Introductory Macroeconomics Notes

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Contents

1 Basic Concepts

1.1 Macroeconomic Indicators

Stocks vs Flows

- Stocks are measured at a specific point in time
  - Existing quantity accumulated in the past
  - I have \$1000 of wealth
- Flows are measured over a period of time
  - Analogous to rate
  - I earn \$300 a week
- Stocks are accumulated or depleted over time by flows (depreciation of capital vs investment)

GDP

There are three ways to measure GDP

- GDP(E): the sum of all expenditure in the economy on finished goods and services
  - Consumption, investment, government purchases, next exports
  - This yields the national income identity: $Y = C + I + G + NX$
- GDP(I): the sum of all income earned in the economy
  - Wages and salary, profits, rents, net interest, production taxes less subsidies
- GDP(O): the sum of all production (output) in the economy
  - the "value added" at each stage of production

The reason that GDP(I) = GDP(E) = GDP(O) is that, under the circular flow of the economy, income = expenditure

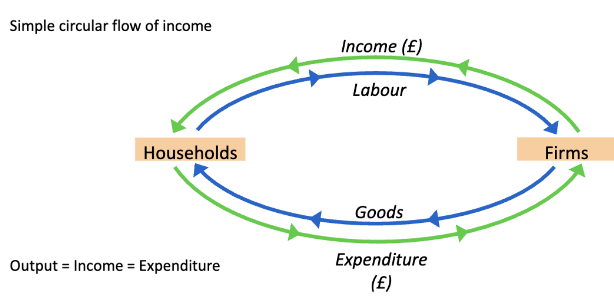


Figure 1: Circular Flow of Income

GDP(E)

Expenditure identity: $Y = C + I + G + NX$

- Consumption (C)
  - About 66% of the UK GDP
  - Durable goods, non-durable goods and services purchased by households
- Investment (I)
  - About 17% of UK GDP (including government investment)

    - Non residential investment (machinery, buildings etc) by firms
    - Residential investment (new homes/renovations) by firms and households
    - Changes in firms' inventories

- Government (G)
    - About 20% of the UK GDP (excluding government investment)
    - Goods and services purchased by all layers of government
    - Excludes social security transfers, which are not exchanged for new goods and services

- Net Exports ($NX = X - M$)
    - about -3% of the UK GDP
    - Expenditure by foreigners on domestically produced goods and services (exports), minus expenditure by UK residents on foreign goods and services (imports)

GDP vs GNI

- Gross Domestic Product: denotes the value of output of goods and services produced within the country (including nationals and resident foreigners). Measures value added in the country

- Gross National Income: denotes value of output of goods and services produced by the nationals of a country (excluding foreign residents remittances but including remittances from nationals resident abroad). Measures income of the country's citizens

- GNI = GDP + transfers from overseas - transfers to overseas countries
    - Transfers from overseas: remitted profits from domestic firms' foreign operations; interest payments and dividends received from overseas investments; remittances from country's residents based overseas; grants received from foreign governments
    - Transfers to overseas countries: remitted profits by foreign firms operating in the country; interest payments and dividends received by foreign investments in the country; remittances by overseas residents based in the country; grants paid by the country's government to foreign nations

Some Issues with GDP

- Not all market activity recorded in official statistics (black market or informal services)
- Market prices may not reflect social values so may not be measure of welfare
- GDP is a "Gross" concept: no allowance for depreciation and no concept of sustainability
- GDP may not relate to consumption which is surely closer to a measure of welfare
- GDP makes no consideration for inequality
- Second hand good sales are not included - change of ownership but not production
- Capital gains on houses do not enter GDP
- Inventories count as a firm buying its own production
- Even though most government services are not sold, they are included in GDP. But price is measured by cost, e.g. public education as sum of teachers' salaries, cost of equipment etc

Unemployment

Measuring unemployment

- Labour force survey

- **Employed:** those 16+ who did paid work in the survey week or were temporarily away from paid work
- **Unemployed:** those without a job who (i) want a job, have been seeking work and can start in the next two weeks; or (ii) have found a job and will start in the next two weeks
- Economically inactive: the rest

- Claimant count measure: those claiming unemployment related benefits
- Unemployment rate = No. of unemployed divided by the total number of economically active people

Inflation

Inflation is a general increase in prices of goods and services in an economy. (relative price changes of particular goods are not inflation)

Measuring inflation

- Inflation rate is the (annual) percentage change in a general price index, based on a basket of goods and services (equivalently, the rate of decay in the value of money)
- Indices (level only for time-series comparisons)

$$Index_t = \frac{Variable_t}{Variable_{Base}} \times 100$$

- Consumer Price Index: CPI is based on a weighted average of prices of goods and services that appear in an average consumer's basket of purchase.
  Laspeyres index (base-weighted index) uses historical quantities as weights:

$$CPI(Base = 0)_t = \frac{P_{1,t} \times q_{1,0} + P_{2,t} \times q_{2,0} + \cdots + P_{N,t} \times q_{N,0}}{P_{1,0} \times q_{1,0} + P_{2,0} \times q_{2,0} + \cdots + P_{N,0} \times q_{N,0}} = \frac{\sum_{i=1}^{N} P_{i,t} \times q_{i,0}}{\sum_{i=1}^{N} P_{i,0} \times q_{i,0}}$$

  - $q_{i,0}$ is the base-period quantity of good i
  - $P_{i,0}$ is the base-period price of good i
  - $P_{i,t}$ is the price of good i at time t

  CPI inflation is the growth rate of the CPI.
  Year-on-Year inflation:

$$\pi_t = \frac{CPI_t}{CPI_{t-1}} - 1$$

- Choosing baskets: Many different baskets of goods and services
  - Consumer Price Index
  - CPIH – includes owner occupiers' housing costs
  - Retail Price Index – basis for many contracts (e.g. pensions and index-linked bonds) but no longer a 'national statistic'
- Upward bias of CPI from uing past basket
  - Substitution bias: More expensive goods are substituted out for cheaper items in basket
  - Outlet substitution bias: Consumers tend to switch from more expensive to expensive outlets (even if purchase same basket)
  - New product bias: New products are often cheaper than the ones they replace. And the quality changes generally not measured well (e.g. cell phones, computers...)

- Calculating contributions by components
  "Group-level price accounting" consists of measuring the contribution of different groups of items to the overall price change.
  We can approximate this with:

$$\pi_t \approx w_1\pi_{1,t} + w_2\pi_{2,t} + w_3\pi_{3,t}\ldots$$

Monetary Policy

Monetary policy is a set of actions (normally by central bank) to influence how much money is in the economy and how much it costs to borrow

- Conventional
  - Manipulate long-term interest rates indirectly by changing short-term interest rates
  - Bank of England's Monetary Policy Committee (MPC) meets each month to set interest rates to hit the Government's 2% inflation target
- Unconventional
  - Manipulate long-term interest rates directly by buying financial assets
  - Quantitative Easing in the UK: Bank of England currently has purchased $375 billion of financial assets, mostly government debt

Fiscal Policy

- Manipulate government spending, taxation and borrowing to achieve macroeconomic goals
- Any deficit has to be financed by borrowing or printing money
- Can cause government deficit if expenditure < tax revenue

2 Growth

2.1 The Long Run

Economic Growth
Growth over the very long run

- Sustained increases in standards of living are a recent phenomenon
- Recent era of increased difference in standards of living across countries (Today per capita GDP differs by a factor of 50 for several countries vs in 1700 only by a factor of 2-3)

Modern Growth around the world

- After WW2, growth in Germany and Japan accelerated
- Convergence: catch-up growth in poor countries to converge with rich countries (China has a growth of 8% vs US 2%)

Benefits and Costs of economic growth

- Benefits: improvements in health, higher incomes, increase in variety of goods and services
- Costs: Environmental problems, income inequality across and within countries, loss of certain types of jobs

Macroeconomic Models
Parts of an Economic Model

- Parameter: an input that is fixed over time, except when the model builder changes it for an experiment
- Exogenous variable: An input that can change over time, but determined ahead by time of the model builder and determined outside the model (a constant)
- Endogenous variable: Outcome of the model, something that is explained by it and determined within the model

Production Models
Summary of the model

- Capital: $K^* = \bar{K}$ & Labour: $L^* = \bar{L}$
  Firms employ all supplied capital and labour in the economy
- Output: $Y^* = F(\bar{K}, \bar{Y}) = \bar{Y}$
  The production function is evaluated with the given supply of inputs
- Rental rate: $r^* = MPK = \frac{\partial F(K,L)}{\partial K}$
  The rental rate is the MPK evaluated at the equilibrium values of Y, K & L
- Wage: $w^* = MPL = \frac{\partial F(K,L)}{\partial L}$

In the production model, we have a closed economy (no foreign trade) and we assume that output is market clearing.

Output Y (real GDP) is assumed to be a function $F(K, L)$ of two factor inputs such that $F : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$

- K is capital
- L is labour

Production functions have returns to scale because they are in the long run (increasing, constant, decreasing)
Allocating resources

Suppose firms operate in competitive conditions taking prices as given
Profit maximisation problem is

$$\max_{K,L} \prod = F(K,L) - rK - wL$$

The factor demands are

$$\frac{\partial F(K,L)}{\partial K} \equiv MPK = r \qquad \frac{\partial F(K,L)}{\partial L} \equiv MPL = w$$

Hire extra inputs if and only if it increases profits

- More capital if MPK > r
- More labour if MPL > w

If a production function has constant returns to scale in both capital and labour, it will exhibit decreasing marginal returns in labour alone.
Factor Supplies and Equilibrium

- Assume fixed supplies of capital $K = \bar{K}$ and labour $L = \bar{L}$
- Given production function, output is also fixed $Y = F(\bar{K}, \bar{L}) = \bar{Y}$
- Equilibrium factor prices are given by the interplay of fixed supply and demand

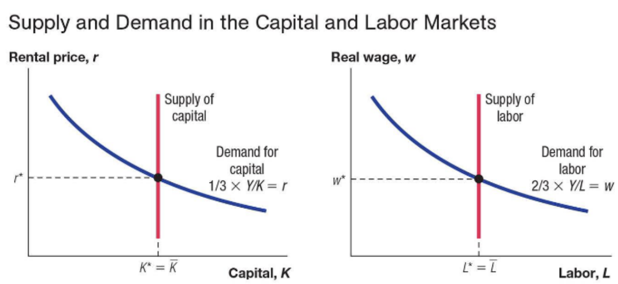


Figure 2: Supply and Demand in the Capital and Labour Markets

Factor Incomes

With constant returns, capital and labour precisely exhaust the output if paid according to their marginal products

$$Y = F(K,L) = MPK \times K + MPL \times L$$

So with CRS, $\pi = Y - MPK \times K - MPL \times L = 0$

- Verifies the assumption of perfect competition
- Verifies that production equals spending equals income $w^*L^* + r^*K^* = Y^*$

Worked Example: Cobb-Douglas and Constant returns to scale production function

The Cobb-Douglas Function is, where A, K, L are the parameters regarding productivity, capital and labour respectively

$$F(K,L) = \bar{A}K^{\alpha}L^{1-\alpha}$$

Input share of income equals its exponent
Labour share of income is

$$\text{Labour share of income} = \frac{wL}{Y} = \frac{\partial F(K,L}{\partial d}\frac{L}{F(K,L)} = \bar{A}(1-\alpha)K^{\alpha}L^{-\alpha} \times \frac{L}{\bar{A}K^{\alpha}L^{1-\alpha}} = 1-\alpha$$

Analysing the production model

Output per person equals the productivity parameter times capital per person to the power α (small letters indicate parameter per person)

$$y \equiv \frac{Y}{L} = \frac{\bar{A}K^{\alpha}L^{1-\alpha}}{L} = \frac{\bar{A}K^{\alpha}}{L^{\alpha}} = \bar{A}k^{\alpha}$$

The production model implies that output per person in equilibrium is the product of two key forces

- Productivity $\bar{A}$
- Capital per person $\bar{k}$

Improving fit of the model with empirical data

- $\bar{A}$ measures how efficiently countries are using their factor inputs (Total Factor Productivity)
- If TFP is no longer equal to 1, we can obtain a better fit of the model
- A lower level of TFP implies that workers product less output for any given level of capital per person
- However, data on TFP is not collected but calculated based on data on output and capital per person; TFP is referred to as the 'residual' or a 'meaasure of our ignorance'

Application to income disparity between countries

Differences in capital per person explain about one-quarter of the difference while TFP explains the remaining three-quarters. SO rich countries are rich since they have more capital per person and use labour and capital more efficiently.

Efficient use of capital and labour are determined by

- Human Capital: Stock of skills education and training that make individuals more productive
- Technology: use more modern and efficient technologies
- Institutions: foster human capital and technological growth through property rights, rule of law, contract enforcement and government systems
- Government failure: Misallocation of resources due to political interference or corruption etc

2.2 The Solow-Swan Growth Model

Most basic growth model: idea is that investment makes the capital stock bigger and depreciation makes it smaller

2.2.1 Introducing the Solow Model

The Solow Model begins with the production function, which describes the accumulation of capital over time. Is a Cobb-Douglas production function with constant returns to scale on capital and labour.
The production function is then

$$Y_t = F(K_t, L_t) = \bar{A}K_t^{\alpha}L_t^{1-\alpha}$$

Resource constraint (assuming no imports, exports or government): Output can be used for consumption or investment

$$C_t + I_t = Y_t$$

Capital Accumulation

Goods invested for the future determine the accumulation of capital.
The capital accumulation equation is

$$K_{t+1} = K_t + I_t - \bar{d}K_t$$

where $\bar{d}$ is the depreciation rate (often estimated at 10%), K_t is this year's capital, I_t is this year's capital and K_{t+1} is next year's capital.

A change in capital stock is defined as

$$\Delta K_{t+1} \equiv K_{t+1} - K_t$$

or

$$\Delta K_{t+1} = I_t - \bar{d}K_t$$

This means that the change in the stock of capital is investment less the capital that depreciates in production.

To analyse capital accumulation, assume that the economy begins with a certain amount of capital K_0

Investment and Labour

Agents consume a fraction of output and invest the rest (savings rate)

$$I_t = \bar{s}Y_t$$

where $\bar{s}$ is the savings rate or the fraction of income invested.
Therefore, consumption is the share of output not invested

$$C_t = (1 - \bar{s})Y_t$$

For simplification, labour demand and supply are not included; the amount of labour in the economy is given exogenously at a constant level

$$L_t = \bar{L}$$

2.2.2 Solving the Solow Model

We will now put labour and capital markets back into the model. If we added equations for the wage and rental rate of capital, we realise that because the wage rate and the rental rate equals the marginal products of labour and capital respectively, omitting these markets do not change anything.
We have:

$$\frac{\partial F(K,L)}{\partial L} \equiv MPL = w \qquad \frac{\partial F(K,L)}{\partial K} - \bar{d} \equiv MPK = r$$

The real interest rate is the amount a person can earn by saving one unit of output for a year or the amount a person must pay to borrow one unit of output for a year; measured in constant \$, not nominal \$
A unit of investment becomes a unit of capital

- The reutrn on saving must equal the rental price of capital
- The real interest rate equals the rental rate of capital, which equals the MPK

Two solutions to solve the Solow Model (which needs to be solved at every point in time, which is not amenable to algebraic analysis)

- Graphical solution
- Steady State analysis: solve the model in the long run

Graphical Solution

First, combine the investment allocation and capital accumulation equation

$$\Delta K_{t+1} = \bar{s}Y_t - \bar{d}K_t$$

which essentially says Net investment = Gross investment - Depreciation
Next, we substitute the fixed amount of labour into the production function

$$Y_t = \bar{A} K_t^{\alpha} \bar{L}^{1-\alpha}$$

We have attained a system of two equations and two unknowns: Y_t, K_t.
We now plot the two terms that govern the change in capital stock, $\bar{s}Y_t, \bar{d}K_t$, against Capital, K
Gross investment is the production function scaled by the investment rate

$$\bar{s}Y_t = \bar{s}\bar{A} K_t^{\alpha} \bar{L}^{1-\alpha}$$

The graph will look like this:

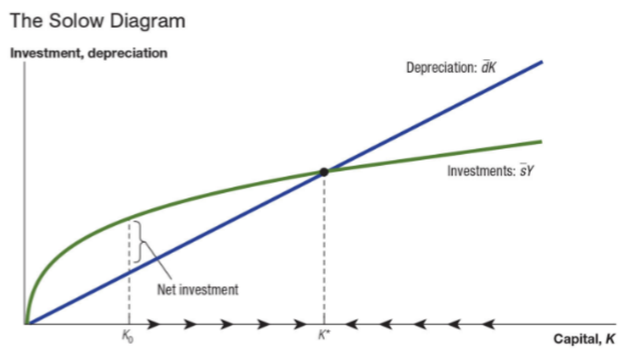


Figure 3: The Solow Diagram

Dynamics of the Model
If $\bar{s}Y_t > \bar{d}K_t$, then $\Delta K_{t+1} > 0$, so Capital stock will increase.
If $\bar{s}Y_t < \bar{d}K_t$, then $\Delta K_{t+1} < 0$, so Capital stock will decrease.
This will happen recursively until the steady state, defined as

$$\bar{s}Y_t = \bar{d}K_t \quad \text{or equivalently,} \quad \Delta K_{t+1} = 0$$

Essentially,

- When not in the steady state, the economy exhibits a movement of capital towards the steady state
- At this rest point of the economy, all endogenous variables are steady
- Transition dynamics take the economy from its initial level of capital to the stead state

Output and consumption in the Solow diagram

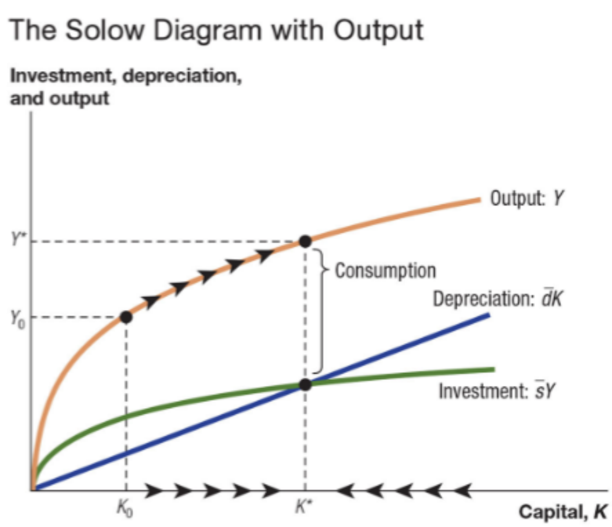


Figure 4: The Solow Diagram with Output

As K moves to its steady state by transition dynamics, output will also move to its steady state at

$$Y_t = \bar{A}K_t^{\alpha}\bar{L}^{1-\alpha}$$

Consumption can also be seen in the diagram as the different between output and investment:

$$C_t = Y_t - I_t$$

Mathematical Solution for Steady State
First, establish that the steady state is when $\Delta K_{t+1} = 0$ or $\bar{s}Y^* = \bar{d}K^*$
Substitute into the production function to yield

$$\bar{s}\bar{A}K^{*\alpha}L^{1-\alpha} = \bar{d}K^*$$

Solve for

$$K^* = \left(\frac{\bar{s}\bar{A}}{\bar{d}}\right)^{\frac{1}{1-\alpha}}\bar{L}$$

From this we can deduce that the steady state of capital is positively correlated with

- the investment rate
$$\frac{\partial K}{\partial s} > 0$$
- the size of the workforce (labour)
$$\frac{\partial K}{\partial L} > 0$$
- the productivity of the economy (TFP)
$$\frac{\partial K}{\partial A} > 0$$

and negatively correlated with depreciation rate

$$\frac{\partial K}{\partial d} < 0$$

Given K^*, we substitute it into the production function to yield

$$Y^* = \left(\frac{\bar{s}}{\bar{d}}\right)^{\frac{\alpha}{1-\alpha}}\bar{A}^{\frac{1}{1-\alpha}}\bar{L}$$

This implies the following relationships with steady-state output

- Higher steady-state production caused by higher productivity and investment rate
- Lower steady-state production caused by faster depreciation

Divide both sides by labour to get output per person, y, in steady state

$$y^* \equiv \frac{Y^*}{L^*} = \left(\frac{\bar{s}}{\bar{d}}\right)^{\frac{\alpha}{1-\alpha}}\bar{A}^{\frac{1}{1-\alpha}}$$

The exponent on productivity is different than in the production model because higher productivity has additional effects in the Solow Model by leading the economy to accumulate more capital

2.2.3 Properties of the Solow Model

Nature of the steady state

- Diminishing returns: the economy reaches a steady state because investment has diminishing returns
- A constant fraction of the capital stock depreciates each period
  Depreciation is not diminishing as capital increases
- There is no long-run economic growth in the Solow Model since net investment tends to 0 eventually
  Output, capital and output per person and consumption per person are call constant

Therefore, capital accumulation cannot be the engine of long run economic growth while savings and investment are only beneficial in the short run

Impact of changing parameters on Solow Model

1. Depreciation rate is exogenously shocked to a higher rate

$$\bar{d} \to \bar{d}'$$

The depreciation curve becomes steeper while the investment curve remains unchanged.

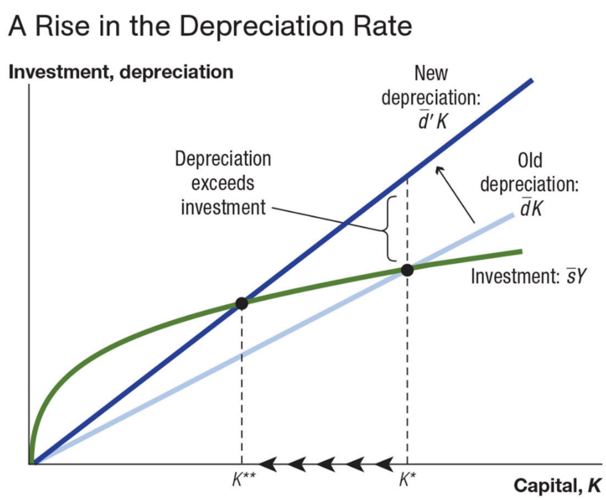


Figure 5: Higher depreciation rate in the Solow Model

The capital stock declines by transition dynamics to reach the new steady state because depreciation exceeds investment. So the new steady state is located to the left:

$$\bar{s}Y = \bar{d}'K$$

The decline in capital reduces output: Output declines rapidly at first, then gradually settles down to its new lower steady state level Y^{**}. We can see this here:

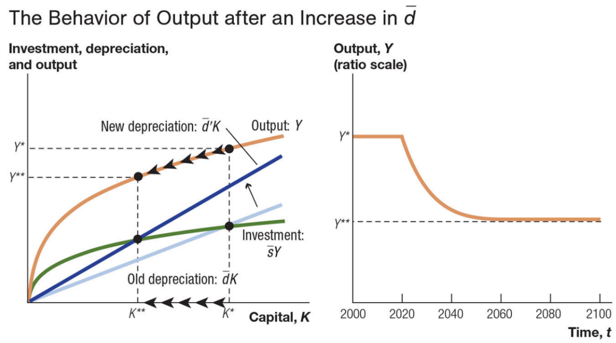


Figure 6: Output after an increase in $\bar{d}$

2. Savings rate/investment rate increases permanently due to exogenous reasons
   Investment curve rotates upwards while the depreciation curve remains unchanged.
   The capital stock increases by transition dynamics to reach the new steady state because investment exceeds depreciation and the new steady state is located to the right (higher output).

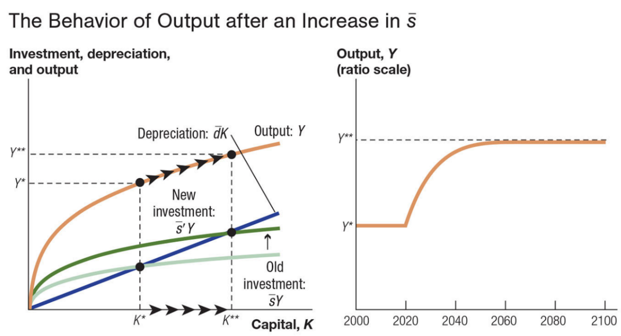


Figure 7: Output after an increase in $\bar{s}$

The rise in investment leads capital to accumulate over time and the higher capital causes output to rise as well. Output increases from its initial steady state level Y^* to Y^{**} as shown.

2.2.4 The Golden Rule in the Solow-Swan Model

Basically, we want to determine which is the optimal savings rate to achieve the best steady state. Since economic well-being depends on consumption, the best steady state has the highest possible value of consumption per person:

$$C^* = (1 - \bar{s})\bar{A}K^{*\alpha}\bar{L}^{1-\alpha} = (1 - \bar{s})f(K^*)$$

given that everything not saved is consumed.
So an increase in $\bar{s}$

- Leads to higher K^* and Y^* which may raise C^*
- Reduces consumption's share of income $(1 - \bar{s})$, which may lower C^*

Our objective is to find the corresponding values of $\bar{s}$ and K^* which maximise C^*
Express C^* in terms of K^* such that

$$C^* = Y^* - I^* = f(K^*) - I^*$$

Since we know that

$$I^* = \Delta K + dK$$

and in the steady state because $\Delta K = 0$, then

$$I^* = dK$$

So,

$$C^* = f(K^*) - dK$$

This implies that we have to look for the point where the gap between output and the depreciation function is the highest, since the steady-state lies on the depreciation function and the investment function and consumption is output - the fraction of output dedicated to investment $\bar{s}Y^*$.

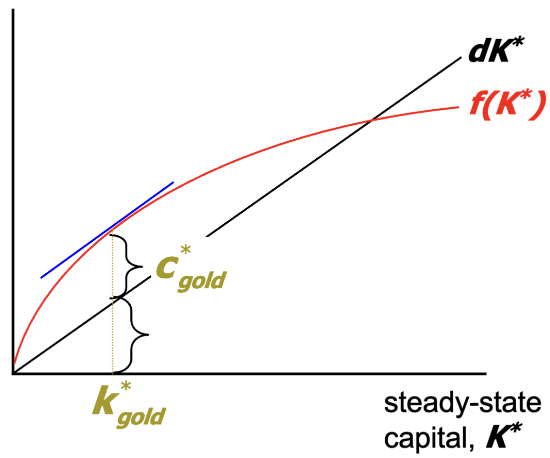


Figure 8: Golden Rule in Solow Model

Or mathematically have

Claim. The golden rule steady state is when $f'(K^*) = dK$ which maximises consumption

Proof.

$$\max C^* = f(K^*) - dK$$

to yield

$$\frac{dC^*}{dK} = f'(K^*) - d = 0 \implies f'(K^*) = d$$

So the golden level of consumption is achieved when the slope of the depreciation function, d, equals the slope of the production function, $f'(K^*)$.

Some properties of the Golden Rule Steady State

- The economy does not have the tendency to move toward the Golden Rule Steady State
- Achieving the Golden Rule requires that policymakers adjust $\bar{s}$
- The adjustment leads to a new steady state with higher consumption

Analysing Consumption during transition to the Golden Rule Steady State

1. Starting with too much capital where $K^* > K^*_G$
   Increasing C^* requires a fall in $\bar{s}$ and consumption is higher at all points in time

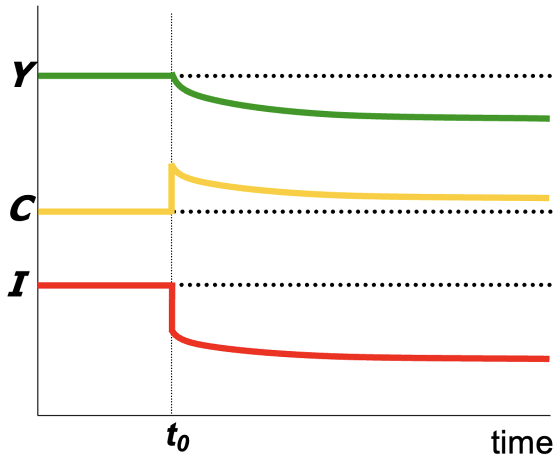


Figure 9: Changes in Output, consumption and investment with too much K

We can see that Y, C, I all decrease over time after t_0 when the change in $\bar{s}$ is made

2. Starting with too little capital where $K^* < K^*_G$
Increasing C^* requires an increase in $\bar{s}$ and future generations enjoy higher consumption but current one experiences an initial drop in consumption

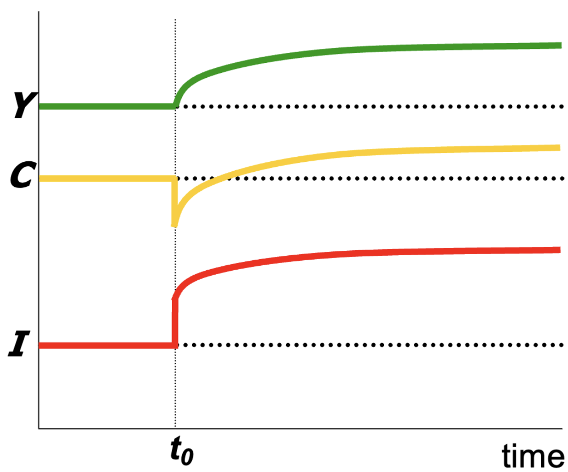


Figure 10: Changes in Output, consumption and investment with too little K

2.2.5 The Solow Swan Model with Population Growth

Now, we include a parameter that dictates the population/labour force growth rate.

$$\frac{\Delta L}{L} = n$$

We now can work directly with per capita variables:

$$k = \frac{K}{L} \qquad y = \frac{Y}{L} = f(k) = Ak^{\alpha}$$

With population growth, the law of motion for capital becomes

$$\Delta k_t = sf(k_t) - (d+n)k$$

where $sf(k_t)$ is actual investment (capital added) and $(d-n)k$ is break-even investment (capital taken out). As usual the steady state is when $\Delta k_t = 0$, and thus

$$sf(k_t) = (d+n)k$$

- dk is to replace capital as it wears out
- nk is to equip new workers with capital
  The amount of capital per worker is now lower if labour force increase; consequently, the per capita output (which varies with capital per worker) would decrease

Impact of Population Growth

If there is an exogenous shock to n where the population growth rate suddenly increases, the break-even curve becomes steeper while investment curve remains unchanged.

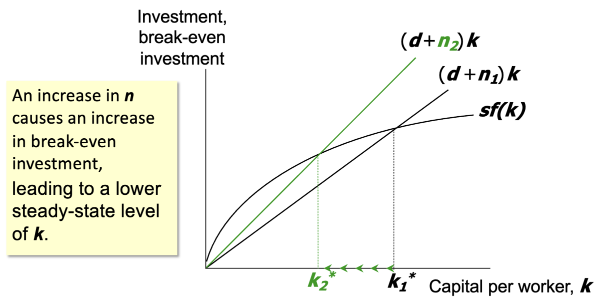


Figure 11: Increase in population growth rate in Solow Model

The capital stock declines by transition dynamics to reach the new steady state since break-even investment exceeds actual investment. The new steady state is located to the left at k_2^*

2.3 Persistent Growth

Some empirical facts regarding the Solow Swan Model: Non-zero long run growth, income disparities between countries, convergence, strengths and weaknesses of the Solow Model. Misc topics include Growth Accounting and the Role of Institutions for Economic Growth.

Does the Solow Model fit with data?

The steady state capital-output ratio is the ratio of investment to depreciation rate

$$\frac{K^*}{Y^*} = \frac{\bar{s}}{\bar{d}}$$

Proof.

$$\Delta K_{t+1} = 0 \implies \bar{s}Y = \bar{d}K$$
$$\frac{K^*}{Y^*} = \frac{\bar{s}}{\bar{d}}$$

So, the data of countries should have a positive increasing gradient of 1 for K/Y against s/d

2.3.1 Differences in output per worker

The Solow model gives more weight to TFP in explaining per capita output than the production model since

$$y^* \equiv \frac{Y^*}{L^*} = \left(\frac{\bar{s}}{\bar{d}}\right)^{\frac{\alpha}{1-\alpha}} \bar{A}^{\frac{1}{\alpha}}$$

so, if we assume $\alpha = \frac{1}{3}$ the saving-depreciation rate is only raised to an exponent of 1/2 while the TFP is raised to an exponent of 3/2.
So, we can use this formula to ascertain differences in output per worker

$$\frac{y_1^*}{y^*2} = \left(\frac{\bar{A}_1}{\bar{A}_2}\right)^{\frac{1}{1-\alpha}} \times \left(\frac{\bar{s}_1}{\bar{s}_2}\right)^{\frac{\alpha}{1-\alpha}}$$

where we assume $\bar{d}$ is the same for both countries 1,2

2.3.2 Convergence and Growth in the Solow Swan Model

There is no long-run growth in the Solow Swan model. This conflicts with empirical evidence where countries grow over time. This can be reconciled by the following:

- Growth in labour force.
  This may lead to growth in aggregate output but not output per person, since diminishing returns leads to capital and output per person to approach the steady state
- The economy is below the steady state
- There is persistent technological progress, such that $\bar{A}$ increases

The Solow Model predicts that poorer countries would grow faster than richer ones because of lower $\frac{K}{L}$. Since the investment/output function exhibits diminishing marginal returns, a country that begins from a lower k would grow at a faster rate than a country with a higher k. This is especially so if the rich country has reached a steady state.
Hence, assuming a constant $\bar{d}, \bar{s}, \bar{A}$, the living standards between rich and poor countries should converge

However, convergence is not seen in empirical evidence. We can explain this by arguing that the savings, depreciation rate and TFP are not equal – so even though countries have all reached their steady states, their steady states are different.

So what the Solow-Swan Model predicts is conditional convergence: countries would converge on income (per capita) only if savings rate, population, education etc are the same. Alternative, each country would converge to their own steady states which in turn depends on their savings rate, population, education etc.

Strengths and Weaknesses of the Solow Model

- Strengths
  - It provides a theory that determines how rich a country is in the long run with its steady state
  - The principle of transition dynamics allows for an understanding of differences in growth rates across countries: a country further from the steady state would grow faster
- Weaknesses
  - It focuses on investment and capital: the much more important factor of TFP is still unexplained
  - It does not explain differences in investment and productivity rates: a more complicated model could endogenise the investment rate
  - The model does not provide a theory of sustained long-run growth

2.3.3 Growth Accounting

Growth accounting determines (i) the sources of growth in an economy (ii) how they may change over time

Consider the a production function which includes both capital and ideas

$$Y_t = A_t K_t^{\alpha} L^{1-\alpha}$$

where A_t is the stock of ideas which refers to TFP

We take the natural log and differentiate implicitly to get

$$\ln(Y_t) = \ln(A_t) + \alpha \ln(K_t) + (1-\alpha)\ln(\bar{L})$$
$$\frac{1}{Y_t}\frac{dY_t}{dt} = \frac{1}{A_t}\frac{dA_t}{dt} + \alpha\frac{1}{K_t}\frac{dK_t}{dt} + (1-\alpha)\frac{1}{\bar{L}}\frac{d\bar{L}}{dt}$$

The insight is that growth rate of GDP = Growth rate of TFP x α x Growth rate of capita + $(1-\alpha)$ x Growth rate of labour

2.3.4 Institutions and impact on Economic Growth

From the Solow model, we know that Economic Growth is determined by

- Investment if you have yet to reach the steady state
- Productivity if you have reached the steady state

Economic institutions affect factors which drive investment such as

- Property rights
- Absence of corruption
- Macroeconomic and Political stability

Obviously, they affect productivity since good governments encourage technological progress

2.4 Romer's Growth Model

Basically, Romer's innovation is in differentiating between physical capital (objects) and ideas which exhibit different returns to scale

2.4.1 Basic Motivation

Growth and ideas

- Objects (physical capital + labour): finite and rivalrous
- Ideas (technology): virtually infinite and non-rivalrous since one person's use of an idea does not reduce the amount left for others

Consider the function

$$Y_t = A_t K_t^{\alpha} \bar{L}^{1-\alpha}$$

For some scalar λ,

$$Y_t(\lambda A_t, \lambda K_t, \lambda \bar{L}) = \lambda^2 Y_t(A_t, K_t, \bar{L})$$

A doubling of all inputs (including ideas) will result in a more than doubling of output, which implies increasing returns to scale since firms pay initial fixed costs to create new ideas but need not reinvent (and pay) later on.

Ideas in perfect competition

- Perfect competition results in Pareto optimality because $P = MC$
  - Fixed initial costs will never be recovered
  - If $P = MC$ under increasing returns, no firm will do research to invent new ideas
  - In order for firms to recoup the fixed cost of research: $P > MC$
- Patents
  - Grant monopoly power over a good for a period
  - Generate positive profits & provide incentive for innovation
  - However, $P > MC$ results in welfare loss
- Other incentives to create ideas so as to avoid welfare loss: government funding and prizes

2.4.2 Production function of the Romer Model

Labour in the Romer model We now have workers who produce ideas and workers who produce output. So our total labour is (given we have a resource constraint of labour)

$$\bar{L} = L_{yt} + L_{at}$$

where at any time t, L_{yt} are the workers producing output while L_{at} are the workers producing ideas.

Definition. $\bar{l}$ is the proportion of workers producing ideas. We then have

$$L_{yt} = (1 - \bar{l})\bar{L}$$
$$L_{at} = \bar{l}\bar{L}$$

Definition. Output is produced using the stock of existing knowledge A_t and workers producing output L_{yt}. We have

$$Y_t = A_t L_{yt}$$

The intuition of this is that the amount of output produced by workers is scaled by technology. There is constant returns to scale to physical capital but increasing returns to both physical capital and ideas.

Definition. Ideas are produced using stock of existing knowledge A_t and workers L_{at}. We further add a productivity parameter $\bar{z}$ to yield

$$\Delta A_{t+1} = \bar{z} A_t L_{at}$$

Stock of knowledge A_t appears in both production of output and ideas – characteristic of nonrivarly

2.4.3 Solving the Romer Model

Theorem. The general solution to the Romer Model is

$$y_t = \bar{A}_0(1 - \bar{l})(1 + \bar{g})^t$$

Proof.
Output per person depends on total stock of knowledge

$$y_t \equiv \frac{Y_t}{\bar{L}} = \frac{A_t L_{yt}}{\bar{L}} = A_t(1 - \bar{l})$$

The growth rate of knowledge is constant, given by

$$\frac{\Delta A_{t+1}}{A_t} = \bar{z} L_{at} = \bar{z}\bar{l}\bar{L}$$

The stock of knowledge depends on its initial value and its growth rate, using the formula for the n-th term of a geometric series to yield

$$A_t = \bar{A}_0(1+\bar{g})^t$$

Output per person grows at a constant rate on a ratio scale (i.e. keeps on doubling: growth rate is constant, not absolute increase).
Combining $y_t \equiv \frac{Y_t}{L} = A_t(1-\bar{l})$ and $A_t = \bar{A}_0(1-\bar{l})$, we yield

$$y_t = \bar{A}_0(1-\bar{l})(1+\bar{g})^t$$

The new level of output per person is now written entirely as a function of the parameters of the model. We can see the balanced growth path:

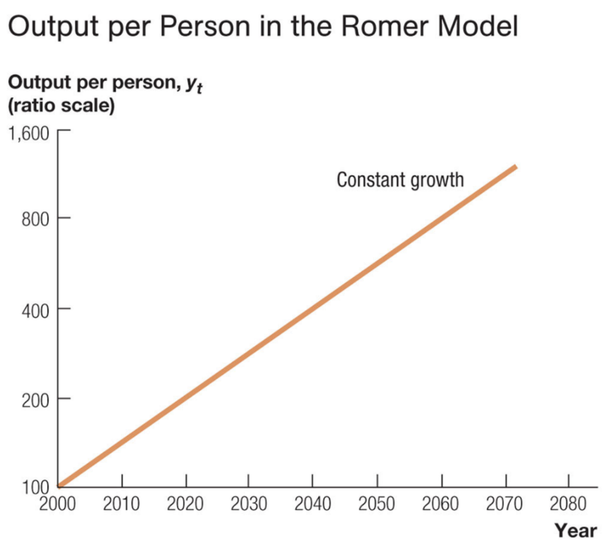


Figure 12: Output per person in the Romer Model

Insights of the Romer Model

- Produces the desired long run growth that Solow did not, since the capital in the Solow model had diminishing returns so capital and income stop growing
- The model does not have diminishing returns to ideas because they are non-rivalrous where

  $$\Delta A_{t+1} = \bar{z}A_t L_{at}$$

  Labour and ideas have increasing returns together, such that if we increase both by the same amount λ, ΔA_{t+1} will be more than λ.
- Balanced growth
  Unlike the Solow Model, the Romer model does not exhibit transition dynamics but instead has a balanced growth path, since the growth rate of all endogenous variables are constant at $\bar{g} = \bar{z}\bar{l}\bar{L}$

2.4.4 Effect of parameters in the Romer model

We shall examine the impacts of changing the number of workers and the number of workers dedicated to producing ideas

- An increase in $\bar{L}$: increase in the number of workers.
  When $\bar{L}$ increases, since $\bar{g} = \bar{z}\bar{l}\bar{L}$, $\bar{g}$ increases.
  Since $y_t = \bar{A}_0(1-\bar{l})(1+\bar{g})^t$, the rate at which y_t increases increases. We can see this here:

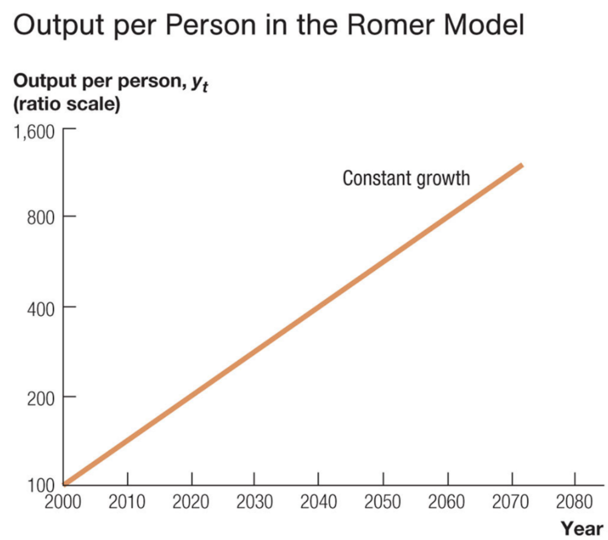


Figure 13: An increase in $\bar{L}$ in the Romer Model

An increase in population will immediately and permanently raise the growth rate of per capita output

- An increase in $\bar{l}$: increase in the number of workers dedicated to producing ideas
  When $\bar{l}$ increases, since $\bar{g} = \bar{z}\bar{l}\bar{L}$, $\bar{g}$ increases.
  But since $y_t = \bar{A}_0(1-\bar{l})(1+\bar{g})^t$, where $(1-\bar{l})$ falls but $\bar{g}$ increases, y_t drops initially but growth rate increases for all future years so output per person will be higher in the long run. We can see this here:

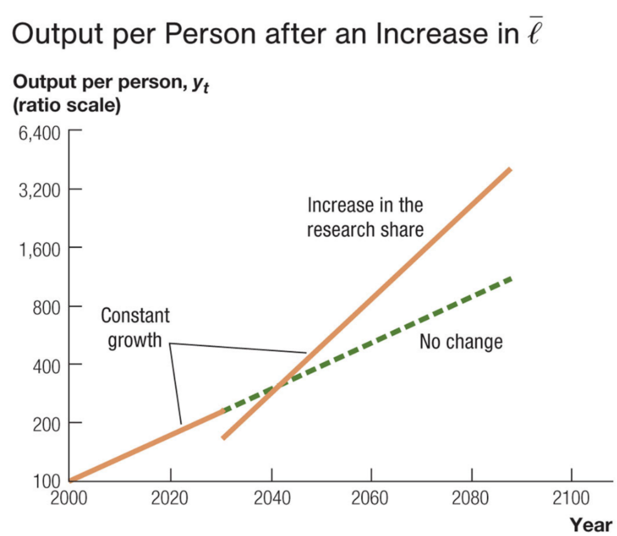


Figure 13: An increase in $\bar{l}$ in the Romer Model

From this we can distinguish between two different effects:

- Growth effects: changes to the rate of growth of per capita output
- Level effects: changes to the level of per capita output

The exponent on ideas in the production function determines the returns to ideas alone.

- If exponent on ideas, A_t, is greater or equal to 1, the Romer Model will generate sustained growth
- If the exponent on ideas is less than 1, the growth effects are eliminated due to diminishing returns on ideas

2.5 Combined Growth Model

2.5.1 Introducing the Combined Model

The Combined model adds capital into the Romer Model production function. The production function has constant returns to scale in objects but increasing returns in ideas and objects together such that

$$Y_t(A_t, K_t, L_t) = A_t K_t^{\alpha} L_t^{1-\alpha}$$

So A_t is no longer an exogenous variable (determined outside the model and held constant) and instead is endogenised and an argument of the function Y_t.

Claim. The function Y_t is constant returns to scale in physical capital L_t, K_t but increasing returns to scale in ideas and physical capital.

Proof. Consider some scalar λ for which,

$$\begin{aligned} Y_t(\lambda K_t, \lambda L_t) &= A_t(\lambda K_t)^{\alpha}(\lambda L_t)^{1-\alpha} \\ &= \lambda A_t K_t^{\alpha} L_t^{1-\alpha} \\ &= \lambda Y_t(K_t, L_t) \end{aligned}$$

The function is homogenous to degree 1 and thus exhibit CRS. Consider the same when we endogenise A_t. We yield

$$\begin{aligned} Y_t(\lambda A_t, \lambda K_t, \lambda L_t) &= \lambda A_t(\lambda K_t)^{\alpha}(\lambda L_t)^{1-\alpha} \\ &= \lambda^2 A_t K_t^{\alpha} L_t^{1-\alpha} \\ &= \lambda^2 Y_t(A_t, K_t, L_t) \end{aligned}$$

The function is homogenous to degree 2 and thus exhibits IRS.

Some Attributes (repetition from before)

- Change in capital stock is investment - depreciation

$$\Delta K_{t+1} = \bar{s}Y - \bar{d}K_t$$

- Researchers are now used to produce ideas

$$\Delta A_{t+1} = \bar{z} A_t L_{at}$$

- Given the resource constraint, the sum of researchers and workers equal to total workforce/population

$$L_{at} + L_{yt} = \bar{L}$$

- A constant fraction of the workforce/population is assumed to work as researchers

$$L_{at} = \bar{l}\bar{L}$$

Insights of the Combined Model

- Romer part: Growth of world knowledge explains non-zero long run economic growth and upward trend in incomes
- Solow part: Transition dynamics imply why some countries may grow faster or slower than others
- The combined model leads to (1) a balanced growth path since A_t grows continually over time and (2) transition dynamics due to capital accumulation

2.5.2 Solving the Combined Model

Theorem. The general solution to the Combined Model with a Cobb-Douglas production function of the form $Y_t = A_t K_t^{\alpha} L_{yt}^{1-\alpha}$ is

$$y_t^* = \left(\frac{\bar{s}}{g_Y^* + \bar{d}}\right)^{\frac{\alpha}{1-\alpha}} A_t^{*\frac{1}{1-\alpha}}(1-\bar{l})$$

Proof. Begin from the production function

$$Y_t = A_t K_t^{\alpha} L_{yt}^{1-\alpha}$$

Apply logarithms and differentiate (implicitly) with respect to t to give growth rates.

$$\ln Y_t = \ln A_t + \alpha \mathrm{K_t} + (1-\alpha)\ln L_{yt}$$

$$\frac{1}{Y_t}\frac{dY_t}{dt} = \frac{1}{A_t}\frac{dA_t}{dt} + \frac{1}{K_t}\frac{dK_t}{dt} + \frac{1}{L_{yt}}\frac{dL_{yt}}{dt}$$

$$g_{yt} = g_{At} + \alpha g_{Kt} + (1-\alpha)g_{Lyt}$$

$g_{Lyt} = 0$ since labour is fixed and the number of workers dedicated to output is fixed, and $g_{At} \equiv \frac{\Delta A_{t+1}}{A_t} = \bar{z}L_{at} = \bar{z}\bar{l}\bar{L}$

Divide the capital accumulation equation by K_t to yield g_{Kt}.

$$g_{Kt} \equiv \frac{\Delta K_{t+1}}{K_t} = \bar{s}\frac{Y_t}{K_t} - \bar{d}$$

So the savings rate and the depreciation rate as constant along a balanced growth path.
If growth is constant then the ratio $\frac{Y_t}{K_t}$ must be constant as well. Hence, let m be some constant. Then,

$$\frac{Y_t}{K_t} = m$$

$$\ln Y_t - \ln K_t = \ln m$$

$$\frac{1}{Y_t}\frac{dY_t}{dt} - \frac{1}{K_t}\frac{dK_t}{dt} = 0$$

Therefore, capital and output grow at the same rate

$$g_K^* = g_Y^*$$

Now, given 4 equations

$$g_{yt} = g_{At} + \alpha g_{Kt} + (1-\alpha)g_{Lyt}$$
$$g_{At} = \bar{z}\bar{l}\bar{L} = \bar{g}$$
$$g_K^* = g_Y^*$$
$$g_{Lyt} = 0$$

We yield

$$g_Y^* = \bar{g} + \alpha g_Y^*$$

$$g_Y^* = \frac{1}{1-\alpha}\bar{g} = \frac{1}{1-\alpha}\bar{z}\bar{l}\bar{L}$$

Key result. The growth rate of output/output per person is

$$g_Y^* = \frac{1}{1-\alpha}\bar{g}$$

The growth rate of output is even larger in the combined model than in the Romer model because

- Ideas have a direct and indirect effect on growth
- Increasing productivity raises output because productivity has increased and higher productivity results in a higher capital stock

Now to solve for Output per capita we have,

$$\frac{\Delta K_{t+1}}{K_t} = \bar{s}\frac{Y_t^*}{K_t^*} - \bar{d}$$

$$g_k = g_y = \bar{s}\frac{Y_t^*}{K_t^*} - \bar{d}$$

$$\frac{Y_t^*}{K_t^*} = \frac{g_Y^* + \bar{d}}{\bar{s}} \implies \frac{K_t^*}{Y_t^*} = \frac{\bar{s}}{g_Y^* + \bar{d}}$$

Substitute the result into the production function and solve

$$y_t^* \equiv \frac{Y_t^*}{\bar{L}} = A_t^* \left(\frac{K_t^*}{\bar{L}}\right)^{\alpha} \left(\frac{L_{yt}^*}{\bar{L}}\right)^{1-\alpha}$$

Since $L_{yt}^* = (1-\bar{l})\bar{L}$ and $Y_t^* = y_t^*\bar{L}$, we yield

$$y_t^* = A_t^* \left(\frac{\bar{s}y_t^*}{g_Y^* + \bar{d}}\right)^{\alpha} (1-\bar{l})^{(1-\alpha)} \implies y_t^* = \left(\frac{\bar{s}}{g_Y^* + \bar{d}}\right)^{\frac{\alpha}{1-\alpha}} A_t^{*\,\frac{1}{1-\alpha}} (1-\bar{l})$$

Some insights

- Growth in A_t leads to sustained growth in output per person along a balanced growth path
- Output per capita y_t depends on the square root (when $\alpha = \frac{1}{3}$) of the investment rate
- A higher investment rate raises the level of output per person along the balanced growth path
- Transition dynamics
  - the Solow and combined model both have diminishing returns to capital (so transition dynamics applies to both models)
  - The farther below its balanced growth path an economy is, the faster it will grow; the farther above its balanced growth path an economy is, the slower it will grow

Examining an increase in $\bar{s}$ or investment rate.

- The balanced growth path is higher via an upward parallel shift
- Current income is unchanged so the economy is now below the new balanced growth path
- The growth rate of income per capita is immediately higher – the slope of the output path is steeper than the balanced growth path

We can see this here:

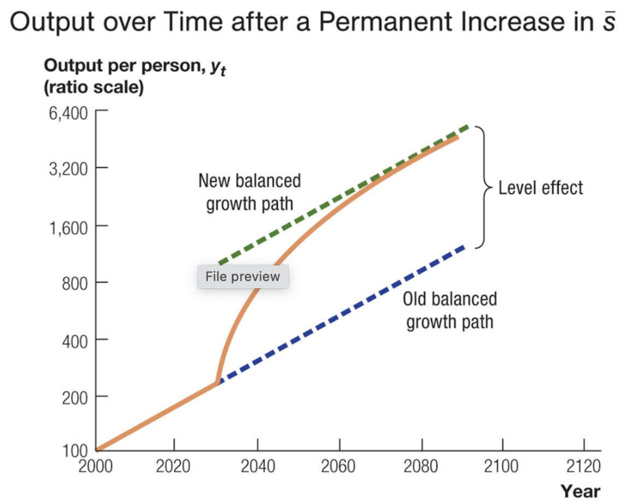


Figure 14: An increase in $\bar{s}$ in the Combined Model

3 Beyond Growth

3.1 Unemployment & the Labour market

Examine Natural Rate of unemployment, wage rigidity and efficiency wages in the labour market
Natural Rate of Unemployment
Model based on flows [Bathtub Model]
States how employment and unemployment evolve over time

$$E_t + U_t = \bar{L}$$
$$\Delta U_{t+1} = \bar{s}E_t = \bar{f}U_t$$

where $\bar{s}$ is the job separation rate and $\bar{f}$ is the job finding rate

Unemployment in the steady state implies a constant labour force, which implies change in unemployment is 0

$$0 = \bar{s}E_t - \bar{f}U_t = \bar{s}(\bar{L} - U_t) - \bar{f}U_t$$

Solving for steady state unemployment we get,

$$U^* = \frac{\bar{s}\bar{L}}{\bar{f} + \bar{s}}$$

and steady state unemployment rate:

$$u^* \equiv \frac{U^*}{\bar{L}} = \frac{\bar{s}}{\bar{f} + \bar{s}}$$

Only way to alter natural rate of unemployment is

- Change the job finding rate $\bar{f}$
- Change the job separation rate $\bar{s}$

These factors are in turn affected by

- Structural, technological and demographic change in the economy and workforce (stochastic volatility in demands and supplies)
- Search costs (costs of gathering information about job vacancies and labour availabilities)
- The costs of mobility (geographical)
- Incentives for job search (welfare systems)
- Employee protection regulations
- Real wage rigidity

Wage Rigidity
Wages may be rigid such that they are invariant with a shift in demand or supply. So there will be unemployment.

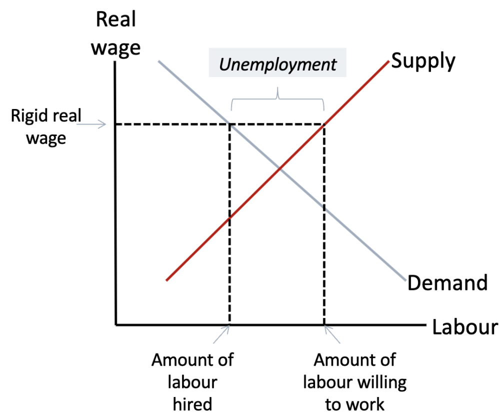


Figure 15: Real wage rigidity

Causes of real wage rigidity

- Market power by incumbent workers via unions and collective bargaining
  Insiders may impose externalities on outsides and with collective bargaining, firms have little incentive to resist
- Unduly high minimum wage laws (moderate minimum wage laws can have an advantage)
- Efficiency wage reasons

Efficiency wages
Reasons for efficiency wages

- Firms could hire alternative workers more cheaply
- Better pay increases worker productivity and greater work effort (reduce shirking) which implies more output/attract better quality workers which imploies more output

So the wage would be higher (to fulfil the incentive role) than the market clearing price and hence there would be workers who want jobs but cannot get them, and thus are involuntarily unemployed

3.2 Inflation

Inflation refers to an increase in price level that is sustained over a significant period of time.
Costs of inflation

- Expected Inflation
  - Shoe leather cost - avoid inflation tax
  - Menu cost - changing prices
  - Relative price distortions
  - Unfair tax treatment
  - Price comparison harder
- Unexpected inflation
  - All of above
  - Arbitrary redistribution of purchasing power
  - Increased uncertainty

Benefits: (i) If nominal wages are sticky downwards but real wages per person need to fall to reduce unemployment, then some price inflation could help (ii) moderate inflation allows more scope for low short-term interest rates, since $r = i - \pi$ and interest rates cannot go (very) negative (iii) government tax revenue from inflation tax by printing more money

3.2.1 Long-Run Determinants of inflation

Quantity Theory of Money

$$M_tV_t = P_tY_t$$

where at time t

- M_t is money supply
- V_t is the velocity of money (how many times each piece of paper currency is used in a transaction)
- P_t is price level
- Y_t is real GDP

The equation implies that the amount of money used in purchases is equal to nominal GDP.

The classical dichotomy is in the long run, the real and nominal sides of the economy are completely separate. Therefore, Y_t, V_t, M_t are instead written as exogenous (and constant) variables determined by forces outside the model (real forces for Y, V is assumed as constant over time, and M is determined by central bank), $\bar{Y}_t, \bar{V}, \bar{M}_t$.
The price level is thus

$$P_t^* = \frac{\bar{M}_t\bar{V}}{\bar{Y}_t}$$

In the long run, the key determinant of price level is money supply.

We can express the quantity theory in terms of growth rates.

$$\bar{M}_t\bar{V} = P_t^*\bar{Y}_t$$
$$\ln\bar{M}_t + \ln\bar{V} = \ln P_t^* + \ln\bar{Y}_t$$
$$\frac{1}{\bar{M}_t}\frac{d\bar{M}_t}{dt} + \frac{1}{\bar{V}}\frac{d\bar{V}}{dt} = \frac{1}{P_t^*}\frac{dP_t^*}{dt} + \frac{1}{\bar{Y}_t}\frac{d\bar{Y}_t}{dt}$$
$$\bar{g}_M + \bar{g}_V = g_P + \bar{g}_Y$$

Realise that $\bar{g}_V = 0$ and g_P is essentially inflation rate, π, and we have

$$\pi^* = \bar{g}_M - \bar{g}_Y$$

Changes in growth rate of money lead one-for-one changes in the inflation rate

3.2.2 Short-Run Determinants of inflation

Short-run determinants

- Cost factors (e.g. unit labour costs and oil prices)
- External factors (e.g. nominal exchange rate)
- Cyclical factors (e.g. output gap)

Short-run shocks and their short-run implications can ignite an inflation spiral: If oil prices increase, then the cost of production rises, leading to shrinking profits. Hence, price of goods and services will rice, which implies demand for higher wages, and if labour cost rise again, the whole cycle repeats.
Inflation expectations play a key role in determining the rise in price of goods and services and demand for higher wages.

Philips Curve

Since inflation is explained by economic activity (output gap or employment), inflationary expectations and external factors like exchange rates, Philips thought that the relationship between employment and wage inflation rate was a trade off:

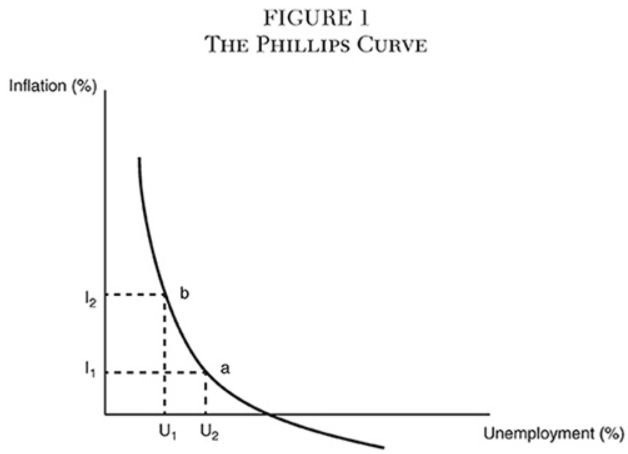


Figure 16: The Phillips Curve

3.2.3 Policy Issues

Hyperinflation and fiscal policy

Fiscal policy may cause high inflation. Consider the government's budget constraint:

$$G = T + \Delta B + \Delta M$$

where T is tax revenue, ΔB is Borrowing and ΔM is change in money stock.
If the government runs a large budget deficit, as debt rises the lenders may worry that the government will have trouble paying back loans and may stop lending to the government all together. Hence the government would raise taxes (depending on political feasibility). The government may resort to printing money to finance its budget and the government creditors will be paid back in currency that is worth less than it lent.

Money and Interest rates

In the quantity theory of money, money growth determines π. The real interest rate r is determined in the market for lonable funds, independently of monetary policy. So the Fischer equation

$$i = r + \pi$$

implies a positive relationship between inflation and interest rates. This is empirically confirmed by data on interest against inflation rates.

3.3 Consumption

Focus on theories of consumption and Ricardian equivalence

3.3.1 Theories of Consumption

Keynesian Consumption Function

Keynes emphasised the role of disposable income as a driver of consumption

$$C_t = a + bY_t$$

where $0 < b < 1$ is the marginal propensity to consume (the proportion of income used to spend on goods and services). Average propensity to consume $\frac{C_t}{Y_t}$ falls as income rises.

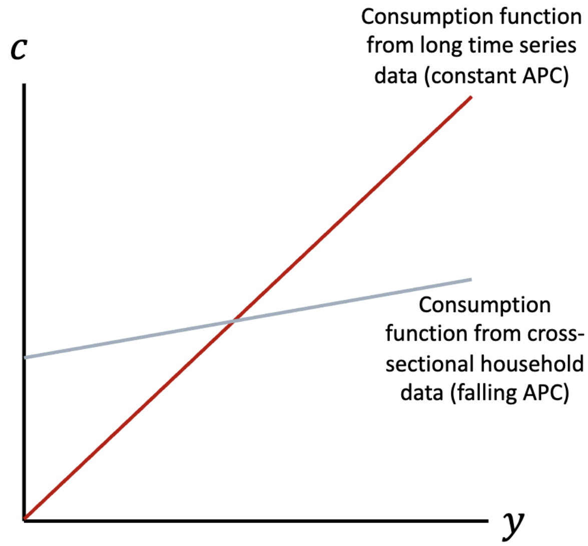


Figure 17: Consumption function and Average Propensity to Consume

Inter-temporal Choice

The Neoclassical consumption model supposes that individuals choose consumption at each moment in time to maximise a lifetime utility function, which depends on current and future consumption. Since future income may be different from today income, we have two time periods: today and the future. So the key decision to make is how much to consume today vs the future?

Intertemporal budget constraint

Let f be financial wealth and we have two budget constraints which say that consumption equals less savings

$$c_{today} = y_{today} - (f_{future} - f_{today})$$
$$c_{future} = y_{future} + (1+R)f_{future}$$

The intuition of the first is that how much you consume today is basically your income today - how much you do not save (difference between future and current financial wealth). The intuition of the second is that, assuming one consumes all income in the future, then your future consumption depends on future income and financial wealth that you saved for the future + interest gained from savings.
Combining the two, we yield the intertemporal budget constraint:

$$c_{today} + \frac{c_{future}}{1+R} = f_{today} + y_{today} + \frac{y_{future}}{1+R}$$

Basically this says, present value of life time consumption = total lifetime wealth (where this is today's wealth and income + present value of future income). This is plotted as a budget set for two goods:

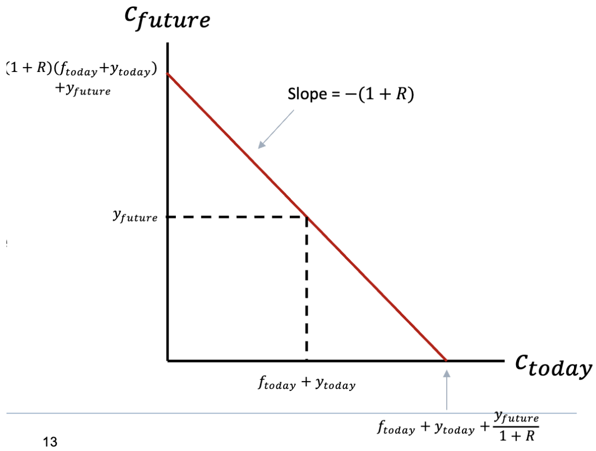


Figure 18: Intertemporal budget set

Intertemporal utility function

Utility increases (diminishingly) as consumption increases. Utility depends on consumption today and in the future. So,

$$U = u(c_{today}) + \beta U(c_{future})$$

If $\beta = 1$ then current and future consumption are weighted equally. If $\beta < 1$ then the future is discounted and today's consumption is valued more than future consumption.

Consumption to maximise lifetime utility

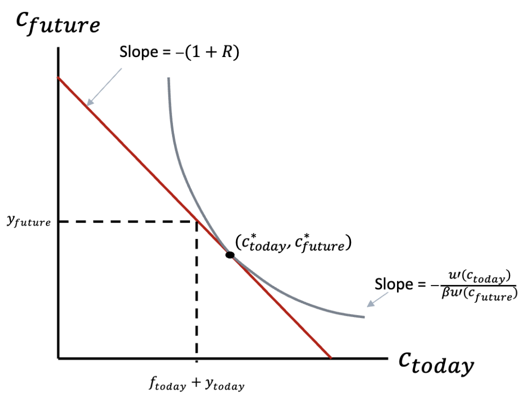


Figure 19: Maximising lifetime utility

So, our values of c_{today} and c_{future} need to satisfy the following equation

$$\frac{u'(c_{today})}{\beta'(c_{future})} = 1 + R$$

From this, we yield the Euler equation

$$u'(c_{today}) = \beta(1+R)u'(c_{future})$$

Applying the inverse function theorem, we yield

$$\frac{1}{c_{today}} = \beta(1+R)\frac{1}{c_{future}}$$

Since we realise that the increase in future consumption over current income (assuming savings rate remain the same) is equivalent to growth, we yield

$$\text{Growth } = \frac{c_{future}}{c_{today}} = \beta(1+R)$$

We can use this budget constraint to solve explicitly for c_{today}
Some Comparative statics

1. Effect of an increase in income (y_{today} or y_{future})
   An outward shift of the budget constraint provided that goods consumed in c_{today} and c_{future} are normal goods, consumption for both present and future increases

2. Effect of an increase in real interest rate R.
   Results in a pivot of the budget constraint around the point $(f_{today}+y_{today}, y_{future})$ of the indifference curve.

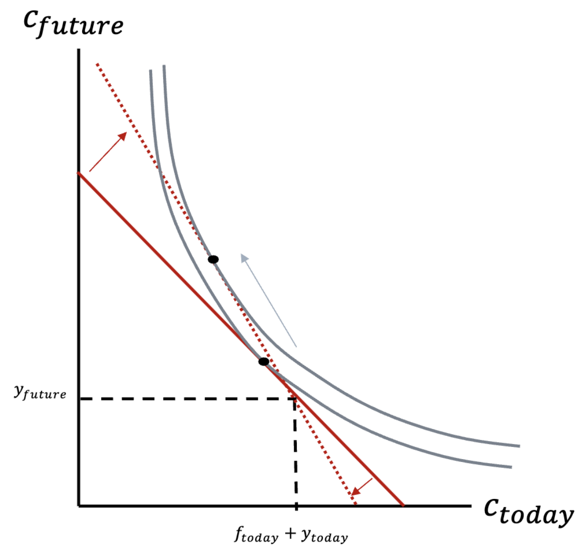


Figure 20: Effect of an increase in R on lifetime consumption

Whether c_{today} and c_{future} falls or rises depends on income and substitution effects as per the Slutsky equation.

Life-cycle hypothesis

Modigliani et al built on the neoclassical consumption model where consumption depends on wealth and income. Income varies systematically over life cycle (since you gain a job and then retire after certain points in time). People rationally smooth their consumption by systematically saving and dis-saving over their life time.

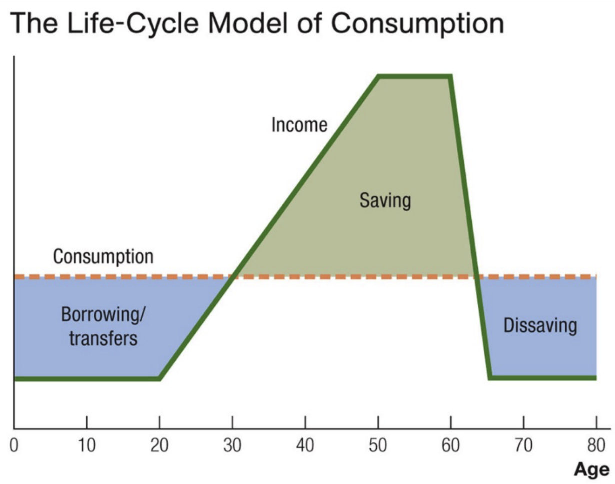


Figure 21: Life cycle hypothesis

Permanent Income Hypothesis

Current income consists of permanent and transitory income:

$$Y_t = \bar{Y}_t + \epsilon_t$$

Smoothing implies that consumption depends only on permanent income $\bar{Y}_t$ – broadly fits the cross-section, short run and long run stylised facts about consumption: transitory income gains do relatively little to boost consumption.

The implication of this is that temporary tax cut stimuli have little effectiveness in boosting demand. The permanent income hypothesis and the Lifecycle hypothesis are complementary: consumption depends on long-run/permanent income and wealth, not today's income.

Consumption as a random walk

Based on rational expectations, where permanent incomes will only be affected by unanticipated changes in income or random shocks since anticipated changes have already been factored according to future incomes and so will not change consumption according to PIH. So consumption will shift only in response to random shocks and thus follow a random walk.

Borrowing constraints and current income

However, data shows that there is sensitivity to current income. The reason for this is that we are unable to freely borrow what we would have earned in the future and pay it back later on. So, we must consider borrowing constraints. More generally, there would be a borrowing constraint if

$$r_{borrowing} > r_{saving}$$

during a credit crunch.

The effect of borrowing constraints can be contrasted based on the indifference curves:

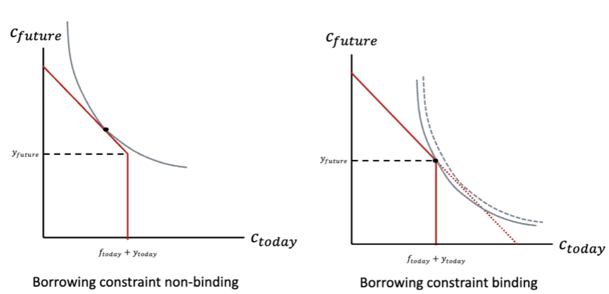


Figure 22: Binding vs non-binding budget constraint

Basically, after a certain point in time, we cannot sacrifice future income for income today (by borrowing so you increase your current income and paying back so you do not have 'future income' so to speak). So there is a limit to how much you can trade-off your future consumption for today's consumption.
But if the indifference curves is tangent with the downward sloping segment then the borrowing constraint does not matter; but if the indifference curve is tangent on the 'kink-ed' part then the borrowing constraint would have an effect.

3.3.2 Ricardian Equivalence

Core question: given the path of government expenditure G, does it matter whether households are taxed now or later? (later means public borrowing now).

Governments also have a lifetime budget constraint (2 period version):

$$g_{today} + \frac{g_{future}}{1+R} = \tau_{today} + \frac{\tau_{future}}{1+R}$$

The household budget constraint is:

$$c_{today} + \frac{c_{future}}{1+R} = f_{today} + y_{today} - \tau_{today} + \frac{y_{future}}{1+R} - \frac{\tau_{future}}{1+R}$$

Which becomes:

$$c_{today} + \frac{c_{future}}{1+R} = f_{today} + y_{today} - g_{today} + \frac{y_{future}}{1+R} - \frac{g_{future}}{1+R}$$

Barro's Ricardian Equivalence: Irrelevance of tax policy – does it matter whether households are taxed now or later?

- With 'perfect' financial markets, it makes no difference because a (lump-sum) tax cut today is merely a postponement and so is neutral for permanent income – there's no effect on the inter-temporal budget constraint of the (infinitely-lived) agents in the economy
- Same can be true for infinitely-lived agents or finitely-lived agents who care about leaving utility-preserving bequests for their children

Problems for Ricardian debt neutrality

- Myopia: Not all consumers thinks so far ahead and so see tax cuts as a windfall
- Borrowing constraints: Many consumers cannot borrow enough to achieve their optimal consumption so they rationally spend much of a tax cut
- Future generations: if consumers expect the burden of repaying to fall on future generations, and Barro's intergenerational linkages do not operate, then a tax cut now makes them better off, so they increase their spending
- Proportional taxes: Affect decisions on work and investment

3.4 Investment

Types of investment

- Business fixed investment
  E.g. Machinery and equipment; buildings and structures
- Residential investment
  E.g. By owner-occupiers and landlords
- Inventory investment
  E.g. Change in value of stocks of finished goods, materials, supplies and work in progress

3.4.1 Determinants of Investments

1. Firm's investment decisions
   A business should keep investing in physical capital until the $MPK = r$ (marginal product of capital falls equal to the rental price of capital, which we equate to interest rate).

   Arbitrage equation

   The basic intuition is that there are two possible ways to invest and if profits are maximised, the two investments must yield the same return because otherwise, we can use the same $ to invest in the investment with a higher return.

   There are two possible ways to invest money: (i) invest in bank and (ii) buy capital.
   The arbitrage equation is
   $$Rp_k = MPK + \Delta p_k$$
   where $\Delta p_K = p_{k+1} - p_k$, R_{p_k} is the return from bank account and $MPK + \Delta p_K$ is the return from capital.
   Rearranging the equation, we yield
   $$MPK = R - \frac{\Delta p_k}{p_k}$$
   Intuition: invest until the MPK of capital falls equal to the differences between the interest rate and the growth rate of the price of the capital.

   User cost of capital

   The user cost of capital is the total cost of the firm of using one more unit of capital.
   If
   $$\frac{\Delta p_k}{p_k} > 0$$
   this implies capital gain. Otherwise, if
   $$\frac{\Delta p_k}{p_k} < 0$$
   this implies capital loss.
   The price of capital changes because of depreciation, wear and tear and technological change (particularly in electronics).
   Price of (physical?) structures usually increase over time since land becomes increasingly scarce.
   Abitrage equation with depreciation:
   $$MPK = R + \bar{d} - \frac{\Delta p_k}{p_k}$$
   where $R + \bar{d} - \frac{\Delta p_k}{p_k}$ refers to the user cost of capital. This is how much the firm should invest: they should invest in capital until the value of the extra output that capital produces falls equal to the user cost.

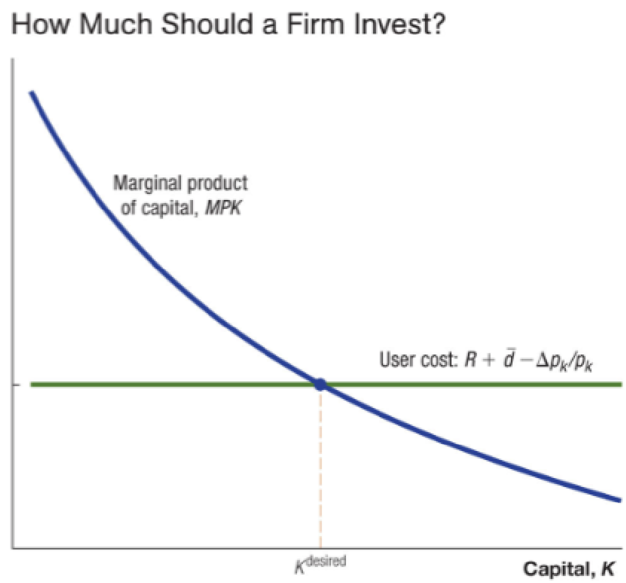


Figure 23: How much a firm should invest

2. Financial markets
   Finance investment options: (i) Put money in savings account, where it pays an interest rate and (ii) Purchase a stock, sell a year later – dividends and possible capital gain.
   "Capital" and "investment" are connected using the arbitrage equation (assuming safe investments):

   $$R \times p_s = \text{dividend} + \Delta p_s$$

   To yield the price of the stock, divide both sides of the arbitrage equation by the price of the stock

   $$R = \frac{\text{dividend}}{p_s} + \frac{\Delta p_s}{p_s}$$

   where R is the % return in the bank account, dividend is the % dividend return, and $\frac{\Delta p_s}{p_s}$ is % capital gain. Solving for p_s (price of stock):

   $$p_s = \frac{\text{dividend}}{R - \frac{\Delta p_s}{p_s}} = \frac{\text{dividend}}{\text{interest rate} - \text{capital gain}}$$

   The price of a stock will equal the present discounted value of the dividends the stock will pay.
   Price/Earning ratios and bubbles: the previous model does not take account for risk or 'bubbles' in the stock market and we can find a price-earnings ratio by dividing the price of a stock by total earnings

   $$\frac{p_s}{\text{earnings}} = \frac{\text{dividend/earnings}}{\text{interest rate - capital gains}}$$

   Efficient markets

   - Informationally efficient market
     Financial prices fully and correctly reflect all available information; it is possible to make economic profits by trading on basis of information; theory states that only unexpected news moves stock prices – prices follow a random walk
   - Efficient markets benchmark is not the final word on understanding financial markets
     Some mutual funds beat the FTSE100 with more persistence than predicted; more volatility in markets than justified by fundamentals in the model; bubbles may be explained by behavioural elements

Tobin's q theory of investment

Tobin considered the adjustment costs in investing in physical capital: the present discounted value of profits can differ from the value of capital + with no adjustment costs, the firm should expand immediately and with adjustment costs, expansion will occur gradually, causing a difference in values. Tobin's q:

$$q = \frac{V}{p_k K} = \frac{\text{stock market value}}{\text{value of capital}}$$

If $q > 1$, market signals that the value of the firm is greater than its capital; firm should invest in more capital.
If $q < 1$, value of the firm is less than the value of its capital; firm should disinvest.
Value of q should be close to 1 as a useful predictor of firm investment.
In practice, this predction has problems

- Some capital is created, patented while other capital is not
- Cash flow and access to loans seem to play a role but is not in theory
- Empirical evidence on the role of q is mixed

3. Inventory investment
Source of inventories

- Firms purchase goods to re-sell later
- Firms are working on products for a while
- Firms are assumed to buy their own unsold products

The stock of these inventories can increase or decrease each period:

$$ds_t = s_t - s_{t-1}$$

The change in inventories matters for GDP; this means that the rate of change matters for GDP growth.
Motives that govern a firm's inventories:

- Production smoothing
Costly to increase production in times of high demand
- Pipeline theory
Firms hold inventory as part of production process itself
- Stockout avoidance
Hold inventories of final goods to make sure they are available if a customer wants to make a purchase

4. Housing market
Arbitrage and housing markets
Arbitrage equation gives two choices: (i) save money and invest in bank or (ii) Use money for deposit, rent out & sell later

$$R \times \text{deposit} = \text{rent} - \bar{d}P_{house} + \Delta P_{house}$$

where the first part of the equation is the return from saving the deposit and the second part is return from investing. Solving, we have

$$P_{house} = \frac{\text{rent}}{R\bar{x} + \bar{d} - \frac{\Delta P_{house}}{P_{house}}}$$

where $\bar{x} = \frac{\text{deposit}}{P_house}$ (fraction of purchase price) and $\frac{\Delta P_{house}}{P_{house}}$ is capital gain.
This shows that the more you expect the house prices to rise, the higher the initial price (so the housing bubble can feed on itself); if deposit is lowered, financial capital can be levered at a lower rate.

5. Uncertainty
   Uncertainty is bad for investment because

   - Default risk is higher which increases credit spreads and costs of capital
   - Typical investment decisions are 'now vs later' and not 'now or never'
   - Uncertainty increases value of wait and see strategy so waiting has an option value and option values increase with uncertainty.

   So not just the cost of capital matters, but expectations of future returns are crucial to investment decisions. This is informed by buisness confidence, herd behaviour, rationality etc

4 IS-MP-PC Model

4.1 The Short Run

On the Short Run (Business Cycle) model: what determines current output and inflation.
About why current output and inflation deviates from long run behaviour.

Definition 4.1 (Potential Output): Amount of goods and services that would be produced if all inputs were utilised at their long run sustainable (or average) levels.
We take the long run as given, so the potential output and long run inflation rate are exogenous. While the short run model takes the current level of output and current inflation as endogenous. Because of economy-wide shocks, current output, for example, may deviate from its potential level.

Trends and Fluctuations

Decompose actual output Y_t in long-run trend and short-run component.

- Long run trend = potential output $\bar{Y}_t$
- Short-run component = % deviations from potential

$$\tilde{Y}_t \equiv \frac{Y_t - \bar{Y}_t}{\bar{Y}_t}$$

Business cycles have procyclical and countercyclical variables.

Measuring Potential Output

Two ways to estimate potential output

- Statistical: estimate a statistical trend on the data
  Example: Linear trend, moving-average, HP trend
- Economic: Construct a model economy and build the "counterfactual" from micro-estimates

Characterisation 4.1 (Recession): Often defined as two consecutive quarters of negative GDP growth, but this is a contentious definition. Ends when output starts to rise again.
US/EU definition: a significant decline in economic activity spread across the market, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production and retail sales.

Rest are not essential information; case studies on Financial Crisis + COVID-19 [Look at the slides]

4.2 The IS curve

Derivation of the IS Curve

Preliminaries. We use the national income accounting identity where

$$Y_t = C_t + I_t + G_t + EX_t - IM_t$$

where at discrete time t, Y_t is GDP, C_t is Consumption, I_t is Investment, G_t is Government Spending, EX_t is Exports, IM_t is Imports.
There are three different approaches from the National Income Accounting Identity

1. Expenditure Approach: Total Purchases
2. Production Approach: Value Added (Net value of goods produced in an economy)

3. Income Approach: Income earned in an economy

All three measures give the same result.

Characterisation 4.1 (IS Curve). ”Investment-Savings” Curve; it captures the negative short-run relationship between the interest rate r, and the output y.

- An increase in interest rate decreases investment and consumption
- In turn, lower investment and consumption decreases output

We rewrite the national income accounting identity, making Investment the subject

$$Y_t - C_t - G_t + IM_t - EX_t = I_t$$

Then we add and subtract tax revenues on the left side

$$(Y_t - T_t - C_t) + (T_t - G_t) + (IM_t - EX_t) = I_t$$

where the first term denotes private savings, the second denotes public savings, the third denotes foreign savings

Setup of the Economy for the IS Curve

Again, we have the National Income Accounting Identity

$$Y_t = C_t + I_t + G_t + EX_t + IM_t$$

Consumption, government spending, exports and imports assumed proportional to potential output. So,

$$\begin{aligned} C_t &= (\bar{a}_c - b_c[R_t - \beta])\bar{Y}_t \\ G_t &= \bar{a}_g \bar{Y}_t \\ EX_t &= \bar{a}_{ex} \bar{Y}_t \\ IM_t &= \bar{a}_{im} \bar{Y}_t \end{aligned}$$

where β is the discount factor. Remarks on Assumptions

- Micro-foundations completely absent (Not a result of household maximisation problems)
- Assumption on consumption consistent with Permanent Income and Life Cycle Theories
- Evidence on GDP shares suggests government spending roughly stable relative to GDP: G_t moves slowly
- Global Imbalances assumed to be temporary
- More generally, the assumptions imply temporary low frequency movements of GDP shares (due to shocks)

Consumption is smooth because of a Risk Averse/Concave Utility Function.

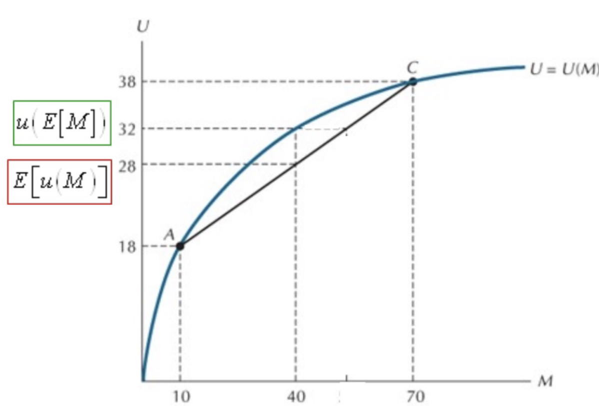


Figure 4.1: Risk Averse Utility function

If other components are smooth, then GDP volatility must depend on investment; empirically I_t is two or three times as volatile as Y_t.
Intuitively, interest rates are key for investment decisions, since they determine

- Cost of Borrowing
- Opportunity costs of funds

The Investment Equation. The Investment equation is

$$\frac{I_t}{\bar{Y}_t} = \bar{a}_i - b_i(R_t - \bar{r})$$

where R_t is the Real Interest Rate and $\bar{r}$ is the marginal product of capital (exogenously determined). If costs of borrowing is higher than the MPK, the investment falls.

- In the Long Run: $R_t = \bar{r}$ because of no arbitrage
- In the Short Run: $R_t \neq \bar{r}$ because it takes time to install capital (among other reasons)

Summary of the Set up of the Economy for the IS curve

Endogenous variables: $Y_t, C_t, I_t, G_t, EX_t, IM_t$

National income identity:	$Y_t = C_t + I_t + G_t + EX_t - IM_t$
Consumption:	$C_t = \bar{a}_c \bar{Y}_t$
Government purchases:	$G_t = \bar{a}_g \bar{Y}_t$
Exports:	$EX_t = \bar{a}_{ex} \bar{Y}_t$
Imports:	$IM_t = \bar{a}_{im} \bar{Y}_t$
Investment:	$\frac{I_t}{\bar{Y}_t} = \bar{a}_i - \bar{b}\,(R_t - \bar{r})$

Exogenous variables/parameters: $\bar{Y}_t, \bar{r}, \bar{a}_c, \bar{a}_i, \bar{a}_g, \bar{a}_{ex}, \bar{a}_{im}, \bar{b}$
Exogenous for now (until next chapter): R_t

Figure 4.2: Setup of the Economy for the IS Curve

Derivation of the IS Curve. We take the national income accounting identity and divide it by $\bar{Y}_t$.

$$\frac{Y_t}{\bar{Y}_t} = \frac{C_t}{\bar{Y}_t} + \frac{I_t}{\bar{Y}_t} + \frac{G_t}{\bar{Y}_t} + \frac{EX_t}{\bar{Y}_t} - \frac{IM_t}{\bar{Y}_t}$$

We substitute assumptions from the setup of the economy for expenditure components to yield

$$\frac{Y_t}{\bar{Y}_t} = \bar{a}_c - b_c[R_t - \beta] + \bar{a}_i - b_i(R_t - \bar{r}) + \bar{a}_g + \bar{a}_{ex} - \bar{a}_{im}$$

We now subtract 1 from each side to yield the Short Run Output fluctuation, since $\frac{Y_t}{\bar{Y}_t} - 1$ would yield the deviation from the long run Output

$$\frac{Y_t}{\bar{Y}_t} - 1 = \bar{a}_c + \bar{a}_i + \bar{a}_g + \bar{a}_{ex} - \bar{a}_{im} - 1 - b_c[R_t - \beta] - b_i(R_t - \bar{r})$$

where the LHS is denoted as $\tilde{Y}_t$ (denoting short-run output fluctuation) and the RHS sans the terms containing b are denoted as $\bar{a}$ collectively. Overall, we yield

$$\tilde{Y}_t = \bar{a} - b_i(R_t - \bar{r}) - b_c[R_t - \beta]$$

Interpretation:

- Short-run output fluctuations depend negatively on gap between real interest rate R_t and the MPK $\bar{r}$. Firms can always earn MPK on new investments

- Short-run output fluctuations depend negatively on gap between real interest rate R_t and the discount factor $\bar{\beta}$.
- Parameter $\bar{a}$ captures aggregate demand shocks; changes in sensitivity of expenditure to potential output
- If $R_t = \bar{r} = \beta$ and $\bar{a} = 0$, then $\tilde{Y}_t = 0$

Graphical Analysis of the IS Curve

Graphical Representation of the IS curve.

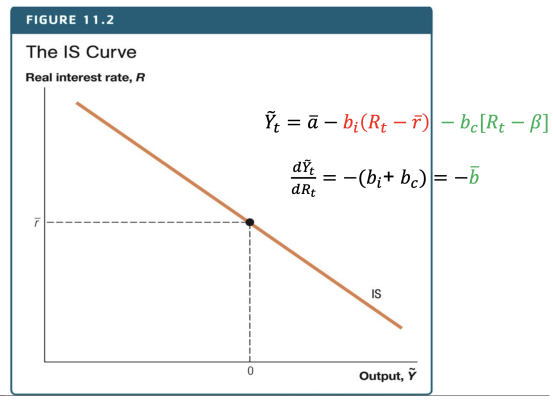


Figure 4.3: The IS Curve

Note: $\tilde{Y}_t$ indicates fluctuations and thus can be negative in value.
Changes in IS Curve when variables or parameters change.

1. **Change in Interest Rate:** When R_t changes, economy moves along the IS curve. Suppose interest rate increases.

   - Economy moves up along the IS Curve
   - Short-run output declines

   Intuition: higher interest rate raises borrowing costs, reduces demand for investment and reduces output below potential. We can see this here

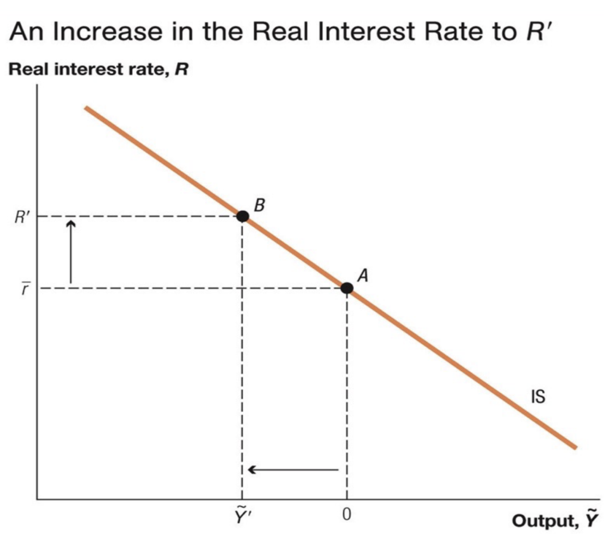


Figure 4.4: Increase in Real Interest Rate

2. **Change in Slope:** The Slope of the IS Curve, in $(\tilde{Y}_t, R_t)$ space, would be

$$\frac{dR_t}{d\tilde{Y}_t} = -\frac{1}{\bar{b}}$$

When $\bar{b}$ increases, there is a higher sensitivity of I_t or C_t to interest rate. So the IS Curve becomes flatter (since a similar rise in interest rate would yield a large fall in output).

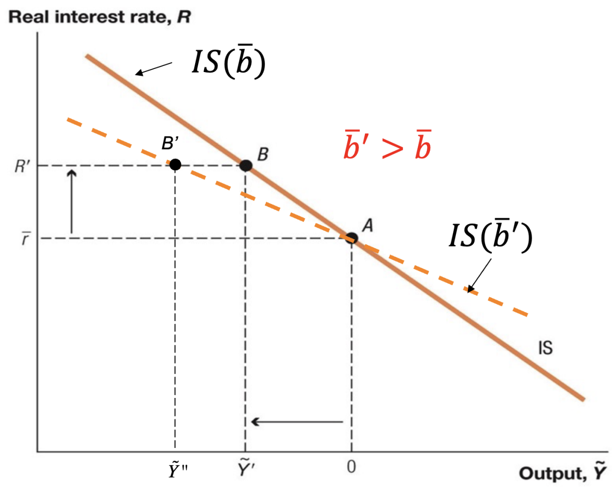


Figure 4.5: Increase in $\bar{b}$

3. **Change in Aggregate Demand:** Parameter $\bar{a}_i$ (or for any shock for some $\bar{a}_x, x = c, i, g, ex, im$ that affects $\bar{a}$ actually) increases, and so does $\bar{a}$. Hence, at every level of interest rate, output is higher. IS curve shifts right.

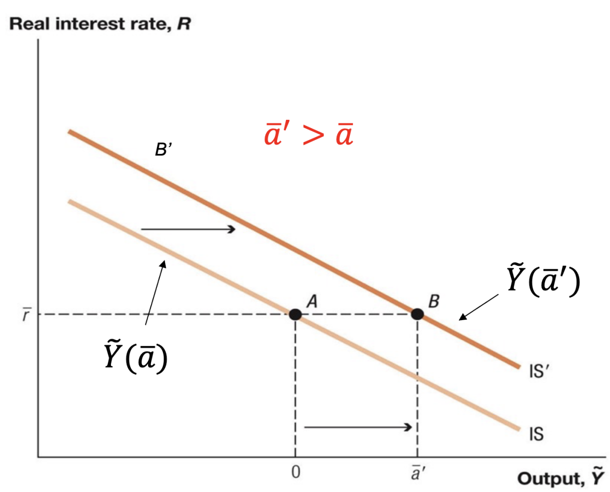


Figure 4.6: Increase in Aggregate Demand

4.3 The MP and Philips curve

The MP Curve

Interest Rates Central Banks set short term nominal interest rates. Shifts away from monetary aggregates both in theory/practice.

Example (Bank of England).
The Bank of England sets interest rates under its control, announcing the rate at which it would be willing to trade (borrow/lend) with banks. Private banks cannot charge higher rates: all other banks would trade with the Central Bank while the Interbank market shuts down. Private banks cannot charger lower rates, so other banks would borrow at lower rates and lend to the Central Bank at higher rates. This is an Arbitrage opportunity (differing prices for the same asset).

The Fisher Equation. MP describes how the central bank sets nominal interest rates. Since the IS curve depends on the real interest rates, we need to connect the nominal and real interest rates.
This is achieved by the Fischer Equation:

$$i_t = R_t + \pi_t$$

where i_t denotes nominal interest rate, R_t denotes real interest rate, and π_t denotes rate of inflation. Hence,

$$\pi_t \equiv \frac{P_t}{P_t - 1} - 1$$

Sticky Inflation and the Fisher Equation

$$R_t = i_t - \pi_t$$

Two assumptions to move from Fisher Equation to Sticky Inflation

- Assumption: Inflation moves slowly over time; implication is that π_t does not respond immediately to i_t.
  So if i_t increases and π_t does not, then R_t would increase
- Assumption: Central Bank effectively controls R_t; at least over the short run

Sticky inflation is crucial for real effects of monetary policy, otherwise we have a classical dichotomy where

- Changes in nominal variables have only nominal effects
- Prices and wages adjust instantaneously
- Monetary policy has only nominal effects

Reasons for sticky prices/inflation:

- Imperfect Information
- "Costs" of changing prices/wages
- Contracts are set in nominal and not real terms

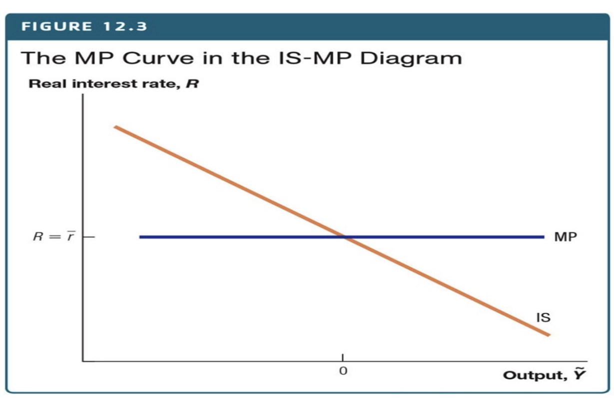


Figure 4.6: IS-MP Diagram

Central Bank sets nominal interest rate but because of sticky prices and inflation, the Central Bank effectively controls real interest rate. The MP is thus a horizontal line in $(\tilde{Y}_t, R_t)$ space.
Changes in parameters and their effects on the IS-MP Diagram.

1. **Monetary Downturn:** Suppose economy starts at potential output, then real interest rate equals $MPK = \beta$ and there are no aggregate demand shocks. Suppose CB raises nominal rate i_t, inflation π_t is slow to adjust and real interest rate r_t rises. Hence Investment I_t and consumption C_t falls, thus short-run output Y_t falls below potential $\bar{Y}_t$.

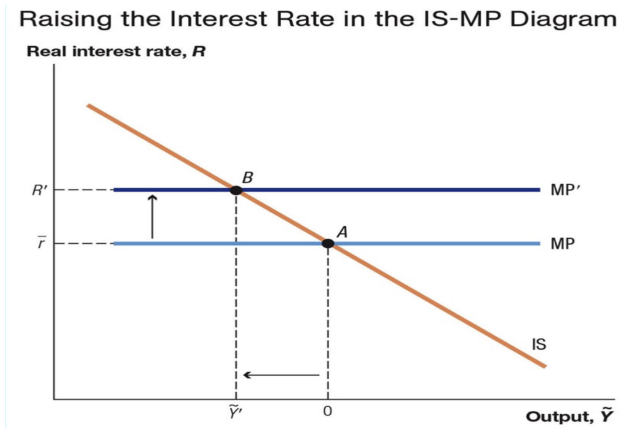


Figure 4.7: IS-MP Curve and Monetary Downturn

2. **Negative aggregate demand shock:** For example, end of Housing Bubble. Affects C_t via wealth effect and deleveraging; Affects I_t: residential investment falls. Thus, IS curve shifts left. CB lowers nominal interest rate i_t which causes real rate r_t to fall because inflation is sticky; if shock is not too large, CB can avoid downturn

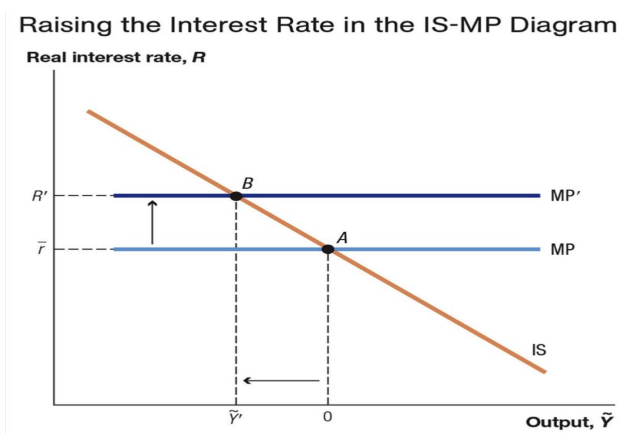


Figure 4.8: Effect of negative aggregate demand shock on IS-MP diagram

The Philips Curve

Derivation of the Philips Curve.

1. Look at how firm set their prices: (i) their expectations of the economy wide inflation and (ii) state of demand for their product. Hence, the price change (denoted by inflation) is

$$\pi_t = E_t\pi_{t+1} + v\tilde{Y}_t$$

where π_t is current inflation, $E_t\pi_{t+1}$ is expected inflation and $\tilde{Y}_t$ is demand conditions (fluctuations in demand)

2. Adaptive Expectations: We adopt a backward-looking model of expectations such that

$$E_t\pi_{t+1} = \pi_{t-1}$$

So there is slow adjustment of inflation expectations; embodies the sticky inflation assumption.

3. Philips Curve: If we replace this into PC we yield

$$\pi_t = \pi_{t-1} + v\tilde{Y}_t$$

Implications

- If output is below potential s.t. $\pi_t < \pi_{t-1}$ (since $\tilde{Y}_t < 0$), then prices rise more slowly than usual
- If output is above potential s.t. $\pi_t > \pi_{t-1}$ (since $\tilde{Y}_t > 0$), then prices rise more rapidly than usual

We can rewrite the equation as

$$\Delta\pi_t = v\tilde{Y}_t$$

since $\Delta\pi_t = \pi_t - \pi_{t-1}$. Graphically we have,

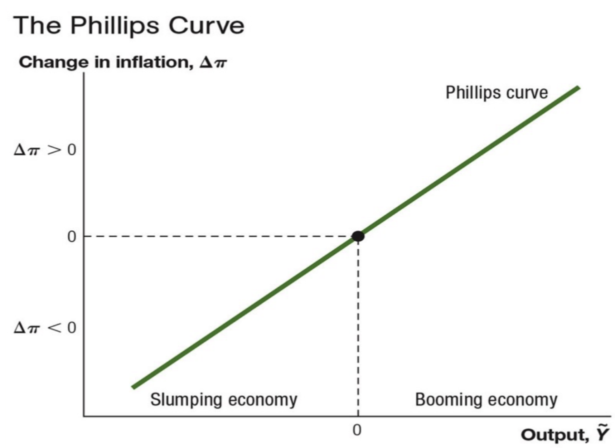


Figure 4.9: The Philips Curve

The PC is historically expressed in terms of the unemployment gap: difference between actual u_t and the potential rate of unemployment (NAIRU) $\bar{u}_t$.

$$\pi_t = \pi_{t-1} - \bar{\alpha}(u_t - \bar{u}_t)$$

Okun's Law: Link between the two formulations of the PC

$$u_t - \bar{u}_t = -\frac{\bar{v}}{\bar{\alpha}}\tilde{Y}_t$$

Shocks to the Philips Curve (temporary shock via e.g. shock to the price of oil)

$$\pi_t = \pi_{t-1} + v\tilde{Y}_t + o_t$$

Rewriting to yield

$$\Delta\pi_t = v\tilde{Y}_t + o_t$$

Graphically,

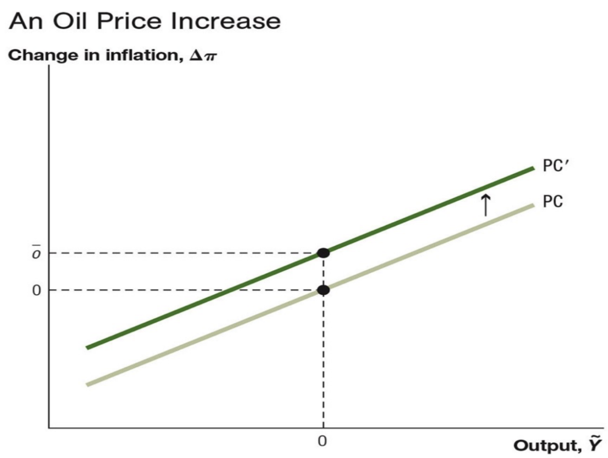


Figure 4.10: Shock to the Philips Curve

4.4 Applications of the Short-Run Models

Fiscal Policy Multipliers

Deriving the Multiplier:

1. Generalised Consumption Function: We continue to assume that consumption depends on long-run output (permanent income) and the real rate (vis-à-vis discount factor), but also on temporary deviation of income. So the Generalised Consumption Function:

$$C_t = (\bar{a}_c - b_c[R_t - \beta])\bar{Y}_t - \chi(Y_t - \bar{Y}_t)$$

where $\chi \in (0,1)$: e.g. credit market frictions. We can rewrite the consumption function as

$$\frac{C_t}{\bar{Y}_t} = \bar{a}_c - b_c[R_t - \beta] + \chi\tilde{Y}_t$$

since $\tilde{Y}_t = \frac{Y_t}{\bar{Y}_t} - 1$

2. Deriving the IS equation as before

$$\tilde{Y}_t = \frac{1}{1-\chi}\{\bar{a} - b_i(R_t - r) - b_c[R_t - \beta]\}$$

where $\frac{1}{1-\chi}$ and the second term is the original IS Curve.

This IS equation

$$\tilde{Y}_t = \frac{1}{1-\chi}\{\bar{a} - b_i(R_t - r) - b_c[R_t - \beta]\}$$

is qualitatively identical to the original specification, and so we can rewrite the IS curve as

$$\tilde{Y}_t = \hat{a} - \hat{b}_i(R_t - r) - \hat{b}_c[R_t - \beta]$$

Thus,

- $\hat{a} \equiv \frac{\bar{a}}{1-\chi}$
- $\hat{b}_i \equiv \frac{b_i}{1-\chi}$
- $\hat{b}_c \equiv \frac{b_c}{1-\chi}$

But this multiplied IS equation has a different quantitative implication since the demand shock has more than a one-to-one impact.

Example (The Government Multiplier). Increase in G_t fraction of $\bar{Y}_t$: a demand shock increases in $\bar{a}_g$ that drives up $\bar{a}$.
With the multiplier, short-run output increases by more than 1:1. Shock "multiplies" through the economy.

1. 1st round: increases short-run output for given C_t, as before
2. 2nd round: consumption increases because C_t depends on $\tilde{Y}_t$
3. Future cycles: Short-run output $\tilde{Y}_t$ increases more (virtuous cycles)

Large positive effects because the economy is in deep recession; but consequences of future tax increases may offset positive effects (since consumption is less overall).
Graphical illustration of initial shocks and fiscal/monetary policy responses:

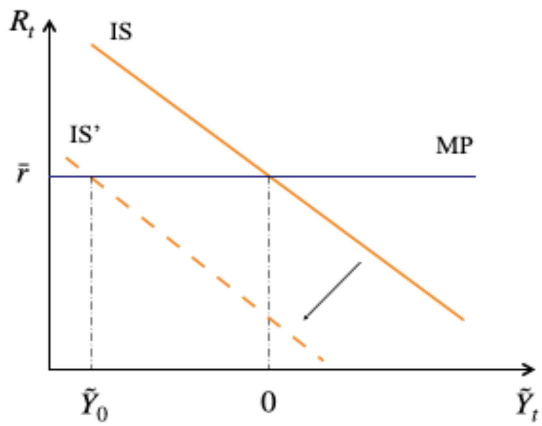


Figure 4.11: Initial Shock

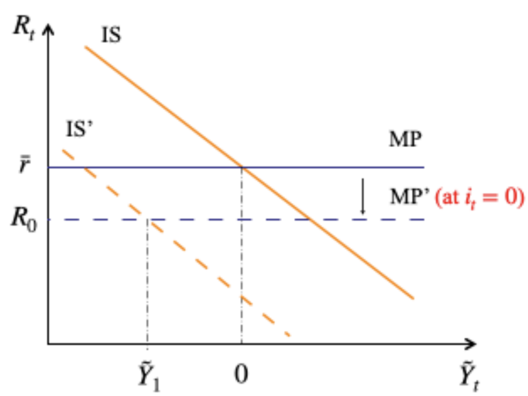


Figure 4.12: Monetary Policy Response

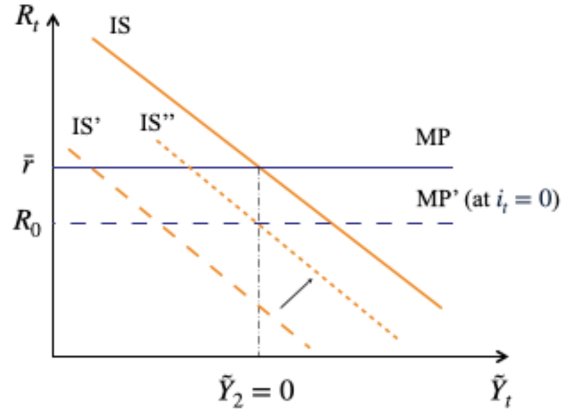


Figure 4.13: Fiscal Policy Response

There can be uncertainty about the size of the multiplier and macro uncertainty may have contributed to overestimates of the effectiveness of monetary policy. Uncertainty may dampen effects of certain policies. If private sector is unsure about future level of taxes/spending (after-tax income).

- Households may postpone consumption
- Firms may postpone investment

Precautionary Savings Motive

1. Monotone: $u' > 0$ (by definition)
2. Risk-adverse: $u'' < 0$ (concave utility function)
3. Prudence: $u''' > 0$

Theoretical debates on the multiplier

- Neoclassical view: If consumption depends only on the present discounted value of income after taxes ("permanent income"), then fiscal expansions unlikely to have large effects
- Keynesian view: If short-run changes to income also matter a lot, fiscal expansion are likely to have sizable positive effects. Frictions matter for the Keynesian View

Fiscal Expansions are effected by (i) Timing of fiscal response (delays); (ii) Timing of tax adjustment; (iii) State of business cycle.

The Great Inflation and the Volcker Disinflation

Reasons for Great Inflation of 1970s

1. OPEC coordinated oil price increases: Oil price shock
2. Loose U.S. monetary policy: Conventional wisdom (Permanent Trade off); R_t low, Y_t above potential, and increasing π_t
3. Federal Reserve: Interpreted slowdown as negative demand shock; but was a permanent fall in potential output

Volcker Disinflation: reducing inflation via tightening of monetary policy

1. Real interest rate increases

2. Economy enters a recession
3. As demand falls, over time firms raise their prices less
4. Eventually, inflation starts to fall

Graphically, we see the real interest rate rise

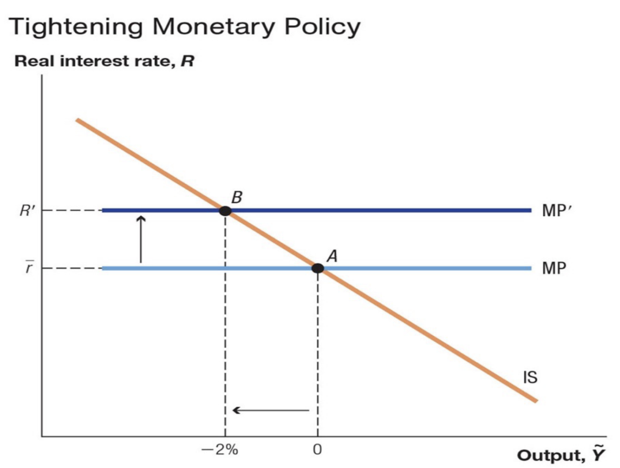


Figure 4.14: Volcker Disinflation and MP Curve

Since $\tilde{Y}_t$ falls, this affects the PC curve

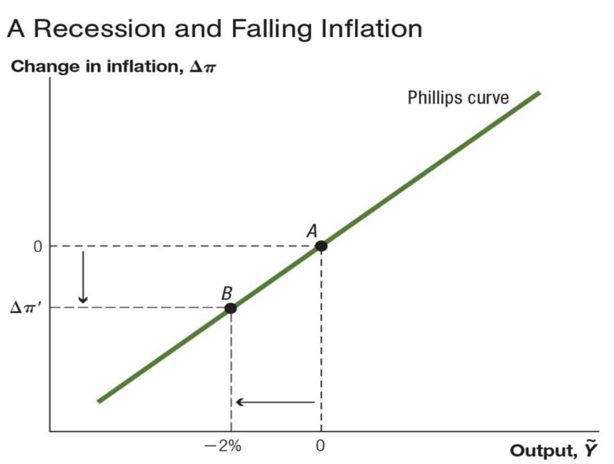


Figure 4.15: Volcker Disinflation and PC Curve

Volcker Disinflation Dynamics can be illustrated as such

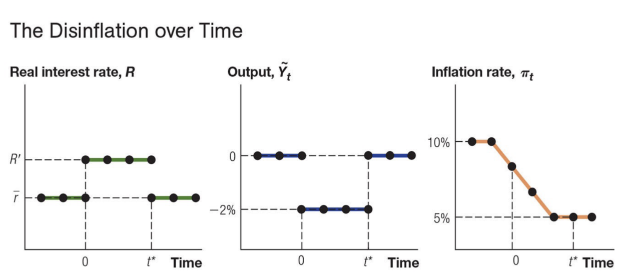


Figure 4.16: Volcker Disinflation Dynamics

Pursuing disinflation led to a fall in short run output. So lowering inflation is costly because it can send the economy into a recession, leading to higher unemployment and lost output. Once inflation has declined sufficiently, we can lower real interest rate back to the MPK/discount rate, allowing output to rise back to potential.

Monetary Policy Instruments

Central banks announce decisions in terms of i_t instead of M_t due to difficulties in estimating the velocity of money.

Money Market.

- Nominal interest rate is opportunity cost of holding money: How much you give up by holding cash in hand because the alternative is to keep in a savings account to accrue interest
- Money demand is downward sloping: Higher interest rates implies lower demand for money; interest rate = price of money
- Money market equilibrium pins down nominal interest rate and quantity of money

The IS-LM Model. The liquidity-money curve: real balances $\frac{M_t}{P_t}$ depend positively on short-run output and negatively on real interest rate

$$\frac{M_t}{P_t} = L(\tilde{Y}_t, R_t)$$

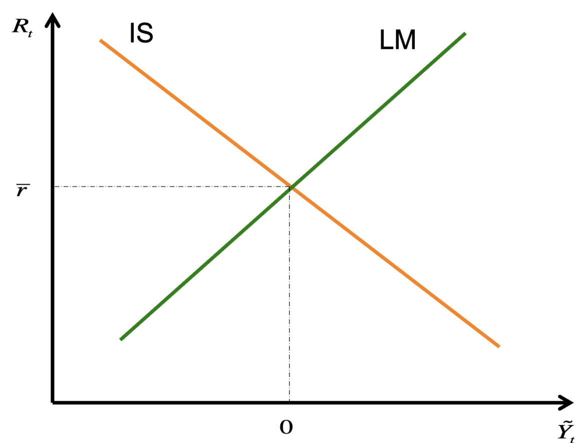


Figure 4.17: IS-LM Model

CBs choose to control i_t instead of M_t because

- Money demand is subject to many shocks: (i) changes in price level, (ii) changes in output, (iii) Financial innovation and velocity
- With constant money supply (i.e. if CB controls it), nominal interest rate is volatile. This leads to (i) output volatility and (ii) financial markets volatility

Two scenarios of the Money Market Equilibrium:

1. When CB controls M_t:

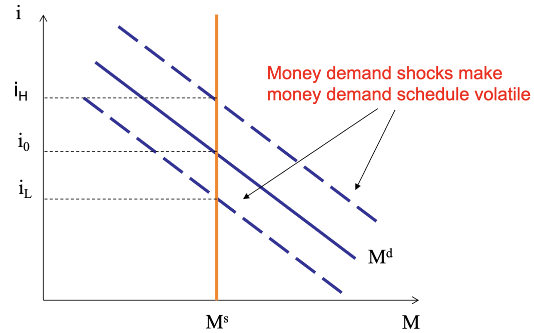


Figure 4.18: CB controlling Money Supply

2. When CB controls i_t:

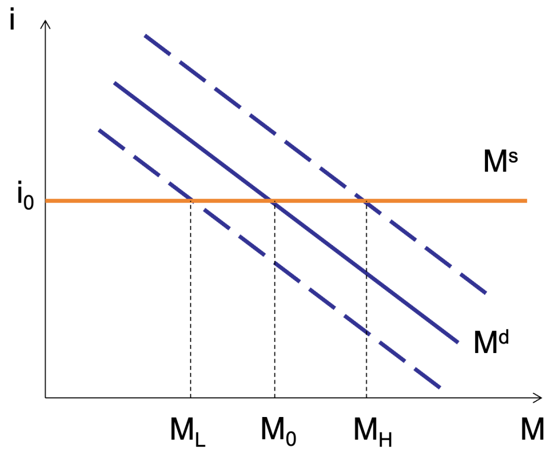


Figure 4.19: CB controlling nominal interest rates

From this, we can see that when CB targets i_t, M_t absorbs money demand shocks and changes in M_t lead to less macroeconomic instability (since interest rates affect investment).

Example. When the US Federal Reserve bank tried to control monetary aggregates early during Volcker's tenure, the Federal Funds Rate became extremely volatile

- Induced financial market volatility: makes arbitrage activity extremely difficult
- Unpredictable revolving credit: adjustable-rate mortgages

5 AD/AS

5.1 The AD/AS Framework

Aggregate Demand Curve

Derivation of the AD Curve.

1. **Monetary Policy Rule:** First, we recall the short-run model where

   - IS Curve: $\bar{Y}_t = \bar{a} - b_i(R_t - \bar{r}) - b_c(R_t - \beta)$
   - MP Curve: The central bank chooses R_t
   - Philips Curve: $\Delta\pi_t = v\bar{Y}_t + o_t$

   High short-run output increases inflation so there is an inflation-output tradeoff; the Central Bank chooses one feasible inflation-output gap combination.
   Definition 5.1 (Monetary Policy Rule). A Monetary Policy rule is a function that determines R_t for any given economic conditions.
   The MP Rule formulates a systematic policy response for any kind of shock that can hit the economy; it depends on the Central Bank Mandate: most central banks worldwide have an inflation mandate while some like the US Fed also have a real activity mandate.

2. **Inflation Targeting:** Next, we start with an inflation mandate π^T. The corresponding rule depends on current inflation (data) and inflation target (objective).
   Rule: If π_t is above π^T, increase R_t; if π_t is below π^T, decrease R_t. This is for stabilisation since when $R_t > \bar{r}$, this results in a fall in I_t, C_t which depend on sensitivity β, which results in a fall in short run output $\tilde{Y}_t$ which leads to a fall in inflation π_t. The limit of this rule is the zero-lower bound on nominal interest rate.
   The Taylor Principle: Determines how CBs should set interest rate

   $$i_t = \phi\pi_t$$

   where the Taylor Principle dictates that $\phi > 1$; i.e. i_t should increase more than 1:1 with π_t. So when we substitute the nominal interest rule into the Fisher equation given the Taylor Principle, then

   $$R_t = i_t - \pi_t = (\phi - 1)\pi_t$$

   If CB does not respond "strongly enough" to inflation

   - $\phi < 1$: a fall in real interest rate R_t would lead to a rise in short run output $\tilde{Y}_t$, which results in a rise in inflation π_t
   - This leads to a destabilising monetary policy

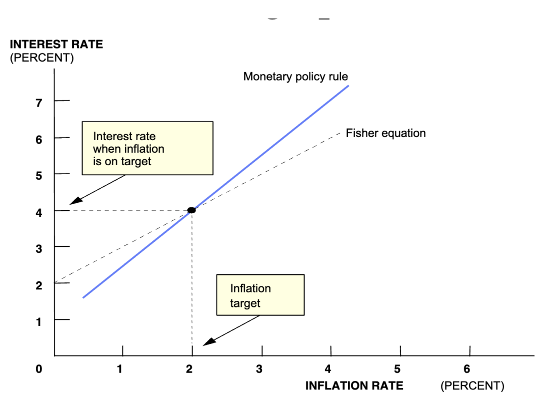


Figure 5.1: Monetary Policy Rule

3. **Inflation targeting Rule:** The rule is

$$R_t - \bar{r} = \psi(\pi_t - \bar{\pi})$$

where $\psi > 0$. Set R_t equal to MPK if inflation is on target ($= \beta$). If $\pi_t > \bar{\pi}$, then set $R_t > \bar{r}$ and vice versa. Coefficient ψ controls CB's response to inflation deviations

4. **Combining IS and MP to yield AD:** the AD curve results from combining IS curve and the MPR; it links short-run output and inflation.
Assumptions and equations

   - Assume: $\bar{r} = \beta$
   - IS: $\tilde{Y}_t = \bar{a} - \bar{b}(R_t - \bar{r})$
   - MP: $R_t - \bar{r} = \psi(\pi_t - \bar{\pi})$

   The when we insert the MPR into the IS, we yield the AD curve

$$\tilde{Y}_t = \bar{a} - \bar{b}\psi(\pi_t - \bar{\pi})$$

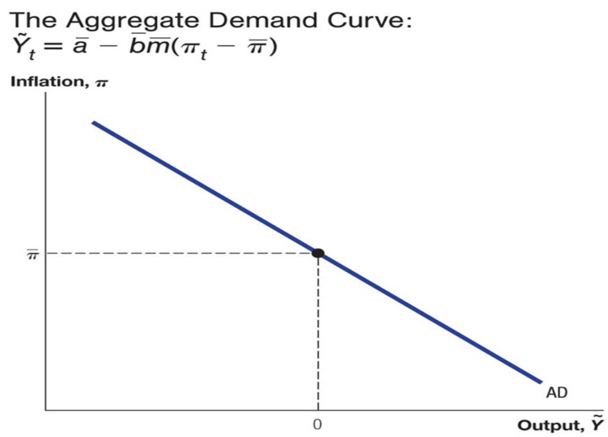


Figure 5.2: The AD Curve

   Changes along the AD Curve

   - If there is an inflation shock ($\pi_t > \bar{\pi}$), the economy moves upward along the AD from π to π'
   - If the CB adopts a more aggressive monetary policy rule s.t. ψ increases, then the AD curve becomes flatter
   - If there is an increase in $\bar{a}$ (shocks) or to inflation target $\bar{pi}$, then there would be a parallel shift of AD

Central Bank Mandate

Federal Reserve: Maximum Employment and stable prices on same level; Bank of England and ECB: Employment and growth objectives subordinated to price stability.

Dual Mandate: We can think of a dual mandate through adding a response to short-run output gap $\tilde{Y}_t$

$$R_t - \bar{r} = \psi(\pi_t - \bar{\pi}) + \phi\tilde{Y}_t$$

This is relevant when shocks push $\tilde{Y}_t$ and π_t in opposite directions; can rewrite the rule in terms of u_t gap through Okun's Law.

Is this a good description of monetary policy? We look at nominal rate again using the Fisher Equation:

$$(i_t - \bar{\iota}) - (\pi_t - \bar{\pi}) = \psi(\pi_t - \bar{\pi}) + \phi\tilde{Y}_t$$

Hence,

$$i_t = \bar{\iota} + (1+\psi)(\pi_t - \bar{\pi}) + \phi\tilde{Y}_t$$

where

- i_t = Effective FFR
- π_t = Quarterly inflation rate of Core CPI
- $\tilde{Y}_t$ = Real GDP - CBO potential output
- Set $\bar{\iota} = 4\%, \bar{\pi} = 2\%, \psi = 0.5, \phi = 0.5$

The AD Curve with the Dual Mandate becomes

1. Combine IS ($\beta = \bar{r}$) and MPR

$$\tilde{Y}_t = \bar{a} - \bar{b}(R_t - \bar{r})$$
$$R_t - \bar{r} = \phi(\pi_t - \bar{\pi}) + \psi\tilde{Y}_t$$

2. Thus,

$$\tilde{Y}_t = \frac{\bar{a}}{1+\bar{b}\psi} - \frac{\bar{b}\phi}{1+\bar{b}\psi}(\pi_t - \bar{\pi}) \equiv \breve{a} - \breve{b}\phi(\pi_t - \bar{\pi})$$

Dual Mandate ($\breve{b} < \bar{b}$) steepens the AD Curve, since $\frac{d\pi_t}{d\tilde{Y}_t} = -\frac{1}{\breve{b}\phi}$

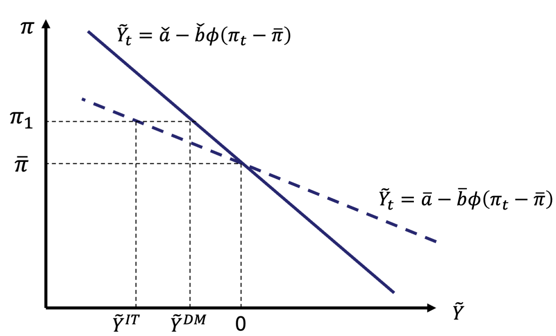


Figure 5.3: AD Curve with Dual Mandate

Aggregate Supply Curve

Derivation of the Aggregate Supply Curve. The Philips Curve represents the Aggregate Supply curve. We use the general form with a cost-push shock

$$\pi_t = \pi_{t-1} + v\tilde{Y}_t + o_t$$

We take inflation in the previous period π_{t-1} as exogenous, and $\frac{\partial\pi_t}{\partial\tilde{Y}_t} = v$

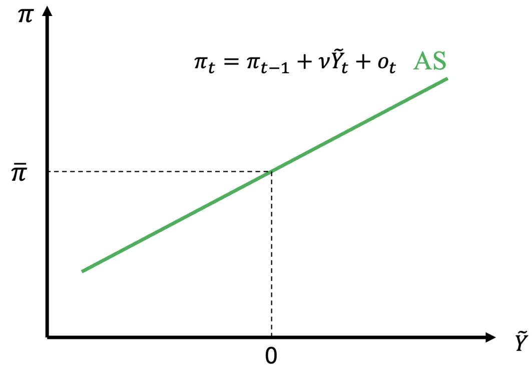


Figure 5.4: the AS Curve

The AD-AS Framework

We have two equations

1. AD Curve:

$$\tilde{Y}_t = \bar{a} - \bar{b}\phi(\pi_t - \bar{\pi})$$

2. AS Curve:

$$\pi_t = \pi_{t-1} + v\tilde{Y}_t + o_t$$

We have a linear system of equations:

$$\tilde{Y}_t = \hat{a} - \hat{b}\phi(\pi_{t-1} - \bar{\pi} + o_t)$$
$$\pi_t - \bar{\pi} = v\bar{a} + (1 - v\hat{b}\phi)(\pi_{t-1} - \bar{\pi} + o_t)$$

The solutions for short-run equilibrium are

$$\hat{a} \equiv \frac{\bar{a}}{1 + \bar{b}\phi v}$$
$$\hat{b} \equiv \frac{\bar{b}}{1 + b\bar{\phi} v}$$

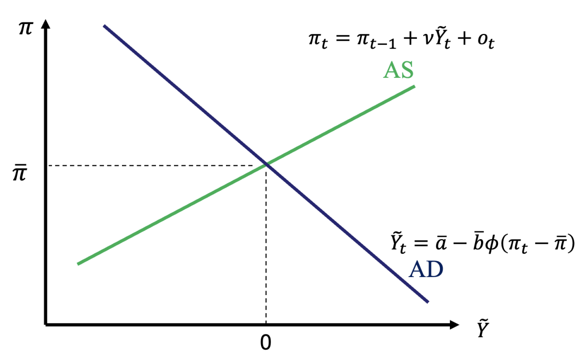


Figure 5.5: AD-AS Graphical Representation

Steady State: Equilibrium in the absence of shocks. It is when the endogenous variables are constant over time

- When inflation equals to its target:

$$\pi_t = \pi_t = \bar{\pi}_t \equiv \pi^*$$

- When output gap equals to zero:

$$\tilde{Y}_t = 0 \equiv \tilde{Y}^*$$

5.2 Shocks and Stabilisation in AD/AS

Types of Shocks in the AD/AS Framework

1. Demand Shocks: Consider a positive, temporary demand shock.
   Modelled as an increase in AD parameter $\bar{a} > 0$. The effect upon impact at $t = 1$ is

   - AD Curve shifts outward $\tilde{Y}_1 > 0$
   - Inflation increases $\pi_1 > \bar{\pi}$
   - Then the adjustment of the AS curve begins at $t = 2$ where the expectation of future inflation is

     $$E_2\pi_3 > \bar{\pi}$$

     And the AS curve shifts upwards as $\pi_1 > \bar{\pi}$.

   Considering an AS Curve with $o_t = 0$ s.t.

   $$\pi_t = \pi_{t-1} + v\tilde{Y}_t$$

   The new AS Curve thus goes through $(\pi_1, 0)$ instead of $(\bar{\pi}, 0)$ as seen

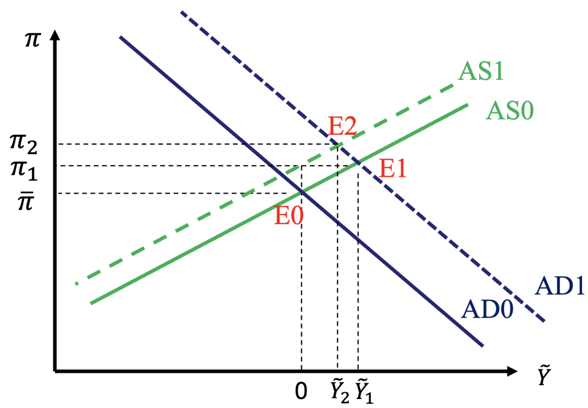


Figure 5.6: Demand Shock on ADAS curves

   So, AD adjusts at $t = 1$ and AS adjusts at $t = 2$.
   The second-period effect:

   - Short-run output gap still positive but smaller $0 < \tilde{Y}_2 < \tilde{Y}_1$
   - Inflation and expected future inflation increase $\pi_2 > \pi_1$

   Adjustment of the AS Curve shifts until $\tilde{Y}_n = 0$ (for simplicity the shock is assumed to last until $\tilde{Y} = 0$

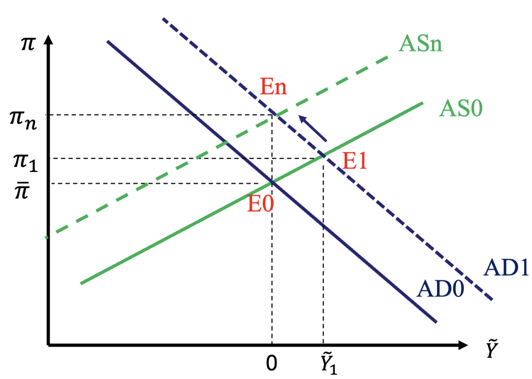


Figure 5.7: Demand Shock until n-th adjustment of AS Curve

After the demand shock is over: AD shifts back; inflation is still above target; economy enters a recession; dynamics in reverse to initial state. This characterises the business cycle where the boom is followed by a bust. Sine the shock ends at $t = n$, the AD curve shifts back at $t = n + 1$

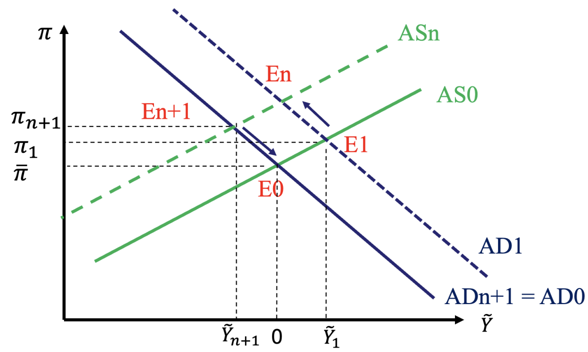


Figure 5.8: AD Curve shifts back after demand shock

Summary (AD Shock: boom followed by a bust)

- In the IS-MP framework, the central bank picks the interest rate that fully stabilises the economy after a demand shock
- In the AD-AS framework, monetary policy rule responds to inflation:
  - (Imperfect) stabilisation encoded in AD curve
  - Dynamics arise because of adaptive expectations
  - There is a question of optimal rule-based stabilisation: limited by imperfect information and lags in transmission mechanism

2. Supply Shocks: Consider a positive, temporary cost-push shock.
   Modelled as $\bar{o}_1 > 0$ and assume the shock lasts one period.
   The impact effect is at $t = 1$:

   - AS curve shifts inward $\pi_1 > \bar{\pi}$
   - Short-run output falls $\tilde{Y}_1 < 0$
   - Characterised as a stagflation shock

   The adjustment starts at $t = 2$, where $\bar{o}$ returns to 0 s.t. $\bar{o}_2 = 0$ but AS curve does not shift back because $\pi_1 > \bar{\pi}$. Therefore the AS2 curve is now

   $$\pi_2 = \pi_1 + v\tilde{Y}_2$$

   and thus passes through $(\pi_1, 0)$ instead of $(\bar{\pi}, 0)$.

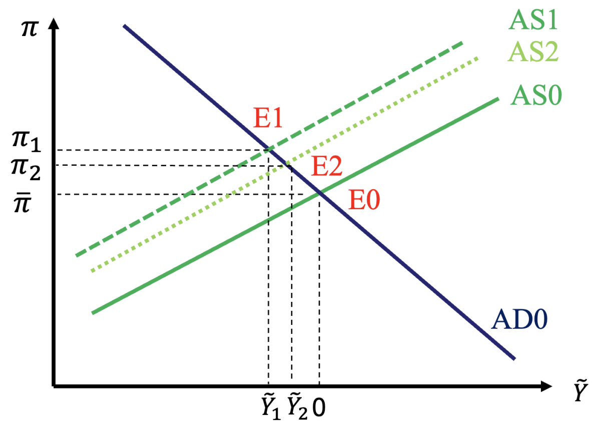


Figure 5.9: Supply shock on ADAS curves, adjustment period

Cost-push shock creates high inflation: raises expected inflation, slows AS adjustment back to initial position, economy (eventually) returns to original steady state.
The adjustment occurs towards steady state

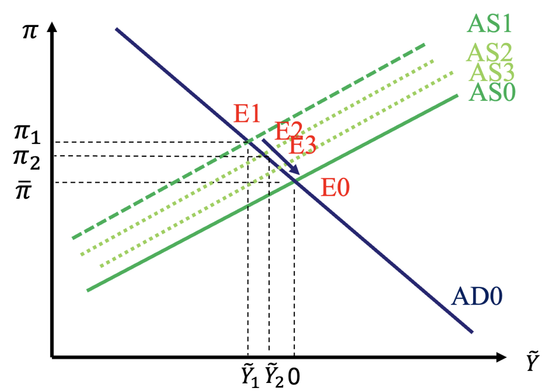


Figure 5.10: Adjustment towards steady state after cost-push shock

Summary (A negative AS shock)

- Higher inflation and lower output (stagflation)
- Transitory shock raises (expected) inflation persistently
  - Inflation remains high for several periods
  - Inflation and the economy back to normal after a downturn
- **Adaptive expectations key to dynamics**

3. Policy Shocks: Consider permanently lower inflation target
   Modelled as $\bar{\pi} \downarrow$. The impact effect at $t = 1$
   - AD curve shifts inward, $\tilde{Y}_1 < 0$
   - Inflation falls relative to initial equilibrium, $\pi_1 \downarrow$
   - But inflation higher than in the "final inflation" which is the new target inflation

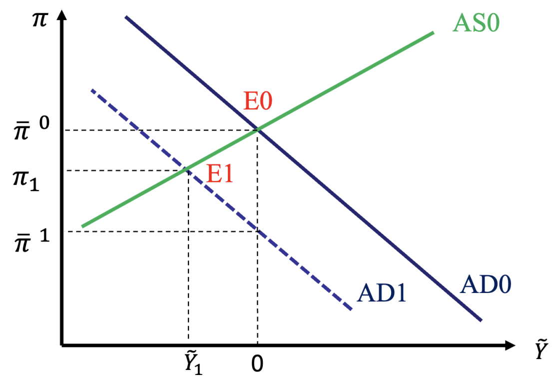


Figure 5.11: Initial response in ADAS for Policy Shock

There will be a new steady-state equilibrium with $\pi_1 = \bar{\pi}^1$ and $\tilde{Y}_t = 0$.
Inflation in initial period above new target

- CB increases interest rates and creates a downturn
- Lower output reduces actual inflation
- In the next period, the AS curve shifts outward

Process takes time since inflation is sticky: $\tilde{Y}_t$ remains below zero until inflation reaches new target.
We can see the eventual outcome

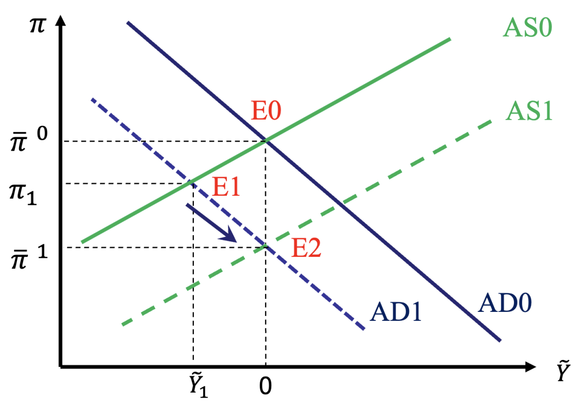


Figure 5.12: Eventual dynamics of disinflation for policy shock

Theory of Expectations
So far, we assumed adaptive expectations s.t.

$$E_t[\pi_{t+1}] = \pi_{t-1}$$

Independent of policy changes + motivated by inflation sluggishness.
With adaptive expectations, the central banks can 'fool' the private sector: pursue policies that boost output at the cost of inflation $\pi_t \uparrow$.

Alternative: Rational Expectations
Definition 5.1 (Rational Expectations): The private sector uses all available information to best forecast all variables of interests.
For inflation, this information may include: (i) costs that make inflation move, (ii) target inflation rate etc.
The rational expectations assumption influences our model in the following ways

- Expected inflation is consistent with realised outcomes
- Shocks still move inflation and output away from targets
- But deviations cannot occur due to systematic CB decisions

If we adopt the assumption of adaptive expectations, and the central bank announces lower inflation target, RE implies

$$E_t[\pi_{t+1}] = \bar{\pi}^1 < \bar{\pi}^0$$

Interpretation: expectation is now the inflation rate which the CB sets (since take into account all information); CB credibility is thus crucial to coordinate expectations. This leads to costless inflation since under RE, AS shifts down immediately to new target (rather than lagging behind). If CB is credible, π_t moves to new target without downturn.

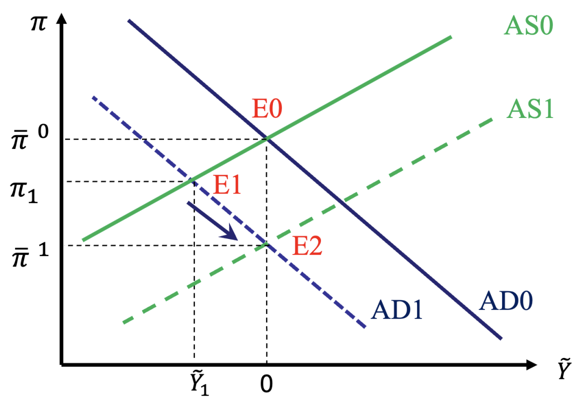


Figure 5.13: Costless disinflation

Are expectations rational?

- RE does not mean zero expectation errors but requires that agents know how the economy works and optimal forecasts are made given information available
- Large literature in behavioural economics where there are discrepancies between RE and expectational data

5.3 Short-Run Models and the Financial Crisis

Financial Frictions and Short-Run Models

Financial Crisis

- Fall in house prices lead to a deterioration of banks' balance sheets: banks cut lending and charge higher interest rates on loans; increase in spread of private sector interest rates and the official rates
- Increase in spreads signals higher risk: even when the central bank cuts the official rate, spread remains high due to risk on bank balance sheets (they don't want to lose money)

To analyse the crisis in short-run model, we introduce **financial frictions**.
Assume actual real interest rate consists of two components

$$R_t = R_t^{ff} + f$$

where

- R_t is borrowing rate for private sector

- R_t^{ff} is real interest rate set by CB
- f is financial friction (extra borrowing cost): in normal times $f = 0$, in crisis $f > 0$

Usually, the financial crisis would lead to a fall in investment and thus a fall in the IS curve. The CB would respond by lowering interest rate (MP curve) and usually, this would close the output gap. However, in a financial crisis, frictions create a deeper downturn caused by higher private sector interest rates, so instead of MP moving down to MP', MP moves up to MP" due to the crisis.

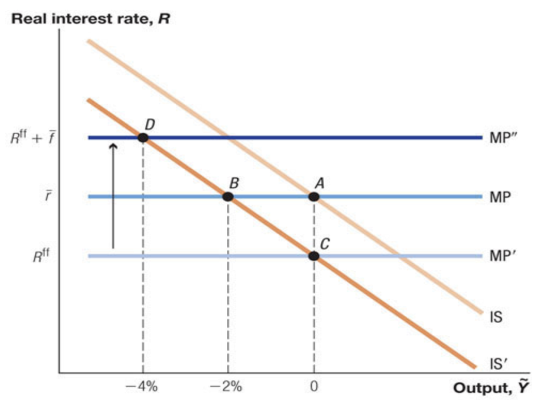


Figure 5.14: Financial Frictions in crsis

Deriving AD-AS with Financial Frictions. We have

- IS Curve:
$$\tilde{Y}_t = a - b(R_t - \bar{r})$$
- MP Rule:
$$R_t^{ff} - \bar{r} = \phi(\pi_t - \bar{\pi})$$
- Financial friction:
$$R_t = R_t^{ff} + f$$

We combined the financial friction equation with the monetary policy rule to yield

$$R_t - \bar{r} = f + \phi(\pi_t - \bar{\pi})$$

Substituting into IS curve yields AD curve with financial frictions

$$\tilde{Y}_t = a - bf - b\phi(\pi_t - \bar{\pi})$$

The financial crisis is a combination of two shocks

1. Negative demand shock due to housing s.t. $a < 0$
2. An increase in spreads charged (private sector i/r) s.t. $f > 0$

We can see the adjustment of the AD-AS curves due to the crisis

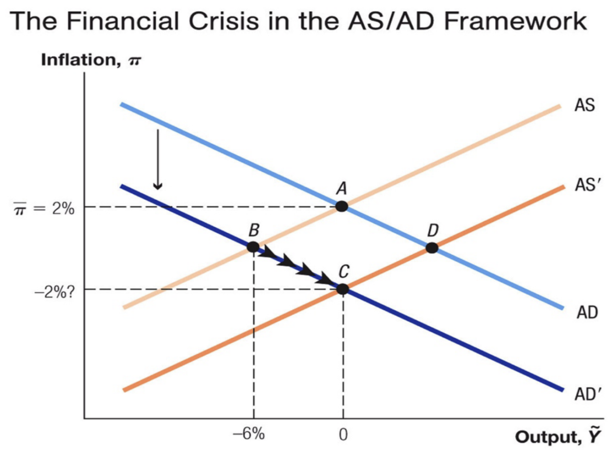


Figure 5.15: AD-AS in financial crisis

According to Fig. 15, we must examine what happens when inflation becomes negative.

Dangers of Deflation. Ceteris paribus, deflation raises real interest rate (from the Fisher equation)

$$R_t = i_t - \pi_t$$

In normal times, the central bank can cut the nominal rate but the nominal rate cannot fall below zero as there is a **Zero Lower Bound** and the liquidity trap; The real interest rate above MPK constitutes a drag on aggregate demand (since the marginal cost of investment is higher than the marginal benefit).

Deflationary Spirals:

- High interest rate (because of deflation and ZLB) worsens initial shock
- Demand falls even more and firms cut their prices
- More deflation and a higher real interest rate
- cycle continues (occurred during the Great Depression)

Policy Response to the Financial Crisis

Due to nominal interest rates hitting the Zero Lower Bound, unconventional monetary policies were adopted: (i) Liquidity provision; (ii) Forward Guidance; (iii) Quantitative Easing. Financial crises damages transmission mechanism of monetary policy which disrupts financial intermediation and flight-to-safety.

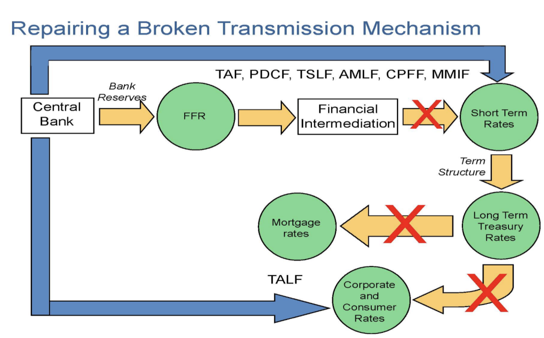


Figure 5.16: Monetary Policy in Normal Times

1. **Liquidity Provision:**

- First phase of crisis characterised by secondary financial market freeze.
  Banks stop lending (to each other) in fear on not being repaid; Value of securities earlier used as collateral becomes highly uncertain.
- The Central Bank buys risky assets in exchange for safe assets.
  Initially government bonds: changes the composition of balance sheets.
  Later reserves: an expansion of the central bank balance sheet.
  The objective is to make the banking system safer.

2. **Forward Guidance:** We recall the Philips Curve

$$\pi_t = E_t[\pi_{t+1}] + v\tilde{Y}_t = \sum_{j=0}^{\infty} vE_t[\tilde{Y}_{t+j}]$$

Forward-looking expectations:

- The entire path of expected future output gaps matter for inflation
- The entire path of expected future interest rates matter for inflation: R_{t+j}

The idea is simple

- Promise low future interest rates
- When the zero lower bound no longer binds

Mechanism

- Create expectations of higher future inflation
- Real interest rate falls, which stimulates real economic activity, which is easily monitored in yield curve:

$$i_{nt} = \frac{i_{1t} + i^e_{1t+1} + i^e_{1t+2} + \cdots + i^e_{1t+n-1}}{n}$$

Phases of forward guidance

- Phase I: Time-contingent forward guidance
  But economic conditions can change in between and it would require adjustment
- Phase II: State-contingent forward guidance
  Uses triggers and thresholds of unemployment rate levels and inflation levels.
  But do rates necessarily increase when $u_t < u^*$

Some related alternatives are to (i) increase inflation target: move from $\bar{\pi} = 2\%$ to $\bar{\pi} = 4\%$ and (ii) Price-level targeting to make up for past errors. But no central bank has experimented with these because the credibility cost is possibly too high

3. **Quantitative Easing:** The central bank buys long-term government bonds. The objective is to reduce long-term interest rates when short-term rate is at ZLB.
Interest rates that matter for most private sector decisions are long-term: like mortgage rates, especially if fixed and corporate bonds/loans.
In normal times, short-term rates changes lead to long-term rate changes.

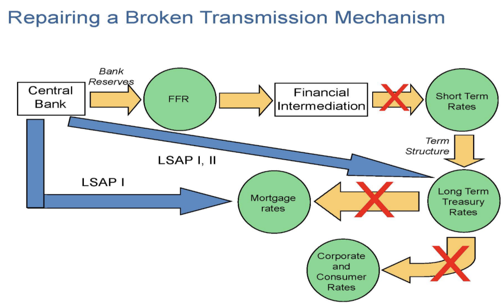


Figure 5.17: Quantitative Easing

CB purchases increase the price of bonds, thus reducing their yield i_{nt}; as private sector sells bonds, resources become available for spending + need private sector not to substitute government bonds: preference for certain type of bonds that become scarcer (Preferred Habitat)

5.4 Exchange Rates & the Open Economy AD-AS Model

Definition 5.2 (Nominal Exchange Rate): Price of domestic currency in units of foreign currency.

Law of One Price: Some tradeable good should sell for the same price in all countries. Suppose countries H and F are trading goods, then

$$EP_H = P_F$$

where E is the nominal exchange rate from the perspective of F since its how much of F$ needed to buy 1 H$. In the long run, same good should sell for same price in all countries; hence domestic and foreign price of same good should be equal. This is the basic no arbitrage relationship; otherwise, we would buy goods where it is cheaper and sell where it is expensive.
Short run failures of Law of One Price:

- Different tax rates across countries
- Tariffs on specific goods
- Transportation costs
- Non-tradeable content of tradeable goods

Long Run Exchange Rates

By the quantity theory of money, where

$$M\bar{v} = P\bar{Y}$$

long run prices are flexible, and monetary policy in the long run pins down price level (quantity of money in circulation determines price level).

Since the long run exchange rate is

$$\bar{E} = \frac{\bar{P}_F}{\bar{P}_H}$$

From the quantity theory of money (since $\bar{v}$ and $\bar{Y}$ are fixed), then

$$\bar{E} = \frac{M_F}{M_H}$$

so cross country quantity of money M pins down $\bar{E}$. So, if F monetary policy is relatively expansionary s.t. $\bar{P}_F$ increases relative to $\bar{P}_H$, then $\bar{E}$ increases and the long run nominal exchanges appreciates.
Nominal exchange rate depreciation to occur in countries with higher inflation. The role of expectations causes persistent inflation divergence but sharp depreciation.

Short Run Exchange Rates

Demand and Supply Analysis for currency (trading in global markets pin down short-run exchange rates):

- Demand: Goods and services across countries
- Supply: Central banks provide supply

Suppose interest rate increases, then UK bonds become more attractive so i_{UK} increases, then demand for GBP increases so GBP appreciates. Hence E from the perspective of the US increases, and USD depreciates against GBP.

Real Exchange Rate

Definition 5.3 (Real Exchange Rate): Price of domestic consumption in units of foreign consumption

$$RER = \frac{EP_H}{P_F}$$

where P_i is the price of one consumption bucket in some country i. E converts 1 GBP into USD.

If law of one price holds for consumption basket, then $R\bar{E}R = 1$.

In the short run however, nominal exchange rates continuously changes and since prices are slow to adjust, LOOP does not necessarily hold. Since prices adjust slowly, RER moves with nominal exchange rate.

Open Economy AD-AS Model

IS curve is derived assuming that trade balance is a constant fraction of potential output $\bar{Y}_t$.

The underlying idea is that RER movements can affect trade. A high RER would make home goods expensive relative to foreign goods so exports are likely to be low and imports to be high. Basic idea is that consumers buy similar goods from cheapest location.

Assumption: trade balance also depends on domestic real interest rate, in deviation from world interest rate

$$\frac{NX_t}{\bar{Y}_t} = a_{nx} - b_{nx}(R_t - \bar{R^w})$$

Take the world real interest rate as exogenous. This captures the role of RER since when i_t increases, then R_t increases, so C_t, I_t decreases. Hence in the open economy, when nominal exchange rate E_t appreciates, RER_t appreciates because prices are slow to adjust, so X_t decreases while IM_t increases, so NX_t decreases from domestic country.

Derivation of IS Curve. We add and subtract the MPK ($\bar{r}$) to net export equation:

$$\frac{NX_t}{\bar{Y}_t} = a_{nx} - b_{nx}(R_t - \bar{r}) + b_{nx}(\bar{R}^w - \bar{r})$$

We can ten obtain the same IS curve

$$\tilde{Y}_t = a - b(R_t - \bar{r})$$

but with different coefficients

$$\begin{aligned} a &\equiv a_c + a_i + a_g + anx + b_{nx}(\bar{R}^w - \bar{r}) - 1 \\ b &\equiv b_i + b_{nx} \end{aligned}$$

Now, suppose that the Fed raises nominal interest rate s.t. $\bar{R}^w$ increases and a increases. The shock to the IS curve creates an outward shift, since investors will invest more in the US and less in the UK, hence demand more USD instead of GBP. So USD appreciates against the GBP. In the UK, RER will depreciate and NX will improve. As per normal, the AD curve shifts up, causing the AS curve to shift up and they begin to return to the steady state once the shock ends.

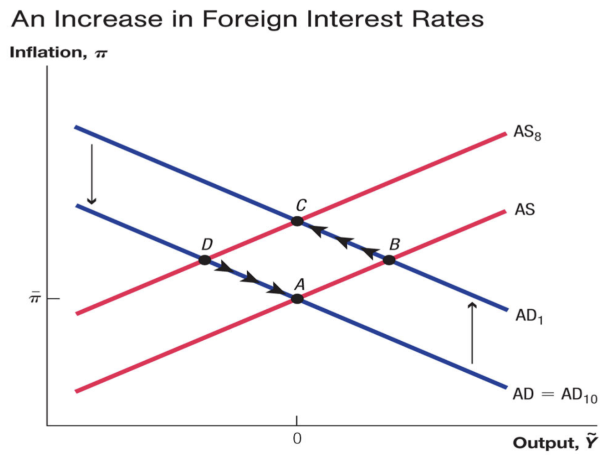


Figure 5.18: A rise in $\bar{R}^w$

5.5 The Open Economy: Applications

Mundell-Flemming Trilemma

Proposition (Mundell-Fleming's Impossible Trilemma). Under the framework of the trilemma, there are three objectives which are impossible for a country to obtain simultaneously:

1. Domestic monetary independence
2. Stable exchange rates
3. Free international capital flows

We can deduce why the three goals are impossible for a country to simultaneously obtain by way of proof by contradiction.

Proof. AFC that all three objectives can be attained at once. We now need to state the relationship between domestic and foreign interest rate, represented by the uncovered interest rate parity equation:

$$i_t = i_t^* + \mathbb{E}_t[E_{t+1}] - E_t$$

where i_t is the domestic interest rate, i_t^* is the foreign interest rate and E_t is the nominal interest rate at time t.
Suppose then I want to fix my exchange rate to achieve a stable exchange rate. This implies that $\mathbb{E}_t[E_{t+1}] = E_t$. Hence, we reach the conclusion that

$$i_t = i_t^*$$

i.e. our interest rate must be pegged to external interest rate, thereby losing monetary policy independence. If we want to restore monetary independence, we have to impose capital controls so that the no arbitrage relationship breaks down and

$$i_t \neq i_t^*$$

and we cannot freely convert domestic currency to foreign currency (this creates a wedge between domestic and foreign interest rates). Similarly, to restore no capital controls, we need to unpeg our exchange rates such that

$$\mathbb{E}_t[E_{t+1}] \neq E_t$$

As we can see, sustaining 2 of the goals requires us to sacrifice another goal. Hence, this contradicts the claim that we can achieve all three goals simultaneously.

Fixed Exchange Rates and IS-MP

Consider negative demand shock: like in Mexico (-ve confidence), SEA (higher cost of borrowing), Argentina (-ve foreign demand shock). IS curve shifts left. With a flexible exchange rate, the central bank retains monetary policy independence and could shift the MP curve down by lowering i_t. But with a fixed ER, the central bank & no capital controls, the nominal interest rate must remain the same as foreign levels. If the shock is deflationary, real interest rates may actually increase, worsening recession and $\tilde{Y} < 0$.

So fixed exchange rates prevent monetary easing and can exacerbate recession if shock is deflationary.

The Euro Debt Crisis

Single Euro solves the problem of credibility of fixed exchange rate system but introduces inflexibility of single monetary policy. However, fiscal problems emerged

- Irresponsible fiscal policy in Greece and Portugal
- high deficits in Ireland and Spain due to bank bailouts
- Low growth in Italy and high (though stable) debt

Compounded by lack of competitiveness. Positive demand shock because of fiscal expansions and housing booms, so IS shifts right. Inflation differentials in periphery amplify boom: ECB sets i_t for all EMU but π_t higher in periphery. So R_t, NX_t falls.

Common element of lack of competitiveness since there is RER appreciation and large trade/current account deficits.

ECB responded with various measures: liquidity provision to banks; commitment to act and avoid disaster scenarios; Quantitative Easing.

Global Imbalances

Motives for external deficits (based on consumption smoothing):

- Risk sharing: country borrows if bad shocks hit
- Permanent income: Country borrows if expectations of future growth are high

Large external deficits in US pre-crisis: fits second explanation of a boom with a period of high expected growth.

External Imbalance Accounting: We can write national income identity as

$$(Y_t - T_t - C_t) + (T_t - G_t) + (IM_t - EX_t) = I_t$$

which is private savings + government savings + foreign savings = investment.
We can define total savings as the sum of private and government savings, implying that trade balance equals net capital outflow

$$NX_t = S_t - I_t$$

because whatever savings not in I_t must be invested elsewhere. Net flow of goods associated with net flow of assets in opposite direction: If high net exports then accumulates financial assets from foreign countries by investing outside ($S_t > I_t$); if negative net exports, then sells financial assets to others by increasing FDI($I_t > S - t$).

What causes global imbalances

- Savings glut hypothesis (Bernanke, 2005): desire to save by foreign countries (China, emerging markets, oil producers), explaining decline in world real interest rate (due to higher supply of funds to lend)
- Financial deregulation and US monetary policy (Taylor, 2007): coherent with housing boom; requires emerging markets to peg to USD to explain low interest rates

Twin Deficits: Large government budget deficits can cause large foreign imbalances:
If private savings roughly equal investment, then government savings must equal (negative) foreign savings i.e.

$$T_t - G_t = -(IM_t - EX_t)$$

Note: But this equation is useless since the US has deficits in both
Policy challenges from Global Imbalances

- Large external imbalances likely to emerge in a globalised world: can fuel asset price boom-bust cycles; can be exacerbated by fixed exchange rates.
- Poses challenges to policy makers: should we put restrictions to free capital mobility or have domestic policies to avoid bubbles or more international policy coordination

6 Appendix

6.1 Full Solow-Swan Model in continuous time

Model Set Up
Let us assume that

$$\begin{aligned} \dot{L}(t) &= nL(t) \\ \dot{A}(t) &= gA(t) \end{aligned}$$

where $L(t), A(t)$ represent labour force and technology with respect to time, and n, g represent population growth rate and technology growth rate respectively.
We can now define the production function as a parametric equation with respect to time, where $A(t)L(t)$ enters the equation multiplicatively as A augments labour. Therefore,

$$Y(K(t), A(t)L(t)) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}$$

It is clear that this production function exhibits constant returns to scale.
Proof.

$$\begin{aligned} Y(\lambda K(t), \lambda A(t)L(t)) &= \lambda K(t)^{\alpha}(A(t)L(t))^{1-\alpha} \\ &= \lambda Y(K(t), A(t)L(t)) \end{aligned}$$

Given that the prodution function exhibits CRS, we want to find output/effective labour, therefore

$$\frac{1}{A(t)L(t)} Y(K(t), A(t)L(t)) = y(k(t), 1) = k^{\alpha}, \text{ where } \frac{K}{AL} = k$$

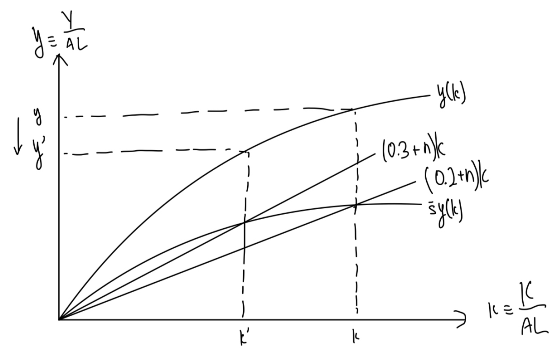
Now, we define the capital accumulation function as actual investment - break-even investment. To find the capital accumulation function, consider the chain rule.

$$\begin{aligned}
\dot{k}(t) &= \frac{d}{dt}\frac{K(t)}{A(t)L(t)} \\
&= \frac{\dot{K}(t)}{A(t)L(t)} - \frac{K(t)}{[A(t)L(t)]^2}[\dot{A}(t)L(t) + \dot{L}(t)A(t)] \\
&= \bar{s}\frac{Y(t)}{A(t)L(t)} - \delta\frac{K(t)}{A(t)L(t)} - \frac{K(t)}{A(t)L(t)}\frac{\dot{L}(t)}{L(t)} - \frac{K(t)}{A(t)L(t)}\frac{\dot{A}(t)}{A(t)} \\
\implies \dot{k}(t) &= \bar{s}y(t) - (\delta + n + g)k(t)
\end{aligned}$$

So the steady state is when $\dot{k}(t) = 0$, therefore when

$$\bar{s}y(t) = (\delta + n + g)k(t)$$

We have described the Solow Swan Model.



So when g increases from 1% to 2%, the break-even investment of capital per effective labourer pivots upwards from $(0.2 + n)k$ to $(o.3 + n)k$. Consequently, the amount of capital per effective labourer falls, since technology is labour augmenting, and a higher technological growth rate increases the number of effective labourers. Consequently, output per effective labourer falls. Of course, we want to examine output per capita, which will be examined next.

Output per capita

Now, reconsider the production function when it is output per capita, such that

$$y_L(\frac{K(t)}{L(t)}, A(t)) = A^{1-\alpha}K^{\alpha}$$

To find growth rates with respect to the change in variables over time, we take the natural logarithms of the equation to get

$$\begin{aligned}
\ln y_L(t) &= (1-\alpha)\ln A(t) + \alpha \ln k(t) \\
\implies \frac{\dot{y_L}(t)}{y_L} &= (1-\alpha)\frac{\dot{A}(t)}{A(t)} + \alpha\frac{\dot{k}(t)}{k(t)}
\end{aligned}$$

In the steady state, initially when $g = 0.01$ and $\dot{k}(t) = 0$,

$$\frac{\dot{y_L}(t)}{y_L(t)} = (1-\alpha)g$$

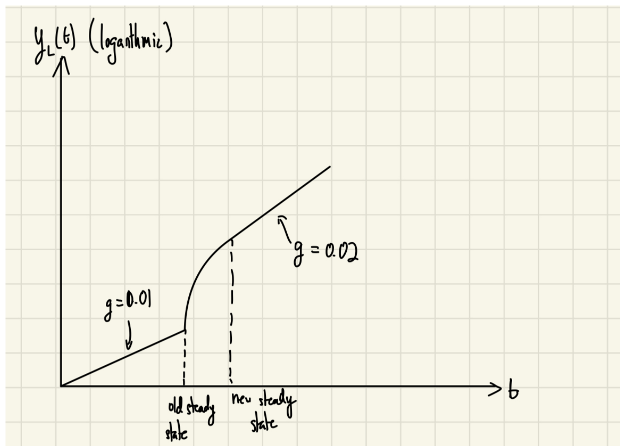
So, the long run output/capital growth follows a balanced path when in steady state.
But when g increased from 1% to 2%, since $k(t)$ is inversely related to $A(t)$ and $y_L(t)$ is positively related to $A(t)$, immediately we see that in transition from the old to new steady state

$$\frac{\dot{y_L}(t)}{y_L(t)} = (1-\alpha)g + \alpha\frac{\dot{k}(t)}{k(t)}$$

We can make two observations,

1. Since $\bar{s}y_L(t) > \delta k(t)$, $\alpha\frac{\dot{k}(t)}{k(t)} > 0$
2. The rate of change of $\alpha\frac{\dot{k}(t)}{k(t)}$ is negative

So if we plot output/capita against time along a logarithmic scale for output/capita (since growth is exponential), we yield the adjustment path



6.2 Stiglitz-Shapiro Model of Efficiency wages

Deriving the No-shirking condition
Suppose each worker has a utility function $u(w, e) = w - e$ and workers maximise the utility function with a discount rate r. Then let b be the probability pr unit time that a worker is dismissed from his job and now there is a lifetime expected utility V_u of an unemployed individual. Then you find the asset value of employment during a short interval $[0, T]$

$$V_e = wT + e^{-rT}[bTV_u + (1 - bT)V_e]$$

because the worker is either dismissed or kept employed during the time. The exponential function appears because the occasion of dismiss in the interval is once and the Poisson distribution is used for the discount rate. Due to the short interval, the exponential function is approximated as $1 - rT$

$$V_e = wT + (1 - rT)[bTV_u + (1 - bT)V_e]$$

and simple calculation yields

$$V_e = \frac{wT + bTV_u - rbT^2V_u}{rT + bT - rbT^2}$$

$$\lim_{t\to 0} V_e = \frac{w + bV_u}{r + b}$$

which gives the fundamental asset equation of a worker

$$rV_e = w + b(V_u - V_e)$$

For a non-shirker, the equation is

$$rV_{e,N} = w - e + b(V_u - V_{e,N})$$

where $b(V_u - V_{e,N}$ is the utility difference between being unemployed and being employed and not shirking
For a shirker it is

$$rV_{e,S} = w + (b+q)(V_u - V_{e,S})$$

where q is the probability per unit time that a worker is caught shirking and sacked.
We then see that

$$V_{e,N} = \frac{w - e + bV_u}{r+b}$$
$$V_{e,S} = \frac{w + (b+q)V_u}{r+b+q}$$

The condition $V_{e,S} < V_{e,N}$ is called the no-shirking condition, which is expressed as

$$\hat{w} = rV_u + \frac{e(r+b+q)}{q} < w$$

where $\hat{w}$ is the critical wage. The worker works hard if and only if the NSC is satisfied (the wage set is higher than the critical wage). So if the workers get sufficiently high wages, then the NSC is met and they will not shirk.

Implications of No-shirking condition

Let a be the probability of getting a job per unit time. In equilibrium, the flow itno the unemployment pool must be equal to the flow out. Thus the probablity is

$$a = \frac{bL}{N-L}$$

where L is the aggregate employment and N is the total labour supply. Assume that the employee is offered his minimum wage $\bar{w}$ by law and the NSC becomes

$$\bar{w} + e + \frac{e(a+b+r)}{q} = \hat{w} < w$$

This is the aggregate NSC. These equations yield

$$e + \bar{w} + \frac{e}{q}(\frac{b}{u} + r) < w$$

where the unemployment rate is $u = \frac{N-L}{N}$. This constraint suggests that full employment always involve shirking.
Now we assume that the aggregate production function $F(L)$ is a function of total effective labour force. A firm's labour demand is given by equating the cost of hiring an additional employee to the marginal product of labour. This cost consists of wages and future unemployment benefits. Now consider the case where $\bar{w} = 0$, then we have

$$\hat{w} = \frac{dF(L)}{dL}$$

In equilibrium, $F'(L) = \hat{w} = w^*$ holds, where w^* is the equilibrium wage. Then, the equilibrium condition becomes

$$F'(L) = \hat{w} = e + \frac{e}{q}(\frac{b}{u} + r) = e\left(1 + \frac{r+b+a}{q}\right)$$

This suggests the following things:

- Demand side approach: if the employer pays less than w^*, the worker's shirking becomes more likely to occur and decreases their productivity. Consequently, wage is unlikely to decrease and this is a microscopic mechanism of nominal rigidity. Thus, wage cannot decrease so that it can stabilise employment level, and hence unemployment must increase during recessions.
- Supply side approach: Jobless persons want to work at w^* or lower, but cannot make a credible promise not to shirk at such wages. As a result, unvoluntary unemployment occurs.