Week 8

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Endogenizing the Job Finding Rates

- We might want to be a little more formal with how we model the job finding rates
- Previously we assumed that agents get a job offer at some rate α
- Similarly, you may have noticed that we have essentially ignored the firm's side of the problem (workers draw from exogenous wage dist.)
- Here we will think about both more seriously; we will put the "match" in "search and matching"

Matching Function

- Two things that are important when thinking about how easy it is to find a job are the number of unemployed persons (u) and the number of job vacancies (v)
- Diamond, Moretensen, and Pissarides: assume that flow of contacts between firms and workers follows matching technology $m = m(u, v)$, where m is number of jobs formed during given time interval
- Assuming that firms and workers are identical, the arrival rates of jobs are then given by

$$\underbrace{\alpha = \frac{m(u,v)}{v}}_{\text{for firms}} \qquad \text{and} \qquad \underbrace{\widetilde{\alpha} = \frac{m(u,v)}{u}}_{\text{for workers}}$$

- $m(\cdot)$ is assumed to be continuous, nonnegative, increasing and concave in both arguments, with

$$m(u,0) = m(0,v) = 0$$

- Another standard assumption is that $m(\cdot)$ is CRS:
$m(cu, cv) = cm(u,v)$
  - increasing returns often yields multiplicity of equilibria
  - CRS is consistent with empirical results
- If, and when, we assume $m(\cdot)$ is CRS, the job finding probabilities only depend on $\theta \equiv v/u$, which is referred to as the *labor market tightness*

$$\alpha = \frac{m(u,v)}{v} = m\left(\frac{1}{\theta}, 1\right) \qquad \widetilde{\alpha} = \frac{m(u,v)}{u} = m\left(1, \theta\right) = \theta\alpha$$

A Note on the Matching Function

- Matching function is a modeling device for economists similar to other aggregate functions in macro (ex: production function)
- Allows us to model frictions in otherwise conventional models with minimum of added complexity
- Can account for information imperfections about potential trading partners, heterogeneities, etc.
- In job search matching function summarizes trading technology between agents who place advertisements, read newspapers, go to employment agencies, and utilize local networks that eventually bring them into productive matches

Firms

- A job is defined as a worker-firm pair
- A filled job is denoted by $J(\pi)$ where $\pi = y - w$ is profits (analogous to an employed worker)
- A vacant job is denoted by V (analogous to an unemployed worker)
- In order for a firm to post a vacancy, it must incur a flow cost $k > 0$ (a "recruitment cost")
- Now let's consider a model with a separation rate of s and a job finding rate (from the firm's perspective) of α

Workers:

$$rU = b + \theta\alpha\Big[V(w) - U\Big]$$

$$rV(w) = w + s\Big[U - V(w)\Big]$$

Firms:

$$rV = -k + \alpha\Big[J(\pi) - V\Big]$$

$$rJ(\pi) = \pi + s\Big[V - J(\pi)\Big]$$

Derivation

Number of Vacancies

- One loose-end that we have to hammer out regards how many firms post vacancies
- The standard way of doing so assumes a free-entry condition: firms enter until the value of a vacancy $V = 0$
- In other words, firms will enter, diluting the probability with which a firm may find a worker, until the value of posting that vacancy is 0

Wage Determination

- We still have yet to specify how wages are determined in this model; before we assumed an exogenous wage distribution $F(w)$
- One way of endogenizing the wage is to model w as the result of a (Nash) bargaining process
- The *generalized Nash bargaining* solution with *threat points* U (for workers) and V (for firms) is given by

$$w \in \operatorname{argmax} \Big[\underbrace{V(w) - U}_{\text{worker surplus}} \Big]^{\beta} \Big[\underbrace{J(y - w) - V}_{\text{firm surplus}} \Big]^{1-\beta}$$

- β: parameter that captures the relative *bargaining power* of each party

The solution will satisfy the following.

$$\beta\Big[J(y-w)-V\Big]V'(w)=(1-\beta)\Big[V(w)-U\Big]J'(y-w)$$

If we recall the expressions from earlier, we can very easily solve for the above derivatives.

$$rV(w)=w+s[U-V(w)]$$
$$V'(w)=\frac{1}{r+s}$$

Same for $J'(y-w)$, so we have:

$$V'(w)=J'(y-w)=\frac{1}{r+s}$$

Plugging this in ...

$$V(w)=U+\beta\Big[\underbrace{J(y-w)-V+V(w)-U}_{S}\Big].$$

The above states that the worker receives her threat point U and some share of the *surplus*:

$$S \equiv J(y - w) - V + V(w) - U.$$

We can use our earlier expressions for $J(\pi)$ and $V(w)$ to plug into the above:

$$S = \frac{y - rU - rV}{r + s}.$$

As we have done in the past, we can solve for the *reservation* strategies of workers and firms. Denote them w_R and π_R, respectively. We can easily determine

$$V(w) - U = \frac{w - w_R}{r + s} \qquad \text{and} \qquad J(\pi) - V = \frac{\pi - \pi_R}{r + s}.$$

Derivation

The Nash bargaining problem can be simplified to

$$w \in \operatorname{argmax} \Big[w - w_R\Big]^{\beta} \Big[y - w - \pi_R\Big]^{1-\beta},$$

which can be solved ...

$$w = w_R + \beta(y - \pi_R - w_R).$$

- Importantly, notice that $w \geq w_R$ iff $y \geq y_R \equiv \pi_R + w_R$

- Similarly, $\pi = y - w \geq \pi_R$ iff $y \geq y_R$

- That is, workers and firms agree to form a relationship iff $y \geq y_R$
    - in other words, a relationship is formed if the match will produce enough for both the worker and firm to make a gain

Equilibrium

- An equilibrium here will be value functions $(J, V(w), U)$, a wage w, and unemployment / vacancy rates (u, v)
- Let's look for a steady-state; from last class we know how to easily find the s.s. unemployment rate:

$$\underbrace{\theta\alpha u}_{\text{mass leaving UE}} = \underbrace{s(1-u)}_{\text{mass entering UE}} \implies u = \frac{s}{\theta\alpha + s}$$

- Notice that there is no $[1 - F(w_R)]$ in the above statement
  - first, there is no exogenous wage distribution
  - second, with bargaining and assuming that matches are beneficial for both parties, whenever someone matches the wage will at least be as high as her reservation wage

- Rewrite the equation giving the surplus as follows (recall free entry implies $V = 0$)

$$(r + s)S = y - rU$$

- Further, we can rewrite the flow value of unemployment utilizing the result from the bargaining process (that the worker will be paid her threat point plus a fraction of the surplus)

$$rU = b + \theta\alpha\beta S$$

- Plug this in to the first equation above

$$(r + s + \theta\alpha\beta)S = y - b \qquad (\star)$$

- For the firm side of things, bargaining implies

$$J(\pi) = (1-\beta)S$$

- Recalling that with free entry we have $V = 0$, we can rewrite the value of a job and plug in the above result

$$\cancel{rV} = -k + \alpha(J(\pi) - \cancel{V}) \implies \alpha J(\pi) = k$$

$$\implies \alpha(1-\beta)S = k \qquad (\star)$$

- Both starred equations completely characterize the equilibrium; indeed, they may combined

$$\frac{r + s + \theta\alpha\beta}{(1-\beta)\alpha} = \frac{y-b}{k} \qquad (\star\star)$$

- Since we know what u is, ($\star\star$) may be solved for v (recall that v is embedded in θ and α):

$$\theta = \frac{v}{u} \qquad\qquad \alpha = m\left(\frac{1}{\theta}, 1\right)$$

- Similarly, we'll be able to solve for a wage:

$$w = y - (r + s)(1 - \beta)S$$

Appendix

Here I give the derivation of the unemployed worker's Bellman equation from discrete time, the derivation of the others is similar.

Discrete time:

$$U = b + \beta[\theta\alpha V(w) + (1-\theta\alpha)U]$$

Generalize length of period to dt and let $\beta = \frac{1}{1+rdt}$

$$\begin{aligned}
U &= bdt + \frac{1}{1+rdt}[\theta\alpha dt V(w) + (1-\theta\alpha dt)U] \\
U(\frac{rdt}{1+rdt}) &= bdt + \frac{\theta\alpha dt}{1+rdt}V(w) - \frac{\theta\alpha dt}{1+rdt}U \\
U &= \frac{(1+rdt)b}{r} + \frac{\theta\alpha}{r}V(w) - \frac{\theta\alpha}{r}U \\
rU &= b + \theta\alpha[V(w) - U]
\end{aligned}$$

Appendix

Here I give the derivation of $V(w) - U$, the derivation of $J(\pi) - V$ is similar. Recall

$$rV(w) = w + s[U - V(w)]$$

Also recall the reservation wage is the wage such that $V(w_R) = U$. Then

$$\begin{aligned}
rV(w) - rU &= rV(w) - rV(w_R) \\
r(V(w) - U) &= w + s[U - V(w)] - (w_R + s[\cancel{U - V(w_R)}]) \\
(r+s)(V(w) - U) &= w - w_R \\
V(w) - U &= \frac{w - w_R}{r + s}
\end{aligned}$$

Reservation Strategies