CSE185
Introduction to Computer Vision
Lab 10: Optical Flow

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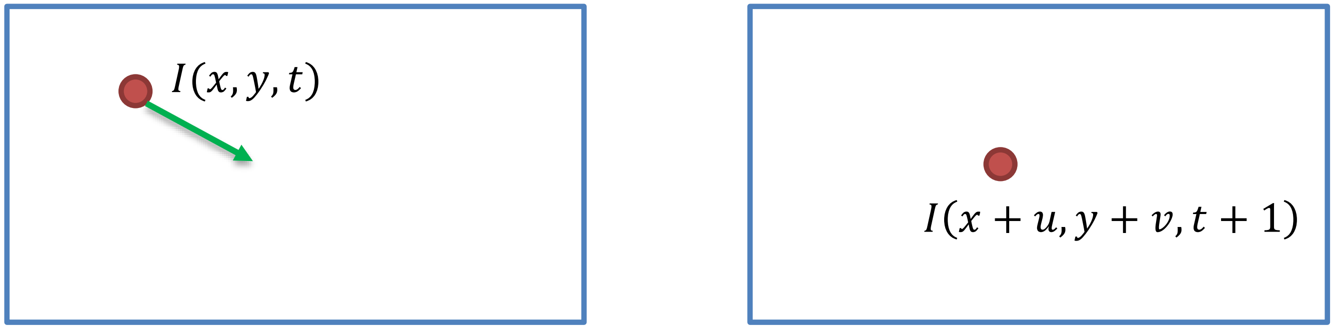
Input 1

Input 2

Optical Flow

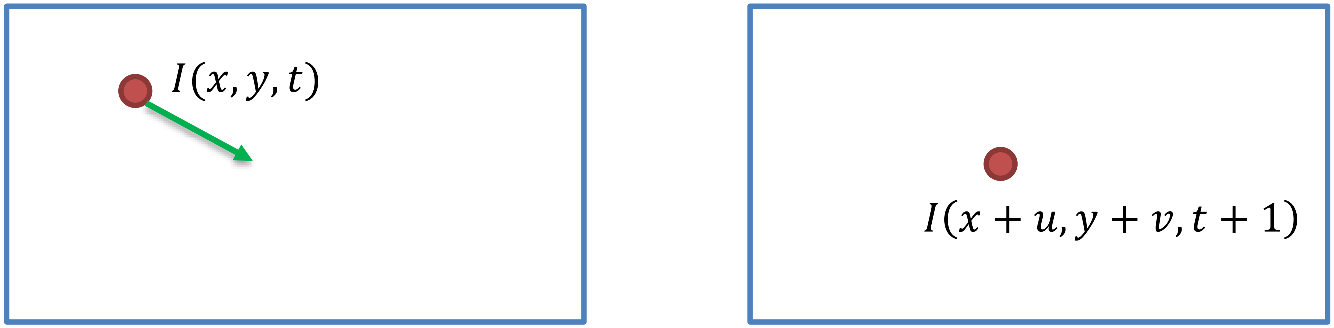
Brightness Constancy

- $I(x+u, y+v, t+1) = I(x, y, t)$



Brightness Constancy

- $I(x+u, y+v, t+1) = I(x, y, t)$



- Tayler expansion:

$$I(x+u, y+v, t+1) \approx I(x, y, t) + I_x \cdot u + I_y \cdot v + I_t$$
$$I(x+u, y+v, t+1) - I(x, y, t) = I_x \cdot u + I_y \cdot v + I_t = 0$$

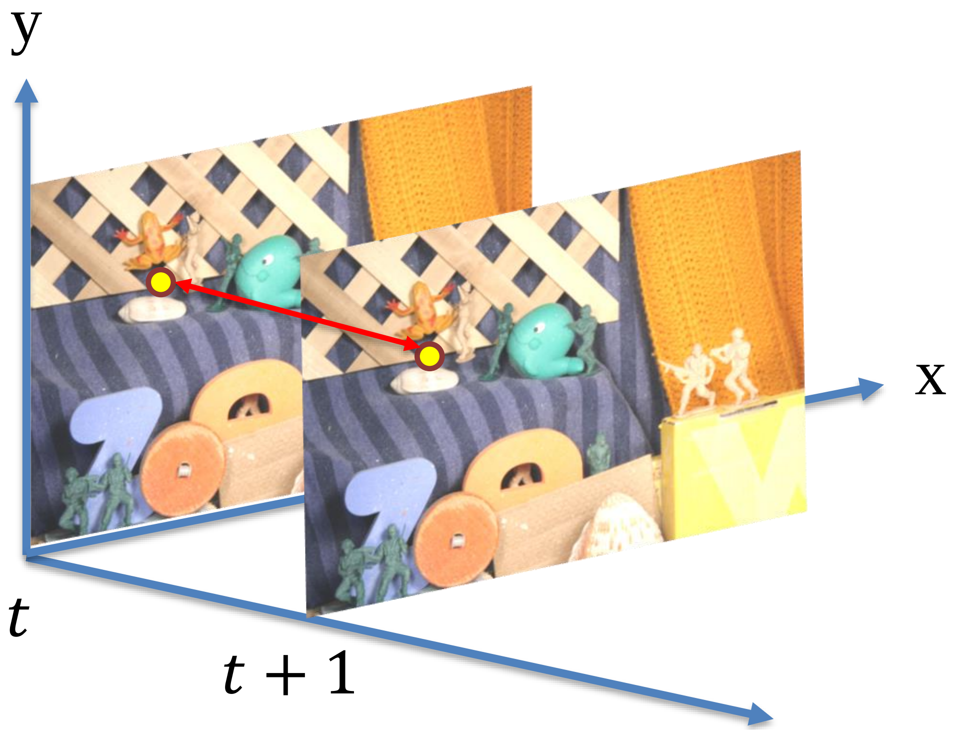
- Brightness constancy equation:

$$I_x \cdot u + I_y \cdot v + I_t = 0$$

  - I_x: x-gradient
  - I_y: y-gradient
  - I_t: pixel difference on time domain

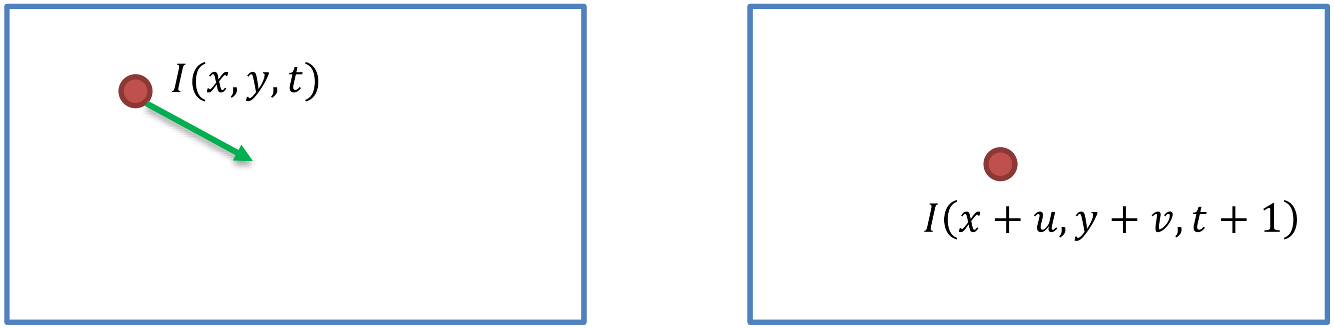
Gradients

- $I_x = I(x+1, y, t) - I(x, y, t)$
- $I_y = I(x, y+1, t) - I(x, y, t)$
- $I_t = I(x, y, t+1) - I(x, y, t)$



Brightness Constancy

- $I(x+u, y+v, t+1) = I(x, y, t)$



- We want to solve (u, v) such that:

$$I_x \cdot u + I_y \cdot v = -I_t$$

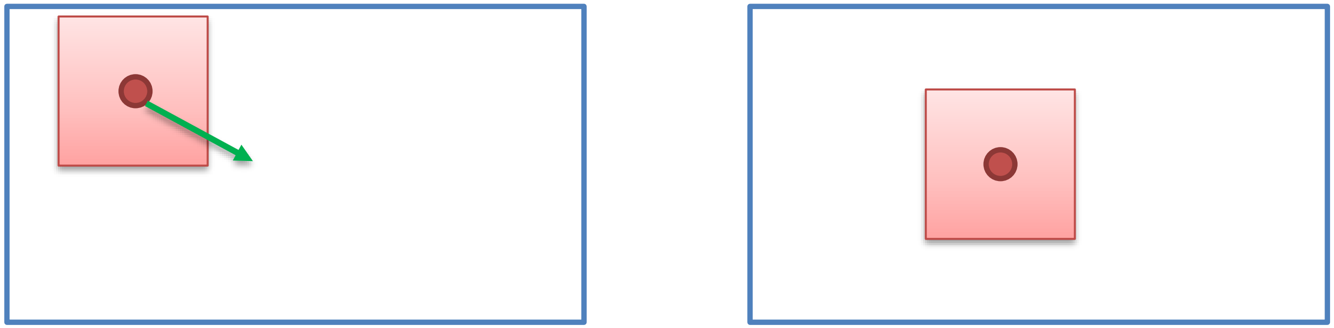
1 equation,
2 variables

- Matrix-vector form:

$$\begin{bmatrix} I_x & I_y \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix} = -I_t$$

Spatial Coherence

- Assume neighbors have the same (u, v)



- A $w \times w$ window gives us w^2 equations:

$$\begin{bmatrix} I_x(p_1) & I_y(p_1) \\ I_x(p_2) & I_y(p_2) \\ \vdots & \vdots \\ I_x(p_{w^2}) & I_y(p_{w^2}) \end{bmatrix} \cdot \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} I_t(p_1) \\ I_t(p_2) \\ \vdots \\ I_t(p_{w^2}) \end{bmatrix}$$

Solve Linear Equation

- Solve $Ax = b$ in MATLAB:
  - left-division: `x = A \ b;`
  - pseudo inverse: `x = pinv(A) * b;`
  - least square solution: `x = inv(A' * A) * A' * b;`

Solve Linear Equation

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- Solve optical flow equation:

$$\underbrace{\begin{bmatrix} I_x(p_1) & I_y(p_1) \\ I_x(p_2) & I_y(p_2) \\ \vdots & \vdots \\ I_x(p_{w^2}) & I_y(p_{w^2}) \end{bmatrix}}_{A} \cdot \underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{x} = - \underbrace{\begin{bmatrix} I_t(p_1) \\ I_t(p_2) \\ \vdots \\ I_t(p_{w^2}) \end{bmatrix}}_{b}$$

Algorithm

- Pseudo code:

```
Input: I1, I2, window size w
Output: flow vector (u, v) for each pixel

u = 0, v = 0 for every pixel
for each pixel in I1:
        compute Ix, Iy, It from w × w window
        convert Ix, Iy, It to vectors
        let A = [Ix, Iy], b = -It
        solve x
        u = x(1), v = x(2)
end
```

Iterative Refinement

- Pseudo code:

```
Input: I1, I2, window size w
Output: flow vector (u, v) for each pixel

u = 0, v = 0 for every pixel
Run k times:
    for each pixel in I1:
        compute Ix, Iy from w × w window
        shift window of I2 by (u, v), compute It
        convert Ix, Iy, It to vectors
        let A = [Ix, Iy], b = -It
        solve x
        u += x(1), v += x(2)
    end
end
```

Algorithm

- lab10.m:

```
window_size = 45;
k = 4;

w = floor(window_size/2);
shift = w + 10;

I1 = rgb2gray(img1);
I2 = rgb2gray(img2);

Ix_m = imfilter(I1, [1 -1;  1 -1], 'replicate');
Iy_m = imfilter(I1, [1  1; -1 -1], 'replicate');

u = zeros(size(I1)); u_next = zeros(size(I1));
v = zeros(size(I1)); v_next = zeros(size(I1));
```

pre-compute Ix and Iy

Algorithm

- lab10.m:

```
for t = 1 : k
    for i = 1 + shift : size(Ix_m, 1) - shift
        for j = 1 + shift : size(Ix_m, 2) - shift
            %% extract Ix, Iy, It from local window

            %% convert Ix, Iy, It to vectors

            %% construct matrix A and vector b

            %% solve A x = b
            x = [0, 0]; % remove this line
            u_next(i, j) = x(1);
            v_next(i, j) = x(2);
        end
    end
    %% update flow
    u = u + u_next;
    v = v + v_next;
end
```

Hints

- Extract `Ix`, and `Iy` from local window:

```
window_size = 45;
w = floor(window_size/2);

Ix = Ix_m(i-w : i+w, j-w : j+w);
```

the same as you did in spatial filtering

- Shift the window of `I2` from `(i, j)` to `(i+v, j+u)` when extract `It` from local window:

```
i2 = i + v(i, j);
j2 = j + u(i, j);
It = I1(i - w : i + w, j - w : j + w)
   - I2(i2 - w : i2 + w, j2 - w : j2 + w);
```

Hints

- Your `Ix` and `Iy` are $w \times w$ matrixs, first convert them to $w^2 \times 1$ vectors, and concatenate them into matrix A

```
A = [Ix, Iy];
```

- A is a $w^2 \times 2$ matrix, b is a $w^2 \times 1$ vector, solve x by
  - left-division: `x = A \ b;`
  - pseudo inverse: `x = pinv(A) * b;`
  - least square solution: `x = inv(A' * A) * A' * b;`

Visualization

- `plot_flow(img2, u, v)`

Iteration 1

Visualization

- `plot_flow(img2, u, v)`

Iteration 2

Visualization

- `plot_flow(img2, u, v)`

Iteration 3

Visualization

- `plot_flow(img2, u, v)`

Iteration 4

Lab Assignment 10

- Complete lab10.m
- Try different window size and different image pairs
- Upload lab10.m and XXX_flow.png (XXX = Army, Backyard, Mequon) separately