*The set $\mathbb{N}$ contains?*

1

*The set $\mathbb{R}$ contains?*

2

*The set $\mathbb{Z}$ contains?*

3

*The set $\mathbb{Q}$ contains?*

4

*What is this?*
$\emptyset$

5

*What is this?*
$\mathbb{S}$

6

*What does $X \subseteq Y$ mean?*

7

*What does $'$ mean after a set (or $^c$)?*

8

*What does $x \in X$ mean?*

9

*What does $x \notin X$ mean?*

10

*For each $a$ in $X$, $a \in X \iff a \in Y$.*
*How is this represented?*

11

*How else could we express:*
$X \subseteq Y \iff Y \subseteq X$

12

*The set of natural numbers (all non-negative integers).*

1

*The set of real numbers (all finite and infinite decimal numbers).*

2

*The set of integers.*

3

*The set of rational numbers.*
*Contains all $m/n$ for $m, n \in \mathbb{Z}$*

4

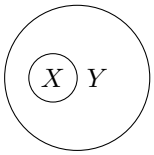
*The null set.*

5

*The universal set, containing all possible elements.*

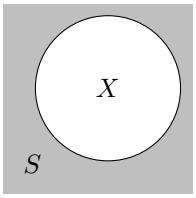
6

*$X$ is a subset of $Y$*
*$Y$ is a superset of $X$*
*$X$ is included in $Y$*
*$Y$ includes $X$*




7

*The complement of the set. E.g. $X'$:*




8

*$x$ is contained in / is a member of $X$*

9

*$x$ is not a member of $X$*

10

$X = Y$

11

$X = Y$

12

*What does $X \cup Y$ mean?*

13

*What does $X \cap Y$ mean?*

14

*The truth table for the* `and` *function is:*

| *Input 1* | *Input 2* | *Input 1* `and` *Input 2* |
|---|---|---|
| $T$ | $T$ | |
| $T$ | $F$ | |
| $F$ | $T$ | |
| $F$ | $F$ | |

15

*The truth table for the* `or` *function is:*

| *Input 1* | *Input 2* | *Input 1* `or` *Input 2* |
|---|---|---|
| $T$ | $T$ | |
| $T$ | $F$ | |
| $F$ | $T$ | |
| $F$ | $F$ | |

16

*The truth table for the* `implies` *function is:*

| *Input 1* | *Input 2* | *Input 1* `implies` *Input 2* |
|---|---|---|
| $T$ | $T$ | |
| $T$ | $F$ | |
| $F$ | $T$ | |
| $F$ | $F$ | |

17

*An operation is* [redacted] *if:*

$$a1 \circledast a2 = a2 \circledast a1$$

18

*An operation is* [redacted] *if:*

$$(a1 \circledast a2) \circledast a3 = a1 \circledast (a2 \circledast a3)$$

19

*Is $(v + w + x)$ a valid expression in the formal language?*

20

*Is $(x + 4)$ a valid expression in the formal language?*

21

*Is $((x \times 0) + (y + z))$ a valid expression in the formal language?*

22

*What expression does this parse tree represent?*

$$\dfrac{x \quad \dfrac{y \quad z}{\cdot}\ (\times)}{\cdot}\ (-)$$

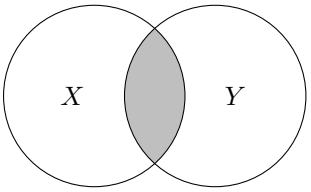
23

*Evaluate the following parse tree*

$$\dfrac{140 \quad \dfrac{10 \quad \dfrac{3 \quad 1}{\cdot}\ (\times)}{\cdot}\ (-)}{\cdot}\ (\div)$$

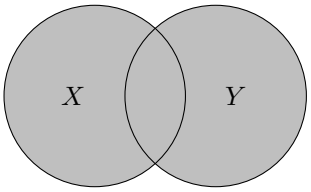
24

*The **intersection** of the sets $X$ and $Y$.*




14

*The **union** of the sets $X$ and $Y$.*




13

*The truth table for the* `or` *function is:*

| *Input 1* | *Input 2* | *Input 1* `or` *Input 2* |
|---|---|---|
| $T$ | $T$ | $T$ |
| $T$ | $F$ | $T$ |
| $F$ | $T$ | $T$ |
| $F$ | $F$ | $F$ |

16

*The truth table for the* `and` *function is:*

| *Input 1* | *Input 2* | *Input 1* `and` *Input 2* |
|---|---|---|
| $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ |
| $F$ | $T$ | $F$ |
| $F$ | $F$ | $F$ |

15

*An operation is commutative if:*

$$a1 \circledast a2 = a2 \circledast a1$$

18

*The truth table for the* `implies` *function is:*

| *Input 1* | *Input 2* | *Input 1* `implies` *Input 2* |
|---|---|---|
| $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ |
| $F$ | $T$ | $T$ |
| $F$ | $F$ | $T$ |

17

*No, there aren't enough brackets.*
$((v + w) + x)$ *would be valid though!*

20

*An operation is associative if:*

$$(a1 \circledast a2) \circledast a3 = a1 \circledast (a2 \circledast a3)$$

19

*No, since there are too many brackets.*
$((x \times 0) + (y + z))$ *would be valid though!*

22

*No, since* 4 *isn't an allowable atom.*
$(x + 0)$ *would be valid though!*

21

$$(140 \div (10 - (3 \times 1))) = 20$$

$$\begin{array}{r} \phantom{140\ \div\ (}\overline{3\quad 1}\ (\times) \\ \phantom{140\ \div\ }\overline{10\quad\ 3}\ (-) \\ \overline{140\quad\ \ 7}\ (\div) \\ 20 \end{array}$$

24

$$(x - (y \times z))$$

23

*Use the fact that $\cup$ is associative to re-arrange:*
$X \cup (Y \cup Z)$

25

*Use the fact that $\cap$ is associative to re-arrange:*
$X \cap (Y \cap Z)$

26

*Use the distributive law on:*
$X \cup (Y \cap Z)$

27

*Use the distributive law on:*
$X \cap (Y \cup Z)$

28

*Use absorbsion on:*
$X \cup (X \cap Y)$

29

*Use absorbsion on:*
$X \cap (X \cup Y)$

30

*What three things happen when De Morgan's law is applied to an expression?*

31

*What does involution mean?*

32

*What is the symbol for logical negation?*

33

*What is the symbol for conjunction?*

34

*What is the symbol for disjunction?*

35

*What is the symbol for logical and?*

36

$$Y \cap (X \cap Z)$$
*or*
$$Z \cap (X \cap Y)$$

26

$$Y \cup (X \cup Z)$$
*or*
$$Z \cup (X \cup Y)$$

25

$$(X \cap Y) \cup (X \cap Z)$$

28

$$(X \cup Y) \cap (X \cup Z)$$

27

$$X$$

30

$$X$$

29

*If an expression is negated twice, they cancel each other out.*
$$X'' = X$$

32

1. *The expression is negated (involution is applied if it's already negated)*
2. *Each sub expression is negated (again, applying involution)*
3. *Each union inside the expression is turned into an intersection and vice versa*

31

$$\wedge$$

34

$$\neg$$

33

$$\wedge$$

36

$$\vee$$

35

*What is the symbol for logical or?*

37

*What is the symbol for implication?*

38

*What is the symbol for bi-implication?*

39

*The truth table for the* **`bi-implication`** *function is:*

| *Input 1* | *Input 2* | *Input 1* $\iff$ *Input 2* |
|---|---|---|
| $T$ | $T$ | |
| $T$ | $F$ | |
| $F$ | $T$ | |
| $F$ | $F$ | |

40

*An expression is a* [redacted] *when all of it's possible outcomes are true*

41

*An expression is* [redacted] *when at least one of it's possible outcomes are true*

42

*An expression is a* [redacted] *when none of it's possible outcomes are true*

43

*What is the notation to say $A$ is a tautology?*

44

*What is the notation to say $A$ is satisfiable?*

45

*What is the notation to say $A$ is a contradiction?*

46

*Use the fact that $\vee$ is associative to re-arrange:*
$X \vee (Y \vee Z)$

47

*Use the fact that $\wedge$ is associative to re-arrange:*
$X \wedge (Y \wedge Z)$

48

$\implies$

38

$\vee$

37

*The truth table for the* `bi-implication` *function is:*

| *Input 1* | *Input 2* | *Input 1* $\iff$ *Input 2* |
|---|---|---|
| $T$ | $T$ | $T$ |
| $T$ | $F$ | $F$ |
| $F$ | $T$ | $F$ |
| $F$ | $F$ | $T$ |

40

$\iff$

39

*An expression is satisfiable when at least one of it's possible outcomes are true*

42

*An expression is a tautology when all of it's possible outcomes are true*

41

$\models A$

44

*An expression is a contradiction when none of it's possible outcomes are true*

43

$\not\models A$

46

$\not\models \neg A$

45

$Y \wedge (X \wedge Z)$
*or*
$Z \wedge (X \wedge Y)$

48

$Y \vee (X \vee Z)$
*or*
$Z \vee (X \vee Y)$

47

*What are the two possible rearrangements of* $A \implies B$*?*

49

*What is the rearrangement of* $\neg(A \implies B)$

50

*What is the rearrangement of* $A \implies \neg B$*?*

51

*Rearrange* $A \iff B$

52

*Rearrange* $A \iff B$

53

*Rearrange* $A \iff B$

54

*Rearrange* $\neg(A \iff B)$

55

*Rearrange* $\neg(A \iff B)$

56

*What two conditions are there for Negation Normal Form?*

57

*What two steps do we do to get an expression into Negation Normal Form?*

58

*What three conditions are there for Conjunctive Normal Form?*

59

*What two steps do we do to get an expression into Conjunctive Normal Form?*

60

$$A \wedge \neg B$$

50

$$\neg A \vee B$$
$$\neg B \implies \neg A$$

49

$$(A \implies B) \wedge (B \implies A)$$

52

$$B \implies \neg A$$

51

$$(A \wedge B) \vee (\neg A \wedge \neg B)$$

54

$$(\neg A \vee B) \wedge (\neg B \vee A)$$

53

$$\neg(A \wedge B) \wedge (A \vee B)$$

56

$$(A \wedge \neg B) \vee (B \wedge \neg A)$$

55

1. *Remove all implication and bi-implication operations by applying the logical identities*
2. *Apply De Morgan's laws to any expressions that are negated*

58

1. *The expression is build up of literals using only conjunction and disjunction*
2. *Negation can be used, but only on literals, not expressions*

*N.b. a literal is a formula that is either atomic or the negation of an atomic formula (i.e. $x$ or $\neg x$)*

57

1. *Get rid of nested brackets using identities*
2. *Use the distributive identities to bring all the disjunctions inside the conjunctions.*

60

1. *The formula must be in NNF already*
2. *There must be no nested brackets*
3. *Conjunction must be used outside of the brackets, and disjunction inside the brackets*

59

*What is the CNF test for tautologies?*

61

*What three conditions are there for Disjunctive Normal Form?*

62

*What is the DNF test for contradictions?*

63

*What is the universal quantifier?*

64

*What is the existential quantifier?*

65

*What can we do to a universal quantifier with a negation such as this:*

$$\neg\forall x P(x)$$

66

*What can we do to an existential quantifier with a negation such as this:*

$$\neg\exists x P(x)$$

67

*What is the arity of a unary symbol?*

68

*Is disjunction inclusive or exclusive?*

69

*What does 'iff' mean?*

70

*What does 'PL' stand for?*

71

*What is a truth valuation?*

72

1. *The formula must be in NNF already*
2. *There must be no nested brackets*
3. *Disjunction must be used outside of the brackets, and conjunction inside the brackets*

62

*Each expression in the formula must have both a literal and the negation of the literal.*

*E.g.* $(p_1 \vee p_2 \vee \neg p_1) \wedge (p_3 \vee \neg p_2 \vee p_2)$

61

$\forall$

64

*Each expression in the formula must have both a literal and the negation of the literal.*

*E.g.* $(p_1 \wedge p_2 \wedge \neg p_1) \vee (p_3 \wedge \neg p_2 \wedge p_2)$

63

*We can turn it into an existential quantifier, such as:*

$\exists x \neg P(x)$

66

$\exists$

65

*1*

*Arity - the number of arguments that a function can take*

68

*We can turn it into a universal quantifier, such as:*

$\forall x \neg P(x)$

67

*If and only if.*

70

*Inclusive.*

69

*A truth valuation is a list of values define the input values for an expression. E.g.:*
$(x = T, y = F)$

72

*Propositional Logic*

71

*If $A \iff B$ is a tautology, what does that mean?*

73

*How can we show that $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$?*

74

*How does $\mathbb{P}(A^c)$ relate to $\mathbb{P}(A)$?*

75

*If $B \subseteq A$ what is the probability of the set difference between $A$ and $B$?*

76

$\mathbb{P}(A \cup B) =$

77

$$\begin{aligned}\mathbb{P}(A \cup B) &= \sum_{i=1}^{n} 1_{A\cup B}(\omega_i)p_i \\ &= \sum_{i=1}^{n} (1_A(\omega_i) + 1_B(\omega_i))p_i \\ &= \sum_{i=1}^{n} (1_A(\omega_i))p_i + \sum_{i=1}^{n} (1_A(\omega_i))p_i \\ &= \mathbb{P}(A) + \mathbb{P}(B)\end{aligned}$$

74

*$A$ and $B$ are logically equivalent.*

73

$$\mathbb{P}(A^c) = 1 - \mathbb{P}(A)$$

75

$$\mathbb{P}(A \ B) = \mathbb{P}(A) - \mathbb{P}(B)$$

*Also,* $\mathbb{P}(B) \geq \mathbb{P}(A)$

76

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(A \cap B)$$

77