MAA-NCS Fall 2018 Meeting

Southwest Minnesota State University

# Leveraging Applications and Cooperative Learning in Mathematics Pedagogy

## Enhancing Student Motivation and Conceptual Understanding

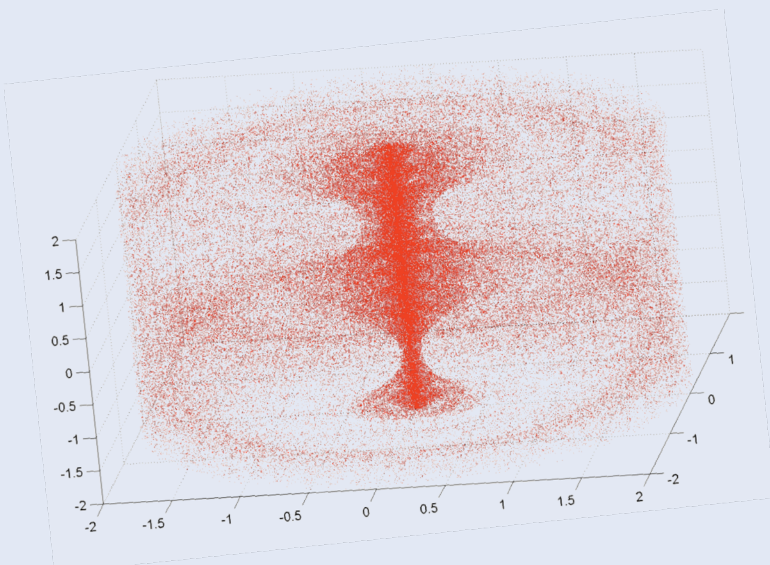
*Dr. Barry A. Peratt*

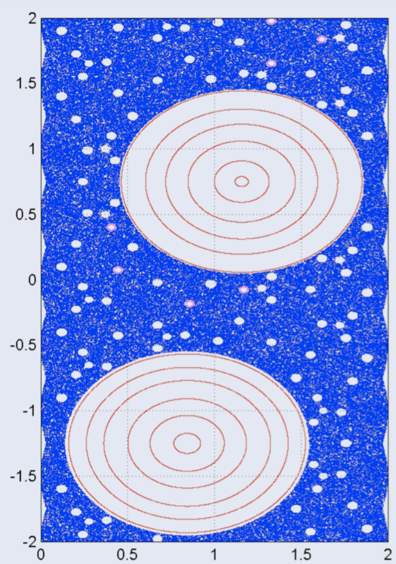
*Winona State University*

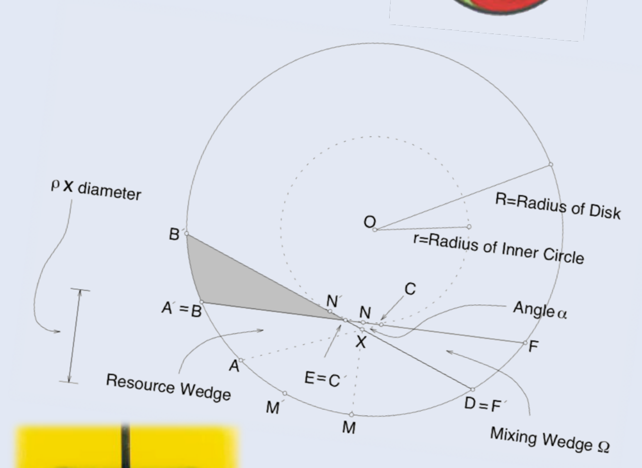
# 1. **Structuring the Talk:** My Research

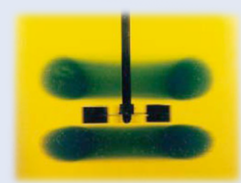
WINONA STATE UNIVERSITY

- Ph.D. in ***Chaos Theory*** (Topology and Complex Analysis)
- Research in ***Granular Mixing*** and ***Stirring of Fluids in a Cylindrical Tank***
  - Structures that Arise in these Processes Under Minimal Assumptions
  - James Yorke and Judy Kennedy
  - Engineers at Rutgers (Troy Shinbrot, Justin Lacombe, etc.)







# 1. **Structuring the Talk:** Emergent Systems

- ***Canonical Example:*** Ant Colony
  - At Individual Level – Mind-Numbingly Stupid
  - Put 100,000 Together – Intelligence & Sophistication Emerge
  - The intelligence belongs to no one individual but rather to the entire colony, as it emerges from the relatively simple interactions among them.
- ***Parker Palmer:*** NASA Lost on Moon Activity
- ***Cooperative Learning:*** Structuring Positive Interdependence.

# 1. **Structuring the Talk:** the Main Catalysts

- ***Dr. Doris Schattschneider:*** Geometer (Moravian College, Bethlehem, PA)
- ***Dr. Robert Mayer:*** Historian and Educator (Moravian College, Bethlehem, PA)
- ***Mrs. Nancy Eckert:*** High School Teacher (Freedom High School, Bethlehem, PA)

THE GEOMETER'S SKETCHPAD®

# 2. Context for Learning

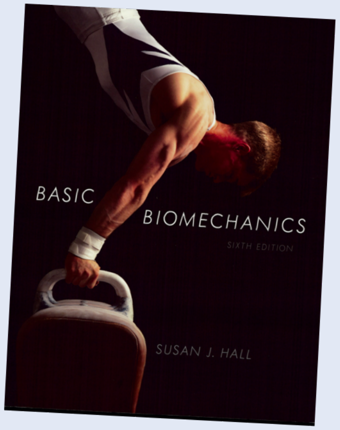
- ***What it Is Not:*** it is not discovery-based learning.
- ***Nancy Eckert Taught Me:*** excellent lecture is vital to
  - provide students with basic knowledge and
  - a vision of what is possible.
- ***Serious Motivation:***
  - Worth 25%-35% of their grade
  - Attendance Required
  - Everyone Responsible for Own Rendering
  - Problems are Too Difficult for Most People in Class to Accomplish Alone and Do Well

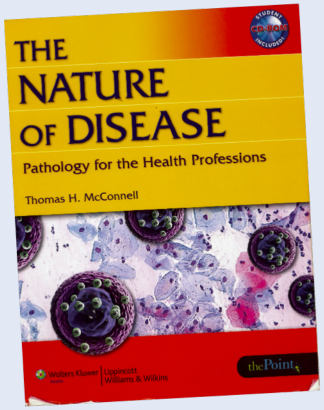
# 2. Context for Learning

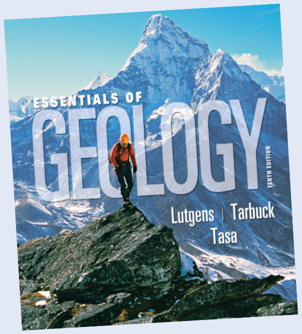
- ***Keep it Light:*** there is room for joking, high fives, etc.
- ***Structuring and Fostering Positive Interdependence:*** look for ways to increase cross-pollination among both students and groups.
- ***Time for Extremely Fruitful Formative Assessment:*** Almost all of my commentary for letters of recommendation come from this time.

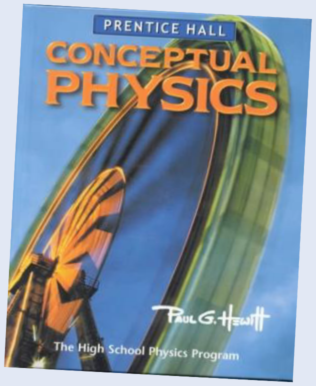
# 3. Precalculus with Modeling

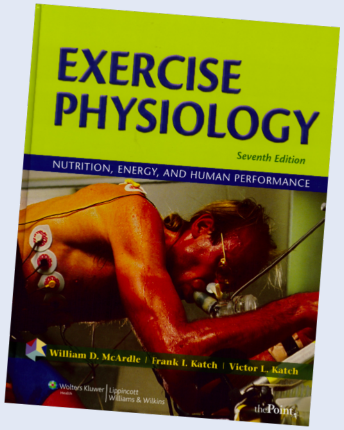
- ***Created from Scratch:*** for Movement Science majors.
- ***Populated Also By:*** Biology and Geoscience majors.
- ***The Process:***
    - Collected Major Level Texts
    - Created a List of Skills and Projects
    - Chose a Text
    - Limited Syllabus to relevant mathematical knowledge and skills, with an emphasis on using mathematics to model phenomena.

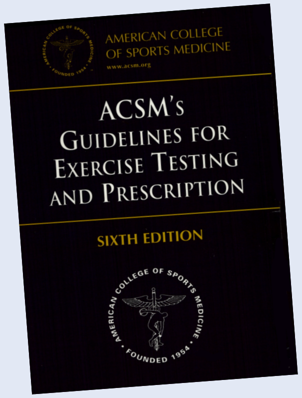


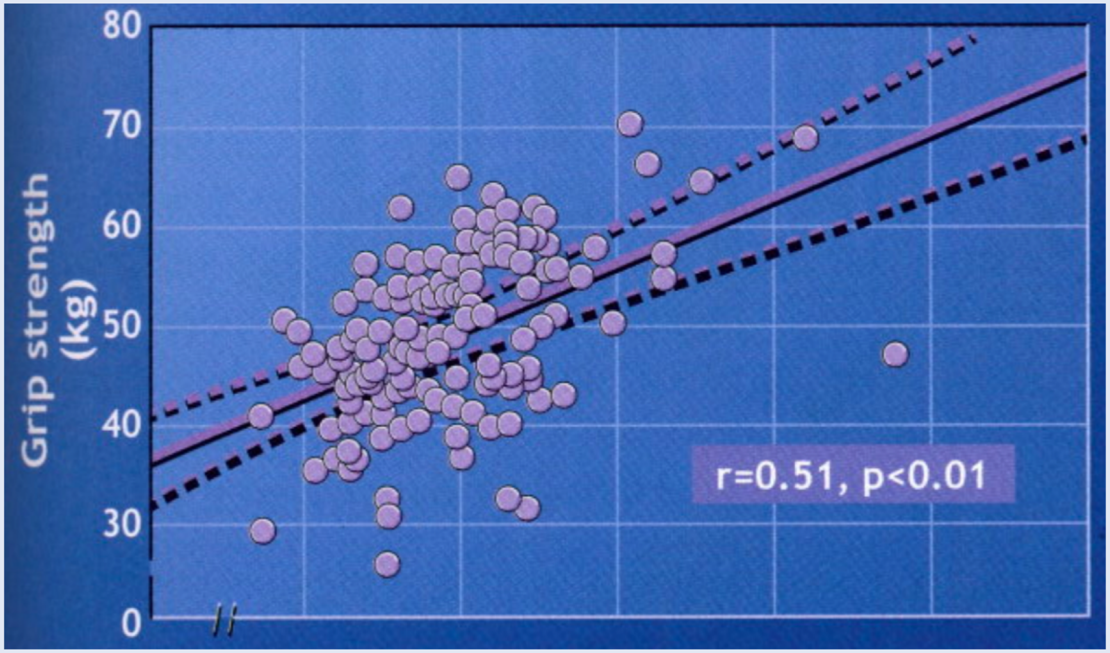



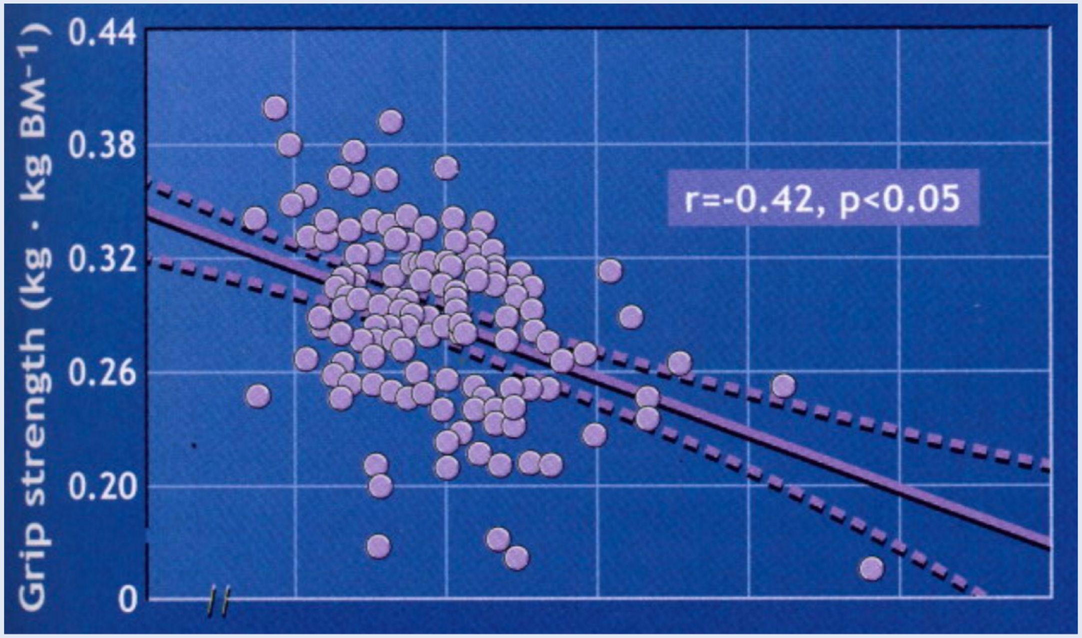
# 3. Precalculus with Modeling: Grip Strength

- ***Grip Strength vs. Body Mass:*** what does this trend tell us?



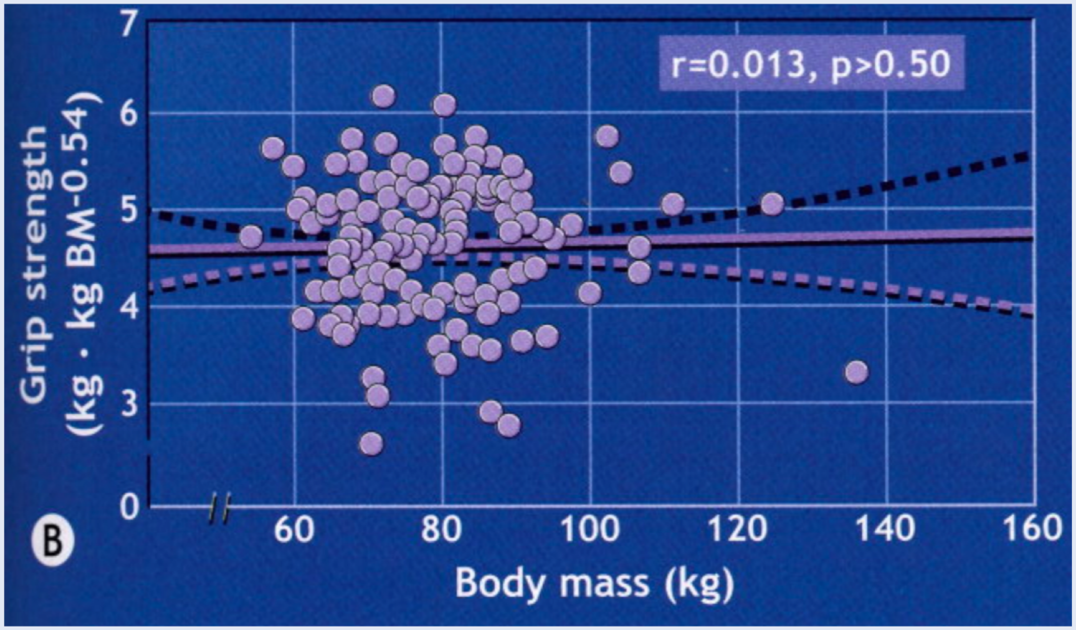
# 3. Precalculus with Modeling: Grip Strength

- ***Grip Strength per Unit Body Mass*** $\frac{GS}{BM}$**:**
  - What does this trend tell us?
  - What does it mean in every day language?



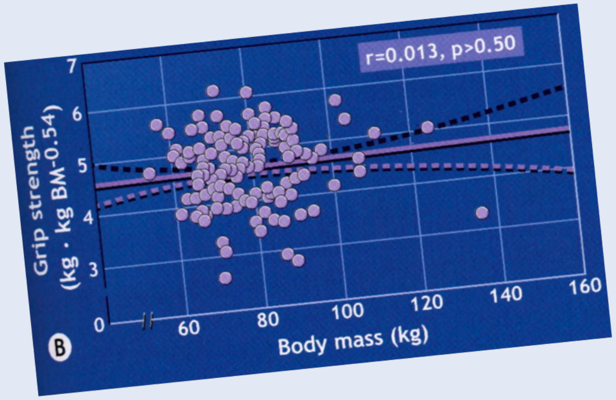
# 3. Precalculus with Modeling: Grip Strength

- ***Grip Strength per Unit Body Mass*** $\frac{GS}{BM^{0.54}}$:
  - Why would they be obsessed with tweaking the power on ***BM*** to get a horizontal line?



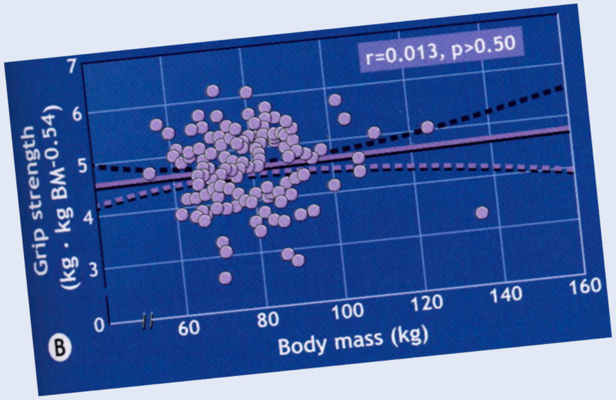
# 3. Precalculus with Modeling: Grip Strength

- ***Grip Strength per Unit Body Mass*** $\dfrac{GS}{BM^{0.54}}$:
  - Why would they be obsessed with tweaking the power on ***BM*** to get a horizontal line?
  - If $\dfrac{GS}{BM^{0.54}} = k$, then $GS = k \cdot BM^{0.54}$. What might change the value of $k$?
  - List of Factors Affecting the Grip Strength:



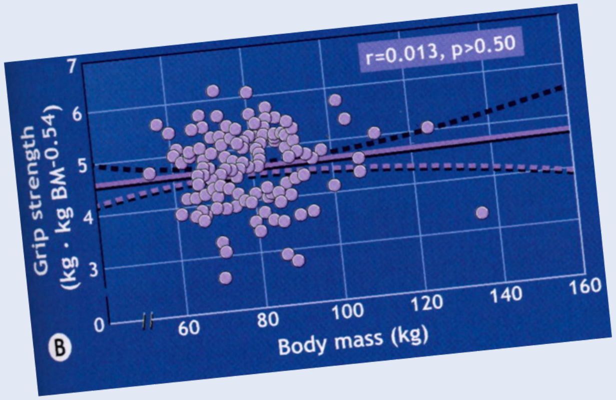
# 3. Precalculus with Modeling: Grip Strength

- ***Grip Strength per Unit Body Mass*** $\frac{GS}{BM^{0.54}}$:
  - Why would they be obsessed with tweaking the power on ***BM*** to get a horizontal line?
  - If $\frac{GS}{BM^{0.54}} = k$, then $GS = k \cdot BM^{0.54}$. What might change the value of $k$?
  - <u>List of Factors Affecting the Grip Strength</u>: Body Mass, Age, Physical Condition, Sex, Species, Health,…



# 3. Precalculus with Modeling: Grip Strength

- ***Grip Strength per Unit Body Mass*** $\frac{GS}{BM^{0.54}}$:
  - Why would they be obsessed with tweaking the power on ***BM*** to get a horizontal line?
  - If $\frac{GS}{BM^{0.54}} = k$, then $GS = k \cdot BM^{0.54}$. What might change the value of $k$?
  - <u>List of Factors Affecting the the Value of $k$</u>: ~~Body Mass~~, Age, Physical Condition, Sex, Species, Health,…
  - When might this model cease to be valid?

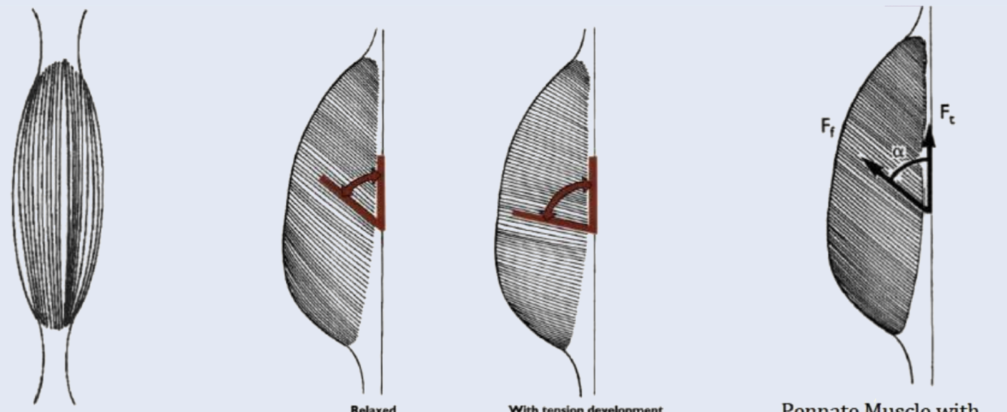


# **3. Precalculus with Modeling:** Muscle Forms

1. **Muscles and Joints:**

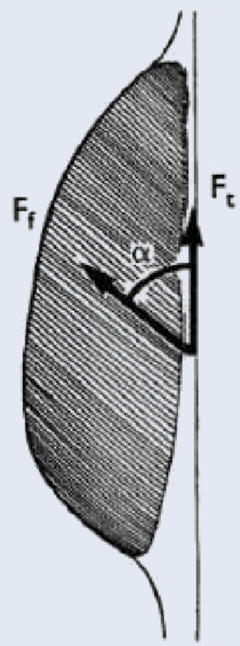
   (a) Muscles come in several forms, depending on which is best suited to their purpose. In the figure below, the muscle on the left is fusiform (longitudinal), because each of the fibers contracts in a direction parallel to the motion. The others are pennate muscles, because the fibers contract at an angle to the motion (called the "pennation angle"). One form of muscle provides a larger range of motion and one provides more power. Explain which is which.

   (b) Suppose that the pennate muscle shown in the middle below exerts a force of $F_f = 100N$ at an angle $a = 40°$. Determine the resulting force $F_t$ that contributes to the motion of the tendon to which the muscle is attached.

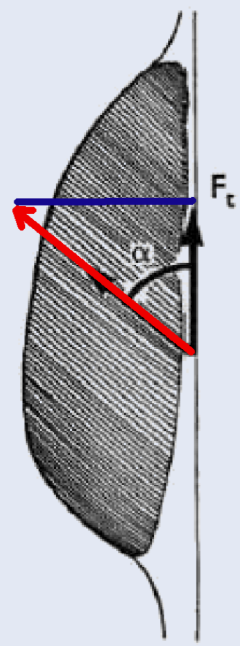



Fusiform (Longitudinal) Pennate Pennate Muscle with Pennation Angle Labeled

# 3. Precalculus with Modeling: Muscle Forms




Pennate Muscle with
Pennation Angle Labeled



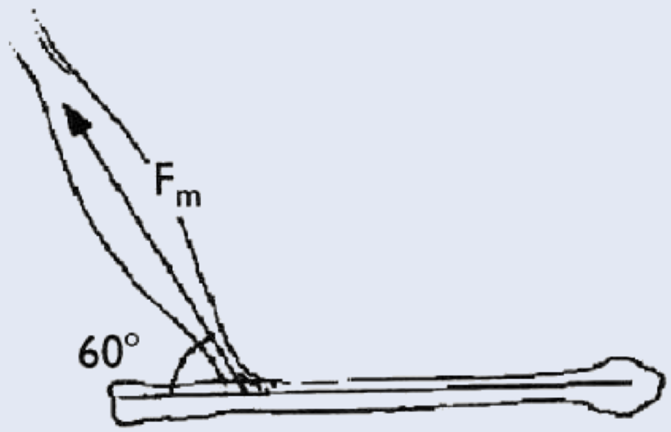

Pennate Muscle with
Pennation Angle Labeled

# 3. **Precalculus with Modeling:** Preserving Torque

WINONA STATE UNIVERSITY

(c) Also in the figure below is a diagram of a joint. Suppose that the force of the muscle fibers is $F_m = 400N$, and the distance from the elbow to the point of attachment is $0.03m$ ($3cm$). Torque about a point is defined as $T = F \times d$, where $F$ is the force applied *in the direction of the motion* and $d$ is the distance from the point at which the force is applied to the pivot point. How much torque will be put on the elbow joint by the muscle?

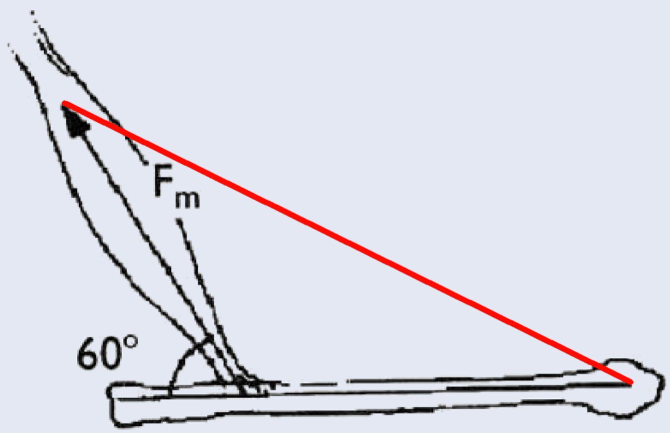
(d) In a situation like this, torque is preserved. If the distance from the elbow joint to the palm of the hand is $34cm$, and a person wants to curl $267N$ (60 lbs), determine the amount of force, $F_m$, that must be applied by the muscles and the torque that this will put on the elbow.



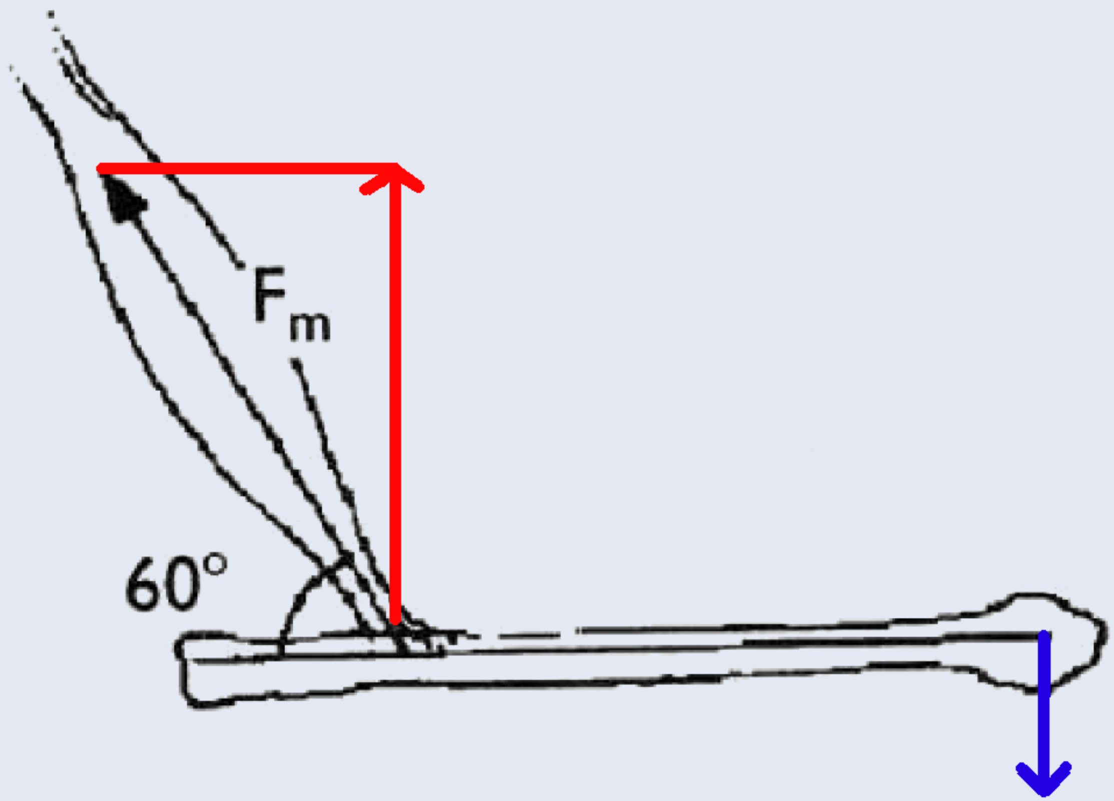
# 3. **Precalculus with Modeling:** Preserving Torque

(c) Also in the figure below is a diagram of a joint. Suppose that the force of the muscle fibers is $F_m = 400N$, and the distance from the elbow to the point of attachment is $0.03m$ ($3cm$). Torque about a point is defined as $T = F \times d$, where $F$ is the force applied *in the direction of the motion* and $d$ is the distance from the point at which the force is applied to the pivot point. How much torque will be put on the elbow joint by the muscle?

(d) In a situation like this, torque is preserved. If the distance from the elbow joint to the palm of the hand is $34cm$, and a person wants to curl $267N$ (60 lbs), determine the amount of force, $F_m$, that must be applied by the muscles and the torque that this will put on the elbow.



# 3. Precalculus with Modeling: Preserving Torque



# 4. **Calculus II:** the Knights Who Say, "Ni!"

# 4. **Calculus II:** the "It" Assignment

WINONA STATE UNIVERSITY

4. **Rewrite the following passage**, replacing each occurrence of the word "it" with the noun that "it" actually refers to. For ease of reading, box in your replacements for each occurrence of the word "it." Do not simply be lazy and write down the words that fill in the blanks, as this is a nightmare to grade (and therefore I won't grade it!).

The passage examines a power series, and in doing so, we can ask two questions. First, given a particular value of $x$, does the series converge for that value of $x$ (i.e. is that particular value of $x$ in the domain of the power series)? Secondly, is there a simple way to find out, using one computation, which values of $x$ are in the domain of the power series and which are not? The passage speaks about the series $\sum_{n=1}^{\infty} \frac{1}{n^2}x^n = x + \frac{1}{4}x^2 + \frac{1}{9}x^3 + \cdots$:

Suppose we want to know whether $x = 1$ is in the domain. Then, the relevant question becomes: Does $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \ldots$ converge? We know that if $\boxed{\text{it}}$ doesn't go to zero, $\boxed{\text{it}}$ doesn't converge. On the other hand, if $\boxed{\text{it}}$ converges, $\boxed{\text{it}}$ must go to zero. Even if $\boxed{\text{it}}$ does go to zero, $\boxed{\text{it}}$ might diverge if $\boxed{\text{it}}$ doesn't go to zero fast enough. In particular, $\boxed{\text{it}}$ must go to zero faster than $\boxed{\text{it}}$ does.

# 4. Calculus II: the "It" Assignment

WINONA STATE UNIVERSITY

Suppose we want to know whether $x = 1$ is in the domain. Then, the relevant question becomes: Does $\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{4} + \frac{1}{9} + \ldots$ converge? We know that if $\boxed{\text{it}}$ doesn't go to zero, $\boxed{\text{it}}$ doesn't converge. On the other hand, if $\boxed{\text{it}}$ converges, $\boxed{\text{it}}$ must go to zero. Even if $\boxed{\text{it}}$ does go to zero, $\boxed{\text{it}}$ might diverge if $\boxed{\text{it}}$ doesn't go to zero fast enough. In particular, $\boxed{\text{it}}$ must go to zero faster than $\boxed{\text{it}}$ does.

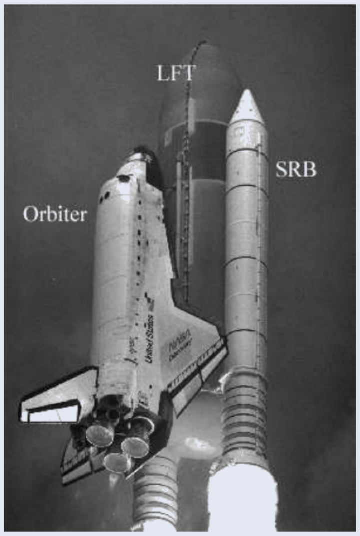
Then, there is the ratio test. In the ratio test, we compute $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$. If $\boxed{\text{it}}$ goes to some value that is less than one, then $\boxed{\text{it}}$ converges, whereas if $\boxed{\text{it}}$ goes to some value greater than one, then $\boxed{\text{it}}$ diverges. In this particular case, $\boxed{\text{it}}$ goes to one, which means the $\boxed{\text{it}}$ fails.

Now, what if we want to determine all of the values of $x$ that make $\boxed{\text{it}}$ converge? If we apply the ratio test to the power series, we find that $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = |x|$. So, $\boxed{\text{it}}$ converges only if $|x| < 1$. This means that, for these values of $x$, $\boxed{\text{it}}$ goes to zero fast enough to make $\boxed{\text{it}}$ converge. If $\boxed{\text{it}}$ is greater than 1, we know that $\boxed{\text{it}}$ diverges. But if $\boxed{\text{it}}$ equals 1, we know nothing about $\boxed{\text{it}}$. In all cases, we don't know what $\boxed{\text{it}}$ converges to, only whether or not $\boxed{\text{it}}$ converges or diverges.

# 4. Calculus II: Space Shuttle

6. **Space Shuttle:** The Space Shuttle has two means of thrust, the two Solid Rocket Boosters (SRBs), and the shuttle engines which burn the liquid fuel in the Liquid Fuel Tank[1] (LFT). See the figure below. In total, the shuttle system has a mass of 2,000,000 kg when it leaves the ground. For the first two minutes the liquid fuel and SRBs burn, until the shuttle reaches a height of 45 km. At this point, the 498,951 kg of solid fuel in each of the SRBs, which have a total loaded mass of 589,670 kg each, is exhausted and the SRBs are ejected. For the next 8 minutes, the shuttle continues to burn the fuel in the LFT. At 112 km, the LFT is empty and ejected, leaving the 84,822 kg orbiter alone in orbit.

   Compute the amount of work that is done by gravity in lifting the Space Shuttle to the height at which it ejects its LFT. Obviously, this value will be huge and no one (including me) will have much intuition about what that much work "means." Therefore, after you have computed the amount of work done, convert it into some unit that will give us some better intuition of how much work is involved (be creative!). It may help for you to note that work is a unit of energy and that energy is also measured in horsepower, kilowatt-hours, etc.[2]



# 4. Calculus II: Space Shuttle

WINONA STATE UNIVERSITY

***Questions that Arise:***

- Why is the a Calculus problem?
- At what rate do the liquid and solid fuel burn?
- Is the burn rate constant (per unit time or per unit height)?
- Do we need to account for the change in the force of gravity?
- How do we account for the solid rocket boosters being ejected?
- What is the mass of the Liquid Fuel Tank?

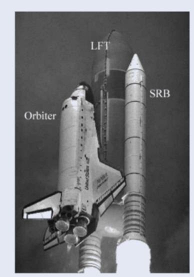
$$\int_a^b M(h)dh$$

or

$$\int_a^b M(h(t)) \cdot \frac{dh}{dt} \cdot dt$$

6. **Space Shuttle:** The Space Shuttle has two means of thrust, the two Solid Rocket Boosters (SRBs), and the shuttle engines which burn the liquid fuel in the Liquid Fuel Tank[1] (LFT). See the figure below. In total, the shuttle system has a mass of 2,000,000 kg when it leaves the ground. For the first two minutes the liquid fuel and SRBs burn, until the shuttle reaches a height of 45 km. At this point, the 498,951 kg of solid fuel in each of the SRBs, which have a total loaded mass of 589,670 kg each, is exhausted and the SRBs are ejected. For the next 8 minutes, the shuttle continues to burn the fuel in the LFT. At 112 km, the LFT is empty and ejected, leaving the 84,822 kg orbiter alone in orbit.

   Compute the amount of work that is done by gravity in lifting the Space Shuttle to the height at which it ejects its LFT. Obviously, this value will be huge and no one (including me) will have much intuition about what that much work "means." Therefore, after you have computed the amount of work done, convert it into some unit that will give us some better intuition of how much work is involved (be creative!). It may help for you to note that work is a unit of energy and that energy is also measured in horsepower, kilowatt-hours, etc.[2]



# **4. Modern Geometry:** Polyhedra Exploration

**Directions:** Using your polygonal faces and the rubber bands provided, complete the following activities. The purpose of this assignment is twofold:

1. To give you hands-on experience in investigating the fascinating world of polyhedra.
2. To give you first-hand experience in the application of van Hiele Levels of geometric understanding.

1. **Investigating Corners:** We will first investigate the types of possible corners that a polyhedron can have. Let's restrict our attention to polyhedra composed entirely of one kind of regular polygon.

   (a) If a polyhedron has all regular triangular faces, how many different types of corners could it have? Build each type and record its vertex configuration and the defect at each vertex in the table below.

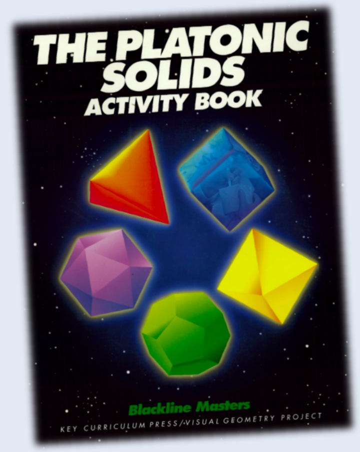
   (b) If a polyhedron has all square faces, how many different types of corners could it have? Build each type and record its vertex configuration and the defect at each vertex in the table below.

   (c) If a polyhedron has all regular pentagonal faces, how many different types of corners could it have? Build each type and record its vertex configuration and the defect at each vertex in the table below.

   (d) If a polyhedron has all regular hexagonal faces, how many different types of corners could it have? Build each type and record its vertex configuration and the defect at each vertex in the table below.

   (e) What can be said about the types of possible corners for polyhedra whose faces are regular polygons with more than six sides? Explain your reasoning.

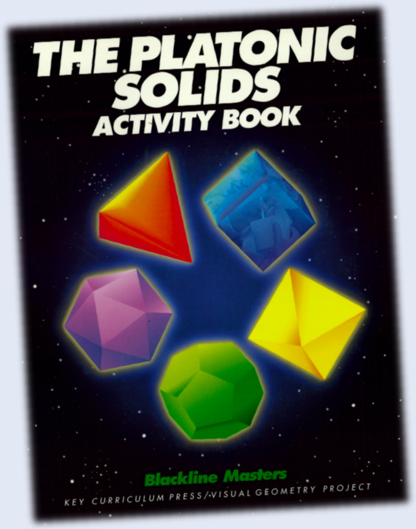
| Type of Face | Vertex Configuration | Defect |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

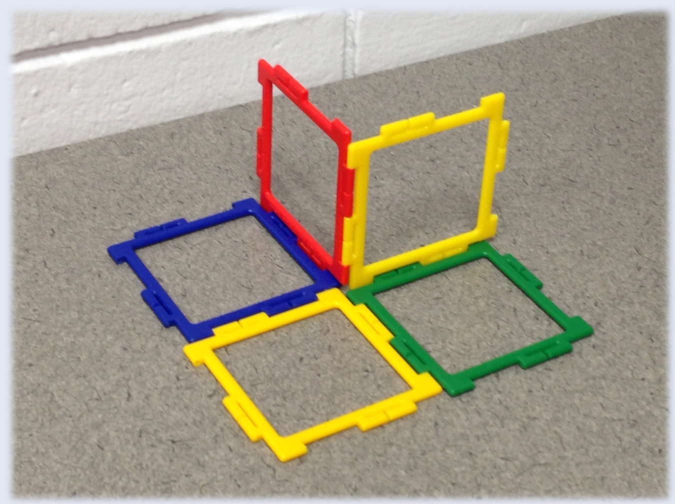



# **4. Modern Geometry:** Polyhedra Exploration

***Questions that Arose:*** more than 30 significant questions arose during the activity.

1. Can you fit more than 4 squares around a vertex? Can you fit 7 triangles around a vertex?
2. Can you make a polyhedron with all concave vertices?
3. What is the easiest way to count the edges of a large polyhedron?
4. What kind of polyhedron can be built with all hexagons?



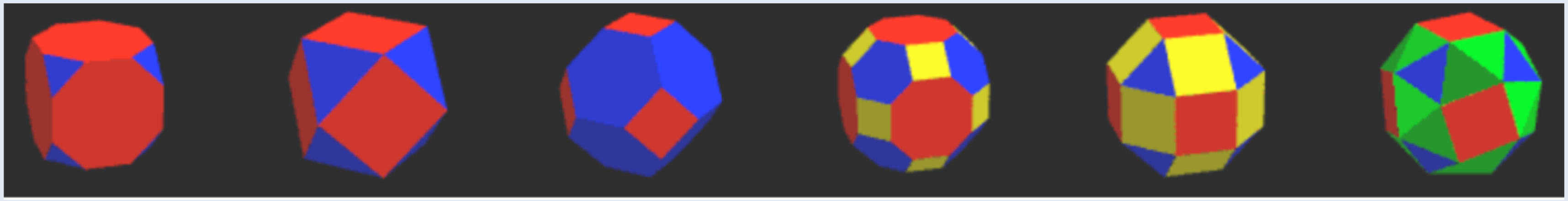



# 4. Modern Geometry: Polyhedra Exploration

WINONA STATE UNIVERSITY

***Questions that Arose:*** more than 30 significant questions arose during the activity.

- Regular Polyhedron: (1) faces are regular polygons, (2) all faces are congruent, and (3) all vertex configurations are the same.
- Semi-Regular Polyhedron: (1) faces are regular polygons, (2) ~~all faces are congruent~~, and (3) all vertex configurations are the same.

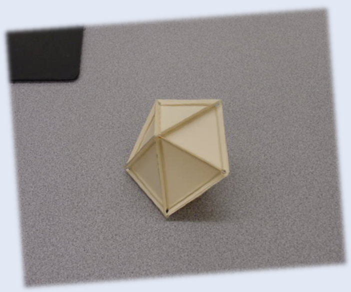


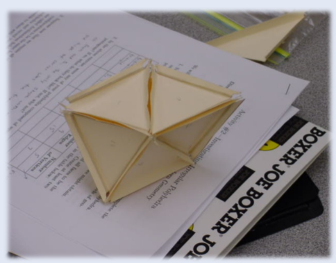
# 4. Modern Geometry: Polyhedra Exploration

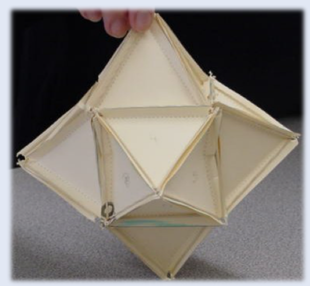
WINONA STATE UNIVERSITY

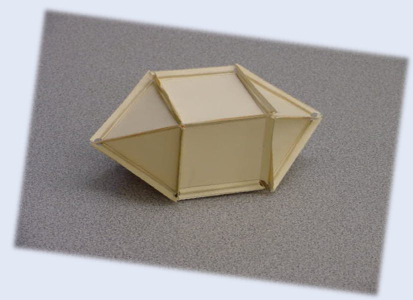
***Questions that Arose:*** more than 30 significant questions arose during the activity.

4. Are the conditions for a regular polyhedron redundant? That is, if we require a polyhedron to be composed of only one type of regular polygon, won't the vertex configurations necessarily be the same?
5. Similarly, if we require that all faces be regular and that only two types of faces be used, won't the vertex configurations automatically be the same?









# 4. Modern Geometry: Polyhedra Exploration

***Questions that Arose:*** more than 30 significant questions arose during the activity.

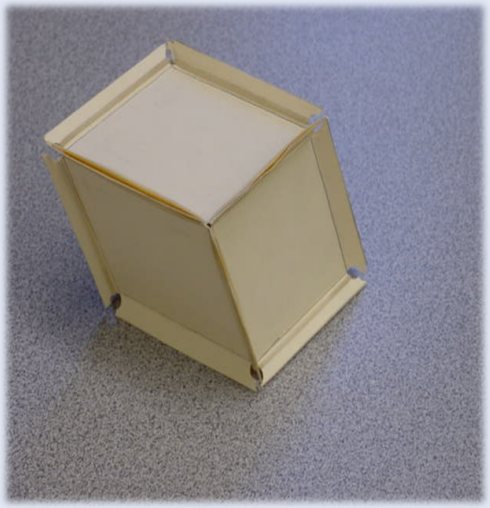
6. Can we construct polyhedra that use 3 different types of faces? How about 4 or 5?
7. Do the faces of a polyhedron need to be flat?
8. What does it mean, exactly, to say that each vertex configuration is "the same"? Are there some cases in which our notation limits our thinking?
9.

.

.

.

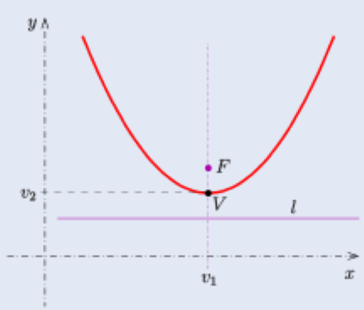
30.

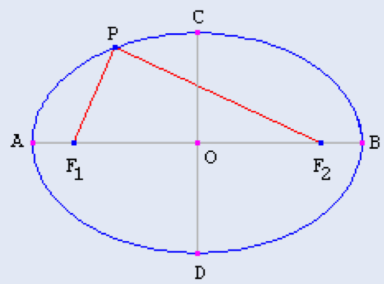


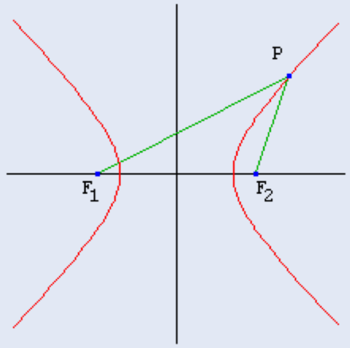
# 4. Modern Geometry: Using GSP to Explore Loci

**_Loci and Conic Sections:_** the ancient Greeks defined the conic sections in terms of loci.

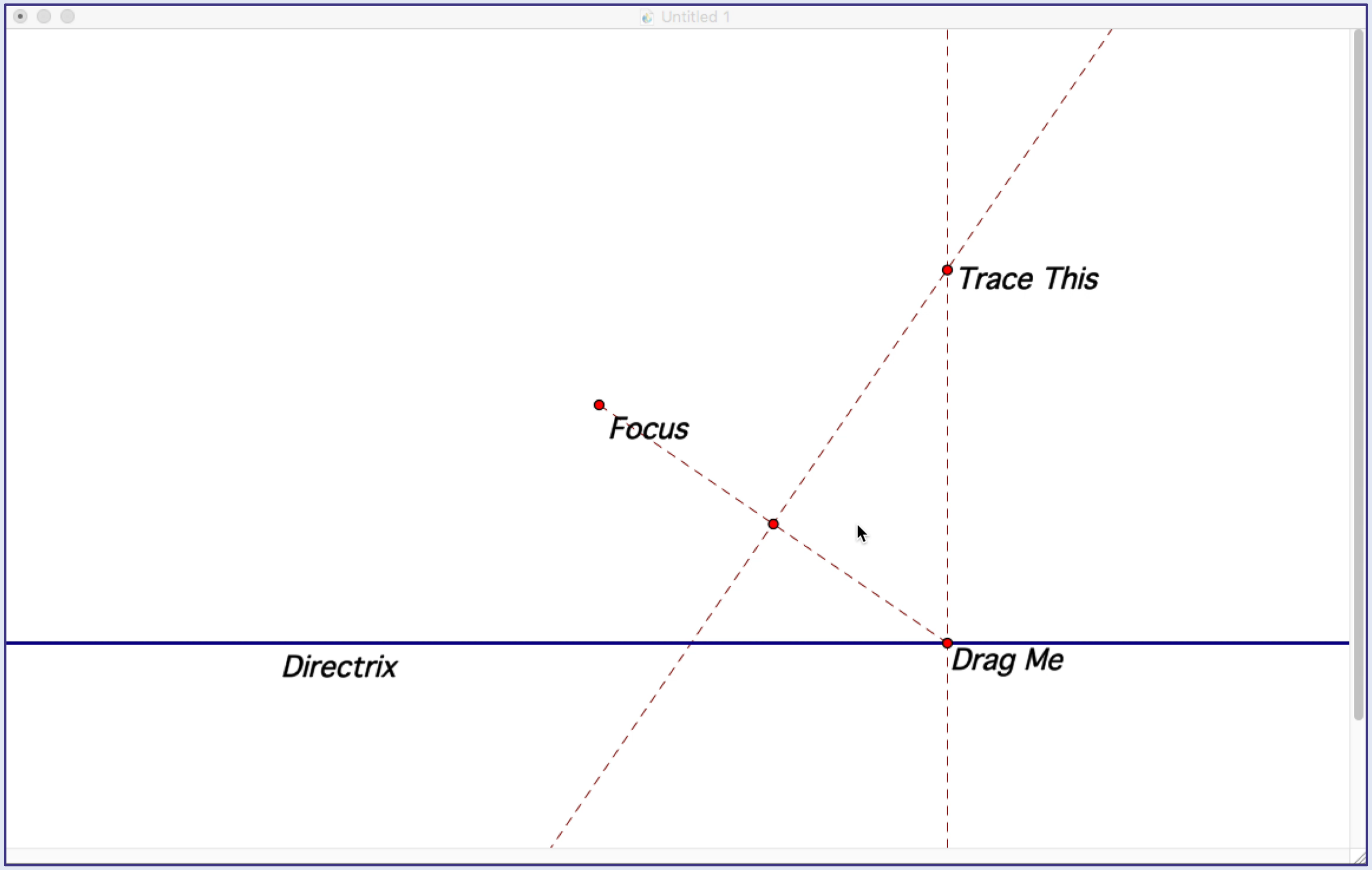
- Parabola: the locus of points equidistant from a given point (focus) and a given line (directrix).
- Ellipse: the locus of points such the the sum of distances from that point to two given points (foci) is constant.
- Hyperbola: the locus of points such the the difference of distances from that point to two given points (foci) is constant.



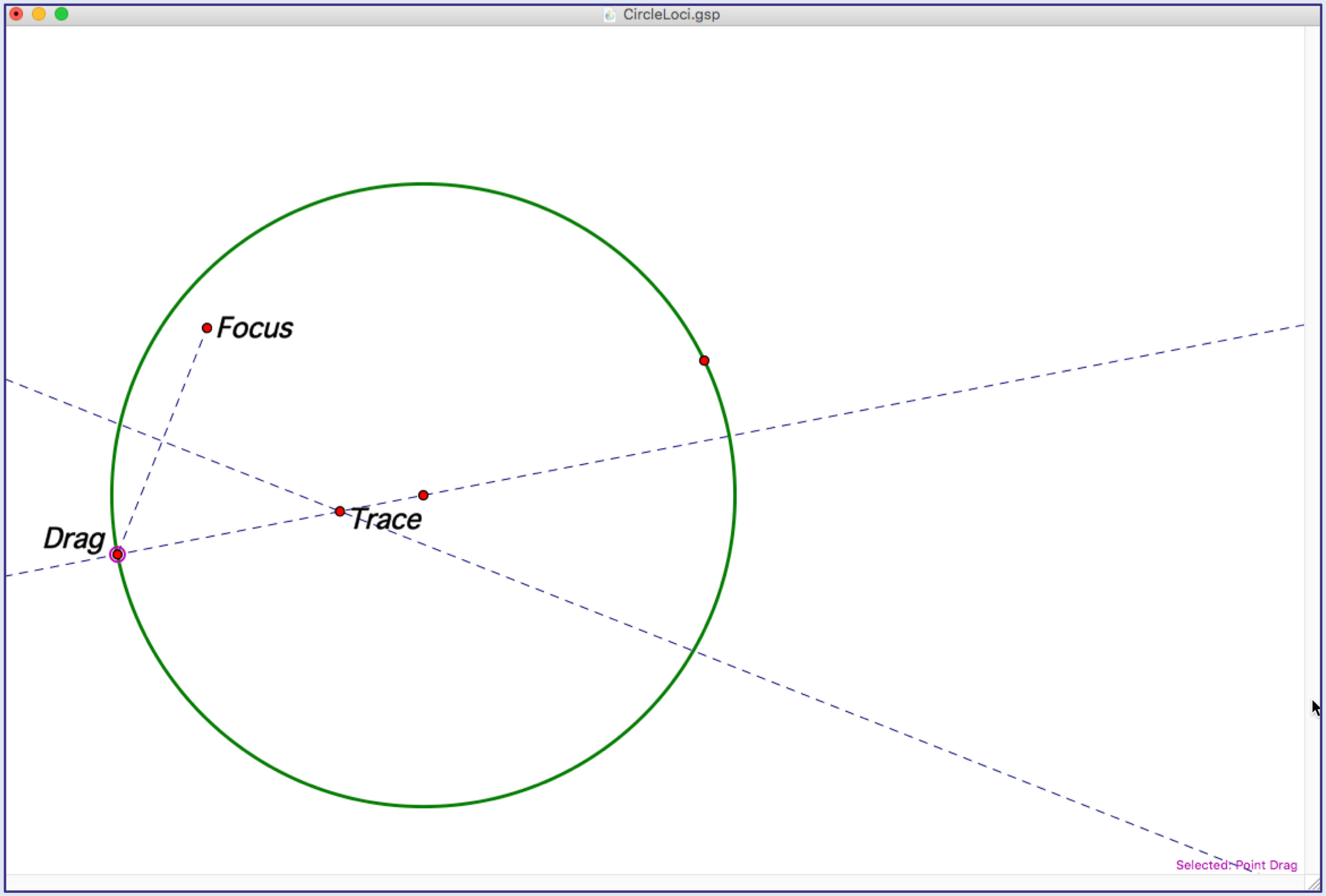



# 4. **Modern Geometry:** Using GSP to Explore Loci



# 4. **Modern Geometry:** Using GSP to Explore Loci



WINONA STATE UNIVERSITY

# 5. Differential Equations: The Main Catalysts

- ***Mr. Christopher Balsley:*** $9^{th}$ grade Earth Science teacher.
- ***Dr. Jim Yorke:*** who coined the term, "Chaos Theory," was director for the Institute for Physical Science and Technology at the University of Maryland, and was one of my thesis advisors.
- ***Debbie:*** was a non-traditional student of mine, worked at DuPont and was returning to school at the University of Delaware to obtain her B.S. in Composite Materials Engineering.

DUPONT

# **5. Differential Equations:** Intuition Behind L, CC DE's

WINONA STATE UNIVERSITY


**2nd Order, Linear, Constant Coefficient, Non-Homogeneous Initial Value Problem**

$$\boldsymbol{ay'' + by' + cy = f(x), \qquad y(0) = y_0, \qquad y'(0) = y_0'}$$

- ***Complementary Solution:*** to the homogeneous DE, $y_c(x) = Ay_1(x) + By_2(x)$
- ***Particular Solution:*** one solution to the non-homogeneous DE, $y_p(x)$
  - Method of Undetermined Coefficients:
    - We guess a solution of the form $\boldsymbol{f(x)}$, because that is what works, or
    - We use the idea of annihilators from the beautiful field of Functional Analysis to motivate our choice of $\boldsymbol{f(x)}$.
  - Method of Variation of Parameters: seek a solution of the form $A(x) \cdot y_1(x) + B(x) \cdot y_2(x)$, because that ends up working.
- ***Solution to the IVP is:*** $y(x) = y_c(x) + y_p(x)$
- ***Initial Conditions:*** we may now invoke the initial conditions to obtain the final particular solution to the IVP.

# **5. Differential Equations:** Intuition Behind L, CC DE's

***Externally Forced Linear Oscillator***

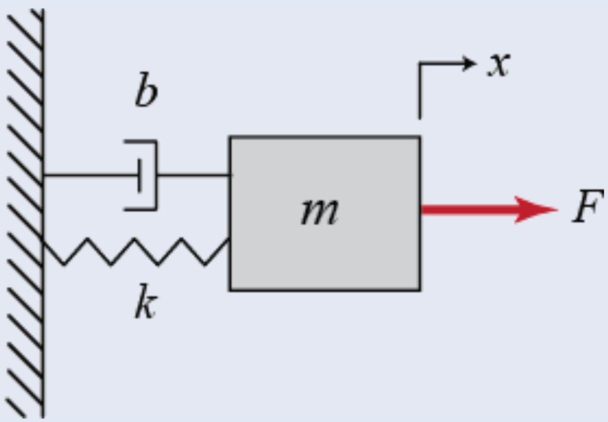
Newton's 2nd Law states: $\sum \boldsymbol{F} = \boldsymbol{m} \cdot \boldsymbol{a}.$ So, $\boldsymbol{F_{damping}} + \boldsymbol{F_{spring}} + \boldsymbol{F_{driving}} = \boldsymbol{m} \cdot \boldsymbol{x''(t)}$, or

$\boldsymbol{m} \cdot \boldsymbol{x''(t)} = -\boldsymbol{\delta} \cdot \boldsymbol{x'(t)} - \boldsymbol{k} \cdot \boldsymbol{x(t)} + \boldsymbol{f(t)}$, which becomes

$$\boldsymbol{m} \cdot \boldsymbol{x''(t)} + \boldsymbol{\delta} \cdot \boldsymbol{x'(t)} + \boldsymbol{k} \cdot \boldsymbol{x(t)} = \boldsymbol{f(t)},$$

$\boldsymbol{x(0)} = \boldsymbol{x_0}$ (Related to Initial Potential Energy Input)

$\boldsymbol{x'(0)} = \boldsymbol{x_0}'$ (Related to Initial Kinetic Energy Input)



# **5. Differential Equations:** Intuition Behind L, CC DE's

***Externally Forced Linear Oscillator***

$$m \cdot x''(t) + k \cdot x(t) + \delta \cdot x'(t) = f(t),$$

$$x(0) = x_0, \qquad x'(0) = x_1$$

# 5. Differential Equations: Intuition Behind L, CC DE's

WINONA STATE UNIVERSITY

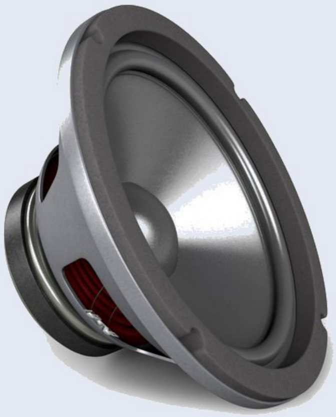
***Non-Forced Linear Oscillator***

$$m \cdot x''(t) + k \cdot x(t) + \delta \cdot x'(t) = 0,$$

$$x(0) = x_0, \qquad x'(0) = x_1$$

$$x_c(t) = A \cdot e^{-rt} \cdot \cos(st) + B \cdot e^{-rt} \cdot \sin(st),$$

The values of A and B are adjusted to accommodate the initial potential and kinetic energy input into the system.



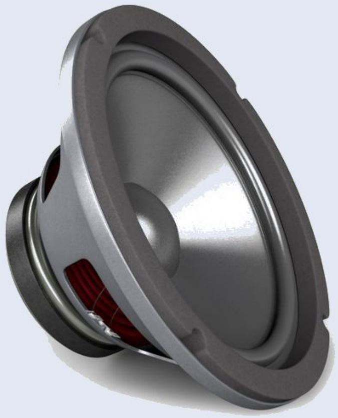
# **5. Differential Equations:** Intuition Behind L, CC DE's

***Forced Linear Oscillator***

$$\boldsymbol{m \cdot x''(t) + k \cdot x(t) + \delta \cdot x'(t) = 5\sin 3t},$$

$$\boldsymbol{x_p(t) = a \cdot \sin(3t) + b \cdot \cos(3t)}$$

And, the first question the students ask is?



# **5. Differential Equations:** Intuition Behind L, CC DE's

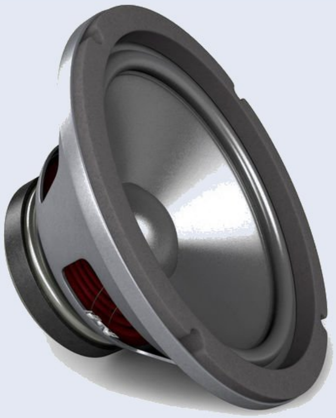
***Forced Linear Oscillator***

$$\boldsymbol{m \cdot x''(t) + k \cdot x(t) + \delta \cdot x'(t) = 5 \sin 3t},$$

$$\boldsymbol{x_p(t) = a \cdot \sin(3t) + b \cdot \cos(3t)}$$

And, the first question the students ask is?

*Why do we guess a sine and cosine when the function in the DE contains only sine?*



# **5. Differential Equations:** Intuition Behind L, CC DE's

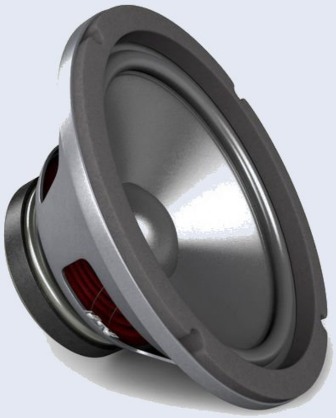
***Forced Linear Oscillator***

$$m \cdot x''(t) + k \cdot x(t) + \delta \cdot x'(t) = 5 \sin 3t \,,$$

$\cancel{x_p(t) = a \cdot \sin(3t) + b \cdot \cos(3t)}$

Call this the ***response*** of the system to the forcing.

If we force a system like $f(t)$, we expect it to behave the way we are forcing it, only with:



# 5. Differential Equations: Intuition Behind L, CC DE's

***Forced Linear Oscillator***

$$m \cdot x''(t) + k \cdot x(t) + \delta \cdot x'(t) = 5\sin 3t\,,$$

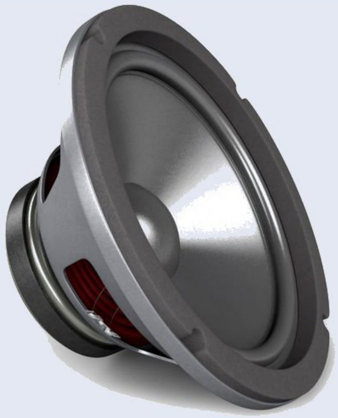
$$\cancel{x_p(t) = a \cdot \sin(3t) + b \cdot \cos(3t)}$$

Call this the ***response*** of the system to the forcing.

If we force a system like $f(t)$, we expect it to behave the way we are forcing it, only with:

1. **Time Delay**
2. **Amplitude Change**

$$\begin{aligned} x_p(t) &= \alpha \cdot \sin 3(t-\beta) \\ &= \alpha \cdot \cos(-3\beta) \cdot \sin(3t) + \alpha \cdot \sin(-3\beta) \cdot \cos(3t) \\ &= a \cdot \sin(3t) + b \cdot \cos(3t) \end{aligned}$$



# 5. Differential Equations: Intuition Behind L, CC DE's

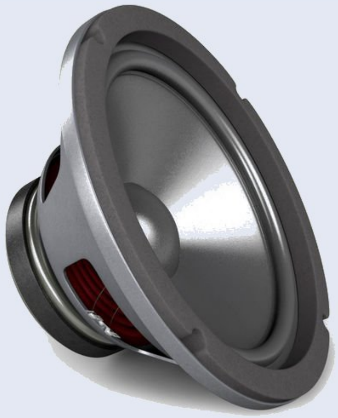
WINONA STATE UNIVERSITY

***Forced Linear Oscillator***

$$m \cdot x''(t) + k \cdot x(t) + \delta \cdot x'(t) = f(t),$$

$$f(t) = 3t^2 \to x_p(t) = \alpha(t-\beta)^2 = \alpha t^2 - 2\alpha\beta \cdot t + \alpha\beta^2 = at^2 + bt + c$$

$$f(t) = 8e^{7t} \to x_p(t) = \alpha e^{7(t-\beta)} = \alpha e^{7t} \cdot e^{-7\beta} = ae^{7t}$$



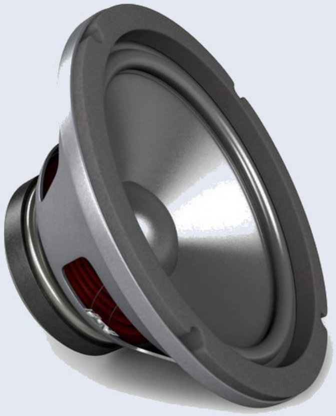
# **5. Differential Equations:** Intuition Behind L, CC DE's

***Forced Linear Oscillator***

$$\boldsymbol{m \cdot x''(t) + k \cdot x(t) + \delta \cdot x'(t) = f(t),}$$

So, how do we process the final solution to the original IVP?

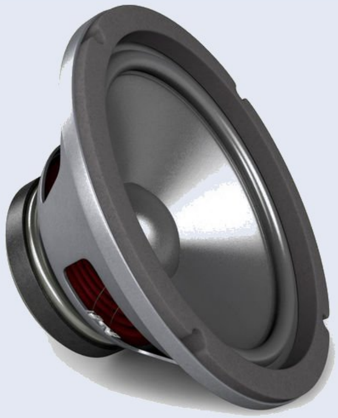
$$x(t) \quad = \quad \underbrace{A \cdot \boxed{e^{-rt}} \cdot \sin(st) + B \cdot \boxed{e^{-rt}} \cdot \cos(st)}_{\text{Initial Energy Response}} \quad + \quad \underbrace{a \cdot \sin(3t) + b \cdot \cos(3t)}_{\text{Response to Forcing}}$$



# 5. Differential Equations: Intuition Behind L, CC DE's

**Some Lingering Issues:**

1. It is in fact true that not every quadratic of the form $\boldsymbol{at^2+bt+c}$ can be written in the form $\boldsymbol{\alpha(t-\beta)^2}$, but rather as $\boldsymbol{\alpha(t-\beta)^2+\gamma}$.
2. Why does this method of amplitude change and time delay not work for forcing functions that are not sine, cosine, exponential, or polynomial in form?
3. Is there a similarly intuitive explanation for the Method of Variation of Parameters?



WINONA
STATE UNIVERSITY