From the UK to GEO: A Lunar Gravity Assisted Journey

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ABSTRACT

The aim of this project is to assess if lunar gravity assist transfers from low earth orbit (LEO) to geostationary orbit (GEO) for a number of UK based launch sites are more effective than the traditional propulsive transfers currently used. Improvement in delta-v expenditure for such missions will support increased payload mass to GEO, potentially expanding the opportunities for the UK launch sector. This comparison was made using a novel mission planning method, the v-infinity globe, which allowed for the calculation of delta-v budgets, required for the calculation of payload mass available to GEO, and the overall mass budget.

Upon comparison with standard propulsive methods, transfers using lunar gravity assist provide highly efficient paths to GEO, with savings of up to 1.25 Km/s for near polar orbits and up to almost 30 percent payload mass increase, but at the cost of longer transfer times of at least 6.2 additional days. This makes the use of lunar gravity assist transfers to GEO highly attractive for all launch sites in the UK, as despite the longer transfer time the increased overall payload mass to GEO makes them very effective, when transfer time is not a major mission design factor but payload mass is at a premium for mission designers.

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1 INTRODUCTION

This report covers the topics of orbital dynamics and mission planning, these denote the equations of motion that model the movements of bodies in orbit of planets and moons, together with the application of changes in velocity known as burns to change the orbit of a satellite, such that it can enter a desired orbit. These principles are used to model the orbits and orbital maneuvers needed to place a satellite into a desired orbit. These techniques have been used to investigate the effective use of lunar gravity assists to launch from a site in the UK to the geostationary ring. Orbital dynamics form the basis of the technical work carried out and provides the majority of the data driven conclusions to this work. The mission planning aspect of this work entailed research into real world aspects such as the positions of launch sites available in the UK as well as the launch capability from said sites. This contextualises and provides comparison to the data provided by the orbital dynamics work to create useful conclusions from this project.

The main body of the technical work in this project is programming a model using MATLAB to produce data that illustrates how both a propulsive approach to orbital insertion and an approach using a lunar gravity assist compares, in terms of the delta-v expenditure and time of flight values needed. The orbital paths have been generated using the model to practically assess the difference and effectiveness of each launch plan.

1.1 Background and Context

The space industry around the world has seen consistent growth over the last five to ten years with a sharp drop in launch cost due to innovations in recoverable rockets and small launch vehicles [15]. This has opened space applications to many companies that were previously costed out of the market. In this light the UK government has deemed it a good investment to expand the space industry and especially launch capability available domestically [8]. The current proposed UK launch sites and air launch systems are focused on high inclination orbits like polar and sun-synchronous [9]. As the market in the UK expands and small launcher technology improves, access to more orbit types will be necessary to attract a larger customer base. The geostationary ring holds many opportunities as satellites age out and need to be replaced over time. If the UK were able to fill that market, launches to the orbit could generate consistent revenue. From the currently planned UK launch sites the available inclinations prohibit direct injection into the zero degree inclination needed for geostationary orbits (GEO). To allow the UK to meet this need trajectories need to be investigated, modelled and planned to calculate the needed delta-v to both extended and circularise the orbit as well as de-inclinate from the near polar orbits that can be achieved from these launch sites. Then using computer modelling these trajectory plans can potentially be improved upon using lunar gravity assists. By using the natural satellite to change orbital elements the delta-v requirements may be reduced, especially for the inclination change which is a major source of expenditure in this mission architecture.

A review of literature has not found any current public research or planning into the capability to launch from the UK to the geostationary ring specifically, but previous studies have identified the use of trajectories for the lunar gravity assist trajectory plan [13]. The aim of this project is to asses if alternative transfer methods will provide a viable system for launching from the UK to the target orbit. This project uses a modified patched-conic approximation approach called the v-infinity globe method to map lunar assist trajectories. This method provides the capability to plan a final orbit from the characteristics of an encounter allowing for the calculation of the assist characteristics needed to go from the encounter orbit to the final target orbit.

1.2 Objectives

The main objectives for this project were as follows:

- Objective 1: Provide an overview of UK launch sites and possible initial orbits
- Objective 2: Find delta-v values for mission plans using solely propulsive methods
- Objective 3: Find delta-v values for mission plans using lunar gravity assist
- Objective 4: Find mass budgets using available launch vehicle's payload to orbit and compare propulsive and lunar mission plans

1.3 Overview of Final Report

In this dissertation the body of work is split into four major sections, the first section details research into the current techniques used to reach GEO and current research similar to that completed in this project. Then research into the available and soon to be available launch sites in the UK to identify both launch capability in terms of the vehicles that can be utilised and the inclinations that each site can insert into. The second section details the work to create baseline values utilising the initial orbits from the launch site research and then implementing impulse maneuvers to desired orbital parameters calculating the delta-v needed for each manoeuvre as the point of comparison for the lunar assist transfers. The third section then covers the derivation of a methodology used to find the parameters of a lunar gravity assist needed to use the moon to attain a target set of orbital parameters, the orbital paths and delta-v needed to carry out such a transfer is then calculated. Then with the delta-v budget the payload mass to GEO is calculated. The final major section analyses the results of this work to compare the effective use of each transfer and discusses their use in the context of launching from the UK.

2 LITERATURE REVIEW AND BACKGROUND RESEARCH

This section is comprised of two parts, firstly a literature review of existing geostationary transfer methods and research into earth-earth transfers using the moon. Secondly background research into the current and future launch sites in the UK to analyse the current orbits, launch vehicles and potential payload masses that could be launched from these sites.

2.1 Literature Review

2.1.1 The Geostationary Orbit (GEO)

With the target of this report being to investigate methods to reach geostationary orbit it is important to first define what GEO is and why it is important for specific satellite applications. This paper [21] describes the technical aspects of the GEO. The primary characteristic of the GEO is that the period of the orbit is equal to that of the sidereal day, that being the time it takes for the earth to turn once on its axis with respect to the stars acting as an fixed reference. An orbit that fulfils this requirement does not move relative to the surface of the earth allowing it to act as a fixed point in space for certain satellite applications. For a satellite to have this property a specific orbital setup is needed, requiring an orbital inclination of zero degrees and an orbital altitude of 35,786 Km. This allows the orbit to have the correct period, and with zero inclination the path of the satellite follows that of the rotation of the earth keeping it fixed relative to a position over the earths surface.

To provide some context on the importance of geostationary satellites this website [2] details some of the main uses. As the geostationary ring allows for the satellite to stay over the same point of the earths surface across time, its main use is in communications. As it stays in a fixed point in the sky a ground station communications dish can be set to look at that point in the sky. As a result it does not require the adjustment that is needed to take into account the relative orbital motion that is present in low earth orbit (LEO) communications satellites, making the ground station infrastructure much less complex. Another benefit of the fixed point characteristic is that it can be used as a relay point between any two points on earth that a geostationary satellite has line of sight with and will provide constant service as it will never cross a horizon. The other major use for these satellites is for weather analysis as their fixed points allow any earth observation to take repeated images from the same position and compare any changes, allowing weather patterns to be more easily seen. Although due to the high altitude of GEO the angular resolution achievable is limited.

2.1.2 Current Geostationary Transfer Methods

2.1.2.1 The Hohmann Transfer

In the current satellite launch market GEO is attained through standard propulsive means, using only an engine to change the orbital velocity of the satellite, without the use of maneuvers such as the gravity assist. This is detailed in the book [19] where the authors outline typical orbital maneuvers for entering GEO launched from Kourou in French Guiana, Cape Canaveral in the US and Baiknour in Kazakhstan.

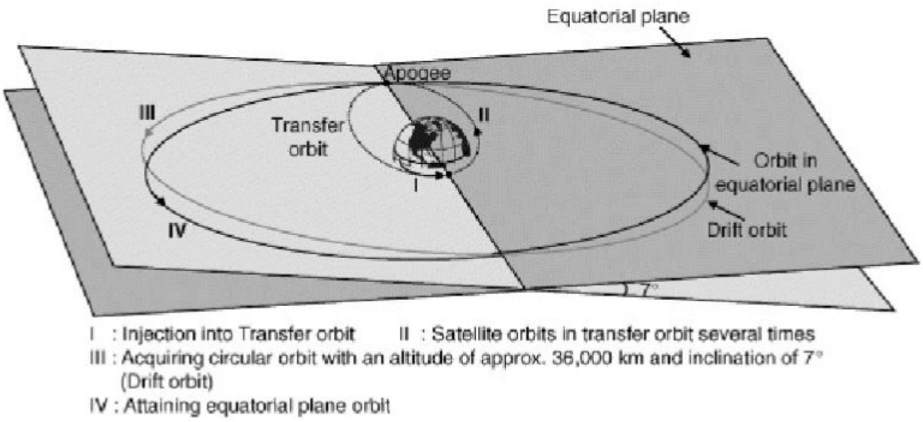

Figure 2.1: A Generalised Bi-Elliptic Transfer [19]

The launches are comprised of a similar set of maneuvers from each site even with differing initial conditions (eg inclination of initial orbit from each site). The first step is to launch the satellite into an initial near circular parking orbit (I in Figure 2.1) with the lowest orbital inclination possible for the site. Secondly mission then executes a burn to raise the maximum altitude of the satellite to near the geostationary altitude, attaining a geostationary transfer orbit (II). Then at the apogee point a second burn is performed to circularise to orbit (III). Thirdly the orbital inclination is changed to zero degrees to attain a near GEO, (IV). Finally corrections are made to trim the orbit such that the satellite enters the correct point over the earth and exact GEO is reached. This technique using two burns, one to raise to a set target altitude and the second to circularise at that altitude is known as a Hohmann transfer, with in this case the addition of a third burn to correct the inclination to zero. This is the standard transfer used by most launches to GEO. As the owners of the launch sites (US, EU and Russia) have access to heavy lift launch vehicles the payloads delivered to those orbits are substantial (upwards of 1 tonne). UK launch sites do not support these heavy lift launch vehicles, therefore more fuel efficient maneuvers are required to launch heavier payloads to GEO.

2.1.2.2 The Bi-Elliptic Transfer

To increase the fuel efficiency of a propulsive launch to GEO a maneuver called the bi-elliptic transfer can be employed [11]. The bi-elliptic transfer is defined by a set of three burns as shown in the Figure below (2.2).

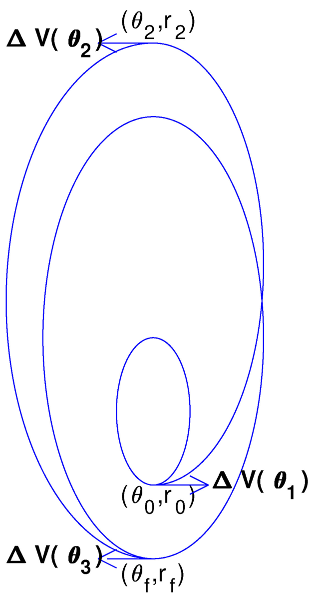

Figure 2.2: Generalised Bi-Elliptic Transfer [11]

The first burn (θ_1) is to raise the apogee of the orbit to an altitude higher than that of the final planned orbit, followed by a coast phase. A second burn at apogee (θ_2) to raise the perigee of the orbit to that of the target altitude is carried out. Finally a burn at perigee (θ_3) is carried out to attain the target orbit characteristics. These transfers can be more effective than the direct Hohmann transfer when the ratio between the initial and final orbit's semi major axis is large enough. This limit is described in Figure 2.3, where $\frac{r_C}{r_A}$ describes this ratio and as shown for values above 11.94, it indicates an efficient bi-elliptic transfer can be designed.

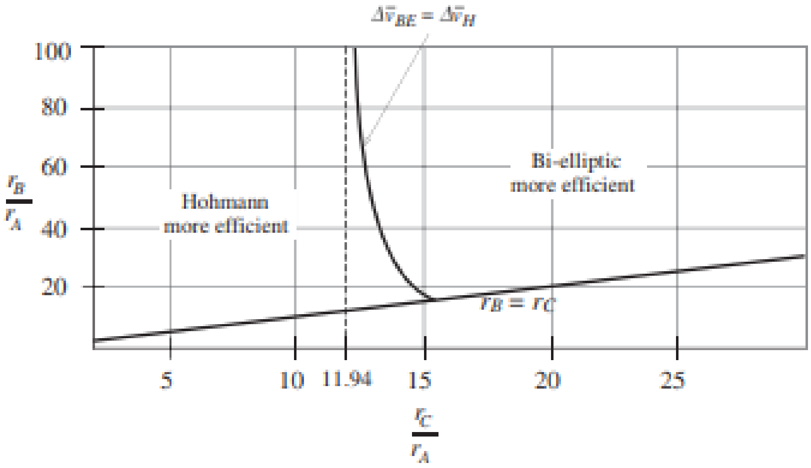

Figure 2.3: Ratio Of Initial to Final Orbits Describing Efficiency Limit [14]

This is because although the first burn costs more delta-v in the bi-elliptic maneuver it is more efficient at adding specific orbital energy to the spacecraft, and as a result the further two burns are limited in delta-v expenditure, resulting in a lower net delta-v than when compared to a Hohmann transfer to the same orbit.

For this project both of the described propulsive methods have been used as baselines to compare to lunar gravity assists but due to launch site limitations the initial orbits will be highly inclined (see Section 2.2) an extra burn would be needed to change the inclination of the final orbit. For this project a paper [13] was used in which it describes a method for integrating the inclination change into the standard description of each set of orbital maneuvers. By splitting the required inclination changes to go from an arbitrary inclination to zero between the burns of each maneuver means the inclination change can be performed in the most efficient way.

2.1.3 Use of Lunar Gravity Assists for Earth-Earth Transfers

The main focus of this project is to investigate the potential efficacy of using lunar gravity assist manoeuvres to place satellites into target orbits. The techniques needed to complete a lunar transfer are investigated in an ESA bulletin published in 2000 [10] describing different techniques to go from initial parking orbits to lunar capture. The bulletin details two main philosophies for attaining lunar orbit;

- **The direct way: fast but expensive;** direct transfers to the moon with one trans-lunar injection burn using an elliptic orbit that encounters the moon at its apogee to then enter the moons sphere of influence, upon which a breaking burn is executed to remain in lunar orbit
- **Indirect ways: slow but cost effective;** using the same direct transfer method but using the moons influence to go into wider orbits that upon a second encounter with the moon provide more favorable conditions, reducing the delta-v needed for the breaking burn and making the transfer more efficient, though the use of these techniques greatly increases the time of flight for the transfer

For this project the main take away from this paper is that for the use of lunar gravity assist to move into a target orbit then the fast but expensive transfer type is the most appropriate, as the more intricate lunar encounters detailed in the slow but cost effective methods try to reduce the relative velocities of the spacecraft and the moon to reduce the capture delta-v. This is a downside compared to the direct way because when leveraging gravity assists the higher relative velocity allows for more flexibility in attainable orbits post gravity assist, this will be further explained in Section 2.1.3.

Other papers have described types of mission plans where the moons gravity is used to complete earth - earth maneuvers.

A first paper [13] covers the use of lunar gravity assists for geostationary transfers using different methods than the ones used in this project, from the work in the paper it identifies that for high inclination changes the use of lunar gravity assists can provide an effective method for transfer from low earth orbit (LEO) to GEO. It makes the same comparisons that are made in this project by comparing the use of lunar assists to propulsive methods, the commonly used Hohmann and more efficient bi-elliptic transfers. The paper draws the conclusion that for orbits with a higher inclination than the moon's the use of lunar gravity assist can provide the most effective method of transfer between LEO and GEO. Though the paper is operating from arbitrary initial orbits and does not integrate the real world aspects of launching from a specific point or payload mass. The paper also uses a different method attain its results so can provide a good point of comparison to validate the results generated from this project.

A second paper [17] considers the use of lunar gravity assists to facilitate high inclination change maneuvers in comparison to other propulsive trajectories. The work also compares the accuracy of different mathematic methodologies using both a 3D patched-conic approximation approach and a spatial circular restricted three-body problem (SCR3BP) to simulate the trajectories and then assesses the accuracy of the patched-conic approach (see Section 2.1.4.1 for detailed description of patched-conic approximation). The results of this work highlight the good agreement of the two models, supporting the accuracy of the patched-conic approximation for delta-v calculations, as the difference between each method is shown to be small across all scenarios. Though the delta-v savings for low altitude final orbits and low inclination changes is shown to be minimal or even less efficient when using the lunar gravity assist approach, the study shows that in cases where the orbits are raised from their initial altitude and have high inclination changes delta-v savings can reach 3 kilometers per second. These findings strongly support the research into the use of lunar assist trajectory design for transfers from LEO to the geostationary ring.

A third article that reports a similar investigation to the one being carried out in this project describes a scenario of launching a geostationary mission from a launch site in the Republic of Korea [12]. This provides similar parameters to the one in this project as initial orbits are heavily restricted due to geographical position, with missions launching into an 80 degree initial orbital inclination. The paper plans and simulates multiple lunar gravity assist paths comparing how each variation changes the delta-v needed to insert into the target GEO. It concludes that lunar gravity assist can execute effective maneuvers for changing the inclination of a high inclination orbit to that of near zero when targeting the specific geostationary target. However this work does not make any comparison to standard maneuvers and only identifies the differences between different types of lunar interaction.

2.1.4 Gravity Assist Modeling and Planning Method

2.1.4.1 Patched-Conic Approximation

The main aim of the project is to asses the viability of the use of a lunar gravity assist in earth-earth transfers to the geostationary ring, and to do this a method is needed to model this encounter. In this case the patched-conic approximation has been used. This method uses the relative velocity between the satellite and assist body (eg the moon) to calculate the velocity vector and position of the satellite after the encounter, with respect to the common central body that both the satellite and assist body are orbiting (eg the earth).

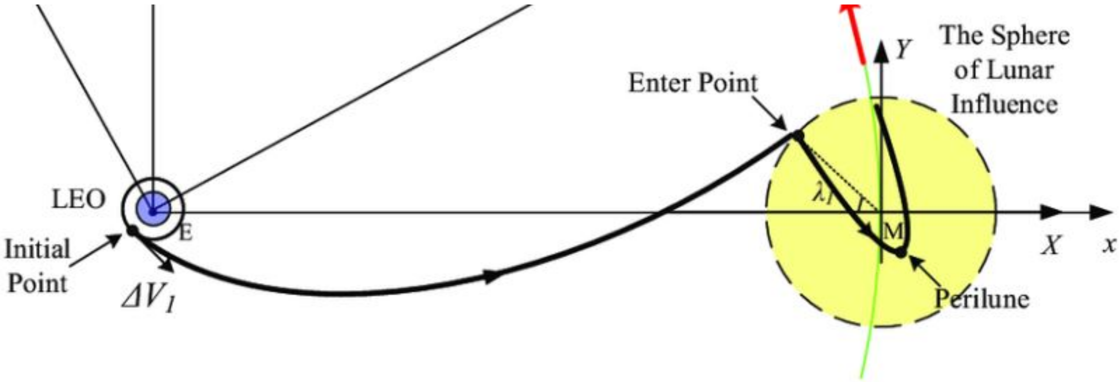

Figure 2.4: Patched-conic example image [22]

In a general case a satellite is put onto an orbit that intercepts the sphere of influence (SOI) of the gravity assist body. This is a theoretical field which when crossed into it is decided that the spacecraft is under the gravitational effect of the assist body more than the central body that is being orbited by them both. As shown in Figure 2.4 at the point at which the satellite enters the SOI of the gravity assist body the velocity of both bodies is measured. At this point the satellite is unaffected by all other bodies and is only under the influence of the gravity assist body. Using the relative position and velocity between the satellite and the assist body at the point it enters the SOI an orbital path around the assist body is plotted. The satellite then follows this orbit to the point at which it leaves the SOI, at which point the position and velocity of the satellite with respect to the assist body is taken for a second time. This is then translated to its position and velocity with respect to the central body, giving it a new set of orbital parameters. The main characteristic of this method is that the magnitude of the relative velocity at the start and end of the encounter is the same this being the v-infinity value.

This method can be used to calculate any encounter with much simpler sets of equations compared to potentially more accurate simulations such as the SCR3BP method used in [17], where Newtonian dynamics are used to model the encounter.

In this project a method was required to plan the encounter, such that a set of target orbital characteristics are met by the orbit after the encounter.

2.1.4.2 The V-infinity Globe Method

To be able to plan an encounter such that a target set of orbital characteristics can be acquired by the final orbit after a gravity assist maneuver the v-infinity globe method [23] was used. The main difference between this and the general case described in the section before is that this method uses zero sphere of influence. This means that the position of the satellite and assist body are assumed to be the same at the time of the encounter. The v-infinity globe provides a system of equations that takes the initial conditions of an encounter and uses them to construct a ”map” of orbits it is possible for the encounter to produce. The method is based on the v-infinity vector, this being the relative velocity between the satellite and gravity assist body at the point of encounter. It then uses the magnitude of v-infinity and creates a sphere of which all points on its surface, denoted by a pump and crank angle, correspond to a specific orbit around the central body. These orbits are bound by having the same v-infinity magnitude at the point of encounter along their orbit, differing in the velocity vector of the satellite with respect to the central body.

This means that as the v-infinity magnitudes are the same for all points on its surface the orbits that they denote are able to be reached from a gravity assist according to the patched-conic approximation. To target a specific set of orbital parameters lines which correspond to orbits with the desired parameters can be plotted across the surface of the sphere. With a number of set parameters these lines will cross at specific points across the sphere, with these points and the point corresponding to the encounter orbit, the side of encounter and the radius needed to move from initial encounter orbit to target orbit through the gravity assist can be calculated. From a given set of encounter parameters the sphere can be constructed and with the target parameter lines plotted all the variables needed for a gravity assist that produces a specific target orbit can be calculated.

2.1.5 Research Gaps

Overall current research shows interest in the use of lunar gravity assists to perform earth-earth maneuvers and they seem to provide effective reduction in delta-v budgets when moving from low earth to medium or high earth orbits as well as changes in inclination. Current research of this type doesn’t employ the v-infinity globe method detailed in Section 2.1.4.2, this report will help highlight its efficacy as a mission planning tool for proof of concept earth-earth lunar gravity assist transfers. The current research does not cover launches specifically from the UK, allowing the work from this paper to be used as a baseline for proof of concept missions designs using launch services from the UK.

2.2 Background Research

The first step of this project's work was to identify the initial orbits that satellites can be inserted into when executing mission plans that launch from the UK. The UK launch sites and launch capability selected are those available currently and planned for the near future. The main limitation to the orbits available at any site is the geography of the site, this is due to the staging that rockets must perform to attain orbit. As these parts fall away from the rocket at high altitude the path of the rocket cannot pass over inhabited land. This greatly reduces the capability to launch into low inclination orbits in the direction of the rotation of the earth (prograde), which is essential for GEO. Continental Europe prevents these low inclination prograde launches, and therefore a launch must depart towards the North Sea at high inclination, dropping the rocket stages into the sea to enable an orbit in the prograde direction.

Current missions in GEO are predominately comprised of multi tonne satellite platforms, however the market for small mass payloads to GEO are a new development with the smallest payload mass for a GEO mission identified as 400 Kg launched in May of 2023 [1]. With trends of reducing size and mass for technical components of satellites, as well as developments in radio communications technology, a market for microsatellites (11-200 Kg) may emerge in the near future allowing for much lower payload masses.

As part of the UK government's recent push to expand the space sector, a brochure describing the different sites has been published [9]. Seven locations are identified, comprising four air launch sites and three ground vertical launch sites. From these sites, three have been chosen as archetypes for the launch market available from the UK.

- Spaceport Cornwall, England, providing the air launch capabilities of Virgin Orbit. Air launch technologies allow for smaller, cheaper rockets to attain good payload to orbit characteristics, as well as launch flexibility
- SaxaVord Spaceport, Scotland represents a medium launch capability, as it provides a higher payload mass to orbit than any other site in the UK, so is the most viable to be used for current geostationary payloads
- Space Hub Sutherland, Scotland represents a small satellite payload launch capability that may become a larger share of the geostationary market in the future

The other UK vertical launch sites were excluded as they did not provide any major difference in capability or initial orbit parameters than those selected. When looking at air launch sites the others were excluded as they would use the same or similar vehicles to that used by Virgin Orbit or its competitor Pegasus (developed by Northrop Grumman), which provide similar payload mass to orbit. No other air launch vehicles are in a phase of development that is near completion with performance characteristics in the public domain. Due to the nature of air launch the position and initial inclinations from any site in the UK will be similar as they launch from the same areas over the North Sea irrespective of their starting airport. The combination of Spaceport Cornwall using the Virgin Orbit vehicle has been chosen, as Cornwall is the only current site with a completed launch test, and although the Virgin Orbit launch was a failure at that site, it has been successfully launched in other missions from the US.

2.2.1 Launch Site Specifications

In this section the specifications of each launch site will be covered, including a brief description of the site, available launch inclinations and a launch vehicle or prospective payload mass to orbit. This informs the initial orbits that each site will provide for the technical phase of the project.

2.2.2 Spaceport Cornwall

Spaceport Cornwall is an air launch site in the southern UK with its main operator being Virgin Orbit [7]. The site and company aim to be the premier air launch site in the UK providing both launch and ground services to prospective clients. The launch service is provided by LauncherOne, this is one of only two air launch to orbit (ATLO) platforms currently operational globally, though its first mission from the UK on 9 January 2023 failed due to a technical fault, the platform has had several successful missions launched from the US starting in 2021. The rocket is designed to be launched from a Boeing 747 aircraft platform from high altitude, allowing flexible launch positions and azimuths as well as increasing payload mass due to lower atmospheric loses.

Due to the flexible nature of the air launch system the position of launch and available inclinations are dependent on mission need, Figure 2.5 indicating the range of launch inclinations available from the UK [16].

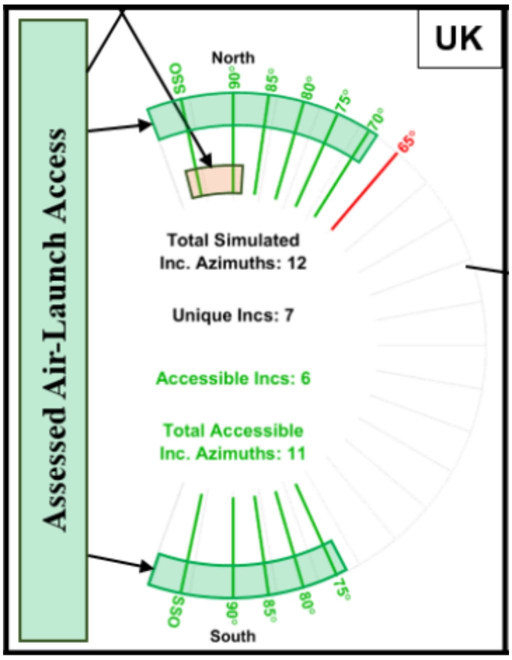

Figure 2.5: Available UK Air Launch Inclinations [16]

From this figure the minimum launch inclination of 70 degrees is identified. The service guide published by Virgin Orbit [7] states a payload mass to LEO of 500 Kg and an orbital altitude of 230 Km circular orbit at zero degrees inclination. Due to not launching directly eastwards at zero degree inclination from the UK, this payload mass should be reduced as it cannot make full use of the rotation energy of the earth. Therefore in order to account for this and to support realistic technical calculations a value of 20 Kg has been removed from the launch mass. In May of 2023 Virgin Orbit filed for bankruptcy. Even though launch operations for this specific launcher have ceased and are unlikely to resume, the values from their technical specifications have been used, and conclusions drawn on the viability of such a mission.

2.2.3 SaxaVord Spaceport

SaxaVord Spaceport is a traditional vertical launch complex located on the island of Unst in the Shetland Islands and is the most northern prospective launch site in the UK. The site aims to provide ground facilities to prepare for launch as well as tracking and ground control capabilities for launch execution. With the possibility to use many different launch vehicles one specific launcher has not been identified from the literature, but from the information available on the website for SaxaVord [5] the launch complex aims to be able to execute missions putting payloads of 1.5 tonnes into high inclination LEO. The website provides launch azimuths of between 330 and 075 degrees true, therefore to calculate the launch inclinations the following equation is used, where a is the launch azimuth in degrees and l is the latitude of the launch site;

$$i = \cos^{-1}(\sin(a)\cos(l)) \tag{2.1}$$

Using the provided azimuths and the published latitude of the site being 60.81 degrees [9], this finds the minimum inclination available as 61.89 degrees. As the information available to the public does not provide a specific orbit that the mass can be inserted into, an assumed initial orbital configuration of 300 Km circular has been chosen to be well clear of any potential atmospheric effects.

2.2.4 Space Hub Sutherland

To represent a potential small satellite mission payload Space Hub Sutherland has been chosen as the final launch site used in the project. The launch complex located in northern Scotland is home to the Orbex company who are the main user of the site and its primary investor [20]. The company aims to leverage 3D printing fabrication of critical components, such as the engine, to reduce construction price and time. From their website [20] the available launch inclinations and payload values are provided with minimum inclination of 83 degrees and 185.6 Kg to a circular 300 Km orbit.

2.3 Work Outputs

From the work completed above, the inclinations, orbit characteristics and payload masses used in the technical section have been identified and will provide the initial masses for fuel mass calculations once delta-v values are found. The information is collated in the table below,

Table 2.1: Collated Research Information

Launch Site	Inclination (Degrees)	Payload Mass (Kg)	Orbit Characteristics (Km)
Spaceport Cornwall	70	480	230x230
SaxaVord Spaceport	62	1,500	300x300
Space Hub Sutherland	83	185	300x300

2.4 Summary

In summary, this chapter describes previous research into earth-earth lunar gravity assists, and using several different calculation methodologies showing that they can provide an effective method for executing high inclination change maneuvers. The research done into UK launch sites provides the basis for both the initial orbital values and the payload mass deliverable to the initial orbits. This data will be utilised to evaluate the viability of missions from each launch site and sets the groundwork for the technical section.

3 MISSION PLANNING WITH STANDARD PROPULSIVE METHODS

3.1 Mission Simulation

3.1.1 Orbital Elements

For this project the orbital paths for the different orbits the satellite will utilize need a way to be defined. This is done with the use of orbital elements, these values are used to define the orbit of any body around a central point.

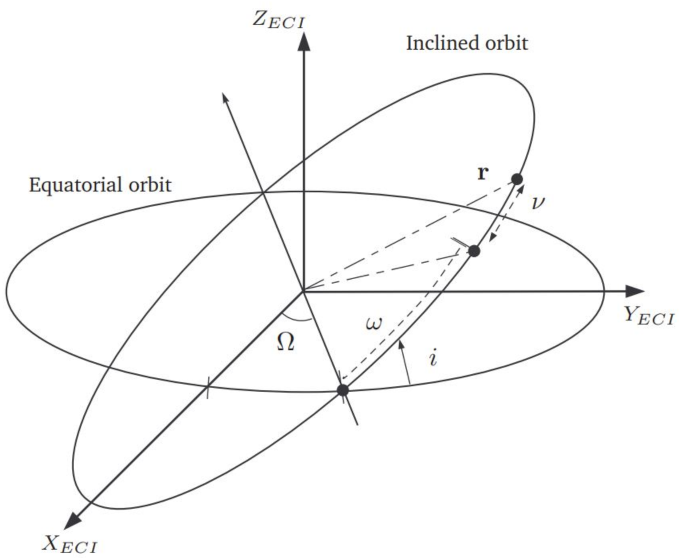

Figure 3.1: Classic Orbital Elements [18]

The first two values determine the orbits size and shape;

- a is the semi major axis which is defined as half of the sum of the distance to the maximum and minimum points on the orbit which defines the overall size of the orbit
- e is eccentricity which describes how elongated the orbit is compared to a circle at zero as the value increases becoming more stretched

The next two values define its position around the body it is orbiting;

- Ω is the right ascension of the ascending node which defines the angle between the point on the orbit at which the orbit crosses the line of the equator and a set reference direction
- i is the inclination of the orbit denoting the angle between the plane of the orbit and the equator
- ω is the argument of periapsis which defines the orientation of the ellipsis about the central body

These values allow for MATLAB to draw orbits about the central body and with the final value ν the true anomaly which changes through time between 0 and 360 defining a point along the orbit at which the spacecraft resides.

3.1.2 Calculating Delta-v

For each mission set multiple changes in speed are required to move between an initial orbit and a new target orbit. This is known as delta-v and each maneuver is called a burn. By using the system of equations in Appendix A, a set of position and velocity vectors across time can be calculated from the orbital elements of each orbit. For each burn the two orbits will share a position at which the burn is performed, as for this project the burns are assumed to be instantaneous changes in velocity. With the known shared position the absolute difference in velocity is measured, this is taken as the delta-v needed to be used to move between the orbits.

3.1.3 The Rocketry Equation

Once the delta-v values for each path have been found the amount of fuel required to execute the path with a given rocket engine will need to be calculated this is then used to fulfil objective four calculation of the mass budgets. This can be carried out using the rocket equation as shown below,

$$\Delta V = I_{sp} g_0 \ln(\frac{m_{initial}}{m_{final}}) \tag{3.1}$$

With a known delta-v requirement from the simulation work, the specific impulse (ISP) of a theoretical engine that is identified in further work and the payload mass deliverable to LEO found when researching the launch site capability. From these values the mass of fuel needed to complete a mission plan can be identified. As such final deliverable payload to the GEO can identified as the total mass delivered to LEO minus the fuel required to reach GEO for comparison between the different mission plans as well as an assessment of the viability of a mission from the UK.

3.2 Propulsive Transfer to Geostationary Orbit

3.2.1 Novel Method For Inclination Change

As the initial orbits are highly inclined a change in inclination to zero degrees is needed, using the equations from [13] a method for integrating the inclination change into the standard burns of either the Hohmann or bi-elliptic transfer can be found,

$$i_i = \frac{1}{x^{\frac{3}{2}}} \frac{(\sqrt{\frac{2x}{1+x}} - 1)}{H(x,i)} \sin i \tag{3.2}$$

$$H(x,i) = \left[\frac{2}{x(x+1)} + \frac{1}{x} - \frac{2}{x}\sqrt{\frac{2}{1+x}} \cos i \right] \tag{3.3}$$

Where i is the initial orbit's inclination and x is the ratio between the final and initial orbits radii, this will provide i_i which when subtracted from the initial orbit's inclination will give the change in inclination in the first burn. The remaining inclination will be removed in the second burn when the satellite is at its maximum point in the transfer. This method will be used to remove the need for a specific inclination change maneuver and by using the ratio between orbital radius values this means the majority of the change in inclination is performed at high altitude. This makes the inclination change more efficient as higher the altitude it is performed at the lower the delta-v required to make such a maneuver, using this method allows for the most efficient inclination change with the fewest maneuvers.

3.2.2 Hohmann Transfer

To move from the initial orbits that the satellite would be placed into post launch to the target GEO a transfer method using two major burns called the Hohmann transfer is the most commonly implemented. The first being a burn to raise the apogee of the initial orbit to the altitude of the final target orbit this being the transfer orbit. Then another at the apogee of the transfer orbit to circularise the orbit such that it meets the characteristics of the final target orbit. In this case as mentioned in Section 3.2.1 these burns will have a change in inclination integrated into them. With the initial orbits found in the research section and a transfer orbit with a known target altitude the orbital elements for each can be found. Using the previous methods described to find the delta-v values needed the delta-v budget needed to move from the initial orbit to GEO can be calculated.

An example of the Hohmann transfer orbits used for moving from the initial orbit provided by Space Hub Sutherland to GEO is graphed in Figure 3.2.

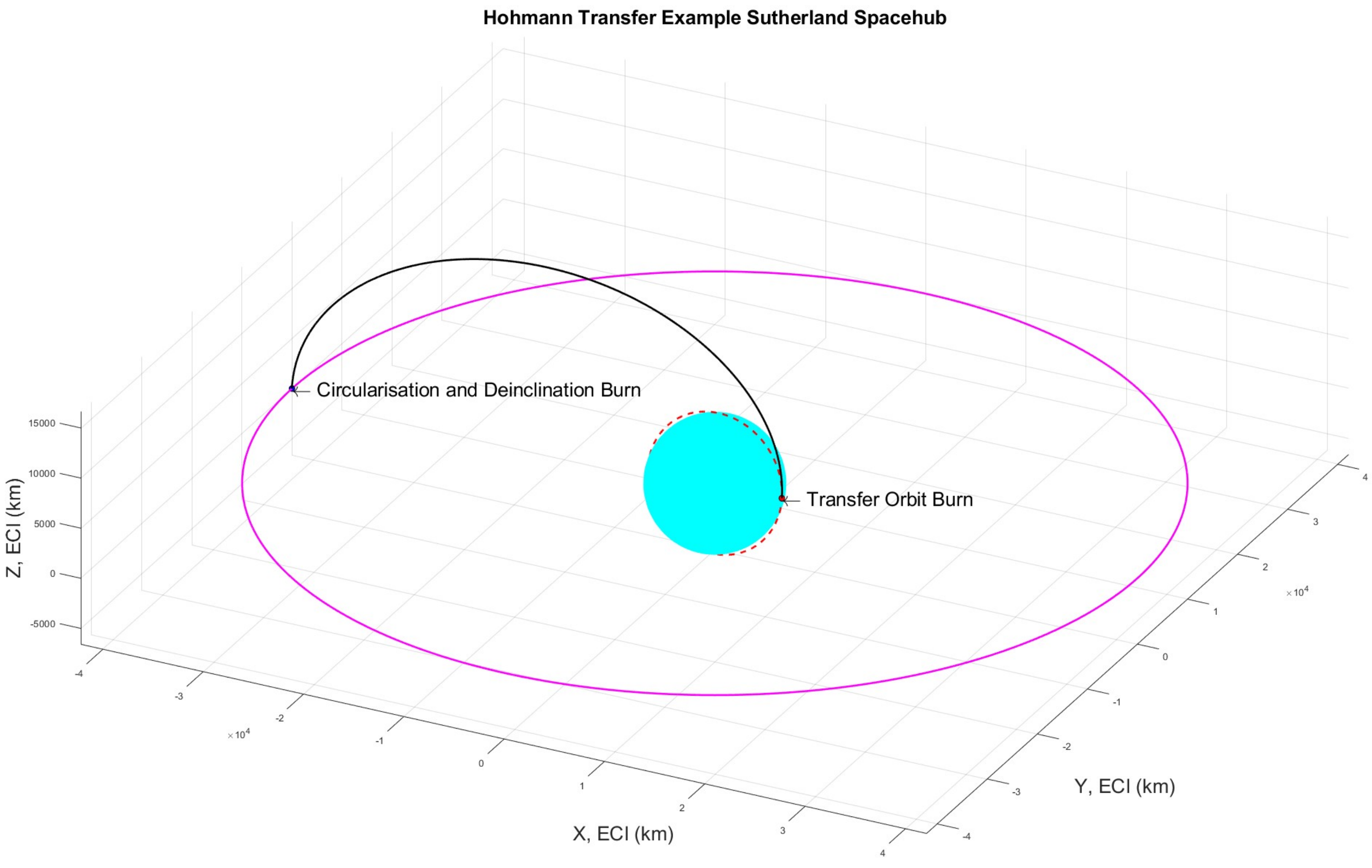

Figure 3.2: Hohmann Transfer from LEO to GEO

Where the red dotted orbit is the initial orbit for the Space Hub Sutherland launch site, the black orbit is the transfer path taken to the geostationary ring and the magenta orbit is GEO. The values for the delta-v budget calculated are shown in Table 3.1 where delta-v1 is for transfer orbit burn and delta-v2 for the circularisation and declination burn as shown on Figure 3.2. The time of flight needed to complete the transfer is calculated using the method described in Appendix C.

Table 3.1: Hohmann Transfer ΔV budget

Launch Site	ΔV_1 (Km/s)	ΔV_2 (Km/s)	Total ΔV (Km/s)	Time of flight (Days)
Cornwall Spaceport	2.4837	2.3281	4.8118	0.2192
SaxaVord Spaceport	2.4652	2.3980	4.8633	0.2198
Space Hub Sutherland	2.4599	3.0221	5.4819	0.2198

3.2.3 Bi-Elliptic Transfer

As a second point of comparison to the lunar gravity assists the bi-elliptic transfer method has been implemented, as shown in the theory section this method is compromised of three burns. These are the initial burn to a transfer ellipse with its maximum altitude much higher than the final target, second burn to raise the periapsis to the target altitude at apogee and in this case this is coupled with a de-inclination burn within the same maneuver. Finally a burn at periapsis to circularise at the target altitude. These maneuvers can be more efficient than the two impulse Hohmann when the ratios between the initial and final orbit semi major axis values are high enough, when calculated between the initial LEOs and GEO the ratio is found to be 6.32 (SaxaVord and Sutherland) and 6.39 (Cornwall). While these are below the limit shown in Figure 2.3 this may not exclude the use of bi-elliptic entirely, as the transfers in this work require high inclination changes. The high maximum altitude of the transfer orbit can be utilised to make the inclination change much more efficient as the satellites orbital velocity becomes low at apogee. Due to this the delta-v needed to change the inclination of the orbit is greatly reduced as shown in Equation 3.4, where the amount of delta-v used is highly dependent on orbital velocity.

$$\Delta v = 2v \sin\left(\frac{\Delta i}{2}\right) \tag{3.4}$$

So by using the high altitude transfer and executing the plane change at the maximum point the delta-v expenditure can be reduced compared to the Hohmann system. As in this case, the trade-off between the extra delta-v used to boost the maximum altitude to beyond the target then reduce it again to circularise and the delta-v reduced in the plane change may reduce the delta-v total. The orbital path of such a transfer method is shown in Figure 3.3.

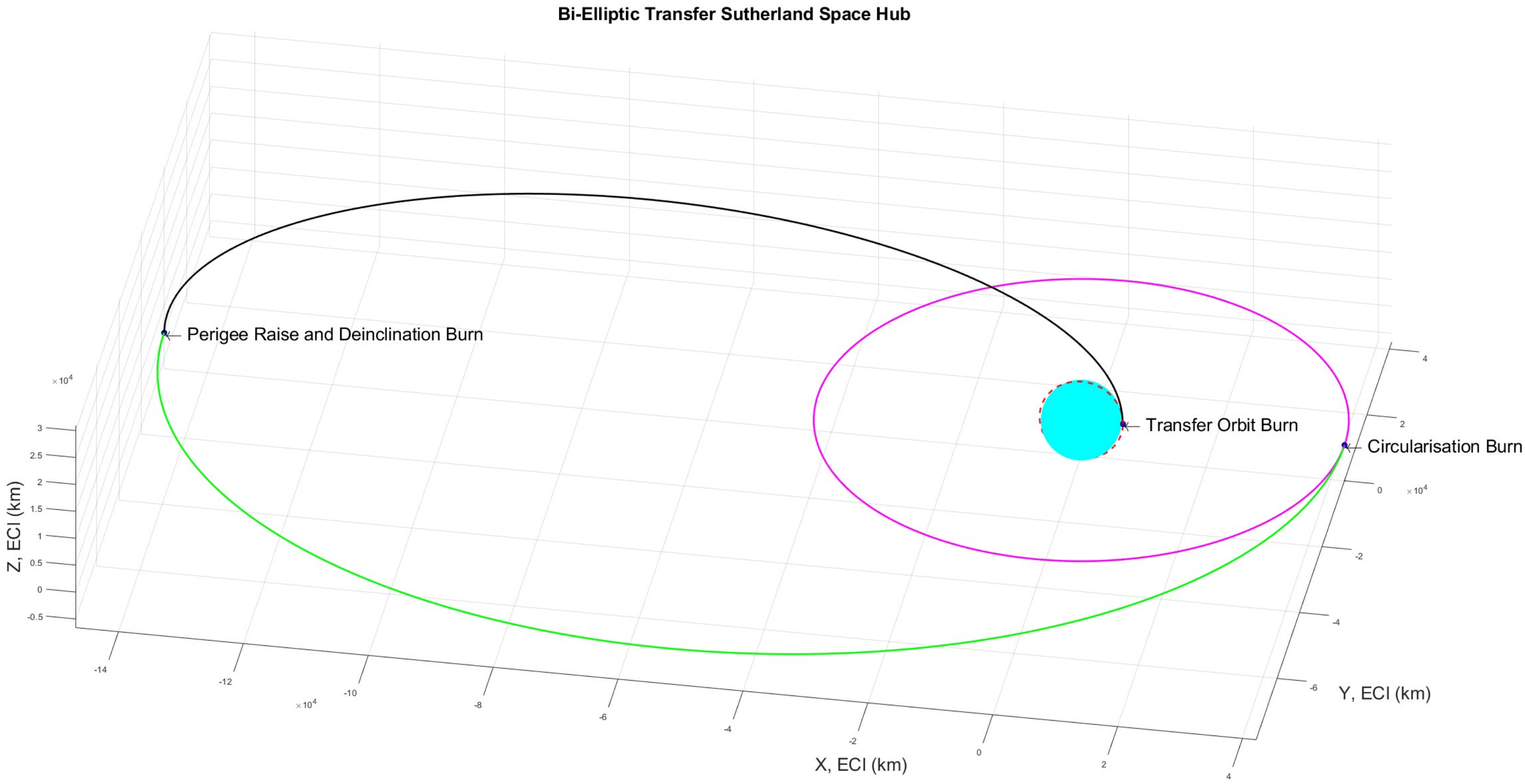

Figure 3.3: Bi-Elliptic Transfer from LEO to GEO

The main choice in the planning of this maneuver is the apoapsis of the transfer ellipsis. To find the most efficient path the delta-v needed to complete the transfer to geostationary using a set of transfer apoapsis radius values from 45000 Km to 350000 Km is calculated. Then to find potential improvement in delta-v expenditure the difference between each transfer path and its corresponding Hohmann transfer for the launch site is found. The bounding of the radius values are chosen as the minimum point 45000 Km is a starting point just above the target orbit, so will show only minor improvement and the maximum is close to the minimum orbital altitude of the moon and in a real world scenario the satellite would want to avoid any major gravitational influences from the moon when executing its paths, to prevent changes in the return orbital path. The differential values are plotted against transfer radius in Figure 3.4.

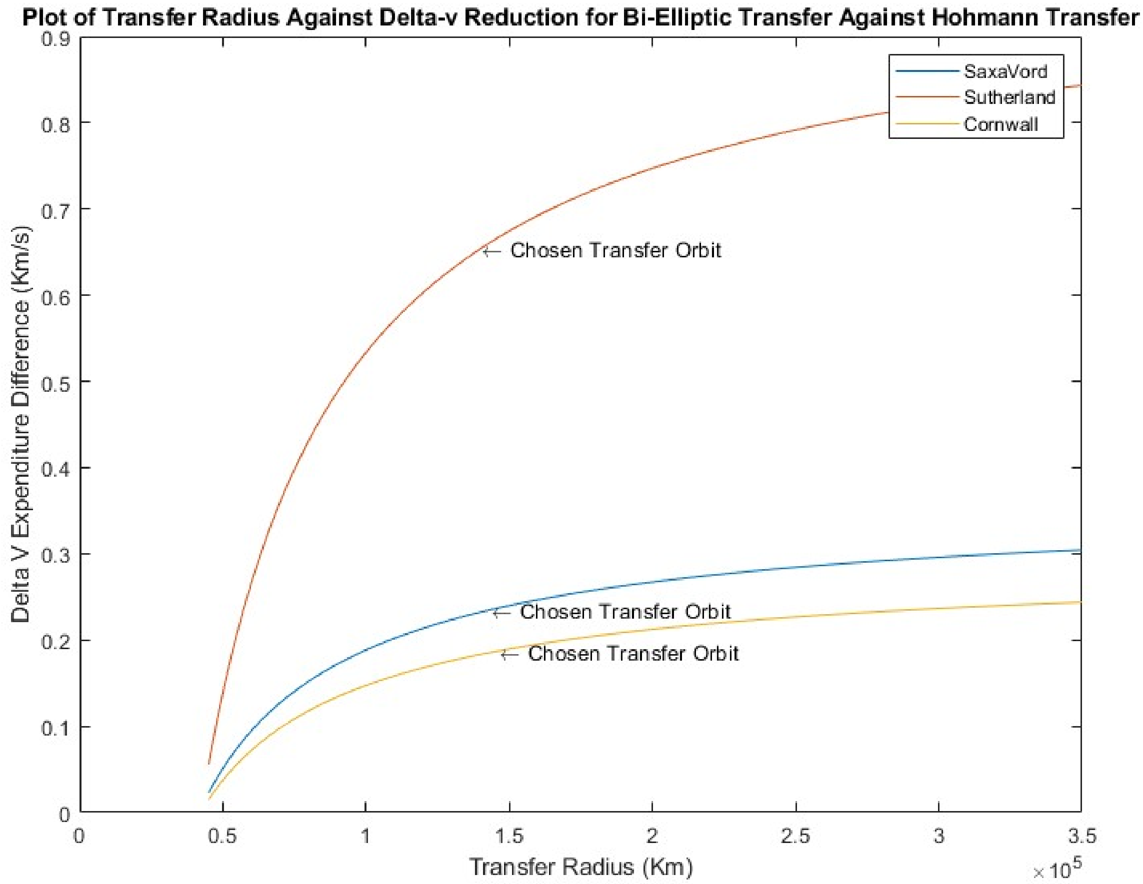

Figure 3.4: Bi-Elliptic Delta-v difference compared to Hohmann Transfer For Set of Transfer Radii

As shown on the graph, the higher the transfer orbit apoapsis radius the higher the difference in delta-v expenditure against Hohmann transfer is seen, this means the bi-elliptic method can provide an effective mission plan when moving from the initial orbit to GEO compared to Hohmann. The graph also shows that as the transfer radius increases the difference in delta-v expenditure per Km plateaus. This effect can be used to choose an effective transfer path as the higher the radius the longer the time of flight needed to reach GEO, increasing almost linearly as shown in Figure 3.5.

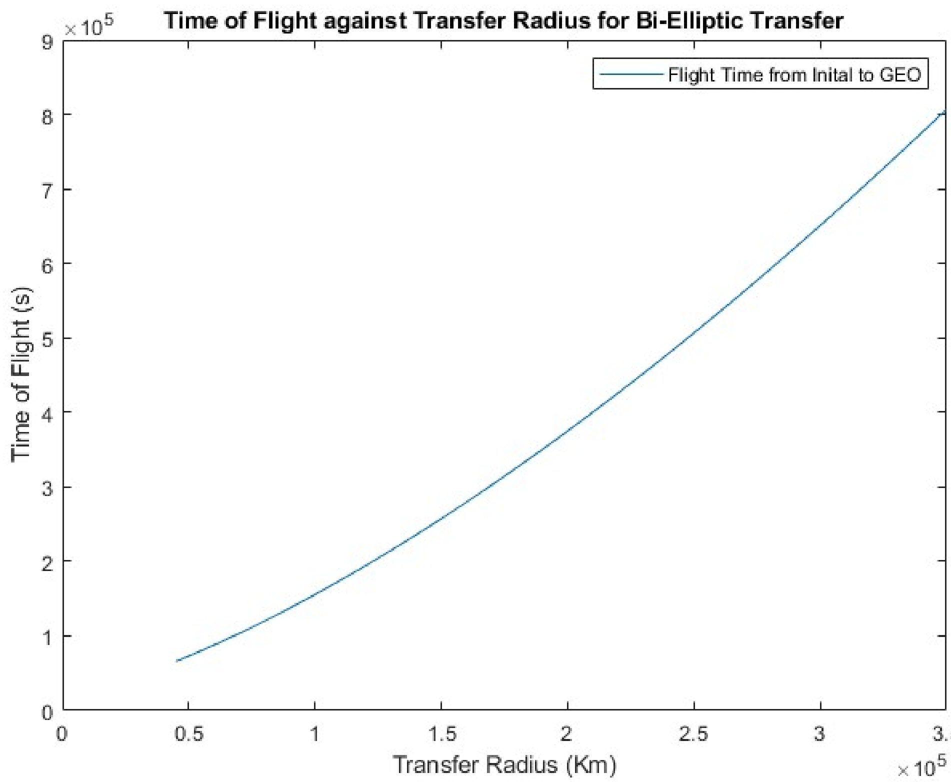

Figure 3.5: Time of Flight Against Transfer Radius For Bi-Elliptic Transfer

With this relationship the transfer paths chosen for the project were those where the difference between each successive delta-v reduction is below one percent, finding the most delta-v reduction for the least transfer radius and thus shortest time of flight. With the transfer radius chosen the delta-v values and specific time of flight for the transfer path for each launch site can be found and shown in Table 3.2where delta-v1 is the transfer orbit burn, delta-v2 is raising the perigee and deinclination burn and delta-v3 is the circularisation burn. The time of flight needed to complete the transfer is calculated using the method described in Appendix C,

Table 3.2: Bi-Elliptic Transfer Transfer Radius and ΔV (Km/s) budget

Launch Site	ΔV_1	ΔV_2	ΔV_3	Total ΔV	Time of flight (Days)
Spaceport Cornwall	3.0104	0.8563	0.7575	4.6242	2.8927
SaxaVord Spaceport	2.9872	0.8926	0.7483	4.6281	2.8162
Space Hub Sutherland	2.9821	1.1059	0.7389	4.8269	2.7395

4 MISSION PLANNING USING LUNAR GRAVITY ASSISTS

4.1 Gravity Assist Methodology

4.1.1 Simulation Assumptions

To be able to calculate the effect of a lunar gravity assist the orbital parameters of the moon need to be set so that the positions and velocities at encounter can be calculated. For this project the parameters of the orbit of the moon have been simplified to make calculations more streamlined. The real parameters of the moon [24] include a non circular orbit with a mean eccentricity of 0.0549 and an inclination to the ecliptic of 5.145 degrees. This gives the moon a variation in its inclination to the equator of 23.5 $\pm$ 5.145 degrees across time. As this project defines no time for the simulation of orbital paths and as no specific date is set for the epoch of the specific inclination chosen is arbitrary, in this project the maximum inclination of 28.64 degrees is used. Further simplifications are made as the orbit used set to zero eccentricity with an orbital radius of 400,000 Km, as the orbit is now circular the argument of periapsis is undefined and set to zero. The right ascension of the ascending node is also set to zero as in a real world scenario these values could be matched between the satellite and the moon by launching the rocket a specific date such that the line of nodes for both are aliened meaning that when the burn to transfer is completed the maximum point of the transfer both intersects the moons obit and is at either the ascending or descending node. As a real world launch plan could attain this condition the value for the ascending nodes for both the satellite and moon are arbitrary so are set to zero. The v-infinity globe method contains one major assumption of using a zero SOI patched-conic approximation. This is where the effect of the lunar gravity is only used at the direct point of encounter rather than using field around the moon to represent its gravitational influence. This means that the systems takes the parameters at the point of encounter uses them to calculated a new velocity vector for the satellite which is then used in conjunction with the fixed point of the moon at encounter to calculate new orbital parameters, meaning the encounter is performed over zero time entering and exiting the moons orbit at a fixed point.

4.1.2 The V-infinity Globe method

4.1.2.1 Technical Concept

To plan the gravity assist maneuvers the v-infinity globe method [23] is implemented. The underlining principle of this system is that it can take the v-infinity magnitude, that being the absolute difference in the velocity of a satellite and a gravity assist body at the point of encounter, and find any other orbit that would have the same v-infinity magnitude at the same point of lunar encounter in its own orbit. This meaning the new orbit is reachable by gravity assist from the initial orbit according to the patched-conic approximation. The system can then be used to find the radius of encounter around the moon needed to complete the gravity assist moving from one orbit to another. In this case an encounter orbit is set and at the point of encounter the position and velocity of the spacecraft and moon is found, from this the v-infinity vector is calculated using Equation 4.1.

$$\vec{v_\infty} = \vec{v_{sc}} - \vec{v_{ga}} \tag{4.1}$$

Where $\vec{v_{sc}}$ is the velocity vector of the spacecraft at intercept and $\vec{v_{ga}}$ is the velocity vector of the gravity assist body in this case the moon at intercept. This v-infinity vector and its magnitude is the essential value that is needed to relate all orbits to all other orbits and is the main value used to construct the globe on which all the orbits are plotted. The globe provides a map that is used to plot encounters, this map is constructed of two angular values the pump and the crank angle. The pump angle describes the angle between the v-infinity vector and the gravity assist body vector made on the outside of the vector triangle made by the three vectors.

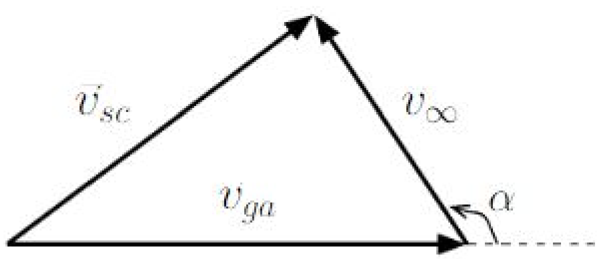

Figure 4.1: Velocity Vector Triangle [23]

Where alpha is the pump angle, this angle describes the latitude on the globe and is responsible for calculating the incoming orbit's energy as it can be used to calculate the semi major axis of any post encounter orbit from just the velocity vectors and an arbitrary pump angle. This mechanism is independent of inclination however, so a second value is needed to resolve the inclination of an orbit plotted on the globe, this is where crank angle is introduced to describe how inclination changes the component values of the v-infinity vector to describe an orbit around the central body.

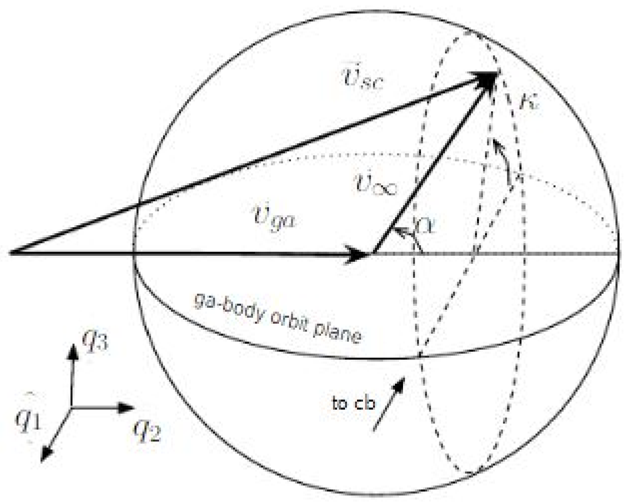

Figure 4.2: Crank Sphere [23]

The crank angle describes the angular orientation of the velocity triangle with respect to the plane of the gravity assist body this denotes the longitude values on the sphere. Using the pump and crank angle to be able to point to any position on the sphere these values can be reversed to a set of orbital elements. By taking any arbitrary set of pump and crank angles a new v-infinity vector with unique components but the same magnitude as the v-infinity calculated when constructing the sphere can be calculated. With the new v-infinity vector the known position and velocity of the moon at encounter Equation 4.1 can be implemented to find the spacecraft's new velocity vector. With the spacecraft velocity and moons position the orbital elements of the orbit can be found by using the method in Appendix B.

To use this method a set of lines corresponding to desired orbital characteristics such as inclination and a target perigee altitude in this project. These lines can be drawn across the surface of that sphere and the point at which they cross will give the pump and crank angle needed to produce an orbit with the two target characteristics. Using the values the v-infinity vector for said orbit can be calculated, by taking the angle between the two velocity vectors the bending angle for the encounter can be found and from this the radius of lunar encounter needed to move from the initial encounter orbit to the final target orbit.

4.1.2.2 Mathematics System

For the initial conditions of the sphere the v-infinity value is calculated using Equation 4.1. Then the initial pump and crank angles for the intercept orbit are calculated by using the equations,

$$\cos(\alpha) = \frac{v_{sc}^2 - v_{\infty}^2 - v_{ga}^2}{2v_{\infty}v_{ga}} \tag{4.2}$$

$$\cos(\kappa) = \frac{v_{sc}\sin(\gamma_{sc})}{v_{\infty}\sin(\alpha)} \tag{4.3}$$

where γ_{sc} is the flight path angle of the satellite about the central body calculated using the equation,

$$\cos(\gamma_{sc}) = \sqrt{\frac{\frac{a_{sc}}{r_{enc}}(1 - e_{sc}^2)}{2 - \frac{r_{enc}}{a_{sc}}}} \tag{4.4}$$

where r_{enc} is the radius to the central body at the point of encounter, e_{sc} is the eccentricity of the intercept orbit and a_{sc} is the semi major axis of the intercept orbit. This allows for the plotting of the corresponding pump and crank angles for the intercept.

To construct the sphere a frame of reference is constructed that allows the system to convert pump and crank angles to a v-infinity vector using the magnitude of the v-infinity used to construct the sphere.

$$\hat{p_1} = \hat{r_{enc}} \tag{4.5}$$

$$\hat{p_3} = \frac{\hat{r_{enc}} \times \hat{v_{ga}}}{\cos(\gamma_{ga})} \tag{4.6}$$

$$\hat{p_2} = \hat{p_3} \times \hat{p_1} \tag{4.7}$$

Where the value $\hat{r_{enc}}$ is the unit vector of the radius from the central body at the point of encounter, $\hat{v_{ga}}$ is the unit vector of the velocity of the gravity assist body and γ_{ga} is the flight path angle of the gravity assist body relative to the central body. As in this case due to the simplifications the moons orbit is taken as circular γ_{ga} is always zero and thus $\cos\gamma_{ga}$ is always one. From this set of unit vectors an equation can be constructed to relate pump, crank and the magnitude of the v-infinity vector at encounter to a new v-infinity vector and thus specific orbit attainable from a lunar encounter given then initial values from the intercept. The equation used is as follows,

$$\vec{v_\infty} = v_\infty \sin(\alpha)\cos(\kappa)\hat{p_1} + v_\infty \cos(\alpha)\hat{p_2} - v_\infty \sin(\alpha)\sin(\kappa)\hat{p_3} \tag{4.8}$$

With this equation the v-infinity vector of any set of pump and crank angles can be found and when used with the Equation 4.1 as the moons velocity vector is known from the encounter parameters the velocity of the spacecraft can be found and thus the post intercept orbit. The pump angles are bounded by 0 to 180 degrees as shown by the velocity triangle (Figure 4.1) and crank angles are bounded by the 360 degrees in circle that can be corresponded around the sphere for each pump angle at each crank angle value.

The project aims to use a lunar encounter to reach the geostationary ring, this orbit type has two main considerations zero inclination and an orbital radius of 42,164 Km. To target these specific orbits lines are drawn across the surface of the pump and crank sphere corresponding to each of these target parameters. When implemented into Matlab a 2D grid search is performed calculating the orbital characteristics of all possible pump and crank angles then plotting the points with the correct characteristics to find these lines, due to limitations with Matlab values the line for 0.2 degrees inclination are plotted. With the crossing points of the lines of target characteristic the v-infinity vectors are found. Then by using intercept v-infinity vector the bending angle of the encounter required to produce the target orbit can be calculated as the angle between it and the v-infinity vector for the target orbit. This corresponds to the bending angle δ, then using the Equation 4.9

$$\sin(\frac{\delta}{2}) = \frac{\mu_{ga}}{\mu_{ga} + r_{pfb} v_\infty^2} \tag{4.9}$$

where μ_{ga} is the standard gravitational parameter of the gravity assist body in this case the moon and r_{pfb} is the radius of encounter. Rearranging the equation to make r_{pfb} the subject allows the system to find the radius of the encounter which can be used to check the viability of the maneuver as if the radius required is less than the radius of the moon, then the gravity assist maneuver to move from the encounter to the target orbits is impossible.

4.1.3 Lunar Gravity Assist Maneuvers

To allow the maneuvers to reach the desired orbits the assist must allow for low inclination orbits this means the orbits must intercept in such a way that both bodies are at the nodes of the satellites orbits. This allows for the low inclinations as the change in velocity provided by the encounter is performed directly at a node, which is the point of intersection between the orbit and the equator, allowing the orbit to acquire a new orbit that is inline with the equator and therefore at a zero inclination with respect to the earths equator. From the initial orbits a burn to a transfer orbit is performed to form an orbit that will intercept the lunar orbit at the descending node of both orbits. As the simplifications for the lunar orbit give it a right ascension of the ascending node of zero as-well as an argument of perigee of zero as a circular orbit, the RAAN is reflected in the initial orbits such that when the apogee of the orbit is extended to the orbital altitude of the moon the intercept point is the descending node of both orbits. As per the patched-conic approximation the two main characteristics of a gravity assist are the radius of encounter, which is set by the requirements of the final orbit and the side of the moon at which the encounter is performed. These are the planet and anti planet hemispheres of the moon, denoting the side facing to or from the earth respectively. This will change the characteristics of satellite's orbit after the gravity assist these can be compared to find the most effective maneuver setup based on mission parameters. As the crank angles for the v-infinity method are bounded between 0 and 360 this will prescribe a ring on the graph for the orbits that denote zero inclination, this will cross the line which defines the target periapsis twice denoting the two possible encounter types of which side the assist is performed. As shown in Figure 4.2 the crank angle is denoted by the angle between the velocity triangle and a line from the central body that is perpendicular to the orbital plane of the gravity assist body, in this case the zero degree crank angle is when the velocity triangle is pointed directly towards the central body. As this is the case crank angles from 0 to 90 and 270 to 360 are planet hemisphere transfers and 90 through 270 degrees are anti-planet hemisphere transfers. For another point of comparison the time of flight to complete the transfer is needed to differentiate and identify the most effective mission plan, as the planet and anti planet encounters will produce differing orbital paths but may not need differing delta-v requirements to each other. With all parameters of the orbits known, method in Appendix C will be used to calculate the flight times for the transfers.

4.2 Results

4.2.1 Spaceport Cornwall

For Spaceport Cornwall an intercept orbit is plotted and the encounter characteristics calculated. This allows for the production of a pump crank angle graph with the lines denoting the target characteristics.

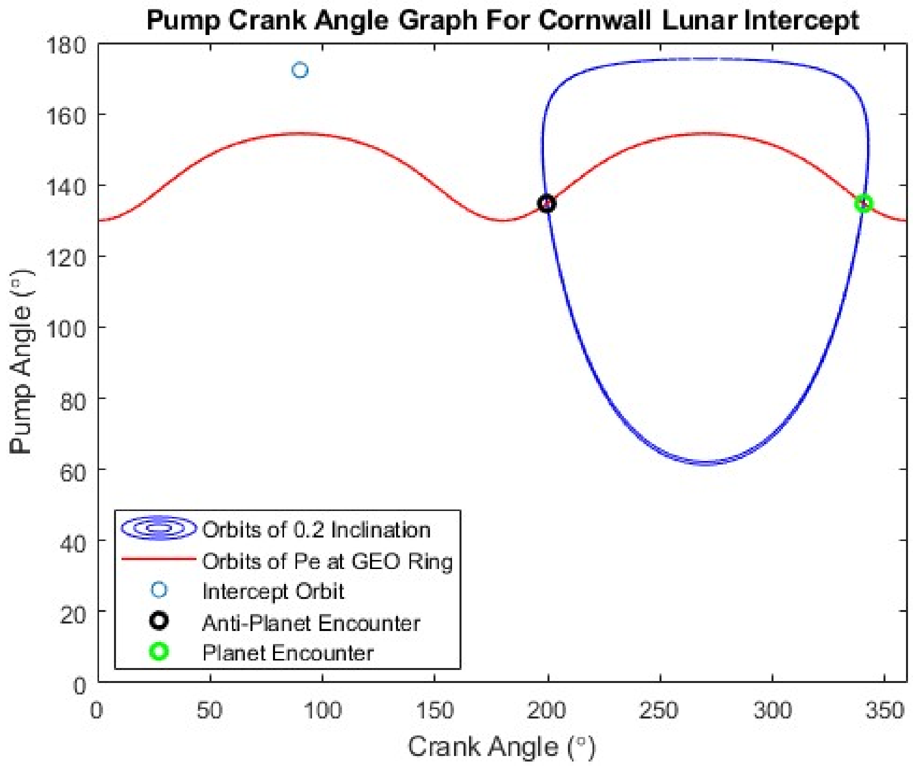

Figure 4.3: Graph of Pump and Crank Angles for Cornwall Launch Site Lunar Gravity Assist

From this graph the following results are extracted and the initial lunar intercept plotted in Figure 4.4.

Table 4.1: Pump and Crank Angles For Cornwall Launch Site Lunar Gravity Assist

Orbit	Pump (°)	Crank (°)
Intercept	172.217	90
Anti-Planet	134.705	199.578
Planet	134.705	340.422

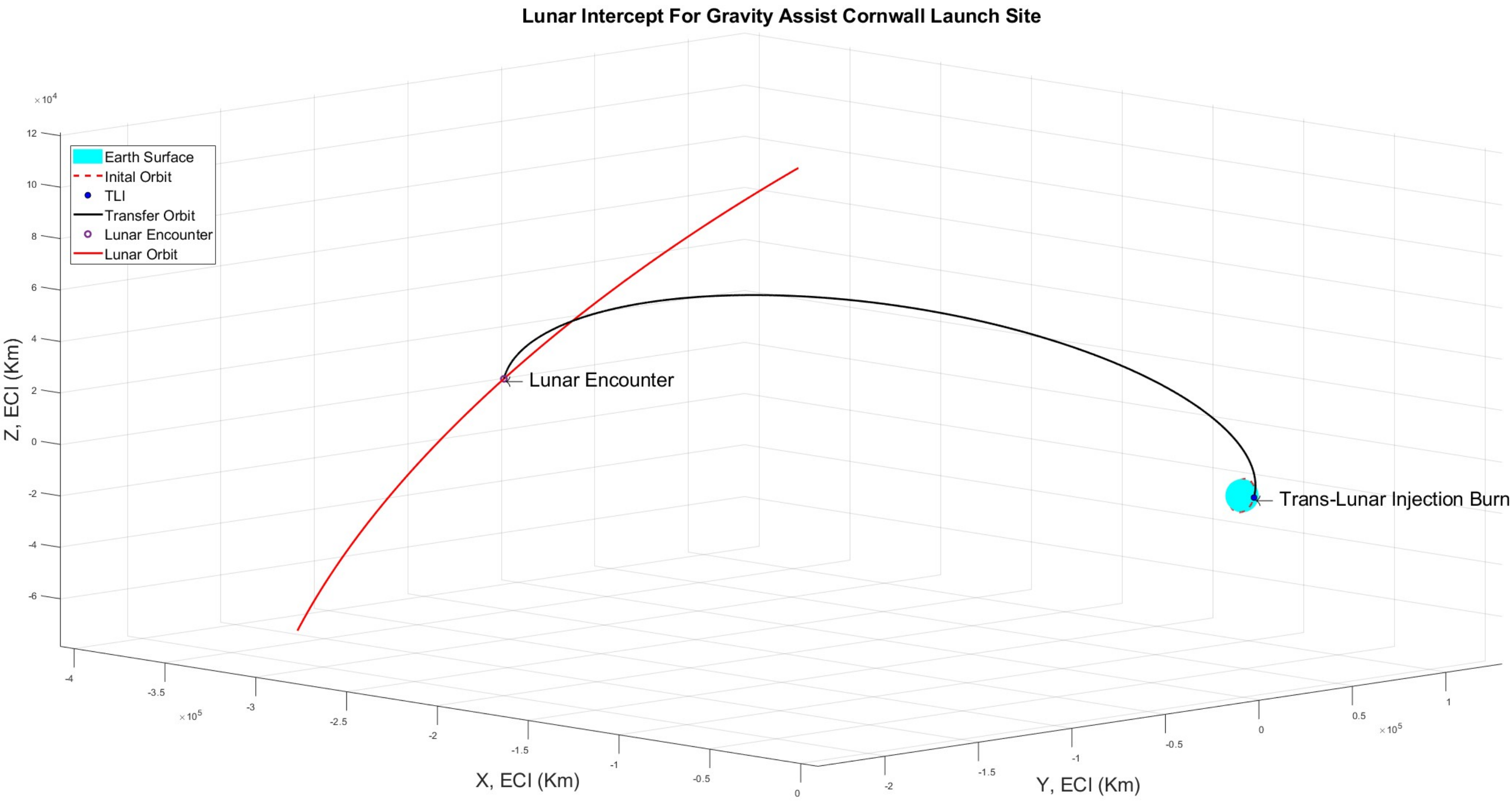

Figure 4.4: Lunar Intercept Orbit for Cornwall Launch Site

Using the results for the planet and anti-planet encounter set ups, the orbits that correspond to those pump and crank angles are calculated and rendered into orbital path diagrams as shown in Figure 4.5.

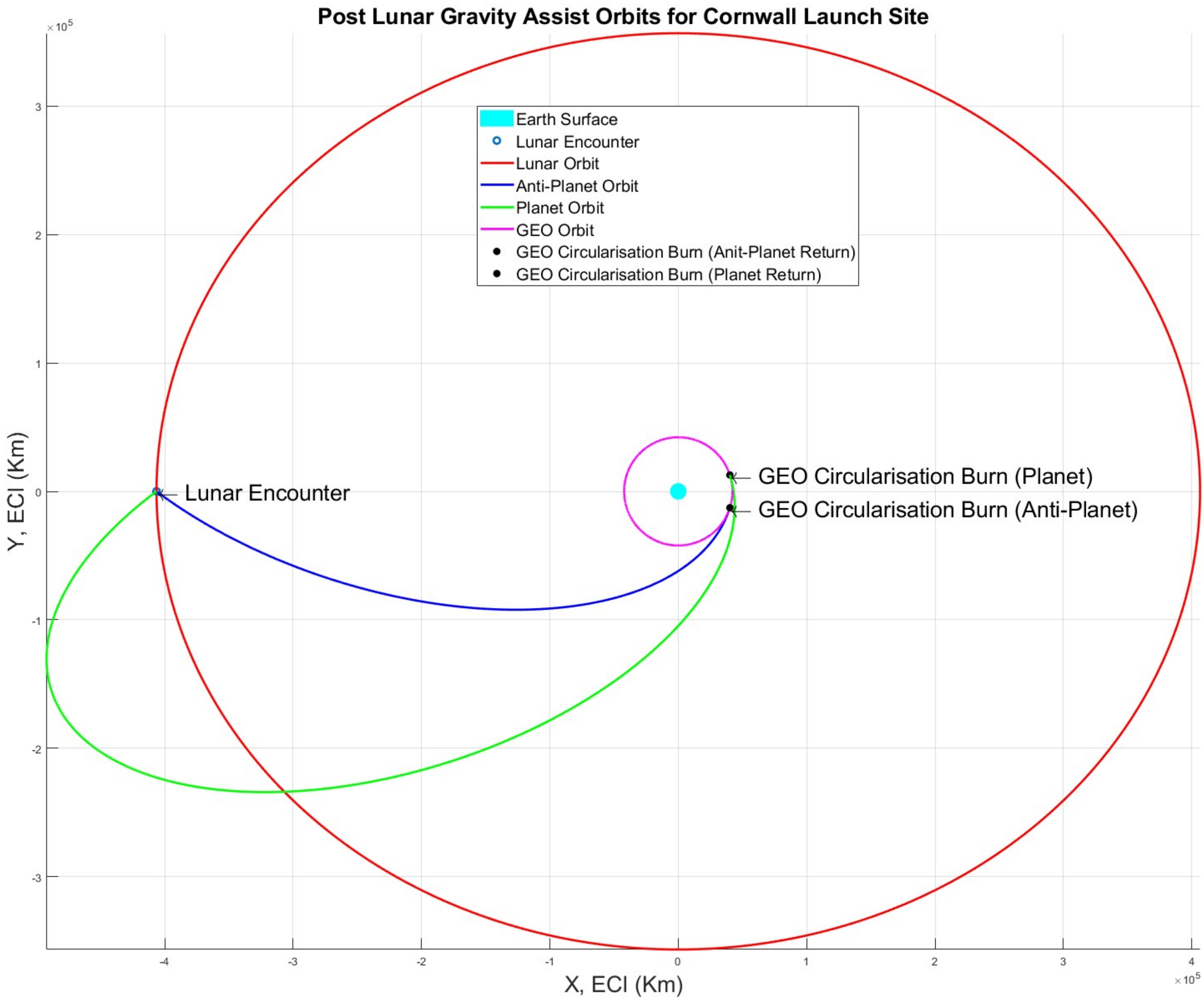

Figure 4.5: Post Lunar Gravity Assist Orbits for Cornwall Launch Site

Then using the methods in Section 3.1.2, Equation 4.9 and the method in Appendix C the delta-v needed for each burn, the radius of encounter needed to execute the gravity assist and time of flight for each path are calculated and shown in Tables 4.2,4.3 and 4.4.

Table 4.2: Cornwall Space Port Lunar Gravity Assist ΔV budget

Return Type	ΔV_1 (Km/s)	ΔV_2 (Km/s)	Total ΔV (Km/s)	Encounter Altitude (Km)
Anti-Planet Return	3.1288	1.1050	4.2338	7697.6
Planet Return	3.1288	1.1050	4.2338	7697.6

Table 4.3: Time of Flight For LEO to GEO Transfer Through Lunar Gravity Assist, Cornwall

Return Type	Time of Flight (s)	Time of Flight (Days)
Anti-Planet Return	8.0076×10^5	9.2681
Planet Return	1.5858×10^6	18.3541

Table 4.4: Orbital Parameters for Cornwall Lunar Gravity Assist Transfer

Orbit	Semi-Major Axis (Km)	Eccentricity	Inclination (°)	Ω (°)	ω (°)
Transfer	2.0649×10^5	0.9680	70	0	0
Anti-Planet Return	2.7724×10^5	0.8479	0.1999	9.5589×10^-13	-17.5902
Planet Return	2.7724×10^5	0.8479	0.1999	9.5589×10^-13	17.5902

4.2.2 SaxaVord Spaceport

For SaxaVord Spaceport an intercept orbit is plotted and the encounter characteristics calculated. This allows for the production of a pump crank angle graph with the lines denoting the target characteristics.

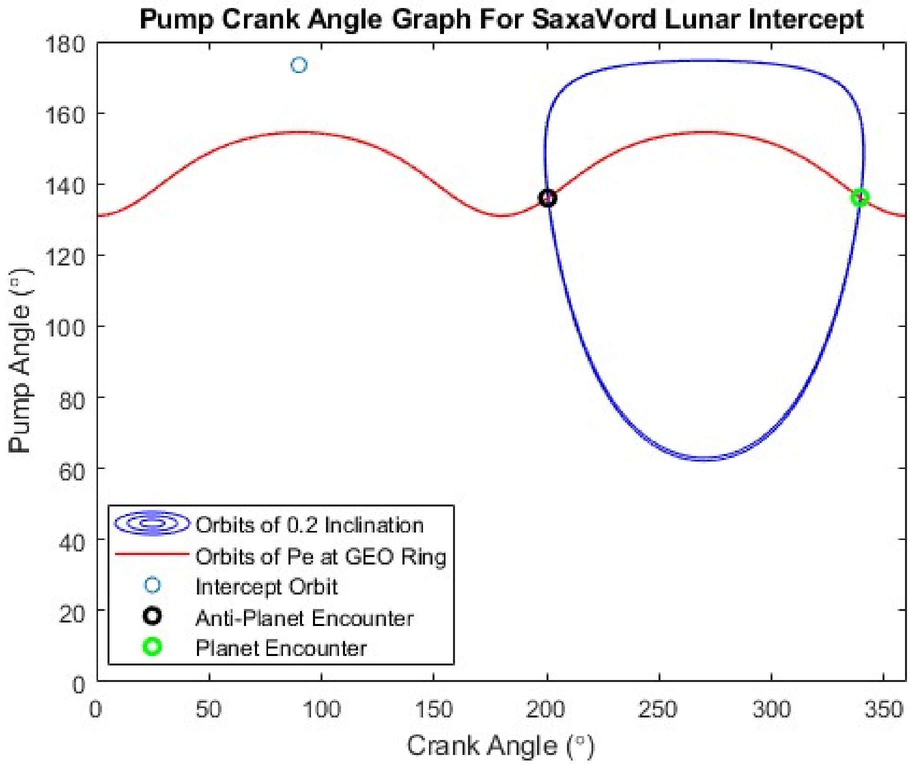

Figure 4.6: Graph of Pump and Crank Angles for SaxaVord Launch Site Lunar Gravity Assist

From this graph the following results are extracted and the initial lunar intercept plotted in Figure 4.7.

Table 4.5: Pump and Crank Angles For SaxaVord Launch Site Lunar Gravity Assist

Orbit	Pump (°)	Crank (°)
Intercept	173.353	90
Anti-Planet	135.961	200.462
Planet	136.105	339.207

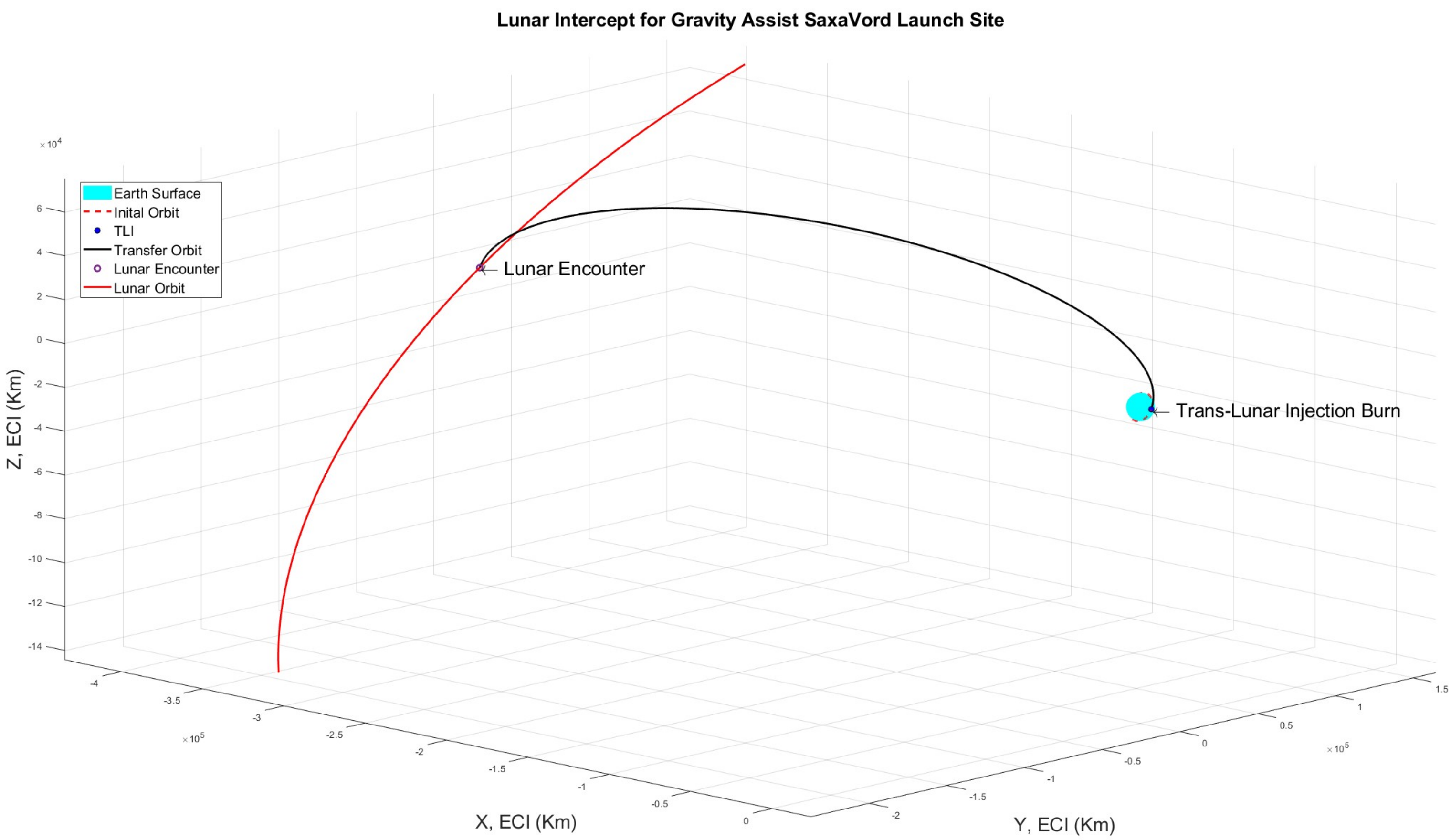

Figure 4.7: Lunar Intercept Orbit for SaxaVord Launch Site

Using the results for the planet and anti-planet encounter set ups, the orbits that correspond to those pump and crank angles are calculated and rendered into orbital path diagrams as shown in Figure 4.8.

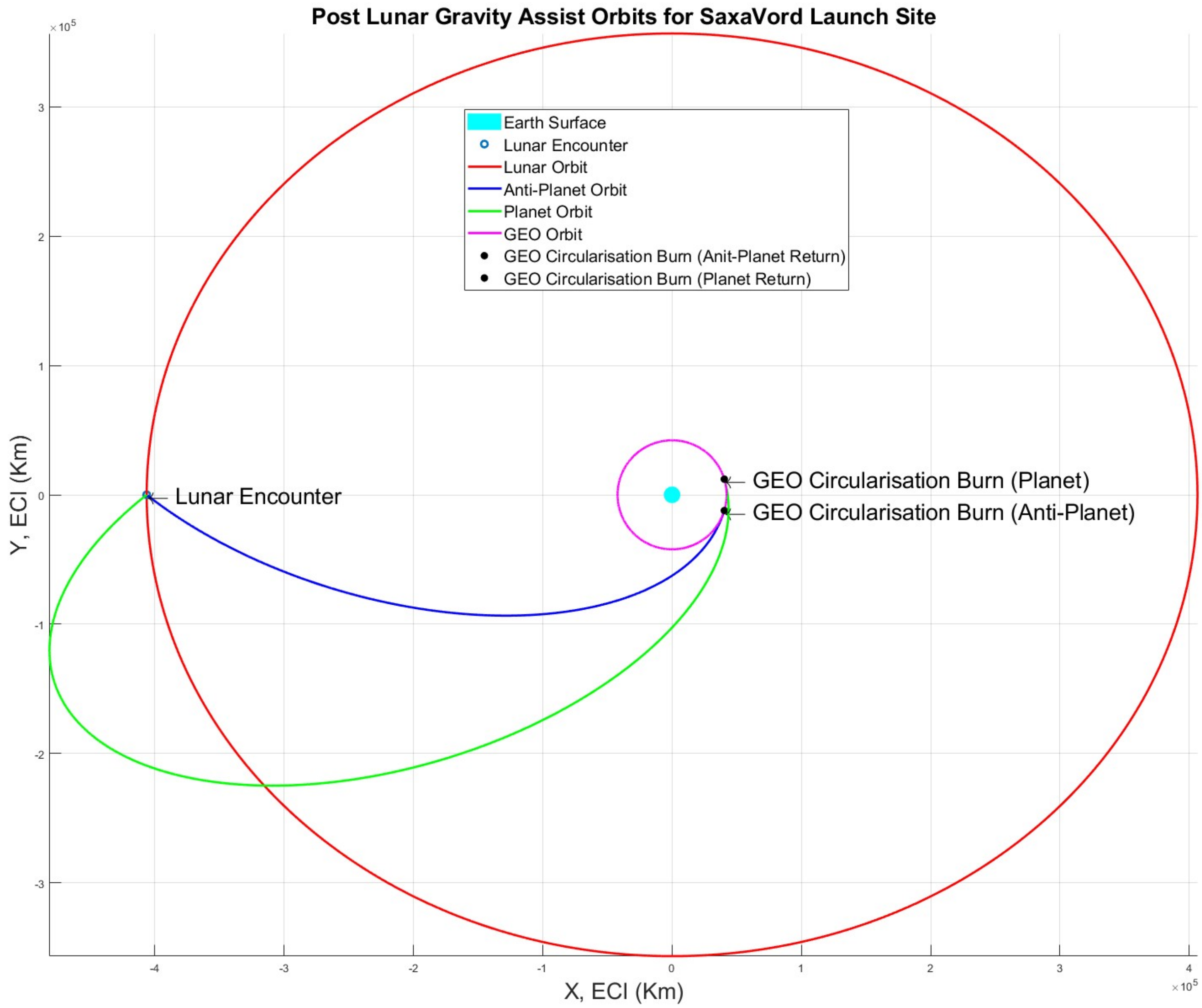

Figure 4.8: Post Lunar Gravity Assist Orbits for SaxaVord Launch Site

Then using the methods in Section 3.1.2, Equation 4.9 and the method in Appendix C the delta-v needed for each burn, the radius of encounter needed to execute the gravity assist and time of flight for each path are calculated and shown in Tables 4.6,4.7 and 4.8.

Table 4.6: SaxaVord Space Port Lunar Gravity Assist ΔV budget

Return Type	ΔV_1 (Km/s)	ΔV_2 (Km/s)	Total ΔV (Km/s)	Encounter Altitude (Km)
Anti-Planet Return	3.1114	1.1013	4.2127	8656.6
Planet Return	3.1114	1.1013	4.2127	8656.6

Table 4.7: Time of Flight For LEO to GEO Transfer Through Lunar Gravity Assist, SaxaVord

Return Type	Time of Flight (s)	Time of Flight (Days)
Anti-Planet Return	8.0706×10^5	9.3410
Planet Return	1.5331×10^6	17.7442

Table 4.8: Orbital Parameters for SaxaVord Lunar Gravity Assist Transfer

Orbit	Semi-Major Axis (Km)	Eccentricity	Inclination (°)	Ω (°)	ω (°)
Transfer	2.0653×10^5	0.9677	62	0	0
Anti-Planet Return	2.7127×10^5	0.8446	0.1999	9.5573×10^-13	-16.7781
Planet Return	2.7127×10^5	0.8446	0.1999	9.5604×10^-13	16.7781

4.2.3 Space Hub Sutherland

For Sutherland Space Hub an intercept orbit is plotted and the encounter characteristics calculated. This allows for the production of a pump crank angle graph with the lines denoting the target characteristics.

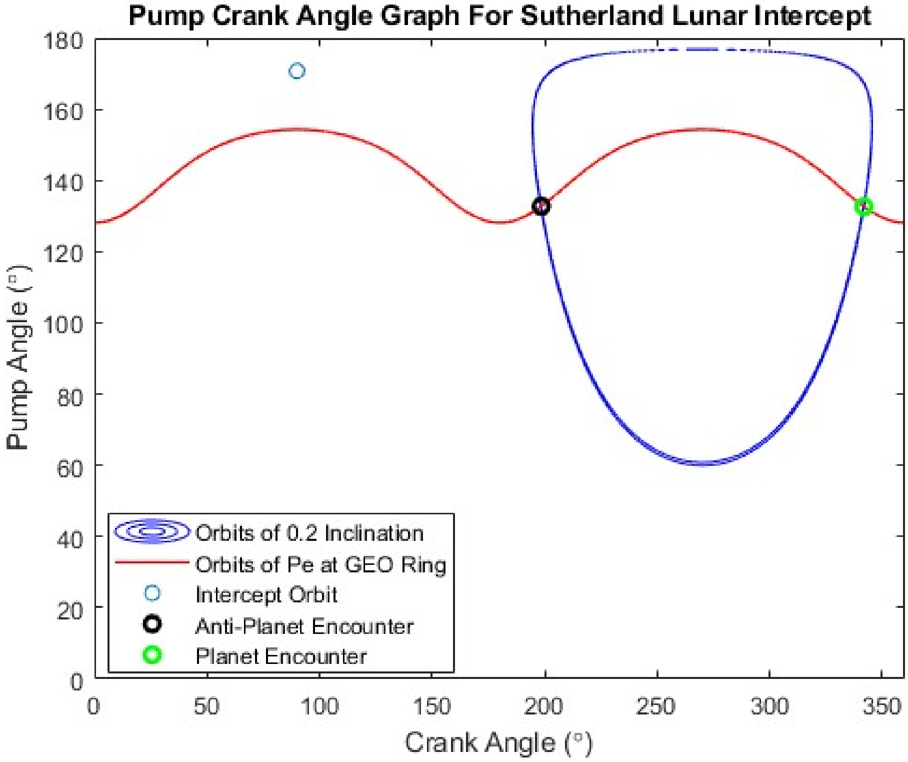

Figure 4.9: Graph of Pump and Crank Angles for Sutherland Launch Site Lunar Gravity Assist

From this graph the following results are extracted and the initial lunar intercept plotted in Figure 4.10.

Table 4.9: Pump and Crank Angles For Sutherland Launch Site Lunar Gravity Assist

Orbit	Pump (°)	Crank (°)
Intercept	170.723	90
Anti-Planet	132.601	198.191
Planet	132.601	341.809

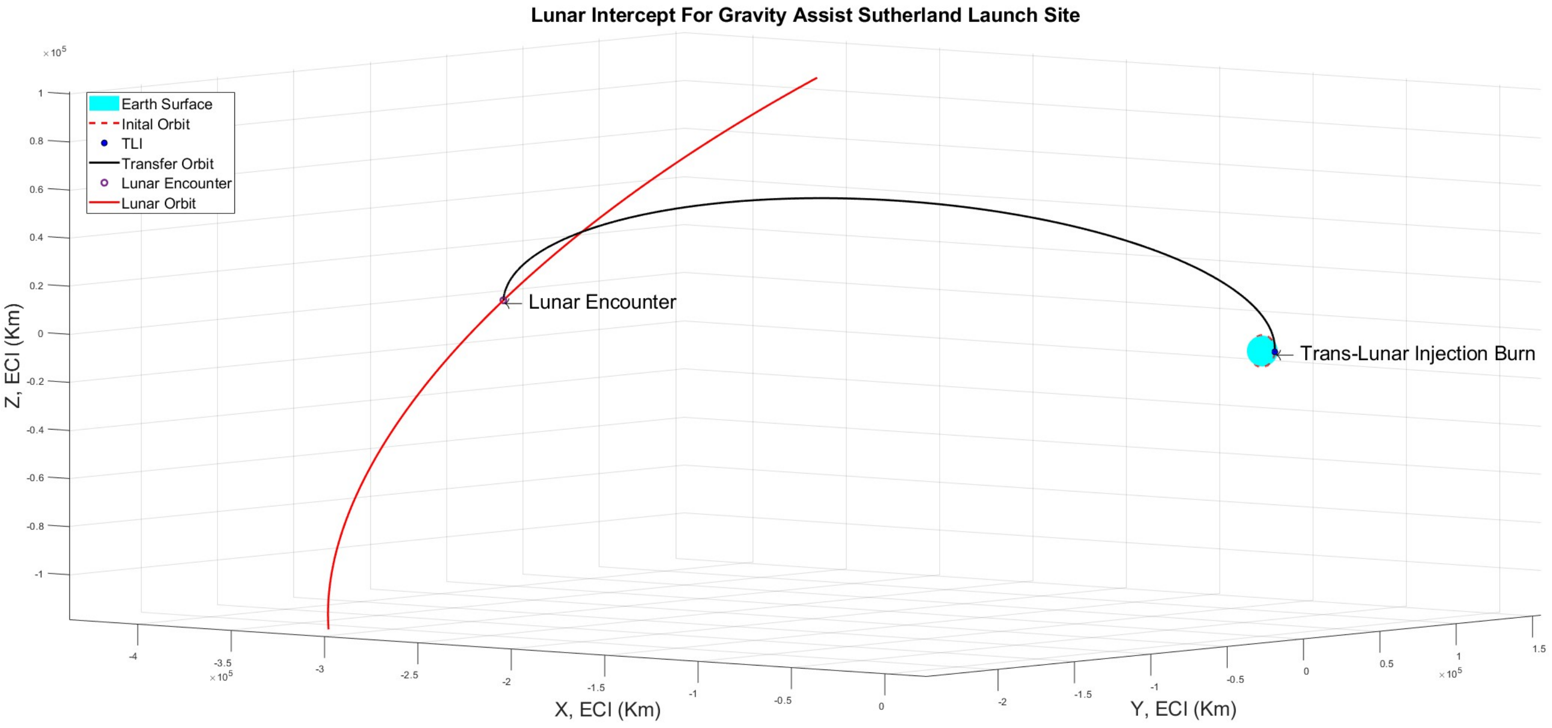

Figure 4.10: Lunar Intercept Orbit for Sutherland Launch Site

Using the results for the planet and anti-planet encounter set ups, the orbits that correspond to those pump and crank angles are calculated and rendered into orbital path diagrams as shown in Figure 4.11.

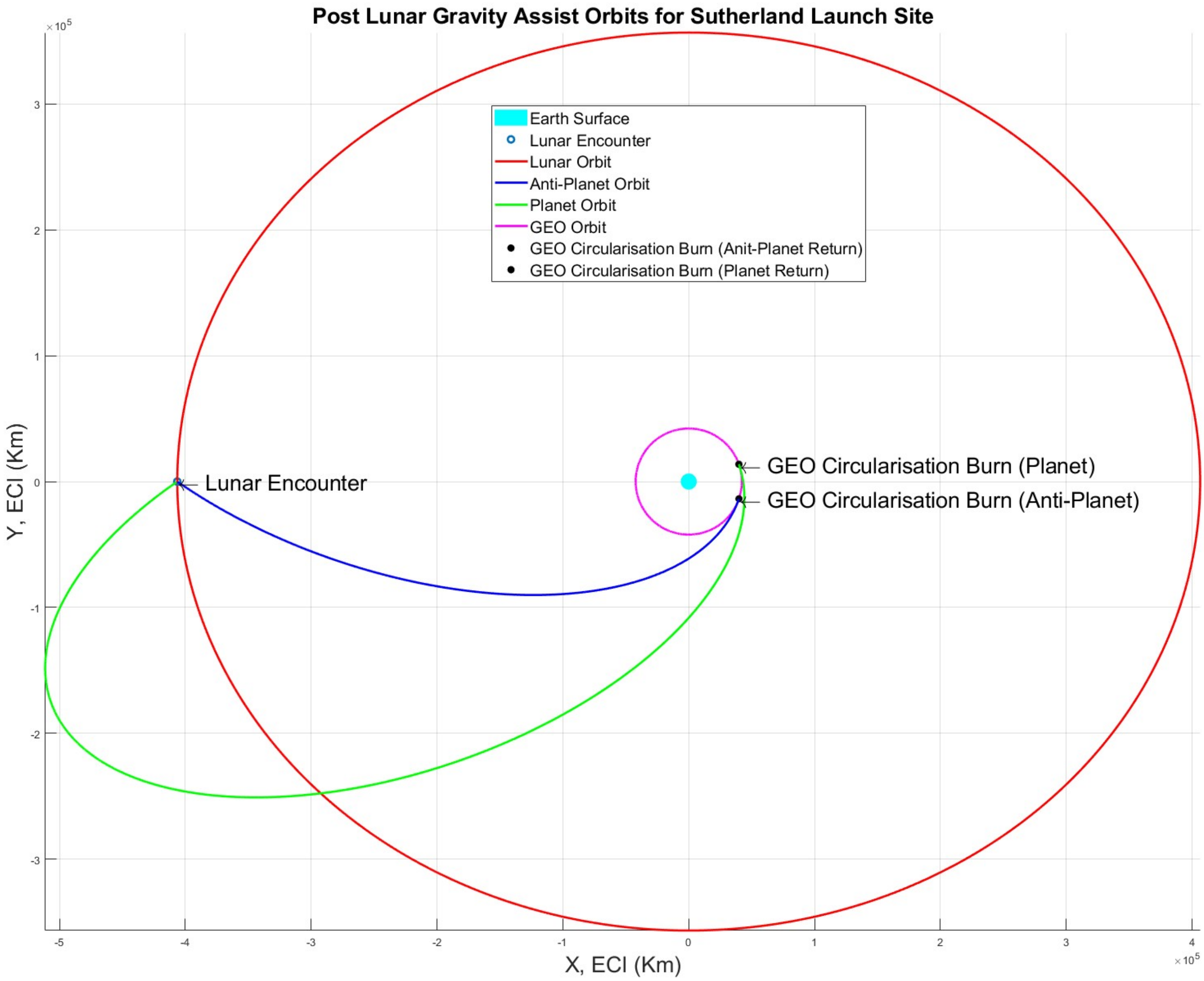

Figure 4.11: Post Lunar Gravity Assist Orbits for Sutherland Launch Site

Then using the methods in Section 3.1.2, Equation 4.9 and the method in Appendix C the delta-v needed for each burn, the radius of encounter needed to execute the gravity assist and time of flight for each path are calculated and shown in Tables 4.10,4.11 and 4.12.

Table 4.10: Space Hub Sutherland Lunar Gravity Assist ΔV budget

Return Type	ΔV_1 (Km/s)	ΔV_2 (Km/s)	Total ΔV (Km/s)	Encounter Altitude (Km)
Anti-Planet Return	3.1114	1.1122	4.2236	6323.2
Planet Return	3.1114	1.1122	4.2236	6323.2

Table 4.11: Time of Flight For LEO to GEO Transfer Through Lunar Gravity Assist, Space Hub Sutherland

Return Type	Time of Flight (s)	Time of Flight (Days)
Anti-Planet Return	7.9047×10^5	9.1490
Planet Return	1.6915×10^6	19.5775

Table 4.12: Orbital Parameters for Space Hub Sutherland Lunar Gravity Assist Transfer

Orbit	Semi-Major Axis (Km)	Eccentricity	Inclination (°)	Ω (°)	ω (°)
Transfer	2.0653×10^5	0.9677	83	0	0
Anti-Planet Return	2.8921×10^5	0.8542	0.1998	9.5622×10^-13	-19.0118
Planet Return	2.8921×10^5	0.8542	0.1998	9.5637×10^-13	19.0118

5 RESULTS ANALYSIS

5.1 Results Comparison Tables

Table 5.1: ΔV Savings for Lunar Assists (Km/s)

Launch Site	Lunar Assist ΔV Total	Savings wrt Hohmann	Savings wrt Bi-Elliptic
Spaceport Cornwall	4.2338	0.5780	0.3904
SaxaVord Spaceport	4.2123	0.6510	0.4158
Space Hub Sutherland	4.2236	1.2583	0.4006

Table 5.2: Time of Flight For Transfer Methods

Launch Site	Hohmann (Days)	Bi-Elliptic (Days)	Anti-Planet (Days)	Planet (Days)
Spaceport Cornwall	0.2192	2.8927	9.2681	18.3541
SaxaVord Spaceport	0.2198	2.8162	9.3410	17.7442
Space Hub Sutherland	0.2198	2.7395	9.1490	19.5775

5.2 Results Validation

As the technical work presented in this report is calculated based on a specific methodology the values can be compared to one another, however the absolute accuracy of the values produced may be incorrect. To check the accuracy of the individual values presented, comparison to real world missions or other papers will confirm the validity of these results. As such referring to paper [13] which performs the same investigation into the comparison of propulsive and gravity assist earth-earth maneuvers to the geostationary ring, provides a useful point of reference. The paper uses a different methodology to generate it's results for the lunar encounter, if a commonality is found between the values from that paper it strengths the conclusions drawn from the results generated in this project. Table 4 of the paper [13] provides delta-v values for transfers using the same transfer methods investigated in this project with a replication of the table below (Table 5.3).

Table 5.3: Cerci 2001, Total ΔV and Transfer Time for Geostationary Transfer Using Four Different Strategies, t_o = Jan. 7 [13]

Type of Transfer	Total ΔV (Km/s)	Transfer time (days)
Hohmann	4.825	0.219
Bielliptic ($x_a = 4 \times 10^5$)	4.5232	11.02
Lunar swing-by ($x_a = 4 \times 10^5$)	4.234	8.69
Far lunar assist ($x_a = 3.7 \times 10^5$)	4.224	18.08

Table 5.4: Total ΔV (Km/s,$\%$) compared to Circi 2001 (Table 5.3)

Type of Transfer	Cornwall	SaxaVord	Sutherland
Hohmann	4.8118 / 0.29	4.8633 / 0.78	5.482 / 12.75
Anti-Planet/Lunar swing-by	4.2338 / 0.00	4.2123 / 0.49	4.2236 / 0.24
Planet/Far lunar assist	4.2338 / 0.24	4.2123 / 0.28	4.2236 / 0.00

In the main, the values calculated using the methods in this report show minimal differences to those of the paper (Table 5.4). When comparing the lunar gravity assist data, the variation between the total delta-v budgets is less than one per cent. This is expected as even with different initial orbits to that in the paper and across different launch sites, the burns needed to complete the lunar gravity assist transfer to GEO perform the same actions, irrespective of starting point. The small difference in the delta-v totals between each launch site and the paper demonstrates that the methodology used is accurate.

The Hohmann transfer total delta-v showed small differences for the Cornwall and SaxaVord sites, however a larger difference of 12.75 percent was calculated for Sutherland Space Hub. This difference may be expected based upon the difference in initial conditions for the orbits presented in this report, and the orbit presented in the paper [13]. An analysis of bi-elliptic transfer values was not carried out, as the transfer orbits used in the report are different to that in the paper [13].

In addition to total delta-v budgets, comparison of trans-lunar injection delta-v values (a component of the total budget) to published data [10] and [13] has been made. This maneuver has been performed in multiple flown missions and can show the effect of any variation made by the assumptions used for the lunar orbit. The paper [10] calculates a delta-v for trans-lunar injection of 3.1 Km/s and the second paper [13] calculates a burn delta-v of 3.122 Km/s. This is in line with the delta-v value of 3.11 Km/s calculated for TLI in this report (see results section). This can also be compared to real world data to gain a point of comparison outside that of the simulation work presented in the papers, to define if the calculated values are within a realistic range. The data used is the TLI burn used in the Apollo 16 mission [4], where a delta-v of 10,373 feet per second (3.1617 Km/s) for TLI was implemented, in line with the simulated values.

5.3 Mass Budgets

To asses the potential use of these transfer maneuvers an analysis of the potential payload mass that could be delivered to the target orbit is a main design constraint for satellite engineering and the limitations on this value inform many of the design decisions made when developing a mission. Therefore the mass budget is a useful tool of comparison, as any increase in it's value may justify the use of a mission plan despite other downsides, such as time of flight. The mass budgets for the different transfer methods can be calculated from the delta-v budgets for each. Using Equation 3.1 and the known values of initial mass and delta-v requirement, the final usable payload mass that can be injected into GEO can be calculated. The only unknown value is the specific impulse (ISP) of whichever engine could be used on the payload to execute the burns.

As for a mission to GEO the system is operating on orbit and for an extended mission duration an engine that is low complexity, can perform multiple restarts and uses a fuel with low boil-off on orbit over a long duration, will be the most effective for this mission scenario. From those requirements a hypergolic type engine fulfils these requirements and they are relatively common in orbital maneuvering applications. For the mass budget calculations the ISP of a AJ10-190 engine [6] (Aerojet) has been used (316 seconds) as it is widely used and currently in service, so could be chosen to execute the mission. It's fuel combination of nitrogen tetroxide and monomethyl hydrazine are non-cryogenic, removing the effects of fuel boil off.

Using these specifications the deliverable payload masses for each method can be calculated. The initial payload mass to LEO for Spaceport Cornwall, SaxaVord Spaceport and Space Hub Sutherland, are 480 Kg, 1500 Kg and 185 Kg respectively.

Table 5.5: Payload And Fuel Mass Values for Spaceport Cornwall (Kg)

Transfer Type	Propellant Mass	Deliverable Payload	Mass Increase wrt Hohmann
Hohmann	378.40	101.60	n/a
Bi-Elliptic	372.06	107.94	6.34
Lunar Gravity Assist	357.57	122.43	20.83

Table 5.6: Payload And Fuel Mass Values for SaxaVord Spaceport (Kg)

Transfer Type	Propellant Mass	Deliverable Payload	Mass Increase wrt Hohmann
Hohmann	1187.73	312.27	n/a
Bi-Elliptic	1163.11	336.89	24.62
Lunar Gravity Assist	1114.78	385.22	72.95

Table 5.7: Payload And Fuel Mass Values for Space Hub Sutherland (Kg)

Transfer Type	Propellant Mass	Deliverable Payload	Mass Increase wrt Hohmann
Hohmann	153.46	31.54	n/a
Bi-Elliptic	146.03	38.97	7.34
Lunar Gravity Assist	137.66	47.34	15.8

5.4 Mission Plan Comparison

5.4.1 Hohmann Transfer

The Hohmann transfer is the simplest and most widely used method for orbital transfers, although in this analysis the least effective transfer maneuver for moving from the initial orbits to the geostationary ring. As the baseline being compared to the other methods, the delta-v expenditure is the highest, and as a result the deliverable payload mass to GEO is appreciably lower. An upside to this method is the much shorter transfer time, it being the only method that has transfer times less than one day. If mission parameters dictate a very short time to the implementation to GEO, then the Hohmann method may be useful, although such urgency is considered unlikely in real world scenarios. When assessing launching from the UK, this mission plan provides very little in advantage when compared to the other two methods, as the low payload mass of each launcher (calculated range 31.54 - 312.27 Kg) compared to the current multi tonne payload that is common for geostationary satellites means its use would be actively detrimental to missions where the payload mass is a premium.

5.4.2 Bi-Elliptic Transfer

The bi-elliptic transfer provides a marked improvement over the Hohmann transfer saving between 4 and 11 percent delta-v across the different sites, in the scenarios calculated in this report. This improvement allows for better payload mass to GEO allowing for around a 7 percent increase from each launch site (6.051 for Cornwall and 21.075 for Sutherland). This improvement is offset by an increase in transfer time to around three days for the transfer. Although the time of transfer is increased over Hohmann, the reduction in delta-v need and corresponding increase in payload capacity provides an improved option comparatively while still only using current widely used propulsive methods, without the added complexity of a lunar transfer. However, the increased time in high orbit outside some of the protections of the earths magnetic field provides different technical challenges, such as high energy particle radiation. Although not as efficient in terms of delta-v expenditure as lunar gravity assist transfer, bi-elliptic transfer provides a simpler option to mission planners, than the difficulty of executing highly accurate flyby maneuvers required to attain the correct orbit for a lunar gravity assist transfer.

5.4.3 Lunar Gravity Assist Transfer

Lunar gravity assists have two different variations, the anti-planet encounter where the spacecraft crosses behind the moon and planet encounter where the spacecraft passes on the side of the moon that faces the earth. The two version provided the same delta-v characteristics with the only difference being the time of flight needed to complete the transfer, planet-side encounters adding an additional almost ten days for each launch site. With the same delta-v values irrespective of which variation is used, the anti-planet encounter provides the most overall effective transfer. As described in the results section, all lunar assist maneuvers demonstrated valid encounter altitudes, meaning that for each of the sites the implementation of such a transfer is possible. The delta-v values for the lunar transfer provide the most efficient transfer maneuver investigated, improving upon bi-elliptic transfer by around 400 m/s (range 390 - 416 m/s) resulting in improvements in deliverable payload mass to GEO of up to 33.38 percent (Space Hub Sutherland).

Compared to Hohmann and bi-elliptic, lunar transfers greatly increase the viability of the all UK launch sites to operate geostationary launches, although the increase in payload mass to GEO is at the cost of mission time as transfers using the anti-planet maneuver add at a minimum 6.3 extra days to the transfer (Cornwall anti-planet comparison to bi-elliptic, Table 5.2). Much of this additional time is spend in high orbit meaning that the effects of high energy particle radiation are much more of a factor that must be accounted for. This can result in the use of heavier shielding or larger radiation hardened electronics that may be needed to survive the transfer with limited damage. As geostationary missions tend to be long duration any damage incurred on the transfer may limit the lifespan of the mission below the target for that mission. However, even with the potential increases in mass used for radiation protection reducing mission payload, the increased overall payload mass to GEO makes the use of lunar assist very effective, when transfer time is not a major design factor.

5.5 Mission Viability

With the results of the comparison of the different mission plans, the lunar gravity assist prove to be the most effective improving the payload mass to target orbit by a considerable amount, even with this boost the mass to target available may not provide adequate payload mass to GEO. Looking at the deliverable payload masses available from Spaceport Cornwall and Space Hub Sutherland, 122.43 and 47.35 Kg respectively, they do not provide enough mass to lift payload that is close to the lowest mass active geostationary satellite identified (400 Kg) [1]. With future launches such as Intelsat 45 reported to be ”one-tenth the size of conventional satellite” [3] launching in 2025, a market for geostationary satellites with the low masses calculated in this report may develop, but at the time of writing these types of payloads do not appear to be under development. Although when considering the reported payload mass to LEO of 1.5t from SaxaVord Spaceport [5] the payload mass to target orbit using lunar gravity assist approaches the 400 Kg of current smallsat GEO payloads. With this payload mass to orbit and the site aiming to perform test launches by the end of the 2023 the development time needed for both payload and site a lunar mission plan could be feasibly implemented, with a payload launching from the UK within the next five years.

6 CONCLUSIONS

When executing a mission to launch to GEO from a site in the UK, the mission requires a high inclination change due to the available inclinations that the current sites can provide due to geographical restrictions. The launch sites of Spaceport Cornwall, SaxaVord Spaceport and Space Hub Sutherland were chosen in this project to provide initial orbits that a mission could start from to then move to GEO. Specific launch vehicles that could execute the mission were identified so that the mass budgets available for such missions could be calculated and used to investigate the real world advantages of more delta-v efficient transfer paths.

As a baseline standard propulsive methods were investigated to provide a comparison point to the use of lunar gravity assist transfers. The two propulsive methods identified, Hohmann and bi-elliptic, as possible mission plans used simulation on MATLAB to calculate delta-v budgets and time of flight. It is concluded that of the propulsive methods assessed, bi-elliptic is the most efficient way to inject a satellite into the target orbit. Using the delta-v budgets calculated, a payload mass that could reach the target orbit was found. A novel patched-conic approximation method for planning lunar gravity assists called the v-infinity globe method was implemented, allowing the production of a map that plots all possible orbits available from a specific intercept orbit using a gravity assist, and the parameters needed to move from the intercept to an orbit with targeted parameters. Using this method, lunar gravity assist transfers were identified and the delta-v and time of flight needed to execute them calculated.

Upon comparison with standard propulsive methods, transfers using lunar gravity assist provide highly efficient paths to GEO, with savings of up to 1.25 Km/s for near polar orbits (Table 5.1) and up to almost 30 percent payload mass increase (Tables 5.5, 5.6 and 5.7), but at the cost of longer transfer times of at least 6.2 additional days (Table 5.2). This makes the use of lunar gravity assist transfers to GEO highly attractive for all launch sites in the UK, as despite the longer transfer time the increased overall payload mass to GEO makes the use of lunar assist very effective, when transfer time is not a major design factor but payload mass is at a premium for mission designers.

However, even with this increase in potential payload mass, the UK may not provide adequate launch mass. Looking at the deliverable payload masses available from Spaceport Cornwall and

Space Hub Sutherland, 122.43 and 47.35 Kg respectively, they do not provide enough launch mass to lift payload that is close to the lowest mass active geostationary satellite identified (400 Kg) [1]. With future launches such as Intelsat 45 reported to be ”one-tenth the size of conventional satellite” [3] launching in 2025, a market for geostationary satellites with the low masses calculated in this report may develop, but at the time of writing these types of payloads do not appear to be under development. In contrast, when considering the reported payload mass to LEO of 1.5t from SaxaVord Spaceport [5] the payload mass to target orbit using lunar gravity assist approaches the 400 Kg of current smallsat GEO payloads. With this payload mass to GEO and the site aiming to perform test launches by the end of the 2023, a lunar gravity assist transfer based mission plan could feasibly be implemented, with a payload launching from the UK within the next five years.

6.1 Future Work

With additional time to improve upon the work completed, the addition of a time epoch to the simulation would allow for the calculation of launch windows, to allow for specificity in time and date of a launch plan. Further improvements to the simulation could be made with the implementation of luni-solar effects to investigate their use to reduce delta-v requirements.

The data in this report could be used to plan proof of concept missions from one of the UK sites investigated with specific payloads. In addition, mission payloads could be designed for use in launches from these site to the geostationary ring, using the payload mass to GEO results.

6.2 Evaluation

All objectives have been fully met by the work carried out and the research laid out in this report. Objective one was met, by comprehensive research into UK launch sites providing a complete overview of the available sites and rationale for those chosen and identification of the initial orbits forming the basis of the technical work for the manoeuvre plans. The technical work then generated the delta-v budgets for mission plans for each launch site and identified the most effective transfer method when using only propulsive methods completing objective two. These were then used as baseline comparison values for the values generated with the development of a MATLAB program that can plan and plot lunar gravity assists to find the delta-v budgets needed to execute the mission plans, completing objective three. Finally the mass budgets for each plan were calculated with a comparison made to the effective use of each of the mission plans completing objective four.

At the start of the project a timeline was developed using a Gantt chart, with milestones being agreed with the project supervisor. The project was completed in the planned time and the majority of milestones met, however during the implementation of the v-infinity sphere method, additional time to that originally planned was required to generate the data required. This resulted in a reduction of the planned report preparation time from three weeks to two. As all objectives have been met and valuable results generated therefore the project is deemed a success.

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A APPENDIX A: COE TO POSITION AND VELOCITY METHODOLOGY

To simulate orbits through time, a series of equations that can take initial orbital elements then find the position and velocity a the satellite around earth across the total orbit needs to be established. In this project a system that takes a set of classical orbital elements (COE) and using the mean, true and eccentricity anomaly through time is used. The system take the orbital elements given, these being eccentricity (e), semi major axis (a), inclination (i), argument of periapsis (ω) and right ascension of the ascending node (Ω). From these elements and the standard gravitational parameter (μ) an orbit can be constructed through time.

The first step of this process is to calculate the orbital period of the satellite using the equation A1.

$$T = \frac{2\pi}{\sqrt{\mu}} a^{\frac{3}{2}} \tag{A.1}$$

Then calculate the mean motion of the given orbit this being the radians per second the satellite moves around the ellipse prescribed by the orbit using the equation A2.

$$n = \sqrt{\frac{\mu}{a^3}} \tag{A.2}$$

From the mean motion value and the array of time values each representing a step through the orbital period, the value of the mean anomaly at each point can be calculated by using the equation,

$$M(t) = M_0 + n(t - t_0) \tag{A.3}$$

Where M_0 is the mean anomaly value at the corresponding t_0 time (if starting the propagation at the periapsis both values can be set to 0). After finding the mean anomaly values the eccentricity anomaly for each point can be found, which must be calculated using the Newton-Raphson method, as there is no analytical solution to find the eccentricity directly from the mean anomaly, equations A4, A5 and A6.

$$E = E_0 - \frac{f(E_0)}{f'(E_0)} \tag{A.4}$$

Where $f(E)$ and $f'(E)$ are,

$$f(E) = E - e \sin E - M \tag{A.5}$$

$$f'(E) = 1 - e\cos E \tag{A.6}$$

Using this method where in the initial run for each loop E_0 is set as the mean anomaly value that is being calculated, the resulting value for eccentricity anomaly is then used in the next iteration of the calculation, producing a new value of E based on equation A.4. This is then repeated until the change in the value from one loop to the next is almost negligible, signifying a root to the equation has been found and the actual eccentricity anomaly for the given mean anomaly has been calculated.

Next the true anomaly for each point in time is found using equation A7.

$$\theta = 2\tan^{-1}(\sqrt{\frac{1+e}{1-e}}\tan\frac{E}{2}) \tag{A.7}$$

This calculation is carried out for every eccentricity anomaly value through time to establish a true anomaly value for every point in time.

Finally to calculate the position and velocity vectors through time, the following set of equations are used (A8 to A12). The initial COE and position and velocity vectors describe different frames of reference, as the values are being transferred between frames a direction cosign matrix (DCM) is needed to translate the values. In this case using the inclination, argument of periapsis and longitude of the ascending node the DCM is constructed as such,

$$[JP] = \begin{bmatrix} \cos\Omega\cos\omega - \sin\omega\cos i\cos\omega & -\cos\Omega\sin\omega - \sin\Omega\cos i\cos\omega & \sin\Omega\sin i \\ \sin\Omega\cos\omega + \cos\Omega\cos i\sin\omega & -\sin\Omega\sin\omega + \cos\Omega\cos i\cos\omega & -\cos\Omega\sin i \\ \sin i\sin\omega & \sin i\cos\omega & cosi \end{bmatrix} \tag{A.8}$$

This is used across all points in the orbit but for each point values r, the distance from the center of the earth and h, the specific angular momentum, are needed to calculate the final vector, this is done using the following equations.

$$r = \frac{a(1-e^2)}{1+e\cos\theta} \tag{A.9}$$

$$h = \sqrt{\mu a(1-e^2)} \tag{A.10}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = [JP] \begin{pmatrix} r\cos\theta \\ r\sin\theta \\ 0 \end{pmatrix} \tag{A.11}$$

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = [JP] \begin{pmatrix} -\frac{\mu}{h}\sin\theta \\ \frac{\mu}{h}(\cos\theta + e) \\ 0 \end{pmatrix} \tag{A.12}$$

The system of equations described in this Appendix then provides the position and velocity vector values needed to calculate the delta-v required to move from an initial to a target orbit using a rocket engine.

B APPENDIX B: POSITION AND VELOCITY TO COE METHODOLOGY

The v-infinity method provides the post encounter velocity vector for the satellite and with the known position of the moon at encounter, a system of equations is needed to convert these two vectors into a set of classical orbital elements. This is required so the path of the new orbit can be defined throughout the path and the position and velocity of the satellite calculated at another point along its orbit. This is done with the following set of equations (B1-17).

As a general case, starting with two 3 position vectors for position $\vec{r}$ and velocity $\vec{v}$,

$$\vec{r} = \begin{bmatrix} r_x \\ r_y \\ r_z \end{bmatrix} \tag{B.1}$$

$$\vec{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \tag{B.2}$$

Taking the magnitude of each as r and v, the specific energy of the system is calculated with,

$$E = \frac{1}{2}v^2 - \frac{\mu}{r} \tag{B.3}$$

This can then be used to find the semi major axis of the orbit,

$$a = -\frac{\mu}{2E} \tag{B.4}$$

A further value is required to calculate the eccentricity and inclination of the orbit, this is the specific angular momentum of the orbit $\vec{h}$ calculated by,

$$\vec{h} = \vec{r} \times \vec{v} \tag{B.5}$$

from this the eccentricity vector is calculated.

$$\vec{e} = \frac{1}{\mu}\vec{r} \times \vec{v} - \frac{\vec{r}}{r} \tag{B.6}$$

The further values describe the position of the orbit around the central body, as such a set of unit vectors are used to convert to convert these values into a common reference frame, shown below as,

$$\hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{B.7}$$

$$\hat{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{B.8}$$

$$\hat{I} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{B.9}$$

Using these the inclination of the orbit can be calculated.

$$i = \cos^{-} 1(\frac{\vec{h} \cdot \hat{k}}{h}) \tag{B.10}$$

A further derivations of the unit vectors with the addition of the specific angular momentum is needed to calculate the RAAN, with the equation below

$$\hat{n} = \frac{\hat{k} \times \vec{h}}{||\hat{k} \times \vec{h}||} \tag{B.11}$$

then used in the next equation to find RAAN,

$$\Omega = \tan^{-} 1(\frac{\hat{n} \cdot \hat{j}}{\hat{n} \cdot \hat{i}}) \tag{B.12}$$

To find the argument of periapsis ω a further unit vector is required, calculated using equation B13,

$$\hat{n}_{\perp} = \frac{\vec{h} \times \hat{n}}{||\vec{h} \times \hat{n}||} \tag{B.13}$$

with this the argument of periapsis ω is calculated

$$\omega = \tan^{-} 1(\frac{\vec{e} \cdot \hat{n}_{\perp}}{\vec{e} \cdot \hat{n}}) \tag{B.14}$$

With the calculated values a, e, i, Ω, ω the path of the orbit can be defined.

To calculate the time of flight of the path, the true anomaly of the point on the orbit it is calculated from can be found. Starting with a set of unit vectors calculated as,

$$\hat{i}_e = \frac{\vec{e}}{e} \tag{B.15}$$

$$\hat{i}_p = \frac{\vec{h} \times e}{||\vec{h} \times e||} \tag{B.16}$$

with these and the position vector the true anomaly can be calculated.

$$\theta = \tan^{-} 1(\frac{\vec{r} \cdot \hat{i}_p}{\vec{r} \cdot \hat{i}_e}) \tag{B.17}$$

With the defined set of equations as presented here, a position and velocity vector can be converted to a set of orbital elements for use in a MATLAB simulation of the orbits.

C APPENDIX C: TIME OF FLIGHT THROUGH MEAN MOTION METHODOLOGY

To calculated the time of flight two methods are implemented, for the propulsive methods as the satellite travels half orbits to perform burns at either apogee or perigee the equation C1 is used as the semi major axis for each transfer orbit is known, then taking half that value of each orbit travelled gives the time of flight.

$$T = \frac{2\pi}{\sqrt{\mu}} a^{\frac{3}{2}} \tag{C.1}$$

For lunar gravity assist methods it is more complex as the orbital paths do not travel exact half orbits back from the moon. The half orbit travel of the transfer orbit is added to the time of flight calculated for the return. This is done through a set of equations starting with the true anomaly θ calculated from the output of the position and velocity of the flyby, as described in Appendix B. The mean anomaly for the same point in the orbit is then calculated using the equations C2 and C3,

$$E = 2\tan^{-}1\left(\sqrt{\frac{1+e}{1-e}} * \tan\left(\frac{\theta}{2}\right)\right) \tag{C.2}$$

where E is the eccentricity anomaly allowing for the transfer from true anomaly to the target mean anomaly value calculated as,

$$M = E - e\sin E \tag{C.3}$$

With the mean anomaly value of the point in the orbit found, the angular distance between that and any other point on the orbit is then derived (labelled as x in Equation C5). The mean motion of the orbit, that being the rate of angular change of the satellite, is calculated using equation C4.,

$$n = \sqrt{mu/a^3} \tag{C.4}$$

With an angular distance x and the mean motion of the orbit n the time needed to travel the orbit path is calculated using Equation C5.,

$$ToF = x/n \tag{C.5}$$

D APPENDIX D: CODE REPOSITORY

For all the code used in the project the Git-Lab Repository is linked below, `https://gitlab.surrey.ac.uk/msc_astrodynamics/msc_matthew.git`