# COMP11120 Notes

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# 1 Discrete Structures

## 1.1 Terminology

A *structure* consists of certain *sets*. It also contains *elements* of these sets, *operations* on these sets and *relations* on these sets.

## 1.2 Number systems to learn

The following number must be learnt:

| | |
|---|---|
| $\mathbb{N}$ | The set of natural numbers (all whole numbers from 0 to $\infty$) |
| $\mathbb{Z}$ | The set of integers (all whole numbers from $-\infty$ to $\infty$) |
| $\mathbb{Q}$ | The set of rational numbers (any integer divided by any other integer e.g. $\frac{5}{4} = 1.25$) |
| $\mathbb{R}$ | The set of real numbers (all finite and infinite decimal numbers) |

### 1.2.1 Operations

Each number system has a set of valid operations that can be performed on elements in that system. Number systems only contain operations that will produce an output that is still within the number system.

For example, the number system $\mathbb{N}$ contains the operations of addition and multiplication. This is because the summation of any two positive integers will *always* be a member of $\mathbb{N}$, and the same goes for multiplication.

However, you may be wondering why subtraction and division aren't included in this number system. This is because for some numbers, the result of subtraction or division won't be inside the set $\mathbb{N}$. An example of this would be subtracting 4 from 2. Even though both of the operands are inside $\mathbb{N}$, the answer isn't.

Different sets may have different operations available. For example, you can concentrate any two members of the set $\mathbb{S}tring$ and end up with another $\mathbb{S}tring$.

## 1.3 Bases

Conventionally, we count using base 10. Base 10 includes, you guessed it, ten different symbols from 0 through to 9.

Sometimes however, it is convenient to count using different bases. Popular bases include:

| **Base** $n$ | **Member symbols** | **Name** |
|---|---|---|
| $n = 2$ | $\mathbb{Z}_2 = \{0, 1\}$ | Binary |
| $n = 8$ | $\mathbb{Z}_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ | Octal |
| $n = 10$ | $\mathbb{Z}_{10} = \{0, 1, 2, 3, 4, 5, 6, 7\}$ | Decimal/Denary |
| $n = 16$ | $\mathbb{Z}_{16} = \{0, 1, 2, \ldots, 9, A, B, C, D, E, F\}$ | Hexadecimal |

### 1.3.1 How to read numbers in any given base

The formula for reading a number in a given base is as follows:

$$\sum_{i=0}^{k} a_i b^i$$

Where the number you're trying to read takes the form $a_k, a_{k-1}, \ldots, a_2, a_1, a_0$ and $b$ is the base you're using.

**Example 1** Lets apply the formula to the base 10 number 27385:

$$\begin{aligned}
27385 &= (5 \times 10^0) + (8 \times 10^1) + (3 \times 10^2) + (7 \times 10^3) + (2 \times 10^4) \\
&= (5 \times 1) + (8 \times 10) + (3 \times 100) + (7 \times 1000) + (2 \times 10000) \\
&= 5 + 80 + 300 + 7000 + 20000 \\
&= 27385
\end{aligned}$$

**Example 2** Lets apply the formula to the base 16 number $F00BA4$:

$$\begin{aligned}
F00BA4 &= (4 \times 16^0) + (A \times 16^1) + (B \times 16^2) + (0 \times 16^3) + (0 \times 16^4) + (F \times 16^5) \\
&= (4 \times 16^0) + (10 \times 16) + (11 \times 256) + (0 \times 4096) + (0 \times 65536) + (15 \times 1048576) \\
&= 4 + 160 + 2816 + 0 + 0 + 15728640 \\
&= 15731620
\end{aligned}$$

### 1.3.2 Changing from base 10 to base $n$

In order to change into base $n$ from base 10, we just repeatedly divide by $n$ and use the remainder as the value for base $n$. Here are a few examples:

**Example 1** Convert 893 into base 2.

$$\begin{array}{rcll} 893 \div 2 & = & 446 & \text{r1} \\ 446 \div 2 & = & 223 & \text{r0} \\ 223 \div 2 & = & 111 & \text{r1} \\ 111 \div 2 & = & 55 & \text{r1} \\ 55 \div 2 & = & 27 & \text{r1} \\ 27 \div 2 & = & 13 & \text{r1} \\ 13 \div 2 & = & 6 & \text{r1} \\ 6 \div 2 & = & 3 & \text{r0} \\ 3 \div 2 & = & 1 & \text{r1} \\ 1 \div 2 & = & 0 & \text{r1} \end{array}$$

Reading up from the bottom, we can see that the binary (base 2) representation is 1101111101.

**Example 2** Convert 893 into base 9.

$$\begin{array}{rcll} 893 \div 9 & = & 99 & \text{r2} \\ 99 \div 9 & = & 11 & \text{r0} \\ 11 \div 9 & = & 1 & \text{r2} \\ 1 \div 9 & = & 0 & \text{r1} \end{array}$$

Reading up from the bottom, we can see that the nonal (base 9) representation is 1202.

**Example 2** Convert 893 into base 16.

$$\begin{array}{rcll} 893 \div 16 & = & 55 & \text{r13} \\ 55 \div 16 & = & 3 & \text{r7} \\ 3 \div 16 & = & 0 & \text{r3} \end{array}$$

Reading up from the bottom, we can see that the hexadecimal (base 16) representation is $3, 7, 13$ or $37D$.