (1-\eta)y$, and solve for η. Write out the policy functions for l and c.

Once again let's rewrite our first order conditions from part (b):

$$u_l(c,l) + (1-\alpha)u_c(c,l)(zk^{\alpha}l^{-\alpha}) = 0 \tag{1}$$
$$u_c(c,l) = \beta\mathbb{E}[u_{c'}(c',l')\alpha z' k'^{\alpha-1}l'^{1-\alpha}|z] \tag{2}$$

Now we can plug in the derivatives of the utility function. Again let's start with (2):

$$\begin{aligned}
\frac{1}{c-\frac{1}{2}l^2} &= \beta\mathbb{E}[\frac{1}{c'-\frac{1}{2}l'^2}\alpha z' k'^{\alpha-1}l'^{1-\alpha}|z] \\
\frac{1}{c-\frac{1}{2}l^2} &= \alpha\beta\mathbb{E}[\frac{1}{c'-\frac{1}{2}l'^2} z' k'^{\alpha-1}l'^{1-\alpha}|z] \\
\frac{1}{\eta y-\frac{1}{2}l^2} &= \alpha\beta\mathbb{E}[\frac{1}{\eta y'-\frac{1}{2}l'^2} z' k'^{\alpha-1}l'^{1-\alpha}|z] && \text{(Using } c = \eta y\text{)} \\
\frac{1}{\eta y-\frac{1}{2}l^2} &= \alpha\beta\mathbb{E}[\frac{1}{\eta y'-\frac{1}{2}l'^2} z' k'^{\alpha-1}l'^{1-\alpha}|z] \\
\frac{1}{\eta y-\frac{1}{2}l^2} &= \alpha\beta\mathbb{E}[\frac{1}{\eta y'-\frac{1}{2}l'^2} \frac{y'}{k'}|z] && (♪)
\end{aligned}$$

Note that here we have a somewhat complicated expression, so let's see if we can work with (1) to get something that can simplify (♪):

$$\begin{aligned}
\frac{1}{c-\frac{1}{2}l^2}(-l) + (1-\alpha)\frac{1}{c-\frac{1}{2}l^2}(zk^{\alpha}l^{-\alpha}) &= 0 \\
-l + (1-\alpha)\frac{y}{l} &= 0 \\
l^2 &= (1-\alpha)y && (☼)
\end{aligned}$$

Now let's plug (☼) into (♪):

$$\begin{aligned}
\frac{1}{\eta y - \frac{1}{2}(1-\alpha)y} &= \alpha\beta\mathbb{E}[\frac{1}{\eta y' - \frac{1}{2}(1-\alpha)y'}\frac{y'}{k'}|z] \\
\frac{1}{\eta y - \frac{1}{2}(1-\alpha)y} &= \alpha\beta\mathbb{E}[\frac{1}{\eta y' - \frac{1}{2}(1-\alpha)y'}\frac{y'}{k'}|z] \\
\frac{1}{\eta y - \frac{1}{2}(1-\alpha)y} &= \alpha\beta\mathbb{E}[\frac{1}{[\eta - \frac{1}{2}(1-\alpha)]k'}|z] \\
\frac{1}{\eta y - \frac{1}{2}(1-\alpha)y} &= \alpha\beta\mathbb{E}[\frac{1}{[\eta - \frac{1}{2}(1-\alpha)](1-\eta)y}|z]
\end{aligned}$$

Note that we can now drop the expectation operator:

$$\frac{1}{\eta y - \frac{1}{2}(1-\alpha)y} = \alpha\beta \frac{1}{[\eta - \frac{1}{2}(1-\alpha)](1-\eta)y}$$
$$\eta = 1 - \alpha\beta$$

We can now plug this result back in to (☼):

$$l^2 = (1-\alpha)zk^{\alpha}l^{1-\alpha}$$
$$l = [(1-\alpha)zk^{\alpha}]^{\frac{1}{1+\alpha}}$$

Thus we have:

$$l = [(1-\alpha)zk^{\alpha}]^{\frac{1}{1+\alpha}}$$
$$k' = \alpha\beta[(1-\alpha)zk^{\alpha}]^{\frac{1-\alpha}{1+\alpha}}zk^{\alpha}$$
$$c = (1-\alpha\beta)[(1-\alpha)zk^{\alpha}]^{\frac{1-\alpha}{1+\alpha}}zk^{\alpha}$$

Combining terms we can write:

$$l = [(1-\alpha)zk^{\alpha}]^{\frac{1}{1+\alpha}}$$
$$k' = \alpha\beta(1-\alpha)^{\frac{1-\alpha}{1+\alpha}}z^{\frac{2}{1+\alpha}}k^{\frac{2\alpha}{1+\alpha}}$$
$$c = (1-\alpha\beta)(1-\alpha)^{\frac{1-\alpha}{1+\alpha}}z^{\frac{2}{1+\alpha}}k^{\frac{2\alpha}{1+\alpha}}$$

(e) [5] Looking back at your answers to (c) and (d), which of the utility functions resulted in a constant labor supply? Why does this occur?

The utility function from part (c) resulted in a constant labor supply. This is due to having additive separability in consumption and labor along with log utility on consumption, which results in the income and substitution effects of a technology shock z exactly canceling each other with respect to labor decisions. Thus while consumption decisions are impacted by productivity shocks, there is no labor response to these shocks.

Problem 2

Consider an infinite horizon cake eating problem where the cake eating agent is given a cake of size k_0 and she has to determine how to optimally consume it over her infinite lifespan. Assume the agent has instantaneous utility $u(c)$ and has discount factor $0 < \beta < 1$.

Part I: Consider the case when the individual has multiplicative taste shocks. In other words, the individual's instantaneous utility becomes $zu(c)$ where z is stochastic. Further, assume the shocks take on one of two values, z_{L} or z_{H} (where $z_{\mathrm{L}} < z_{\mathrm{H}}$), and that the shocks are a first order Markov process with the following transition matrix:

$$\pi = \begin{bmatrix} \pi_{LL} & \pi_{LH} \\ \pi_{HL} & \pi_{HH} \end{bmatrix}$$

(Recall: $\pi_{ij} = p(z' = z_j | z = z_i)$ for $i \in \{L, H\}$, $j \in \{L, H\}$)

(a) [10] Write down the cake eater's Bellman equation. What is (are) the choice variable(s) and what is (are) the state variable(s)?

Choice Variables: c, k'
State Variables: k, z

$$
\begin{aligned}
V(k,z) = \max_{c,k'} zu(c) + \beta \mathbb{E}[V(k',z')|z] \\
\text{s.t. } c + k' = k \\
c, k' \geq 0
\end{aligned}
$$

(b) [10] Derive the Euler equation when the cake eater receives shock z_i where $i \in \{L, H\}$. Next write out the Euler equation when the cake eater receives a shock of z_H using the appropriate transition probabilities from the transition matrix given above. Finally, write out the Euler equation when the cake eater receives a shock of z_L using the appropriate transition probabilities from the transition matrix given above.

Assuming the utility function satisfies the Inada conditions and plugging in our budget constraint we have

$$V(k,z) = \max_{k'} zu(k - k') + \beta \mathbb{E}[V(k',z')|z]$$

Our FOC is:

$$zu'(k - k') = \beta \mathbb{E}[V_{k'}(k',z')|z]$$

Using the Benveniste-Scheinkman Theorem we have:

$$
\begin{aligned}
V_k(k,z) &= zu'(k - k') \\
V_{k'}(k',z') &= z'u'(k' - k'') && \text{(Pushing forward)}
\end{aligned}
$$

Thus our Euler equation is

$$zu'(c) = \beta \mathbb{E}[z'u'(c')|z]$$

Assuming that the cake eater received a shock of z_H this period our Euler equation becomes

$$
\begin{aligned}
z_H u'(c) &= \beta \mathbb{E}[z'u'(c')|z_H] \\
z_H u'(c) &= \beta(\pi_{HL}\ z_L u'(c') + \pi_{HH}\ z_H u'(c'))
\end{aligned}
$$

Assuming that the cake eater received a shock of z_L this period our Euler equation becomes

$$
\begin{aligned}
z_L u'(c) &= \beta \mathbb{E}[z' u'(c')|z_L] \\
z_L u'(c) &= \beta(\pi_{LL}\ z_L u'(c') + \pi_{LH}\ z_H u'(c'))
\end{aligned}
$$

(c) [5] Imagine that the cake eater received a low shock (z_L) this period. How does her optimal cake eating behavior differ when her current shock is z_L and π_{LL} is near 1 versus when her current shock is z_L and π_{LL} is near 0? In other words, given that she gets a shock of z_L today, compare the cake eater's optimal consumption policy when π_{LL} is near 1 and when π_{LL} is near 0. Explain the intuition.

The cake eater gets a higher current period payoff when she gets the high shock (z_H), therefore she should want to consume more cake when she gets the high shock and less cake when she gets the low shock. If she gets the low shock today and π_{LL} is near 1, this means that the probability that she gets the low shock again tomorrow is very high. If π_{LL} is near 0, then there is a high probability that she gets the high shock tomorrow. Therefore, relative to when π_{LL} is near 1, when π_{LL} is near 0 the cake eater will eat less cake today and save more for tomorrow in hopes of getting the high shock. To understand this, imagine that $\pi_{LL} = 0.9999999999$. Then you are very likely get the low shock again tomorrow, and therefore you have less of a reason to wait to consume (relative to the case when π_{LL} is near 0). Now imagine that $\pi_{LL} = 0.0000000001$. Then since you are very likely to get the high shock tomorrow, you might as well save more cake and consume it tomorrow when you will "enjoy it more" (again relative to the case when π_{LL} is near 1.

Part II: Consider the same setup as in Part I, only now the cake eater doesn't know the current period shock when she makes her consumption decision.

(a) [10] Write down the cake eater's Bellman equation. What is (are) the choice variable(s) and what is (are) the state variable(s)?

Choice Variables: c, k'
State Variables: k, z_{-1}

$$
\begin{aligned}
V(k, z_{-1}) = \max_{c,k'} & \ \mathbb{E}[zu(c) + \beta V(k', z)|z_{-1}] \\
\text{s.t. } & \ c + k' = k \\
& \ c, k' \geq 0
\end{aligned}
$$

Of course you can also split up the expectation and write

$$
\begin{aligned}
V(k, z_{-1}) = \max_{c,k'} & \ \mathbb{E}[zu(c)|z_{-1}] + \mathbb{E}[\beta V(k', z)|z_{-1}] \\
\text{s.t. } & \ c + k' = k \\
& \ c, k' \geq 0
\end{aligned}
$$

(b) [10] Derive the Euler equation.

Assuming the utility function satisfies the Inada conditions and plugging in the budget constraint we have

$$V(k, z_{-1}) = \max_{k'} \mathbb{E}[zu(k - k') + \beta V(k', z)|z_{-1}]$$

Our FOC is

$$\mathbb{E}[zu'(k - k')|z_{-1}] = \beta \mathbb{E}[V(k', z)|z_{-1}]$$

Applying the Benveniste-Scheinkman Theorem we have

$$\begin{aligned} V_k(k, z_{-1}) &= \mathbb{E}[zu'(k - k')|z_{-1}] \\ V_{k'}(k', z) &= \mathbb{E}[z'u'(k' - k'')|z] && \text{(Pushing forward)} \end{aligned}$$

Thus our Euler equation becomes

$$\begin{aligned} \mathbb{E}[zu'(k - k')|z_{-1}] &= \beta \mathbb{E}[\mathbb{E}[z'u'(k' - k'')|z]|z_{-1}] \\ \mathbb{E}[zu'(k - k')|z_{-1}] &= \beta \mathbb{E}[z'u'(k' - k'')|z_{-1}] && \text{(Law of Iterated Expectations)} \end{aligned}$$

(c) [5] Will the optimal policy depend on this period's shock? Briefly explain why or why not.

No, the optimal policy will not depend on this period's shock. It will, however, depend on last period's shock. This period's shock is unobserved when the cake eater is deciding how much to consume, but since shocks follow a Markov process, last period's shock will tell her about the probability of getting the high shock or the low shock this period.

Part III: Consider the same setup as in Part I (the cake eater once again sees the current period shock before making her consumption decision), only now the cake eater cannot eat a fraction of the cake but instead each period must decide to either consume the entire cake or save the entire cake. Assume that if the cake is not consumed in the current period, then the size of the cake is θk in the next period. If $\theta < 1$, then the cake depreciates each period it is not eaten (this would be the case if a portion of the cake goes bad each period or if you have a sibling that eats a fraction of the cake each night while you are asleep. If $\theta > 1$, then the cake grows each period that it is kept. Finally, if $\theta = 1$, then the cake does not change in size from one period to the next.

(a) [10] What is V_s, the value for the cake eater if she saves the cake? What is V_e, the value for the cake eater if she consumes the cake? Use V_s and V_e to write the value function for the cake eater. What is (are) the choice variable(s) and what is (are) the state variable(s)?

$$\begin{aligned} V_s(k, z) &= \beta \mathbb{E}[V(\theta k, z')|z] \\ V_e(k, z) &= zu(k) \end{aligned}$$

Then the value function for the cake eater is:

$$V(k,z) = \max_{\text{save, eat}} \{V_e(k,z), V_s(k,z)\}$$
$$V(k,z) = \max_{\text{save, eat}} \{zu(k), \beta\mathbb{E}[V(\theta k, z')|z]\}$$

Note that here the choice is to save the cake or eat it. We can formalize this by creating the variable d where $d \in \{0,1\}$ and $d = 0$ if the cake eater saves the cake this period and $d = 1$ if the cake eater eats the cake this period. The state variables are the current cake size k and the current period shock z.

(b) [5] Write the value function for the cake eater when the current period shock is z_H, using the appropriate transition probabilities from the transition matrix given above. Write the value function for the cake eater when the current period shock is z_L, using the appropriate transition probabilities from the transition matrix given above.

The value function for the cake eater when the current period shock is z_H is

$$V(k, z_H) = \max_{\text{save, eat}} \{z_H u(k), \beta(\pi_{HL}\ V(\theta k, z'_L) + \pi_{HH}\ V(\theta k, z'_H))\}$$

The value function for the cake eater when the current period shock is z_L is

$$V(k, z_L) = \max_{\text{save, eat}} \{z_L u(k), \beta(\pi_{LL}\ V(\theta k, z'_L) + \pi_{LH}\ V(\theta k, z'_H))\}$$

(c) [5] Assume $\theta = 1$, under what conditions does the cake eater consume the entire cake this period? Briefly explain the intuition.

In the case where $\theta = 1$ note that when the current period shock is z_H, then the cake eater will consume the cake. This is because there is no larger shock, meaning the current period payoff from eating the cake cannot be higher, so if it is not optimal to eat the cake now then it never will be. Thus the cake will never be eaten or it will be eaten this period. If the cake is never eaten the return would be 0, and if the cake is eaten this period then the return would be $z_H u(k) > 0$. Therefore if the cake eater gets the high shock this period, she will eat the whole cake this period. Thus we have

$$V(k, z_H) = z_H u(k)$$

If, on the other hand, the current period shock is z_L, then the cake eater will wait to eat the cake if β is close to 1 or π_{LH} is large. If β is close to 1 then the cake eater cares a lot about tomorrow, and therefore is willing to wait for the high shock to eat the cake. If π_{LH} is large, then it is likely that the cake eater will get the high shock tomorrow and thus will wait to eat the cake.

(d) [5] If the current shock is z_L, how large does π_{LH} need to be in order to make the cake eater wait to consume the cake (i.e. to not consume the cake in this period)?

The cake eater's value function given that she got the low shock this period is

$$\begin{aligned} V(k, z_L) &= \max_{\text{save, eat}} \{V_e(k, z'), V_s(\theta k, z')\} \\ V(k, z_L) &= \max_{\text{save, eat}} \{z_L u(k), \beta(\pi_{LL}\ V(\theta k, z'_L) + \pi_{LH}\ V(\theta k, z'_H))\} \end{aligned}$$

Clearly we need

$$\begin{aligned} & z_L u(k) \leq \beta(\pi_{LL}\ V(\theta k, z'_L) + \pi_{LH}\ V(\theta k, z'_H)) \\ & \frac{z_L u(k) - \beta \pi_{LL}\ V(\theta k, z'_L)}{\beta V(\theta k, z'_H)} \leq \pi_{LH} \qquad \text{(Assuming } V(\theta k, z'_H) > 0\text{)} \end{aligned}$$

At this point there isn't much more that we can do, so let's go through the three cases. First, assume $\theta > 1$. Then the cake is growing each period, and therefore getting the high shock today does not imply that the cake eater will consume the cake today. Thus there is a tradeoff between the gain from waiting θ and future discounting β. Second, when $\theta < 1$, it is again optimal to consume the cake when the cake eater receives the high shock, and thus we can rewrite the condition above as

$$\frac{z_L u(k) - \beta \pi_{LL}\ V(\theta k, z'_L)}{\beta z_H u(\theta k))} \leq \pi_{LH}$$

Here the gain from waiting is only the chance to get the high shock next period. Finally, when $\theta = 1$ we can go farther. When $\theta = 1$ we have

$$V(k, z_L) = \max_{\text{save, eat}} \{z_L u(k), \beta(\pi_{LL}\ V(k, z'_L) + \pi_{LH}\ V(k, z'_H))\}$$

Note that since we know the cake eater will always eat the whole cake when she gets the high shock, we can rewrite the line above as:

$$V(k, z_L) = \max_{\text{save, eat}} \{z_L u(k), \beta(\pi_{LL}\ V(k, z'_L) + \pi_{LH}\ z_H u(k))\}$$

Then the value for the cake eater if she gets the low shock in the current period and saves the cake is

$$V_s(k, z_L) = \beta(\pi_{LL}\ V_s(k, z'_L) + \pi_{LH}\ z_H u(k))$$

Now we can solve for $V_s(k, z_L)$

$$V_s(k, z_L) = \frac{\beta \pi_{LH}\ z_H u(k)}{1 - \beta \pi_{LL}}$$

Therefore the cake eater will wait to eat the cake if

$$\begin{aligned} z_L u(k) &\leq \frac{\beta \pi_{LH}\ z_H u(k)}{1 - \beta \pi_{LL}} \\ \frac{z_L(1 - \beta \pi_{LL})}{\beta z_H} &\leq \pi_{LH} \end{aligned}$$

Noting that $\pi_{LL} = (1 - \pi_{LH})$ we have

$$\frac{(1-\beta)z_L}{\beta(z_H - z_L)} \leq \pi_{LH}$$

Part IV: Consider the same setup as in Part 1 except now the cake eater does not have any preference shocks and her instantaneous utility is once again $u(c)$. Instead, the cake size receives an additive shock each period. The cake magically grows each period by z_t **where z_t is i.i.d.** and can take two values z_H and z_L (where $z_H > z_L$). Therefore, in each period t, you observe the additive shock to the cake size z_t and then make your consumption decision.

(a) [10] Write down the cake eater's Bellman equation. What is (are) the state variable(s) and what is (are) the control variable(s)?

Choice Variables: c, k'
State Variables: k, z

$$\begin{aligned} V(k,z) = \max_{c,k'} & \; u(c) + \beta \mathbb{E}[V(k',z)] \\ \text{s.t. } & \; c + k' = k + z \\ & \; c, k' \geq 0 \end{aligned}$$