Finding All Max-Complexity Patterns in a 3×3 Grid

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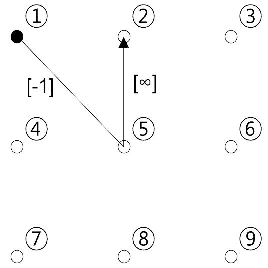
1. Defining notations

Let us name the 9 dots ①~⑨ starting from the top left.

[Fig 1] Pattern ‘152’



When indicating a pattern, we can either only list the order of dots, or also list slopes such as ‘(dot) [slope] (dot) [slope] (dot)⋯’.
For example, the pattern ‘152’ is the same as ‘① [−1] ⑤ [∞] ②’.

When choosing the next dot in the middle of drawing a pattern,
(1) Do not go to any dot using a slope that’s already been used
(2) Check if there is a slope needed to be used right now

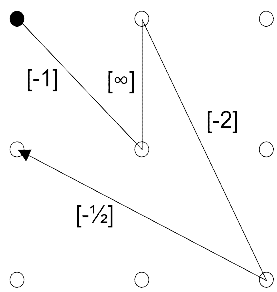
Suppose we are finding a max-complexity pattern starting with ‘152’. We can use (1) to exclude ⑥ and ⑧ from the candidates of the next dot, because if we go to ⑥ we use [−1], and if we go to ⑧ we use [∞].

On the other hand, we can use (2) to determine that the next dot is ⑨. The slope [−2] can only go between ①-⑧ or ②-⑨, and if we do not go to ⑨, it is not possible to use [−2] at all. After going to ⑨, we need to move to ④ because slope [−½] is needed to be used.

If we need to use a certain slope like this, or if there is only one possible path we can take, we denote a ‘!’ beside the slope.

① [−1] ⑤ [∞] ②
→ [−2]! ⑨ [−½]! ④

[Fig 2] ‘15294’



Suppose we are finding a max-complexity pattern starting with ‘15294’. Now the next dot can be either ③ or ⑥.

First, if we go to ③, then we need to go to ⑧ because slope [2] is needed to be used. Now the left slopes are [0] and [1], and the left dots are ⑥ and ⑦. If we go to ⑥ then slope [0] is impossible to use, and if we go to ⑦ then slope [1] is impossible to use. When we cannot make a max-complexity pattern like this, we write ‘(X)’, and write the reason such as ‘([0] impossible)’. If it is not hard to know why it is impossible, it can be omitted.

Second, if we go to ⑥, then we need to go ⑦ because slope [$\frac{1}{2}$] is needed to be used. Now the left slopes are [1] and [2], and the left dots are ③ and ⑧. If we go to ③ and then ⑧, we can use all 8 different slopes. When we succeeded in making a max-complexity pattern like this, we write '(O)'. The entire notation for this example is as follows.

```
① [-1] ⑤ [∞] ②
   → [-2]! ⑨ [-½]! ④
      → [½] ③ [2]! ⑧ (X)
      → [0] ⑥ [½]! ⑦ [1]! ③ [2]! ⑧ (O)
```

Now we use this notation to find all max-complexity patterns in a 3×3 grid.

2. Starting from the corner

Dots ①, ③, ⑦ or ⑨ are corners of a 3×3 grid. If we find all patterns starting from dot ①, we can find the rest with rotation. Dots ②, ④, ⑤, ⑥ or ⑧ can come after ①, but if we find the cases for ②, ⑤ and ⑥, the rest can be found through symmetry.

A. Starting with '12'

```
① [0] ②
   → [-2]! ⑨ [-½]! ④ [-1]! ⑧ [2]! ③ [∞]! ② [½]! ⑦ [1]! ⑤ (O)
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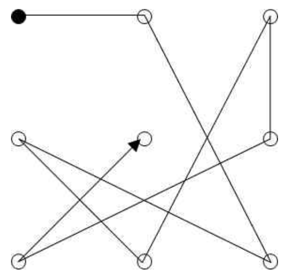
[Fig 3] '129483675'



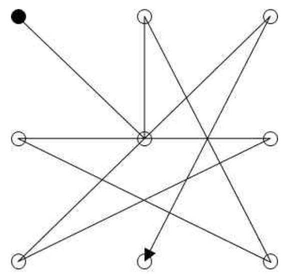
B. Starting with '15'

Next, if we find the cases for ②, ③ and ⑥, the rest can be found through symmetry.

1) Starting with '152'

```
① [-1] ⑤ [∞] ②
   → [-2]! ⑨ [-½]! ④
      → [½] ③ [2]! ⑧ (X)
      → [0] ⑥ [½]! ⑦ [1]! ③ [2]! ⑧ (O)
```

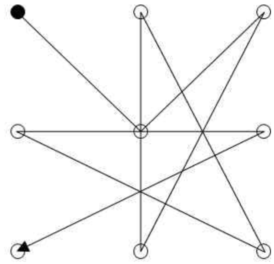
[Fig 4] '152946738'



2) Starting with ‘153’

① [-1] ⑤ [1] ③
→ [0] ② (X) (either [2] or [-2] impossible)
→ [$\frac{1}{2}$] ④ [$-\frac{1}{2}$]! ⑨ [-2]! ② [2]! ⑦ (X)
→ [∞] ⑥ [$\frac{1}{2}$]! ⑦ [2]! ② [-2]! ⑨ (X)
→ [2] ⑧
→ [∞] ② [-2]! ⑨ [$-\frac{1}{2}$]! ④ [0]! ⑥ [$\frac{1}{2}$]! ⑦ (O)
→ [0] ⑦ [$\frac{1}{2}$]! ⑥ [∞]! ⑨ (X)
→ [0] ⑨ [$-\frac{1}{2}$]! ④ (X) ([-2] impossible)

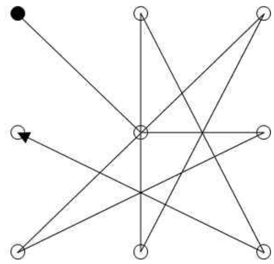
[Fig 5] ‘153829467’



3) Starting with ‘156’

① [-1] ⑤ [0] ⑥
→ [∞] ③ [$\frac{1}{2}$]! ④ (X) ([1] impossible)
→ [$\frac{1}{2}$] ⑦
→ [2] ② [-2]! ⑨ [∞]! ③ (X)
→ [1] ③ [2]! ⑧ [∞]! ② [-2]! ⑨ [$-\frac{1}{2}$]! ④ (O)
→ [∞] ④ [$-\frac{1}{2}$]! ⑨ (X) ([1] impossible)
→ [1] ⑧ [2]! ③ [$\frac{1}{2}$]! ④ [$-\frac{1}{2}$]! ⑨ (X)
→ [∞] ⑨ [-2]! ② (X) ([$-\frac{1}{2}$] impossible)

[Fig 6] ‘156738294’



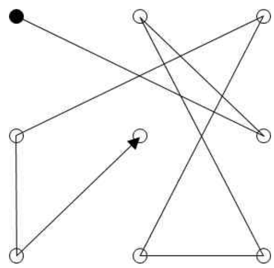
C. Starting with ‘16’

Dots ②, ③, ⑤, ⑦, ⑧ or ⑨ can come afterwards.

1) Starting with ‘162’

① [$-\frac{1}{2}$] ⑥ [-1] ②
→ [-2]! ⑨
→ [∞] ③ [$\frac{1}{2}$]! ④ (X)
→ [0] ⑧ [2]! ③ [$\frac{1}{2}$]! ④ [∞]! ⑦ [1]! ⑤ (O)

[Fig 7] ‘162983475’



2) Starting with '163'

① [−½] ⑥ [∞] ③
- → [½]! ④
  - → [1] ② [2]! ⑦ (X) ([−2] impossible)
  - → [0] ⑤ [1]! ⑦ (X) ([−1] impossible)
  - → [−1] ⑧
    - → [0] ⑦ [2]! ② (X)
    - → [0] ⑨ [−2]! ② [2]! ⑦ [1]! ⑤ (O)

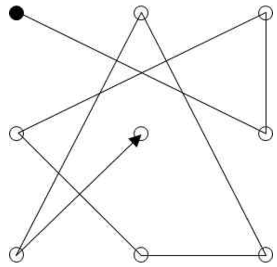
3) Starting with '165'

① [−½] ⑥ [0] ⑤
- → [∞] ② [−2]! ⑨ (X)
- → [1] ③ [½]! ④ [−1]! ⑧ [∞]! ② (X)
- → [1] ⑦
  - → [2] ② [−2]! ⑨ [∞]! ③ [½]! ④ [−1]! ⑧ (O)
  - → [∞] ④ [½]! ③ (X) ([−1] impossible)
- → [∞] ⑧ [−1]! ④ [½]! ③ [1]! ⑦ [2] ② [−2] ⑨ (O)
- → [−1] ⑨ [−2]! ② [2]! ⑦ [1]! ③ (X)

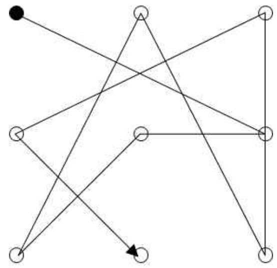
4) Starting with '167'

① [−½] ⑥ [½] ⑦
- → [2] ② [−2]! ⑨
  - → [∞] ③ [1]! ⑤ [0]! ④ [−1]! ⑧ (O)
  - → [−1] ⑤ [1]! ③ (X)
  - → [0] ⑧ [−1]! ④ (X)
- → [∞] ④
  - → [1] ② [−2]! ⑨ [−1]! ⑤ (X)
  - → [0] ⑤ [1]! ③ (X) ([−1] impossible)
  - → [−1] ⑧ [2]! ③ [0]! ② (X)
- → [1] ⑤
  - → [∞] ② [−2]! ⑨ [0]! ⑧ (X)
  - → [0] ④ [−1]! ⑧ [2]! ③ [∞]! ⑨ [−2]! ② (O)
  - → [∞] ⑧ [2]! ③ (X) ([−1] impossible)
  - → [−1] ⑨ [−2]! ② [∞]! ⑧ (X)
- → [0] ⑧ [2]! ③
  - → [1] ⑤ [−1]! ⑨ (X)
  - → [∞] ⑨ [−2]! ② (X)

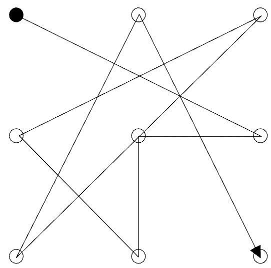
[Fig 8] '163489275'



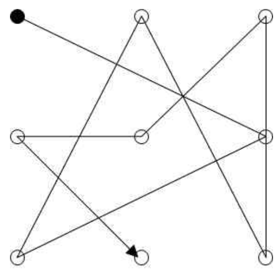
[Fig 9] '165729348'



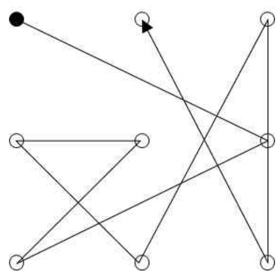
[Fig 10] '165843729'



[Fig 12] '167293548'



[Fig 11] '167548392'



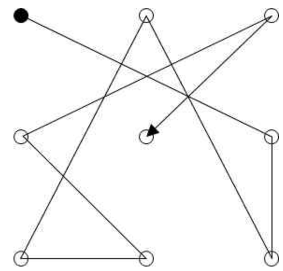
5) Starting with ‘168’

- ① [−½] ⑥ [1] ⑧
  - → [2] ③ [½]! ④
    - → [0] ⑤ [−1]! ⑨ (X)
    - → [∞] ⑦ [0]! ⑨ (X)
  - → [−1] ④ [½]! ③
    - → [0] ② (X) (either [2] or [−2] impossible)
    - → [∞] ⑨ [−2]! ② (X)
  - → [∞] ⑤
    - → [0] ④ [½]! ③ (X)
    - → [−1] ⑨ [−2]! ② [2]! ⑦ (X)
  - → [0] ⑦ [2]! ② [−2]! ⑨ [−1]! ⑤ (X)
  - → [0] ⑨ [−2]! ② (X) ([−1] impossible)

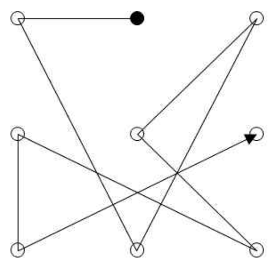
6) Starting with ‘169’

- ① [−½] ⑥ [∞] ⑨
  - → [−2]! ②
    - → [0] ③ [2]! ⑧ (X) ([½] impossible)
    - → [1] ④ [½]! ③ (X) ([−1] impossible)
    - → [2] ⑦
      - → [1] ⑤ [0]! ④ (X)
      - → [0] ⑧ [−1]! ④ [½]! ③ [1]! ⑤ (O)

[Fig 13] ‘169278435’



[Fig 14] ‘218359476’



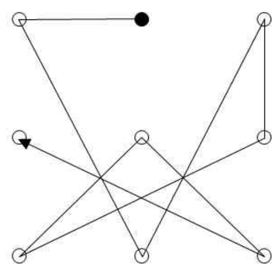
3. Starting from the edge

Dots ②, ④, ⑥ or ⑧ are edges of a 3×3 grid. If we find all patterns starting from dot ②, we can find the rest with rotation. Dots ①, ③, ④, ⑤, ⑥, ⑦ or ⑨ can come after ②, but if we find the cases for ①, ④, ⑤ and ⑦, the rest can be found through symmetry.

A. Starting with ‘21’

- ② [0] ①
  - → [−2]! ⑧ [2]! ③
    - → [½] ④ [−½]! ⑨ [∞]! ⑥ (X)
    - → [1] ⑤ [−1]! ⑨ [−½]! ④ [∞]! ⑦ [½]! ⑥ (O)
    - → [∞] ⑥ [½]! ⑦ [1]! ⑤ [−1]! ⑨ [−½]! ④ (O)

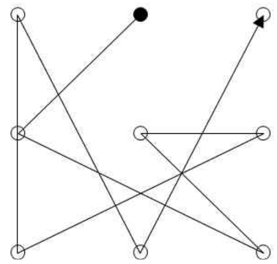
[Fig 15] ‘218367594’



B. Starting with '24'

② [1] ④
→ [∞] ① [−2]! ⑧ (X) ([$-\frac{1}{2}$] impossible)
→ [$\frac{1}{2}$] ③ [2]! ⑧ [−2]! ① [$-\frac{1}{2}$]! ⑥ [∞]! ⑨ (X)
→ [0] ⑤
  → [−1] ① [−2]! ⑧ (X) ([$-\frac{1}{2}$] impossible)
  → [∞] ⑧ [−2]! ① (X) ([2] impossible)
  → [−1] ⑨ [∞]! ⑥ [$-\frac{1}{2}$]! 1 (X) ([$\frac{1}{2}$] impossible)
→ [∞] ⑦
  → [$\frac{1}{2}$] ⑥ [$-\frac{1}{2}$]! ① [−2]! ⑧ [2]! ③ (X)
  → [0] ⑧ [−2]! ① (X) ([2] impossible)
→ [1] ⑧ [−2]! ① (X) ([2] impossible)
→ [$-\frac{1}{2}$] ⑨
  → [−1] ⑤
    → [0] ⑥ [$\frac{1}{2}$]! ⑦ [∞]! ① [−2]! ⑧ [2]! ③ (O)
    → [∞] ⑧ [−2]! ① (X) ([2] impossible)
  → [∞] ⑥ [$\frac{1}{2}$]! ⑦ [0]! ⑧ [−2]! ① (X)
  → [0] ⑧ [−2]! ① (X) ([2] impossible)

[Fig 16] '249567183'



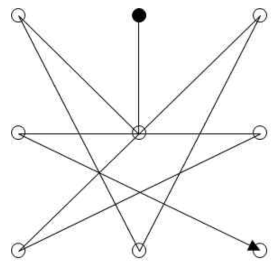
C. Starting with '25'

Next, if we find the cases for ①, ④ and ⑦, the rest can be found through symmetry.

1) Starting with '251'

② [∞] ⑤ [−1] ①
→ [−2]! ⑧ [2]! ③ [1]! ⑦ [$\frac{1}{2}$]! ⑥ [0]! ④ [$-\frac{1}{2}$]! ⑨ (O)

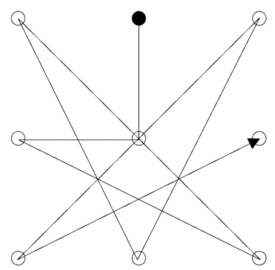
[Fig 17] '251837649'



2) Starting with '254'

② [∞] ⑤ [0] ④
→ [$\frac{1}{2}$] ③ [2]! ⑧ [−2]! ① (X) ([1] impossible)
→ [−1] ⑧ [−2]! ① (X) ([2] impossible)
→ [$-\frac{1}{2}$] ⑨ [−1]! ① [−2]! ⑧ [2]! ③ [1]! ⑦ [$\frac{1}{2}$] ⑥ (O)

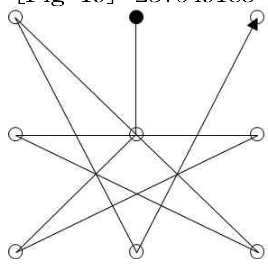
[Fig 18] '254918376'



3) Starting with ‘257’

② [∞] ⑤ [1] ⑦
→ [$\frac{1}{2}$] ⑥
→ [$-\frac{1}{2}$] ① [−2]! ⑧ [2]! ③ (X)
→ [0] ④ [$-\frac{1}{2}$]! ⑨ [−1]! ① [−2]! ⑧ [2]! ③ (O)

[Fig 19] ‘257649183’



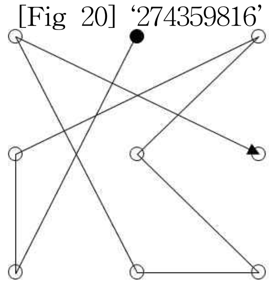
D. Starting with ‘27’

Dots ④, ⑤, ⑥ or ⑧ can come afterwards.

1) Starting with ‘274’

② [2] ⑦ [∞] ④
→ [$\frac{1}{2}$]! ③
→ [0] ① [−2]! ⑧ (X) ([$-\frac{1}{2}$] impossible)
→ [1] ⑤
→ [−1] ① [−2]! ⑧ (X)
→ [0] ⑥ [$-\frac{1}{2}$]! ① (X)
→ [−1] ⑨ [0]! ⑧ [−2]! ① [$-\frac{1}{2}$]! ⑥ (O)

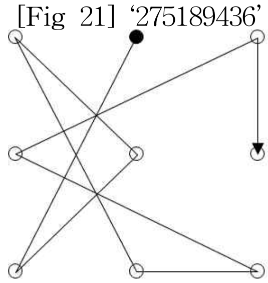
[Fig 20] ‘274359816’



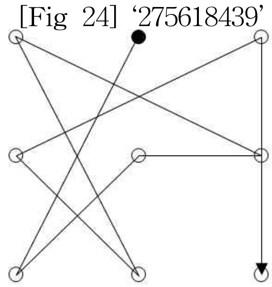
2) Starting with ‘275’

② [2] ⑦ [1] ⑤
→ [−1] ① [−2]! ⑧ [0]! ⑨ [$\frac{1}{2}$]! ④ [$-\frac{1}{2}$]! ③ [∞]! ⑥ (O)
→ [0] ④ [$\frac{1}{2}$]! ③ [∞]! ⑥ [$-\frac{1}{2}$]! ① (X)
→ [0] ⑥
→ [$-\frac{1}{2}$] ① [−2]! ⑧ [−1]! ④ [$\frac{1}{2}$]! ③ [∞]! ⑨ (O)
→ [∞] ③ [$\frac{1}{2}$]! ④ [$-\frac{1}{2}$]! ⑨ [−1]! ① [−2]! ⑧ (O)
→ [∞] ⑨ [$-\frac{1}{2}$]! ④ (X) ([−1] impossible)
→ [∞] ⑧ [−2]! ① [$-\frac{1}{2}$]! ⑥ (X)
→ [−1] ⑨
→ [$-\frac{1}{2}$] ④ [$\frac{1}{2}$]! ③ (X)
→ [∞] ⑥ [$-\frac{1}{2}$]! ① [−2]! ⑧ (X)
→ [0] ⑧ [−2]! ① [$-\frac{1}{2}$]! ⑥ [∞]! ③ [$\frac{1}{2}$]! ④ (O)

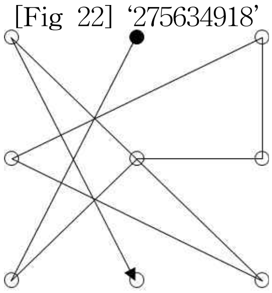
[Fig 21] ‘275189436’



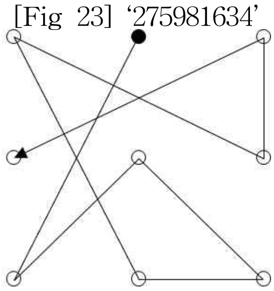
[Fig 24] ‘275618439’



[Fig 22] ‘275634918’



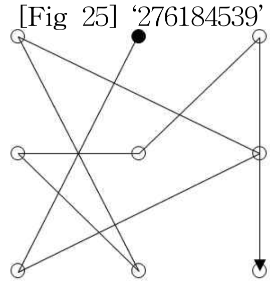
[Fig 23] ‘275981634’



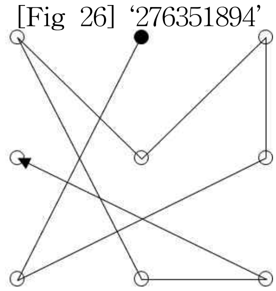
3) Starting with ‘276’

② [2] ⑦ [$\frac{1}{2}$] ⑥
- → [$-\frac{1}{2}$] ① [-2]! ⑧
  - → [-1] ④ [0]! ⑤ [1]! ③ [∞]! ⑨ (O)
  - → [∞] ⑤ [1]! ③ (X)
  - → [0] ⑨ [-1]! ⑤ (X)
- → [∞] ③ [1]! ⑤
  - → [-1] ① [-2]! ⑧ [0]! ⑨ [$-\frac{1}{2}$]! ④ (O)
  - → [0] ④ [$-\frac{1}{2}$]! ⑨ [-1]! ① [-2]! ⑧ (O)
- → [0] ⑤ [1]! ③ [∞]! ⑨ [$-\frac{1}{2}$]! ④ [-1]! ⑧ [-2]! ① (O)
- → [1] ⑧ [-2]! ①
  - → [0] ③ [∞]! ⑨ (X)
  - → [∞] ④ [$-\frac{1}{2}$]! ⑨ (X)
  - → [-1] ⑤ [0]! ④ [$-\frac{1}{2}$]! ⑨ [∞]! ③ (O)
- → [∞] ⑨ [$-\frac{1}{2}$]! ④
  - → [0] ⑤ [1]! ③ (X)
  - → [-1] ⑧ [$-\frac{1}{2}$]! ① [0]! ③ [1]! ⑤ (O)

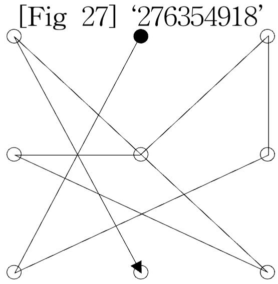
[Fig 25] ‘276184539’



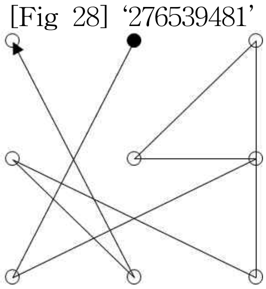
[Fig 26] ‘276351894’



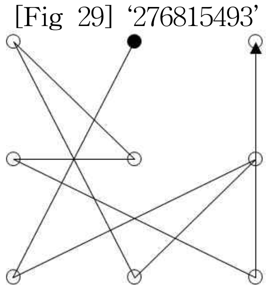
[Fig 27] ‘276354918’



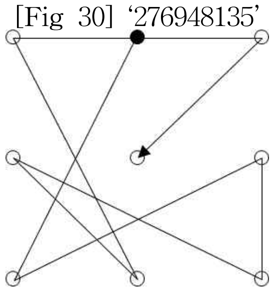
[Fig 28] ‘276539481’



[Fig 29] ‘276815493’



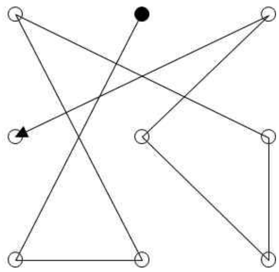
[Fig 30] ‘276948135’



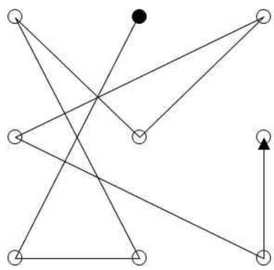
4) Starting with ‘278’

② [2] ⑦ [0] ⑧
- → [-2]! ①
  - → [∞] ④ [$\frac{1}{2}$]! ③ (X) ([$-\frac{1}{2}$] impossible)
  - → [-1] ⑤ [1]! ③ [$\frac{1}{2}$]! ④ [$-\frac{1}{2}$]! ⑨ [∞]! ⑥ (O)
  - → [$-\frac{1}{2}$] ⑥
    - → [∞] ③ [$\frac{1}{2}$]! ④ (X)
    - → [∞] ⑨ [-1]! ⑤ [1]! ③ [$\frac{1}{2}$]! ④ (O)

[Fig 31] ‘278169534’



[Fig 32] ‘278153496’



4. Starting from the center

The dot ⑤ is the center of a 3×3 grid. For the next dot, if we find the cases for ① and ②, the rest can be found through symmetry.

A. Starting with ‘51’

Next, if we find the cases for ② and ⑥, the rest can be found through symmetry.

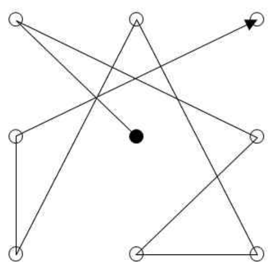
1) Starting with ‘512’

⑤ [−1] ① [0] ②
 → [−2]! ⑨ [$-\frac{1}{2}$]! ④
  → [$\frac{1}{2}$] ③ [2]! ⑧ (X)
  → [∞] ⑦ [$\frac{1}{2}$]! ⑥ [1]! ⑧ [2]! ③ (O)

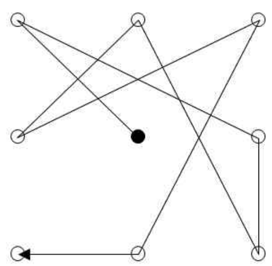
2) Starting with ‘516’

⑤ [−1] ① [$-\frac{1}{2}$] ⑥
 → [∞] ③ [2]! ⑧ (X) ([$\frac{1}{2}$] impossible)
 → [0] ④ [$\frac{1}{2}$]! ③ [1]! ⑦ [2]! ② (X)
 → [$\frac{1}{2}$] ⑦
  → [2] ② [−2]! ⑨ (X) ([1] impossible)
  → [1] ③ [2]! ⑧ [∞]! ② (X)
  → [∞] ④ [1]! ② [−2]! ⑨ [0]! ⑧ [2]! ③ (O)
  → [0] ⑧ [2]! ③ [∞]! ⑨ [−2]! ② [1]! ④ (O)
 → [1] ⑧
  → [∞] ② [−2]! ⑨ (X) ([2] impossible)
  → [2] ③ [$\frac{1}{2}$]! ④ [∞]! ⑦ [0]! ⑨ [−2]! ② (O)
  → [0] ⑦ [2]! ② [−2]! ⑨ [∞]! ③ [$\frac{1}{2}$]! ④ (O)
  → [0] ⑨ [−2]! ② [2]! ⑦ [∞]! ④ [$\frac{1}{2}$]! ③ (O)
 → [∞] ⑨ [−2]! ②
  → [0] ③ [2]! ⑧ (X)
  → [1] ④ [$\frac{1}{2}$]! ③ [2]! ⑧ [0]! ⑦ (O)
  → [2] ⑦ [1]! ③ (X)

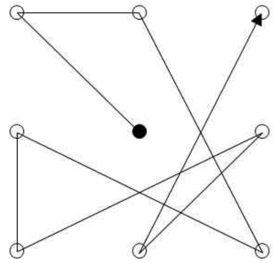
[Fig 38] ‘516892743’



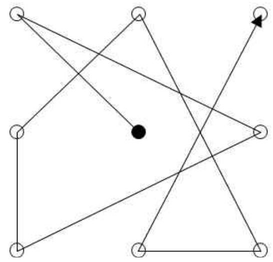
[Fig 39] ‘516924387’



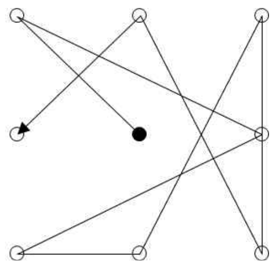
[Fig 33] ‘512947683’



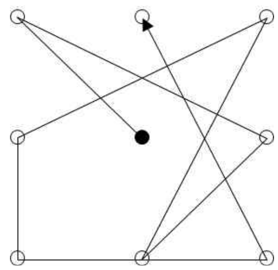
[Fig 34] ‘516742983’



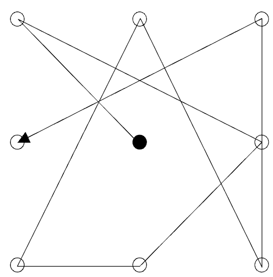
[Fig 35] ‘516783924’



[Fig 36] ‘516834792’



[Fig 37] ‘516872934’



B. Starting with ‘52’

Next, if we find the cases for ①, ④ and ⑦, the rest can be found through symmetry.

1) Starting with ‘521’

⑤ [∞] ② [0] ①
→ [−2]! ⑧ (X) ([−1] impossible)

2) Starting with ‘524’

⑤ [∞] ② [1] ④
→ [½] ③ [2]! ⑧ [−2]! ① [−½]! ⑥ (X)
→ [0] ⑥ [−½]! ① (X) ([½] impossible)
→ [−1] ⑧ [−2]! ① (X) ([2] impossible)
→ [−½] ⑨
→ [−1] ① [−2]! ⑧ [2]! ③ (X)
→ [0] ⑧ [−2]! ① (X) ([2] impossible)

3) Starting with ‘527’

⑤ [∞] ② [2] ⑦
→ [1] ③ [½]! ④
→ [0] ⑥ [−½]! ① (X)
→ [−1] ⑧ [−2]! ① (X)
→ [−½] ⑨ [−1]! ① (X)
→ [½] ⑥ [1]! ⑧ [−2]! ① [−1]! ⑨ (X)
→ [0] ⑧ [−2]! ① (X) ([1] impossible)

5. Results

From chapters 2-4, the number of max-complexity patterns are: 11 that start from the corner, 19 that start from the edge, and 7 that start from the center. This does not include rotated/reflected patterns. If we include the reflection and 4 rotations, the number of max-complexity patterns are $(11+19+7)\times 8 = 296$.