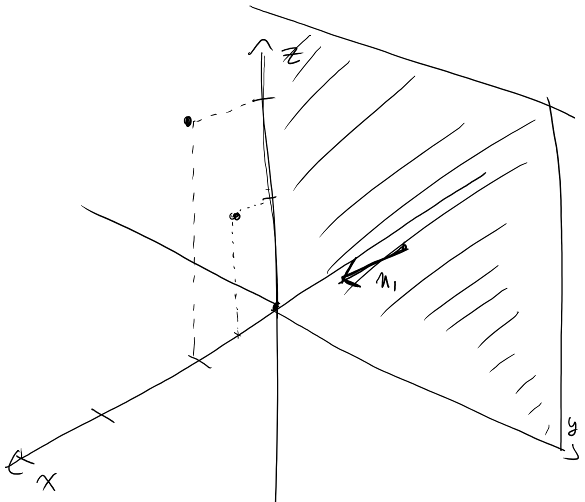
Homework 2

September 27, 2017 7:11 PM



equation of x axis point + dir

$= (0,0,0) + t\langle 1, 0, 0\rangle$

equation of zy plane $= s\vec{b}$ + point

$= (0,0)$

$\vec{b} = t_2 \langle 1, \overset{n}{0}, 0\rangle$

point on plane $= \langle 0, 0, 0\rangle$

$1(x-0) + 0 + 0 = 0$

$x - 0 = 0$

$x = 0$

distance from line from point $P = (P_1, P_2, P_3)$

use $(0,0,0)$

distance is denoted by $d_1 = \dfrac{\left|(P - \text{any point on line}) \text{ cross with dir line}\right|}{\text{len dir}}$

$d_1 = \dfrac{|P \times \langle 1, 0, 0\rangle|}{1}$

distance from P to $x=0$

1 (point − point on plane) $\overset{\text{Scalar}}{\text{proj}}$ onto $\vec{n}$.

distance from

$$d_2 = (\text{point} - \underset{\substack{\downarrow \\ 0,0,0}}{\text{point on plane}}) \overset{\text{Scalar}}{\text{proj}} \text{ onto } \vec{n}_1$$

$$d_2 = \frac{P \cdot \langle 1,0,0 \rangle}{1}$$

$$d_1 = 2 d_2$$

$$|P \times \langle 1,0,0 \rangle| = 2\, P \cdot \langle 1,0,0 \rangle$$

$$= 2P_1$$

$$\begin{vmatrix} & 0 & 0 & 1 & 0 & 0 \\ P_1 & P_2 & P_3 & P_1 & P_2 & P_3 \end{vmatrix}$$

$$= 0, -P_3, P_2$$

$$\text{Len} = \sqrt{P_3^2 + P_2^2} = 2P_1$$

$$P_3^2 + P_2^2 = (2P_1)^2$$

$$\boxed{z^2 + y^2 = x^2}$$

b) point $x^2 = 3^2 + 2^2$

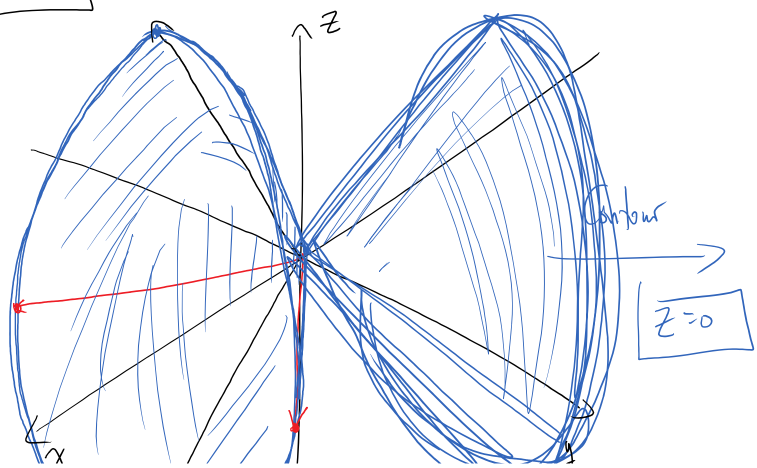
$$x = \sqrt{9+4}$$

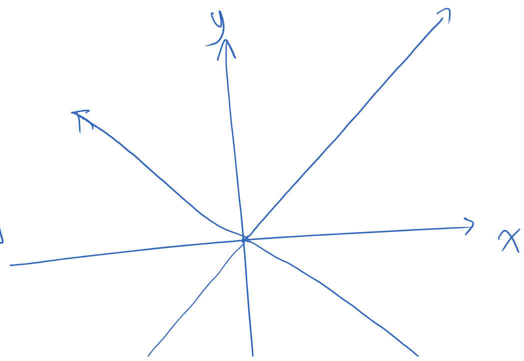
$$= \sqrt{13}$$

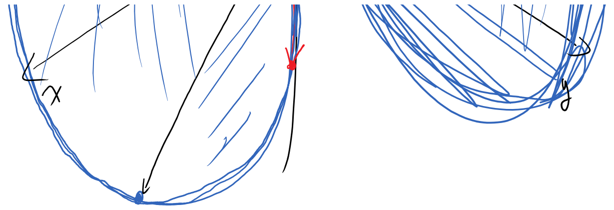
$$\boxed{\text{point is } (\sqrt{13}, 2, 3)}$$

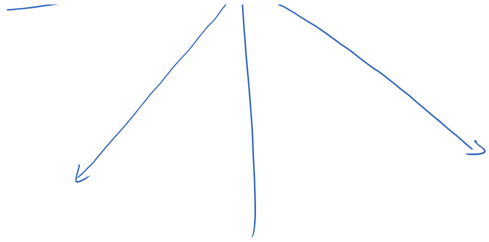
c) $\boxed{\text{Cone}}$

d)



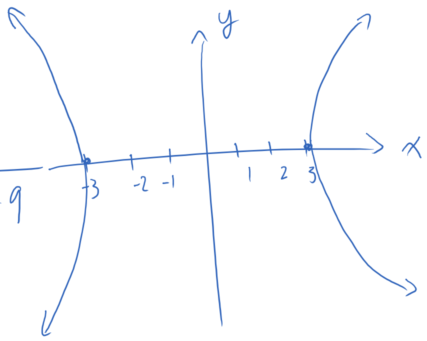




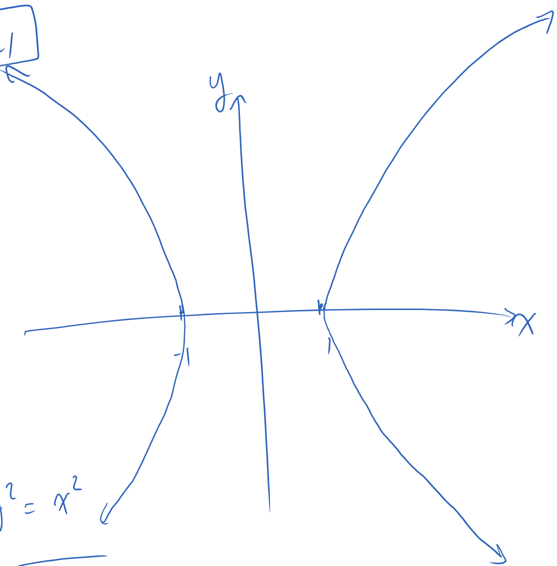


$\boxed{Z=3}$

$y=\sqrt{x^2-9}$



$\boxed{Z=1}$

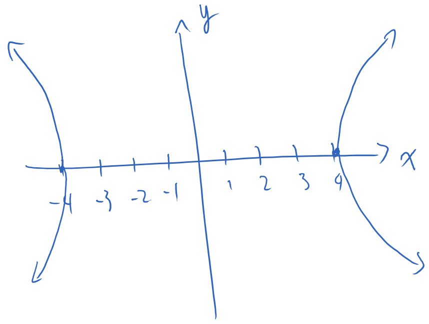


If $y^2=x^2$

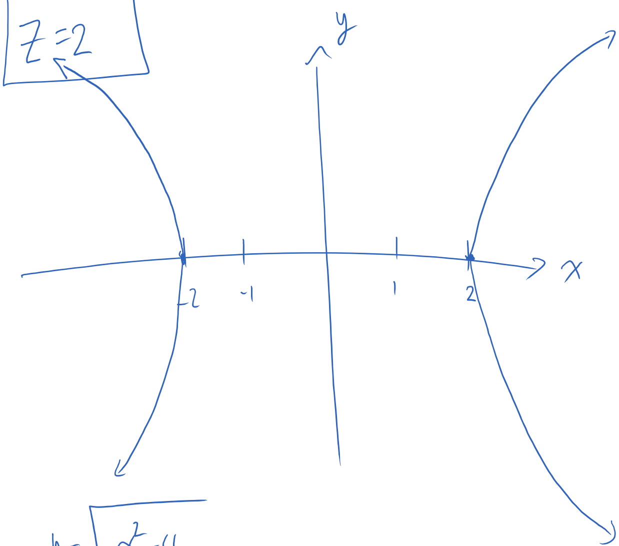
$y=\sqrt{x^2-1}$

$\boxed{Z=4}$

$y=\sqrt{x^2-16}$



$\boxed{Z=2}$



$y=\sqrt{x^2-4}$

2.

a) find the eq of the path of the vector

↳ starts at $(e^4, 0, 0)$

↳ travels along $r(t) = \langle e^{-t+4}, 2t, t \rangle$

$$\underbrace{e^{-t+4}}_{x},\ \underbrace{2t}_{y},\ \underbrace{t}_{z}$$

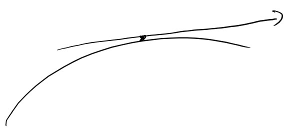
the path he follows is TANGENT to $r(t)$ @ $t=4$

$r'(t) = \frac{d}{dt}(e^{-t+4}) + \frac{d}{dt}2t + \frac{d}{dt}t$

$r'(t) = -e^{-t+4} + 2+1$ or $\langle -e^{-t+4}, 2, 1 \rangle$

$r'(4) = -e^{-4+4} + 2+1$ or $\langle -1, 2, 1 \rangle$

↳ VECTOR OF THE TANGENT



point is the pos of rocket @ $t=4$

$= \langle 1,8,4 \rangle$

because time evaluated @ $t=4$

point @ $t=4$

equation of path $\boxed{\vec{l}(t) = \langle -1, 2, 1 \rangle (t-4) + (1, 8, 4)}$

b) If these two agree at vector pos $t=4$

then, we can plug in

at $t=4$

$\vec{l}(4) = \vec{r}(4)$

$\vec{l}(4) = \langle 1,8,4 \rangle + 0$

$\vec{r}(4) = \langle 1,8,4 \rangle$

these are the same!

c)

Distance formula

$$= \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$$

We can find a fn. of t, $d(t)$ and minimize it.

for $\vec{r}(t)$

$$d(t) = \sqrt{\left(e^{-t+4} - (5-t)\right)^2 + (2t-t)^2 + \left(t - (t^2-18t)\right)^2}$$

position of marlin is pos of rocket.

$$\sqrt{(\ \ {}^{-t+4}\ \ )^2 + \ \ {}^2 + (\ \ t^2+18t)^2}$$

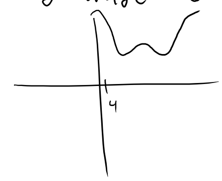
marlin is pos at rocket.

$$d(t) = \sqrt{\left(e^{-t+4} - 5+t\right)^2 + t^2 + \left(-t^2+19t\right)^2}$$

graph to find smallest t value.

minimum $t = 18.9078$ s and $d(t) = 23.5366$ units

↳ Cannot use value because $t>4$

Graph is of form  so all other t values too big.

for $\vec{l}(t) = \langle -1, 2, 1\rangle(t-4) + (1,8,4)$

$(-t+4+1), (2t \cancel{-8+8}), (t \cancel{-4+4})$

$$d(t) = \sqrt{(-t+5-\underbrace{(5-t)}_{-5\ +t})^2 + (2t-t)^2 + (t-t^2+18t)^2}$$

$$d(t) = \sqrt{0 + t^2 + (-t^2+19t)^2}$$

graph to find smallest t.

$t = 18.947077$ $\boxed{d(t) = 18.973592}$ units

d) using method above

$\boxed{t = 18.9471 \text{ s}}$

3. $f(x,y) = 0$ when $(x,y) = (0,0)$

$= \frac{x^2y}{4\,.\,.\,2}$ when $(x,y) =$ anything else.

$= \dfrac{x^2 y}{x^4+y^2}$ when $(x,y) =$ anything else.

Hospital @ $(0,0,\frac{1}{2})$

a) for $(x,y) \neq (0,0)$

$\displaystyle\lim_{(x,0)\to(0,0)} \frac{x^2 y}{x^4+y^2} =$ x

$\dfrac{x^2(0)}{x^4+(0)} = 0$

$\displaystyle\lim_{(0,y)\to(0,0)} \frac{x^2 y}{x^4+y^2} = \frac{0(y)}{0+y^2} = 0$

if $y = mx$ where $m=1$

$\displaystyle\lim_{(x,x)\to(0,0)} \frac{x^2 x}{x^4+x^2} = \frac{\cancel{x^2}x}{\cancel{x^2}(x^2+1)} = \frac{0}{0+1} = 0$

$\displaystyle\lim_{(x,mx)\to(0,0)} \frac{x^2(mx)}{x^4+(mx)^2} = \lim \frac{m\cancel{x^3}\,x}{\cancel{x^2}(x^2+m)} = \frac{mx}{x^2+m} = \frac{0}{0+m} = 0$

$\boxed{\text{lim along } y=mx = 0}$

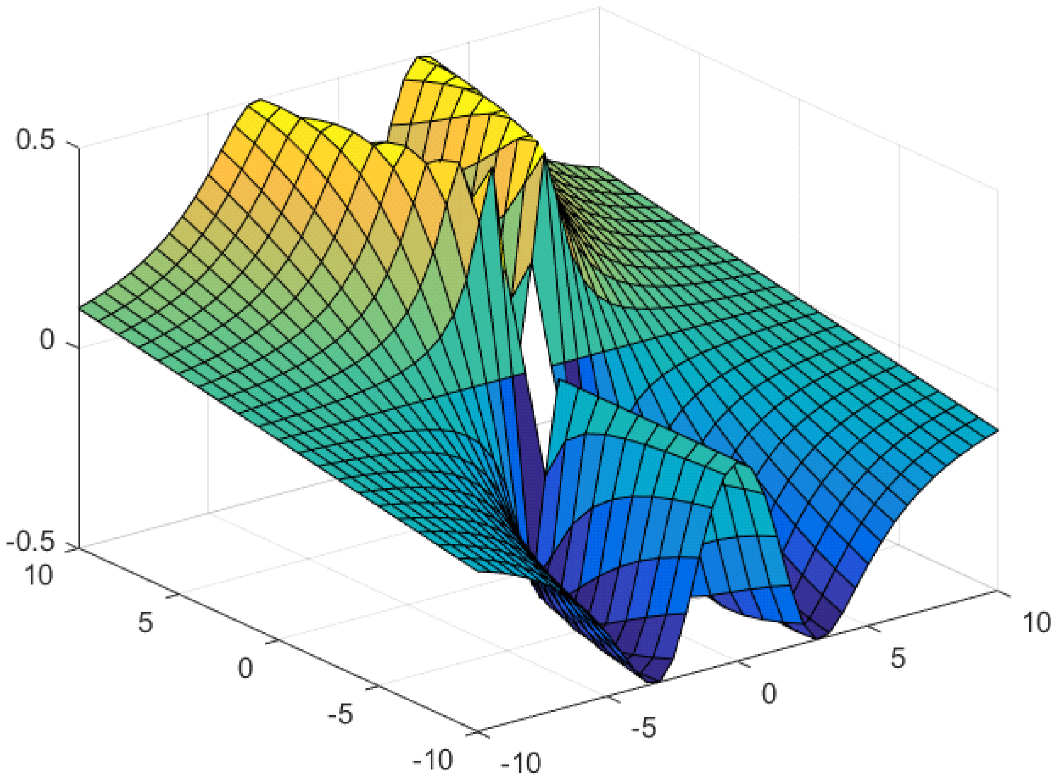
b) $\displaystyle\lim_{(x,x^2)\to(0,0)} \frac{\cancel{x^2(x^2)}^{\,x^4}}{\cancel{x^4+x^4}_{\,2x^4}} = \frac{1}{2}$

$\boxed{\text{limit is } \tfrac{1}{2} \text{ along } y=x^2}$

c) No it isn't because a value (0) exists and the limit as $(x,y)\to(0,0)$ does not evaluate to 0 for all paths.

d) yes it will be continuous because the dominator $\neq 0$ and with x^4 and y^2, no + or −'ve value of will be negative, and thus, the dominator will never

= 0. The top is a polynomial that is defined everywhere.
Thus, the fn. is continuous at any point, P.



1.a) plane can be defined by a point and normal
↳ $ax + by + cz = 0$

$\downarrow$ $x - x_0$ $\quad$ $\downarrow$ $y - y_0$ $\quad$ $\downarrow$ $z - z_0$

$\Delta F = \langle 2x, 2y, 2z \rangle$

$$\boxed{4(x-2) + 4(y-2) + 2(z-1) = 0}$$

b) Using point and normal form:

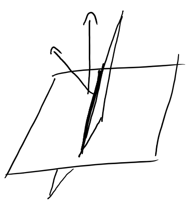
$$4(x-2) + 4(y-2) - 16(z-1) = 0$$

$\Delta F = \langle 2x, 2y, -16z \rangle$

$$\boxed{4(x-2)+4(y-2)-16(z-1)=0}$$

c) we have a point, just need a direction of the line.

direction given by the $\vec{v} = \langle x, y, z \rangle$



vector dir = cross of 2 normals

$$= \langle 4, 4, 2 \rangle \times \langle 4, 4, -16 \rangle$$

$$\begin{vmatrix} 8 & 8 & 4 & 8 & 8 & 4 \\ 8 & 8 & 256 & 8 & 8 & 256 \end{vmatrix} \qquad \begin{vmatrix} 4 & 4 & 2 & 4 & 4 & 2 \\ 4 & 4 & -16 & 4 & 4 & -16 \end{vmatrix}$$

$$\langle -64-8,\ 8+64,\ 16-16 \rangle$$

$$\langle -72, 72, 0 \rangle t \quad \text{if } t = \frac{1}{72}$$

$$= \langle -1, 1, 0 \rangle t$$

↳ dir

line is $\boxed{\langle -1, 1, 0 \rangle t + (2, 2, 1)}$ $\qquad \langle -1, 1, 0 \rangle t + (2,2,1) = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 1 0\right)$

d) $P - (2, 2, 1) =$ Length 1

get length 1 by using unit vector normalization

line $= \langle -t, t, 0 \rangle + (2, 2, 1)$

↳ point.

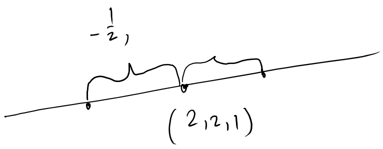
So length of $\langle -t, t, 0 \rangle$ must be 0.

$$\sqrt{t^2 + t^2 + 0} = 1$$

$$\sqrt{t^2+t^2+0} = 1$$

$$\sqrt{2t^2} = 1$$

$$t = \pm\frac{1}{\sqrt{2}}$$



so 2 points at

$$\boxed{\left(2-\frac{1}{\sqrt{2}},\ 2+\frac{1}{\sqrt{2}},\ 1\right)}$$

and

$$\boxed{\left(2+\frac{1}{\sqrt{2}},\ 2-\frac{1}{\sqrt{2}},\ 1\right)}$$