Modeling the Black Death: A Medieval Disease in Modern Times

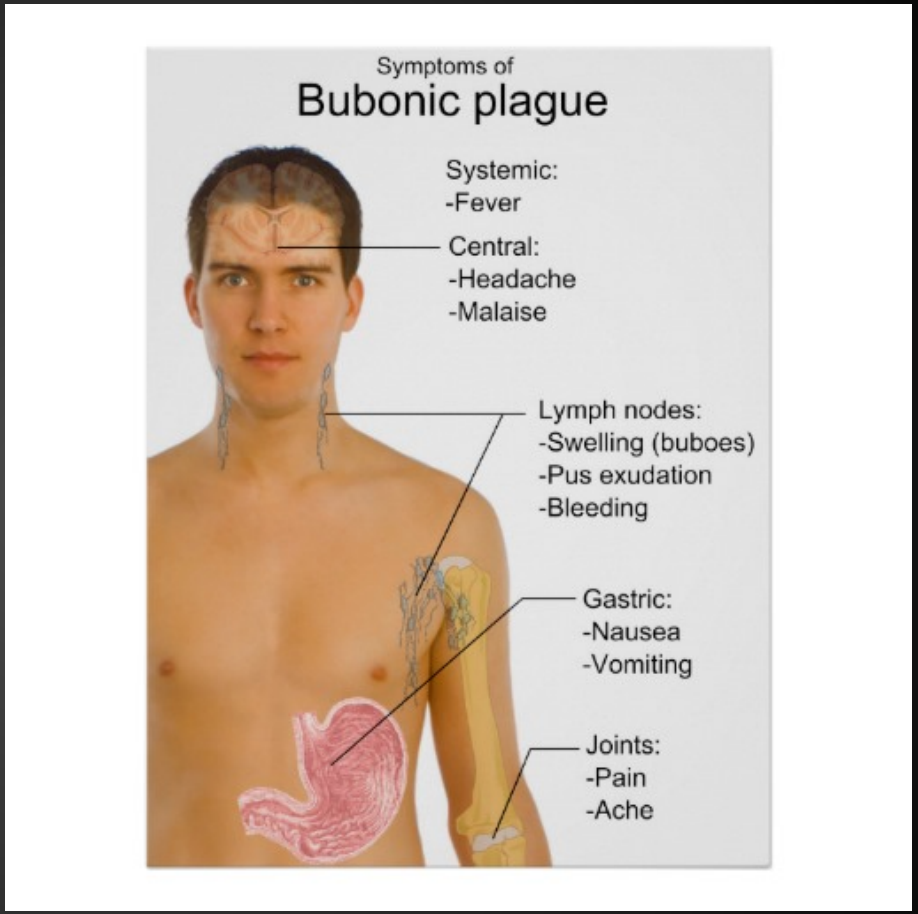
By Cortney Dodge, Matt Strassman, Matthew Taylor

http://blog.brianseitel.com/wp-content/uploads/2012/11/plague.png

Background

- Bacteria, *Yersinia Pestis*
- Flea carriers
- Brought to Europe in 14th century
- Killed 30% - 60% of population

http://juliamallen.com/wp-content/uploads/2012/04/Holbein-death.png



http://www.zazzle.co.nz/main_symptoms_of_black_death_bubonic_plague_chart_poster-228357466391861157
http://jl10ll.wordpress.com/2013/02/11/not-all-of-the-plagues-outcomes-were-deadly/

- Why is the Black Plague important to us now?
  - 1000 - 3000 cases annually
  - Recent outbreaks
    - India
    - Madagascar
    - Mozambique
  - Discovery of multi-drug-resistant strains
  - Widespread in rodent populations

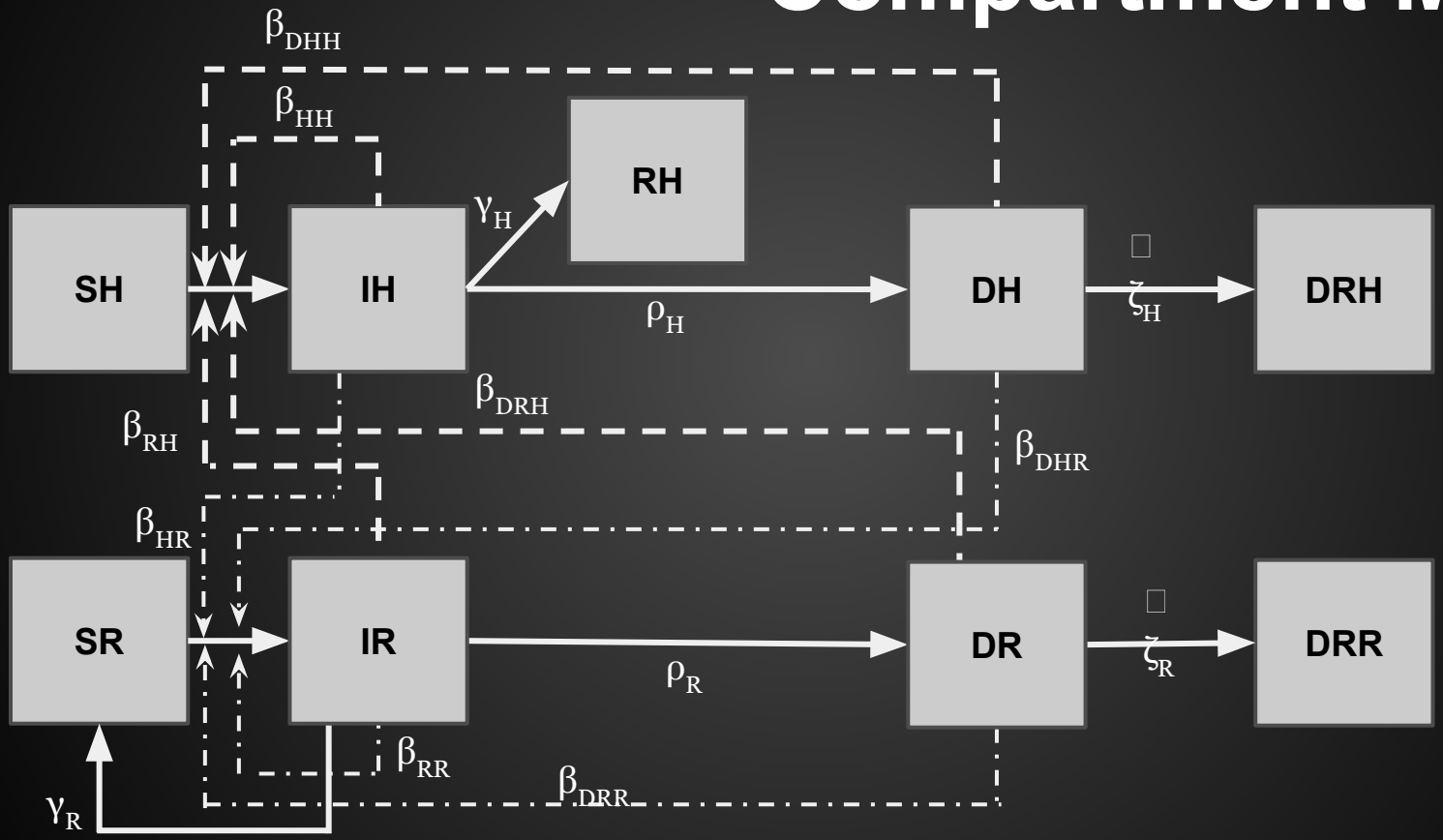
Goals

- Examine potential outbreak of disease in current day
- Test possible treatment strategies
- See which treatment is most effective

Methods

- Differential equations model
- Spatial model
  - Interactions between rats and humans.
  - Simulates space and time together.
- Treatments:
  - Rat removal
  - Quarantine infected humans

Compartment Model



Differential Equations

$$\frac{dSH}{dt} = -\beta_{HH}*SH*IH - \beta_{DHH}*SH*DH - \beta_{DRH}*SH*DR - \beta_{RH}*SH*IR$$

$$\frac{dIH}{dt} = \beta_{HH}*SH*IH + \beta_{DHH}*SH*DH + \beta_{DRH}*SH*DR + \beta_{RH}*SH*IR - \gamma_H*IH - \rho_H*IH$$

$$\frac{dRH}{dt} = \gamma_H*IH$$

$$\frac{dDH}{dt} = \rho_H*IH - \zeta_H*DH$$

$$\frac{dDRH}{dt} = \zeta_H*DH$$

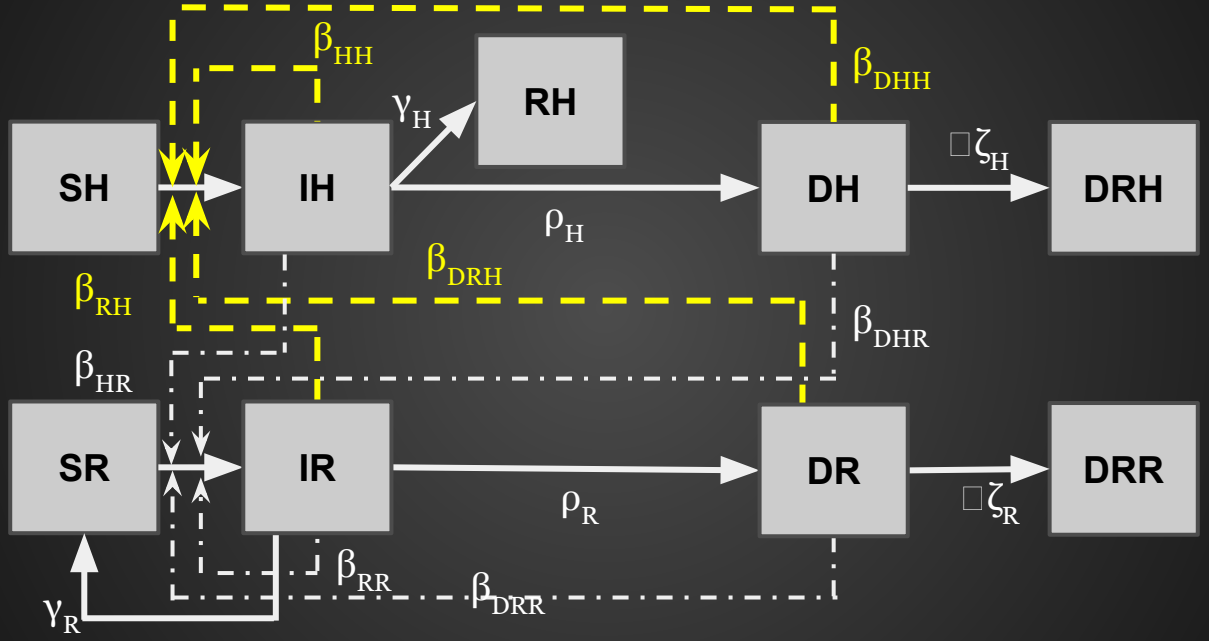
$$\frac{dSR}{dt} = -\beta_{RR}*SR*IR - \beta_{DRR}*SR*DR - \beta_{HR}*SR*IH - \beta_{DHR}*SR*DH + \gamma_R*IR$$

$$\frac{dIR}{dt} = \beta_{RR}*SR*IR + \beta_{DRR}*SR*DR + \beta_{HR}*SR*IH + \beta_{DHR}*SR*DH - \gamma_R*IR - \rho_R*IR$$

$$\frac{dIR}{dt} = \rho_R*IR - \zeta_R*DR$$

$$\frac{dDRR}{dt} = \zeta_R*DR$$

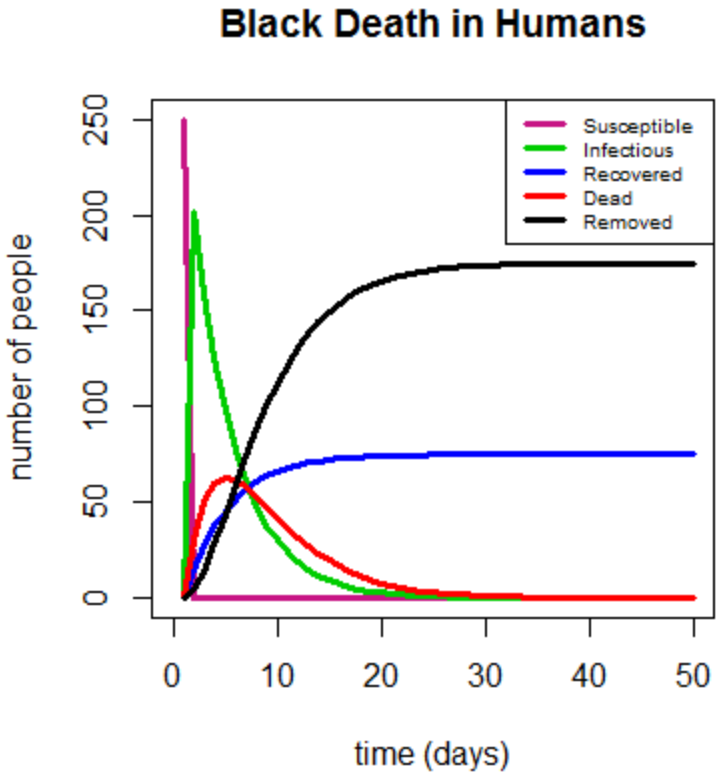
Understanding the Infectious Class

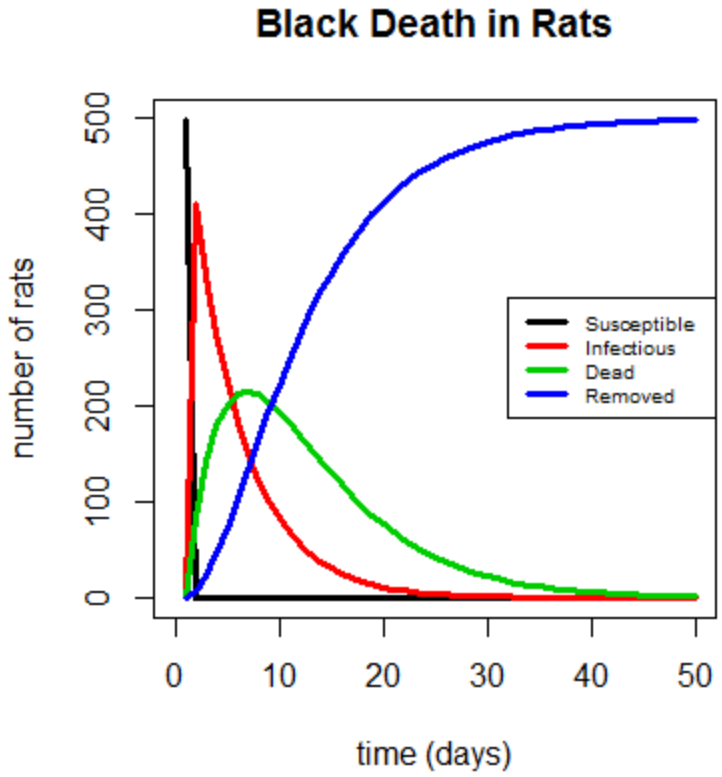


β_{HH} = 0.02
β_{RH} = 0.02
β_{DHH} = 0.30
β_{DRH} = 0.30

$$\frac{dIH}{dt} = \beta_{HH}*SH*IH + \beta_{RH}*SH*IR + \beta_{DRH}*SH*DR + \beta_{DHH}*SH*DH - \gamma_H*IH - \rho_H*IH$$

Differential Equations Results

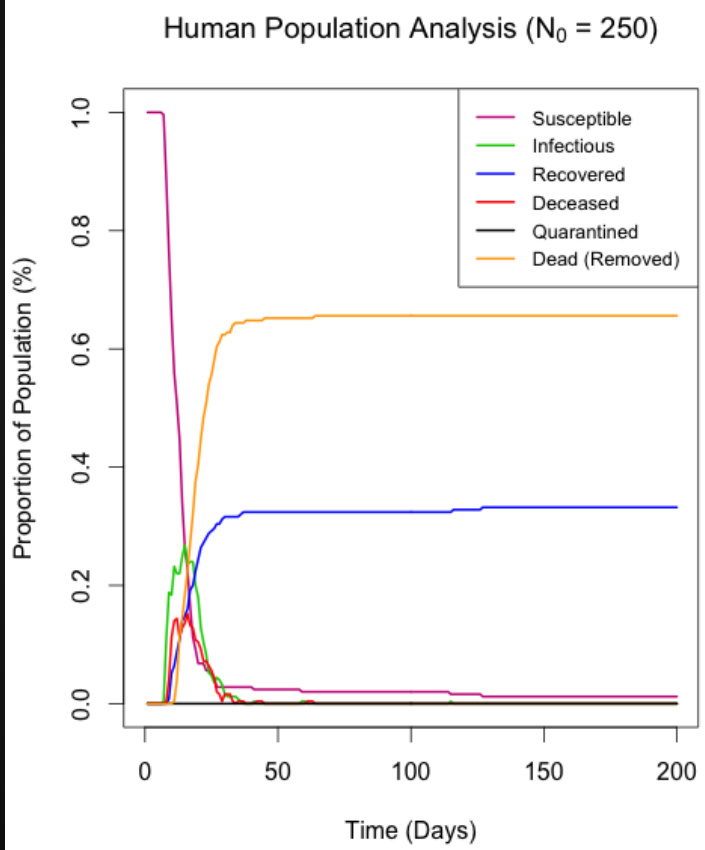


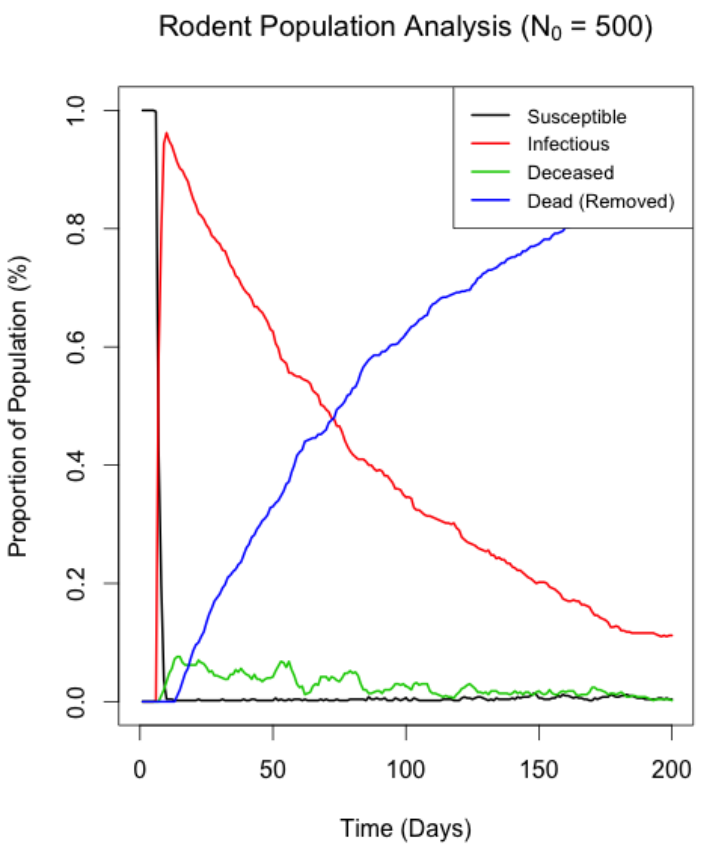


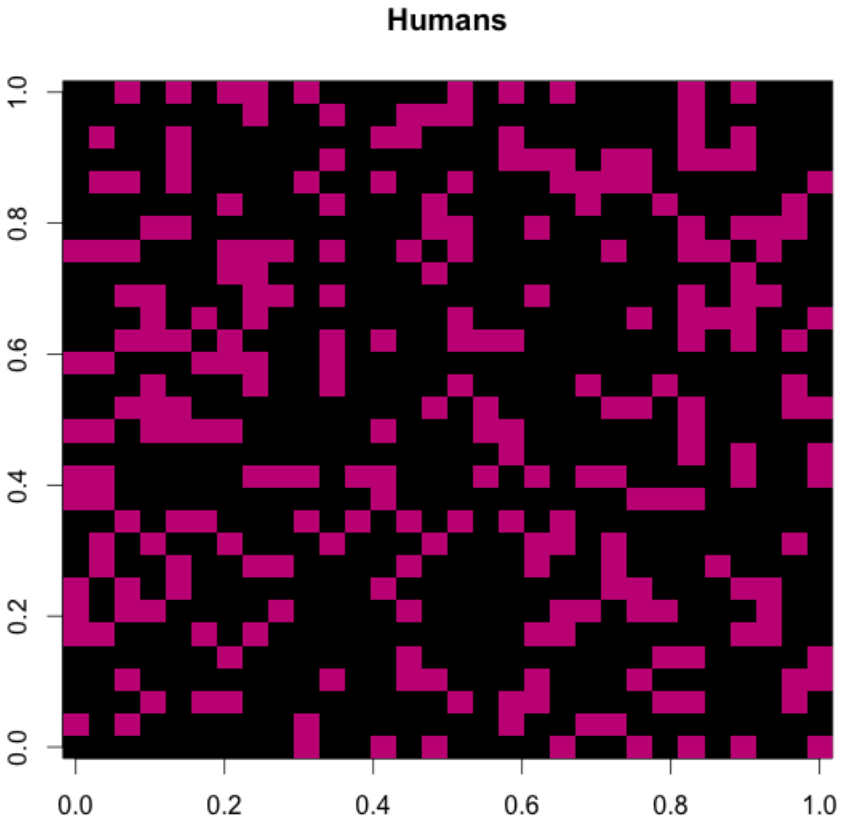
Equilibria

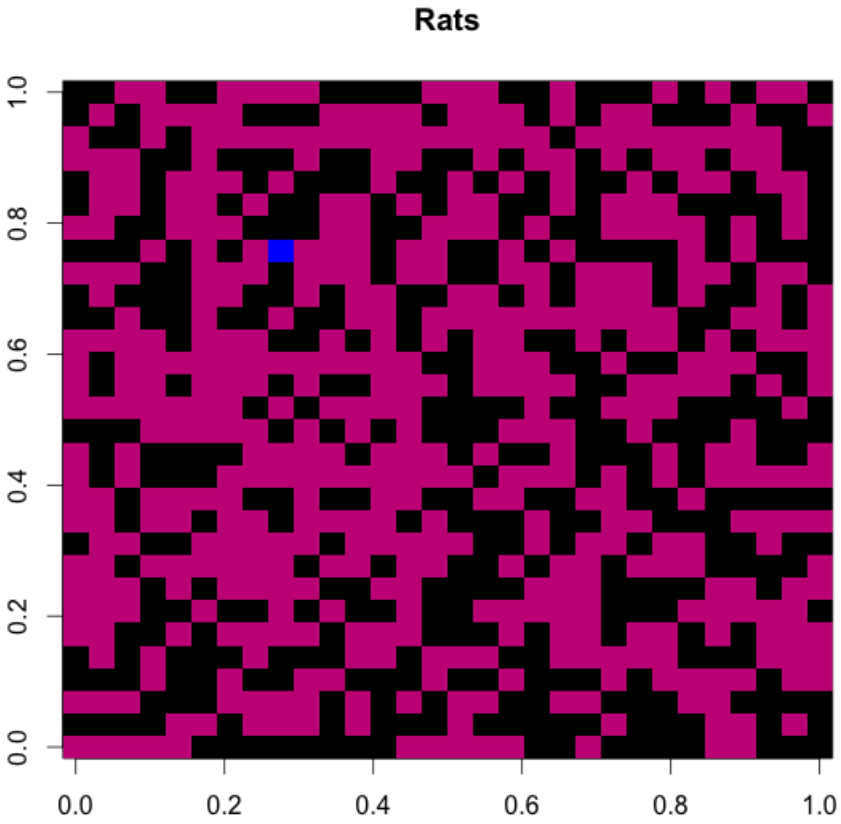
- The states are absorbing, thus there are no equilibria present
- final states dependent on initial population
  - changes in initial populations do not see a return to any equilibrium

Spatial Model Results






Humans
0.0
0.2
0.4
0.6
0.8
1.0


Rats
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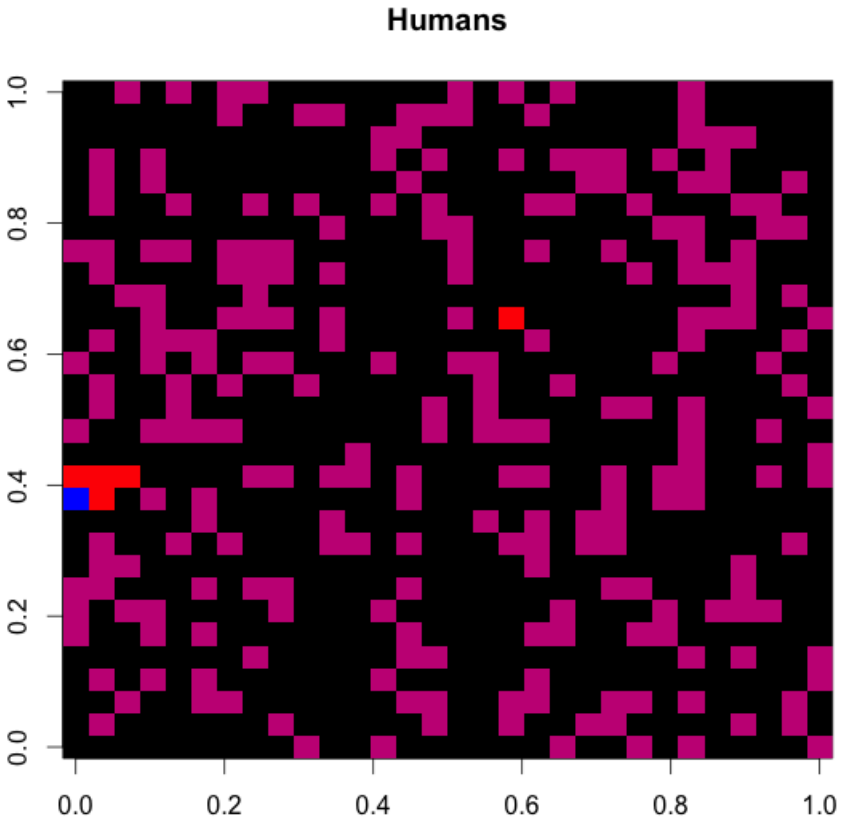
0.2
0.4
0.6
0.8
1.0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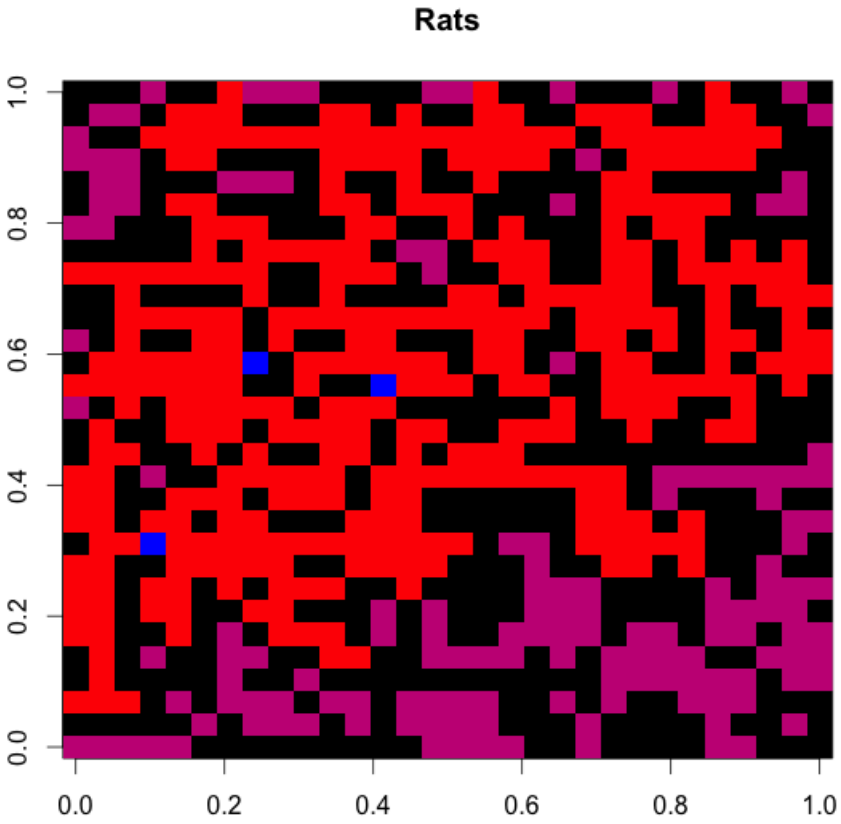
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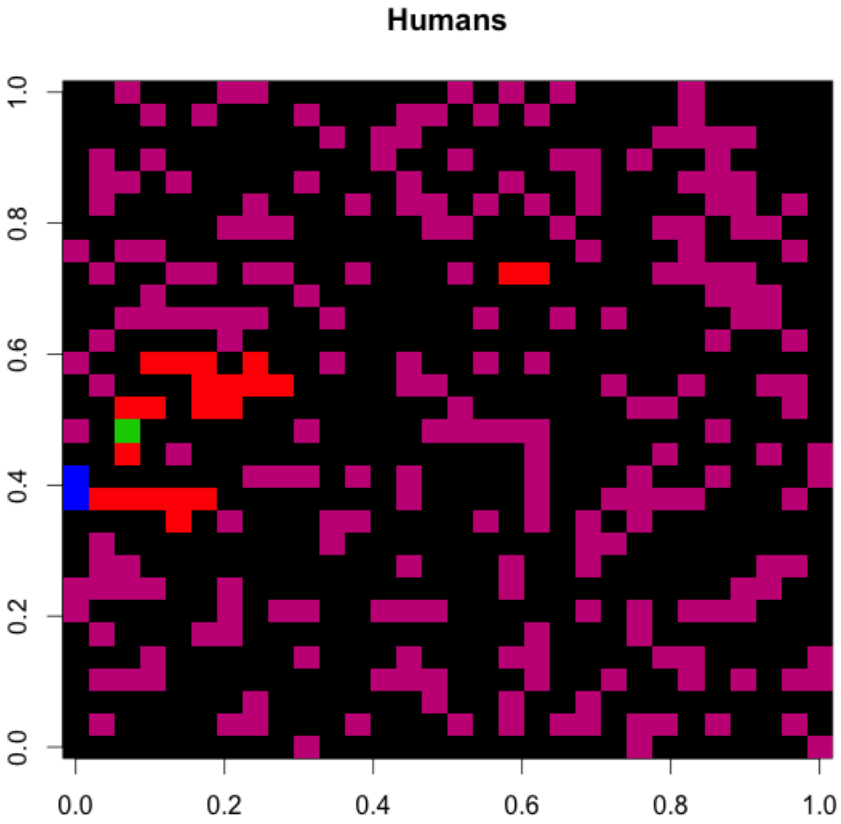
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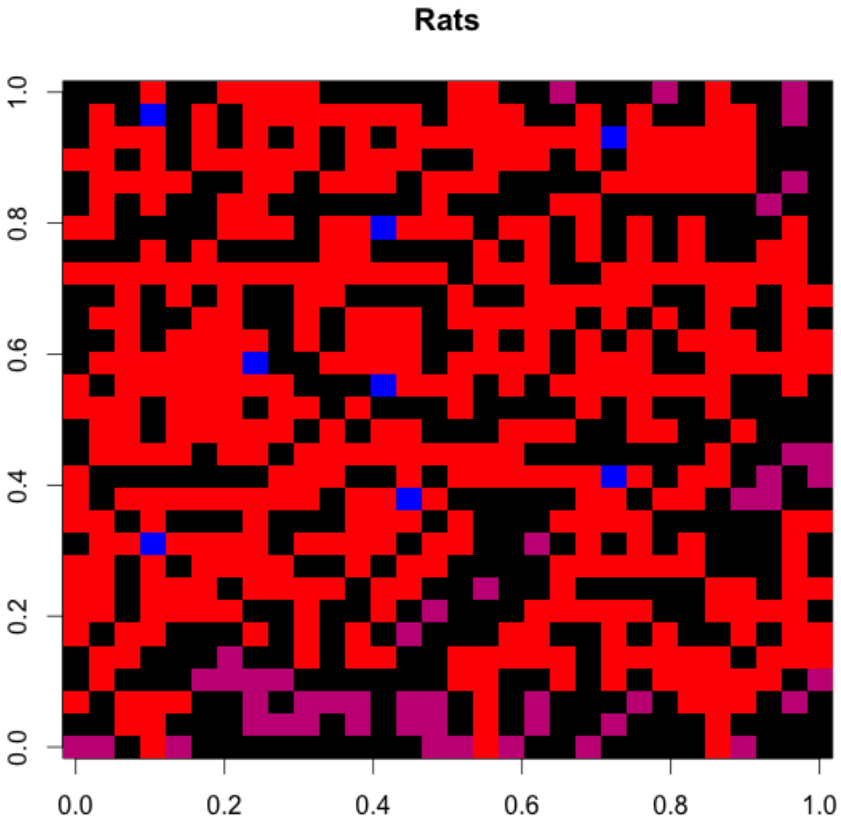
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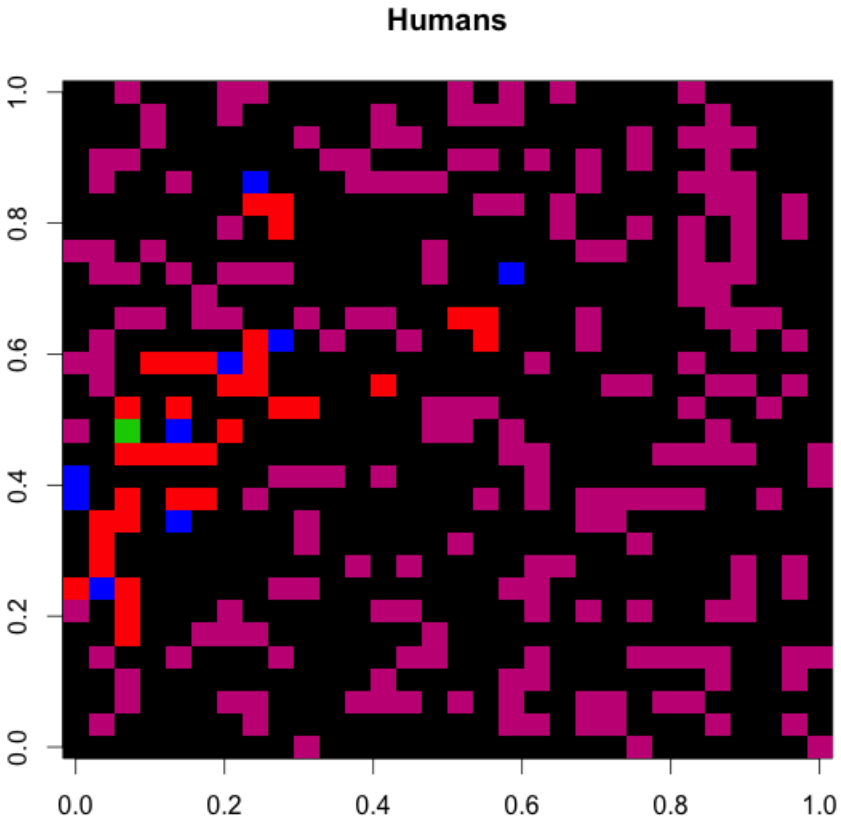
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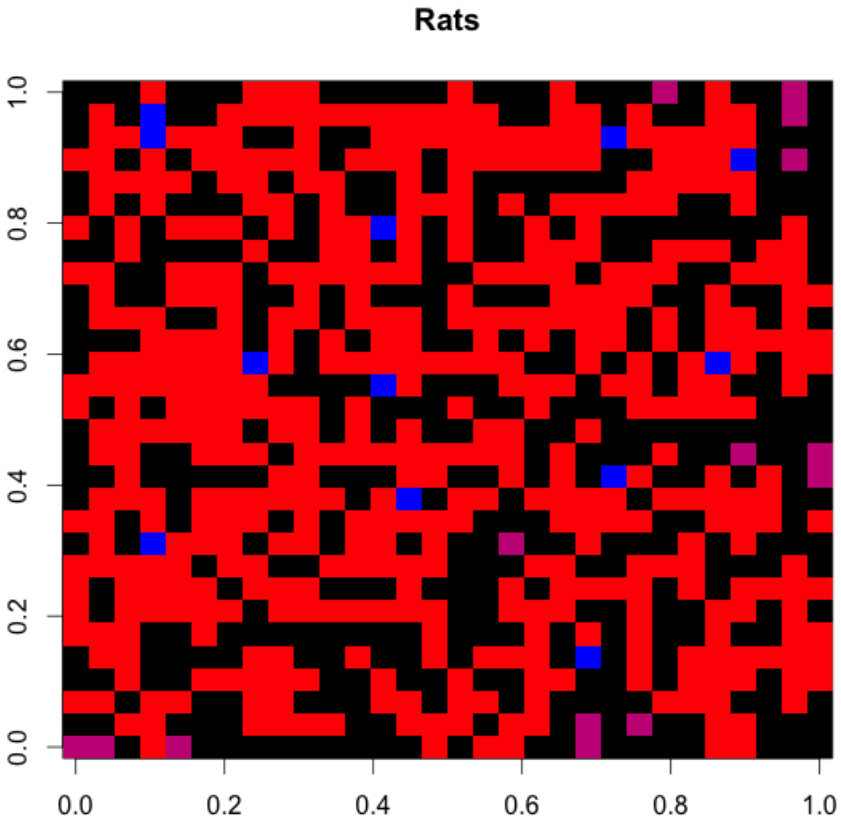
Rats
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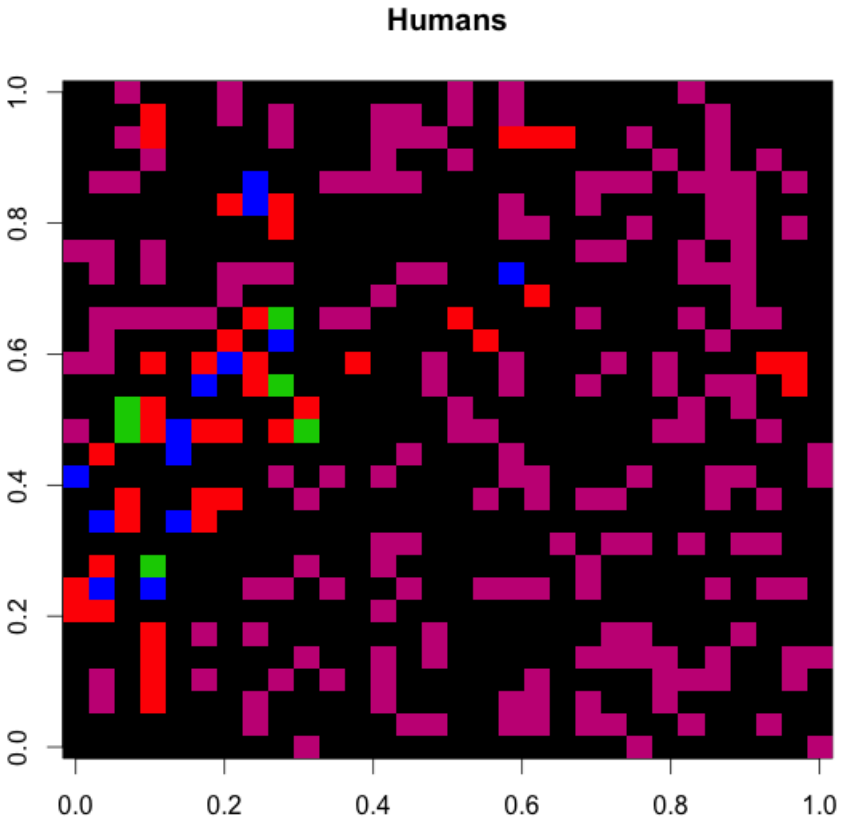
Humans
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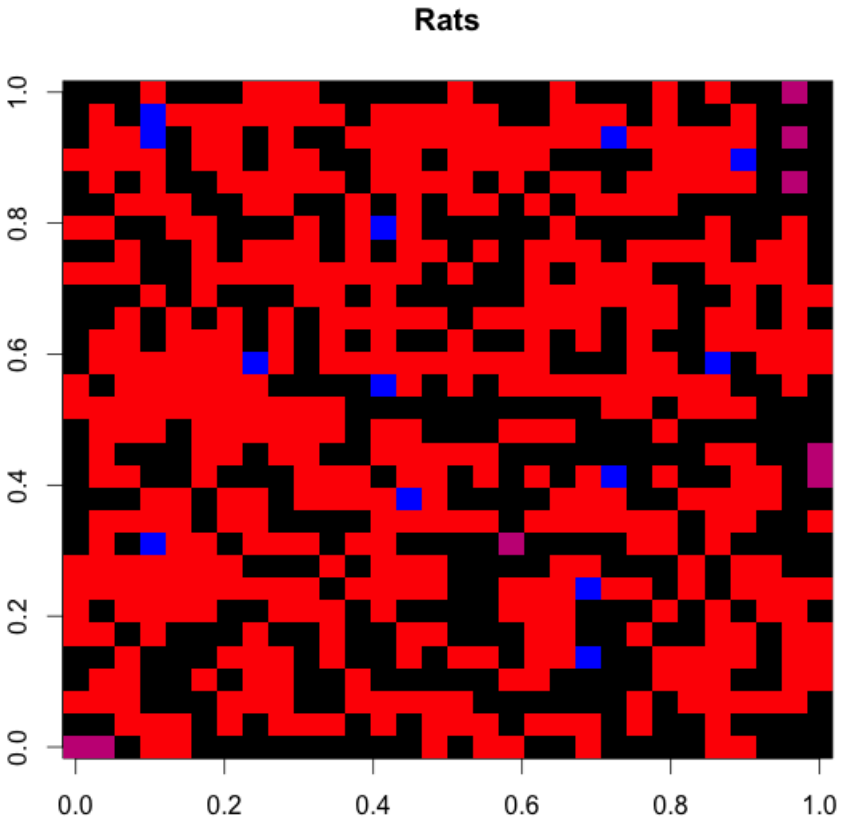
Rats
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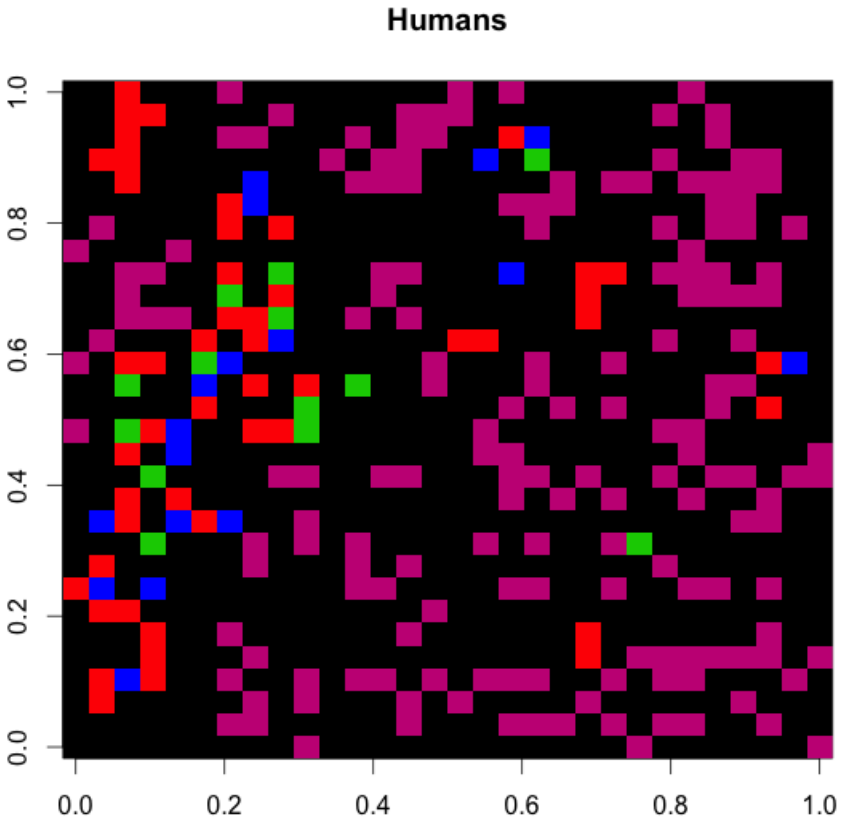
Humans
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0.2
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0.6
0.8
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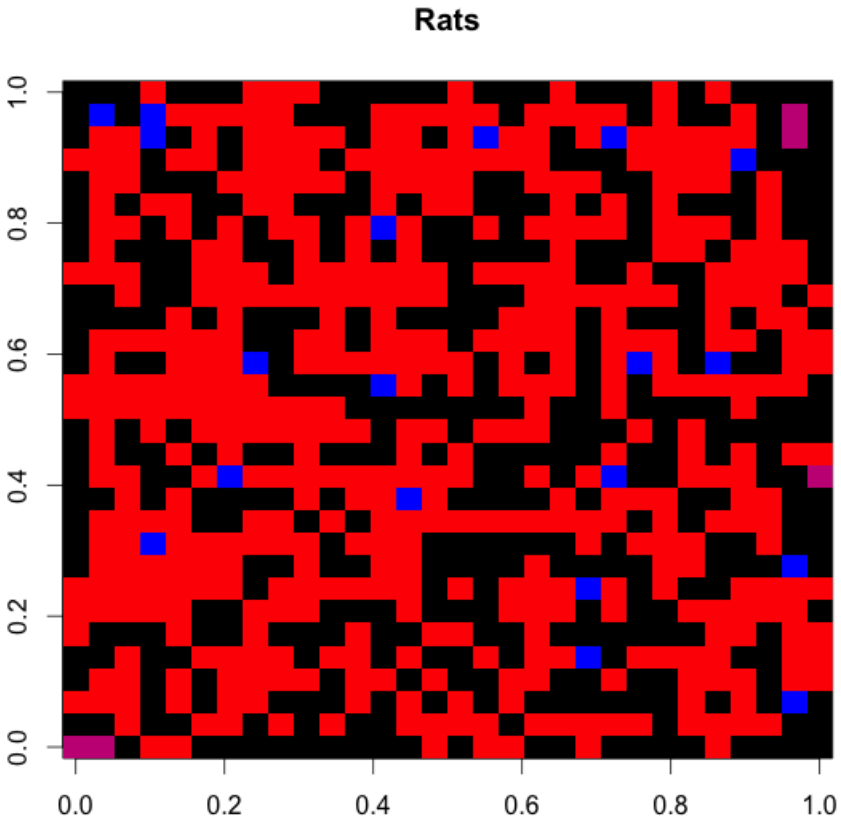
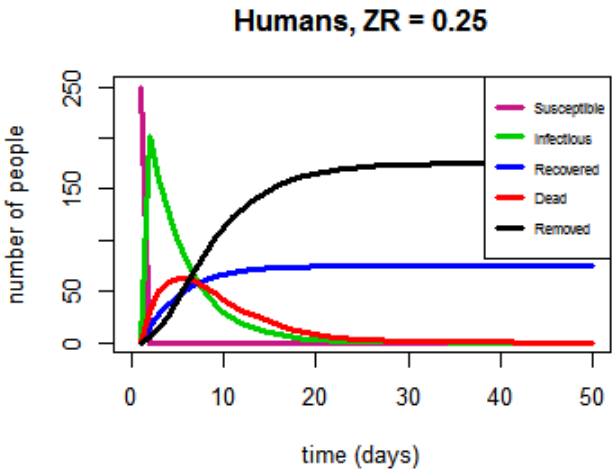
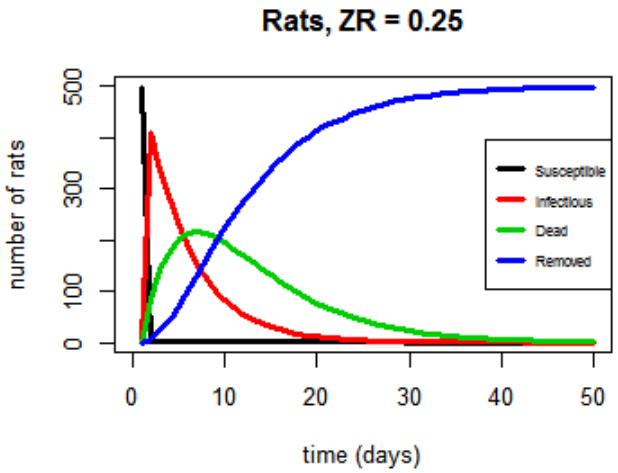
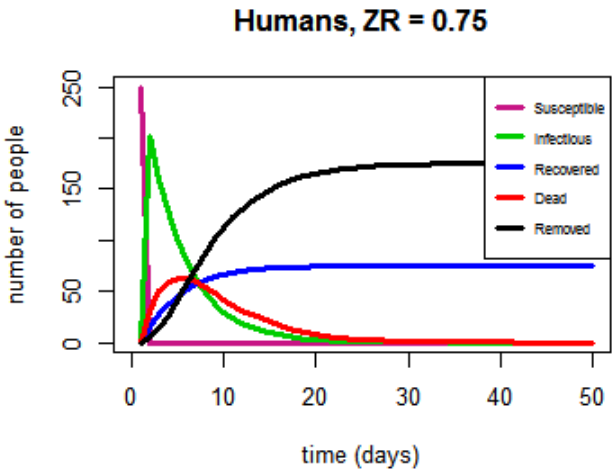

Rats
0.0
0.2
0.4
0.6
0.8
1.0

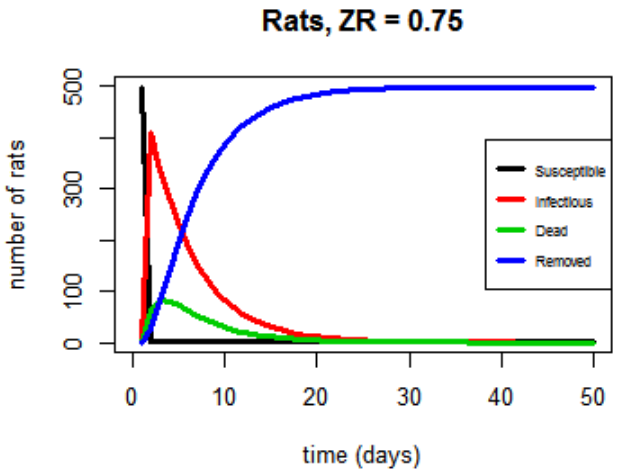
Treatments: Diff. Eqs. Rat Removal

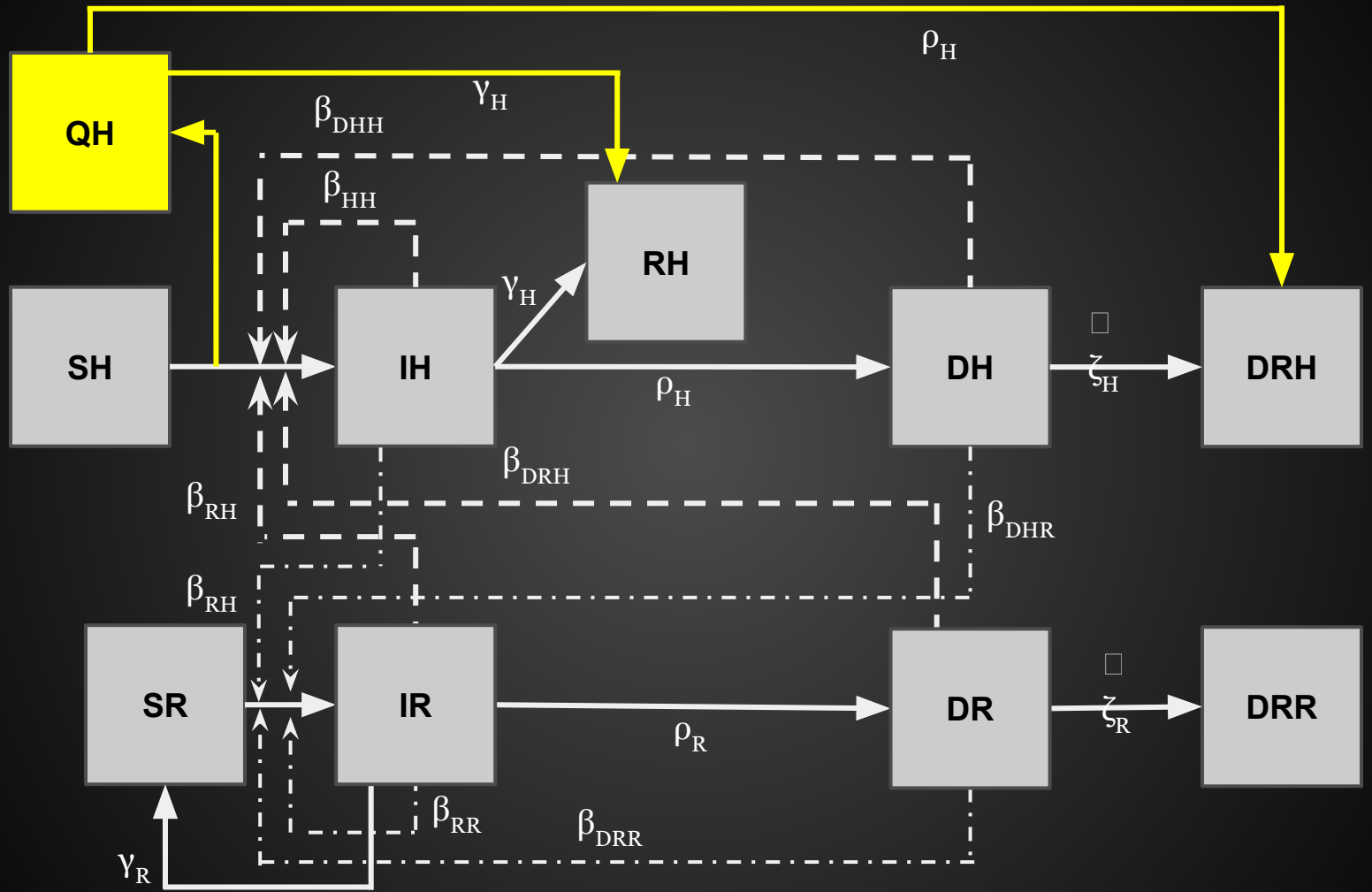
- rat population goes more rapidly to removed class
- total infectious humans at one time decreases by 1 individual










QH
SH
IH
RH
DH
DRH
SR
IR
DR
DRR
ρH
γH
βDHH
βHH
γH
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βDHR
βRH
ρR
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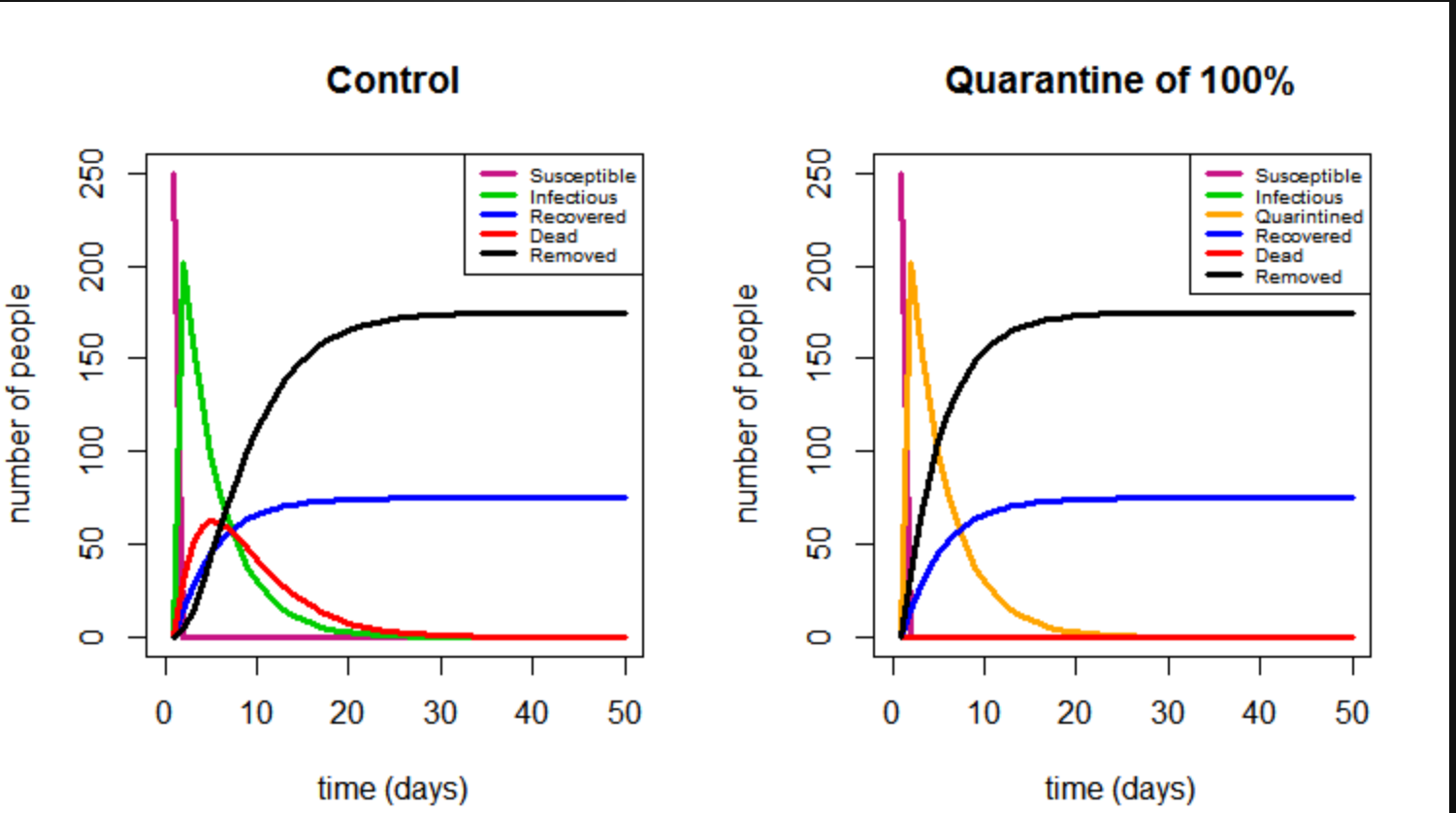
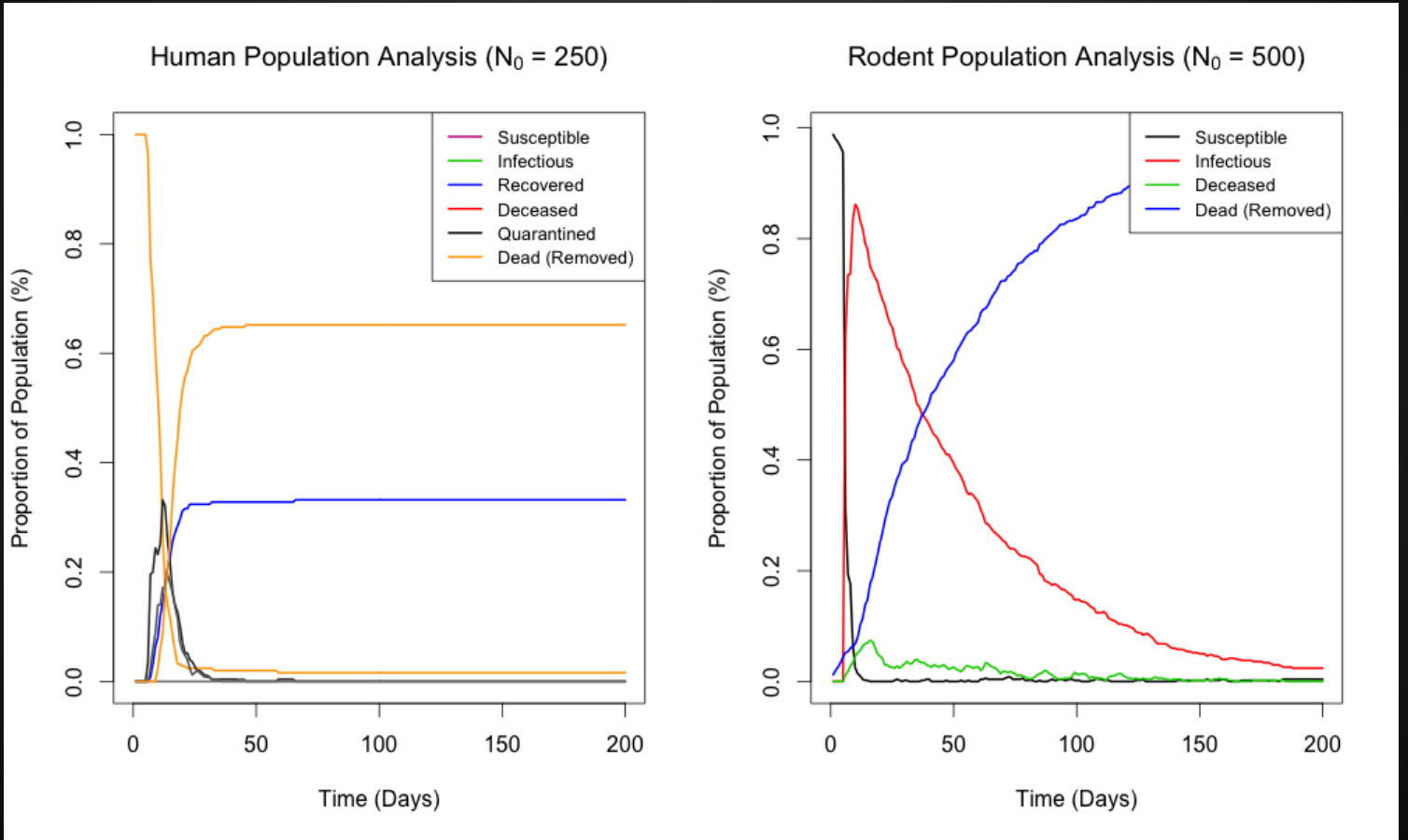
βRR
βDRR
γR

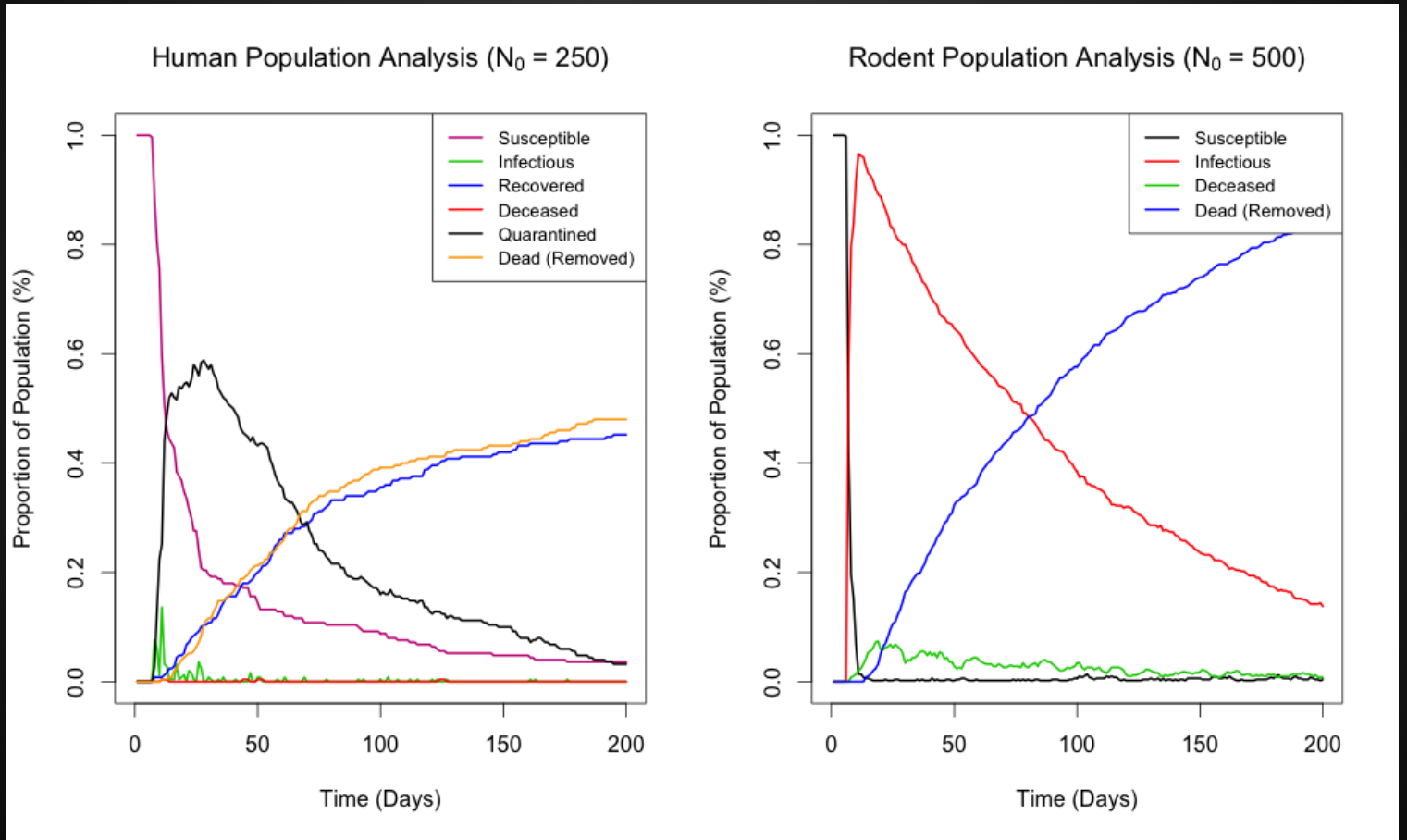
Treatment: Diff. Eqs. Quarantine



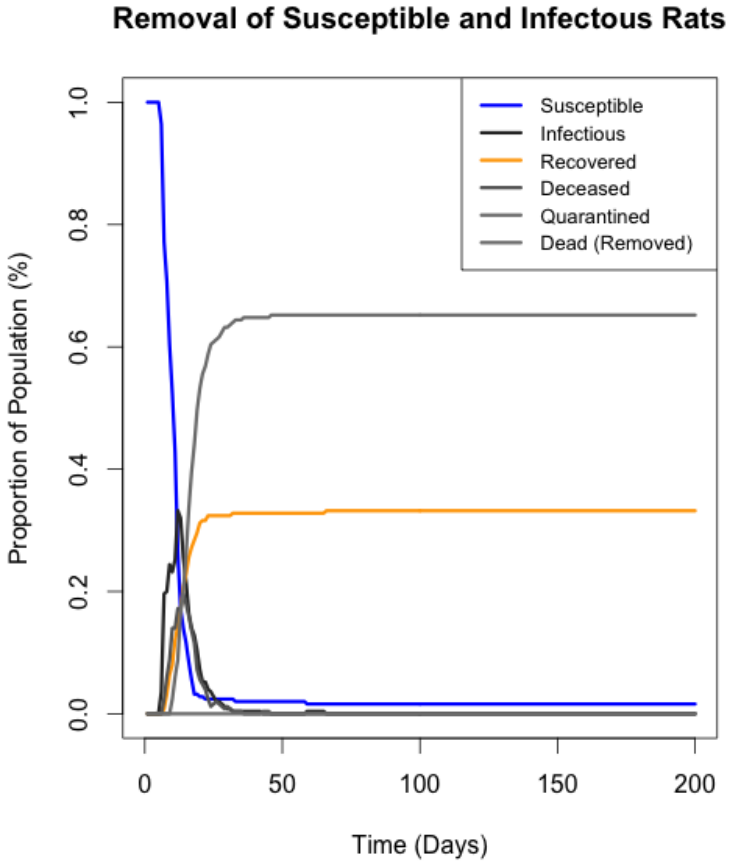
Treatment: Spatial Model Rat Removal

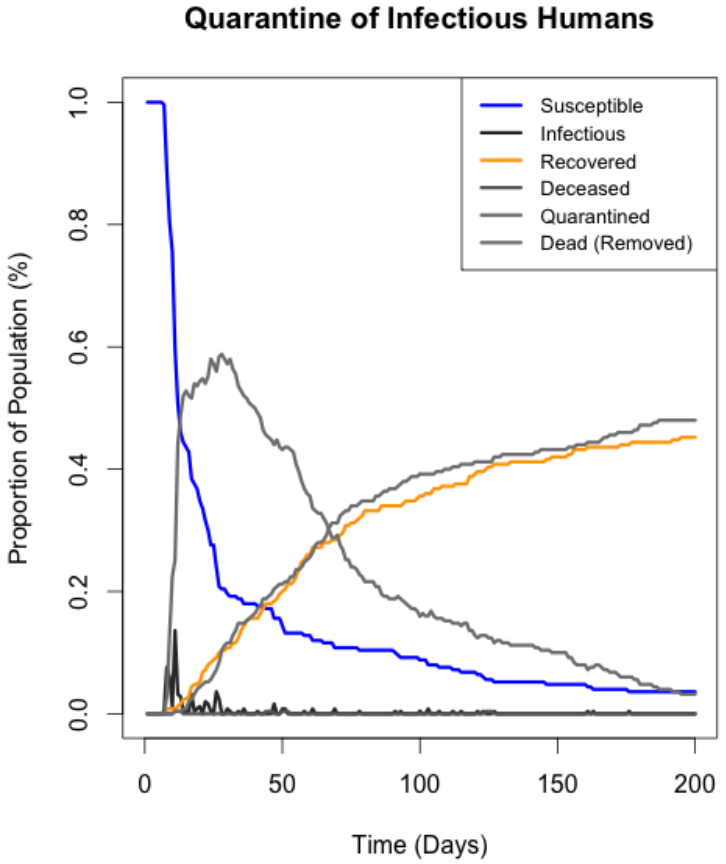


Treatment: Spatial Model Quarantine



Comparison of Treatments





Discussion

- Rat removal has virtually no effect on human population
- Quarantine results in Differential Equations vs. Spatial Model
- Problems with quarantine implementation
  - Draft?
  - Isolation of personnel in quarantine
  - Needs adequate time and preparation

Conclusions

In the event of an outbreak of the plague, or a similar disease, we would suggest the implementation of a quarantine strategy depending on the amount of preparation time.

Works Cited

Keeling, M. J., and C. A. Gilligan. "Bubonic plague: A metapopulation model of a zoonosis." Proceedings of the Royal Society B: Biological Sciences 267 (2000): 2219-230.

Acknowledgements

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- MBS Class

Questions?