Matthew Fitzgerald
Econ 204B
March 13, 2020

Problem Set # 5

Problem 6.8 from S&L: Wage growth and the reservation wage

An unemployed worker receives each period an offer to work for wage w_t forever, where $w_t = w$ in the first period and $w_t = \phi^t w$ after t periods on the job. Assume $\phi > 1$, that is, wages increase with tenure. The initial wage offer is drawn from a distribution $F(w)$ that is constant over time (entry-level wages are stationary); successive drawings across periods are independently and identically distributed.

The worker's objective is to maximize

$$\mathbb{E}\sum_{t=1)}^{\infty} \beta^t y_t, \quad \text{where } 0 < \beta < 1$$

and $y_t = w_t$ if the worker is employed and $y_t = c$ is the worker is unemployed, where c is unemployment compensation. Let $v(w)$ be the optimal value of the objective function for an unemployed worker who has offer w in hand. Write the Bellman equation for this problem. Argue that, if two economies differ only in the growth rate of wages of employed workers, say $\phi_1 > \phi_2$, the economy with the higher growth rate has the smaller reservation wage. *Note:* Assume that $\phi_i\beta < 1$, $i = 1, 2$.

For an unemployed worker with an offer or w in hand, the value of staying unemployed is

$$c + \beta \int_{\underline{w}}^{\overline{w}} V(w')dF(w')$$

To examine the value of accepting the job with wage w, we need to think about wage growth over time. Note that:

$$\begin{aligned} w + \phi\beta w + \phi^2\beta^2 w + ... + \phi^j\beta^j w &= w(1 + \phi\beta + \phi^2\beta^2 + ... + \phi^j\beta^j) \\ &= \frac{w}{1-\phi\beta} \qquad \text{(Sum of infinite geometric sequence)} \end{aligned}$$

Then the Bellman equation for an unemployed worker with offer w in hand is

$$V(w) = \max_{\text{accept, reject}} \left\{ \frac{w}{1-\phi\beta},\ c + \beta \int_{\underline{w}}^{\overline{w}} V(w')dF(w') \right\}$$

Then at the reservation wage we have:

$$\begin{aligned}
\frac{w_R}{1-\phi\beta} &= c + \beta \int_{\underline{w}}^{\overline{w}} V(w')dF(w') \\
\frac{w_R}{1-\phi\beta} &= c + \beta \int_{\underline{w}}^{\overline{w}} \max\left\{\frac{w'}{1-\phi\beta}, \frac{w_R}{1-\phi\beta}\right\} dF(w') \\
\frac{w_R}{1-\phi\beta} &= c + \beta \int_{\underline{w}}^{w_R} \frac{w_R}{1-\phi\beta} dF(w') + \beta \int_{w_R}^{\overline{w}} \frac{w'}{1-\phi\beta} dF(w') \\
w_R &= (1-\phi\beta)c + \beta \int_{\underline{w}}^{w_R} w_R dF(w') + \beta \int_{w_R}^{\overline{w}} w' dF(w') \\
(1-\phi\beta)c &= w_R - \beta \int_{\underline{w}}^{w_R} w_R dF(w') - \beta \int_{w_R}^{\overline{w}} w' dF(w') \\
(1-\phi\beta)c &= w_R - \beta \int_{\underline{w}}^{w_R} w_R dF(w') - \beta \int_{w_R}^{\overline{w}} w' dF(w')
\end{aligned}$$

<u>Problem 6.9 from S&L: Search with a finite horizon</u>

Consider a worker who lives two periods. In each period the worker, if unemployed, receives an offer of lifetime work at wage w, where w is drawn from a distribution F. Wage offers are identically and independently distributed over time. The worker's objective is to maximize $\mathbb{E}\{y_1 + \beta y_2\}$, where $y_t = w$ if the worker is employed and is equal to c – unemployment compensation – if the worker is not employed.

Analyze the worker's optimal decision rule. In particular, establish that the optimal strategy is to choose a reservation wage in each period and to accept any offer with a wage at least as high as the reservation wage and to reject offers below that level. Show that the reservation wage decreases over time.