Lab 5: NumPy and Basic Plotting

Name: _____ University Number: _____

Exercise 1: Chebyshev Polynomial of the First Kind

AIM:

Write a Python program that uses the NumPy `Polynomial` class to print a table of the first ten Chebyshev polynomials of the first kind. Here is the table generated by this program:

```
T_0(x) = 1
T_1(x) = x
T_2(x) = 2*x^2 - 1
⋮
```

ALGORITHM:

PROGRAM:

OUTPUT:

Exercise 2: Sound Intensity from a Point Source

AIM:

In an experiment, Mary measured the variation of the intensity I of the sound produced by a point source with the distance r from the source. Here is her measurement result:

r (m)	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4
I (10^{-5} W/m^2)	0.987	0.662	0.525	0.373	0.308	0.262	0.191	0.184

On the other hand, physical theories tell us that for a point source of sound of power P, the sound intensity I and the distance r from the source are related by:

$$I = \frac{P}{4\pi r^2}$$

Write a Python program that uses the `np.linalg` method `lstsq` to find the best least-square fit of

$$\ln I = m \ln r + k$$

for the given data and then display the fitting result together with the theoretical prediction. Your program should output a table of the values of the fitting parameters m and k found from the fitting and their theoretical values as well as the root-mean-square of the residual of the fitting. Assume that the point source emits sound with a power of $P = 4\pi\times10^{-5}$ W.

ALGORITHM:

PROGRAM:

OUTPUT:

Exercise 3: Forced Vibration with Damping

AIM:

A small block of mass m suspended vertically by a spring with spring constant k is driven by an external force $F(t) = F_0 \cos(\omega t)$. The block is moving in a viscous medium with a damping force of the form $-bv$ where $b > 0$ is the damping constant and v is its instantaneous velocity. Taking downward as the positive direction, the vibration of the block is modeled by the differential equation:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \cos(\omega t)$$

where $x(t)$ is the displacement of the block from its equilibrium position at time t. It can be shown that the steady state solution (i. e. $x(t)$ when time $t \to \infty$) is

$$x_s(t) = (MF_0/k)\cos(\omega t - \phi)$$

In this formula, M is the magnification ratio and ϕ is the phase lag defined by

$$M = \frac{1}{\sqrt{(1-\omega^2/\omega_0{}^2)^2 + 4\zeta^2(\omega/\omega_0)^2}}, \quad \phi = \tan^{-1}\left[\frac{2\zeta(\omega/\omega_0)}{1-(\omega/\omega_0)^2}\right],$$

where $\omega_0 = \sqrt{k/m}$ is the natural frequency and $\zeta = b/(2\sqrt{km})$ is the damping ratio. Write a Python program that uses the Matplotlib `Axes` class method `plot` to plot the magnification ratio M over the interval of frequency ratio ω/ω_0 from 0 to 2.0 for damping ratio $\zeta = 0.1$, 0.2, 0.4, 0.6, and 0.8, respectively, on the same graph. You should label your graph with proper axis labels, title, and legends. From your graph, you can observe how the peak value of M depends on ζ, i. e. the effect of damping on the resonance frequency of the block.

ALGORITHM:

PROGRAM:

OUTPUT:

Exercise 4: Employees in Hong Kong's Construction Industry

AIM:

Below is the table of the employment statistics in Hong Kong's construction industry from 2011 to 2020 (source: https://www.censtatd.gov.hk/tc/scode200.html by Census and Statistics Department, HKSAR).

Year	Number of Employees in Thousands	Share of the Employees in the Labour Force
2011	277.0	7.75 %
2012	290.1	7.93 %
2013	309.0	8.30 %
2014	309.7	8.27 %
2015	316.7	8.39 %
2016	328.4	8.67 %
2017	342.0	8.95 %
2018	351.6	9.09 %
2019	337.5	8.77 %
2020	310.0	8.47 %

Write a Python program that uses Matplotlib `Axes` class method `twinx` to produce a bar chart of the number of employees in Hong Kong's construction industry and a line plot of the percentage share of these employees in the labour force as a function of year on the same graph. You should label your graph with proper axis labels, title, and legends.

ALGORITHM:

PROGRAM:

OUTPUT: