EEL 3470: ELECTROMAGNETIC FIELDS

Solution of Homework 2

Solve the following problems in your textbook

1. Problem 2.15

P2.15: A line charge with charge density 2.00 nC/m exists at y = -2.00 m, x = 0.00. (a) A charge Q = 8.00 nC exists somewhere along the y-axis. Where must you locate Q so that the total electric field is zero at the origin? (b) Suppose instead of the 8.00 nC charge of part (a) that you locate a charge Q at (0.00, 6.00m, 0.00). What value of Q will result in a total electric field intensity of zero at the origin?

Solution

(a) The contributions to **E** from the line and point charge must cancel, or $\mathbf{E} = \mathbf{E}_L + \mathbf{E}_Q$.

For the line: $\mathbf{E}_L = \dfrac{\rho_L}{2\pi\varepsilon_o \rho}\mathbf{a}_\rho = \dfrac{(2nC/m)}{2\pi\left(10^{-9}\,F/36\pi m\right)(2m)}\mathbf{a}_y = 18\dfrac{V}{m}\mathbf{a}_y$

and for the point charge, where the point is located a distance y along the y-axis, we have:

$$\mathbf{E}_Q = \frac{Q}{4\pi\varepsilon_o y^2}(-\mathbf{a}_y) = \frac{(8nC)(-\mathbf{a}_y)}{4\pi\left(10^{-9}\,F/36\pi m\right)y^2} = \frac{72}{y^2}(-\mathbf{a}_y)$$

Therefore:

$$\frac{72}{y^2} = 18, \text{ or } y = \sqrt{\frac{72}{18}} = 2m.$$

So $\boxed{\mathrm{Q}(0,2.0\mathrm{m},0)}$

(b)

$$\frac{Q}{4\pi\varepsilon_o (6)^2} = 18,$$

$$Q = \frac{(18)(36)}{9} = 72nC.$$

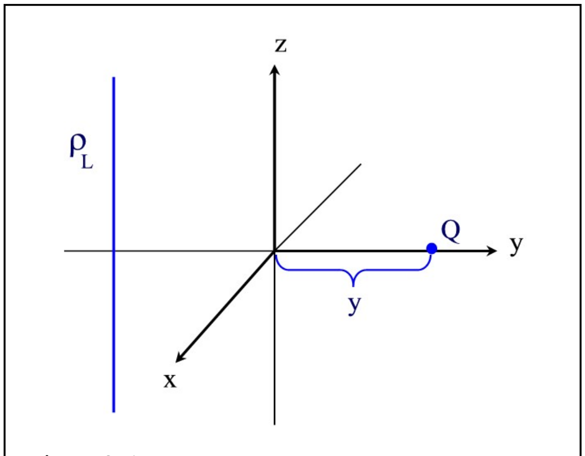


Fig. P2.15

2. Problem 2.16

P2.16: You are given two z-directed line charges of charge density +1 nC/m at x = 0, y = -1.0 m, and charge density –1.0 nC/m at x = 0, y = 1.0 m. Find **E** at P(1.0m,0,0).

Solution

The situation is represented by Figure P2.16a. A better 2-dimensional view in Figure P2.16b is useful for solving the problem.

$$\mathbf{E}_1 = \frac{\left(1x10^{-9}C\right)}{2\pi\left(10^{-9}F/_{36\pi m}\right)\left(\sqrt{2}m\right)}\frac{\left(\mathbf{a}_x+\mathbf{a}_y\right)}{\sqrt{2}}\frac{FV}{C} = 9\left(\mathbf{a}_x+\mathbf{a}_y\right)\frac{V}{m}, \text{ and}$$

$$\mathbf{E}_2 = 9\left(\mathbf{-a}_x+\mathbf{a}_y\right)\frac{V}{m}.$$

So $\mathbf{E}_{TOT} = 18\,\mathbf{a}_y$ V/m.

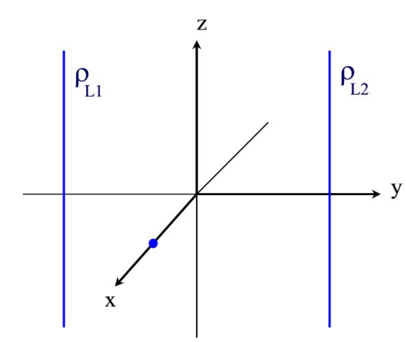


Fig. P2.16a

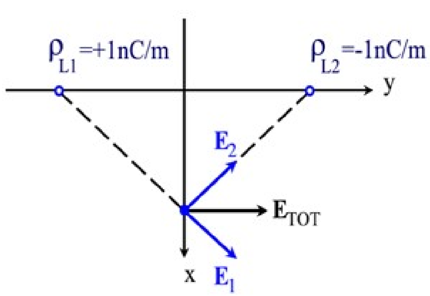


Fig. P2.16b

3. Problem 2.19

In free space, there is a point charge Q = 8.0 nC at (-2.0,0,0)m, a line charge ρ_L = 10 nC/m at y = -9.0m, x = 0m, and a sheet charge ρ_s = 12. nC/m^2 at z = -2.0m. Determine **E** at the origin.

Solution

The situation is represented by Figure P2.19, and the total field is $\mathbf{E}_{TOT} = \mathbf{E}_Q + \mathbf{E}_L + \mathbf{E}_S$.

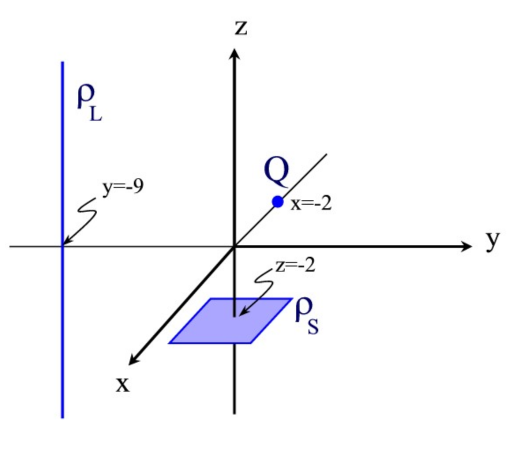


Fig. P2.19

$$\mathbf{E}_Q = \frac{Q}{4\pi\varepsilon_o R^2}\mathbf{a}_R = \frac{\left(8x10^{-9}C\right)\mathbf{a}_x}{4\pi\left(10^{-9}F/36\pi m\right)\left(2m\right)^2}$$

$$= 18\mathbf{a}_x \frac{V}{m}$$

$$\mathbf{E}_L = \frac{\rho_L}{2\pi\varepsilon_o \rho}\mathbf{a}_\rho = \frac{\left(10x10^{-9}\,C/m\right)\mathbf{a}_y}{2\pi\left(10^{-9}F/36\pi m\right)\left(9m\right)}$$

$$= 20\mathbf{a}_y \frac{V}{m}$$

$$\mathbf{E}_s = \frac{\rho_s}{2\varepsilon_o}\mathbf{a}_N = \frac{\left(12x10^{-9}\,C/m^2\right)}{2\left(10^{-9}F/36\pi m\right)}\mathbf{a}_z$$

$$= 679\mathbf{a}_z \frac{V}{m}$$

So: $\mathbf{E}_{tot}$ = 18 $\mathbf{a}_x$ + 20 $\mathbf{a}_y$ + 680 $\mathbf{a}_z$ V/m.

4. Problem 2.20

An infinitely long line charge (ρ_L = 21π nC/m) lies along the z-axis. An infinite area sheet charge (ρ_s = 3 nC/m^2) lies in the x-z plane at y = 10 m. Find a point on the y-axis where the electric field intensity is zero.

Solution

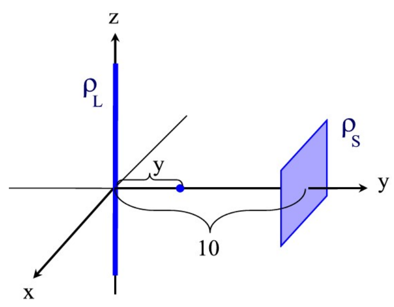


Fig. P2.20

We have $\mathbf{E}_{TOT} = \mathbf{E}_L + \mathbf{E}_S$.

$$\mathbf{E}_L = \frac{\rho_L \mathbf{a}_\rho}{2\pi\varepsilon_o \rho} = \frac{\rho_L \mathbf{a}_y}{2\pi\varepsilon_o y}$$

$$= \frac{\left(21\pi x 10^{-9} C/m\right)}{2\pi\left(10^{-9} F\big/36\pi m\right) y}\mathbf{a}_y = \frac{378\pi}{y}\mathbf{a}_y$$

$$\mathbf{E}_s = \frac{\rho_s \mathbf{a}_N}{2\varepsilon_o} = \frac{\left(3x10^{-9}\, C/m^2\right)\left(-\mathbf{a}_y\right)}{2\left(10^{-9} F\big/36\pi m\right)}$$

$$= -54\pi\mathbf{a}_y$$

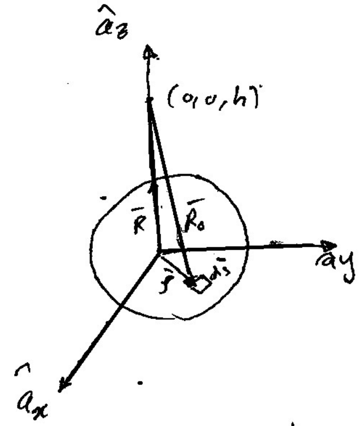
So

$$\frac{378\pi}{y} - 54\pi = 0, \text{ or } y = 7.$$

Therefore, P(0, 7m, 0).

5. **Problem 2.22**

Consider a circular disk in the x-y plane of radius 5.0 cm. Suppose the charge density is a function of radius such that $\rho_s = 12\rho$ nC/cm^2 (when ρ is in cm). Find the electric field intensity a point 20.0 cm above the origin on the z-axis.



$$\boxed{\bar{E} = \frac{1}{4\pi\epsilon_0}\iint_S \frac{\rho_s}{R_o^2}\,\hat{a}_{R_o}\,ds'}$$

$$\overline{ds'} = \rho'\,d\rho'\,d\phi'$$

$$\bar{R} = h\,\hat{a}_z$$

$$\overline{R'} = \rho'\,\hat{r}$$

$$\bar{R}_o = h\hat{a}_z - \rho'\,\hat{r}$$

$$|R_o| = \sqrt{\rho'^2 + h^2}$$

$$\hat{a}_{R_o} = \frac{R - R'}{|R - R'|} = \frac{-\rho'\,\hat{r} + h\,\hat{a}_z}{\sqrt{\rho'^2 + h^2}}$$

$$\boxed{\rho_s = 12\,\rho'} \quad (nC)$$

$$\therefore\ \bar{E} = \frac{1}{4\pi\epsilon_0}\int_{\rho'=0}^{a}\int_{\phi'=0}^{2\pi} \frac{12\,\rho'\,(-\rho'\hat{r} + h\,\hat{a}_z)}{(\rho'^2 + h^2)^{3/2}}\,\rho'\,d\rho'\,d\phi'$$

$$\bar{E} = 10^{-9}\left(\frac{12}{4\pi\epsilon_0}\right)\int_{\rho'=0}^{a}\int_{\phi'=0}^{2\pi}\left[\frac{h\rho'^2}{(\rho'^2+h^2)^{3/2}}\,\hat{a}_z - \frac{\rho'^3\,\hat{r}}{(\rho'^2+h^2)^{3/2}}\right]d\rho'\,d\phi'$$

$\because\ \hat{r} = \cos\phi'\,\hat{a}_x + \sin\phi'\,\hat{a}_y$

$\therefore$ The $\hat{a}_x$ and $\hat{a}_y$ terms in the electric field integrate to zero with respect to $d\phi'$

Therefore,

$$\bar{E} = 10^{-9}\left(\frac{12(2\pi)}{4\pi\epsilon_0}\right)\int_{\rho'=0}^{a} \frac{h\rho'^2}{(\rho'^2+h^2)^{3/2}}\, d\rho' \;\hat{a}_z$$

$$= 10^{-9}\left(\frac{6h}{\epsilon_0}\right)\int_{\rho'=0}^{a} \frac{\rho'^2}{(\rho'^2+h^2)^{3/2}}\, d\rho' \;\hat{a}_z$$

$$\int \frac{\rho'^2}{(\rho'^2+h'^2)^{3/2}}\, d\rho'$$

Let $u = \rho'$ $\qquad du = d\rho'$

$$dv = \frac{\rho' d\rho'}{(\rho'^2+h'^2)^{3/2}}$$

$$v = \frac{-1}{\sqrt{\rho'^2+h^2}}$$

$$\therefore \int u\,dv = uv - \int v\,du$$

$$= -\frac{\rho'}{\sqrt{\rho'^2+h^2}} + \int \frac{1}{\sqrt{\rho'^2+h^2}}\, d\rho'$$

$$= \left.\frac{-\rho'}{\sqrt{\rho'^2+h^2}}\right|_0^a + \left.\ln\left[\rho' + \sqrt{\rho'^2+h^2}\right]\right|_0^a$$

$$= \frac{-a}{\sqrt{a^2+h^2}} + \ln\left[a + \sqrt{a^2+h^2}\right] - \ln[h]$$

$$= \frac{-a}{\sqrt{a^2+h^2}} + \ln\left[\frac{a+\sqrt{a^2+h^2}}{h}\right]$$

we have $h = 20$ cm , $a = 5$ cm

$\epsilon_0 = 8.854\ 10^{-14}$ F/cm

$$\therefore \bar{E} = \frac{(6)(20)(10^{-9})}{8.854 * 10^{-14}} \left[\ln \frac{\sqrt{5^2+20^2} + 5}{20} - \frac{5}{\sqrt{5^2+20^2}} \right]$$

$$= 6682.9\ \hat{a}_z \quad \text{V/cm}.$$