EEL 3470: ELECTROMAGNETIC FIELDS

Solution of Homework 3

1. Problem 2.29

Given $\mathbf{D} = 2\rho\,\mathbf{a}_\rho + \sin\phi\,\mathbf{a}_z$ C/m^2, find the electric flux passing through the surface defined by $2.0 \le \rho \le 4.0$ m, $90.° \le \phi \le 180°$, and $z = 4.0$ m.

$$\Psi = \int \mathbf{E}\bullet d\mathbf{S},\quad d\mathbf{S} = \rho\, d\rho\, d\phi \mathbf{a}_z$$

$$\Psi = \int \left(2\rho\mathbf{a}_\rho + \sin\phi\mathbf{a}_z\right)\bullet\rho\, d\rho\, d\phi\mathbf{a}_z = \int_2^4 \rho\, d\rho \int_{\pi/2}^{\pi} \sin\phi d\phi = 6C$$

2. Problem 2.30

Suppose the electric flux density is given by $\mathbf{D} = 3r\,\mathbf{a}_r - \cos\phi\,\mathbf{a}_\theta + \sin^2\theta\,\mathbf{a}_\phi$ C/m^2. Find the electric flux through both surfaces of a hemisphere of radius 2.00 m and $0.00° \le \theta \le 90.0°$.

Solution

$$\Psi_1 = \int \mathbf{D}\bullet d\mathbf{S},$$
$$d\mathbf{S}_1 = r^2 \sin\theta d\theta d\phi \mathbf{a}_r$$

$$\Psi_1 = \int \left(3r\mathbf{a}_r - \cos\phi\mathbf{a}_\theta + \sin^2\theta\mathbf{a}_\phi\right)\bullet\left(r^2\sin\theta d\theta d\phi\mathbf{a}_r\right)$$
$$= 3r^3 \int_0^{\pi/2} \sin\theta d\theta \int_0^{2\pi} d\phi = 48\pi C$$

$$d\mathbf{S}_2 = rdrd\phi\mathbf{a}_\theta$$
$$\Psi_2 = \int -\cos\phi\mathbf{a}_\theta \bullet rdrd\phi\mathbf{a}_\theta$$
$$= -\left(\left.\frac{r^2}{2}\right|_0^2\right)\sin\phi\Big|_0^{2\pi} = 0$$
$$\therefore \Psi = 48\pi C$$

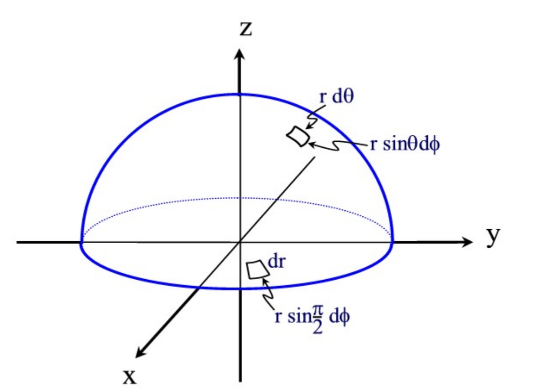


Fig. P2.30

3. Problem 2.36

P2.36: A thick-walled spherical shell, with inner radius 2.00 cm and outer radius 4.00 cm, has an evenly distributed 12.0 nC charge. Plot D_r as a function of radial distance from the origin over the range $0 \le r \le 10$ cm.

Here we'll let a = inner radius and b = outer radius. Then

$Q_{enc} = \oint \mathbf{D} \cdot d\mathbf{S} = \int D_r \mathbf{a}_r \cdot r^2 \sin\theta d\theta d\phi \mathbf{a}_r = 4\pi r^2 D_r$; This is true for each Gaussian surface.

The volume containing charge is

$$v = \int_a^b r^2 dr \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi = \frac{4}{3}\pi\left(b^3 - a^3\right).$$

So

$$\rho_v = \frac{Q}{v} = \frac{3Q}{4\pi\left(b^3 - a^3\right)}.$$

Now we can evaluate Q_{enc} for each Gaussian surface.

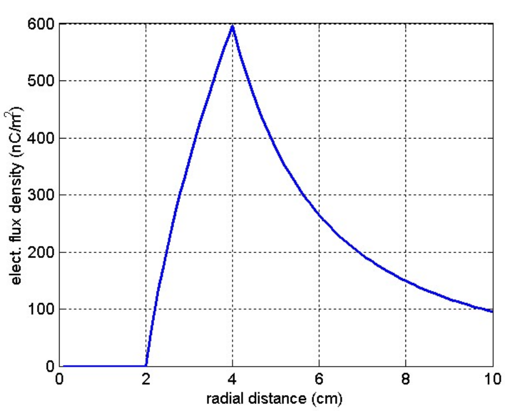
GS1 ($r < a$): $Q_{enc} = 0$ so $D_r = 0$.

GS2 ($a < r < b$): $Q_{enc} = \int \rho_v dv = \rho_v \int_a^r r^2 dr \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi = \frac{\rho_v 4\pi}{3}\left(r^3 - a^3\right).$

Inserting our value for ρ_v, we find

$$D_r = \frac{Q}{4\pi r^2}\frac{\left(r^3 - a^3\right)}{\left(b^3 - a^3\right)} \quad \text{for } a \le r \le b.$$

GS3 ($r > b$): $Q_{enc} = Q$, $D_r = \frac{Q}{4\pi r^2}$, for $r \ge b$.



4. Problem 2.44

P2.44: For the following potential distributions, use the gradient equation to find **E**.

(a) $V = x+y^2z$ (V)

(b) $V = \rho^2\sin\phi$(V)

(c) $V = r\sin\theta\cos\phi$ (V).

(a) $\mathbf{E} = -\nabla V = -\mathbf{a}_x - 2yz\mathbf{a}_y - y^2\mathbf{a}_z$

(b) $\mathbf{E} = -\nabla V = -\left(\frac{\partial V}{\partial \rho}\mathbf{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi + \frac{\partial V}{\partial z}\mathbf{a}_z\right) = -2\rho\sin\phi\mathbf{a}_\rho - \rho\cos\phi\mathbf{a}_\phi$

(c)

$$\mathbf{E} = -\nabla V = -\left(\frac{\partial V}{\partial r}\mathbf{a}_\rho + \frac{1}{r}\frac{\partial V}{\partial \theta}\mathbf{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}\mathbf{a}_\phi\right) = -\sin\theta\cos\phi\mathbf{a}_\rho - \cos\theta\cos\phi\mathbf{a}_\theta + \sin\phi\mathbf{a}_\phi$$

5. Problem 2.45

P2.45: A 100 nC point charge is located at the origin. (a) Determine the potential difference V_{BA} between the point A(0.0,0.0,-6.0)m and point B(0.0,2.0,0.0)m. (b) How much work would be done to move a 1.0 nC charge from point A to point B against the electric field generated by the 100 nC point charge?

(a) $V_{BA} = -\int_A^A \mathbf{E}\bullet d\mathbf{L}$.

The potential difference is only a function of radial distance from the origin. Letting r_a = 6m and r_b = 2m, we then have

$$V_{BA} = -\int_{r_a}^{r_b} \frac{Q}{4\pi\varepsilon_o r^2}\mathbf{a}_r \bullet dr\mathbf{a}_r = \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{r_b} - \frac{1}{r_a}\right) = 300V.$$

(b) $W = Q_2V_{BA} = \left(10^{-9}C\right)\left(300V\right)\frac{J}{CV} = 300nJ$