

Table of Integrals*

Basic Forms

$$\int x^n dx = \frac{1}{n+1} x^{n+1} \quad (1)$$

$$\int \frac{1}{x} dx = \ln|x| \quad (2)$$

$$\int u dv = uv - \int v du \quad (3)$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln|ax+b| \quad (4)$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2} dx = -\frac{1}{x+a} \quad (5)$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1 \quad (6)$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \quad (7)$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x \quad (8)$$

$$\int \frac{1}{a^2+x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} \quad (9)$$

$$\int \frac{x}{a^2+x^2} dx = \frac{1}{2} \ln|a^2+x^2| \quad (10)$$

$$\int \frac{x^2}{a^2+x^2} dx = x - a \tan^{-1} \frac{x}{a} \quad (11)$$

$$\int \frac{x^3}{a^2+x^2} dx = \frac{1}{2} x^2 - \frac{1}{2} a^2 \ln|a^2+x^2| \quad (12)$$

$$\int \frac{1}{ax^2+bx+c} dx = \frac{2}{\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \quad (13)$$

$$\int \frac{1}{(x+a)(x+b)} dx = \frac{1}{b-a} \ln \frac{a+x}{b+x}, a \neq b \quad (14)$$

$$\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln|x+a| \quad (15)$$

$$\begin{aligned} \int \frac{x}{ax^2+bx+c} dx &= \frac{1}{2a} \ln|ax^2+bx+c| \\ &\quad - \frac{b}{a\sqrt{4ac-b^2}} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \end{aligned} \quad (16)$$

Integrals with Roots

$$\int \sqrt{x-a} dx = \frac{2}{3} (x-a)^{3/2} \quad (17)$$

$$\int \frac{1}{\sqrt{x \pm a}} dx = 2\sqrt{x \pm a} \quad (18)$$

$$\int \frac{1}{\sqrt{a-x}} dx = -2\sqrt{a-x} \quad (19)$$

$$\int x \sqrt{x-a} dx = \frac{2}{3} a (x-a)^{3/2} + \frac{2}{5} (x-a)^{5/2} \quad (20)$$

$$\int \sqrt{ax+b} dx = \left(\frac{2b}{3a} + \frac{2x}{3} \right) \sqrt{ax+b} \quad (21)$$

$$\int (ax+b)^{3/2} dx = \frac{2}{5a} (ax+b)^{5/2} \quad (22)$$

$$\int \frac{x}{\sqrt{x \pm a}} dx = \frac{2}{3} (x \mp 2a) \sqrt{x \pm a} \quad (23)$$

$$\int \sqrt{\frac{x}{a-x}} dx = -\sqrt{x(a-x)} - a \tan^{-1} \frac{\sqrt{x(a-x)}}{x-a} \quad (24)$$

$$\int \sqrt{\frac{x}{a+x}} dx = \sqrt{x(a+x)} - a \ln [\sqrt{x} + \sqrt{x+a}] \quad (25)$$

Integrals with Logarithms

$$\int \ln ax dx = x \ln ax - x \quad (42)$$

$$\int \frac{\ln ax}{x} dx = \frac{1}{2} (\ln ax)^2 \quad (43)$$

$$\int \ln(ax+b) dx = \left(x + \frac{b}{a} \right) \ln(ax+b) - x, a \neq 0 \quad (44)$$

$$\int \ln(x^2 + a^2) dx = x \ln(x^2 + a^2) + 2a \tan^{-1} \frac{x}{a} - 2x \quad (45)$$

$$\int \ln(x^2 - a^2) dx = x \ln(x^2 - a^2) + a \ln \frac{x+a}{x-a} - 2x \quad (46)$$

$$\begin{aligned} \int \ln(ax^2 + bx + c) dx &= \frac{1}{a} \sqrt{4ac-b^2} \tan^{-1} \frac{2ax+b}{\sqrt{4ac-b^2}} \\ &\quad - 2x + \left(\frac{b}{2a} + x \right) \ln(ax^2 + bx + c) \end{aligned} \quad (47)$$

$$\begin{aligned} \int x \ln(ax+b) dx &= \frac{bx}{2a} - \frac{1}{4} x^2 \\ &\quad + \frac{1}{2} \left(x^2 - \frac{b^2}{a^2} \right) \ln(ax+b) \end{aligned} \quad (48)$$

$$\begin{aligned} \int x \ln(a^2 - b^2 x^2) dx &= -\frac{1}{2} x^2 + \\ &\quad \frac{1}{2} \left(x^2 - \frac{d^2}{b^2} \right) \ln(a^2 - b^2 x^2) \end{aligned} \quad (49)$$

Integrals with Exponentials

$$\int e^{ax} dx = \frac{1}{a} e^{ax} \quad (50)$$

$$\begin{aligned} \int \sqrt{x} e^{ax} dx &= \frac{1}{a} \sqrt{x} e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}} \operatorname{erf}(i\sqrt{ax}), \\ \text{where } \operatorname{erf}(x) &= \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \end{aligned} \quad (51)$$

$$\int x e^x dx = (x-1)e^x \quad (52)$$

$$\int x e^{ax} dx = \left(\frac{x}{a} - \frac{1}{a^2} \right) e^{ax} \quad (53)$$

$$\int x^2 e^x dx = (x^2 - 2x + 2) e^x \quad (54)$$

$$\int x^2 e^{ax} dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3} \right) e^{ax} \quad (55)$$

$$\int x^3 e^x dx = (x^3 - 3x^2 + 6x - 6) e^x \quad (56)$$

$$\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx \quad (57)$$

$$\int x^n e^{ax} dx = \frac{(-1)^n}{a^{n+1}} \Gamma[1+n, -ax],$$

$$\text{where } \Gamma(a, x) = \int_x^\infty t^{a-1} e^{-t} dt$$

$$\int e^{ax^2} dx = -\frac{i\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}(ix\sqrt{a}) \quad (59)$$

$$\int e^{-ax^2} dx = \frac{\sqrt{\pi}}{2\sqrt{a}} \operatorname{erf}(x\sqrt{a}) \quad (60)$$

$$\int x e^{-ax^2} dx = -\frac{1}{2a} e^{-ax^2} \quad (61)$$

$$\int x^2 e^{-ax^2} dx = \frac{1}{4} \sqrt{\frac{\pi}{a^3}} \operatorname{erf}(x\sqrt{a}) - \frac{x}{2a} e^{-ax^2} \quad (62)$$

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Integrals with Trigonometric Functions

$$\int \sin ax dx = -\frac{1}{a} \cos ax \quad (63)$$

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \quad (64)$$

$$\begin{aligned} \int \sin^n ax dx &= \\ &- \frac{1}{a} \cos ax {}_2F_1 \left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax \right] \end{aligned} \quad (65)$$

$$\int \sin^3 ax dx = -\frac{3 \cos ax}{4a} + \frac{\cos 3ax}{12a} \quad (66)$$

$$\int \cos ax dx = \frac{1}{a} \sin ax \quad (67)$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \quad (68)$$

$$\begin{aligned} \int \cos^p ax dx &= -\frac{1}{a(1+p)} \cos^{1+p} ax \times \\ &{}_2F_1 \left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax \right] \end{aligned} \quad (69)$$

$$\int \cos^3 ax dx = \frac{3 \sin ax}{4a} + \frac{\sin 3ax}{12a} \quad (70)$$

$$\int \cos ax \sin bx dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b \quad (71)$$

$$\begin{aligned} \int \sin^2 ax \cos bx dx &= -\frac{\sin[(2a-b)x]}{4(2a-b)} \\ &+ \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)} \end{aligned} \quad (72)$$

$$\int \sin^2 x \cos ax dx = \frac{1}{3} \sin^3 x \quad (73)$$

$$\begin{aligned} \int \cos^2 ax \sin bx dx &= \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b} \\ &- \frac{\cos[(2a+b)x]}{4(2a+b)} \end{aligned} \quad (74)$$

$$\int \cos^2 ax \sin ax dx = -\frac{1}{3a} \cos^3 ax \quad (75)$$

$$\begin{aligned} \int \sin^2 ax \cos^2 bx dx &= \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)} \\ &+ \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} \end{aligned} \quad (76)$$

$$\int \sin^2 ax \cos^2 ax dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \quad (77)$$

$$\int \tan ax dx = -\frac{1}{a} \ln \cos ax \quad (78)$$

$$\int \tan^2 ax dx = -x + \frac{1}{a} \tan ax \quad (79)$$

$$\begin{aligned} \int \tan^n ax dx &= \frac{\tan^{n+1} ax}{a(n+1)} \times \\ &{}_2F_1 \left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax \right) \end{aligned} \quad (80)$$

$$\int \tan^3 ax dx = \frac{1}{a} \ln \cos ax + \frac{1}{2a} \sec^2 ax \quad (81)$$

$$\int \sec x dx = \ln |\sec x + \tan x| = 2 \tanh^{-1} \left(\tan \frac{x}{2} \right) \quad (82)$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax \quad (83)$$

$$\int \sec^3 x dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| \quad (84)$$

$$\int \sec x \tan x dx = \sec x \quad (85)$$

$$\int \sec^2 x \tan x dx = \frac{1}{2} \sec^2 x \quad (86)$$

$$\int \sec^n x \tan x dx = \frac{1}{n} \sec^n x, n \neq 0 \quad (87)$$

$$\int \csc x dx = \ln \left| \tan \frac{x}{2} \right| = \ln |\csc x - \cot x| + C \quad (88)$$

$$\int \csc^2 ax dx = -\frac{1}{a} \cot ax \quad (89)$$

$$\int \csc^3 x dx = -\frac{1}{2} \cot x \csc x + \frac{1}{2} \ln |\csc x - \cot x| \quad (90)$$

$$\int \csc^n x \cot x dx = -\frac{1}{n} \csc^n x, n \neq 0 \quad (91)$$

$$\int \sec x \csc x dx = \ln |\tan x| \quad (92)$$

Products of Trigonometric Functions and Monomials

$$\int x \cos x dx = \cos x + x \sin x \quad (93)$$

$$\int x \cos ax dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax \quad (94)$$

$$\int x^2 \cos x dx = 2x \cos x + (x^2 - 2) \sin x \quad (95)$$

$$\int x^2 \cos ax dx = \frac{2x \cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3} \sin ax \quad (96)$$

$$\begin{aligned} \int x^n \cos x dx &= -\frac{1}{2} (i)^{n+1} [\Gamma(n+1, -ix) \\ &+ (-1)^n \Gamma(n+1, ix)] \end{aligned} \quad (97)$$

$$\begin{aligned} \int x^n \cos ax dx &= \frac{1}{2} (ia)^{1-n} [(-1)^n \Gamma(n+1, -iax) \\ &- \Gamma(n+1, ixa)] \end{aligned} \quad (98)$$

$$\int x \sin x dx = -x \cos x + \sin x \quad (99)$$

$$\int x \sin ax dx = -\frac{x \cos ax}{a} + \frac{\sin ax}{a^2} \quad (100)$$

$$\int x^2 \sin x dx = (2 - x^2) \cos x + 2x \sin x \quad (101)$$

$$\int x^2 \sin ax dx = \frac{2 - a^2 x^2}{a^3} \cos ax + \frac{2x \sin ax}{a^2} \quad (102)$$

$$\int x^n \sin x dx = -\frac{1}{2} (i)^n [\Gamma(n+1, -ix) - (-1)^n \Gamma(n+1, -ix)] \quad (103)$$

Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x dx = \frac{1}{2} e^x (\sin x - \cos x) \quad (104)$$

$$\int e^{bx} \sin ax dx = \frac{1}{a^2 + b^2} e^{bx} (b \sin ax - a \cos ax) \quad (105)$$

$$\int e^x \cos x dx = \frac{1}{2} e^x (\sin x + \cos x) \quad (106)$$

$$\int e^{bx} \cos ax dx = \frac{1}{a^2 + b^2} e^{bx} (a \sin ax + b \cos ax) \quad (107)$$

$$\int x e^x \sin x dx = \frac{1}{2} e^x (\cos x - x \cos x + x \sin x) \quad (108)$$

$$\int x e^x \cos x dx = \frac{1}{2} e^x (x \cos x - \sin x + x \sin x) \quad (109)$$

Integrals of Hyperbolic Functions

$$\int \cosh ax dx = \frac{1}{a} \sinh ax \quad (110)$$

$$\begin{aligned} \int e^{ax} \cosh bx dx &= \\ \begin{cases} \frac{e^{ax}}{a^2 - b^2} [a \cosh bx - b \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} + \frac{x}{2} & a = b \end{cases} \end{aligned} \quad (111)$$

$$\int \sinh ax dx = \frac{1}{a} \cosh ax \quad (112)$$

$$\begin{aligned} \int e^{ax} \sinh bx dx &= \\ \begin{cases} \frac{e^{ax}}{a^2 - b^2} [-b \cosh bx + a \sinh bx] & a \neq b \\ \frac{e^{2ax}}{4a} - \frac{x}{2} & a = b \end{cases} \end{aligned} \quad (113)$$

$$\begin{aligned} \int e^{ax} \tanh bx dx &= \\ \begin{cases} \frac{e^{(a+2b)x}}{(a+2b)^2} {}_2F_1 \left[1 + \frac{a}{2b}, 1, 2 + \frac{a}{2b}, -e^{2bx} \right] \\ -\frac{1}{a} e^{ax} {}_2F_1 \left[\frac{a}{2b}, 1, 1E, -e^{2bx} \right] \\ \frac{e^{ax} - 2 \tan^{-1}[e^{ax}]}{a} & a \neq b \\ a = b \end{cases} \end{aligned} \quad (114)$$

$$\int \tanh ax dx = \frac{1}{a} \ln \cosh ax \quad (115)$$

$$\begin{aligned} \int \cos ax \cosh bx dx &= \frac{1}{a^2 + b^2} [a \sin ax \cosh bx \\ &+ b \cos ax \sinh bx] \end{aligned} \quad (116)$$

$$\begin{aligned} \int \cos ax \sinh bx dx &= \frac{1}{a^2 + b^2} [b \cos ax \cosh bx \\ &- a \sin ax \sinh bx] \end{aligned} \quad (117)$$

$$\begin{aligned} \int \sin ax \cosh bx dx &= \frac{1}{a^2 + b^2} [-a \cos ax \cosh bx \\ &+ b \sin ax \sinh bx] \end{aligned} \quad (118)$$

$$\begin{aligned} \int \sin ax \sinh bx dx &= \frac{1}{a^2 + b^2} [b \cosh bx \sin ax \\ &- a \cos ax \sinh bx] \end{aligned} \quad (119)$$

$$\int \sinh ax \cosh bx dx = \frac{1}{4a} [-2ax + \sinh 2ax] \quad (120)$$

$$\begin{aligned} \int \sinh ax \cosh bx dx &= \frac{1}{b^2 - a^2} [b \cosh bx \sinh ax \\ &- a \cosh ax \sinh bx] \end{aligned} \quad (121)$$