1. $f(x) = \cos\left(\frac{2\pi x}{5p} - \frac{2\pi}{3}\right)$

This is in the form $\cos(\omega x - \phi)$ where ω is the angular frequency. $\omega = 2\pi f$ and ϕ is phase shift. So if we find the period $\left(\frac{1}{f}\right)$ we would have found the wavelength. Thus we see that $\frac{2\pi}{\omega}$ is the wavelength. So $\rightarrow$

$\frac{2\pi}{\left(\frac{2\pi}{5p}\right)} = 5p$ so the wavelength of this function is $\boxed{5p.}$

2. $Y_1(x,t) = 8\cos(6kx - 3\omega t)$ $\quad Y_2(x,t) = 2\cos(3kx + \omega t)$

These equations are in the form $A\cos(kx - \omega t)$ where $k = \frac{2\pi}{\lambda}$ and $\omega = 2\pi f$. note that $\frac{\omega}{k} = \frac{f}{\lambda}$ which give the units of velocity, and this is called the phase velocity, so $\longrightarrow$

$V_1 = \frac{-3\omega}{6k} = -\frac{1}{2}\frac{\omega}{k}$ $\quad V_2 = \frac{\omega}{3k}$ so $\left|\frac{V_1}{V_2}\right| = \left|\frac{-3}{2}\right| = \frac{3}{2}$

$\boxed{\text{So the ratio of the speeds is } \frac{3}{2}}$

It is clear that the ratio of the velocities $\left(V_{x,1} = \frac{-1}{2}\frac{\omega}{k},\ V_{x,2} = \frac{\omega}{3k}\right)$ is: $\boxed{\frac{V_{x,1}}{V_{x,2}} = \frac{-3}{2}}$

3. Phase Velocity - The phase velocity is the velocity that a wave is travelling through space. This velocity is defined through the equation $V_p = \frac{\lambda}{T}$ where λ = wavelength and T = period, this is also equal to $V_p = \frac{\omega}{k}$ where $\omega = \frac{2\pi}{T}$ and $k = \frac{2\pi}{\lambda}$.

Group velocity - The group velocity is the velocity at which the amplitudes of the wave move. In other words its the velocity of the shape of the wave. The group velocity is defined as follows: $V_g = \frac{\partial \omega}{\partial k}$. This is the partial derivative of the angular velocity (ω) with respect to the angular wave number.

4. a_0 - This must be a negative number because the graph itself crosses the y axis at a negative number.

a_1 - This must be negative because the slope of the line is only negative thus the a_1 coefficient must be negative since the slope is only negative.

a_2 - This coefficient will be zero because during the second derivative at $x=0$ there is an inflection point which means that the second derivative must be 0.

a_3 - This will be positive because if we draw out the first derivative of this function we will see that at $x=0$ it is concave up, and by the second derivative test it will be positive and so a_3 will be positive since we are looking at the first derivative graph.

5. $M = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$

a. To find eigen values we subtract λ from main diagonal and take determinant.

$$\det\left(\begin{bmatrix} -\lambda & i \\ -i & -\lambda \end{bmatrix}\right) = \lambda^2 - 1$$

we now set equal to 0 and solve for λ

$\lambda^2 - 1 = 0$ $\boxed{\lambda = \pm 1}$

b.

$\lambda=1$ $\begin{bmatrix} -1 & i \\ -i & -1 \end{bmatrix} \sim \begin{bmatrix} -1 & i \\ 0 & 0 \end{bmatrix}$ Eigen vector $\begin{bmatrix} 1 \\ i \end{bmatrix}$

$\lambda=-1$ $\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & i \\ 0 & 0 \end{bmatrix}$ Eigenvectors $\begin{bmatrix} 1 \\ -i \end{bmatrix}$

C. The eigenvalues of a matrix are related to the trace in the fact that the addition of the eigenvalues should result in the trace. The trace of this matrix is 0, and the addition of both eigenvalues also give 0.

D. The eigen values of of a matrix are related to the determinant as the multiplication of the eigen values should give the same value as the determinant. Clearly $1 \cdot -1 = -1$ and $0-(-i)(i) = -1$ so these are the same showing that my solution satisfies this condition.

6. a. This answer is not correct as the combinations of terms listed do not form a square wave (only sines do), as it only needs odd harmonics

b. This answer is not correct because the combination of terms doesn't form a square wave. They must be all sines to form it, since it needs to be all odd harmonics

C. This answer is very close to the correct one but is not correct, This does have all the needed terms (sine only term with "n" being odd), This forms an odd square wave, but the picture given is not odd and is shifted up by 1.

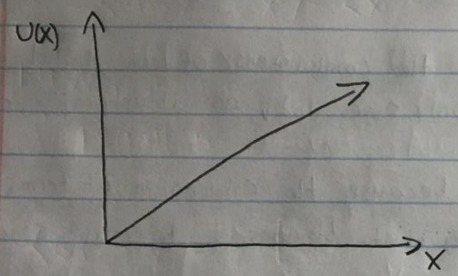
d. This is the correct answer as it accounts for where answer 'C' fails and shifts the graph up by one resulting in the correct answers.

e. This answer is wrong as it creates an even function but what we have is neither odd nor even

F. This answer is wrong because it forms an even function but the graph given is neither even nor is it odd

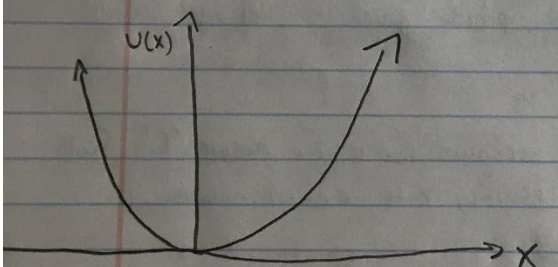
7. A potential energy function in classical physics is a function that describes how much energy is stored within an object. A potential energy function is useful because the negative derivative of the function with respect to distance gives you the force acting on the object.

Example 1.
Gravitational Potential Energy function



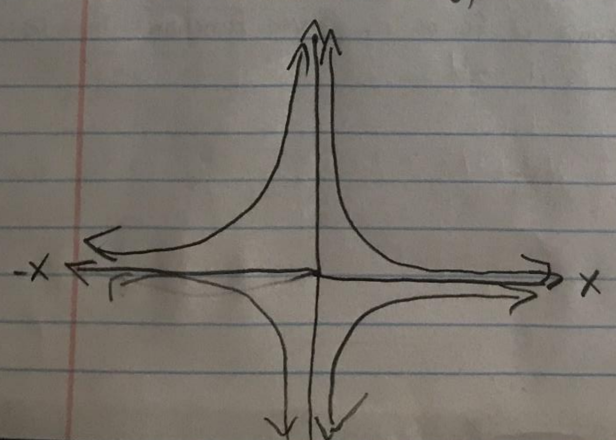
Potential Energy of moving a ball up from Earth's surface is represented by $U(x) = mgx$. Note that this is a linear relationship, and that the -derivative gives $-mg$ which is the force towards earth.

2. Spring potential Energy $U = \frac{1}{2}kx^2$



Potential Energy Stored in a spring is represented by $U = \frac{1}{2}kx^2$. As the spring is stretched or compressed from equilibrium the amount of stored potential increases quadratically.

3. Electrical Potential Energy : $U(x) = \frac{kQq}{x}$



8. $\langle x^2 \rangle$ is average of the squares
$\langle x \rangle^2$ is Square of the averages

$$\sigma_x^2 \equiv \langle (\Delta x)^2 \rangle$$
$$= \int (\Delta x)^2 \rho(x)dx$$
$$= \int (x - \langle x \rangle)^2 \rho(x)dx$$
$$= \int (x^2 - 2x\langle x \rangle + \langle x \rangle^2) \rho(x)dx$$
$$= \int x^2 \rho(j) - 2\langle x \rangle \int x \rho(x) + \langle x \rangle^2 \Sigma \rho(x)$$

Note that by definition $\Sigma x^2 \rho(x)$ is the average of x^2, $\Sigma x \rho(x)$ is average of j and $\Sigma \rho(x) = 1$ because the summation of all probabilities must be 1 thus we see that

$\sigma_x^2 = \langle x^2 \rangle - 2\langle x \rangle^2 + \langle x \rangle^2 = \boxed{\langle x^2 \rangle - \langle x \rangle^2}$ as we were trying to prove