

A2_DES_next_event_sim Final

March 15, 2019

1 Next-Event Simulation Assignment

1.0.1 Year 2018-2019 - Semester II

1.0.2 CCE3501 / CCE3502

1.1 ##### Developed by - Adrian Muscat, 2019

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In this assignment you will be implementing next-event simulations for systems involving queues and service nodes in resource allocation systems.

Tasks and Questions (repeated below at the appropriate position)

1. You are given a ready-coded single server queuing system simulation. Go through the code and note the event service routines, timing routine, iat and st generators, etc.
2. Develop a function to plot the queue length versus time
3. Develop functions to calculate the average queue length, average interarrival rate and server utilisation.
4. Compare the answers in 3 to the M/M/1 analytical model
5. modify the event routines (re-write new ones) to model a resource allocation system without queuing, i.e if a customer calls and the server is busy, the customer is blocked.
6. Plot the blocking rate versus number of servers ($N=1, 2, 3 \dots 20$)
7. Add a buffer of finite size to the resource allocation system, such that when a customer calls and the server is busy, the customer is queued.
8. Plot the blocking rate versus buffer size.

```
In [16]: import random, math
import numpy as np
import matplotlib.pyplot as plt
# import scipy.stats as stats
```

```
In [17]: # this line plots graphs in line
%matplotlib inline
```

1.2 1. Below is a ready-coded single server queuing system simulation. Go through the code and note the event service routines, timing routine, iat and st generators, etc.

```
In [18]: fileOutA=True # Flag, if true output to file
stateList=[] # List containing [q(t), s(t), time]
```

```

servTimeList=[]
eventList=[]      # Event calendar list
notice=[]         # notice = [ notice number, time, type ]
                  # type = 0, arrival
                  # type = 1, begin service
                  # type = 2, end service
now = 0.0         # Simulation time
Q = 0             # Queue Length, Q=0 when empty
S=0               # Server State 0=idle, 1=busy
EN=0              # Event Notice Number
avgintarrtimeList = [] #list for accumulating interarrival time

```

```

In [19]: def ScheduleEventNotice(notice):
    global eventList
    if len(eventList)==0:
        eventList.append(notice)
    elif len(eventList)==1:
        if eventList[0][1] <= notice[1]:
            eventList.append(notice)
        else:
            eventList.insert(0,notice)
    elif eventList[len(eventList)-1][1] < notice[1]:
        eventList.append(notice)
    else:
        i=0
        while (eventList[i][1]<=notice[1] and i<(len(eventList)-1)):
            i += 1
        eventList.insert(i,notice)

```

```

In [20]: def DisplayEventList(header):
    print (header)
    i=0
    while i<len(eventList):
        print (eventList[i])
        i += 1

    def writeToFile():
        #print 'Q = ',Q, ' S = ',S, ' now = ', now
        stateEntry = [Q, S, now]
        stateList.append(stateEntry)

    def DisplayStateList(header):
        print (header)
        i=0
        while i<len(stateList):
            print (stateList[i])
            i += 1

```

```

In [21]: def InterArrivalTime():

```

```

    #return -10*math.log(random.random())
    return np.random.exponential()*8.001

```

```

def ServiceTime():
    #return -8*math.log(random.random())
    return np.random.exponential()*8

```

```

In [22]: def EventRoutineArrival():
    global EN, Q, S
    EN=EN+1
    intarrivaltime = InterArrivalTime()
    avgintarrtimeList.append(intarrivaltime)
    eventTime = now+intarrivaltime
    #notice=[EN+1, now+InterArrivalTime(), 0]
    notice=[EN, eventTime, 0]
    ScheduleEventNotice(notice)
    Q = Q + 1
    if S==0:
        EN+=1
        notice=[EN, now, 1]
        ScheduleEventNotice(notice)
    if fileOutA: writeToFile()

```

```

In [23]: def EventRoutineBeginService():
    global EN, Q, S
    Q = Q - 1
    S = 1
    EN+=1
    sT = ServiceTime()
    servTimeList.append(sT)
    eventTime = now+sT
    notice=[EN, eventTime, 2]
    ScheduleEventNotice(notice)
    if fileOutA: writeToFile()

```

```

In [24]: def EventRoutineEndService():
    global EN, Q, S
    S=0
    if Q>0:
        EN=EN+1
        eventTime = now
        notice = [EN, eventTime, 1]
        ScheduleEventNotice(notice)
    if fileOutA: writeToFile()

```

```

In [25]: # Schedule arrival event at time NOW;
    notice=[EN, now, 0]
    ScheduleEventNotice(notice)

```

```

print ('Start of Simulation run')

while (eventList!=[] and now<2000): #200000
    now = eventList[0][1]
    #print now, ' ', eventList[0]
    #print 'Q = ', Q, ' S = ', S
    #DisplayEventList('Hello')
    if eventList[0][2]==0:
        EventRoutineArrival()
    elif eventList[0][2]==1:
        EventRoutineBeginService()
    elif eventList[0][2]==2:
        EventRoutineEndService()
    else:
        print ('Unknown event type')
    #Remove first notice
    eventList.pop(0)

print ('End of Simulation run')

```

Start of Simulation run
End of Simulation run

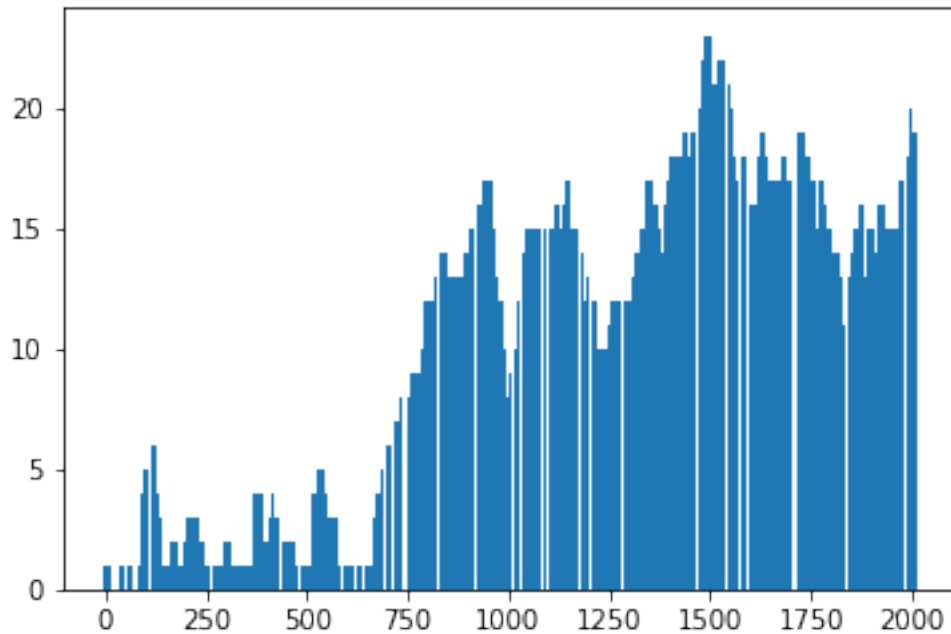
2 GRADED QUESTIONS BELOW: 7 questions, Total = 70, weight =15%

2.1 2. Develop a function to plot the queue length versus time [5 marks]

```

In [26]: x = np.array(stateList)
         plt.bar(x[:,2],x[:,0], width=10,align='center')
         plt.show()

```



In []:

2.2 3. Develop functions to calculate the average queue length, average interarrival rate, server utilisation and mean waiting time. [10 marks]

```
In [39]: def avgQlength():
    x = np.array(stateList)
    extracted = x[:,0]
    b = sum(extracted)
    avgQueueLength = b/len(extracted)
    return avgQueueLength

def avgservice():
    sum = 0
    for i in range(1000):
        x = ServiceTime()
        sum += x

    avgservicerate = sum/1000
    return avgservicerate

def avgIrate():
    sum = 0
    for i in range(1000):
        x = InterArrivalTime()
        sum += x
```

```

        avgArrivalRate = sum/1000
        return avgArrivalRate

def serverUtil(a,b):
    # = lambda/mu = row = 1/mean rate of arrival / 1/mean service rate
    x = (1/a) / (1/b)
    print(x)

def meanWaitingT():
    x = np.array(stateList)
    extracted = x[:,2]
    b = sum(extracted)
    answer = b/len(extracted)
    print(answer)

```

```
avgservice = avgservice()
```

```
In [40]: #Question 1
        print(avgQLen)
```

```
9.789877300613497
```

```
In [41]: #Question 2
        print(avgArrivalRate)
```

```
8.005255367886113
```

```
In [42]: #Question 3
        serverUtil(avgArrivalRate,avgservice)
```

```
0.9926755393901777
```

```
In [43]: #Question4
        meanWaitingT()
```

```
1004.2780049277006
```

2.3 4. Compare the answers in 3 to the M/M/1 analytical model [5 marks]

```
In [44]: #M/M/1:
        #L =  $\rho^2 / (1 - \rho)$ 
        #L = mean no of customers
        # = server utilization = (1/mean rate of arrivals)/(1/mean service rate)
```

```

# lambda = 1/interarrival time
# mu = 1/servicetime

def avgQlength():
    x = np.array(stateList)
    extracted = x[:,0]
    b = sum(extracted)
    avgQueueLength = b/len(extracted)
    return avgQueueLength

def avgIrate():
    sum = 0
    for i in range(1000):
        x = InterArrivalTime()
        sum += x

    avgArrivalRate = sum/1000
    return avgArrivalRate

def avgservice():
    sum = 0
    for i in range(1000):
        x = ServiceTime()
        sum += x

    avgservicerate = sum/1000
    return avgservicerate

def serverUtil(a,b):
    # = lambda/mu = row = 1/mean rate of arrival / 1/mean service rate
    x = (1/a) / (1/b)
    return x

def meanWaitingT():
    x = np.array(stateList)
    extracted = x[:,2]
    b = sum(extracted)
    answer = b/len(extracted)
    return (answer)

#M/M/1 model to compare with average queue length
def MM1meancustomers(b):
    x = (b**2)/(1-b)
    return x

```

```

#M/M/1 model to compare with average waiting time, wait in queue = mean no of customers
def MM1meanwaiting(a,b):
    c = 1/b
    x = a/c
    return x

avgservice = avgservice()
avgArrivalRate = avgIrate()
A = serverUtil(avgArrivalRate,avgservice)
avgQLen = avgQlength()
meanwaitrate = meanWaitingT()

B = MM1meancustomers(A)
C = MM1meanwaiting(B,avgArrivalRate)

```

```
In [47]: print(B)
```

```
20.4117639572627
```

```
In [48]: print(C)
```

```
166.37656658931732
```

2.4 5. Modify the appropriate functions (re-write new ones) to model a resource allocation system (with N servers) without queuing, i.e if a customer calls and all servers are busy, the customer is blocked. Set mean iat = 5, mean service rate =170 and N=8. Run the simulation and compute the blocking rate. [10 marks]

```
In [19]: blockedCustomers = 0
```

```
endedCustomers = 0
```

```
In [20]: fileOutA=True # Flag, if true output to file
stateList=[] # List containing [q(t), s(t), time]
servTimeList=[]
eventList=[] # Event calendar list
notice=[] # notice = [ notice number, time, type ]
# type = 0, arrival
# type = 1, begin service
# type = 2, end service

now = 0.0 # Simulation time
#Q = 0 # Queue Length, Q=0 when empty
S=0 # No of Servers occupied
EN=0 # Event Notice Number
avgintarrtimeList = [] #list for accumulating interarrival time
n=0 #no of servers

```



```

In [21]: def ScheduleEventNotice(notice):
    global eventList
    if len(eventList)==0:
        eventList.append(notice)
    elif len(eventList)==1:
        if eventList[0][1] <= notice[1]:
            eventList.append(notice)
        else:
            eventList.insert(0,notice)
    elif eventList[len(eventList)-1][1] < notice[1]:
        eventList.append(notice)
    else:
        i=0
        while (eventList[i][1]<=notice[1] and i<(len(eventList)-1)):
            i += 1
        eventList.insert(i,notice)

In [22]: def DisplayEventList(header):
    print (header)
    i=0
    while i<len(eventList):
        print (eventList[i])
        i += 1

    def writeToFile():
        #print 'Q = ',Q, ' S = ',S, ' now = ', now
        stateEntry = [Q, S, now]
        stateList.append(stateEntry)

    def DisplayStateList(header):
        print (header)
        i=0
        while i<len(stateList):
            print (stateList[i])
            i += 1

In [23]: def InterArrivalTime():
    #return -10*math.log(random.random())
    return np.random.exponential()*5

    def ServiceTime():
        #return -8*math.log(random.random())
        return np.random.exponential()*170

In [24]: def EventRoutineArrival(n):
    global EN, Q, S, blockedCustomers
    EN=EN+1

```

```

intarrivaltime = InterArrivalTime()
avgintarrtimeList.append(intarrivaltime)
eventTime = now+intarrivaltime
#notice=[EN+1, now+InterArrivalTime(), 0]
notice=[EN, eventTime, 0]
ScheduleEventNotice(notice)
Q = Q + 1
if (S >= n):
    blockedCustomers+=1
else:
    notice=[EN, now, 1]
    ScheduleEventNotice(notice)
if fileOutA: writeToFile()

```

In [25]: `def EventRoutineBeginService():`

```

    global EN, Q, S
    S += 1
    EN+=1
    sT = ServiceTime()
    servTimeList.append(sT)
    eventTime = now+sT
    notice=[EN, eventTime, 2]
    ScheduleEventNotice(notice)
    if fileOutA: writeToFile()

```

In [26]: `def EventRoutineEndService():`

```

    global EN, Q, S, endedCustomers
    endedCustomers += 1
    S-=1
    if fileOutA: writeToFile()

```

In [27]: `# Schedule arrival event at time NOW;`

```

n = 8
notice=[EN, now, 0]
ScheduleEventNotice(notice)

print ('Start of Simulation run')

while (eventList!=[] and now<200): #200000
    now = eventList[0][1]
    #print now, ' ', eventList[0]
    #print 'Q = ', Q, ' S = ', S
    #DisplayEventList('Hello')
    if eventList[0][2]==0:
        EventRoutineArrival(n)
    elif eventList[0][2]==1:
        EventRoutineBeginService()

```

```

elif eventList[0][2]==2:
    EventRoutineEndService()
else:
    print ('Unknown event type')
    #Remove first notice
    eventList.pop(0)

print ('End of Simulation run')
print ('Blocking rate is {b}/{e}'.format(b=blockedCustomers,e = endedCustomers+blockedCustomers))

```

Start of Simulation run

End of Simulation run

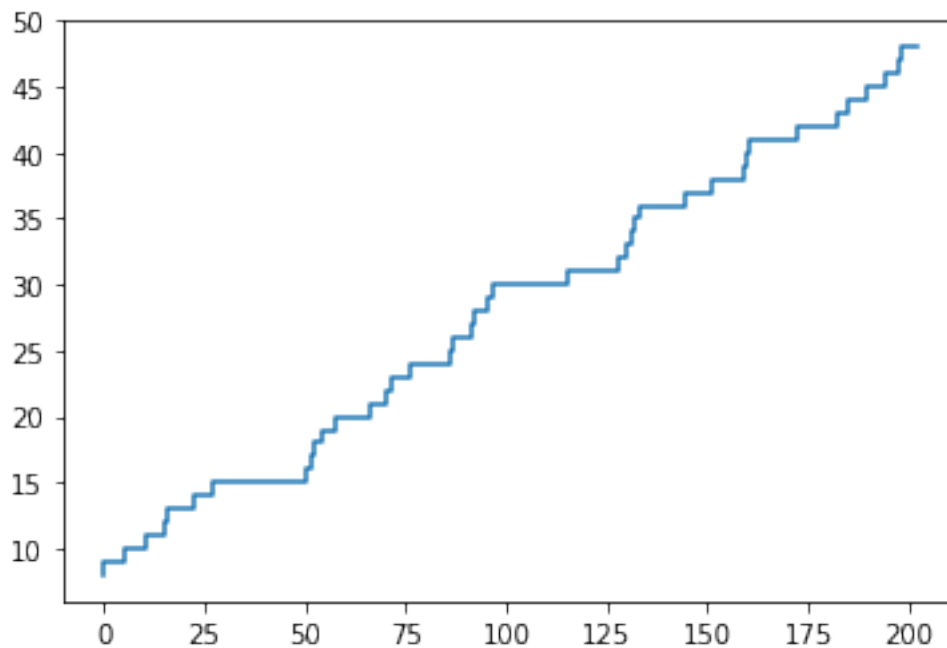
Blocking rate is 27/33

```

In [28]: b = np.asarray(stateList)
        x1=b[:,2]
        y1=b[:,0]
        y12=b[:,1]
        #n=8
        print(np.shape(x1),np.shape(y1),np.shape(y12))
        #PlotQueueLengthVSTime(x1,y1)
        AvgQueueLen(x1,y1)
        AvgIntArrivalTime(x1,y1)

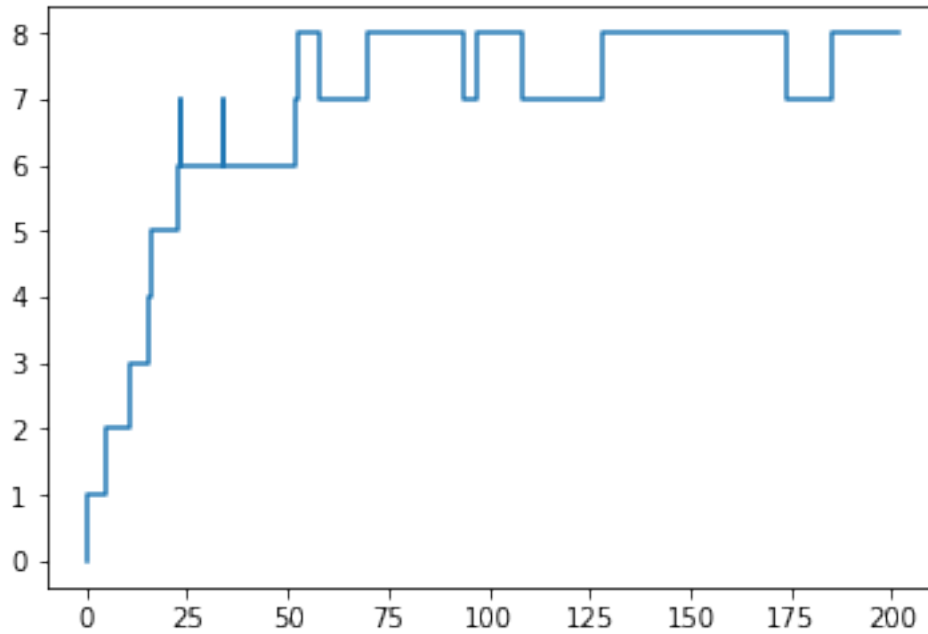
```

(61,) (61,) (61,)



Average queue length is equal to : 27.9015406285575
Average interarrival time: 5.007706514138418

In [29]: ServerUtil(x1,y12,8)



1395.3963946411761
201.84930481368062
Server util is equal to : 0.8641325244649797

In []:

2.5 6. Plot the blocking rate versus number of servers (N=1, 2, 3 50) [10 marks]

```
In [30]: i = 0
blockingRate = []
servers = []
while (i<50):
    fileOutA=True # Flag, if true output to file
    stateList=[] # List containing [q(t), s(t), time]
    servTimeList=[]
    eventList=[] # Event calendar list
    notice=[] # notice = [ notice number, time, type ]
                # type = 0, arrival
                # type = 1, begin service
```

```

                                # type = 2, end service
now = 0.0                        # Simulation time
#Q = 0                          # Queue Length, Q=0 when empty
S=0                             # No of Servers occupied
EN=0                            # Event Notice Number
avgintarrtimeList = []          #list for accumulating interarrival time
n=i
notice=[EN, now, 0]
ScheduleEventNotice(notice)

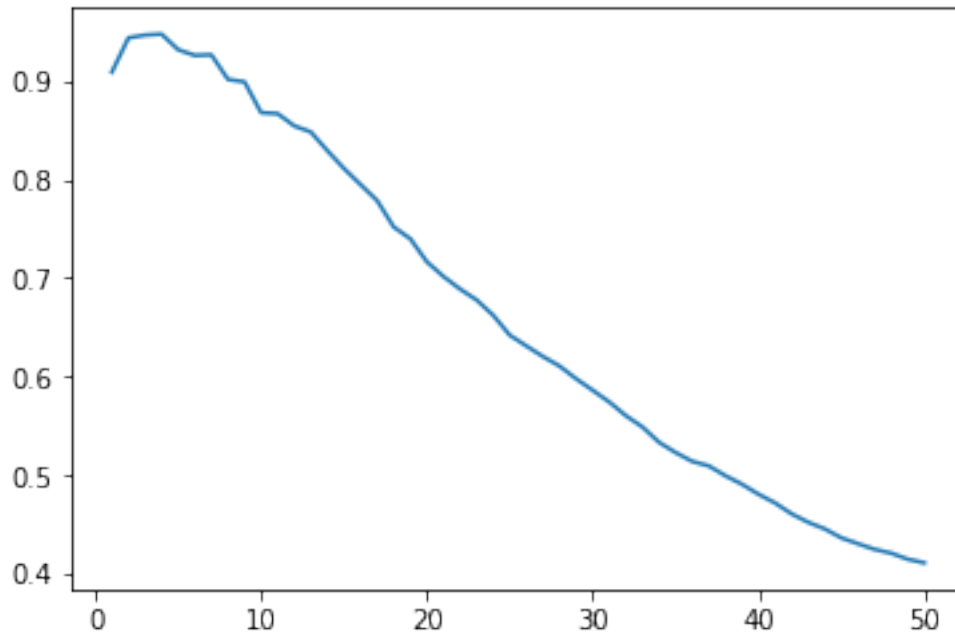
#print ('Start of Simulation run')

while (eventList!=[] and now<200): #200000
    now = eventList[0][1]
    #print now, ' ', eventList[0]
    #print 'Q = ', Q, ' S = ', S
    #DisplayEventList('Hello')
    if eventList[0][2]==0:
        EventRoutineArrival(n)
    elif eventList[0][2]==1:
        EventRoutineBeginService()
    elif eventList[0][2]==2:
        EventRoutineEndService()
    else:
        print ('Unknown event type')
    #Remove first notice
    eventList.pop(0)

#print ('End of Simulation run')
#print ('Blocking rate is {b}/{e}'.format(b=blockedCustomers,e = endedCustomers+b
blockingRate.append(blockedCustomers/(endedCustomers+blockedCustomers))
servers.append(i+1)
i+=1
#print (blockingRate)
#print (servers)
plt.plot(servers,blockingRate)

```

Out[30]: [<matplotlib.lines.Line2D at 0x27ae14b82e8>]



2.6 7. Add a buffer of finite size to the resource allocation system, such that when a customer calls and the server is busy, the customer is queued. [5 marks]

```
In [46]: fileOutA=True      # Flag, if true output to file
stateList=[]               # List containing [q(t), s(t), time]
servTimeList=[]
eventList=[]              # Event calendar list
notice=[]                 # notice = [ notice number, time, type ]
                           # type = 0, arrival
                           # type = 1, begin service
                           # type = 2, end service

now = 0.0                 # Simulation time
Q = 0                     # Queue Length, Q=0 when empty
S=0                       # Server State 0=idle, 1=busy
EN=0                      # Event Notice Number
avgintarrtimeList = []    #list for accumulating interarrival time
buffersize = 20
n=0                       #no of servers
enteredCustomers = 0
```

```
In [47]: def ScheduleEventNotice(notice):
    global eventList
    if len(eventList)==0:
        eventList.append(notice)
    elif len(eventList)==1:
        if eventList[0][1] <= notice[1]:
```

```

        eventList.append(notice)
    else:
        eventList.insert(0,notice)
elif eventList[len(eventList)-1][1] < notice[1]:
    eventList.append(notice)
else:
    i=0
    while (eventList[i][1]<=notice[1] and i<(len(eventList)-1)):
        i += 1
    eventList.insert(i,notice)

```

In [48]: `def DisplayEventList(header):`

```

    print (header)
    i=0
    while i<len(eventList):
        print (eventList[i])
        i += 1

```

`def writeToFile():`

```

    #print 'Q = ',Q, '    S = ',S, '    now = ', now
    stateEntry = [Q, S, now]
    stateList.append(stateEntry)

```

`def DisplayStateList(header):`

```

    print (header)
    i=0
    while i<len(stateList):
        print (stateList[i])
        i += 1

```

In [49]: `def InterArrivalTime():`

```

    #return -10*math.log(random.random())
    return np.random.exponential()*14

```

`def ServiceTime():`

```

    #return -8*math.log(random.random())
    return np.random.exponential()*7

```

In [50]: `def EventRoutineArrival(n,b):`

```

    global EN, Q, S, blockedCustomers, enteredCustomers#buffersize
    EN=EN+1
    enteredCustomers+=1
    intarrivaltime = InterArrivalTime()
    avgintarrtimeList.append(intarrivaltime)
    eventTime = now+intarrivaltime
    #notice=[EN+1, now+InterArrivalTime(), 0]
    notice=[EN, eventTime, 0]

```

```

ScheduleEventNotice(notice)
Q = Q + 1

if S<n and Q<=b: # if idle

    if(b > 0):
        Q += 1
        EN += 1
        notice=[EN, now, 1]
        ScheduleEventNotice(notice)
    elif Q<b:
        Q += 1
    else:
        blockedCustomers += 1

if fileOutA: writeToFile()

```

```

In [51]: def EventRoutineBeginService():
    global EN, Q, S
    Q = Q - 1
    S += 1
    EN+=1
    sT = ServiceTime()
    servTimeList.append(sT)
    eventTime = now+sT
    notice=[EN, eventTime, 2]
    ScheduleEventNotice(notice)
    if fileOutA: writeToFile()

```

```

In [52]: def EventRoutineEndService():
    global EN, Q, S, endedCustomers
    endedCustomers += 1
    S-=1
    if Q>0:
        EN=EN+1
        eventTime = now
        notice = [EN, eventTime, 1]
    if fileOutA: writeToFile()

```

```

In [53]: # Schedule arrival event at time NOW;
    buffersize = 8

```

```

# def WithBufferSize(b):
#     global stateList, servTimeList, eventList, notice, now, Q, S, EN, avgintarrtime
#     fileOutA=True # Flag, if true output to file
#     stateList=[] # List containing [q(t), s(t), time]

```



```

#     servTimeList=[]
#     eventList=[]      # Event calendar list
#     notice=[]         # notice = [ notice number, time, type ]
#                       # type = 0, arrival
#                       # type = 1, begin service
#                       # type = 2, end service
#     now = 0.0         # Simulation time
#     Q = 0             # Queue Length, Q=0 when empty
#     S=0               # Server State 0=idle, 1=busy
#     EN=0              # Event Notice Number
#     avgintarrtimeList = [] #list for accumulating interarrival time
#     buffersize = 0
#     n=0               #no of servers
#     enteredCustomers = 0
#     n = 8

```

```

notice=[EN, now, 0]
ScheduleEventNotice(notice)

```

```

print ('Start of Simulation run')

```

```

while (eventList!=[] and now<2000): #200000
    now = eventList[0][1]
    #print now, ' ', eventList[0]
    #print 'Q = ',Q, ' S = ',S
    #DisplayEventList('Hello')
    if eventList[0][2]==0:
        EventRoutineArrival(n,buffersize)
    elif eventList[0][2]==1:
        EventRoutineBeginService()
    elif eventList[0][2]==2:
        EventRoutineEndService()
    else:
        print ('Unknown event type')
    #Remove first notice
    eventList.pop(0)

```

```

print ('End of Simulation run')

```

```

Start of Simulation run
End of Simulation run

```

```

In [54]: # WithBufferSize(buffersize)

```

```

In [55]: c = np.asarray(stateList)
         x1=c[:,2]
         y1=c[:,0]

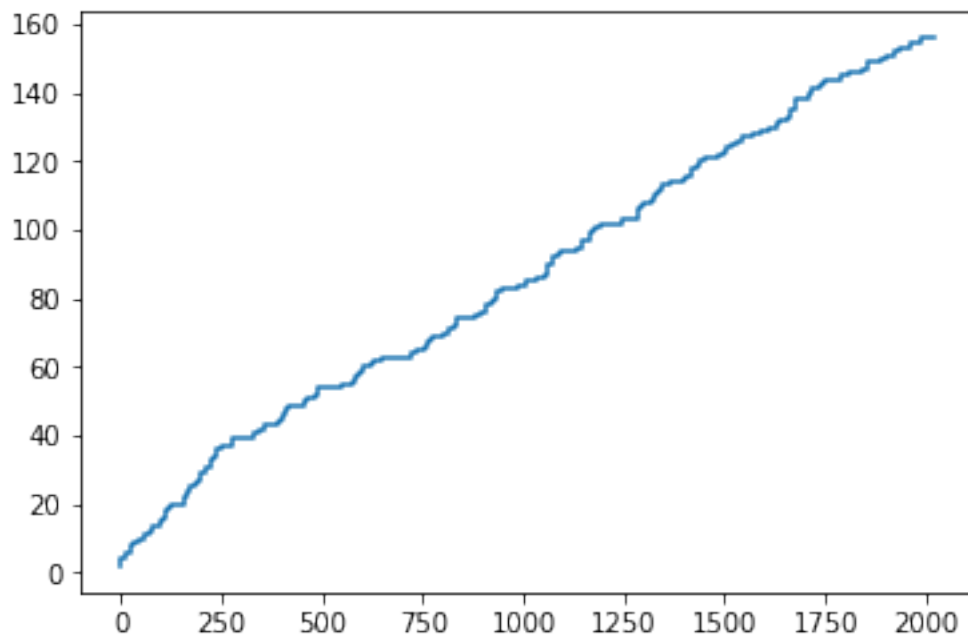
```

```

y12=c[:,1]
#n=8
print(np.shape(x1),np.shape(y1),np.shape(y12))
#PlotQueueLengthVSTime(x1,y1)
AvgQueueLen(x1,y1)
AvgIntArrivalTime(x1,y1)
ServerUtil(x1,y12,n)
#print ((blockedCustomers,(enteredCustomers)))

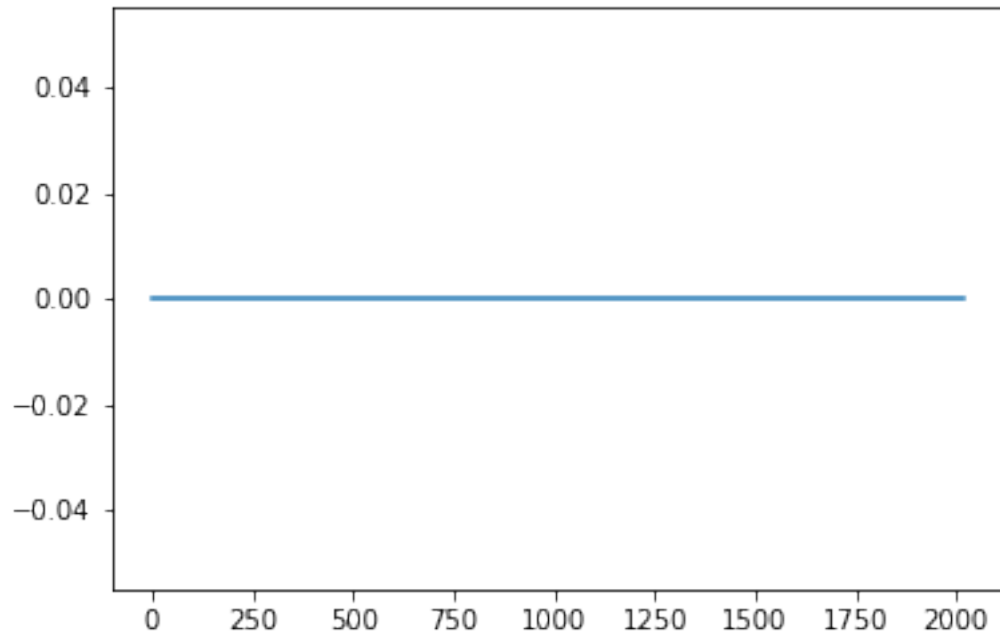
```

(152,) (152,) (152,)



Average queue length is equal to : 86.62430650303934

Average interarrival time: 13.300819866764009



0.0
 2020.1782176634374
 Server util is equal to : nan

C:\Users\matth\Anaconda3\lib\site-packages\ipykernel_launcher.py:40: RuntimeWarning: invalid v

2.7 8. Plot the blocking rate versus buffer size. [5 marks]

```
In [41]: i = 1
         n=8
         buffersize = 0
         #blockedCustomers = 0

         blockingRate = []
         buffersizePerRun = []

         while (i<50):
             fileOutA=True # Flag, if true output to file
             stateList=[] # List containing [q(t), s(t), time]
             servTimeList=[]
             eventList=[] # Event calendar list
             notice=[] # notice = [ notice number, time, type ]
                        # type = 0, arrival
                        # type = 1, begin service
```

```

                                # type = 2, end service
now = 0.0                      # Simulation time
Q = 0                          # Queue Length, Q=0 when empty
S=0                            # No of Servers occupied
EN=0                           # Event Notice Number
avgintarrtimeList = [] #list for accumulating interarrival time
blockedCustomers = 0
enteredCustomers = 0
notice=[EN, now, 0]
ScheduleEventNotice(notice)
#print ('Start of Simulation run')

while (eventList!=[] and now<200): #200000
    now = eventList[0][1]
    #print now, ' ', eventList[0]
    #print 'Q = ', Q, ' S = ', S
    #DisplayEventList('Hello')
    if eventList[0][2]==0:
        EventRoutineArrival(n,bufferSize)
    elif eventList[0][2]==1:
        EventRoutineBeginService()
    elif eventList[0][2]==2:
        EventRoutineEndService()
    else:
        print ('Unknown event type')
    #Remove first notice
    eventList.pop(0)

    #print ('End of Simulation run')
    print ('Blocking rate is {b}/{e}'.format(b=blockedCustomers,e = enteredCustomers))
    blockingRate.append((blockedCustomers/(enteredCustomers)))
    bufferSizePerRun.append(i+1)
    i+=1
    bufferSize+=1
d = np.asarray(stateList)
x1=d[:,2]
y1=d[:,0]
y12=d[:,1]
AvgIntArrivalTime(x1,y1)
AvgQueueLen(x1,y1)
#ServerUtil(x1,y12,50)
# print (blockingRate)
# print (servers)
plt.plot(bufferSizePerRun,blockingRate)

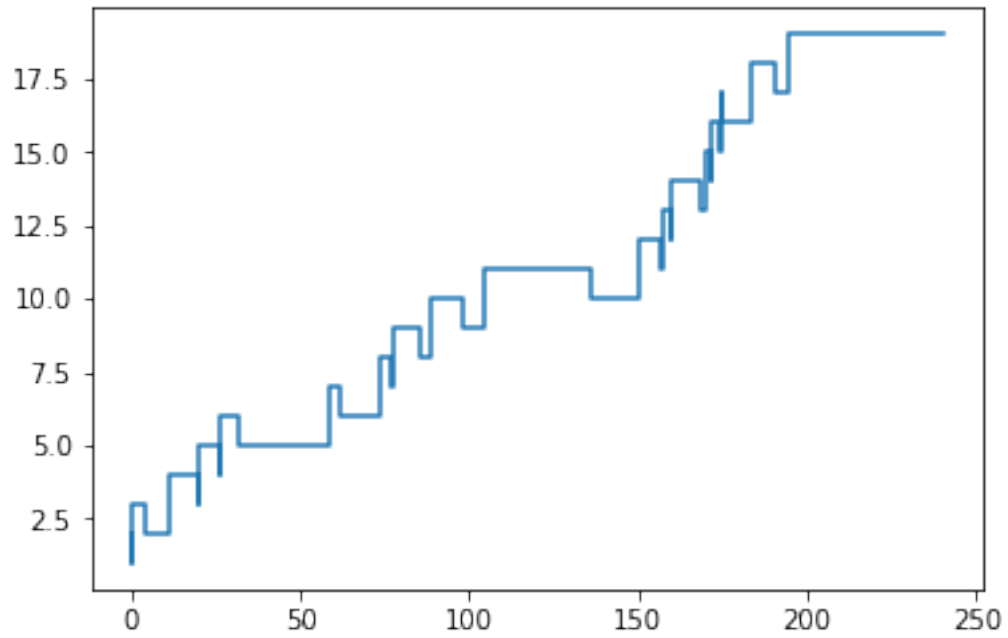
```

Blocking rate is 20/20

Blocking rate is 15/16

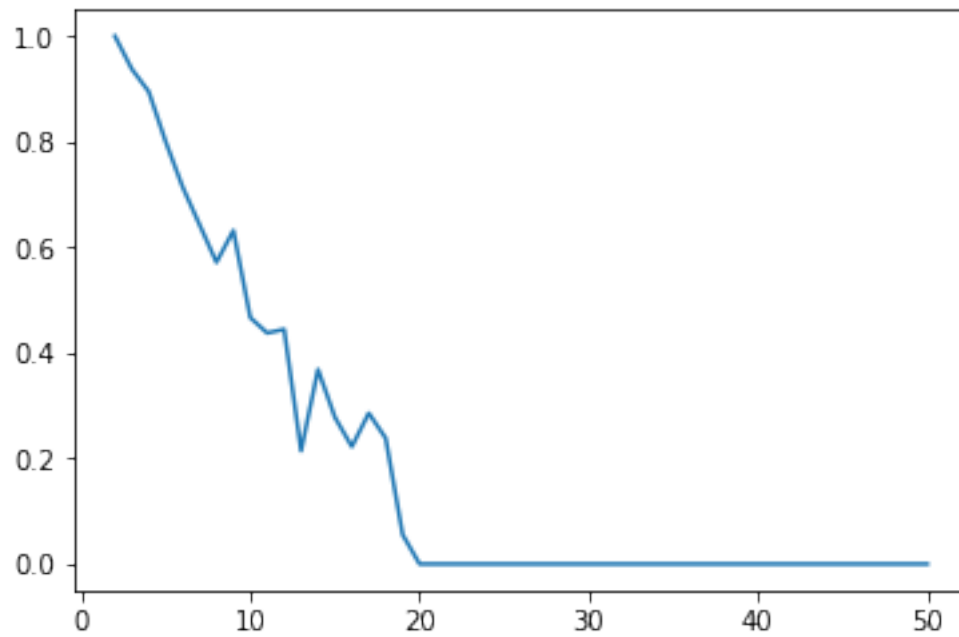
Blocking rate is 17/19

Blocking rate is 12/15
Blocking rate is 10/14
Blocking rate is 9/14
Blocking rate is 8/14
Blocking rate is 12/19
Blocking rate is 7/15
Blocking rate is 7/16
Blocking rate is 8/18
Blocking rate is 3/14
Blocking rate is 7/19
Blocking rate is 5/18
Blocking rate is 4/18
Blocking rate is 6/21
Blocking rate is 5/21
Blocking rate is 1/18
Blocking rate is 0/18
Blocking rate is 0/15
Blocking rate is 0/14
Blocking rate is 0/16
Blocking rate is 0/15
Blocking rate is 0/11
Blocking rate is 0/15
Blocking rate is 0/11
Blocking rate is 0/14
Blocking rate is 0/14
Blocking rate is 0/17
Blocking rate is 0/21
Blocking rate is 0/14
Blocking rate is 0/26
Blocking rate is 0/8
Blocking rate is 0/13
Blocking rate is 0/15
Blocking rate is 0/18
Blocking rate is 0/18
Blocking rate is 0/15
Blocking rate is 0/22
Blocking rate is 0/16
Blocking rate is 0/11
Blocking rate is 0/14
Blocking rate is 0/19
Blocking rate is 0/15
Blocking rate is 0/10
Blocking rate is 0/15
Blocking rate is 0/14
Blocking rate is 0/10
Blocking rate is 0/18
Average interarrival time: 15.565527510891176



Average queue length is equal to : 11.06174509413068

Out[41]: [



```
In [ ]:
```

```
In [ ]:
```

```
In [ ]:
```