The set $\mathbb{N}$ contains?

1

The set $\mathbb{R}$ contains?

2

The set $\mathbb{Z}$ contains?

3

The set $\mathbb{Q}$ contains?

4

What is this?
$\emptyset$

5

What is this?
$\mathbb{S}$

6

What does $X \subseteq Y$ mean?

7

What does $'$ mean after a set (or ^{c})?

8

What does $x \in X$ mean?

9

What does $x \notin X$ mean?

10

For each a in X, $a \in X \iff a \in Y$.
How is this represented?

11

How else could we express:
$X \subseteq Y \iff Y \subseteq X$

12

The set of natural numbers (all non-negative integers).

1

The set of real numbers (all finite and infinite decimal numbers).

2

The set of integers.

3

The set of rational numbers.
Contains all m/n for $m, n \in \mathbb{Z}$

4

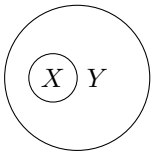
The null set.

5

The universal set, containing all possible elements.

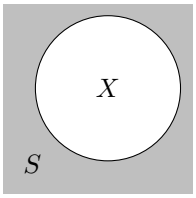
6

X is a subset of Y
Y is a superset of X
X is included in Y
Y includes X



7

The complement of the set. E.g. X':



8

x is contained in / is a member of X

9

x is not a member of X

10

$X = Y$

11

$X = Y$

12

What does $X \cup Y$ mean?

13

What does $X \cap Y$ mean?

14

The truth table for the `and` *function is:*

Input 1	*Input 2*	*Input 1* `and` *Input 2*
T	T	
T	F	
F	T	
F	F	

15

The truth table for the `or` *function is:*

Input 1	*Input 2*	*Input 1* `or` *Input 2*
T	T	
T	F	
F	T	
F	F	

16

The truth table for the `implies` *function is:*

Input 1	*Input 2*	*Input 1* `implies` *Input 2*
T	T	
T	F	
F	T	
F	F	

17

An operation is [redacted] *if:*

$$a1 \circledast a2 = a2 \circledast a1$$

18

An operation is [redacted] *if:*

$$(a1 \circledast a2) \circledast a3 = a1 \circledast (a2 \circledast a3)$$

19

Is $(v + w + x)$ a valid expression in the formal language?

20

Is $(x + 4)$ a valid expression in the formal language?

21

Is $((x \times 0) + (y + z))$ a valid expression in the formal language?

22

What expression does this parse tree represent?

$$\dfrac{x \quad \dfrac{y \quad z}{\cdot}\ (\times)}{\cdot}\ (-)$$

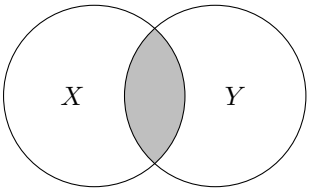
23

Evaluate the following parse tree

$$\dfrac{140 \quad \dfrac{10 \quad \dfrac{3 \quad 1}{\cdot}\ (\times)}{\cdot}\ (-)}{\cdot}\ (\div)$$

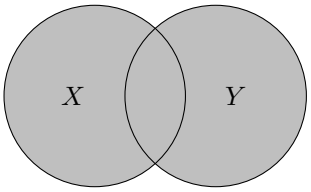
24

*The **intersection** of the sets X and Y.*



14

*The **union** of the sets X and Y.*



13

The truth table for the `or` *function is:*

Input 1	*Input 2*	*Input 1* `or` *Input 2*
T	T	T
T	F	T
F	T	T
F	F	F

16

The truth table for the `and` *function is:*

Input 1	*Input 2*	*Input 1* `and` *Input 2*
T	T	T
T	F	F
F	T	F
F	F	F

15

An operation is commutative if:

$$a1 \circledast a2 = a2 \circledast a1$$

18

The truth table for the `implies` *function is:*

Input 1	*Input 2*	*Input 1* `implies` *Input 2*
T	T	T
T	F	F
F	T	T
F	F	T

17

No, there aren't enough brackets.
$((v + w) + x)$ *would be valid though!*

20

An operation is associative if:

$$(a1 \circledast a2) \circledast a3 = a1 \circledast (a2 \circledast a3)$$

19

No, since there are too many brackets.
$((x \times 0) + (y + z))$ *would be valid though!*

22

No, since 4 *isn't an allowable atom.*
$(x + 0)$ *would be valid though!*

21

$$(140 \div (10 - (3 \times 1))) = 20$$

$$\begin{array}{r} \overline{3\quad 1}\ (\times) \\ \overline{10\quad\ 3}\ (-) \\ \overline{140\quad\ \ 7}\ (\div) \\ 20 \end{array}$$

24

$$(x - (y \times z))$$

23

Use the fact that $\cup$ is associative to re-arrange:
$X \cup (Y \cup Z)$

25

Use the fact that $\cap$ is associative to re-arrange:
$X \cap (Y \cap Z)$

26

Use the distributive law on:
$X \cup (Y \cap Z)$

27

Use the distributive law on:
$X \cap (Y \cup Z)$

28

Use absorbsion on:
$X \cup (X \cap Y)$

29

Use absorbsion on:
$X \cap (X \cup Y)$

30

What three things happen when De Morgan's law is applied to an expression?

31

What does involution mean?

32

What is the symbol for logical negation?

33

What is the symbol for conjunction?

34

What is the symbol for disjunction?

35

What is the symbol for logical and?

36

$$Y \cap (X \cap Z)$$
or
$$Z \cap (X \cap Y)$$

26

$$Y \cup (X \cup Z)$$
or
$$Z \cup (X \cup Y)$$

25

$$(X \cap Y) \cup (X \cap Z)$$

28

$$(X \cup Y) \cap (X \cup Z)$$

27

$$X$$

30

$$X$$

29

If an expression is negated twice, they cancel each other out.
$$X'' = X$$

32

1. *The expression is negated (involution is applied if it's already negated)*
2. *Each sub expression is negated (again, applying involution)*
3. *Each union inside the expression is turned into an intersection and vice versa*

31

$$\wedge$$

34

$$\neg$$

33

$$\wedge$$

36

$$\vee$$

35

What is the symbol for logical or?

37

What is the symbol for implication?

38

What is the symbol for bi-implication?

39

The truth table for the **`bi-implication`** *function is:*

Input 1	*Input 2*	*Input 1* $\iff$ *Input 2*
T	T	
T	F	
F	T	
F	F	

40

An expression is a [redacted] *when all of it's possible outcomes are true*

41

An expression is [redacted] *when at least one of it's possible outcomes are true*

42

An expression is a [redacted] *when none of it's possible outcomes are true*

43

What is the notation to say A is a tautology?

44

What is the notation to say A is satisfiable?

45

What is the notation to say A is a contradiction?

46

Use the fact that $\vee$ is associative to re-arrange:
$X \vee (Y \vee Z)$

47

Use the fact that $\wedge$ is associative to re-arrange:
$X \wedge (Y \wedge Z)$

48

$\implies$

38

$\vee$

37

The truth table for the `bi-implication` *function is:*

Input 1	*Input 2*	*Input 1* $\iff$ *Input 2*
T	T	T
T	F	F
F	T	F
F	F	T

40

$\iff$

39

An expression is satisfiable when at least one of it's possible outcomes are true

42

An expression is a tautology when all of it's possible outcomes are true

41

$\models A$

44

An expression is a contradiction when none of it's possible outcomes are true

43

$\not\models A$

46

$\not\models \neg A$

45

$Y \wedge (X \wedge Z)$
or
$Z \wedge (X \wedge Y)$

48

$Y \vee (X \vee Z)$
or
$Z \vee (X \vee Y)$

47

What are the two possible rearrangements of $A \implies B$*?*

49

What is the rearrangement of $\neg(A \implies B)$

50

What is the rearrangement of $A \implies \neg B$*?*

51

Rearrange $A \iff B$

52

Rearrange $A \iff B$

53

Rearrange $A \iff B$

54

Rearrange $\neg(A \iff B)$

55

Rearrange $\neg(A \iff B)$

56

What two conditions are there for Negation Normal Form?

57

What two steps do we do to get an expression into Negation Normal Form?

58

What three conditions are there for Conjunctive Normal Form?

59

What two steps do we do to get an expression into Conjunctive Normal Form?

60

$$A \wedge \neg B$$

50

$$\neg A \vee B$$
$$\neg B \implies \neg A$$

49

$$(A \implies B) \wedge (B \implies A)$$

52

$$B \implies \neg A$$

51

$$(A \wedge B) \vee (\neg A \wedge \neg B)$$

54

$$(\neg A \vee B) \wedge (\neg B \vee A)$$

53

$$\neg(A \wedge B) \wedge (A \vee B)$$

56

$$(A \wedge \neg B) \vee (B \wedge \neg A)$$

55

1. *Remove all implication and bi-implication operations by applying the logical identities*
2. *Apply De Morgan's laws to any expressions that are negated*

58

1. *The expression is build up of literals using only conjunction and disjunction*
2. *Negation can be used, but only on literals, not expressions*

N.b. a literal is a formula that is either atomic or the negation of an atomic formula (i.e. x or $\neg x$)

57

1. *Get rid of nested brackets using identities*
2. *Use the distributive identities to bring all the disjunctions inside the conjunctions.*

60

1. *The formula must be in NNF already*
2. *There must be no nested brackets*
3. *Conjunction must be used outside of the brackets, and disjunction inside the brackets*

59

What is the CNF test for tautologies?

61

What three conditions are there for Disjunctive Normal Form?

62

What is the DNF test for contradictions?

63

What is the universal quantifier?

64

What is the existential quantifier?

65

What can we do to a universal quantifier with a negation such as this:

$$\neg\forall x P(x)$$

66

What can we do to an existential quantifier with a negation such as this:

$$\neg\exists x P(x)$$

67

What is the arity of a unary symbol?

68

Is disjunction inclusive or exclusive?

69

What does 'iff' mean?

70

What does 'PL' stand for?

71

What is a truth valuation?

72

1. *The formula must be in NNF already*
2. *There must be no nested brackets*
3. *Disjunction must be used outside of the brackets, and conjunction inside the brackets*

62

Each expression in the formula must have both a literal and the negation of the literal.

E.g. $(p_1 \vee p_2 \vee \neg p_1) \wedge (p_3 \vee \neg p_2 \vee p_2)$

61

$\forall$

64

Each expression in the formula must have both a literal and the negation of the literal.

E.g. $(p_1 \wedge p_2 \wedge \neg p_1) \vee (p_3 \wedge \neg p_2 \wedge p_2)$

63

We can turn it into an existential quantifier, such as:

$\exists x \neg P(x)$

66

$\exists$

65

1

Arity - the number of arguments that a function can take

68

We can turn it into a universal quantifier, such as:

$\forall x \neg P(x)$

67

If and only if.

70

Inclusive.

69

A truth valuation is a list of values define the input values for an expression. E.g.:
$(x = T, y = F)$

72

Propositional Logic

71

If $A \iff B$ is a tautology, what does that mean?

73

How can we show that $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$?

74

$$\begin{aligned}\mathbb{P}(A \cup B) &= \sum_{i=1}^{n} 1_{A\cup B}(\omega_i)p_i \\ &= \sum_{i=1}^{n} (1_A(\omega_i) + 1_B(\omega_i))p_i \\ &= \sum_{i=1}^{n} (1_A(\omega_i))p_i + \sum_{i=1}^{n} (1_A(\omega_i))p_i \\ &= \mathbb{P}(A) + \mathbb{P}(B)\end{aligned}$$

A and B are logically equivalent.