Assignment 1 submission

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Question 1

The binary operation $*$ is defined on the set $\mathbb{Q}$ of rational numbers by:

$$p * q = 2p + 2q - pq - 2$$

Part a

Proof of commutativity:

$$\begin{aligned} p * q &= 2p + 2q - pq - 2 \\ &= 2q + 2p - qp - 2 \\ &= q * p \end{aligned}$$

Part b

Proof of associativity:

$$\begin{aligned} (p * q) * r &= (2p + 2q - pq - 2) * r \\ &= 4p + 4q - 2pq - 4 + 2r - 2pr - 2qr + rpq + 2r - 2 \\ &= 4p + 4q - 2pq + 4r - 2pr - 2qr + rpq - 6 \\ &= 4r + 4q - 2pq - 4 + 2p - 2qp - 2rp + pqr + 2p - 2 \\ &= (2r + 2q - rq - 2) * p \\ &= (r * q) * p \end{aligned}$$

Question 2

A small formal language R of expressions is defined by the following rules:

- Each of the lower case letter symbols (from a to z) is an expression.
- If A is an expression, then so is $A!$
- If A and B are expressions then so are $AB\circledcirc$, $AB\ominus$ and $AB\circledast$.

Some examples of expressions in this language may be:

- a
- $a!$
- $ab\circledcirc$
- $ab!\ominus$
- $abc \circledast \circledcirc$

See the attached sheet for the parse tree of the expression $np!q\circledcirc!rs \ominus \circledast \circledcirc t\ominus$:

Question 3

For any two subsets A and B of some set S,

$$(A \cup B) - (A \cap B) = (A - B) \cup (B - A)$$

Where $X - Y = X \cap Y'$

Part a

See attached sheet.

Part b

Algebraic proof:

$$\begin{aligned}
(A \cup B) - (A \cap B) &= (A \cup B) \cap (A \cap B)' \\
&= (A \cup B) \cap (A' \cup B') \\
&= ((A' \cup B') \cap A) \cup ((A' \cup B') \cap B) \\
&= ((A \cap A') \cup (A \cap B')) \cup ((B \cap A') \cup (B \cap B')) \\
&= (\emptyset \cup (A \cap B')) \cup ((B \cap A') \cup \emptyset) \\
&= (A \cap B') \cup (B \cap A') \\
&= (A - B) \cup (B - A)
\end{aligned}$$