Give the identity for:

$$\sum_{i=0}^{n} i$$

1

Give the identity for:

$$\sum_{i=0}^{n} x^i$$

2

Give the identity for:

$$\sum_{i=0}^{n} i(i+1)$$

3

Give the identity for:

$$\sum_{i=0}^{n} i^2$$

4

Define reflexivity in a mathematical way.

5

Define symmetric relations in a mathematical way.

6

Define transitive relations in a mathematical way.

7

Define anti-symmetric relations in a mathematical way.

8

$$\frac{x^{n+1}-1}{x-1}$$

2

$$\frac{n(n+1)}{2}$$

1

$$\frac{n(n+1)(2n+1)}{6}$$

4

$$\frac{n(n+1)(n+2)}{3}$$

3

$$\forall a, b \in A, aRb \implies bRa$$

6

$$\forall a \in A, aRa$$

5

$$\forall a, b, aRb \implies a = b$$

8

$$\forall a, b, c \in A, (aRb \land bRc) \implies aRc$$

7

What three conditions must be satisfied for a relation to be an equivalence relation?

9

Define a partial order.

10

When can we define relational composition?

11

In a right handed axes, your ____ should point in the direction of the ____ axis, your ____ should point in the direction of the ____ axis, and your ____ should point in the direction of the ____ axis.

12

How do you add two matrices together? What are the conditions for matrix addition?

13

How can a matrix be scaled?

14

How is matrix subtraction performed? What are the conditions?

15

What is the condition for matrix multiplication?

16

A relation that is reflexive, anti-symmetric and transitive.

10

The relation bust be reflexive, symmetric and transitive.

9

In a right handed axes, your thumb should point in the direction of the x axis, your fore finger should point in the direction of the y axis, and your middle finger should point in the direction of the z axis.

12

When the target of one relation is the same as the source of another.

11

Just multiply each cell in the matrix by the scaling factor.

14

In order for you to be able to add two matrices together, they must both have the same dimensions. Then, just add value of each position in one matrix to the corresponding position in the other matrix.

$$(A+B)_{ij} = A_{ij} + B_{ij}$$

13

The number of columns in the first matrix must equal the number of rows in the second.

16

In order to subtract one matrix from another, just scale the first one by a factor of -1 and add them together. As with addition, the matrices must have the same dimensions.

$$(A-B)_{ij} = (A + (-1)B)_{ij}$$

15

How do we determine the value of the cell i, j when multiplying a matrix A by another matrix B?

17

What is the identity matrix?

18

How do you work out the transpose of a matrix? What is the notation?

19

What is a symmetric matrix?

20

Define the zero matrix.

21

Define the term **commuting matrices**.

22

Is matrix multiplication associative?

23

Is matrix multiplication a distributive operation?

24

A square matrix where every cell is set to zero, except from those on the diagonal from the top left to the bottom right, where they are set to one. For example:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

18

It depends on the values in the i'th row of A and the j'th column of B.

$$C_{ij} = \sum_{k=1}^{n} A_{ik} B_{kj}$$

17

A matrix M is symmetric when $M = M^T$. This means it is symmetric along the main diagonal.

Alternately, you could say that $M_{ij} = M_{ji}$.

20

To work out the transpose of a matrix, you simply rotate everything clockwise by $90°$. E.g.:

$$A = \begin{pmatrix} 1 & 3 \\ 0 & 5 \\ 8 & 7 \end{pmatrix}$$
$$A^T = \begin{pmatrix} 1 & 0 & 8 \\ 3 & 5 & 7 \end{pmatrix}$$

19

A pair of matrices A, B where $AB = BA$

22

A matrix where all of the cells have a value of zero.

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

21

Yes.

$$A(B + C) = AB + AC$$
$$(B + C)A = BA + CA$$

24

Yes.

$$A(BC) = (AB)C$$

23

Does the following hold between the two matrices A and B:

$$(AB^T) = A^T B^T$$

25

Does the following matrix represent a point or a vector?

$$\begin{bmatrix} 4 \\ 2 \\ 3 \\ 1 \end{bmatrix}$$

26

Does the following matrix represent a point or a vector?

$$\begin{bmatrix} 4 \\ 2 \\ 3 \\ 0 \end{bmatrix}$$

27

What cells are the same for all affine transformation matrices?

28

What is the matrix that will perform an affine translation?

29

What is the matrix that will perform an affine scaling?

30

How is it possible to combine two or more affine transformation matrices?

31

How do you do transformation matrix powers?

32

The point $(4, 2, 3)$.

26

Yes.

25

The bottom three/four.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & b1 \\ a_{21} & a_{22} & a_{23} & b2 \\ a_{31} & a_{32} & a_{33} & b3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \textit{ and } \begin{pmatrix} a_{11} & a_{12} & b1 \\ a_{21} & a_{22} & b2 \\ 0 & 0 & 1 \end{pmatrix}$$

28

The vector $4i + 2j + 3k$.

27

$$\begin{pmatrix} x & 0 & 0 & 0 \\ 0 & y & 0 & 0 \\ 0 & 0 & z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

30

$$\begin{pmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

29

You apply the matrix n *times, where* n *is the power.*

32

Multiply them together in the reverse order for which they are to be applied.

31

What is the identity transformation?

33

What do you get if you multiply a matrix and the inverse of the same matrix together?

34

What is the general formula for a matrix to scale by β_{xyz} times about a point (p, q, r)?

35

How do you rotate about a point in 2D space?

36

What is a matrix to rotate about the origin in 2D space?

37

What is the formula for a 3D rotation about the x axis?

38

What is the formula for a 3D rotation about the y axis?

39

What is the formula for a 3D rotation about the z axis?

40

The identity matrix.

34

A transformation that does nothing.

33

1	*Translate the point to the origin*
2	*Rotate through θ as appropriate*
3	*Translate the origin back to the point to rotate around*

36

$$\begin{pmatrix} \beta_x & 0 & 0 & p(1-\beta_x) \\ 0 & \beta_y & 0 & q(1-\beta_y) \\ 0 & 0 & \beta_z & r(1-\beta_z) \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

35

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

38

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

37

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

40

$$\begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

39

How do you undo rotations?

41

What is the general method for rotating around a line L in three dimensions?

42

What is the matrix to reflect points along a line $ax + by = f$?

43

How is a reflection undone?

44

What is the formula for a 3D reflection along the line $ax + by + cz = f$?

45

What is the formula for projecting a 3D point onto the plane $ax + by + cz = f$?

46

Why is it impossible to reverse a projection?

47

The determinant is defined only for ____ matrices.

48

1. *Translate L so that it goes through the origin.*
2. *Rotate around the x axis so that the line lies in the x-y plane and $z = 0$*
3. *Rotate around the z axis to make the line lie on the x axis.*
4. *Perform the rotation of $\theta°$ on the x axis.*
5. *Reverse step 3*
6. *Reverse step 2*
7. *Reverse step 1*

42

Just do the same rotation as before except rotate through $-\theta$ instead of θ. Make sure you're rotating about the same point or axis.

41

Just re-apply the reflection. An interesting property of any reflection matrix R, is that $R^2 = I$, where I is the identity matrix.

44

$$\begin{bmatrix} 1-\frac{2a^2}{a^2+b^2} & -\frac{2ab}{a^2+b^2} & \frac{2af}{a^2+b^2} \\ -\frac{2ab}{a^2+b^2} & 1-\frac{2b^2}{a^2+b^2} & \frac{2bf}{a^2+b^2} \\ 0 & 0 & 1 \end{bmatrix}$$

43

$$\begin{bmatrix} 1-\frac{a^2}{a^2+b^2+c^2} & -\frac{ab}{a^2+b^2+c^2} & -\frac{ac}{a^2+b^2+c^2} & \frac{af}{a^2+b^2+c^2} \\ -\frac{ab}{a^2+b^2+c^2} & 1-\frac{b^2}{a^2+b^2+c^2} & -\frac{bc}{a^2+b^2+c^2} & \frac{bf}{a^2+b^2+c^2} \\ -\frac{ac}{a^2+b^2+c^2} & -\frac{bc}{a^2+b^2+c^2} & 1-\frac{c^2}{a^2+b^2+c^2} & \frac{cf}{a^2+b^2+c^2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

46

$$\begin{bmatrix} 1-2\frac{a^2}{a^2+b^2+c^2} & -2\frac{ab}{a^2+b^2+c^2} & -2\frac{ac}{a^2+b^2+c^2} & 2\frac{af}{a^2+b^2+c^2} \\ -2\frac{ab}{a^2+b^2+c^2} & 1-2\frac{b^2}{a^2+b^2+c^2} & -2\frac{bc}{a^2+b^2+c^2} & 2\frac{bf}{a^2+b^2+c^2} \\ -2\frac{ac}{a^2+b^2+c^2} & -2\frac{bc}{a^2+b^2+c^2} & 1-2\frac{c^2}{a^2+b^2+c^2} & 2\frac{cf}{a^2+b^2+c^2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

45

The determinant is defined only for square matrices.

48

Projection matrices are examples of singular matrices, and therefore don't have an inverse. If you think about it, multiple points could be mapped onto the same point in the plane anyway, so undoing a projection wouldn't make sense.

47

What is the formula to find the determinant of a matrix M where $n > 2$?

49

How do you find $\overline{M_{ij}}$?

50

How is the determinant defined for a 2×2 matrix such as:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

51

How can you find the determinant of a matrix in row echerlon form?

52

In order to get a matrix into row echerlon form so that you can find the determinant, you can do what two operations? What are their effects?

53

What does the determinant of a matrix signify in a geometric sense?

54

When is a square matrix singular?

55

If M is an $n \times n$ matrix, then M_{ij} is the $(n-1) \times (n-1)$ matrix obtained from M by throwing away the ith row and jth column.

50

$$det\ M = \sum_{j=1}^{n} (-1)^{j+1} \cdot M_{1j} \cdot det\ \overline{M}_{1j}$$

49

Multiply the cells along the main diagonal together.

52

$$ad - bc$$

51

The area scale factor for 2D transformations and the volume scale factor for 3D ones.

54

- *You can swap rows, but doing so negates the determinant.*
- *You can multiply one row by another, this does not affect the determinant.*

53

If and only if its determinant if zero.

55