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Polychronization: Computation with Spikes

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We present a minimal spiking network that can polychronize, that is, exhibit reproducible time-locked but not synchronous firing patterns with millisecond precision, as in synfire braids. The network consists of cortical spiking neurons with axonal conduction delays and spike-timing-dependent plasticity (STDP); a ready-to-use MATLAB code is included. It exhibits sleeplike oscillations, gamma (40 Hz) rhythms, conversion of firing rates to spike timings, and other interesting regimes. Due to the interplay between the delays and STDP, the spiking neurons spontaneously self-organize into groups and generate patterns of stereotypical polychronous activity. To our surprise, the number of coexisting polychronous groups far exceeds the number of neurons in the network, resulting in an unprecedented memory capacity of the system. We speculate on the significance of polychrony to the theory of neuronal group selection (TNGS, neural Darwinism), cognitive neural computations, binding and gamma rhythm, mechanisms of attention, and consciousness as "attention to memories."

1 Introduction

The classical point of view that neurons transmit information exclusively via modulations of their mean firing rates (Shadlen & Newsome, 1998; Mazurek & Shadlen, 2002; Litvak, Sompolinsky, Segev, & Abeles, 2003) seems to be at odds with the growing empirical evidence that neurons can generate spike-timing patterns with millisecond temporal precision in vivo (Lindsey, Morris, Shannon, & Gerstein, 1997; Prut et al., 1998; Villa, Tetko, Hyland, & Najem, 1999; Chang, Morris, Shannon, & Lindsey, 2000; Tetko & Villa, 2001) and in vitro (Mao, Hamzei-Sichani, Aronov, Froemke, & Yuste, 2001; Ikegaya et al., 2004). The patterns can be found in the firing sequences of single neurons (Strehler & Lestienne, 1986; de Ruyter van Steveninck, Lewen, Strong, Koberle, & Bialek, 1997; Reinagel & Reid, 2002; Bryant & Segundo, 1976; Mainen & Sejnowski, 1995) or in the relative timing of spikes of multiple neurons (Prut et al., 1998; Chang et al., 2000) forming a functional neuronal group (Edelman 1987, 1993). Activation of such a neuronal group can be triggered by stimuli or behavioral events (Villa et al., 1999; Riehle,

Grün, Diesmann, & Aertsen, 1997). These findings have been widely used to support the hypothesis of temporal coding in the brain (Buzsaki, Llinas, Singer, Berthoz, & Christen, 1994; Abeles, 1991, 2002; Diesmann, Gewaltig, & Aertsen, 1999; Bienenstock, 1995; Miller, 1996a, 1996b; see also special issue of *NEURON* (September 1999) on the binding problem).

In addition to the growing empirical evidence of precise spike-timing dynamics in the brain, there is growing theoretical interest in the artificial neural networks community to spike timing as an additional variable in the information processing by the brain. See special issue of *IEEE TNN* on pulse-coupled neural networks (May 1999); special issue of *Neural Networks* on spiking neurons (July 2001); and special issue of *IEEE TNN* on temporal coding (July 2004).

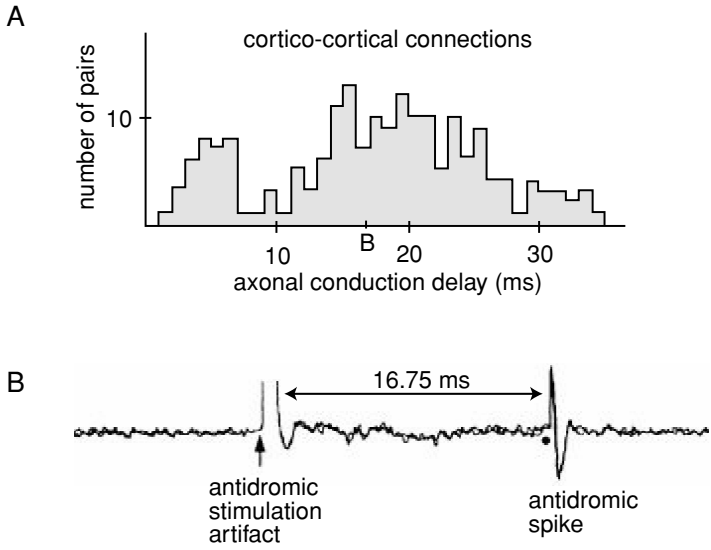
1.1 Spikes. When considering spiking neurons, most researchers emphasize synchrony of firing. Indeed, it is widely believed that if two or more neurons have a common postsynaptic target and fire synchronously, then their spikes arrive at the target at the same time, thereby evoking potent postsynaptic responses. If the neurons fire asynchronously (i.e., randomly), their spikes arrive at the postsynaptic target at different times, evoking only weak or no response. An implicit assumption here is that the axonal conduction delays are negligible or equal.

1.2 Delays. A careful measurement of axonal conduction delays in the mammalian neocortex (Swadlow 1985, 1988, 1992) showed that they could be as small as 0.1 ms and as large as 44 ms, depending on the type and location of the neurons. A typical distribution of axonal propagation delays between different pairs of cortical neurons (depicted in Figure 1A) is broad, spanning two orders of magnitude. Nevertheless, the propagation delay between any individual pair of neurons is precise and reproducible with a submillisecond precision (see Figure 1B; Swadlow 1985, 1994). Why would the brain maintain different delays with such precision if spike timing were not important?

The majority of computational neuroscientists discard delays as a nuisance that only complicates modeling. From a mathematical point of view, a system with delays is not finite- but infinite-dimensional,[1] which poses some mathematical and simulation difficulties.

In this letter, we argue that an infinite dimensionality of spiking networks with axonal delays is not a nuisance but an immense advantage that results

[1]For example, the simplest delay equation $x' = -x(t-1),\ x \in \mathbb{R}$ has infinite dimension because to solve it, we need to specify the initial condition on the entire interval $[-1, 0]$, and not just in the point $t = 0$. Delayed dynamical systems can exhibit astonishingly rich and complex dynamics (e.g., see Foss & Milton, 2000); however, the mathematical theory of such equations is still in its infancy (Wiener & Hale, 1992; Bellen & Zennaro, 2003).



C

connection	delays (ms)	animal	reference
layer 6 → LGN	1 - 44	cat	Ferster & Lindstrom (1983)
	1.7 - 32	rabbit	Swadlow (1994)
cortico-cortical	1 - 35	rabbit	Swadlow (1985)
	1.2 - 19	rabbit	Swadlow (1994)
cortico-(ipsi)cortical	2.2 - 32.5	rabbit	Swadlow (1994)
layer 5 → LGN	0.6 - 2.3	rabbit	Swadlow (1994)
cortico-collicular	0 - 3	cat	Ferster & Lindstrom (1983)
VB → layer 4	2	mice	Salami, Itami, Tsumoto, & Kimura (2003)

Figure 1: (A) Distribution of experimentally measured conduction delays of cortical axons running through the corpus collosum (antidromic stimulation, modified from Figure 3A of Swadlow, 1985). (B) Superposition of two voltage traces recorded in vivo shows the submillisecond precision of axonal conduction delays in the same pair of neurons (modified from Figure 4 of Swadlow, 1994). (C) Summary of experimental evidence of axonal conduction delays in different neurons and species.

in an unprecedented information capacity. In particular, there are stable firing patterns that are not possible without the delays.

1.3 Polychronization. To illustrate the main idea, consider neuron *a* in Figure 2A receiving inputs from neurons b, c, and d with different

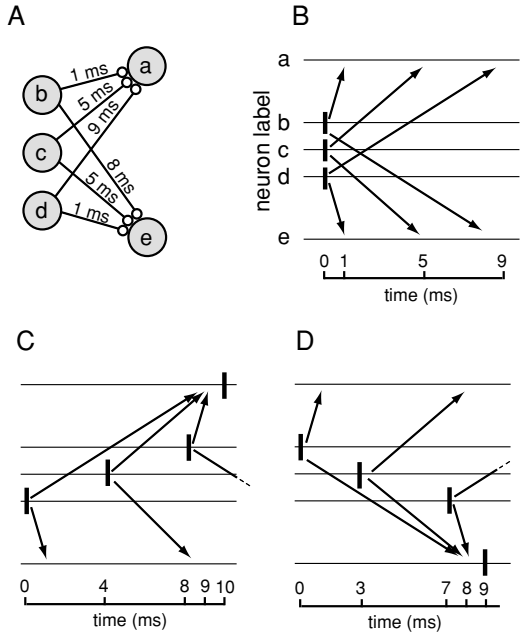


Figure 2: (A) Synaptic connections from neurons b, c, and d to neurons a and e have different axonal conduction delays. (B–D) Firings of neurons are denoted by the vertical bars. Each arrow points to the spike arrival time to the postsynaptic neuron. (B) Synchronous firing is not effective in eliciting a potent postsynaptic response since the spikes arrive at the postsynaptic neurons at different times. (C) The spiking pattern with neuron d firing at 0 ms, neuron c firing at 4 ms, and neuron b firing at 8 ms is optimal to excite neuron a because the spikes arrive at a simultaneously. (D) The reverse order of firing is optimal to excite neuron e.

conduction delays. Synchronous firing, as in Figure 2B, is not effective to excite neuron a, because the spikes arrive at this neuron at different times. To maximize the postsynaptic response in neuron a, the presynaptic neurons should fire with the temporal pattern determined by the delays and depicted in Figure 2C so that the spikes arrive at neuron a simultaneously. A different spike-timing pattern, as in Figure 2D, excites neuron e.

We see that depending on the order and the precise timing of firing, the same three neurons can evoke a spike in either neuron a or neuron e, or possibly in some other neuron not shown in the figure. Notice how the conduction delays make this possible.

If b, c, and d are sensory neurons driven by an external input, then the simple circuit in Figure 2 can recognize and classify simple spatiotemporal patterns (Hopfield, 1995; Seth, Mckinstry, Edelman, & Krichmar, 2004a).

Indeed, sensory input as in Figure 2C results in (d,c,b,a) firing as a group with spike-timing pattern $(0, 4, 8, 10)$ ms. Sensory input as in Figure 2D results in a different set of neurons: (b,c,d,e), firing as a group with a different spike-timing pattern, namely, $(0, 3, 7, 9)$ ms.

Since the firings of these neurons are not synchronous but time-locked to each other, we refer to such groups as polychronous, where *poly* means *many* and *chronous* means *time* or *clock* in Greek. Polychrony should be distinguished from asynchrony, since the latter does not imply a reproducible time-locking pattern, but usually describes noisy, random, nonsynchronous events. It is also different from the notion of clustering, partial synchrony (Hoppensteadt & Izhikevich, 1997), or polysynchrony (Stewart, Golubitsky, & Pivato, 2003), in which some neurons oscillate synchronously while others do not.

1.4 Networks. To explore the issue of spike timing in networks with conduction delays, we simulated an anatomically realistic model consisting of 100,000 cortical spiking neurons having receptors with AMPA, NMDA, $GABA_A$, and $GABA_B$ kinetics and long-term and short-term synaptic plasticity (Izhikevich, Gally, & Edelman, 2004). We found that the network contained large polychronous groups, illustrated in Figure 12, capable of recognizing and classifying quite complicated spatiotemporal patterns.

The existence of such groups, requiring finely tuned synaptic weights and matching (or converging) conduction delays, might seem unlikely in randomly connected networks with distributions of conduction delays. However, spike-timing-dependent plasticity (STDP) can select matching conduction delays and spontaneously organize neurons into such groups, a phenomenon anticipated by M. Abeles (personal communication with his students), Bienenstock (1995) and Gerstner, Kempter, van Hemmen, & Wagner (1996) (see also Changeux & Danchin, 1976, and Edelman, 1987). An unexpected result is that the number of coexisting polychronous groups could be far greater than the number of neurons in the network, sometimes even greater than the number of synapses. That is, each neuron was part of many groups, firing with one group at one time and with another group at another time. This is the main result of this letter. In retrospect, it is not surprising, since the networks we consider have delays, and hence are infinite-dimensional from a purely mathematical point of view.

In this letter we present a minimal model that captures the essence of this phenomenon. It consists of a sparse network of 1000 randomly connected spiking neurons with STDP and conduction delays, thereby representing a cortical column or hypercolumn. The MATLAB code of the model, `spnet`, and its technical description is given in the appendix. In section 2, we demonstrate that despite its simplicity, the model exhibits cortical-like dynamics, including oscillations in the delta (4 Hz) frequency range, 40 Hz gamma oscillations, and a balance of excitation and inhibition. In section 3, we describe polychronous groups in detail. Our definition differs from the

one used by Izhikevich et al. (2004), who relied on the existence of so-called anchor neurons and hence could not have more groups than neurons (that is, most of the groups in that study went undetected). We also illustrate how polychronous groups contribute to cognitive information processing going beyond the Hopfield-Grossberg or liquid state machine paradigms. In sections 4 and 5, we discuss some open problems and present some speculations on the significance of this finding in modeling binding, attention, and primary (perceptual) consciousness.

2 Dynamics

The model neural network, described in the appendix, preserves important ratios found in the mammalian cortex (Braitenberg & Schuz, 1991). It consists of 1000 randomly connected excitatory (80%) and inhibitory (20%) neurons. The network is sparse with 0.1 probability of connection between any two neurons. Behavior of each neuron is described by the simple spiking model (Izhikevich, 2003), which can reproduce 20 of the most fundamental neurocomputational features of biological neurons (summarized in Figure 3). Despite its versatility, the model can be implemented efficiently (Izhikevich, 2004). Since we cannot simulate an infinite-dimensional system on a finite-dimensional digital computer, we substitute the network by its finite-dimensional approximation having time resolution 1 ms.

2.1 Plasticity. Synaptic connections among neurons have fixed conduction delays, which are random integers between 1 ms and 20 ms. Thus, the delays in the model are not as dramatic as those observed experimentally in Figure 1. Excitatory synaptic weights evolve according to the STDP rule illustrated in Figure 4 (Song, Miller, & Abbott, 2000). The magnitude of change of synaptic weight between a pre- and a postsynaptic neuron depends on the timing of spikes: if the presynaptic spike arrives at the postsynaptic neuron before the postsynaptic neuron fires—for example, it causes the firing—the synapse is potentiated. Its weight is increased according to the positive part of the STDP curve in Figure 4 but does not allow growth beyond a cut-off value, which is a parameter in the model. In this simulation, we use the value 10 mV, which means that two presynaptic spikes are enough to fire a given postsynaptic cell. If the presynaptic spike arrives at the postsynaptic neuron after it fired, that is, it brings the news late, the synapse is depressed. Its weight is decreased according to the negative part of the STDP curve. Thus, what matters is not the timing of spiking per se but the exact timing of the arrival of presynaptic spikes to postsynaptic targets.

2.2 Rhythms. Initially, all synaptic connections have equal weights, and the network is allowed to settle down for 24 hours of model time (which takes 6 hours on a 1 GHz PC) so that some synapses are potentiated and

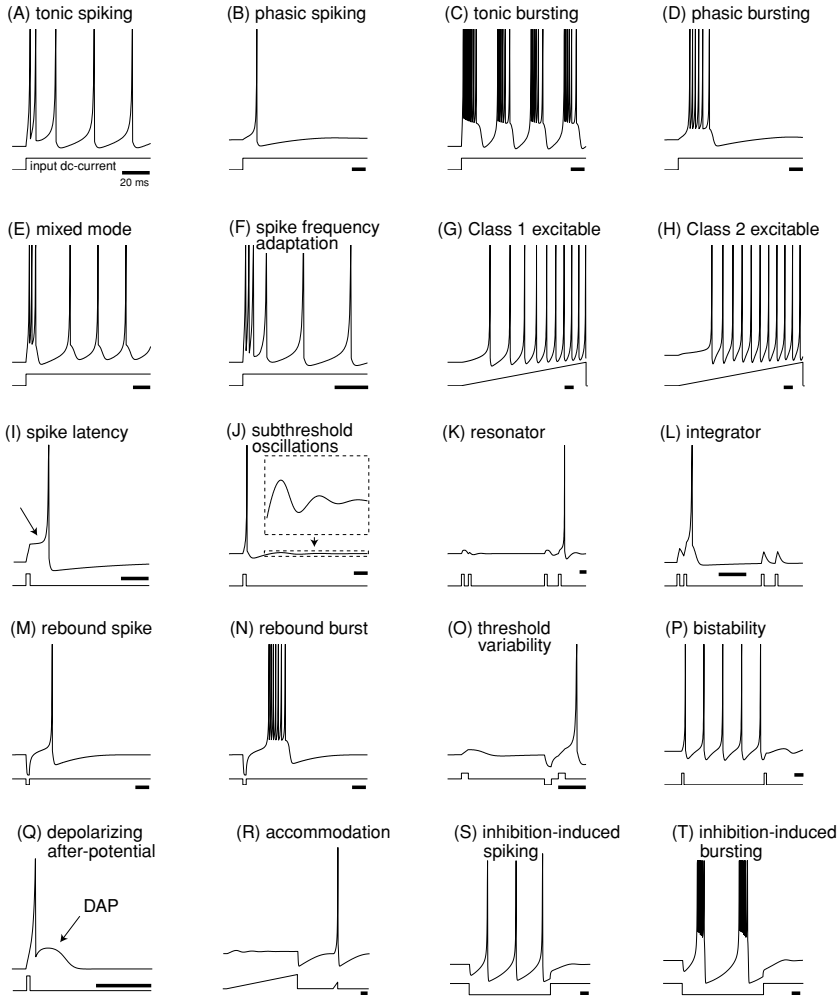


Figure 3: Summary of the neurocomputational properties of biological spiking neurons (Izhikevich 2004). Shown are simulations of the same model (see equations A.1 and A.2) with different choices of parameters. This model is used in this study. Each horizontal bar denotes 20 ms time interval. The MATLAB file generating the figure and containing all the parameters can be downloaded from the author's Web site. This figure is reproduced with permission from www.izhikevich.com. (An electronic version of the figure and reproduction permission are freely available online at www.izhikevich.com.)

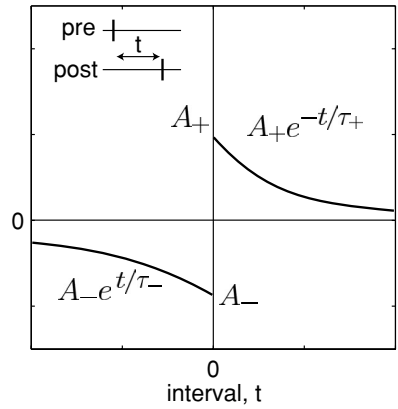


Figure 4: STDP rule (spike-timing-dependent plasticity, or Hebbian temporally asymmetric synaptic plasticity): The weight of synaptic connection from pre- to postsynaptic neuron is increased if the postneuron fired after the presynaptic spike, that is, the interspike interval $t > 0$. The magnitude of change decreases as A_+e^{-t/τ_+}. Reverse order results in a decrease of the synaptic weight with magnitude A_-e^{t/τ_-}. Parameters used: $\tau_+ = \tau_- = 20$ ms, $A_+ = 0.1$, and $A_- = 0.12$.

others are depressed. At the beginning of this settling period, the network exhibits high-amplitude rhythmic activity in the delta frequency range around 4 Hz (see Figure 5, top). This rhythm resembles one of the four fundamental types of brain waves, sometimes called deep sleep waves, because it occurs during dreamless states of sleep, during infancy, and in some brain disorders. Of course, the mechanism of generation of this rhythm in the model is probably different from the one in mammals, since the model does not have thalamus.

As the synaptic connections evolve according to STDP, the delta oscillations disappear, and spiking activity of the neurons becomes more Poissonian and uncorrelated. After a while, gamma frequency rhythms in the range 30 to 70 Hz appear, as one can see in Figure 5 (bottom). The mechanism generating this rhythms is often called PING (pyramidal-interneuron network gamma; Whittington, Traub, Kopell, Ermentrout, & Buhl, 2000): strong firings of pyramidal neurons excite enough inhibitory interneurons, leading to transient reciprocal inhibition that temporarily shuts down the activity. These kinds of oscillations, implicated in cognitive tasks in humans and other animals, play an important role in the activation of polychronous groups, as we describe in the next section.

2.3 Balance of Excitation and Inhibition. There are fewer inhibitory neurons in the network, but their firing rate is proportionally higher, as one can see in Figure 5. As a result, the network converges to a state with

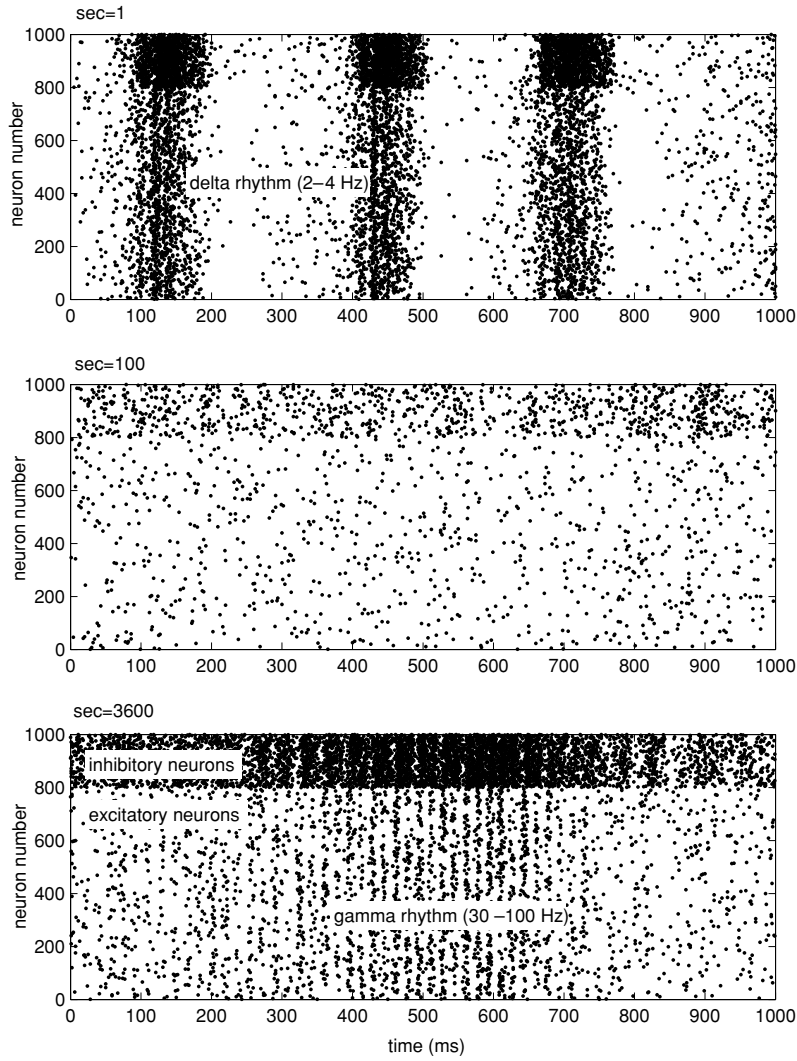


Figure 5: Rhythmic activity of the spiking model is evident from the spike raster. As synaptic weights are evolved according to STDP, initial delta frequency oscillations (top, sec = 1) disappear, relatively uncorrelated Poissonian activity (middle, sec = 100), and then gamma frequency oscillations (bottom, sec = 3600) appear.

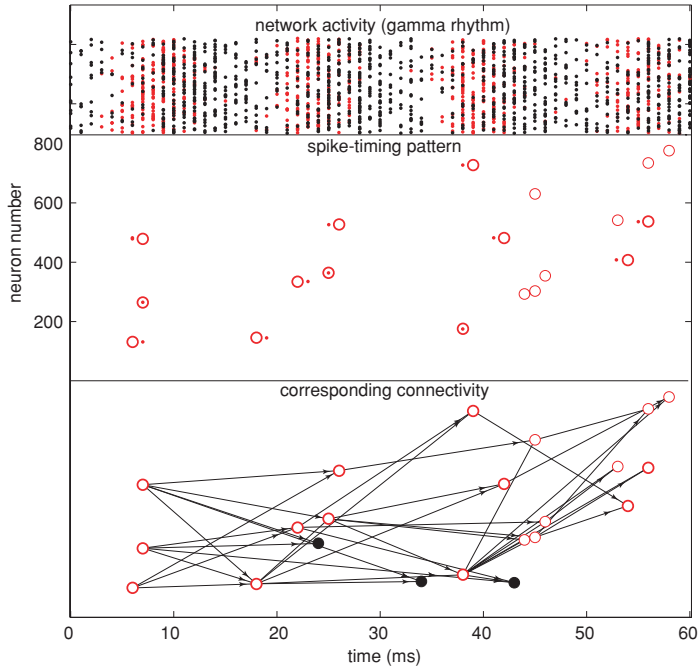


Figure 6: Activation of a polychronous group: Spiking activity of the entire network (top) contains a pattern (middle) that is generated more often than predicted by chance. The pattern occurs because the connectivity between the neurons has matching axonal conduction delays (bottom). See Figures 2 and 7 for more detail. Dots denote actual spikes generated by the model; circles denote the predicted timing of spikes based on the anatomical connectivity and the delays among the neurons. The simulation time step is 1 ms. Red (black) dots denote firings of excitatory (inhibitory) neurons.

an approximate balance of excitation and inhibition (Shadlen & Newsome, 1994; Shadlen & Movshon, 1999; van Vreeswijk & Sompolinsky, 1996; Amit & Brunel, 1997), so that each excitatory neuron fires in the Poissonian manner with the rate fluctuating between 2 and 7 Hz. Even during the episodes of gamma oscillation, such as the one in Figure 5, the spiking activity of excitatory neurons is not synchronized; the neurons skip most of the gamma cycles and fire just a few spikes per second (see the open circles in Figure 6, middle).

Twofold changes of some of the parameters, such as the maximal synaptic weight, the amount of depression in STDP, or the thalamic input, produce only transient changes in network dynamics. Neurons adjust their synaptic weights, balance the excitation and inhibition, and return to the mean firing

rate between 2 and 7 Hz. Thus, the network maintains a certain homeostatic state despite intrinsic or extrinsic perturbations.

3 Polychronous Spiking

Although spiking of excitatory neurons looks random and uncorrelated, there are certain persistent spike-timing patterns that emerge and reoccur with millisecond precision. An example of such a spike-timing pattern is presented in Figure 6. Although no apparent order can be seen in the network activity at the top of the figure, except for a pronounced gamma oscillation, the pattern denoted by circles in the middle of the figure repeats itself a few times per hour with ± 1 ms spike jitter. Statistical considerations standard in the synfire chain literature (not presented here; see, e.g., Prut et al., 1998) suggest that such repetitions are highly unlikely to occur by chance. There must be some underlying connectivity that generates the pattern. Indeed, considering the connections between the neurons, depicted in Figure 6 (bottom), we can see that the neurons are organized into a group, referred here as being polychronous (i.e., multiple timing), which forces the neurons to fire with the pattern.

3.1 Definition. Our definition of polychronous groups is based on the anatomy of the network, that is, on the connectivity between neurons. Let us freeze the simulation and consider the strongest connections among neurons, paying special attention to the conduction delays. In Figure 7 the conduction delays from neurons (125, 275, 490) to neuron 1 are such that when the neurons fire with the timing pattern (0, 3, 7) ms, their spikes arrive at neuron 1 at the same time, thereby making neuron 1 fire at 13 ms. The

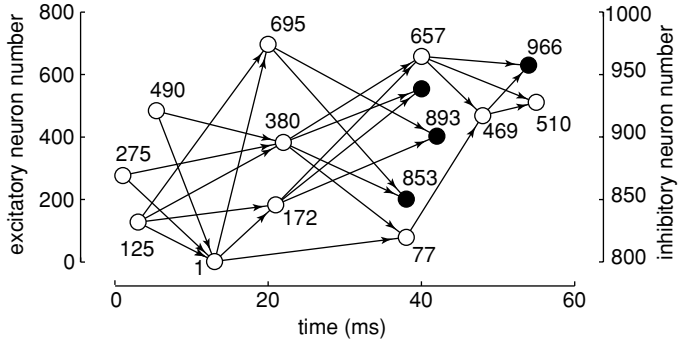


Figure 7: Example of a polychronous group: Firing of neurons (125, 275, 490) with the timing pattern (0, 3, 7) ms results in spikes arriving simultaneously at neuron 1, then at neurons 172, 695, and 380. This multitiming (polychronous) activity propagates farther along the network and terminates at neuron 510.

conduction delays from neurons 125 and 1 to neuron 172 are such that when the neurons fire with the pattern (3, 13) ms, their spikes arrive at neuron 172 simultaneously, thereby making it fire at 21 ms. Since we know all delays in the model, we can continue this procedure spike by spike and untangle the entire group. It consists of 15 neurons, some of them inhibitory; the group ends at neuron 510.

The group in Figure 7 is defined on a purely anatomical basis; there is no firing involved yet. The figure tells only that there is a subgraph in the connectivity matrix of the network so that if neurons (125, 275, 490) fire with the indicated pattern, the stereotypical activity propagates through the subgraph. Thus, the group in the figure is only an anatomical prediction of a possible stereotypical firing pattern. One may never see the pattern, for example, if the first three neurons never fire.

Whenever the neurons in the figure do fire with the spike-timing pattern determined by the connectivity and delays, we say that the group is activated and the corresponding neurons polychronize. Typically, firing of the first few neurons with the right timing is enough to activate most of the group, as it happens in Figure 6. Notice how activation of the group is locked to the gamma oscillation; that is, the first three neurons fire at the first gamma cycle, their spikes travel 10 to 20 ms and arrive at the next four neurons in the next gamma cycle, and so on, resulting in precise stereotypical activity.

We stress here that 1 ms spike-timing precision is the consequence of our definition of the group. Of course, neurons in Figure 6 fire many other spiking patterns with large jitter. We ignore those patterns by calling them noise.

3.2 Emergence of Groups. Considering various triplets, such as neurons (125, 275, 490) in Figure 7 (left), firing with various patterns, we can reveal all polychronous groups emanating from the triplets. In the network of 1000 neurons presented in the appendix, we find over 5000 such groups. The groups did not exist at the beginning of simulation but appear as a result of STDP acting on random spiking (Izhikevich et al., 2004). STDP potentiates some synapses corresponding to connections with converging (matching) delays and depresses (prunes) other synapses. The plasticity takes an initially unstructured network and selects firing patterns that are consistent with the underlying anatomy, thereby creating many strongly connected subgraphs corresponding to polychronous groups. Since STDP is always "ON" in the network, groups constantly appear and disappear; their total number fluctuates between 5000 and 6000. We found a core of 471 groups that appeared and survived the entire duration of 24 hour simulation.

The groups have different sizes, lengths, and time spans, as we summarize in Figure 8. (A few examples are depicted in Figure 12). Since an average group consists of 25 neurons, an average neuron is a member of

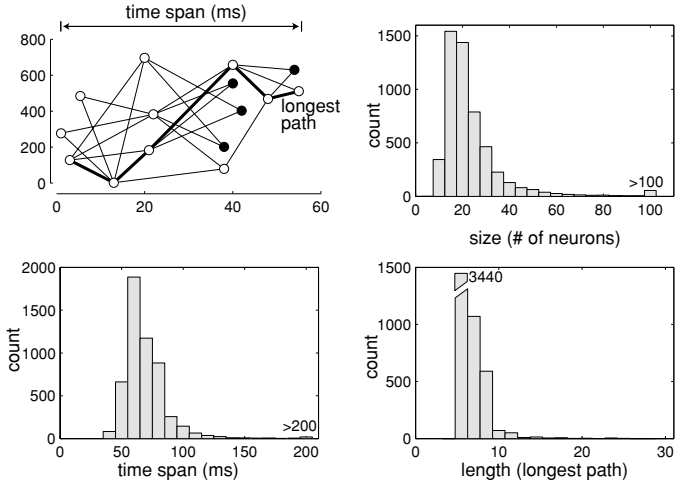


Figure 8: Characteristics of polychronous groups (total number is 5269 in a network of 1000 neurons). (Top left) An example of a polychronous group. (Top right) Distribution of group sizes, that is, the number of neurons that form each group. The example group has size 15. (Bottom left) Distribution of groups' time spans—the time difference between the firing of the first and the last neuron in the group. The example group has time span 56 ms. (Bottom right) Distribution of the longest paths in the groups. The example group has a path of length 5.

131 different groups. Because different groups activate at different times, the neuron can fire with one group at one time and with another group at another time.

3.3 Groups Share Neurons. Quite often, different polychronous groups can share more than one neuron. Two such cases are illustrated in Figure 9. Neurons (8, 103, 351, 609, 711, 883) belong to two different groups in the upper half of the figure. However, there is no ambiguity because their firing order is different; the neurons fire with one spike-timing pattern at one time (when the first group is activated), with the other pattern at some other time (when the second group is activated), and with no pattern most of the time. The lower half of the figure depicts two groups having eight neurons in common and firing with different spike-timing patterns. In addition, neurons 838 and 983 fire twice during activation of the second group. Again, there is no ambiguity here because each polychronous group is defined not only by its constituent neurons but also by their precise spiking time.

As an extreme illustration of this property, consider a toy fully connected network of 5 neurons in Figure 10A. In principle, such a network can exhibit $5! = 240$ different firing patterns if we require that each neuron fires only

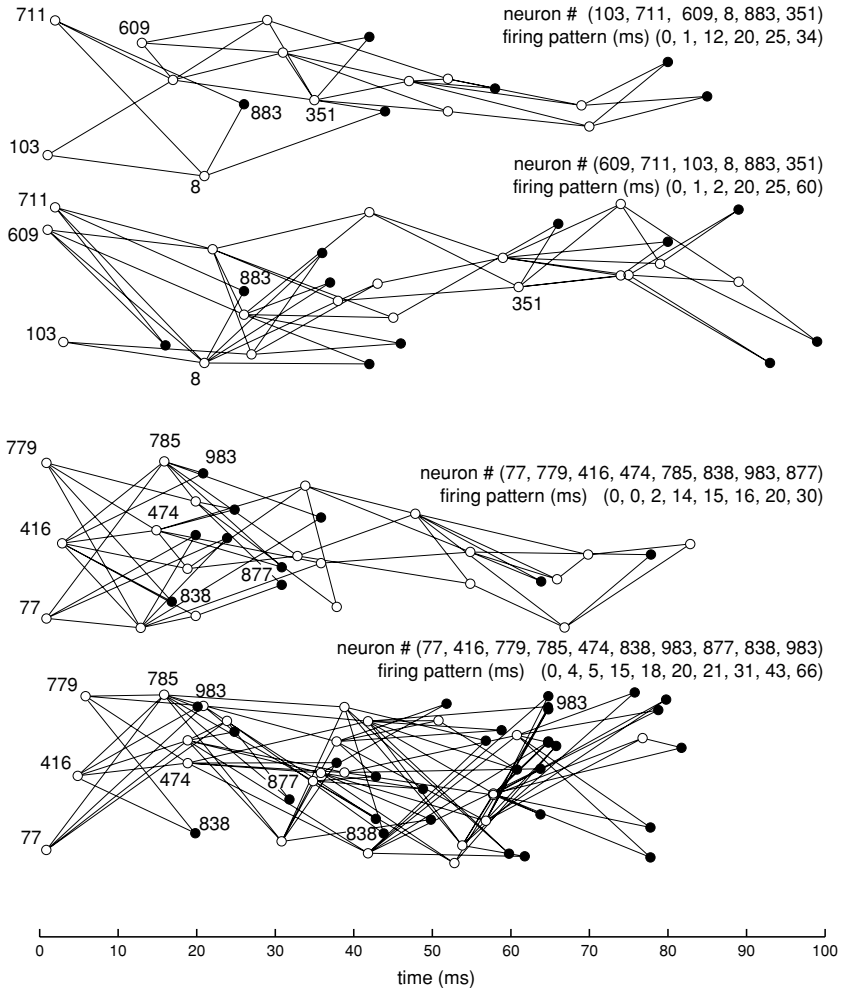


Figure 9: Two examples of pairs of groups consisting of essentially the same neurons but firing with different spike-timing patterns; see also Figure 10. Neurons of interest are marked with numbers. The list of neurons and their firing times are provided for each group in the upper-right corners.

once and we distinguish the patterns only on the basis of the order of firing of the neurons. If we allow for multiple firings, then the number of patterns explodes combinatorially. However, the connectivity among the neurons imposes a severe restriction on the possible sustained and reproducible firing patterns, essentially excluding most of them. Random delays in the network would result in one, and sometimes two, polychronous groups.

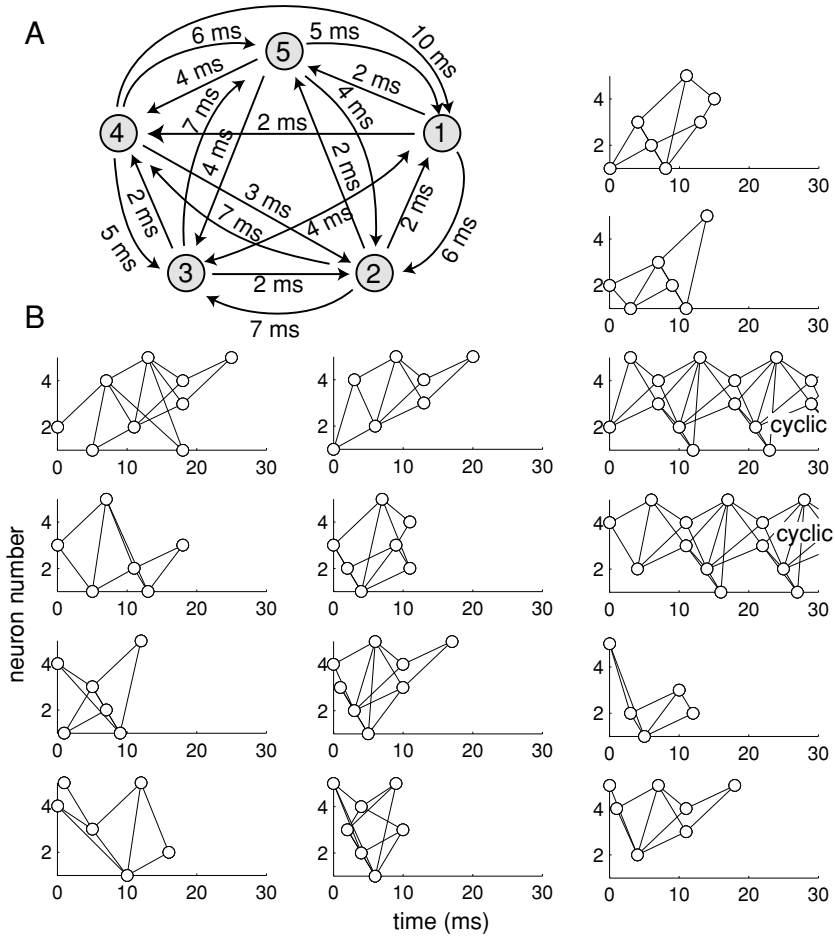


Figure 10: (A) A toy network of five neurons with axonal conduction delays and strong synapses (two simultaneous spikes is enough to fire any neuron). (B) The delays in the network are optimized so that it has 14 polychronous groups, including two cyclic groups that exhibit reverberating activity with a time (phase) shift.

The delays in Figure 10A are not random; they were constructed to maximize the number of polychronous groups. Although there are only five neurons, the network has 14 polychronous groups shown in Figure 10B. Adding a sixth neuron triples the number of groups so that there are more groups than synapses in the network. Considering toy examples like this one, it would not be surprising that a network of 10^{11} neurons (which is the size of the human brain) would have more groups than the number of particles in the universe.

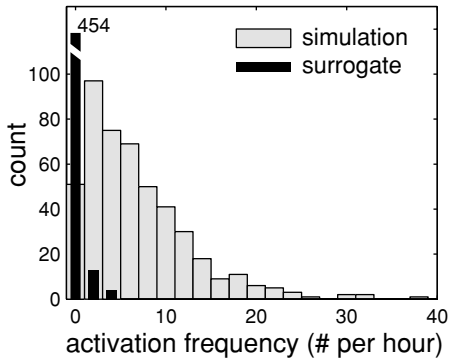


Figure 11: Distribution of frequencies of activation of groups in the simulated and surrogate (inverted time) spike trains. Each group is found using anatomical data (connectivity and delays) and then used as a template to scan through the spike train. The group is said to activate when more than 50% of its neurons polychronize, that is, fire with the prescribed spike-timing pattern with ± 1 ms jitter, as in Figure 6. Surrogate data emphasize the statistical significance of these events.

3.4 Activation of Groups. Our definition of a polychronous group relies on the anatomy of the network, not on its dynamics. (Of course, the former and the latter are dependent via STDP.) We say that a group is half-activated when at least 50% of its constituent excitatory neurons polychronize (i.e., fire according to the prescribed spike-timing pattern with ± 1 ms spike jitter). For example, the group in Figure 6 is 63% activated because 16 of 19 excitatory neurons polychronized. Once all the groups are found using the anatomical data (connectivity and delays), we use each group as a template, scan the spiking data recorded during a 24 hour period, and count how many times the group is half-activated. We apply this procedure only to those groups (total 471) that persist during the 24-hour period.

In Figure 11 we plot the distribution histogram of the averaged frequency of half-activation of polychronous groups. The mean activation frequency is 7 times per hour, that is, every 8 minutes, implying that there is a spontaneous activation of a group every 1 second ($8 \times 60/471 \approx 1$ sec). Since an averaged neuron is a member of 131 different groups, $131 \times 7 = 917$ of its spikes per hour are part of activation of a group, which is less than 4% of the total number of spikes (up to 25,000) fired during the hour. Thus, the majority of the spikes are noise, and only a tiny fraction is involved in polychrony. The only way to tell which is which is to consider these spikes with relation to the spikes of the other neurons.

To test the significance of our finding, we use surrogate data obtained from the spike data by inverting the time. Such a procedure does not change the mean firing rates, interspike histograms, magnitude of

cross-correlations, and other meaningful statistics of the spike trains. In particular, this approach is free from the criticism (Oram, Wiener, Lestienne, & Richmond, 1999; Baker & Lemon, 2000) that precise firing sequences appear exclusively by chance in spike rasters with covarying firing rates. Activation frequency of (noninverted) groups in the surrogate (inverted) spike raster, depicted as black bars in Figure 11, is much lower, indicating that group activations are statistically significant events.

We emphasize that our method of analysis of spike trains is drastically different from the one used to search for synfire chains in in vivo data. We do not search for patterns in spike data; we know what the patterns are (using the connectivity matrix and delays); we just scan the spikes and count the occurrences of each pattern. Apparently such an approach is feasible only in models.

3.5 Representations. What is the significance of polychronous groups? We hypothesize that polychronous groups could represent memories and experience. In the simulation above, no coherent external input to the system was present. As a result, random groups emerge; that is, the network generates random memories not related to any previous experience.

However, coherent external stimulation builds certain groups that represent this stimulation in the sense that the groups are activated when the stimulation is present. Different stimuli build different groups even when the same neurons are stimulated, as we illustrate in Figure 12. Every second during a 20-minute period, we stimulate 40 neurons, $1, 21, 41, 61, \ldots, 781$, either with the pattern $(1, 2, \ldots, 40)$ ms or with the inverse pattern $(40, \ldots, 2, 1)$ ms, as we show in the top of Figure 12. Initially, no groups starting with stimulated neurons existed (we did not explore whether the stimulation activated any of the existing groups consisting of other neurons). However, after 20 minutes of simulation 25 new groups emerged. Fifteen of them correspond to the first stimulus; they can be activated when the network is stimulated with the first pattern. The other 10 correspond to the second stimulus; that is, they can be activated when the network is stimulated with the second pattern. Thus, the groups represent the memory of the two input patterns, and their activation occurs when the network "sees" the corresponding patterns.

In Figure 13 we depict the time evolution of the largest group corresponding to the first pattern in Figure 12. Notice how the group annexes neurons, probably at the expense of the other groups in the network. Further simulation shows that the initial portion of the group is relatively stable, but its tail expands and shrinks in an unpredictable manner.

Finally, not all groups corresponding to a pattern activate when the network is stimulated. Because the groups share some neurons and have excitatory and inhibitory interconnections, they are in a constant state of competition and cooperation. As a result, each presentation of a stimulus activates only two to three groups (15%) in a random manner.

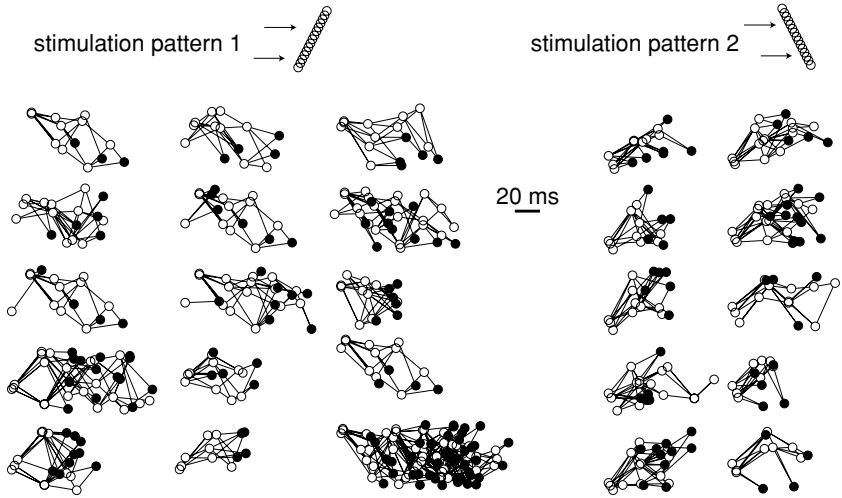


Figure 12: Persistent stimulation of the network with two spatiotemporal patterns (top) result in the emergence of polychronous groups that represent the patterns; the first few neurons in each group are the ones being stimulated, and the rest of the group activates (polychronizes) whenever the patterns are present.

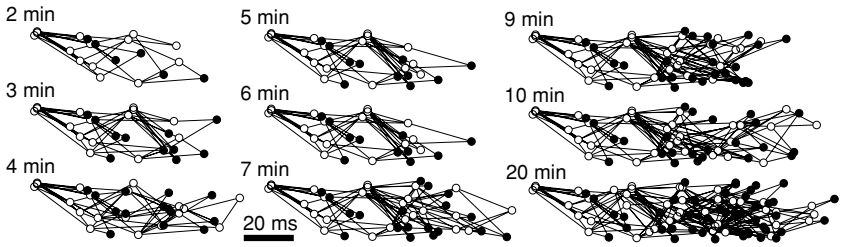


Figure 13: Time evolution (growth) of the last (largest) polychronous group in Figure 12 corresponding to stimulation pattern 1.

3.6 Rate to Spike-Timing Conversion. Neurons in the model use a spike-timing code to interact and form groups. However, the external input from sensory organs, such as retinal cells and hair cells in cochlear, arrives as a rate code, that is, encoded into the mean firing frequency of spiking. How can the network convert rates to precise spike timings?

It is easy to see how rate to spike-timing conversion could occur at the onset of stimulation. As the input volley arrives, the neurons getting stronger excitation fire first, and the neurons getting weaker excitation fire later or not at all. This mechanism relies on the fact that there is a clear onset

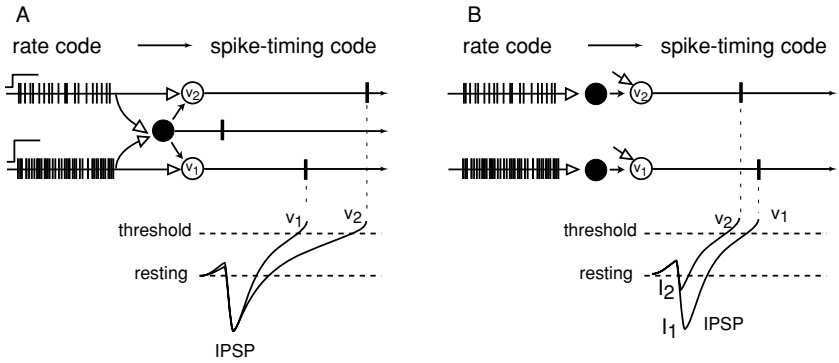


Figure 14: Rate code to spike-timing code conversion by spiking network with fast inhibition. (A) The firing rate input (rate code) induces a phasic inhibition in the network. While the excitatory neurons recover from the inhibition, the ones that get the strongest input fire first, and the ones getting the weakest input fire last (or not fire at all). (B) The strength of the rate code input determines the degree of hyperpolarization of excitatory neurons. Those inhibited less will fire sooner. This mechanism has the desired logarithmic scaling that makes the spike-timing code insensitive to the strength of the input (see the text for detail). Open circles—excitatory neurons; filled circles—populations of inhibitory neurons.

of stimulation, for example, after a visual saccade. What if stimulation is tonic without a clear beginning or end? We hypothesize that intrinsic rhythmicity generates internal "saccades" that can parse the rate input into spike timing, and we discuss three possible mechanisms how this could be accomplished.

First, the intrinsic rhythmic activity can chop tonic input into "mean-firing-rate" packets. Since each packet has a well-defined onset, it can be converted into spike timings according to the mechanism described above: the neuron receiving the strongest input fires first, and so forth. This mechanism is similar, but not equivalent, to the mechanism proposed by Hopfield (1995).

The other two mechanisms rely on the action of inhibitory interneurons. In one case, depicted in Figure 14A, inhibitory neurons, being faster, fire first and inhibit excitatory neurons, thereby resulting in a long inhibitory postsynaptic potential (IPSP). The rate with which excitatory neurons recover from the IPSP depends on their intrinsic properties and the strength of the overall external input. The stronger the input, the sooner the neuron fires after the IPSP. Again, the neuron receiving the strongest input fired first, and the neuron receiving the weakest input fired last.

In the third mechanism depicted in Figure 14B, inputs with different firing rates produce IPSPs of different amplitudes in the excitatory neurons downstream. As the excitatory neurons recover from the inhibition, the neurons are ready to fire spikes (due to some other tonic stimulation) at different moments: the neuron inhibited less would fire first, and the neuron inhibited more would fire last or not at all.

Since the recovery from inhibition is nearly exponential, the system is relatively insensitive to the input scaling. That is, a stronger input results in firing with the same spike-timing pattern but with an earlier onset. Similarly, a weaker input does not change the spike-timing pattern but only delays its onset. Let us illustrate this point using two linear dimensionless equations, $v_i' = -v_i$, $i = 1, 2$, that model the recovery of the membrane potential from the inhibition $v_i(0) = -I_i$, where each I_i denotes the amplitude (peak) of IPSP. The recovery is exponential, $x_i(t) = -I_i e^{-t}$, so the moment of time each membrane reaches a certain threshold value, say, $v = -1$, is $t_i = \log I_i$. If we scale the input by any factor (e.g., kI_i), we translate the threshold moment by a constant because $\log kI_i = \log k + \log I_i$, which is the same for both neurons. Thus, regardless of the scaling of the input, the time difference $\log kI_1 - \log kI_2 = \log I_1 - \log I_2$ is invariant. Thus, in contrast to Hopfield (1995), we do not need to postulate that the input is already somehow converted to the logarithmic scale. Synchronized inhibitory spiking implements the logarithmic conversion and makes spike-timing response relatively insensitive to the input scaling.

3.7 Stimulus-Triggered Averages. Notice that synchronized inhibitory activity occurs during gamma frequency oscillations (see Figure 5). Thus, the network constantly converts rate code to spike-timing code (and back) via gamma rhythm. Each presentation of a rate code stimulus activates an appropriate polychronous group or groups that represent the stimulus. This activation is locked to the phase of the gamma cycle but not to the onset of the stimulus. We explain this point in Figure 15, which illustrates results of a typical in vivo experiment in which a visual, auditory, or tactile stimulus is presented to an animal (we cannot simulate this with the present network because, among many other things, we do not model thalamus, the structure that gates inputs to the cortex). Suppose that we record from neuron A belonging to a polychronous group representing the stimulus. Since the input comes at a random phase of the internal gamma cycle, the activation of the group occurs at random moments, resulting in a typical smeared stimulus-triggered histogram. Such histograms were interpreted by many biologists as "the evidence" of absence of precise spike-timing patterns in the brain, since the only reliable effect that the stimulus evokes is the increased probability of firing of neuron A (i.e., increase in its mean-firing rate). Even recording from two or more neurons belonging to different groups would result in broad histograms and weak correlations among the neurons, because the groups rarely activate together, and when they do,

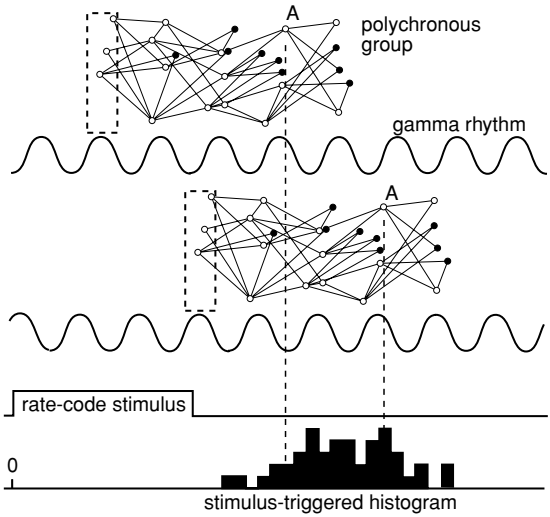


Figure 15: Noisy, unreliable response of neuron A is due to the unreliable activation of the group representing the stimulus. The activation is locked to the intrinsic gamma cycle, not to the onset of the stimulation, resulting in the smeared stimulus-triggered histogram. One needs to record from two or more neurons belonging to the same group to see the precise spike-timing patterns in the network (the group and the associated gamma rhythm are drawn by hand).

they may activate at different cycles of the gamma rhythm. We see that "noisy," unreliable responses of individual neurons to stimuli are the result of noisy and unreliable activations of polychronous groups. Recordings of two or more neurons belonging to the same group are needed to see the precise patterns (relative to the gamma rhythm).

4 Discussion

Simulating a simple spiking network (the MATLAB code is in the appendix), we discovered a number of interesting phenomena. The most striking one is the emergence of polychronous groups—strongly interconnected groups of neurons having matching conduction delays and capable of firing stereotypical time-locked spikes with millisecond precision. Thus, such groups can be seen not only in anatomically detailed cortical models (Izhikevich et al., 2004) but also in simple spiking networks. Changing some of the parameters of the model twofold changes the number of groups that can be supported by the network but does not eliminate them completely. The self-organization of neurons into polychronous groups is a robust phenomenon that occurs despite the experimentalist's efforts to prevent it.

Our model is minimal; it consists of spiking neurons, axonal conduction delays, and STDP. All are well-established properties of the real brain. We hypothesize that unless the brain has an unknown mechanism that specifically prevents polychronization, the real neurons in the mammalian cortex must also self-organize into such groups. In fact, all the evidence of reproducible spike-timing patterns (Abeles, 1991, 2002; Lindsey et al., 1997; Prut et al., 1998; Villa et al., 1999; Chang et al., 2000; Tetko & Villa, 2001; Mao et al., 2001; Ikegaya et al., 2004; Riehle et al., 1997; Beggs & Plenz, 2003, 2004; Volman, Baruchi, & Ben-Jacob, 2005) can be used as the evidence of the existence and activation of polychronous groups.

4.1 How Is It Different from Synfire Chains? The notion of a synfire chain (Abeles, 1991; Bienenstock, 1995; Diesmann et al., 1999; Ikegaya et al., 2004) is probably the most beautiful theoretical idea in neuroscience. Synfire chains describe pools of neurons firing synchronously, not polychronously. Synfire activity relies on synaptic connections having equal delays or no delays at all. Though easy to implement, networks without delays are finite-dimensional and do not have rich dynamics to support persistent polychronous spiking. Indeed, in the context of synfire activity, the groups in Figure 9 could not be distinguished, and the network of five neurons in Figure 10 would have only one synfire chain showing reverberating activity (provided that all the delays are equal and sufficiently long). Bienenstock (1995) referred to polychronous activity as a synfire braid.

Synfire chain research concerns the stability of a synfire activity. Instead, we employ here population thinking (Edelman, 1987). Although many polychronous groups are short-lived, there is a huge number of them constantly appearing. And although their activation is not reliable, there is a spontaneous activation every second in a network of 1000 neurons. Thus, the system is robust not in terms of individual groups but in terms of populations of groups.

4.2 How Is It Different from Hopfield-Grossberg Attractor Networks? Polychronous groups are not attractors from dynamical system point of view (Hopfield, 1982; Grossberg, 1988). When activated, they result in stereotypical but transient activity that typically lasts three to four gamma cycles (less than 100 ms; see Figure 8). Once the stimulation is removed, the network does not return to a "default" state but continues to be spontaneously active.

4.3 How Is It Different from Feedforward Networks? The anatomy of the spiking networks that we consider is not feedforward but reentrant (Edelman, 1987). Thus, the network does not "wait" for stimulus to come but exhibits an autonomous activity. Stimulation perturbs only the intrinsic activity, as it happens in mammalian brain. As a result, the network does not have a rigid stimulus-response function. The same stimulus can elicit

quite different responses because it activates a different (random) subset of polychronous groups representing the stimulus. Thus, the network operates in a highly degenerate regime (Edelman & Gally, 2001).

4.4 How Is It Different from Liquid-State Machines? Dynamics of a network implementing liquid-state-machine paradigm (Maass, Natschlaeger, & Markram, 2002) is purely stimulus driven. Such a network does not have short-term memory, and it cannot place the input in the context of the previous inputs. The simple model presented here implements some aspects of the liquid-state computing (e.g., it could be the liquid), but its response is not quite stimulus driven; it depends on the current state of the network, which in turn depends on the short-term and long-term experience and previous stimuli. This could be an advantage or a drawback, depending on the task that needs to be solved.

Let us discuss some interesting open problems and implementation issues that are worth exploring further:

- Finding groups: Our algorithm for finding polychronous groups considers various triplets firing with various spiking patterns and determines the groups that are initiated by the patterns. Because of the combinatorial explosion, it is extremely inefficient. In addition, we probably miss many groups that do not start with three neurons.
- Training: Our training strategy is the simplest and probably the least effective one: choose a set of "sensory" neurons, stimulate them with different spike-timing sequences, and let STDP form or select/reinforce appropriate groups. It is not clear whether this strategy is effective when many stimuli are needed to be learned.
- Incomplete activation: When a group is activated, whether in response to a particular stimulation or spontaneously, it rarely activates entirely. Typically, neurons at the beginning of the group polychronize, that is, fire with the precise spike-timing pattern imposed by the group connectivity, but the precision fades away as activation propagates along the group. As a result, the connectivity in the tail of the group does not stabilize, so the group as a whole changes.
- Stability: Because of continuous plasticity, groups appear, grow (see Figure 13), live for a certain period of time, and then could suddenly disappear (Izhikevich et al., 2004). Thus, spontaneous activity of the network leads to a slow degradation of the memory, and it is not clear how to prevent this.
- Sleep states: The network can switch randomly between different states. Some of them correspond to "'vigilance" with gamma oscillations, and others resemble "sleep" states, similar to the one in Figure 5

(top). It is not clear whether such switching should be prevented or whether it provides certain advantages for homeostasis of connections.

- Optimizing performance: Exploring the model, we encounter a regime when the number of polychronous groups was greater than the number of synapses in the network. However, the network was prone to epileptic seizures, which eventually lead to uncontrolled, completely synchronized activity. More effort is required to fine-tune the parameters of the model to optimize the performance of the network without inducing paroxysmal behavior.
- Context dependence: Propagation delays are assumed to be constant in the present simulation. In vivo studies have shown that axonal conduction velocity has submillisecond precision, but it also depends on the prior activity of the neuron during last 100 ms; hence, it can change with time in a context-dependent manner (Swadlow, 1974; Swadlow & Waxman, 1975; Swadlow, Kocsis, & Waxman, 1980). Thus, a polychronous group may exist and be activated in one time, but can temporarily disappear at another time because of the previous activity of its constituent neurons.
- Synaptic scaling: We assumed here that the maximal cut-off synaptic value is 10 mV, which is slightly more than half of the threshold value of the pyramidal neuron in the model. Since the average neuron in the network has 100 presynaptic sources, it implies that 2% of presynaptic spikes is enough to make it fire. It is interesting, but computationally impossible at present, to estimate the number of different polychronous groups when each neuron has, say, 400 presynaptic sources, each having maximal value of 2.5 mV. In this case, each group would be "wider," since at least four neurons (the same 2%) are needed to fire any given postsynaptic cell.
- Network scaling: We simulated a network of 10^3 neurons and found 10^4 polychronous groups. How does the number of groups scale with the number of neurons? In particular, how many polychronous groups are there in a network of 10^{11} neurons, each having 10^4 synapses? This is a fundamental question related to the information capacity of the human brain.
- Closing the loop: An important biological observation is that organisms are part of the environment. The philosophy at the Neurosciences Institute (the author's host institute) is "the brain is embodied, the body is embedded." Thus, to understand and simulate the brain, we need to give the neural network a body and put it into real or virtual environment (Krichmar & Edelman, 2002). In this case, the network becomes part of a closed loop: the environment stimulates "sensory" neurons via sensory organs. Firings of these neurons combined with

the current state of the network (i.e., the context) activate appropriate polychronous groups, which excite "motor" neurons and produce appropriate movements in the environment (i.e., response). The movements change the sensory input and close the causality loop.

- Reward and reinforcement learning: Some stimuli bring the reward (not modeled here) and activate the value system (Krichmar & Edelman, 2002). It strengthens recently active polychronous groups—the groups that resulted in the reward. This increases the probability that the same stimuli in the future would result in activation of the same groups and thereby bring more reward. Thus, in addition to passively learning input stimuli, the system can actively explore those stimuli that bring the reward.

5 Cognitive Computations

Let us discuss possible directions of this research and its connection to studies of neural computation, attention, and consciousness. This section is highly speculative; it is motivated by, but not entirely based on, the simulations described above.

5.1 Synchrony: Good or Bad? Much research on the dynamics of spiking and oscillatory networks is devoted to determining the conditions that ensure synchronization of the network activity. Many researchers (including this author until a few years ago) are under the erroneous assumption that synchronization is something good and desirable. What kind of information processing could possibly go on in a network of synchronously firing neurons? Probably none, since the entire network acts as a single neuron. Here we treat synchronization (or polychronization) of all neurons in the network as being an undesirable property that should be avoided. In fact, synchronization (or polychronization) should be so rare and difficult to occur by chance that when it happens, even transiently in a small subset of the network, it would signify something important, something meaningful, e.g., a stimulus is recognized, two or more features are bound, attention is paid. All these cognitive events are related to the activation of polychronous groups, as we discuss in this section.

5.2 Novel Model of Neural Computation. Most of artificial neural network research concerns supervised or unsupervised training of neural nets, which consists in building a mapping from a given set of inputs to a given set of outputs. For example, the connection weights of Hopfield-Grossberg model (Hopfield, 1982; Grossberg, 1988) are modified so that the given input patterns become attractors of the network. In these approaches, the network is "instructed" what to do.

In contrast, we take a different approach in this article. Instead of using the instructionist approach, we employ a selectionist approach, known as the theory of neuronal group selection (TNGS) and neural Darwinism (Edelman 1987). There are two types of selection constantly going on in the spiking network:

- Selection on the neuronal level: STDP selects subgraphs with matching conduction delays in initially unstructured network, resulting in the formation of a large number of groups, each capable of polychronizing, that is, generating a reproducible spike-timing pattern with millisecond precision. The number of coexisting polychronous groups, called repertoire of the network, is potentially unlimited.
- Selection on the group level: Polychronous groups are representations of possible inputs to the network, so that each input selects groups from the repertoire. That is, every time the input is presented to the network, a polychronous group (or groups) whose spike-timing pattern resonates with the input is activated (i.e., the neurons constituting the group polychronize).

Using the analogy with the immune system, where we have antibodies for practically all possible antigens, even those that do not exist on earth, we can take our point of view to the extreme and say that the network "has memories of all past and future events," with the past events corresponding to certain groups with assigned representations and the future events corresponding to the large, amorphous, potentially unlimited cloud of available groups with no representation. Learning of a new input consists of selecting and reinforcing an appropriate group (or groups) that resonates with the input. Assigning the representation (meaning) to the group consists of potentiating weak connections that link this group with other groups coactive at the same time, that is, putting the group in the context of the other groups that already have representations (see Figure 16). In this sense, each polychronous group represents its stimulus and the context. In addition, persistent stimuli may create new groups, as we show in section 3. In any case, the input constantly shapes the landscape of the groups present in the network, selecting and amplifying some groups and suppressing and destroying others.

The major result of this article is that spiking networks with delays have more groups than neurons. Thus, the system has potentially enormous memory capacity and will never run out of groups, which could explain how networks of mere 10^{11} neurons (the size of the human neocortex) could have such a diversity of behavior. Of course, we need to learn how to use this extraordinary property in models.

5.3 Binding and Gamma Rhythm. Binding is discussed in detail by Bienenstock (1995) in the context of synfire activity (see also the special

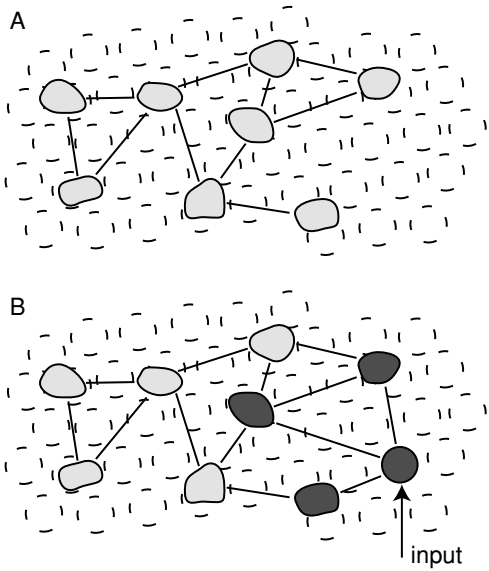


Figure 16: Due to the potentially unlimited number of coexisting polychronous groups, the system "has memories of all past and future events," denoted by shaded and empty figures, respectively. (A) Past events are represented by the groups with assigned representations; they activate in response to specific inputs. Connections between the groups provide the context. (B) Experiencing a new event consists of selecting a group out of the amorphous set of "available" groups that resonates with the input. The context is provided by the potentiated connections between the group and recently active groups.

issue of *NEURON* (September 1999) on the binding problem). Bienenstock's major idea is that dynamic binding of various features of a stimulus corresponds to the synchronization of synfire waves propagating along distinct chains. The synchronization is induced by weak reentrant synaptic coupling between these chains (see also Seth et al. 2004b). This idea is equally applicable to polychronous activity.

In Figure 17 we illustrate what could happen when different groups representing different features of a stimulus are activated asynchronously (left) or time locked (right). In the former case, no specific temporal relationship would exist between firings of neurons belonging to different groups, except that the firings would be correlated (they are all triggered by the common input). The dotted lines in Figure 17 (right) are the reentrant connections between groups that establish the context for each group. These connections would coordinate activations of the groups and would be responsible for the time locking (polychronization) in Figure 17 (right). In essence, the four groups in the figure would act as a single meta-group whose reproducible

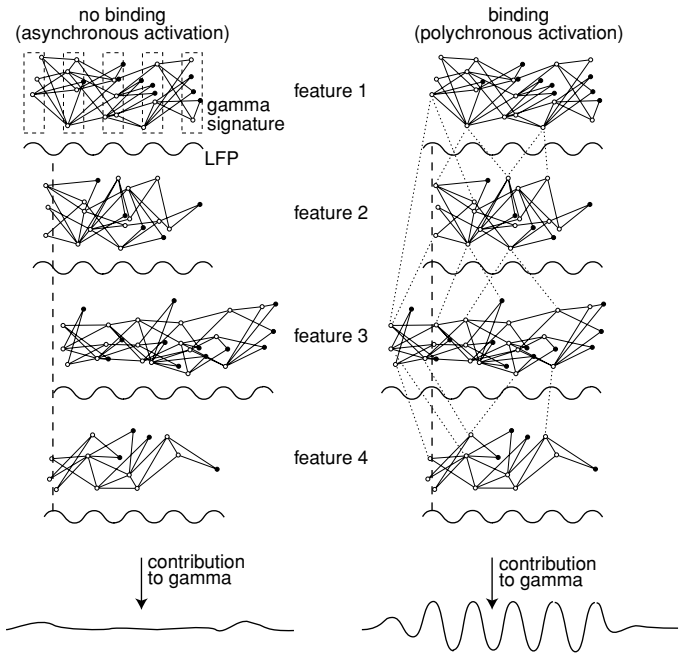


Figure 17: Time-locked activation of groups representing various features of a stimulus results in binding of the features and increased gamma rhythm. Each group contributes small gamma oscillation to the network gamma. (Left) The oscillations average out during the asynchronous activation. (Right) The oscillations add up during the time-locked activation. Dotted lines: weak reentrant connections between the groups that synchronize (or polychronize) their activation (the groups and the associated gamma rhythm are drawn by hand).

spike-timing pattern represents all features of the stimulus bound together into a whole.

Each group has a gamma signature indicated by dashed boxes in Figure 17 (top left) and discussed in section 3 (see Figure 6). Activation of such a group produces a small oscillation of the local field potential (LFP) in the gamma frequency. When groups activate asynchronously, their LFPs would have random phases and cancel each other. When groups activate polychronously during binding, their LFPs would add up, resulting in the noticeable network gamma rhythm and increased synchrony (Singer & Gray, 1995).

5.4 Modeling Attention. The small size of the system does not allow us to explore other cognitive functions of spiking networks. In September 2005, the author simulated a detailed thalamo-cortical system having 10^{11}

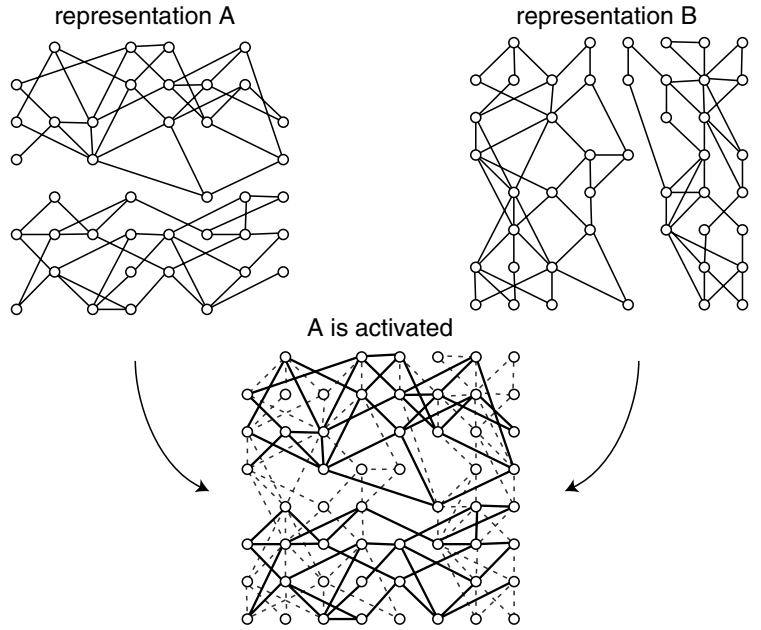


Figure 18: Stimuli A and B are both represented by pairs of polychronous groups with overlapping neurons. Selective attention to representation A (both groups representing A are active) does not inhibit neurons involved in representation B. Because the neurons are shared, they just fire with the spike-timing pattern corresponding to A.

spiking neurons (i.e., the size of the human brain), 6-layer cortical microcircuitry, specific, non-specific, and reticular thalamic nuclei. One second of simulation took more than one month on a cluster of 27 3-GHz processors. In a large-scale network, there could be many groups (more than the 15 depicted in Figure 12) that represent any particular input stimulus. The stimulus alone could activate only a small subset of the groups. However, weak reentrant connections among the groups may trigger a regenerative process leading to explosive activation of many other groups representing the stimulus, resulting in its perception (and possibly increases gamma rhythm). These groups take up most of the neurons in the network so that only a relatively few neurons are available for activation of any other group not related to the stimulus. We might say that the stimulus is the focus of attention. If two or more stimuli are present, then activation of groups representing one stimulus essentially precludes the other stimuli from being attended. Remarkably, the groups corresponding to the unattended stimuli are not inhibited. The neurons constituting the groups fire, but with a different spike-timing pattern (see Figure 18). We hypothesize that this mutual exclusion is related to the phenomenon of selective attention.

In our view, attention is not a command that comes from the "higher" or "executive" center and tells which input to attend to. Instead, we view attention as an emerging property of simultaneous and regenerative activation (via positive feedback) of a large subset of groups representing a stimulus, thereby impeding activation of other groups corresponding to other stimuli. Multiple stimuli compete for the focus of attention, and the winner is determined by many factors, mostly the context.

5.5 Consciousness as Attention to Memory. When no stimulation is present, there is a spontaneous activation of polychronous groups, as in Figure 11. We hypothesize that if the size of the network exceeds a certain threshold, a random activation of a few groups representing a previously seen stimulus may activate other groups representing the same stimulus so that the total number of activated groups is comparable to the number of activated groups that occurs when the stimulus is actually present. Not only would such an event exclude all the other groups not related to the stimulus from being activated, but from the network's point of view, it would be similar to the event when the stimulus is actually present and it is the focus of attention. One can say that the network "thinks" of the stimulus—that is, it pays attention to the memory of the stimulus. Such "thinking" resembles "experiencing" the stimulus. A sequence of spontaneous activations corresponding to one stimulus, then another, and so on may be related to the stream of primary (perceptual or sensory) consciousness (Edelman, 2004), which can be found in many nonhuman animals. Of course, it does not explain the high-order (conceptual) consciousness of humans.

Appendix: The Model

The MATLAB code simulating the network activity is in Figure 19. The upper half of the program initializes the network, and it takes approximately 30 sec on a 1 GHz Pentium PC. The lower half of the program executes the model, and it takes 5 seconds to simulate 1 second of network activity. The actual time may vary depending on the firing rate of the neurons.

The MATLAB code and an equivalent 20-times-faster C++ code are also available on the author's Web page. Let us describe the details of the model.

A.1 Anatomy. The network consists of `N=1000` neurons with the first `Ne=800` of excitatory RS type, and the remaining `Ni=200` of inhibitory FS type (Izhikevich, 2003). The ratio of excitatory to inhibitory cells is 4 to 1, as in the mammalian neocortex. Each excitatory neuron is connected to `M=100` random neurons, so that the probability of connection is `M/N=0.1`, again as in the neocortex. Each inhibitory neuron is connected to `M=100` excitatory neurons only. The indices of postsynaptic targets are in the `N`×`M`-matrix `post`. Corresponding synaptic weights are in the `N`×`M`-matrix `s`. Inhibitory weights are not plastic, whereas excitatory weights evolve according to the STDP

```
% spnet.m: Spiking network with axonal conduction delays and STDP
% Created by Eugene M.Izhikevich.                February 3, 2004
M=100;                  % number of synapses per neuron
D=20;                   % maximal conduction delay
% excitatory neurons    % inhibitory neurons      % total number
Ne=800;                 Ni=200;                   N=Ne+Ni;
a=[0.02*ones(Ne,1);     0.1*ones(Ni,1)];
d=[   8*ones(Ne,1);     2*ones(Ni,1)];
sm=10;                  % maximal synaptic strength

post=ceil([N*rand(Ne,M);Ne*rand(Ni,M)]);
s=[6*ones(Ne,M);-5*ones(Ni,M)];         % synaptic weights
sd=zeros(N,M);                          % their derivatives
for i=1:N
  if i<=Ne
    for j=1:D
      delays{i,j}=M/D*(j-1)+(1:M/D);
    end;
  else
    delays{i,1}=1:M;
  end;
  pre{i}=find(post==i&s>0);             % pre excitatory neurons
  aux{i}=N*(D-1-ceil(ceil(pre{i}/N)/(M/D)))+1+mod(pre{i}-1,N);
end;
STDP = zeros(N,1001+D);
v = -65*ones(N,1);                      % initial values
u = 0.2.*v;                             % initial values
firings=[-D 0];                         % spike timings

for sec=1:60*60*24                      % simulation of 1 day
  for t=1:1000                          % simulation of 1 sec
    I=zeros(N,1);
    I(ceil(N*rand))=20;                 % random thalamic input
    fired = find(v>=30);                % indices of fired neurons
    v(fired)=-65;
    u(fired)=u(fired)+d(fired);
    STDP(fired,t+D)=0.1;
    for k=1:length(fired)
      sd(pre{fired(k)})=sd(pre{fired(k)})+STDP(N*t+aux{fired(k)});
    end;
    firings=[firings;t*ones(length(fired),1),fired];
    k=size(firings,1);
    while firings(k,1)>t-D
      del=delays{firings(k,2),t-firings(k,1)+1};
      ind = post(firings(k,2),del);
      I(ind)=I(ind)+s(firings(k,2), del)';
      sd(firings(k,2),del)=sd(firings(k,2),del)-1.2*STDP(ind,t+D)';
      k=k-1;
    end;
    v=v+0.5*((0.04*v+5).*v+140-u+I);    % for numerical
    v=v+0.5*((0.04*v+5).*v+140-u+I);    % stability time
    u=u+a.*(0.2*v-u);                   % step is 0.5 ms
    STDP(:,t+D+1)=0.95*STDP(:,t+D);     % tau = 20 ms
  end;
  plot(firings(:,1),firings(:,2),'.');
  axis([0 1000 0 N]); drawnow;
  STDP(:,1:D+1)=STDP(:,1001:1001+D);
  ind = find(firings(:,1) > 1001-D);
  firings=[-D 0;firings(ind,1)-1000,firings(ind,2)];
  s(1:Ne,:)=max(0,min(sm,0.01+s(1:Ne,:)+sd(1:Ne,:)));
  sd=0.9*sd;
end;
```

Figure 19: MATLAB code of the spiking network with axonal conduction delays and spike-timing-dependent plasticity (STDP). It is available on the author's Web page: www.izhikevich.com.

rule discussed in the next section. Their derivatives are in the N×M-matrix sd , though only the Ne×M-block of the matrix is used.

Each synaptic connection has a fixed integer conduction delay between 1 ms and D=20 ms, where D is a parameter (M/D must be integer in the model). We do not model modifiable delays (Huning, Glunder, & Palm, 1998; Eurich, Pawelzik, Ernst, Cowan, & Milton, 1999) or transmission failures (Senn, Schneider, & Ruff, 2002). The list of all synaptic connections from neuron i having delay j is in the cell array delays{i, j} . Our MATLAB implementation assigns 1 ms delay to all inhibitory connections, and 1 to D ms delay to all excitatory connections. Although the anatomy of the model is random, reflecting the connectivity within a cortical minicolumn, one can implement an arbitrarily sophisticated anatomy by specifying the matrices post and delays. The details of the anatomy do not matter in the rest of the MATLAB code and do not slow the simulation.

Once the matrices post and delays are specified, the program initializes cell arrays pre and aux. The former contains indices of all excitatory neurons presynaptic to a given neuron, and the latter is an auxiliary table of indices needed to speed up STDP implementation.

A.2 Spiking Neurons. Each neuron in the network is described by the simple spiking model (Izhikevich, 2003)

$$v' = 0.04v^2 + 5v + 140 - u + I \tag{A.1}$$

$$u' = a(bv - u) \tag{A.2}$$

with the auxiliary after-spike resetting

$$\text{if } v \geq +30 \text{ mV, then } \begin{cases} v \leftarrow c \\ u \leftarrow u + d. \end{cases} \tag{A.3}$$

Here variable v represents the membrane potential of the neuron, and u represents a membrane recovery variable, which accounts for the activation of K^+ ionic currents and inactivation of Na^+ ionic currents, and it provides negative feedback to v. After the spike reaches its apex at +30 mV, which is not to be confused with the firing threshold, the membrane voltage and the recovery variable are reset according to equation A.3. Depending on the values of the parameters, the model can exhibit firing patterns of all known types of cortical neurons (Izhikevich, 2003). It can also reproduce all of the 20 most fundamental neurocomputational properties of biological neurons summarized in Figure 3, (see Izhikevich, 2004).

We use $(b, c) = (0.2, -65)$ for all neurons in the network. For excitatory neurons, we use the values $(a, d) = (0.02, 8)$ corresponding to cortical pyramidal neurons exhibiting regular spiking (RS) firing patterns. For inhibitory neurons, we use the values $(a, d) = (0.1, 2)$ corresponding to cortical

interneurons exhibiting fast spiking (FS) firing patterns. Better values of parameters corresponding to different types of cortical neurons, as well as the explanation of the model, can be found in Izhikevich (2006).

Variable I in the model combines two kinds of input to the neuron: (1) random thalamic input and (2) spiking input from the other neurons. This is implemented via `N`-dimensional vector `I`.

A.3 Spike-Timing-Dependent Plasticity. The synaptic connections in the model are modified according to the spike-timing-dependent plasticity (STDP) rule (Song et al., 2000). We use the simplest and the most effective implementation of this rule, depicted in Figure 4. If a spike from an excitatory presynaptic neuron arrives at a postsynaptic neuron (possibly making the postsynaptic neuron fire), then the synapse is potentiated (strengthened). In contrast, if the spike arrives right after the postsynaptic neuron fired, the synapse is depressed (weakened).

If pre- and postsynaptic neurons fire uncorrelated Poissonian spike trains, there are moments when the weight of the synaptic connection is potentiated, and there are moments when it is depressed. We chose the parameters of the STDP curve so that depression is stronger than potentiation and the synaptic weight goes slowly to zero. Indeed, such a connection is not needed and should be eliminated. In contrast, if the presynaptic neuron often fires before the postsynaptic one, then the synaptic connection slowly potentiates. Indeed, such a connection causes the postsynaptic spikes and should be strengthened. In this way, STDP strengthens causal interactions in the network.

The magnitude of potentiation or depression depends on the time interval between the spikes. Each time a neuron fires, the variable `STDP` is reset to 0.1. Every millisecond, `STDP` decreases by `0.95*STDP`, so that it decays to zero as $0.1e^{-t/20(\mathrm{ms})}$, according to the parameters in Figure 4. This function determines the magnitude of potentiation or depression.

For each fired neuron, we consider all its presynaptic neurons and determine the timings of the last excitatory spikes arrived from these neurons. Since these spikes made the neuron fire, the synaptic weights are potentiated according to the value of `STDP` at the presynaptic neuron adjusted for the conduction delay. This corresponds to the positive part of the STDP curve in Figure 4. Notice that the largest increase occurs for the spikes that arrived right before the neuron fired, that is, the spikes that actually caused postsynaptic spike.

In addition, when an excitatory spike arrives at a postsynaptic neuron, we depress the synapse according to the value of `STDP` at the postsynaptic neuron. This corresponds to the negative part of the STDP curve in Figure 4. Indeed, such a spike arrived after the postsynaptic neuron fired, and hence the synapse between the neurons should be weakened. (The same synapse will be potentiated when the postsynaptic neuron fires.)

Instead of changing the synaptic weights directly, we change their derivatives `sd`, and then update the weights once a second according to the rule `s` ← `s + 0.01 + sd`, and `sd` ← `0.9sd`, where 0.01 describes activity-independent increase of synaptic weight needed to potentiate synapses coming to silent neurons (Turrigiano, Leslie, Desai, Rutherford, & Nelson, 1998; Desai, Cudmore, Nelson, & Turrigiano, 2002). Thus, the synaptic change is not instantaneous but slow, taking many seconds to develop. We manually keep the weights in the range between 0 and `sm`, where `sm` is a parameter of the model, typically less than 10 (mV).

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References

Abeles, M. (1991). *Corticonics: Neural circuits of the cerebral cortex.* Cambridge: Cambridge University Press.

Abeles, M. (2002). Synfire chains. In M. A. Arbib (Ed.), *The handbook of Brain theory and neural networks.* (pp. 1143–1146). Cambridge, MA: MIT Press.

Amit, D. J., & Brunel N. (1997). Model of global spontaneous activity and local structured activity during delay periods in the cerebral cortex. *Cereb. Cortex*, *7*, 237–252.

Baker, S. N., & Lemon R. N. (2000). Precise spatiotemporal repeating patterns in monkey primary and supplementary motor areas occur at chance levels. *J. Neurophysiol.*, *84*, 1770–1780.

Beggs, J. M., & Plenz, D. (2003). Neuronal avalanches in neocortical circuits. *J. Neuroscience*, *23*, 11167–11177.

Beggs, J. M., & Plenz, D. (2004). Neuronal avalanches are diverse and precise activity patterns that are stable for many hours in cortical slice cultures. *J. Neuroscience*, *24*, 5216–5229.

Bellen, A., & Zennaro, M. (2003). *Numerical methods for delay differential equations.* Oxford: Clarendon Press.

Bienenstock, E. (1995). A model of neocortex. *Network: Comput. Neural Syst.*, *6*, 179–224.

Braitenberg, V., & Schuz, A. (1991). *Anatomy of the cortex: Statistics and geometry.* Berlin: Springer-Verlag.

Bryant, H., & Segundo, J. (1976). Spike initiation by transmembrane current: A white-noise analysis. *J. Physiol. (Lond.)*, *260*, 279–314.

Buzsaki, G., Llinas, R., Singer, W., Berthoz, A., & Christen, Y. (Eds.). (1994). *Temporal coding in the brain*. New York: Springer-Verlag.

Chang, E. Y., Morris, K. F., Shannon, R., & Lindsey, B. G. (2000). Repeated sequences of interspike intervals in baroresponsive respiratory related neuronal assemblies of the cat brain stem. *J. Neurophysiol.*, *84*, 1136–1148.

Changeux, J. P., & Danchin, A. (1976). Selective stabilization of developing synapses as a mechanism for the recall and recognition. *Cognition*, *33*, 25–62.

de Ruyter van Steveninck, R. R., Lewen, G. D., Strong, S. P., Koberle, R., & Bialek, W. (1997). Reproducibility and variability in neural spike trains. *Science*, *275*, 1805–1808.

Desai, N. S., Cudmore, R. H., Nelson, S. B., & Turrigiano, G. G. (2002). Critical periods for experience-dependent synaptic scaling in visual cortex. *Nature Neuroscience*, *5*, 783–789.

Diesmann, M., Gewaltig, M.-O., & Aertsen, A. (1999). Stable propagation of synchronous spiking in cortical neural networks. *Nature*, *402*, 529–533.

Edelman, G. M. (1987). *Neural Darwinism: The theory of neuronal group selection.* New York: Basic Books.

Edelman, G. M. (1993). *Neural Darwinism: Selection and reentrant signaling in higher brain function. Neuron*, *10*, 115–125.

Edelman, G. M. (2004). *Wider than the sky: The phenomenal gift of consciousness.* New Haven, CT: Yale University Press.

Edelman, G. M., & Gally, J. (2001). Degeneracy and complexity in biological systems. *PNAS*, *98*, 13763–13768.

Eurich, C., Pawelzik, K., Ernst, U., Cowan, J., & Milton, J. (1999). Dynamics of self-organazed delay adaptation. *Phys. Rev. Lett.*, *82*, 1594–1597.

Ferster, D., & Lindstrom, S. (1983). An intracellular analysis of geniculocortical connectivity in area 17 of the cat. *Journal of Physiology*, *342*, 181–215.

Foss, J., & Milton, J. (2000). Multistability in recurrent neural loops arising from delay. *J. Neurophysiol.*, *84*, 975–985.

Gerstner, W., Kempter, R., van Hemmen, J. L., & Wagner, H. (1996). A neuronal learning rule for sub-millisecond temporal coding *Nature*, *383*, 76–78.

Grossberg, S. (1988). Nonlinear neural networks: Principles, mechanisms, and architectures. *Neural Networks*, *1*, 17–61.

Hopfield, J. J. (1982). Neural networks and physical systems with emergent collective computational abilities. *PNAS*, *79*, 2554–2558.

Hopfield, J. J. (1995). Pattern recognition computation using action potential timing for stimulus representation. *Nature*, *376*, 33–36.

Hoppensteadt, F. C., & Izhikevich, E. M. (1997). *Weekly connected neural networks.* New York: Springer-Verlag.

Huning, H., Glunder, H., & Palm, G. (1998). Synaptic delay learning in pulse-coupled neurons. *Neural Computation*, *10*, 555–565.

Ikegaya, Y., Aaron, G., Cossart, R., Aronov, D., Lampl, I., Fester, D., & Yuste, R. (2004). Synfire chains and cortical songs: Temporal modules of cortical activity. *Science*, *304*, 559–564.

Izhikevich, E. M. (2003). Simple model of spiking neurons. *IEEE Transactions on Neural Networks*, *14*, 1569–1572.

Izhikevich, E. M. (2004). Which model to use for cortical spiking neurons? *IEEE Transactions on Neural Networks*, *15*, 1063–1070.

Izhikevich, E. M. (2006). Dynamical systems in neuroscience: The geometry of excitability and bursting. Cambridge, MA: The MIT Press.

Izhikevich, E. M., Gally, J. A., & Edelman, G. M. (2004). Spike-timing dynamics of neuronal groups. *Cerebral Cortex*, *14*, 933–944.

Krichmar, J. L., & Edelman, G. M. (2002). Machine psychology: Autonomous behavior, perceptual categorization and conditioning in a brain-based device. *Cerebral Cortex*, *12*, 818–830.

Lindsey, B. G., Morris, K. F., Shannon, R., & Gerstein, G. L. (1997). Repeated patterns of distributed synchrony in neuronal assemblies. *J. Neurophysiol.*, *78*, 1714–1719.

Litvak, V., Sompolinsky, H., Segev, I., & Abeles, M. (2003). On the transmission of rate code in long feed-forward networks with excitatory-inhibitory balance. *J. Neurosci.*, *23*, 3006–3015.

Maass, W., Natschlaeger, T., & Markram, H. (2002). Real-time computing without stable states: A new framework for neural computation based on perturbations. *Neural Computation*, *14*, 2531–2560.

Mainen, Z. F., & Sejnowski, T. J. (1995). Reliability of spike timing in neocortical neurons. *Science*, *268*, 1503–1506.

Mao, B.-Q., Hamzei-Sichani, F., Aronov, D., Froemke, R. C., & Yuste, R. (2001). Dynamics of spontaneous activity in neocortical slices. *Neuron*, *32*, 883–898.

Mazurek, M. E., & Shadlen, M. N. (2002). Limits to the temporal fidelity of cortical spike rate signals. *Nat. Neurosci.*, *5*, 463–471.

Miller, R. (1996a). Neural assemblies and laminar interactions in the cerebral cortex. *Biol. Cybern.*, *75*(3), 253–261.

Miller, R. (1996b). Cortico-thalamic interplay and the security of operation of neural assemblies and temporal chains in the cerebral cortex. *Biol. Cybern.*, *75*(3), 263–275.

Oram, M. W., Wiener, M. C., Lestienne, R., & Richmond, B. J. (1999). Stochastic nature of precisely timed spike patterns in visual system neuronal responses. *J. Neurophysiol.*, *81*, 3021–3033.

Prut, Y., Vaadia, E., Bergman, H., Haalman, I., Slovin, H., & Abeles, M. (1998). Spatiotemporal structure of cortical activity: Properties and behavioral relevance. *J. Neurophysiol.*, *79*, 2857–2874.

Reinagel, P., & Reid, R. C. (2002). Precise firing events are conserved across neurons. *J. Neurosci.*, *22*, 6837–6841.

Riehle, A., Grün, S., Diesmann, M., & Aertsen, A. (1997). Spike synchronization and rate modulation differentially involved in motor cortical function. *Science*, *278*, 1950–1953.

Salami, M., Itami, C., Tsumoto, T., & Kimura, F. (2003). Change of conduction velocity by regional myelination yields constant latency irrespective of distance between thalamus and cortex. *PNAS*, *100*, 6174–6179.

Senn, W., Schneider, M., & Ruf, B. (2002). Activity-dependent development of axonal and dendritic delays, or, why synaptic transmission should be unreliable. *Neural Computation*, *14*, 583–619.

Seth, A. K., McKinstry, J. L., Edelman, G. M., & Krichmar, J. L. (2004a). Spatiotemporal processing of whisker input supports texture discrimination by a brain-based device. In S., Schaal, A., Billard, S., Vijayakumar, J., Hallam, & J.-A., Meyer (Eds.), *From animals to animats 8: Proceedings of the Eighth International Conference on the Simulation of Adaptive Behavior.* Cambridge, MA: MIT Press.

Seth, A. K., McKinstry, J. L., Edelman, G. M., & Krichmar, J. L. (2004b). Visual binding through reentrant connectivity and dynamic synchronization in a brain-based device. *Cerebral Cortex*, *14*, 1185–1199.

Shadlen, M. N., & Newsome, W. T. (1994). Noise, neural codes and cortical organization. *Curr. Opin. Neurobiol.*, *4*, 569–579.

Shadlen, M. N., & Newsome, W. T. (1998). The variable discharge of cortical neurons: Implications for connectivity, computation and information coding. *J. Neurosci.*, *18*, 3870–3896.

Shadlen, M. N., & Morshon, J. A. (1999). Synchrony unbound: A critical evaluation of the temporal binding hypothesis. *Neuron*, *24*, 67–77.

Singer, W., & Gray, C. M. (1995). Visual feature integration and the temporal correlation hypothesis. *Annual Review of Neuroscience*, *18*, 555–586.

Song, S., Miller, K. D., & Abbott, L. F. (2000). Competitive Hebbian learning through spike-timing-dependent synaptic plasticity. *Nature Neurosci.*, *3*, 919–926.

Stewart, I., Golubitsky, M., & Pivato, M. (2003). Symmetry groupoids and patterns of synchrony in coupled cell networks. *SIAM J. Appl. Dynam. Sys.*, *2*, 606–646.

Strehler, B. L., & Lestienne, R. (1986). Evidence on precise time-coded symbols and memory of patterns in monkey cortical neuronal spike trains. *PNAS*, *83*, 9812–9816.

Swadlow, H. A. (1974). Systematic variations in the conduction velocity of slowly conducting axons in the rabbit corpus collosum. *Experimental Neurology*, *43*, 445–451.

Swadlow, H. A. (1985). Physiological properties of individual cerebral axons studied in vivo for as long as one year. *J. Neurophysiology*, *54*, 1346–1362.

Swadlow, H. A. (1988). Efferent neurons and suspected interneurons in binocular visual cortex of the awake rabbit: Receptive fields and binocular properties. *J. Neurophysiol.*, *88*, 1162–1187.

Swadlow, H. A. (1992). Monitoring the excitability of neocortical efferent neurons to direct activation by extracellular current pulses. *J. Neurophysiol.*, *68*, 605–619.

Swadlow, H. A. (1994). Efferent neurons and suspected interneurons in motor cortex of the awake rabbit: Axonal properties, sensory receptive fields, and subthreshold synaptic inputs. *J. Neurophysiology*, *71*, 437–453.

Swadlow, H. A., Kocsis, J. D., & Waxman, S. G. (1980). Modulation of impulse conduction along the axonal tree. *Ann. Rev. Biophys. Bioeng.*, *9*, 143–179.

Swadlow, H. A., & Waxman, S. G. (1975). Observations on impulse conduction along central axons. *PNAS*, *72*, 5156–5159.

Tetko, I. V., & Villa, A. E. P. (2001). A pattern grouping algorithm for analysis of spatiotemporal patterns in neuronal spike trains. 2: Application to simultaneous single unit recordings. *Journal of Neuroscience Methods*, *105*, 15–24.

Turrigiano, G. G., Leslie, K. R., Desai N. S., Rutherford, L. C., & Nelson, S. B. (1998). Activity-dependent scaling of quantal amplitude in neocortical neurons. *Nature*, *391*, 892–896.

van Vreeswijk, C., & Sompolinsky, H. (1996). Chaos in neuronal networks with balanced excitatory and inhibitory activity. *Science*, *274*, 1724–1726.

Villa, A. E., Tetko, I. V., Hyland, B., & Najem, A. (1999). Spatiotemporal activity patterns of rat cortical neurons predict responses in a conditioned task. *Proc. Natl. Acad. Sci. USA*, *96*, 1106–1111.

Volman, V., Baruchi, I., & Ben-Jacob, E. (2005). Manifestation of function-follow-form in cultured neuronal networks. *Physics Biology*, *2*, 98–110.

Wiener, J., & Hale, J. K. (1992). Ordinary and delay differential equations. New York: Wiley.

Whittington, M. A., Traub, R. D., Kopell, N., Ermentrout, B., & Buhl, E. H. (2000). Inhibition-based rhythms: Experimental and mathematical observations on network dynamics. *Int. J. Psychophysiol.*, *38*, 315–336.

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