EEL 3470: ELECTROMAGNETIC FIELDS

Solution of Homework 1

Solve the following problems (problems 1 to 5 are in your textbook)

1. **Problem 2.9 (a and b)**

(a) $r=\sqrt{6^2+2^2+6^2}=8.7, \theta=\cos^{-1}\left(\frac{6}{8.7}\right)=47^o, \phi=\tan^{-1}\left(\frac{2}{6}\right)=18^o$

(b) $r=\sqrt{0^2+4^2+3^2}=5, \theta=\cos^{-1}\left(\frac{3}{5}\right)=53^o, \phi=\tan^{-1}\left(\frac{-4}{0}\right)=-90^o$

2. **Problem 2.10 (a and b)**

(a)

$x=r\sin\theta\cos\phi=3\sin 30^o\cos 45^o=1.06$

$y=r\sin\theta\sin\phi=3\sin 30^o\sin 45^o=1.06$

$z=r\cos\theta=3\cos 30^o=2.6$

so $P(1.1,1.1,2.6)$.

(b)

$x=r\sin\theta\cos\phi=5\sin 45^o\cos 270^o=0$

$y=r\sin\theta\sin\phi=5\sin 45^o\sin 270^o=-3.5$

$z=r\cos\theta=5\cos 45^o=3.5$

so $P(0,-3.5,3.5)$.

3. Problem 2.11

(a)

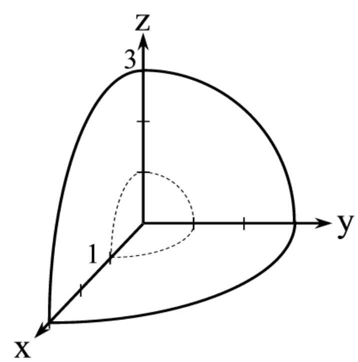

Fig. P2.11

(b)

$$V=\iiint r^2\sin\theta dr d\theta d\phi=\int_1^3 r^2 dr\int_0^{90^o}\sin\theta d\theta\int_0^{\pi/2}d\phi=\frac{13\pi}{3}=13.6m^3.$$

So volume V = 14 m^3.

(c) There are 5 surfaces: an inner, an outer, and 3 identical sides.

$$S_{side}=\iint r dr d\phi=\int_1^3 r dr\int_0^{\pi/2}d\phi=2\pi m^2;\quad S_{sides}=6\pi m^2$$

$$S_{outer}=\iint r^2\sin\theta d\theta d\phi=3^2\int_0^{90^o}\sin\theta d\theta\int_0^{\pi/2}d\phi=\frac{9\pi}{2}m^2$$

$$S_{inner}=\frac{\pi}{2}m^2; S_{TOT}=11\pi m^2=34.6m^2$$

So S_{total} = 35 m^2.

4. Problem 2.12 (a and b)

(a) $\rho=\sqrt{0^2+4^2}=4,\phi=\tan^{-1}\left(\frac{4}{0}\right)=90^o, z=3,\ so\ P(4.0,90^o,3.0)$

(b) $\rho=\sqrt{2^2+3^2}=3.6,\phi=\tan^{-1}\left(\frac{3}{-2}\right)=124^o, z=2,\ so\ P(3.6,120^o,2.0)$

5. Problem 2.13 (a and b)

(a)

$$x = \rho\cos\phi = 2.83\cos 45^o = 2.00$$
$$y = \rho\sin\phi = 2.83\sin 45^o = 2.00$$
$$z = z = 2.00$$
$$so\ P(2.00, 2.00, 2.00).$$

(b)

$$x = \rho\cos\phi = 6.00\cos 120^o = -3.00$$
$$y = \rho\sin\phi = 6.00\sin 120^o = 5.20$$
$$z = z = -3.00$$
$$so\ P(-3.00, 5.20, -3.00).$$

6. Given the field $\mathbf{D} = (x^2+y^2)^{-1}(x\,\mathbf{a}_x + y\,\mathbf{a}_y)$, find its equivalent field in the cylindrical coordinate system.

$$\mathbf{D} = (x^2+y^2)^{-1}(x\mathbf{a}_x + y\mathbf{a}_y)$$

$$x = \rho\cos\phi,\ y = \rho\sin\phi,\ \text{and } x^2+y^2 = \rho^2.$$

Therefore $\mathbf{D} = \frac{1}{\rho}(\cos\phi\mathbf{a}_x + \sin\phi\mathbf{a}_y)$

Then

$$D_\rho = \mathbf{D}\cdot\mathbf{a}_\rho = \frac{1}{\rho}[\cos\phi(\mathbf{a}_x\cdot\mathbf{a}_\rho) + \sin\phi(\mathbf{a}_y\cdot\mathbf{a}_\rho)] = \frac{1}{\rho}\left[\cos^2\phi + \sin^2\phi\right] = \frac{1}{\rho}$$

and

$$D_\phi = \mathbf{D}\cdot\mathbf{a}_\phi = \frac{1}{\rho}[\cos\phi(\mathbf{a}_x\cdot\mathbf{a}_\phi) + \sin\phi(\mathbf{a}_y\cdot\mathbf{a}_\phi)] = \frac{1}{\rho}[\cos\phi(-\sin\phi) + \sin\phi\cos\phi] = 0$$

Therefore

$$\mathbf{D} = \frac{1}{\rho}\mathbf{a}_\rho$$