Lab 5: NumPy and Basic Plotting

Name: Shaheer Ziya University Number: 3035946760

Exercise 1: Chebyshev Polynomial of the First Kind

AIM:

Write a Python program that uses the NumPy `Polynomial` class to print a table of the first ten Chebyshev polynomials of the first kind. Here is the table generated by this program:

```
T_0(x) = 1
T_1(x) = x
T_2(x) = 2*x^2 - 1
⋮
```

ALGORITHM:

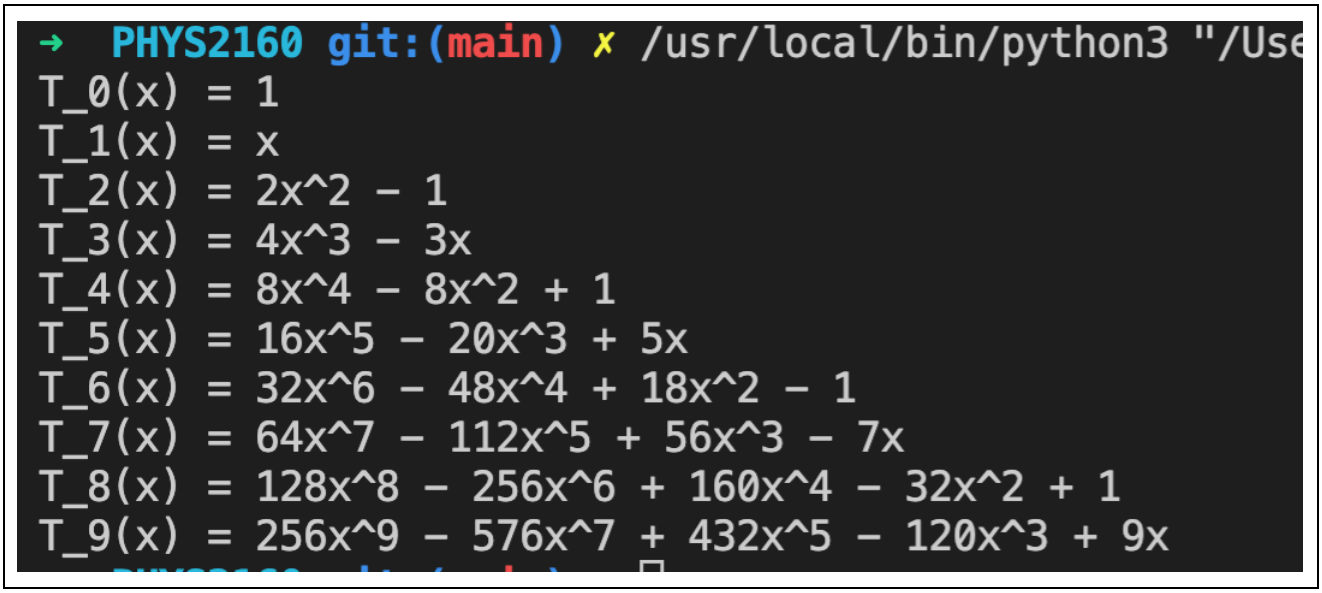
- Obtain the coefficients for the first ten Chebyshev polynomials and store them in a tuple
- Loop over each list containing the coefficients for that order Chebyshev polynomial in reverse and adjust the coefficients for printing in descending order
- Append the coefficients to a polynomial string that is printed at the end of the iteration

PROGRAM:

```
# Chebyshev Polynomials
# Created by Shaheer Ziya

import numpy as np

def main():

  coeff_table = tuple()

  chebyshev_coefficents = [1]
  for i in range(10):
    coeff_table += list(np.polynomial.Chebyshev.convert(np.polynomial.Chebyshev(coef=chebyshev_coefficents),
      kind = np.polynomial.Polynomial).coef),
    chebyshev_coefficents.insert(0, 0)
```

```
 # Print the polynomial
 for i, pol in enumerate(coeff_table):
   polynomial_str = ""
   for idx, coeff in enumerate(pol[::-1]):
     # Determine the sign of the coefficient
     if coeff > 0: sign = " + "
     else: sign = " - "

     # Determine the power of the coefficient
     power = len(pol) - idx - 1

     # Determine the string representation of the coefficient
     if (coeff == 1 and power != 0): coeff_str = ""
     else: coeff_str = str(abs(int(coeff)))



     if coeff == 0:
       continue
     elif power == 0:
       polynomial_str += sign + coeff_str
     elif power == 1:
       polynomial_str += sign + coeff_str + "x"
     else:
       polynomial_str += sign + coeff_str + "x^" + str(power)

   print(f"T_{i}(x) = " + polynomial_str[3:])

main()
```

OUTPUT:


➜ PHYS2160 git:(main) ✗ /usr/local/bin/python3 "/Use
T_0(x) = 1
T_1(x) = x
T_2(x) = 2x^2 - 1
T_3(x) = 4x^3 - 3x
T_4(x) = 8x^4 - 8x^2 + 1
T_5(x) = 16x^5 - 20x^3 + 5x
T_6(x) = 32x^6 - 48x^4 + 18x^2 - 1
T_7(x) = 64x^7 - 112x^5 + 56x^3 - 7x
T_8(x) = 128x^8 - 256x^6 + 160x^4 - 32x^2 + 1
T_9(x) = 256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x

Exercise 2: Sound Intensity from a Point Source

AIM:

In an experiment, Mary measured the variation of the intensity I of the sound produced by a point source with the distance r from the source. Here is her measurement result:

r (m)	1.0	1.2	1.4	1.6	1.8	2.0	2.2	2.4
I (10^{-5} W/m^2)	0.987	0.662	0.525	0.373	0.308	0.262	0.191	0.184

On the other hand, physical theories tell us that for a point source of sound of power P, the sound intensity I and the distance r from the source are related by:

$$I = \frac{P}{4\pi r^2}$$

Write a Python program that uses the `np.linalg` method `lstsq` to find the best least-square fit of

$$\ln I = m \ln r + k$$

for the given data and then display the fitting result together with the theoretical prediction. Your program should output a table of the values of the fitting parameters m and k found from the fitting and their theoretical values as well as the root-mean-square of the residual of the fitting. Assume that the point source emits sound with a power of $P = 4\pi\times10^{-5}$ W.

ALGORITHM:

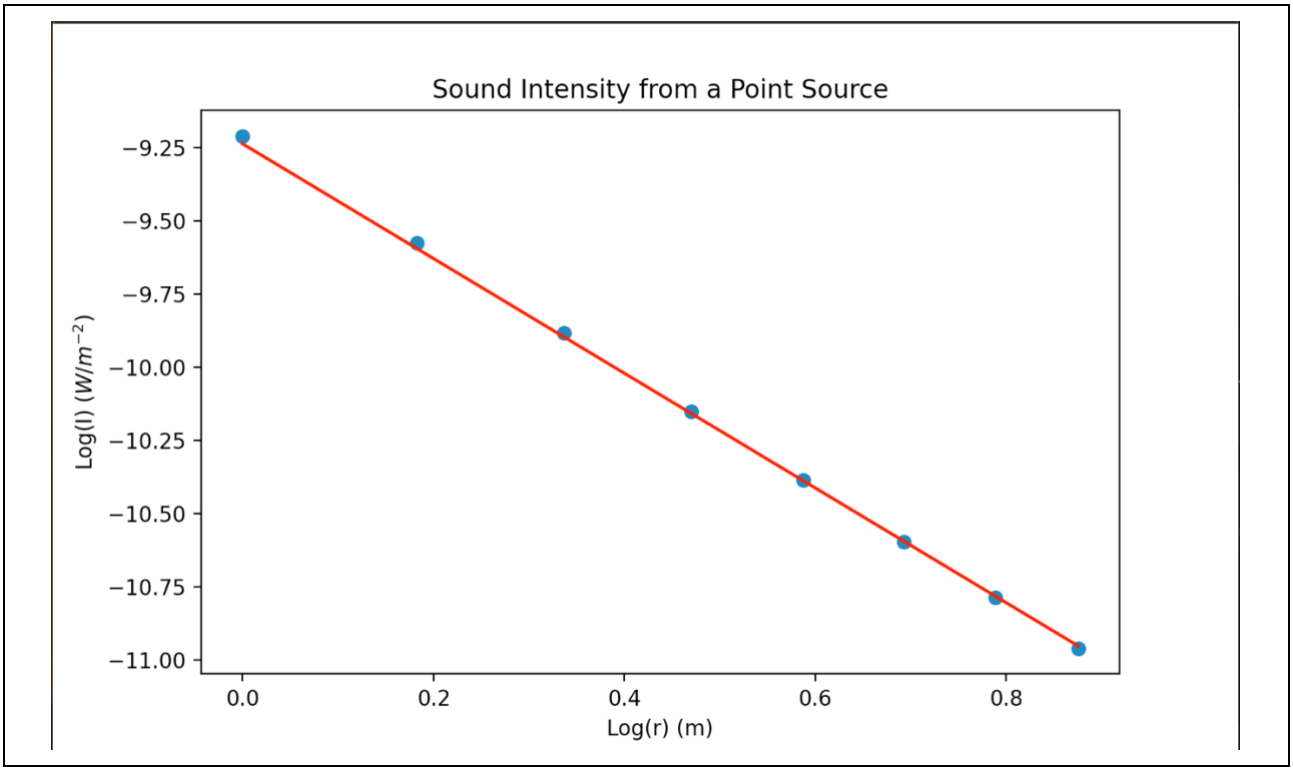
- Feed the values for the data (r and I)
- Prepare the data to be modelled using lstq
- Fit it with np.linalg.lstq()
- Print the fitted values for m, k and residue
- Print both the fitted data and the theoretical redictions

PROGRAM:

```
# Data Fit
# Created by Shaheer Ziya

import matplotlib.pyplot as plt
import numpy as np

P = 4 * np.pi * 10e-5

# Radii & Intensity
```

```
r = np.array([1, 1.2, 1.4, 1.6, 1.8, 2, 2.2, 2.4])
I = np.array([0.987,0.662,0.525,0.373,0.308,0.262,0.191,0.184]) * 10e-5

def main():
  A = np.vstack((np.log(r), np.ones(len(r)))).T
  B = np.log(I)

  x, resid = np.linalg.lstsq(A, B)[0:2]
  m, k = x

  print(m, k, resid)


  plt.subplots()

  # plt.plot(np.log(r), np.log(I), 'o', label='Data')
  plt.plot(np.log(r), np.log(P / (4 * np.pi * r**2)), 'o', label='Theory')

  plt.plot(np.log(r), m*np.log(r) + k, 'r', label='Fit')

  plt.title("Sound Intensity from a Point Source")
  plt.xlabel("Log(r) (m)")
  plt.ylabel("Log(I) $(W/m^{-2})$")

  plt.show()

main()
```

OUTPUT:



Exercise 3: Forced Vibration with Damping

AIM:

A small block of mass m suspended vertically by a spring with spring constant k is driven by an external force $F(t) = F_0 \cos(\omega t)$. The block is moving in a viscous medium with a damping force of the form $-bv$ where $b > 0$ is the damping constant and v is its instantaneous velocity. Taking downward as the positive direction, the vibration of the block is modeled by the differential equation:

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \cos(\omega t)$$

where $x(t)$ is the displacement of the block from its equilibrium position at time t. It can be shown that the steady state solution (i. e. $x(t)$ when time $t \to \infty$) is

$$x_s(t) = (MF_0/k)\cos(\omega t - \phi)$$

In this formula, M is the magnification ratio and ϕ is the phase lag defined by

$$M = \frac{1}{\sqrt{(1-\omega^2/\omega_0{}^2)^2 + 4\zeta^2(\omega/\omega_0)^2}}, \quad \phi = \tan^{-1}\left[\frac{2\zeta(\omega/\omega_0)}{1-(\omega/\omega_0)^2}\right],$$

where $\omega_0 = \sqrt{k/m}$ is the natural frequency and $\zeta = b/(2\sqrt{km})$ is the damping ratio. Write a Python program that uses the Matplotlib `Axes` class method `plot` to plot the magnification ratio M over the

interval of frequency ratio ω/ω_0 from 0 to 2.0 for damping ratio $\zeta = 0.1$, 0.2, 0.4, 0.6, and 0.8, respectively, on the same graph. You should label your graph with proper axis labels, title, and legends. From your graph, you can observe how the peak value of M depends on ζ, i. e. the effect of damping on the resonance frequency of the block.

ALGORITHM:

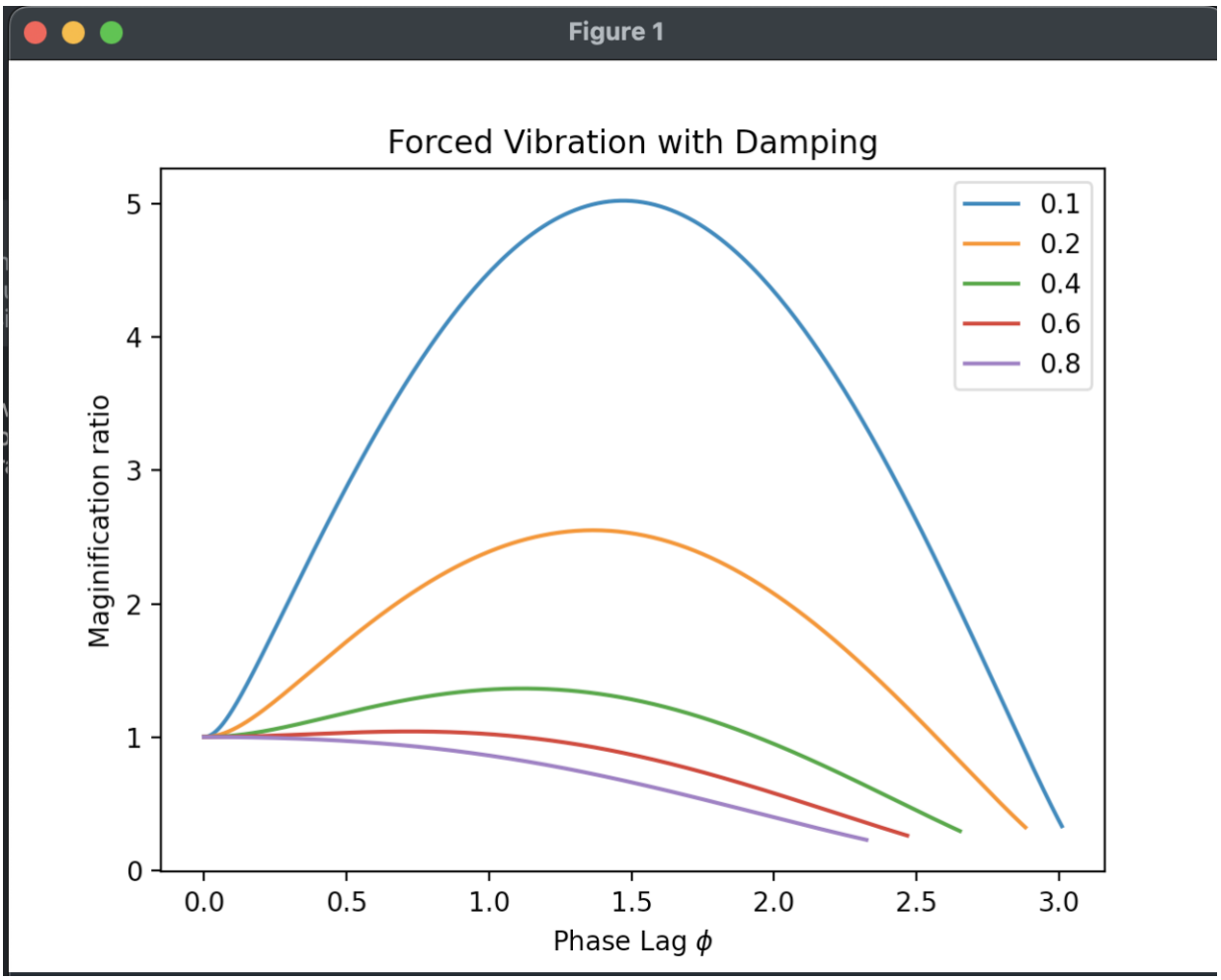
- Prepare the data
- Loop over each xi value and find the magnification ratio and phase-lag, then plot it on the subplot
- Add titles and show legend

PROGRAM:

```
# Plot Damped Oscillator
# Created by Shaheer Ziya

import matplotlib.pyplot as plt
import numpy as np

wOverw0 = np.linspace(0, 2, 1000)
xiS = np.array([0.1, 0.2, 0.4, 0.6, 0.8])

def main():

  plt.subplots()

  for xi in xiS:
    M = 1 / (np.sqrt(
      ((1 - wOverw0**2)**2) + (4 * xi**2 * wOverw0**2)
    ))
    phi = np.arctan2(2*xi*wOverw0, 1 - wOverw0**2)

    plt.plot(phi, M, label=f"{xi}")

  plt.legend()
  plt.xlabel("Phase Lag $\phi$")
  plt.ylabel("Maginification ratio")
  plt.title("Forced Vibration with Damping")
```

```
    plt.show()


main()
```

OUTPUT:



Exercise 4: Employees in Hong Kong's Construction Industry

AIM:

Below is the table of the employment statistics in Hong Kong's construction industry from 2011 to 2020 (source: https://www.censtatd.gov.hk/tc/scode200.html by Census and Statistics Department, HKSAR).

Year	Number of Employees in Thousands	Share of the Employees in the Labour Force
2011	277.0	7.75 %
2012	290.1	7.93 %
2013	309.0	8.30 %
2014	309.7	8.27 %
2015	316.7	8.39 %
2016	328.4	8.67 %
2017	342.0	8.95 %
2018	351.6	9.09 %
2019	337.5	8.77 %
2020	310.0	8.47 %

Write a Python program that uses Matplotlib `Axes` class method `twinx` to produce a bar chart of the number of employees in Hong Kong's construction industry and a line plot of the percentage share of these employees in the labour force as a function of year on the same graph. You should label your graph with proper axis labels, title, and legends.

ALGORITHM:

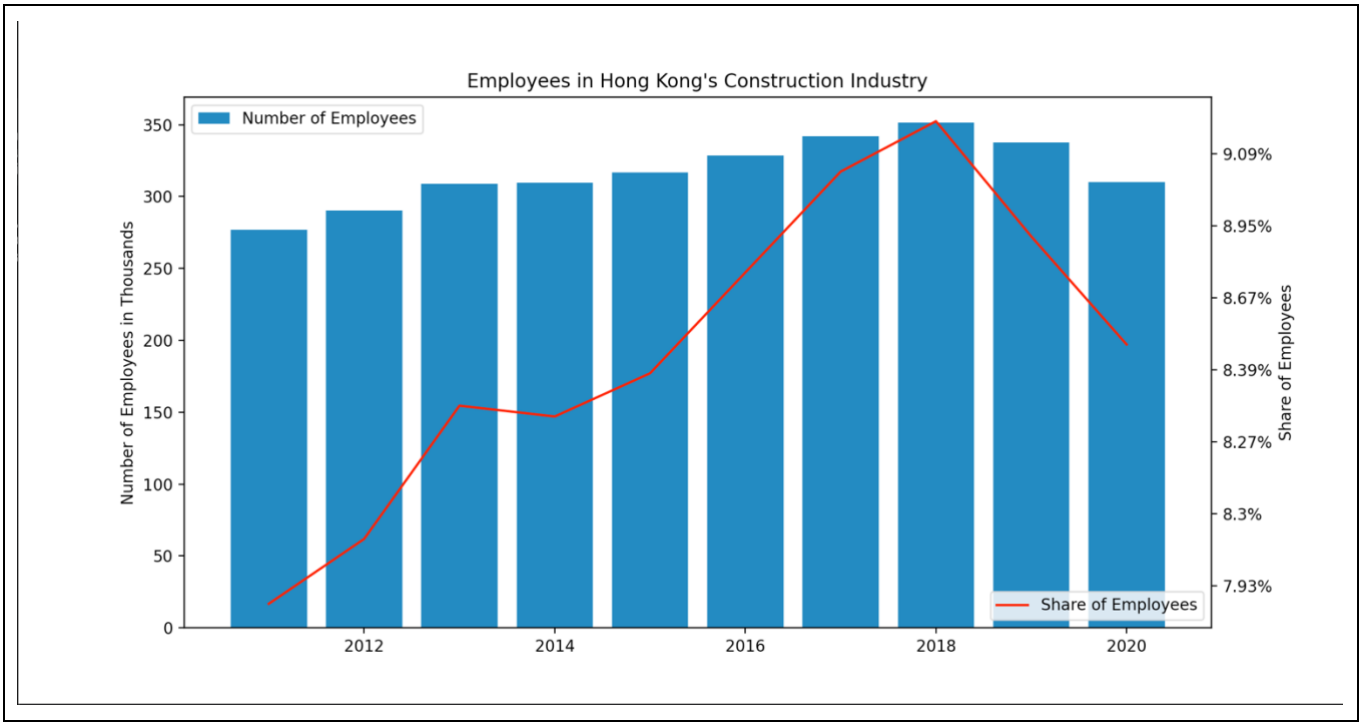
- Initialize the data to be printed
- Plot the first set of data on axs1, labelling the data as we go along
- Instantiate the other twin axis
- Plot its data, labelling the axis and showing its legend
- Show the final plot

PROGRAM:

```
# Twin Bar Chart
# Created by Shaheer Ziya


import matplotlib.pyplot as plt
```

```
import numpy as np

years = [2011, 2012, 2013, 2014, 2015, 2016, 2017, 2018, 2019, 2020]
numEmployees = [277.0, 290.1, 309.0, 309.7, 316.7, 328.4, 342.0, 351.6, 337.5, 310.0]
shareEmployees = [7.75, 7.93, 8.30, 8.27, 8.39, 8.67, 8.95, 9.09, 8.77, 8.47]


def main():
  fig, ax1 = plt.subplots()
  ax1.bar(years, numEmployees, label = 'Number of Employees')
  ax1.set_ylabel('Number of Employees in Thousands')

  ax2 = ax1.twinx()
  ax2.plot(years, shareEmployees, label = 'Share of Employees', color = 'r')
  ax2.set_ylabel('Share of Employees')
  ax2.set_yticklabels(['{}%'.format(x) for x in shareEmployees])


  ax1.legend(loc='upper left')
  ax2.legend(loc='lower right')
  plt.title('Employees in Hong Kong\'s Construction Industry')
  plt.show()


main()
```

OUTPUT:


Employees in Hong Kong's Construction Industry
Number of Employees
Share of Employees
Number of Employees in Thousands
Share of Employees
0
50
100
150
200
250
300
350
7.93%
8.3%
8.27%
8.39%
8.67%
8.95%
9.09%
2012
2014
2016
2018
2020