HW4 #2 and 3

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2. a) $x_0 = -3$ $\quad A = 350 \quad C = 78 \quad F = -3$

$y_0 = 11 \quad B = -30 \quad E = 3 \quad G = 15$

$$g_0(x,y) = A + (x-x_0)B + (y-y_0)C + \frac{(x-x_0)^2}{2}E + (y-y_0)(x-x_0)F + \frac{(y-y_0)^2}{2}G$$

$$g_0(x,y) = A + \begin{bmatrix} B & C \end{bmatrix}\begin{bmatrix} x-x_0 \\ y-y_0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} x-x_0 & y-y_0 \end{bmatrix}\begin{bmatrix} E & F \\ F & G \end{bmatrix}\begin{bmatrix} x-x_0 \\ y-y_0 \end{bmatrix}$$

$$\boxed{g_0(x,y) = 350 + \begin{bmatrix} -30 & 78 \end{bmatrix}\begin{bmatrix} x+3 \\ y-11 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} x+3 & y-11 \end{bmatrix}\begin{bmatrix} 3 & -3 \\ -3 & 15 \end{bmatrix}\begin{bmatrix} x+3 \\ y-11 \end{bmatrix}}$$

b) Minimum occurs at $g_{xx} > 0$, $D(x,y) > 0$ and $g_x = g_y = 0$

$$D(x,y) = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix} = f_{xx}f_{yy} - f_{xy}^2$$

$$E \times G - F^2$$

Using

$$\begin{bmatrix} \dfrac{GB}{EG-F^2} - \dfrac{FC}{EG-F^2} \\ \\ \dfrac{-FB+CE}{EG-F^2} \end{bmatrix} + \begin{bmatrix} x_1 - x_0 \\ y_1 - y_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 = \frac{FC - GB}{EG-F^2} + x_0$$

$$= \frac{(-3)78 - 15(-30)}{3(15) - (-3)^2} - 3 = 6 - 3 = 3$$

$$y_1 = \frac{FB - CE}{EG - F^2} + y_0 = 7$$

$$\boxed{(x_1, y_1) = (3, 7)}$$

$$g_0(3,7) = 350 + \underbrace{\begin{bmatrix} -30 & 78 \end{bmatrix} \begin{bmatrix} 3+3 \\ 7-11 \end{bmatrix}}_{-492} + \frac{1}{2} \underbrace{\begin{bmatrix} 3+3 & 7-11 \end{bmatrix} \begin{bmatrix} 3 & -3 \\ -3 & 15 \end{bmatrix} \begin{bmatrix} 3+3 \\ 7-11 \end{bmatrix}}_{246}$$

a b c d e

$$\boxed{g_0(3,7) = 104}$$

c) $f(3,7) = 80$

This is different from $g_0(x_1, y_1)$ because $g_0(x_1, y_1)$ is just an approximation.

d) $$g_2(x,y) = A + \begin{bmatrix} B & C \end{bmatrix} \begin{bmatrix} x - x_1 \\ y - y_1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} x - x_1 & y - y_1 \end{bmatrix} \begin{bmatrix} E & F \\ F & G \end{bmatrix} \begin{bmatrix} x - x_1 \\ y - y_1 \end{bmatrix}$$

e) The minimizer still occurs at the critical point @ (x_2, y_2) with $(x_1, y_1) = (3,7)$

$$x_2 = \frac{FC - GB}{EG - F^2} + x_1 = 4$$

$$y_2 = \frac{FB - CE}{EG - F^2} + y_1 = 3$$

$$\frac{\qquad}{EG - F^2} + y_1 = 3$$

$$\boxed{(x_2, y_2) = (4, 3)}$$

f) Local minimum determined by

$g_{xx} > 0$

↳ calculated

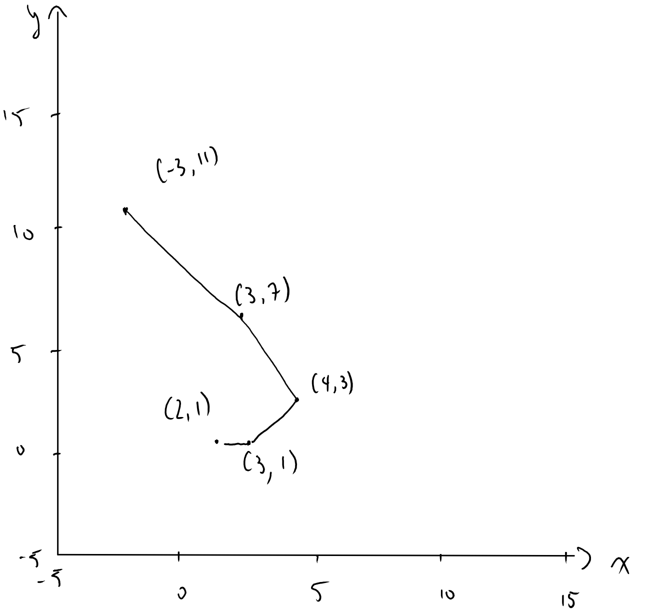
$g_3(x, y)$ @ $(4, 3)$

$= (x_3, y_3) = (3, 1)$

$g_4(x, y)$ @ $(3, 1)$

$= (x_4, y_4) = (2, 1) \longrightarrow$ $f_x = f_y = 0$ and $f_{xx} = 1$

Local min!



g) Looking at the phantom power 98 information, $(x, y) = (99, 88)$ is not a max or min but has a -'ve f_{xx} and $D(x, y) > 0$ so it is approaching a local max. Thus to decrease or go "downhill" we should go towards the local min, $(x,y) = (2,1)$.

3.a) This describes a local maximum.

Thus we need $f_x = f_y = 0$

$f_{xx} < 0 \quad D(x,y) > 0$

i) $\boxed{\text{Mika should go to point } E}$

ii)

This asks for a local min.

$f_{xx} > 0$ and $D(x,y) > 0$

$f_x = f_y = 0$

This occurs at G

$\boxed{\text{Xiao fei should go to } G}$

iii)

This is asking for a saddle point.

$D < 0$

$f_{xx} f_{yy} - f_{xy}^2 < 0$

at B $\quad 4 \times 3 - (-4)^2 = -4 < 0$

$\boxed{\text{Donnie. Should go to point } B}$

iv) Suprises happen at $D(x,y)=0$

$\boxed{\text{Akshat should go to point F}}$