

Group Work 11.A.1 Solution

A)

Recall that the frequency of oscillation *in radians per second* for a simple harmonic oscillator is given by:

$$\omega = \sqrt{\frac{k}{m}}. \quad (1)$$

Plugging in numbers to equation 1 gives:

$$\omega = \sqrt{\frac{200 \frac{N}{m}}{1.0kg + 1.0kg}} = 10 \frac{rad}{s}$$

B)

The equation of motion for a simple harmonic oscillator starting at rest is given by:

$$y(t) = y_0 \cos(\omega t). \quad (2)$$

Where y_0 is the initial position, and ω is the frequency *in radians per second*. Recalling that acceleration is the second derivative of position, we find that:

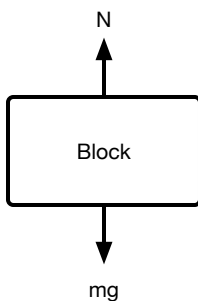
$$a(t) = -\omega^2 y_0 \cos(\omega t) \quad (3)$$

Plugging in values to equation 3 gives:

$$a(0) = -\left(10 \frac{rad}{s}\right)^2 (-0.2m) \cos(0) = 20 \frac{m}{s^2} \quad (4)$$

C)

Let's begin by drawing a free body diagram *of the block*.



It is important to note that the the scale will read the magnitude of the normal force. Now we must find the acceleration of the block at time $\frac{T}{4}$. To do this is is worth noting that:

$$T = \frac{2\pi}{\omega} \implies \frac{T}{4} = \frac{\pi}{2\omega}. \quad (5)$$

Referring back to equation 3 gives:

$$a\left(\frac{T}{4}\right) = (0.2m) \left(10\frac{rad}{s}\right)^2 \cos\left(\omega\frac{\pi}{2\omega}\right) 0 \quad (6)$$

Summing the forces on the block gives:

$$\Sigma F_y = N - mg = m(0) \implies N = mg = (1.0kg) \left(10\frac{m}{s^2}\right) = 20N \quad (7)$$

D)

Asking whether the block stays on the scale at all times is equivalent to asking whether the block is accelerating fast enough that the normal force is 0. Thus, we refer again to the free-body diagram:

$$\Sigma F_y = 0 - mg = ma \implies a = -g = -10\frac{m}{s^2} \quad (8)$$

So the question becomes: is there a time when:

$$a(t) = -\omega^2 y_0 \cos(\omega t) = -g \quad (9)$$

is satisfied? Rearranging equation 9 gives:

$$\cos(\omega t) = \frac{g}{\omega^2 y_0} = \frac{10\frac{m}{s^2}}{\left(10\frac{rad}{s^2}\right)^2 (-0.2m)} = -0.5 \quad (10)$$

Since:

$$|-0.5| \leq 1 \quad (11)$$

A solution exists. Further rearranging gives:

$$t = \frac{\cos^{-1}\left(\frac{g}{\omega^2 y_0}\right)}{\omega} = 0.21s \quad (12)$$