PHYS2160 Introductory Computational Physics
2021/22 Exercise 5

1. Write a Python program that uses the relaxation method to find a root of the function $f(x) = x^3 - 2x - 2$ to an accuracy of 10^{-12} starting from the initial value $x_0 = 1$. Your program should output each successive estimate of x with the magnitude of its error obtained by this method.

2. Write a Python program that uses the relaxation method to find a solution of the equation $x = f(x) = \exp(\exp(-x)\cos x)$ to an accuracy of 10^{-12} starting from the initial value $x_0 = 1$ without using the formula of $f'(x)$. Your program should output each successive estimate of x with the magnitude of its error obtained by this method.

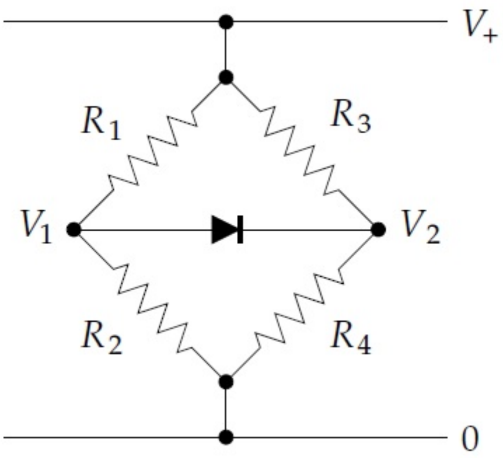
3. Write a Python program that uses the bisection method to find a root of the function $f(x) = x\tan x - \sqrt{100 - x^2}$ on the interval $6.5 \leq x \leq 7.5$ to an accuracy of 10^{-12}. Your program should output each successive estimate of x with its absolute difference from the previous estimate obtained by this method.

4. Write a Python program that uses the Newton's method to find a root of the function $f(x) = 2\sin(3x) - e^x$ to an accuracy of 10^{-12} starting from the initial value $x_0 = 0$. Your program should output each successive estimate of x with the magnitude of its error obtained by this method.

5. Write a Python program that uses the Secant method to find a root of the function $f(x) = e^{x^2}\ln x^2 - x$ to an accuracy of 10^{-12} starting from the initial values $x_0 = 1$ and $x_1 = 2$. Your program should output each successive estimate of x with the magnitude of its error obtained by this method.

6. Write a Python program that uses the relaxation method to find a solution of the simultaneous equations
$$x^2 - 2x + y^4 - 2y^2 + y = 0 \quad \text{and} \quad x^2 + x + 2y^3 - 2y^2 - 1.5y - 0.05 = 0$$
starting from the initial values $x_0 = 0$ and $y_0 = 1$. Like the relaxation method for a single variable, the method will converge to a solution if the equations have appropriate forms. Your program should output each successive estimate of x and y until they both have absolute change less than 10^{-10}.

7. The figure on the top of the next page shows a simple circuit which is a variation on the classic Wheatstone bridge. Note that the resistors obey the normal Ohm's law, but the diode obeys the diode equation:
$$I = I_0(e^{V/V_T} - 1)$$
where V is the voltage across the diode and I_0 and V_T are constants.



The Kirchhoff current law states that the total net current flowing into or out of each point in a circuit must be zero. Applying this law to the points with voltage V_1 and V_2 in the above circuit, we obtain

$$\frac{(V_1 - V_+)}{R_1} + \frac{V_1}{R_2} + I_0[e^{(V_1 - V_2)/V_T} - 1] = 0$$
$$\frac{(V_2 - V_+)}{R_3} + \frac{V_2}{R_4} - I_0[e^{(V_1 - V_2)/V_T} - 1] = 0$$

Write a Python program that uses the Newton's method to solve the above simultaneous equations with the conditions $R_1 = 1\,\mathrm{k\Omega}$, $R_2 = 4\,\mathrm{k\Omega}$, $R_3 = 3\,\mathrm{k\Omega}$, $R_4 = 2\,\mathrm{k\Omega}$, $V_+ = 5\,\mathrm{V}$, $V_T = 0.05\,\mathrm{V}$, $I_0 = 3\,\mathrm{nA}$ to an accuracy of $10^{-8}\,\mathrm{V}$ starting from the initial values $V_{1,0} = 2\,\mathrm{V}$ and $V_{2,0} = 1\,\mathrm{V}$. Note that the error of the estimates of V_1 and V_2 in the ith iteration of this method is approximately given by the norm $\delta(\mathbf{x}_i) = \|(\nabla \mathbf{f})^{-1} \mathbf{f}(\mathbf{x}_i)\|$. Your program should output each successive estimate of V_1 and V_2 with the error $\delta(\mathbf{x}_i)$ obtained by this method.