



Q11. $e = (4+8 \times (1-1)) + (16 \times 3)$

The roman numerals describe the order of derivation.

$$\begin{array}{c}
 \text{VI} \\
 \text{I} \rightarrow \text{I} \quad \text{E-} \\
 \text{IV} \quad \frac{(1-1) \rightarrow 0 \quad 0 \rightarrow 0}{\text{ExRo}} \quad \frac{16 \rightarrow 16 \quad 3 \rightarrow 3}{\text{Ex II.}} \\
 \text{III.} \quad \frac{(4+8) \times (1-1) \rightarrow 0}{\text{E+ I.}} \quad \frac{16 \times 3 \rightarrow 48}{\text{VIII.}}
 \end{array}$$

Q12. It is possible for different derivations of the same expression.
Consider the expression

Mult (Num o) (Num o)

i.e. 0×0 .

Then,

$$\begin{array}{c}
 0_L \rightarrow 0 \quad 0 = 0 \\
 \hline
 \text{ExLo}
 \end{array}$$

$$\begin{array}{c}
 0_R = 0 \quad 0 = 0 \\
 \hline
 \text{ExRo.}
 \end{array}$$

where we label the left & right zeroes for convenience
as 0_L & 0_R resp.

I do not believe that it is possible to modify these inference rules to make sure derivations are unique (unless we apply some restriction on what ~~the~~ rules we can use at a given time).

In this case the cause of multiple derivations being possible is that ExLo is identical to ExRo w/ the commutative law for multiplication. So no matter what rules we modify or replace if the modified inference system is ~~is~~ equivalent to this original then some sequence of inference rules would give ExLo & some sequence of inference rules would be equivalent to ExRo & thus the expression " 0×0 " would yet again have multiple derivations.