Multiple equilibrium states and the abrupt transitions in a dynamical system of soil water interacting with vegetation

Xiaodong Zeng and Samuel S. P. Shen
University of Alberta, Edmonton, Alberta, Canada

Xubin Zeng
University of Arizona, Tucson, Arizona, USA

Robert E. Dickinson
Georgia Institute of Technology, Atlanta, Georgia, USA

Received 23 October 2003; revised 6 January 2004; accepted 2 February 2004; published XX Month 2004.

[1] In semi-arid areas, multiple equilibrium states of an ecosystem (e.g., grassland and desert) are found to coexist, and the transition from grassland to desert is often abrupt at the boundary. A simple ecosystem model is developed to provide the biophysical explanation of this phenomenon. The model has three variables: living biomass, wilted biomass, and soil wetness. The moisture index, which is the ratio of the annual precipitation to potential evaporation, is the only external climate driving force, and the key mechanism is the vegetation-soil interaction. It is found that the maintenance of a grassland requires a minimum moisture index, and the abrupt transition occurs when the moisture index is around this critical value. These results are robust within a wide range for most model parameters, suggesting that the model may be applicable to other temperate grasslands. The characteristics of the wilted biomass also strongly influence the ecosystem's dynamics. *INDEX TERMS:* 4815 Oceanography: Biological and Chemical: Ecosystems, structure and dynamics; 4842 Oceanography: Biological and Chemical: Modeling; 1809 Hydrology: Desertification. **Citation:** Zeng, X., S. S. P. Shen, X. Zeng, and R. E. Dickinson (2004), Multiple equilibrium states and the abrupt transitions in a dynamical system of soil water interacting with vegetation, *Geophys. Res. Lett.*, *31*, LXXXXX, doi:10.1029/2003GL018910.

1. Introduction

[2] The composition of an ecosystem is determined primarily by latitude (sunlight and temperature), altitude (temperature) and precipitation (soil moisture). Over the regions with favorable sunlight and temperature conditions, the so-called "moisture index", i.e., the ratio of annual precipitation to potential evaporation, is the most important parameter. The vegetation distribution is usually continuous in space, but can also be discontinuous, even in regions with a smooth variation of the moisture index. For example, in Inner Mongolia, China, a typical steppe has ample vegetation, e.g., *Stipa grandis* and *Stipa krelovii*, while the desert or semi-desert zone grows only sparse arid grass or shrubs, with a biomass that is about one to two orders of magnitude smaller than that in the steppe. Figure 1 indicates that the boundary between the grassland and desert is sharp and roughly along the line of a moisture index of 0.3.

[3] Such transitions between these two ecosystems can occur in both spatial and temporal domains. Over the Sahara and Sahel regions of Africa, the temporal evolution of ecosystem has been investigated by using simple to intermediate-level models [*Brovkin et al.*, 1998; *Claussen et al.*, 1999; *Zeng et al.*, 1999; *Wang and Eltahir*, 2000]. These models demonstrated that multiple equilibrium states could coexist in the biosphere-atmosphere system. Under certain conditions, subtle variations of climate could be strongly amplified by atmosphere-vegetation feedbacks, triggering an abrupt switch of the biosphere-atmosphere system from one equilibrium state to another.

[4] In the study of the spatial variation of the ecosystem, the climate driven vegetation model of *Zeng et al.* [1994] and *Zeng and Zeng* [1996] demonstrated that, with the vegetation-soil interaction, the land system alone could possess multiple equilibrium states and display spatial discontinuity between grassland and desert. However, that model was of maximum simplification, consisting of only two variables, i.e., the total biomass and soil wetness. Therefore the difference in the functions of the living and wilted (dead) vegetation as well as the difference in the mechanisms of evaporation and transpiration could not be distinguished. This highly idealized model requires an unusually strong interaction between biomass and soil wetness to produce the abrupt change from grassland to desert. The current study improves the realism of that previous model through the introduction of a new variable: wilted biomass. With this third variable, the observed abrupt transition between grassland and desert in Inner Mongolia can be better interpreted. This new model can be used in stand-alone mode or coupled to climate models to explore the general behavior of the self-organization and vegetation-soil interaction in a grassland ecosystem.

2. The Model

[5] Our new model of grassland considers a single vertical column of soil and one species of grass. It has three dimensionless state variables: the mass density of living leaves, x, the available soil wetness, y, and the mass density of wilted (dead) leaves, z. The actual values of these variables are described by $x' = xx^*$, $y' = yy^*$, and $z' = zz^*$,

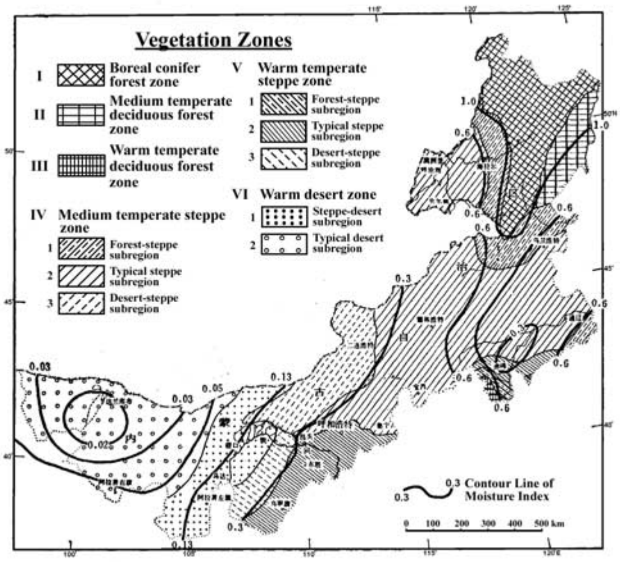

Figure 1. Distribution of vegetation zone and moisture index in Inner Mongolia. In the desert or semi-desert zones (IV3; V3; VI1, 2) only sparse arid grass or shrubs grow, and the biomass is about one to two orders of magnitudes smaller than that in the steppe regions (IV1, 2; V1, 2). The boundary between the grassland and desert is roughly along the line of a moisture index of 0.3. (This figure is reconstructed with permission from data and maps provided by G. Zhao of Inner Mongolian Agricultural University.)

respectively, where dimensional constants x^*, y^*, and z^* are the corresponding characteristic values. This model is built on the earlier idealized model of *Zeng et al.* [1994] and *Zeng and Zeng* [1996] but differs from it by more realistic biophysical considerations in two aspects: (a) the number of prognostic variables is increased from two to the current three so that the impact of wilted biomass on the vegetation-soil interaction can be explicitly considered; and (b) the surface evaporation and transpiration are computed separately in the current model (rather than mixed together as in the 2-variable model).

[6] For simplicity, it is assumed that the living grass covers part of the soil surface, while the wilted grass is distributed uniformly over the soil surface. Precipitation, other atmospheric conditions, and the soil properties are all prescribed. The system is assumed to be horizontally homogeneous. Following the mass conservation law, the dynamics of the ecosystem can be written as

$$dx'/dt = x^* dx/dt = F_1 = G(x,y) - D(x,y) - C(x), \quad (1)$$

$$dy'/dt = y^* dy/dt = F_2 = P - E_s(x,y,z) - E_r(x,y) - R(x,y,z), \quad (2)$$

and

$$dz'/dt = z^* dz/dt = F_3 = G_z(x,y) - D_z(z) - C_z(z), \quad (3)$$

where terms G, D, and C are the growth (photosynthesis subtracts plant respiration), wilting, and consumption (grazing) of the living leaves, G_z, D_z, and C_z are the accumulation, decomposition, and consumption of the wilted grass, P is the atmospheric precipitation (system input), E_s is the evaporation from the soil surface, E_r is the transpiration, and R is runoff. Note that, more accurately, P is the through fall, i.e., precipitation minus intercepted water by live and wilted leaves. Intercepted precipitation is relatively small, and our sensitivity tests show that conclusions remain the same if this term is included.

[7] Similar to the method used in *Zeng and Zeng* [1996], all these terms are formulated and parameterized based on mathematical constraints, and/or ecological and biophysical considerations [e.g., *Campbell and Norman*, 1998; *Dickinson et al.*, 1998].

[8] Briefly speaking, many of the dependences of terms on any particular state variable u are of the form of $(C/k)(1 - e^{-ku})$ which asymptotes to the linear decay term Cu for small u and saturates with $u \to \infty$. For example, processes in equations (1) and (3) are formulated as

$$G = \alpha^*(1 - e^{-\varepsilon_g x})\left(1 - e^{-\varepsilon'_g y}\right), \quad (4)$$

$$D = \alpha^*\beta(e^{\varepsilon_d x} - 1)\left(1 - e^{-\varepsilon'_d y}\right)^{-1}, \quad (5)$$

$$C = \alpha^*\gamma(1 - e^{-\varepsilon_c x}), \quad (6)$$

$$G_z = \alpha_z D = \alpha^*\alpha_z\beta(e^{\varepsilon_d x} - 1)\left(1 - e^{-\varepsilon'_d y}\right)^{-1}, \quad (7)$$

$$D_z = \alpha^*\beta_z(e^{\varepsilon_{dz} z} - 1), \quad (8)$$

and

$$C_z = \alpha^*\gamma_z(1 - e^{-\varepsilon_{cz} z}), \quad (9)$$

where dimensional parameter α^* is the maximum growth rate, the dimensionless coefficients β, γ, β_z, and γ_z are ratios of the maximum or characteristic rates of the corresponding process over α^*, α_z is the rate of wilted biomass accumulation, and ε's with different subscripts are exponential attenuation coefficients.

[9] Now consider the vegetation-soil interaction terms in equation (2). Because the latent heat for evaporation at a wet surface is approximately in balance with the radiation energy, and also because the attenuation of the solar radiation by the living and wilted biomass follows the exponential law, the evaporation from the soil surface shaded by the living and wilted biomass can be expressed as

$$E_s = e^*(1 - e^{-\varepsilon_2 y})e^{-\varepsilon_3 z}\left(\left(1 - \sigma_f\right) + \sigma_f(1 - \kappa_1(1 - e^{-\varepsilon_1 x}))\right), \quad (10)$$

where e^* is the potential evaporation, κ_1 is the correction due to the non-opaque cover of the living biomass for the diffusive radiation, and the fraction of living grass coverage, σ_f, is described by $\sigma_f = 1 - e^{-\varepsilon_f x}$.

Table 1. Values of Parameters and Coefficients Adjusted to the Inner Mongolia Grassland

Dimensional Parameters		Dimensionless Coefficients	
x^*	0.1 kg m^{-2}	β, β_z, γ	0.1
y^*	240 mm	γ_z	0
z^*	0.1 kg m^{-2}	α_z	0.5
α^*	0.4 kg m^{-2} yr^{-1}	ε_f, ε_g, ε_g', ε_d, ε_d', ε_c	1.0
e^*	1000 mm yr^{-1}	ε_{dz}, ε_{cz}	1.0
		φ_{rs}	0.6
		λ	0.015
		κ_1, κ_1''	0.4
		κ_1'	1.0
		ε_1, ε_1''	0.7
		ε_1', ε_2, ε_2', ε_2'', ε_3, ε_3''	1.0

[10] Transpiration occurs simultaneously with photosynthesis, so in a general form, let

$$E_r = e^*\varphi_{rs}\left(1 - e^{-\varepsilon_2' y}\right)\sigma_f\left(1 - \kappa_1' e^{-\varepsilon_1' x}\right), \tag{11}$$

where φ_{rs} is the ratio of potential transpiration to e^*, and κ_1' is coefficient. For the convenience of analysis, the precipitation term P is substituted by a dimensionless coefficient μ, called "moisture index", as $\mu = P/e^*$. Finally, the runoff term considers not only the impact of precipitation and soil wetness but also of living and wilted leaves as follows:

$$R = \lambda e^*\mu\left(e^{\varepsilon_2'' y} - 1\right)e^{-\varepsilon_3'' z}\left(\left(1 - \sigma_f\right) + \sigma_f\left(1 - \kappa_1''\left(1 - e^{-\varepsilon_1^x}\right)\right)\right), \tag{12}$$

where λ and κ_1'' are coefficients.

[11] All these parameters and coefficients are determined by observational data in the Inner Mongolia grassland [*Chinese Academy of Sciences*, 1985; *Jiang*, 1988] and shown in Table 1. For example, x^* and y^* are the average of the corresponding values over the vast grassland, the wilting rate β is subjected to the requirement of balancing the growth and wilting of living biomass at the case of full coverage, and κ_1 and φ_{rs} are influenced by factors such as relative humidity, saturation vapor pressure, and stomatal resistance of leaves, etc.

3. Equilibrium States and the Abrupt Transition Between Ecosystems

[12] The quantitative behaviors of the ecosystem can be investigated by numerically integrating equations (1) to (3). The equilibrium states of the system are of most interest and can be determined by setting $F_1 = F_2 = F_3 = 0$. The stability of the equilibrium states can be determined by means of a general mathematical method [e.g., see *Shen*, 1993, Section 1.2]. Briefly speaking, equations (1) to (3) are first linearly expanded around each equilibrium, and the eigenvalues of the corresponding Jacob matrix are calculated. If the real part of each of the three eigenvalues is negative, the equilibrium is stable, and the system will return to this equilibrium state from any small perturbations. Otherwise, the equilibrium is unstable (a saddle), and a small perturbation will normally cause the system to diverge from this state. Each stable equilibrium state corresponds to a possible ecosystem, either grassland or desert, whereas the unstable state is not expected in nature.

[13] Figure 2 shows the equilibrium states under different values of the prescribed moisture index μ. When μ is less than a critical value μ_1, there is only one equilibrium state, and it is stable with no living grass; that is, the ecosystem is a desert. When μ is larger than another critical value $\mu_2 > \mu_1$, there are two equilibrium states, one stable with a sufficient amount of living biomass, and another unstable with no grass; that is, the ecosystem is a grassland. For $\mu_1 < \mu < \mu_2$, there are three equilibrium states, one unstable, and two stable with one for grassland and another for desert. This implies that (a) a minimum moisture index (i.e., μ_1) is required to maintain a grassland; and (b) an abrupt transition between grassland and desert occurs around μ_1, and the biomass increases continuously with μ when $\mu > \mu_1$.

[14] The biophysical explanations of the ecosystem dynamics shown in Figure 2 are as follows. Qualitatively, the increase of soil wetness always promotes the growth of living biomass. The living vegetation not only extracts soil water through transpiration but also conserves it through reducing the surface evaporation, while the wilted leaves (as well as other non-photosynthesizing surfaces) provide the benefits of shading without the use of the soil water. This principle of "mulching" is well known to farmers and gardeners. For small enough precipitation, the balance of precipitation with evaporation maintains soil water at too low a level for vegetation to succeed, so there is only one solution of bare soil (desert). With enough precipitation, this solution is unstable, and a solution with finite vegetation, i.e., grassland, is possible. Because of the stabilizing effects of the shading by living and wilted vegetation, the grassland can maintain its stability over a range of precipitation for which the bare soil solution is also stable.

[15] To check the robustness of the above results, sensitivity tests have been done by changing the value of a particular coefficient. For most of the coefficients, the

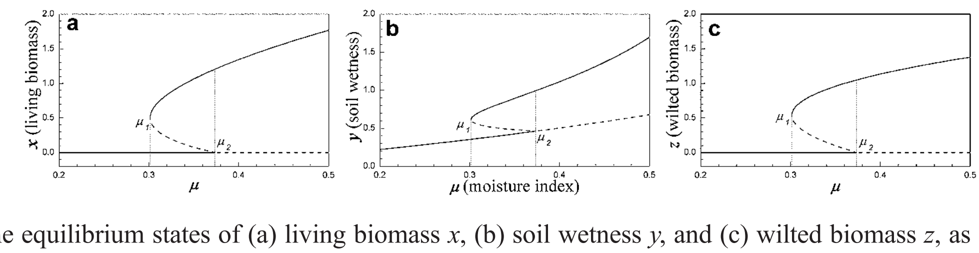

Figure 2. The equilibrium states of (a) living biomass x, (b) soil wetness y, and (c) wilted biomass z, as a function of the moisture index μ. The two critical values are indicated as μ_1 and μ_2. Solid and dash lines refer to stable and unstable equilibrium states respectively. Values of parameters and coefficients are shown in Table 1.

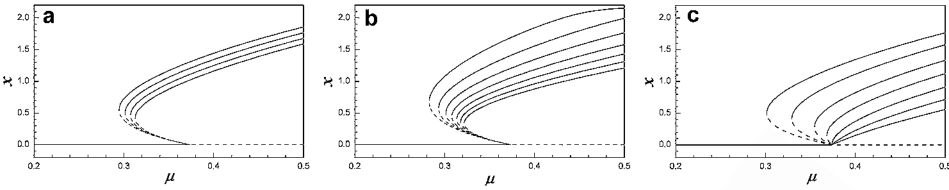

Figure 3. Sensitivity of the equilibrium states to the coefficient κ_1 (shading effect of living biomass), φ_{rs} (the potential transpiration rate), and ε_3 (the exponential attenuated coefficient of wilted biomass shading). From left to right, (a) $\kappa_1 = 0.6$, 0.4, 0.2, and 0; (b) $\varphi_{rs} = 0.4$, 0.5, 0.6, 0.7, 0.8, 0.9, and 1.0; (c) $\varepsilon_3 = 1.0$, 0.8, 0.6, 0.45, 0.3, 0.15 and 0. All other coefficients are kept the same as those in Figure 2.

bifurcation and abrupt change still occur, although the critical values of μ_1 and μ_2 could be different. Figures 3a and 3b show the dependence of x on κ_1 and φ_{rs}. The abrupt change persists even for the extreme cases of $\kappa_1 = 0$ (i.e., no shading by living grass) or $\varphi_{rs} = 1$ (i.e., maximum potential transpiration).

[16] However, the dynamics of the system can change with the wilted-biomass-related coefficients, e.g., ε_3, α_z, and β_z. For instance, Figure 3c shows the existence of abrupt transitions for the shading effect coefficient $\varepsilon_3 = 0.45$ to 1.0. As ε_3 is further reduced below 0.3, however, abrupt transitions no longer occur, and biomass varies smoothly with μ. This finding implies the wilted biomass' significant influence on the ecosystem's dynamics, which was not explicitly considered in our previous study [*Zeng and Zeng*, 1996]. Our previous 2-variable model, which can correspond to the case of $\varphi_{rs} = 0$ (i.e., no transpiration) and $\varepsilon_3 = 0$ (i.e., no shading by the wilted biomass) in this model, requires $\kappa_1 > 0.8$ (i.e., very strong shading by the total biomass) for the existence of an abrupt transition. As shown here, the wilted biomass not only enhances the shading effect, but also reduces the evaporation from the soil surface not covered by living leaves. Hence, the ecosystem with plenty of wilted biomass behaves like the strong vegetation-soil interaction case addressed in our previous study. The abrupt transition could occur with a small κ_1 even when the process of transpiration is included, and disappear only in the cases of a very weak interaction between wilted grass and soil wetness.

4. Conclusions and Further Discussion

[17] A simple 3-variable ecosystem model is developed to study grassland ecosystem. The three variables are living biomass, wilted biomass, and soil wetness. It is found that multiple equilibrium states of ecosystem coexist in semi-arid areas, and the minimum moisture index for the existence of grassland is around 0.3 which is in agreement with observations in the Inner Mongolia grassland. The sufficient condition for the existence of grassland is that the moisture index μ be greater than another critical value $\mu_2 > \mu_1$. For $\mu_1 < \mu < \mu_2$, there exist two stable equilibrium states, one for grassland and another for desert. In other words, the change of biomass is abrupt even when μ changes smoothly between μ_1 and μ_2.

[18] The key mechanism of the system analyzed is the shading of otherwise sunlight soil by living and wilted vegetation. The introduction of wilted biomass as a prognostic variable is especially important. In the ecosystem with soil surface shaded by plenty of wilted biomass, soil evaporation is reduced significantly, and hence the soil wetness is conserved. This shading may be important for the occurrence of vegetation in semi-arid areas. Many suggestions have been made in the past to improve desertified landscapes through revegetation based on the belief that the introduction of the vegetation would increase precipitation enough to lead to a stable regime with more vegetation and precipitation. Such efforts may seem fruitless because of the likely smallness of the increase of precipitation. However, the mechanisms considered in this study suggest that the presence of vegetation alone may stabilize soil moisture at a high enough level to maintain the vegetation. This possibility should be further studied with accurate ecological data for semi-arid areas.

[19] The variation of μ can be discussed in both the spatial and temporal domains, and the former has been emphasized so far. In the spatial domain, the behavior of the multiple equilibrium states should be a concern in grassland maintenance in semi-arid areas. For the zone of $\mu_1 < \mu < \mu_2$, a certain amount of living and wilted biomass, determined by the corresponding unstable equilibrium state, is required to form the grassland, and an overuse of the living or wilted grass of the existing grassland might eventually lead to the stable equilibrium state of a desert. The range of parameter space over which these equilibriums can coexist may be increased by positive feedbacks of evapotranspiration on the precipitation [e.g., *Wang and Eltahir*, 2000]. The precipitation forcing and various coefficients may be taken to vary periodically and stochastically in time. With such variation, the simple system we consider may be viewed as a characterization of a complete nonlinear climate system with interactive vegetation. As such it provides insight into "attractor basins", i.e., states of the complete system such as drought (or desert) regimes that maintain their stability for long periods of time.

[20] **Acknowledgments.** This work was supported by the NOAA, NASA (NAGS-13322) and the grant from China Natural Science Foundation (No. 40233027). Shen also thanks the support from the MITACS (Mathematics of Information Technology and Complex Systems) for a research grant, and the Chinese Academy of Sciences for an Overseas Assessor's research grant and for the Well-Known Overseas Chinese Scholar award. The authors thank Gang Zhao, Inner Mongolian Agricultural University, for providing the data and maps for Figure 1. Guiling Wang and an anonymous reviewer are also thankful for their helpful comments.

References

Brovkin, V., M. Claussen, V. Petoukhov, and A. Ganopolski (1998), On the stability of the atmosphere-vegetation system in the Sahara/Sahel region, *J. Geophys. Res.*, *103*, 31,613–31,624.

Campbell, G. S., and J. M. Norman (1998), *An Introduction to Environmental Biophysics*, 2nd ed., 286 pp., Springer-Verlag, New York.

Chinese Academy of Sciences Comprehensive Survey Group of Inner Mongolia and Ningxia Autonomous Regions (1985), *The Inner Mongolia Vegetation*, 884 pp., Science Press, Beijing.
Claussen, M., C. Kubatzki, V. Brovkin, A. Ganopolski, P. Hoelzmann, and H. Pachur (1999), Simulation of an abrupt change in Saharan vegetation in the mid-Holocene, *Geophys. Res. Lett.*, *24*, 2037–2040.
Dickinson, R. E., M. Shaikh, R. Bryant, and L. Graumlich (1998), Interactive canopies for a climate model, *J. Clim.*, *11*, 2823–2836.
Jiang, S. (1988), *Methodology for Grassland Ecosystem Investigation*, 301 pp., Agriculture Press, Beijing.
Shen, S. S. P. (1993), *A Course on Nonlinear Waves*, 327 pp., Kluwer Academic Publishers.
Wang, G., and E. A. B. Eltahir (2000), Biosphere-atmosphere interactions over West Africa, 2. Multiple climate equilibria, *Q. J. R. Meteorol. Soc.*, *126*, 1261–1280.
Zeng, N., J. D. Neelin, K-M. Lau, and C. J. Tucker (1999), Enhancement of interdecadal climate variability in the Sahel by vegetation interaction, *Science*, *286*, 1537–1540.
Zeng, Q.-C., P.-S. Lu, and X. D. Zeng (1994), Maximum simplified dynamic model of grass field ecosystem with two variables, *Sci. China Ser. B*, *37*, 94–103.
Zeng, Q.-C., and X. D. Zeng (1996), An analytical dynamic model of grass field ecosystem with two variables, *Ecol. Model.*, *85*, 187–196.

R. E. Dickinson, School of Earth and Atmospheric Sciences, Georgia Institute of Technology, Atlanta, GA 30332, USA. (robted@eas.gatech.edu)
S. S. P. Shen, Department of Mathematical and Statistical Sciences, University of Alberta, Edmonton, AB T6G 2G1, Canada. (shen@ualberta.ca)
X. Zeng, Department of Atmospheric Sciences, University of Arizona, Tucson, AZ 85721, USA. (xdzeng@atmo.arizona.edu)
X. Zeng, Department of Atmospheric Sciences, University of Arizona, Tucson, AZ 85721, USA. (xubin@atmo.arizona.edu)