Lecture 2

Review of Vector Analysis
and
Electrostatic Field Intensity

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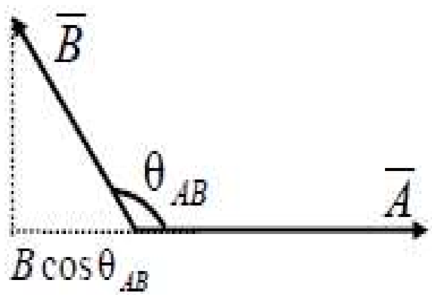
The Dot Product

- Given two vectors **A** and **B**, the dot product, is defined as the product of the magnitude of A, the magnitude of B, and the cosine of the smaller angle between them,



$$\boxed{\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}|\,|\mathbf{B}|\cos\theta_{AB}}$$

Orthogonal vectors condition?

If $\boldsymbol{A} = A_x\boldsymbol{a}_x + A_y\boldsymbol{a}_y + A_z\boldsymbol{a}_z$ *and* $\boldsymbol{B} = B_x\boldsymbol{a}_x + B_y\boldsymbol{a}_y + B_z\boldsymbol{a}_z$

Then the dot product can be calculated as follows:

$$\mathbf{A}\cdot\mathbf{B} = A_xB_x + A_yB_y + A_zB_z$$

$\boldsymbol{a}_x$, $\boldsymbol{a}_y$, and $\boldsymbol{a}_z$ *are the unit vectors of the Cartesian coordinate system*

Evaluation of the dot product in the Cartesian space

Let $\bar{A} = A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z$ and

$\bar{B} = B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z$

$$\therefore \bar{A} \cdot \bar{B} = (A_x \hat{a}_x + A_y \hat{a}_y + A_z \hat{a}_z) \cdot (B_x \hat{a}_x + B_y \hat{a}_y + B_z \hat{a}_z)$$

$$= A_x B_x \hat{a}_x \cdot \hat{a}_x + A_x B_y \cancelto{0}{\hat{a}_x \cdot \hat{a}_y} + A_x B_z \cancelto{0}{\hat{a}_x \cdot \hat{a}_z}$$

$$+ A_y B_x \cancelto{0}{\hat{a}_y \cdot \hat{a}_x} + A_y B_y \hat{a}_y \cdot \hat{a}_y + A_y B_z \cancelto{0}{\hat{a}_y \cdot \hat{a}_z}$$

$$+ A_z B_x \cancelto{0}{\hat{a}_z \cdot \hat{a}_x} + A_z B_y \cancelto{0}{\hat{a}_z \cdot \hat{a}_y} + A_z B_z \hat{a}_z \cdot \hat{a}_z$$

$$\boxed{\therefore \bar{A} \cdot \bar{B} = A_x B_x + A_y B_y + A_z B_z}$$

Properties of the dot product

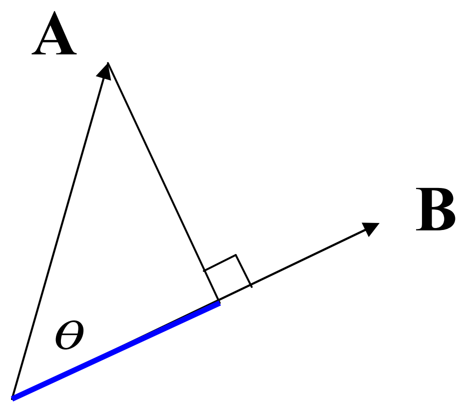
- **The dot product is commutative**
  - $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$
- **The dot product is distributive over addition**
  - $\mathbf{C} \cdot (\mathbf{A} + \mathbf{B}) = \mathbf{C} \cdot \mathbf{A} + \mathbf{C} \cdot \mathbf{B}$
- **The dot product of a vector with itself**
  - $\mathbf{A} \cdot \mathbf{A} = |\mathbf{A}|^2$
- **Scaling the dot product by a constant**
  - $k\,(\mathbf{A} \cdot \mathbf{B}) = k\mathbf{A} \cdot \mathbf{B} = \mathbf{A} \cdot k\mathbf{B}$

Projection of **A** on **B**

$$\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}|\,|\mathbf{B}|\, cos\,(\theta)$$

$$\mathbf{A}\cdot\mathbf{B}/\,|\mathbf{B}| = |\mathbf{A}|\, cos\,(\theta)$$

$$\mathbf{A}\cdot\boldsymbol{a}_{\mathbf{B}} = |\mathbf{A}|\, cos\,(\theta)$$



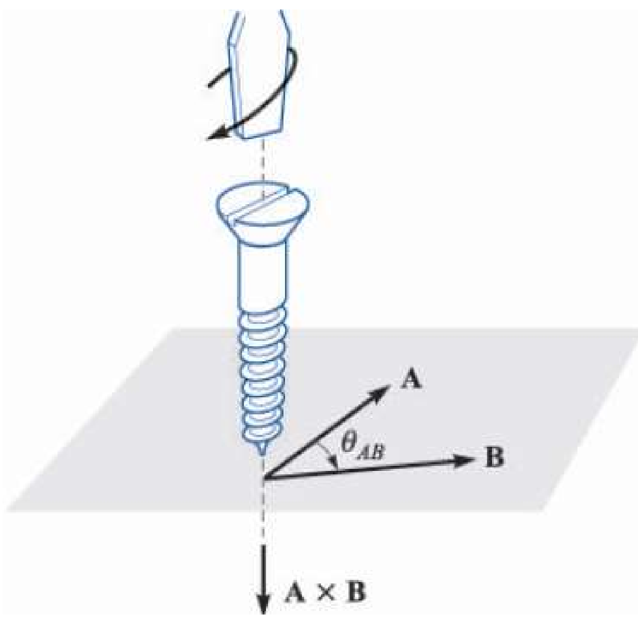
The projection of vector **A** on vector **B** is the dot product of vector **A** and the **unit vector** in the direction of vector **B**

The Cross Product

- Given two vectors **A** and **B**, the cross product of A and B is a vector defined as:

$$\mathbf{A} \times \mathbf{B} = \mathbf{a}_N |\mathbf{A}|\,|\mathbf{B}| \sin \theta_{AB}$$

- The magnitude of **A** × **B** equals the product of the magnitudes of **A**, **B**, and the sine of the smaller angle between **A** and **B**.
- The direction of **A** × **B** is perpendicular to the plane containing **A** and **B** and is along the direction of advance of a right-handed screw as A is turned into **B**.



The cross product is not commutative

$$\mathbf{B} \times \mathbf{A} = -(\mathbf{A} \times \mathbf{B})$$

Properties of the cross product

- The cross product is anticommutative,

$$\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a},$$

- Distributive over addition,

$$\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) + (\mathbf{a} \times \mathbf{c}),$$

- and compatible with scalar multiplication so that

$$(r\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (r\mathbf{b}) = r(\mathbf{a} \times \mathbf{b}).$$

- It is not associative, but satisfies the Jacobi identity:

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) + \mathbf{b} \times (\mathbf{c} \times \mathbf{a}) + \mathbf{c} \times (\mathbf{a} \times \mathbf{b}) = \mathbf{0}.$$

- If $\mathbf{a} \times \mathbf{b} = \mathbf{0}$, then
  - Either or both of them is the zero vector, ($\mathbf{a} = \mathbf{0}$ and/or $\mathbf{b} = \mathbf{0}$) , or
  - $\mathbf{a}$ and $\mathbf{b}$ are parallel ($\mathbf{a} \parallel \mathbf{b}$), so that the sine of the angle between them is zero, ($\theta = 0°$ or $\theta = 180°$ and $\sin\theta = 0$).
- The self cross product of a vector is the zero vector, i.e., $\mathbf{a} \times \mathbf{a} = \mathbf{0}$.

The cross product of unit vectors using Cartesian coordinates

$$\mathbf{a}_x \times \mathbf{a}_y = \mathbf{a}_z$$

$$\mathbf{a}_y \times \mathbf{a}_z = \mathbf{a}_x$$

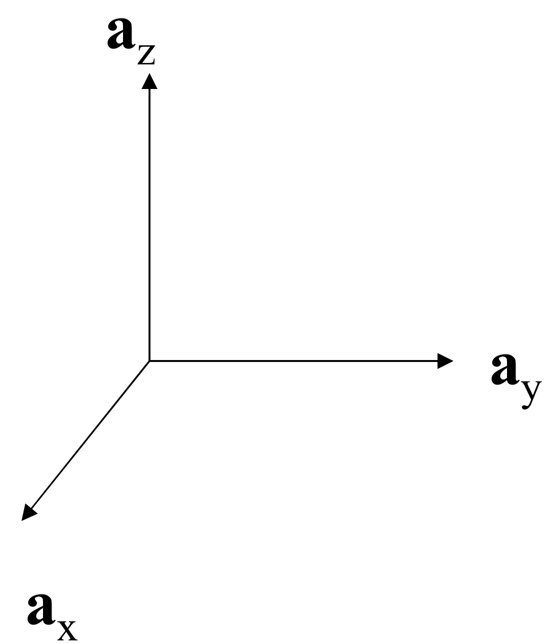
$$\mathbf{a}_z \times \mathbf{a}_x = \mathbf{a}_y$$

and

$$\mathbf{a}_x \times \mathbf{a}_z = -\ \mathbf{a}_y$$

$$\mathbf{a}_z \times \mathbf{a}_y = -\ \mathbf{a}_x$$

$$\mathbf{a}_y \times \mathbf{a}_x = -\ \mathbf{a}_z$$



Unit vectors of a right-handed Cartesian coordinate system.

Example 1: Evaluation of the Cross product of two vectors using the Cartesian Coordinates

- Given two vectors **A** and **B** in a right-handed Cartesian coordinates as follows:

  $\mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$ and

  $\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$

Then

$$\begin{aligned}\mathbf{A}\times\mathbf{B} = {} & \cancelto{0}{A_xB_x\mathbf{a}_x\times\mathbf{a}_x} + A_xB_y\mathbf{a}_x\times\mathbf{a}_y + A_xB_z\mathbf{a}_x\times\mathbf{a}_z \\ & + A_yB_x\mathbf{a}_y\times\mathbf{a}_x + \cancelto{0}{A_yB_y\mathbf{a}_y\times\mathbf{a}_y} + A_yB_z\mathbf{a}_y\times\mathbf{a}_z \\ & + A_zB_x\mathbf{a}_z\times\mathbf{a}_x + A_zB_y\mathbf{a}_z\times\mathbf{a}_y + \cancelto{0}{A_zB_z\mathbf{a}_z\times\mathbf{a}_z}\end{aligned}$$

Which can be reduced to the following:

$$\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y)\mathbf{a}_x + (A_z B_x - A_x B_z)\mathbf{a}_y + (A_x B_y - A_y B_x)\mathbf{a}_z$$

or written as a determinant in a more easily remembered form,

$$\boxed{\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}}$$

Thus, if $\mathbf{A} = 2\mathbf{a}_x - 3\mathbf{a}_y + \mathbf{a}_z$ and $\mathbf{B} = -4\mathbf{a}_x - 2\mathbf{a}_y + 5\mathbf{a}_z$,

we have

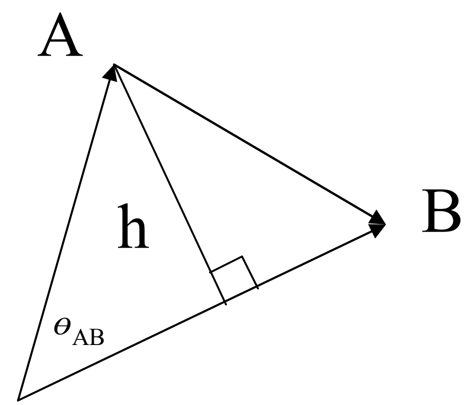
$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ 2 & -3 & 1 \\ -4 & -2 & 5 \end{vmatrix}$$

$$= [(-3)(5) - (1(-2)]\mathbf{a}_x - [(2)(5) - (1)(-4)]\mathbf{a}_y + [(2)(-2) - (-3)(-4)]\mathbf{a}_z$$

$$= -13\mathbf{a}_x - 14\mathbf{a}_y - 16\mathbf{a}_z$$

Example 2:

Use the result obtained in the previous example to find the area of the triangle formed by the two vectors **A** and **B**.



Solution

The Area of the triangle = 0.5 h |**B**|

$$= 0.5\ |\mathbf{A}|\ \sin\theta_{AB}\ |\mathbf{B}|$$

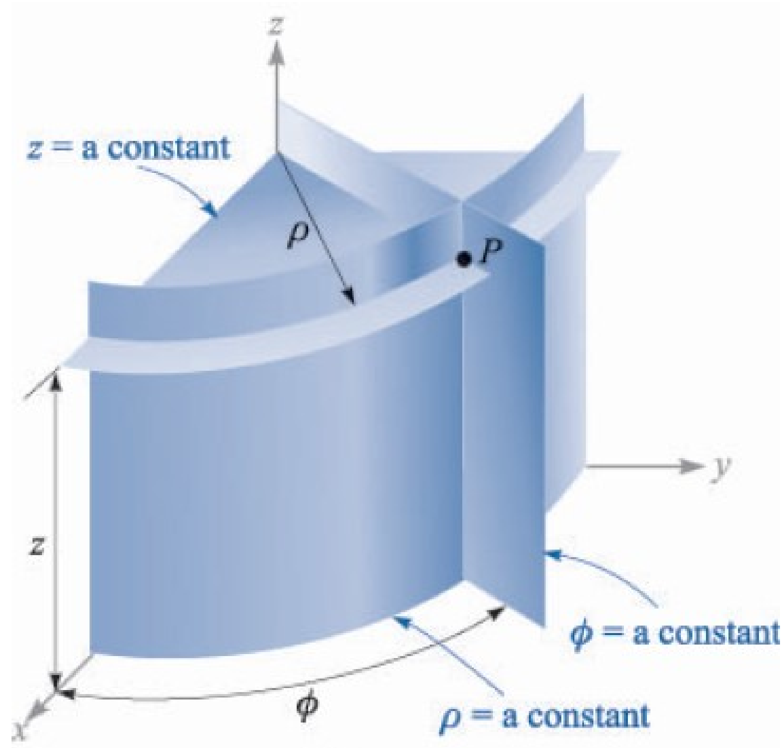
$$= 0.5\ |\mathbf{A}\times\mathbf{B}|$$

$$= 0.5\sqrt{13^2 + 14^2 + 16^2}$$

$$= 12.459 \text{ square units}$$

The Cylindrical Coordinate System

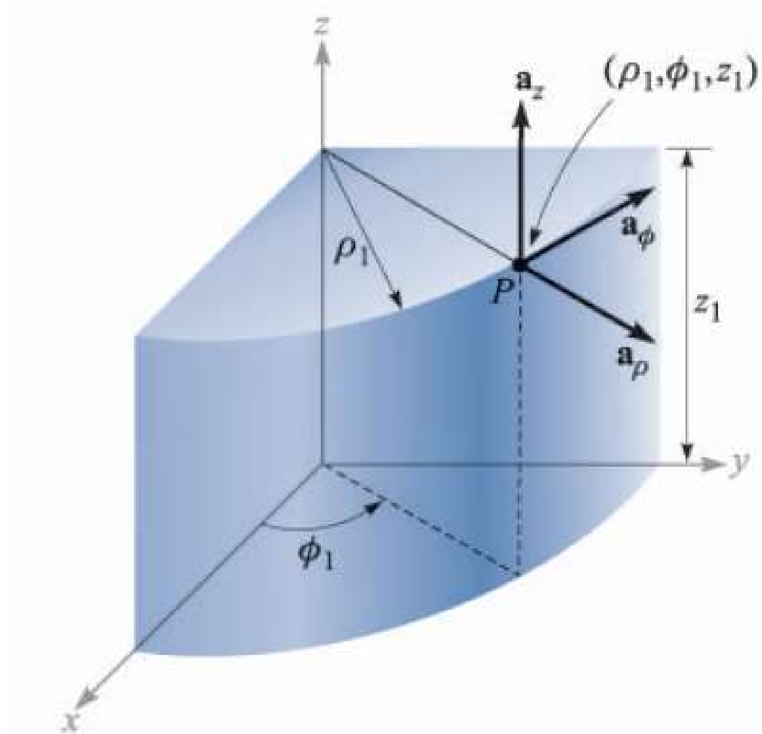
- Is a three-dimensional coordinate system, in which a point is specified by the intersection of three mutually perpendicular surfaces:
  - Constant ρ plane
  - Constant φ plane
  - Constant z plane



This corresponds to the location of a point in a Cartesian coordinate system by the intersection of three planes (x = constant, y = constant, and z = constant).

The Cylindrical Coordinate System

- Three unit vectors are utilized in the cylindrical coordinate system:
  - $\mathbf{a}_\rho$: unit vector in the direction of increasing ρ;
  - $\mathbf{a}_\varphi$: unit vector in the direction of increasing φ; and
  - $\mathbf{a}_z$: unit vector in the direction of increasing z.



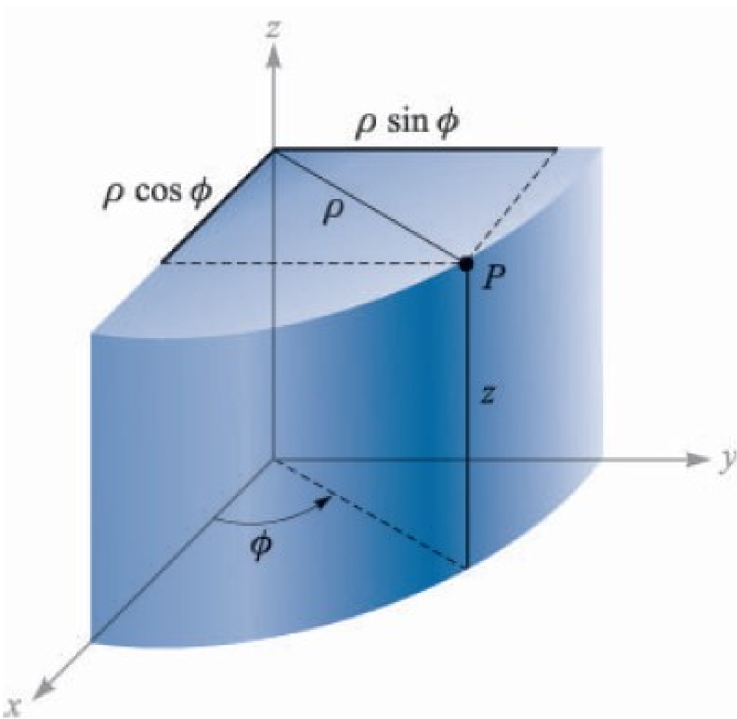
$(0 \le \rho < \infty)$

$(0 \le \phi < 2\pi)$

$(-\infty < z < \infty)$

Transformation between Cartesian and Cylindrical Coordinates

- 1- A point



From Cartesian to Cylindrical:

$$\rho = \sqrt{x^2 + y^2} \quad (\rho \geq 0)$$

$$\phi = \tan^{-1}\frac{y}{x}$$

$$z = z$$

From Cylindrical to Cartesian:

$$x = \rho\cos\phi$$

$$y = \rho\sin\phi$$

$$z = z$$

2- Vector transformation

Given a Cartesian vector $\mathbf{A} = A_x \mathbf{a}_x + A_y \mathbf{a}_y + A_z \mathbf{a}_z$

where vector components are represented in terms of x; y, and z, and we need a vector in Cylindrical coordinates

$$\mathbf{A} = A_\rho \mathbf{a}_\rho + A_\phi \mathbf{a}_\phi + A_z \mathbf{a}_z$$

where vector components are represented in terms of ρ, φ, and z.

Thus, we need to find the components of **A** in the directions $\mathbf{a}_\rho$, $\mathbf{a}_\varphi$, and $\mathbf{a}_z$

How can we achieve that?

Recall that:

$$A_\rho = \mathbf{A} \cdot \mathbf{a}_\rho \qquad \text{and} \qquad A_\phi = \mathbf{A} \cdot \mathbf{a}_\phi$$

Expanding these dot products, we have

$$A_\rho = (A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\underline{\mathbf{a}_z) \cdot \mathbf{a}_\rho} = A_x\mathbf{a}_x \cdot \mathbf{a}_\rho + A_y\mathbf{a}_y \cdot \mathbf{a}_\rho$$

$$A_\phi = (A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\underline{\mathbf{a}_z) \cdot \mathbf{a}_\phi} = A_x\mathbf{a}_x \cdot \mathbf{a}_\phi + A_y\mathbf{a}_y \cdot \mathbf{a}_\phi$$

Similarly

$$A_z = (A_x\mathbf{a}_{\underline{x}} + A_y\mathbf{a}_{\underline{y}} + A_z\mathbf{a}_z) \cdot \mathbf{a}_{\underline{z}} = A_z\mathbf{a}_z \cdot \mathbf{a}_z = A_z$$

Dot products of unit vectors in cylindrical and cartesian coordinate systems

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x \cdot$	$\cos\phi$	$-\sin\phi$	0
$\mathbf{a}_y \cdot$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z \cdot$	0	0	1

- Transforming vectors from Cartesian to Cylindrical coordinates or vice versa is therefore accomplished in two steps:
    - Converting variables (as in slide 14), and
    - Converting components using the dot products of the unit vectors (as in slide 16).
- The two steps may be taken in either order.

Example 3

Transform the vector $\mathbf{B} = y\mathbf{a}_x - x\mathbf{a}_y + z\mathbf{a}_z$ into cylindrical coordinates.

Solution. The new components are

$$B_\rho = \mathbf{B} \cdot \mathbf{a}_\rho = y(\mathbf{a}_x \cdot \mathbf{a}_\rho) - x(\mathbf{a}_y \cdot \mathbf{a}_\rho)$$
$$= y \cos\phi - x \sin\phi = \rho \sin\phi \cos\phi - \rho \cos\phi \sin\phi = 0$$
$$B_\phi = \mathbf{B} \cdot \mathbf{a}_\phi = y(\mathbf{a}_x \cdot \mathbf{a}_\phi) - x(\mathbf{a}_y \cdot \mathbf{a}_\phi)$$
$$= -y \sin\phi - x \cos\phi = -\rho \sin^2\phi - \rho \cos^2\phi = -\rho$$

$$B_z = \mathbf{B} \cdot \mathbf{a}_z = z\mathbf{a}_z$$

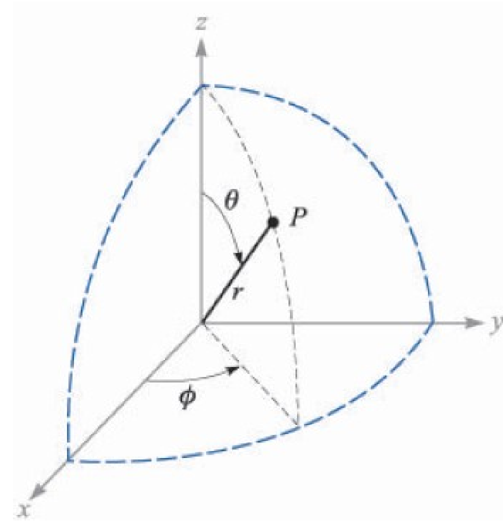
Thus,

$$\boxed{\mathbf{B} = -\rho\mathbf{a}_\phi + z\mathbf{a}_z}$$

The Spherical Coordinate System

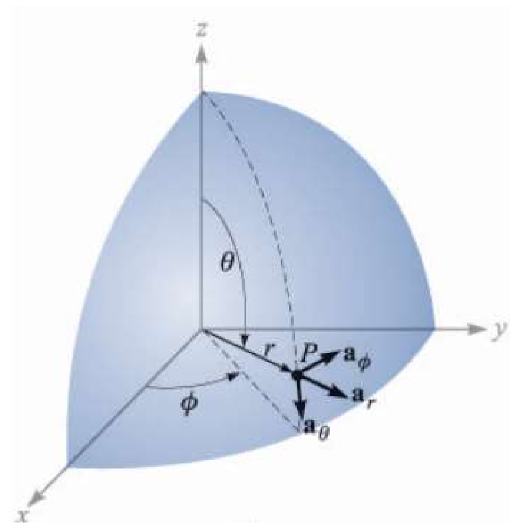
From Spherical to Cartesian:

$$x = r \sin\theta \cos\phi$$
$$y = r \sin\theta \sin\phi$$
$$z = r \cos\theta$$



From Cartesian to Spherical:

$$r = \sqrt{x^2 + y^2 + z^2} \qquad (r \geq 0)$$
$$\theta = \cos^{-1} \frac{z}{\sqrt{x^2 + y^2 + z^2}} \qquad (0^\circ \leq \theta \leq 180^\circ)$$
$$\phi = \tan^{-1} \frac{y}{x}$$



Dot products of unit vectors in spherical and cartesian coordinate systems

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x \cdot$	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y \cdot$	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z \cdot$	$\cos\theta$	$-\sin\theta$	0

Example 4:

transform the vector field $\mathbf{G} = (xz/y)\mathbf{a}_x$ into spherical components and variables.

Solution.

$$G_r = \mathbf{G} \cdot \mathbf{a}_r = \frac{xz}{y}\mathbf{a}_x \cdot \mathbf{a}_r = \frac{xz}{y}\sin\theta\cos\phi$$

$$= r\sin\theta\cos\theta\frac{\cos^2\phi}{\sin\phi}$$

$$G_\theta = \mathbf{G} \cdot \mathbf{a}_\theta = \frac{xz}{y}\mathbf{a}_x \cdot \mathbf{a}_\theta = \frac{xz}{y}\cos\theta\cos\phi$$

$$= r\cos^2\theta\frac{\cos^2\phi}{\sin\phi}$$

$$G\phi = \mathbf{G} \cdot \mathbf{a}_\phi = \frac{xz}{y}\mathbf{a}_x \cdot \mathbf{a}_\phi = \frac{xz}{y}(-\sin\phi)$$

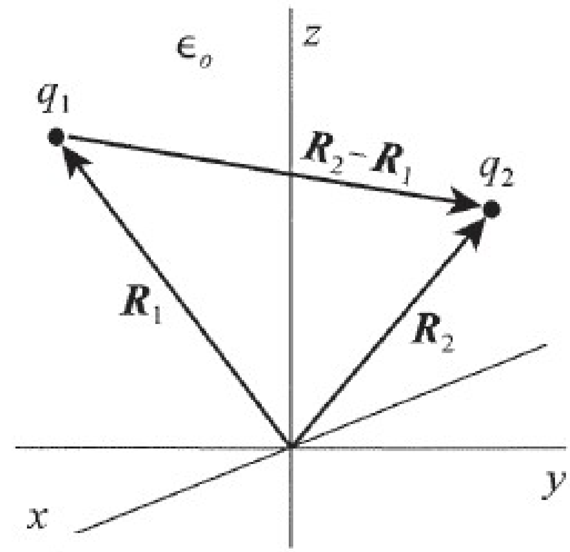
$$= -r\cos\theta\cos\phi$$

Collecting these results, we have

$$\mathbf{G} = r\cos\theta\cos\phi\,(\sin\theta\cot\phi\,\mathbf{a}_r + \cos\theta\cot\phi\,\mathbf{a}_\theta - \mathbf{a}_\phi)$$

Electrostatic Fields

- Electrostatic fields are time invariant electric fields produced by stationary charges.



Coulomb's Law

Given point charges [q_1, q_2 (units=C)] in air located by vectors $\boldsymbol{R}_1$ and $\boldsymbol{R}_2$, respectively, the vector force acting on charge q_2 due to q_1 [$\boldsymbol{F}_{12}$ (units=N)] is defined by Coulomb's law as

$$\boldsymbol{F}_{12} = \frac{q_1 q_2}{4\pi\epsilon_o |\boldsymbol{R}_2 - \boldsymbol{R}_1|^2} \hat{\boldsymbol{a}}_{12} \quad \text{(Coulomb's law)}$$

where $\hat{\boldsymbol{a}}_{12}$ is a unit vector pointing from q_1 to q_2 and ϵ_o is the free-space *permittivity* [$\epsilon_o = 8.854 \times 10^{-12}$ F/m].

Coulomb's Law (Alternative form)

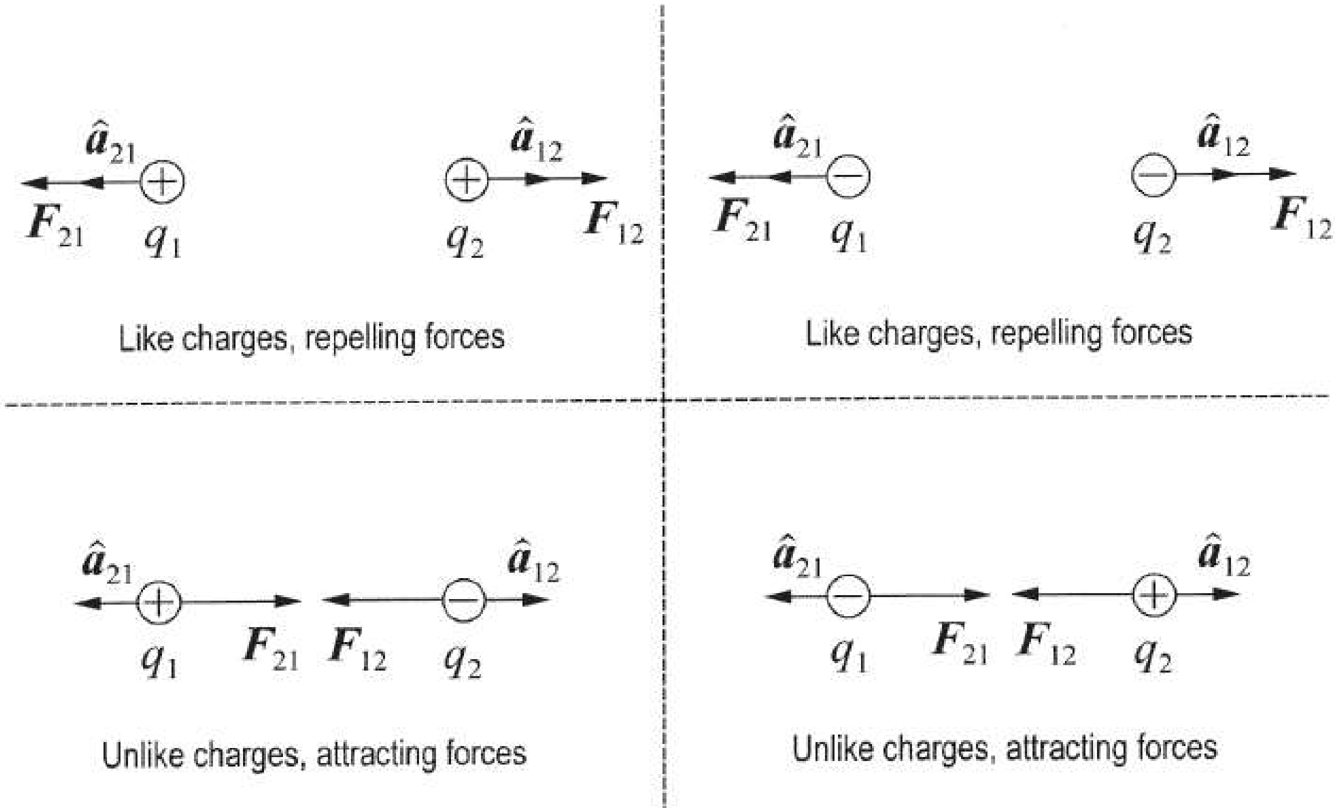
We have
$$\boxed{\boldsymbol{F}_{12} = \frac{q_1 q_2}{4\pi\epsilon_o |\boldsymbol{R}_2 - \boldsymbol{R}_1|^2}\hat{\boldsymbol{a}}_{12}}$$

And the unit vector
$$\hat{\boldsymbol{a}}_{12} = \frac{\boldsymbol{R}_2 - \boldsymbol{R}_1}{|\boldsymbol{R}_2 - \boldsymbol{R}_1|}$$

Therefore,

$$\boxed{\boldsymbol{F}_{12} = \frac{q_1 q_2(\boldsymbol{R}_2 - \boldsymbol{R}_1)}{4\pi\epsilon_o |\boldsymbol{R}_2 - \boldsymbol{R}_1|^3} \qquad \text{(Coulomb's law)}}$$

Illustration of forces.



Example 5

A point charge $Q_1 = 2nC$ is located in free space at $P_1(-3,7,-4)$, while $Q_2 = -5nC$ is at $P_2(2,4,-1)$. Find the force vector on (a) Q_2; (b) Q_1

$$\overline{F}_{12} = \frac{q_1 q_2}{4\pi\varepsilon_o R_{12}^2} \hat{a}_{R_{12}}$$

$$\hat{a}_{R_{12}} = \frac{\overline{R}_{12}}{|\overline{R}_{12}|} = \frac{(x_2 - x_1)\hat{a}_x + (y_2 - y_1)\hat{a}_y + (z_2 - z_1)\hat{a}_z}{|\overline{R}_{12}|}$$

$$= \frac{5\hat{a}_x - 3\hat{a}_y + 3\hat{a}_z}{\sqrt{25+9+9}}$$

$$\overline{F}_{12} = \frac{-10\times 10^{-18}}{4\pi\times 8.854\times 10^{-12}} \frac{5\hat{a}_x - 3\hat{a}_y + 3\hat{a}_z}{43\sqrt{43}}$$

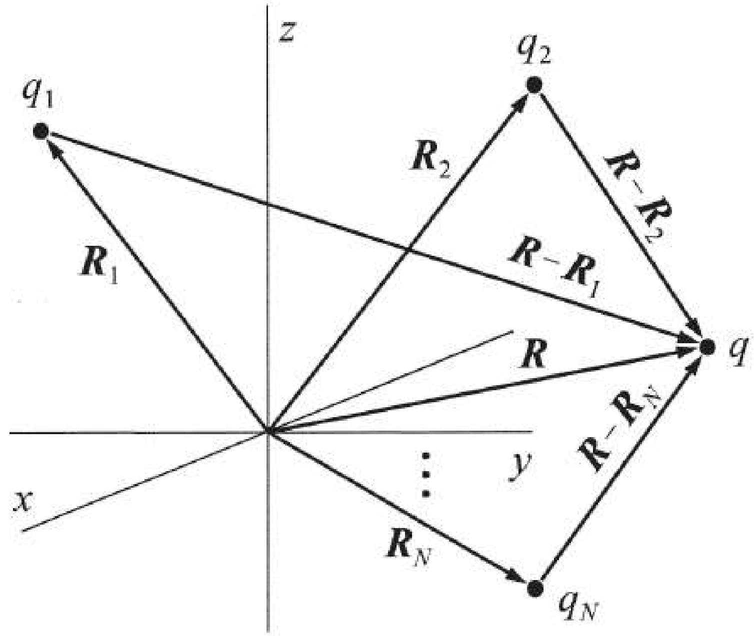
$$\overline{F}_{12} = -1.59\hat{a}_x + 0.957\hat{a}_y - 0.957\hat{a}_z \quad \text{nN}$$

(b) $\overline{F}_{21} = -\overline{F}_{12}$

Force due to multiple point charges (Superposition).

Given a point charge q in the vicinity of a set of N point charges (q_1, $q_2, \ldots, q_N$), the total vector force on q is the vector sum of the individual forces due to the N point charges.

$$\boxed{\boldsymbol{F} = \frac{q}{4\pi\epsilon_o} \sum_{k=1}^{N} q_k \frac{(\boldsymbol{R} - \boldsymbol{R}_k)}{|\boldsymbol{R} - \boldsymbol{R}_k|^3}}$$



$$\boldsymbol{F} = \frac{q q_1 (\boldsymbol{R} - \boldsymbol{R}_1)}{4\pi\epsilon_o |\boldsymbol{R} - \boldsymbol{R}_1|^3} + \frac{q q_2 (\boldsymbol{R} - \boldsymbol{R}_2)}{4\pi\epsilon_o |\boldsymbol{R} - \boldsymbol{R}_2|^3} + \ldots + \frac{q q_N (\boldsymbol{R} - \boldsymbol{R}_N)}{4\pi\epsilon_o |\boldsymbol{R} - \boldsymbol{R}_N|^3}$$

Electric Field Intensity

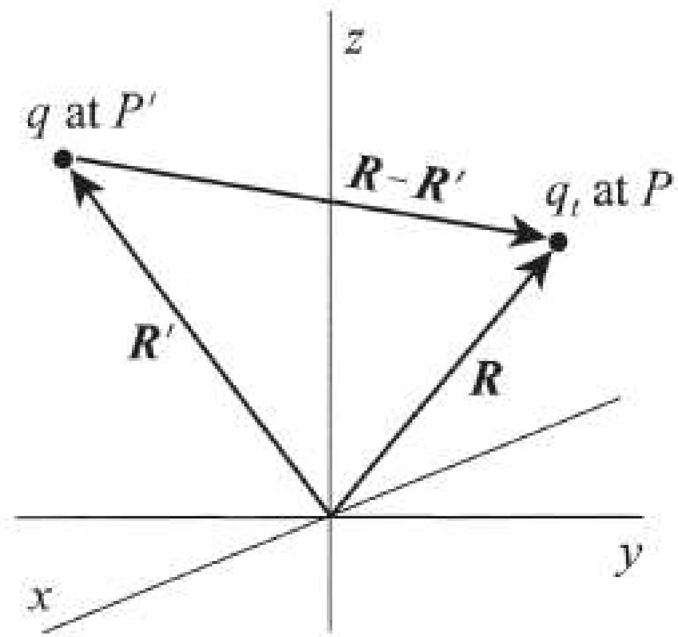
Using a positive test charge to measure the electric field, the electric field intensity is defined as the *vector force per unit charge* experienced by the test charge.

q - point charge producing the electric field

q_t - positive test charge used to measure the electric field of q

$\mathbf{R}'$ - locates the *source point* P' (location of source charge q)

$\mathbf{R}$ - locates the *field point* P (location of test charge q_t)



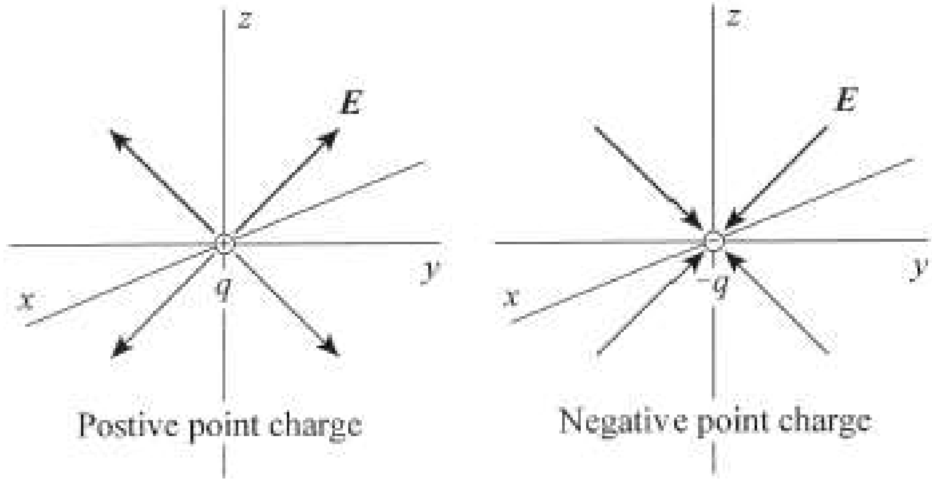
$$\mathbf{E} \equiv \frac{\mathbf{F}}{q_t} = \frac{q(\mathbf{R}-\mathbf{R}')}{4\pi\epsilon_o|\mathbf{R}-\mathbf{R}'|^3} \quad (\mathrm{N/C})$$

$$\frac{\text{N}}{\text{C}} = \frac{\text{J/m}}{\text{C}} = \frac{\text{J/C}}{\text{m}} = \frac{\text{V}}{\text{m}}$$

For the special case of a point charge at the origin ($\boldsymbol{R}' = \mathbf{0}$), the electric field reduces to the following spherical coordinate expression:

$$\boldsymbol{E} = \frac{q\boldsymbol{R}}{4\pi\epsilon_o|\boldsymbol{R}|^3} = \frac{q}{4\pi\epsilon_o|\boldsymbol{R}|^2}\frac{\boldsymbol{R}}{|\boldsymbol{R}|} = \frac{q}{4\pi\epsilon_o R^2}\hat{\boldsymbol{R}} \quad \text{(V/m)}$$

Radial unit vector



Note that the electric field points radially outward given a positive point charge at the origin and radially inward given a negative point charge at the origin. In either case, the electric field of the a point charge at the origin is spherically symmetric and the magnitude of the electric field varies as $\boldsymbol{R}^{-2}$.

Electric field due to multiple point charges

The electric field due to multiple point charges can be determined using the principle of superposition. The vector force on a test charge q_t at $\boldsymbol{R}$ due to a system of point charges $(q_1, q_2, \ldots, q_N)$ at $(\boldsymbol{R}_1', \boldsymbol{R}_2', \ldots, \boldsymbol{R}_N')$ is, by superposition,

$$\boldsymbol{F} = \frac{q_t}{4\pi\epsilon_o} \sum_{k=1}^{N} q_k \frac{(\boldsymbol{R} - \boldsymbol{R}_k')}{|\boldsymbol{R} - \boldsymbol{R}_k'|^3}$$

The resulting electric field is

$$\boxed{\boldsymbol{E} = \frac{\boldsymbol{F}}{q_t} = \frac{1}{4\pi\epsilon_o} \sum_{k=1}^{N} q_k \frac{(\boldsymbol{R} - \boldsymbol{R}_k')}{|\boldsymbol{R} - \boldsymbol{R}_k'|^3}}$$

Example 6

Determine the vector electric field at (1,−3,7) m due to point charges $q_1 = 5$ nC at (2,0,4) m and $q_2 = -2$ nC at (−3,0,5) m.

$$\boldsymbol{R} = \hat{x} - 3\hat{y} + 7\hat{z}$$

$$\boldsymbol{R}_1' = 2\hat{x} + 4\hat{z}$$

$$\boldsymbol{R}_2' = -3\hat{x} + 5\hat{z}$$

$$\boldsymbol{E} = \frac{1}{4\pi\epsilon_o}\left[q_1\frac{(\boldsymbol{R}-\boldsymbol{R}_1')}{|\boldsymbol{R}-\boldsymbol{R}_1'|^3} + q_2\frac{(\boldsymbol{R}-\boldsymbol{R}_2')}{|\boldsymbol{R}-\boldsymbol{R}_2'|^3}\right]$$

$$= \frac{1}{4\pi\epsilon_o}\left[(5\times10^{-9})\frac{(-\hat{x}-3\hat{y}+3\hat{z})}{[1^2+3^2+3^2]^{3/2}}\right.$$

$$\left.+(-2\times10^{-9})\frac{(4\hat{x}-3\hat{y}+2\hat{z})}{[4^2+3^2+2^2]^{3/2}}\right]$$

$$= 8.988\left[\frac{5}{(19)^{3/2}}(-\hat{x}-3\hat{y}+3\hat{z}) - \frac{2}{(29)^{3/2}}(4\hat{x}-3\hat{y}+2\hat{z})\right]$$

$$\boxed{\boldsymbol{E} = (-1.004\hat{x} - 1.284\hat{y} + 1.399\hat{z})\ \text{V/m}}$$

Announcements

- Homework 1 is assigned (Check Webcourses).

- Next lecture:
  - Electrostatic Field Intensity (continued)
  - Introduction to Matlab