EEL 3470: ELECTROMAGNETIC FIELDS

Solution of Homework 5

1. Problem 2.62

P2.62: For y < 0, $\varepsilon_{r1} = 4.0$ and $\mathbf{E}_1 = 3\mathbf{a}_x + 6\pi\mathbf{a}_y + 4\mathbf{a}_z$ V/m. At y = 0, $\rho_s = 0.25$ nC/m^2. If $\varepsilon_{r2} = 5.0$ for y > 0, find $\mathbf{E}_2$.

Solution

$\mathbf{E}_1 = 3\mathbf{a}_x + 6\pi\mathbf{a}_y + 4\mathbf{a}_z$ V/m	(g) $\mathbf{E}_2 = 3\mathbf{a}_x + 20.7\mathbf{a}_y + 4\mathbf{a}_z$ V/m
(a) $\mathbf{E}_{N1} = 6\pi\mathbf{a}_y$	(f) $\mathbf{E}_{N2} = \mathbf{D}_{N2}/5\varepsilon_o = 20.7\mathbf{a}_y$
(b) $\mathbf{E}_{T1} = 3\mathbf{a}_x + 4\mathbf{a}_z$	(c) $\mathbf{E}_{T2} = \mathbf{E}_{T1} = 3\mathbf{a}_x + 4\mathbf{a}_z$
(d) $\mathbf{D}_{N1} = \varepsilon_{r1}\varepsilon_o\mathbf{E}_{N1} = 24\pi\varepsilon_o\,\mathbf{a}_y$	(e) $\mathbf{D}_{N2} = 0.92\,\mathbf{a}_y$

(e) $\mathbf{a}_{21}\bullet(\mathbf{D}_1 - \mathbf{D}_2) = \rho_s, \quad -\mathbf{a}_y\bullet(D_{N1} - D_{N2})\mathbf{a}_y = \rho_s, \quad D_{N2} - D_{N1} = \rho_s$

$$D_{N2} = \rho_s + D_{N1} = 0.25\frac{nC}{m^2} + 24\left(\frac{10^{-9}F}{36\pi m}\right)\pi\frac{nC}{m^2} = 0.92\frac{nC}{m^2}$$

2. Problem 2.63

P2.63: For $z \le 0$, $\varepsilon_{r1} = 9.0$ and for $z > 0$, $\varepsilon_{r2} = 4.0$. If $\mathbf{E}_1$ makes a 30° angle with a normal to the surface, what angle does $\mathbf{E}_2$ make with a normal to the surface?

Solution

Refer to Figure P2.63.

$E_{T1} = E_1 \sin\theta_1, \quad E_{T2} = E_2 \sin\theta_2,$ and $E_{T1} = E_{T2}$

also

$D_{N1} = \varepsilon_{r1}\varepsilon_o E_1 \cos\theta_1, \quad D_{N2} = \varepsilon_{r2}\varepsilon_o E_2 \cos\theta_2,$ and $D_{N1} = D_{N2}$ (since $\rho_s = 0$)

Therefore

$\dfrac{E_{T1}}{D_{N1}} = \dfrac{E_{T2}}{D_{N2}}$, and after routine math we find $\theta_2 = \tan^{-1}\left(\dfrac{\varepsilon_{r2}}{\varepsilon_{r1}}\tan\theta_1\right)$

Using this formula we obtain for this problem $\theta_2 = 14°$.

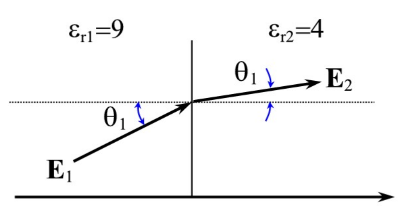

Fig. P2.63

3. Problem 2.73

P2.73: An inhomogeneous dielectric fills a parallel plate capacitor of surface area 50. cm^2 and thickness 1.0 cm. You are given $\varepsilon_r = 3(1 + z)$, where z is measured from the bottom plate in cm. Determine the capacitance.

Solution

Place $+Q$ at $z = d$ and $-Q$ at $z = 0$.

$$\rho_s = \frac{Q}{S},$$

$$\mathbf{D} = -\frac{Q}{S}\mathbf{a}_z,$$

$$\mathbf{E} = -\frac{Q}{\varepsilon_r \varepsilon_o S}\mathbf{a}_z$$

$$V_{do} = -\int_0^d \mathbf{E} \bullet d\mathbf{L} = -\int_0^d \frac{-Q}{\varepsilon_r \varepsilon_o S}\mathbf{a}_z \bullet dz\mathbf{a}_z = \frac{Q}{\varepsilon_o S}\int_0^d \frac{dz}{\varepsilon_r}$$

evaluating the integral:

$$\int_0^d \frac{dz}{\varepsilon_r} = \int_0^d \frac{dz}{3(1+z)} = \frac{1}{3}\ln(1+z)\Big|_0^1 = \frac{1}{3}\ln 2\ cm$$

$$C = \frac{Q}{V_{do}} = \frac{3\varepsilon_o S}{\ln 2} = \frac{3(8.854x10^{-12}\ F/m)(50cm^2)}{(\ln(2)cm)}\left(\frac{m}{100cm}\right)^2 = 19\,pF$$

4. Problem 3.5

$\mathbf{H}_o = \mathbf{H}_1 + \mathbf{H}_2$

Referring to the figure,

$\mathbf{R} = 3\mathbf{a}_y - 4\mathbf{a}_z$, $R = 5$,

$\mathbf{a}_R = 0.6\mathbf{a}_y - 0.8\mathbf{a}_z$

$\mathbf{a}_\phi = \mathbf{a}_x \times \mathbf{a}_R = 0.80\mathbf{a}_y + 0.60\mathbf{a}_z$

$$\mathbf{H}_1 = \frac{I_1}{2\pi\rho}\mathbf{a}_\phi$$

$$= \frac{2A}{2\pi}\frac{(0.80\mathbf{a}_y + 0.60\mathbf{a}_z)}{5m}$$

$$= 51\mathbf{a}_y + 38\mathbf{a}_z\ {}^{mA}\!/_{m}$$

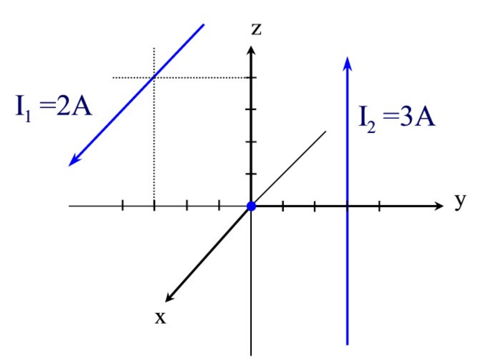

Fig. P3.5a

$$\mathbf{H}_2 = \frac{I_2}{2\pi\rho}\mathbf{a}_\phi$$

$$= \frac{3A}{2\pi}\frac{\mathbf{a}_x}{3m} = 159\mathbf{a}_x \, mA/m$$

$\mathbf{H}_o = 159\mathbf{a}_x + 51\mathbf{a}_y + 38\mathbf{a}_z$ mA/m

Or with 2 significant digits

$\mathbf{H}_o = 160\mathbf{a}_x + 51\mathbf{a}_y + 38\mathbf{a}_z$ mA/m

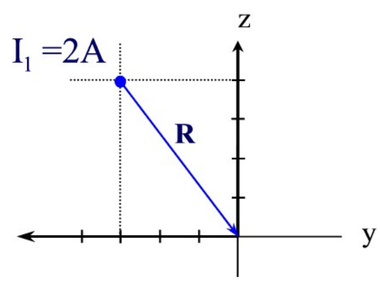

Fig. P3.5b

5. Problem 3.8

By inspection of the figure, we see that only the arc portions of the loop contribute to **H**.
From a ring example we have:

$$\mathbf{H} = \frac{Ia^2\mathbf{a}_z}{4\pi\left(h^2+a^2\right)^{3/2}}\int_0^{2\pi} d\phi$$

For h = 0 and $\rho = a$ segment of the loop:

$$\mathbf{H}_a = \frac{Ia^2\mathbf{a}_z}{4\pi a^3}\int_0^{\pi/2} d\phi = \frac{I}{8a}\mathbf{a}_z$$

For h =0 and ρ = b: $\mathbf{H}_b = \dfrac{Ib^2\mathbf{a}_z}{4\pi b^3}\displaystyle\int_{\pi/2}^{0} d\phi = \dfrac{-I}{8b}\mathbf{a}_z$;

So $\mathbf{H}_{TOT} = \dfrac{I}{8}\left(\dfrac{1}{a}-\dfrac{1}{b}\right)\mathbf{a}_z = \dfrac{1}{8}\left(\dfrac{1}{0.02}-\dfrac{1}{0.06}\right)\mathbf{a}_z = 4.2\mathbf{a}_z \dfrac{A}{m}$

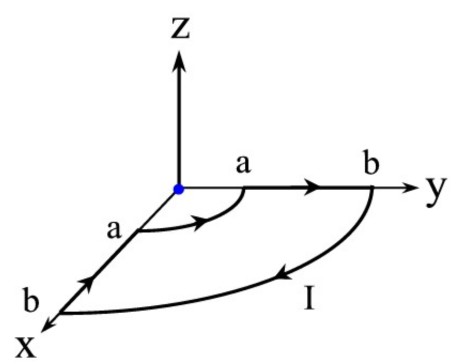

Fig. P3.8

6. Problem 3.11

$$\mathbf{H} = \frac{NI}{2h}\left[\frac{h-z}{\sqrt{(h-z)^2 + a^2}} + \frac{z}{\sqrt{z^2 + a^2}}\right]\mathbf{a}_z$$

$$\mathbf{H} = \frac{200(1A)}{2(0.1m)}\left[\frac{0.1-0.05}{\sqrt{(0.1-0.05)^2 + 0.01^2}} + \frac{0.05}{\sqrt{0.05^2 + 0.01^2}}\right]\mathbf{a}_z = 1961\frac{A}{m}\mathbf{a}_z$$

Or $\mathbf{H}$ = 1960 A/m $\mathbf{a}_z$

The approximate solution, assuming 10cm >> 1cm, is

$$\mathbf{H} = \frac{NI}{h}\mathbf{a}_z = \frac{200(1A)}{0.1m}\mathbf{a}_z = 2000\frac{A}{m}\mathbf{a}_z$$