Divide and Conquer Master Theorem Problem

1) $T(n) = 8T(n/3) + n^2$

$a = 8$, $b = 3$, $f(n) = n^2$, $\log_3 8 < 2$
$n^{\log_b a} = n^{\log_3 8} < n^2 \Rightarrow n^2 = n^{\log_3 8 + \epsilon}$, where $\in$ is equal to $(2 - \log_3 8)$, therefore > 0.
$n^2 = \Omega(n^{\log_3 8 + \epsilon})$
$af(n/b) <= c*f(n)$, where $c<1$
$8f(n/3)^2 <= c*f(n)^2$
$8(n^2/9) <= c*n^2$, therefore $c >= 8/9$
Thus, case 3 applies and $T(n) = \Theta(n^2)$

2) $T(n) = 10T(n/3) + n^2$

$a = 10$, $b = 3$, $f(n) = n^2$, $\log_3 10 > 2$
$n^{\log_b a} = n^{\log_3 10} > n^2 \Rightarrow n^2 = n^{\log_3 10 - \epsilon}$, where $\in$ is equal to $(\log_3 10 - 2)$, therefore > 0.
Thus, case 1 applies and $T(n) = \Theta(n^{\log_3 10})$

3) $T(n) = 16T(n/4) + n^2\log^3 n$

$a = 16$, $b = 4$, $f(n) = n^2\log^3 n$, $\log_4 16 = 2$
$n^{\log_b a} = n^{\log_4 16} = n^2 = f(n) = n^2$.
Therefore, $T(n) = \Theta(n^{\log_b a} * \log^{k+1}(n))$, where $k = 3$.
Thus, case 2 applies and $T(n) = \Theta(n^2\log^4(n))$

4) $T(n) = 9T(n/3) + n^3$

$a = 9$, $b = 3$, $f(n) = n^3$, $\log_3 9 = 2$
$n^{\log_b a} = n^{\log_3 9} < n^3 \Rightarrow n^{2 + \epsilon}$, where $\in$ is equal to 1, therefore > 0.
$af(n/b) <= c*f(n)$, where $c<1$
$9f(n/3)^3 <= c*f(n)^3$
$9(n^3/27) <= c*n^3$, therefore $c >= 9/27 = 1/3$
Thus, case 3 applies and $T(n) = \Theta(n^3)$