Financial Crises and Shadow Banks: A Quantitative Analysis

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Abstract

Motivated by the build-up of shadow bank leverage prior to the financial crisis of 2007-2008, I develop a nonlinear macroeconomic model featuring excessive leverage accumulation and endogenous financial crises to capture the observed dynamics and to quantify the build-up of financial fragility. I use the model to illustrate that extensive leverage makes the shadow banking system runnable, thereby raising the vulnerability of the economy to future financial crises. The model is taken to U.S. data with the objective of estimating and analyzing the probability of a run in the years preceding the financial crisis of 2007-2008. According to the model, the estimated risk of a run was already considerable in 2005 and kept increasing due to the upsurge in leverage. Using counterfactual scenarios, I assess the impact of alternative monetary and macroprudential policy strategies on the estimated build-up of financial fragility.

Keywords: Financial crises, leverage, credit boom, nonlinear estimation

JEL classification: E32, E44, G23

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1 Introduction

The financial crisis of 2007-2008 was, at the time, the most severe economic downturn in the US since the Great Depression. Although the origins of the financial crisis are complex and various, the financial distress in the shadow banking sector has been shown to be one of the key factors.[1] The shadow banking sector, which consists of financial intermediaries operating outside normal banking regulation, expanded considerably before the crisis. Crucially, there was an excessive build-up of leverage (asset to equity ratio) for these unregulated banks. The collapse of the highly leveraged major investment bank Lehman Brothers in September 2008 intensified then a run on the short-term funding of many financial intermediaries, with very severe repercussions for the real economy in the fourth quarter of 2008. Figure 1 documents these stylized facts about GDP growth and shadow bank leverage using balance sheet data from the Flow of Funds and Compustat.

In this paper, I build a new nonlinear quantitative macroeconomic model with financial intermediaries and runs to capture the observed dynamics as well as to quantify the endogenous build-up of financial fragility. The possibility of a run on the financial system is state-dependent and relies on economic circumstances. Specifically, risk-shifting incentives for intermediaries can result in extensive leverage accumulation, which makes the financial system runnable and thereby raises the vulnerability of the economy to a financial crisis. I then take the model to the data to obtain a novel structural estimate of the endogenous financial fragility around the financial crisis of 2007-2008. For this purpose, I first fit the framework to salient features of the U.S. economy and the shadow banking sector. I then condition the model on selected data to estimate the endogenous probability of a run through the lens of a structural nonlinear model. My results suggest that the estimated financial fragility and the economic downside risk increase considerably from 2005 onwards and peak in 2008 due to rising shadow bank leverage. Using counterfactual scenarios, I assess the impact of alternative monetary and macroprudential policy strategies on the estimated build-up of financial fragility and the occurrence of a financial crisis in 2007-2008. The counterfactual suggests that macroprudential policy limiting leverage sufficiently would have avoided the run on the financial sector in 2008 itself.

The framework is designed to evaluate and quantify the build-up of financial fragility because it features endogenous boom-bust dynamics and reconciles key macroeconomic as well as financial features of the great financial crisis. The dynamics rely on the interaction among two features that correspond well to the shadow banking sector. First, the financial intermediaries face risk-shifting incentives and volatility shocks, which allows me to account for extensive leverage accumulation similar to Adrian and Shin (2014) and Nuño and Thomas (2017). Second, runs on the financial sector are endogenous since they are state-dependent as in Gertler, Kiyotaki and Prestipino (2020*b*).

The boom-bust mechanism is as follows. First, a period of low volatility reduces the risk-shifting incentives of financial intermediaries, which results in substantially elevated leverage.

[1] See e.g. Adrian and Shin (2010), Bernanke (2018), Brunnermeier (2009) and Gorton and Metrick (2012).

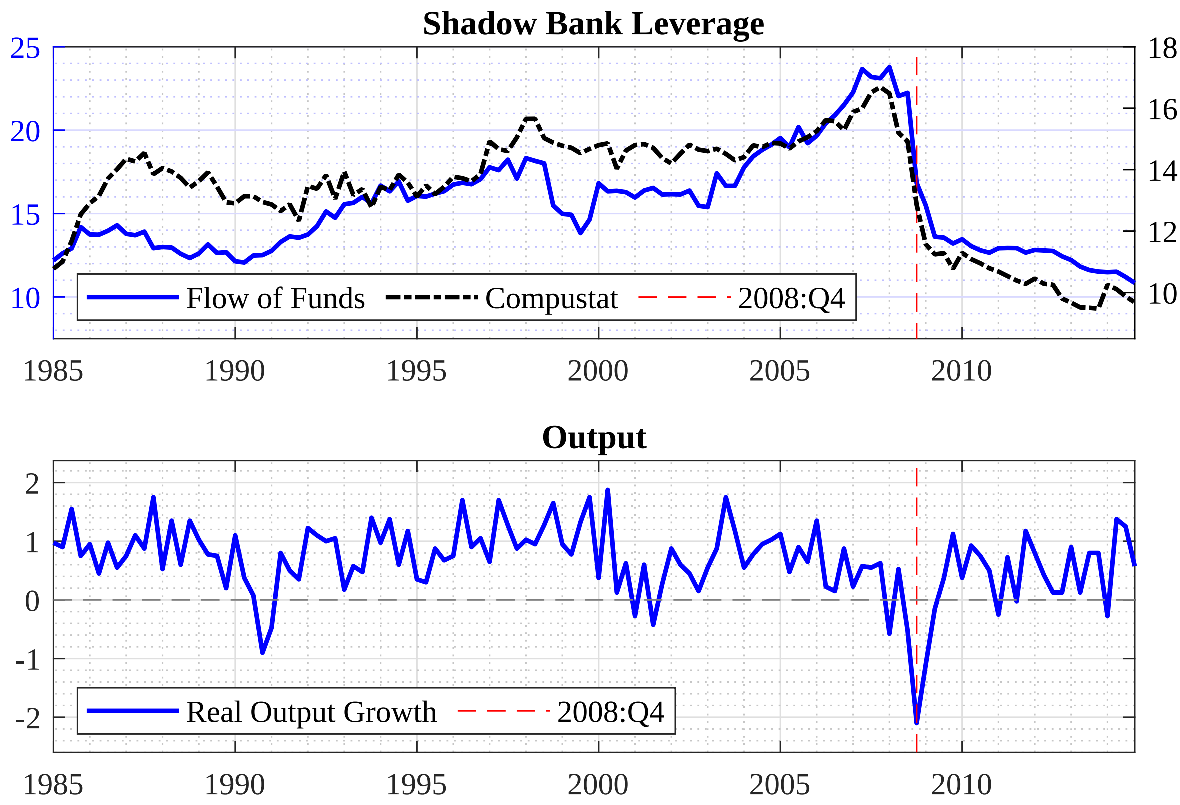

Figure 1: The upper graph shows two measures of U.S. shadow bank book leverage. The first measure is based on balance sheet data from the Flow of Funds (left axis). The alternative one uses Compustat data (right axis). The leverage series rely on the book value of equity. Appendix A shows the details. The lower graph shows the quarter on quarter real output growth rate in percent.

Subsequently, credit and output also expand. At the same time, the financial sector's loss absorbing capacities are diminished as the intermediaries have relatively low equity buffers. A negative shock can then cause a self-fulfilling roll-over crisis. In particular, an abrupt stop to the roll over of deposits forces intermediaries to liquidate their balance sheet for a firesale price, which then pushes the intermediaries into bankruptcy. This run on the financial sector results, then, in a sharp contraction in output as observed in the great financial crisis in the fourth quarter of 2008.

Importantly, the framework accounts for key empirical observations concerning financial crises as a run is preceded by a credit boom (Schularick and Taylor, 2012), low pre-crisis credit spreads (Krishnamurthy and Muir, 2017) and elevated shadow bank leverage as observed around 2008 (Adrian and Shin, 2010). The boom-bust dynamics also feature a volatility paradox, in the spirit of Brunnermeier and Sannikov (2014). Another important implication is that not every boom ends in a bust in line with recent empirical evidence of Gorton and Ordonez (2020). The quantitative model also sheds light on the connection between monetary policy and financial crisis in the current low interest rate environment. The zero lower bound, which limits potential rate cuts during a run, increases the likelihood of an endogenous financial crises as well as the real repercussions of such a crisis. The build-up of financial fragility in combination with the threat of encountering the zero lower bound results in deflationary pressure during a boom, and creates substantial downside risk for inflation in line with López-Salido and Loria (2020).

The model is taken to the data to obtain a structural estimate of the endogenous build-up of financial fragility and economic downside risk in the U.S. around the great financial crisis. The estimation relies on a two step procedure. In a first step, the nonlinear model is calibrated to key features of the U.S. and the shadow banking sector. In a second step, the filter estimates the endogenous probability of a run over time conditional on the path of shadow bank leverage and real output growth. In particular, I employ a particle filter because such a filter can account for the nonlinear setup with endogenous financial crisis and the zero lower bound.

I estimate that the probability of a financial crisis starts to increase significantly from 2005 onwards and peaks in 2008. The framework predicts in 2008:Q3 that the risk of a roll-over crisis in the next quarter is close to 5%, which would correspond to 20% in annual terms. The estimation highlights the importance of low volatility because it causes the rise in leverage and makes the financial system prone to instability. While variations in total factor productivity explain most output deviations, the large contraction in economic activity in 2008:Q4 is explained by a run on the financial sector. This emphasizes the importance of considering nonlinear dynamics in accounting for the data. The framework also formalizes the evolution of macroeconomic downside risk over time. Specifically, the possibility of an endogenous run on the financial system creates a multimodal distribution for output forecasts.

The quantitative analysis also offers a new angle for appraising policy counterfactuals with regard to financial fragility. Using the results from the estimation, the counterfactual paths under alternative policies can be constructed. I investigate the impact of a monetary as well as macroprudential policy instrument on the build-up of financial fragility prior to 2008 and the occurrence of the financial crisis. With regard to monetary policy, I evaluate a "leaning against the wind" policy, which prescribes a tighter monetary policy stance during a credit boom to lower financial fragilities. The analysis shows that a "leaning against the wind" would have reduced the probability of a run slightly during its peak in 2008. However, I find that it would not have succeeded in averting the run on the financial system. The macroprudential instrument follows the idea of a leverage tax for shadow banks as proposed in the "Minneapolis Plan to End Too Big to Fail" drawn up by the Minneapolis Federal Reserve Bank. The leverage tax, which is a tax on deposit holdings, reduces financial fragility more substantially. The counterfactual analysis suggests that such a macroprudential policy could have reduced the build-up of financial fragility and could have averted the run on the financial sector in 2008:Q4 itself.

The approach outlined in this paper for estimating the endogenous financial fragility based on a structural model with financial crises is general and can be applied to related models or other countries. It provides a model-based growth-at-risk approach that links current macrofinancial conditions to the distribution of future output growth using a microfounded nonlinear framework with endogenous financial fragility. The developed structural approach also provides a new tool to evaluate different policy counterfactuals. It allows me to analyze the counterfactual path of the estimated build-up of financial fragility and economic downside risk under alternative scenarios.

Related Literature Even though runs on the financial system and leverage cycles have both been analyzed independently, I connect these approaches to explain the run on the financial sector in the financial crisis of 2007-2008. Gertler, Kiyotaki and Prestipino (2020*b*) pioneer the incorporation of self-fulfilling runs into macroeconomic models to explain financial crises.[2] My paper differs in that I focus on the dynamics of leverage and show how elevated leverage endogenously creates the scenario of a boom going bust. Introducing risk-shifting incentives and volatility shocks along the lines of Adrian and Shin (2014) and Nuño and Thomas (2017) as a new channel makes it possible to account for the build-up of leverage prior to the financial crisis and to connect it to the financial collapse.[3] Another strength is that the developed framework provides boom-bust dynamics in a parsimonious way that lends itself to a quantitative analysis and an estimation of financial fragility.

The paper also adds to the literature on the connection between endogenous financial crises and monetary policy (e.g. Boissay et al., 2021; Gertler, Kiyotaki and Prestipino, 2020*b*). The novelty lies in the focus on the low interest rate environment and the zero lower bound in connection with financial crises. My work is also related to other papers that incorporate runs into quantitative macroeconomic frameworks such as Amador and Bianchi (2021), Faria-e-Castro (2019), Ferrante (2018), Gertler, Kiyotaki and Prestipino (2020*a*) De Groot (2021), Ikeda and Matsumoto (2021), Hakamada (2021), Mikkelsen and Poeschl (2019), Paul (2020) and Poeschl (2020). Other approaches capturing credit booms that go bust include asymmetric information (e.g. Boissay, Collard and Smets, 2016), optimistic beliefs (e.g. Bordalo, Gennaioli and Shleifer, 2018) and learning (e.g. Boz and Mendoza, 2014; Moreira and Savov, 2017). While this paper is about the distress in the financial sector, other studies such as Justiniano, Primiceri and Tambalotti (2015), Guerrieri and Lorenzoni (2017) and Kaplan, Mitman and Violante (2020) emphasize the role of housing.

I further complement the literature by providing a (real-time) estimate for the probability of a financial crisis and downside risk. To capture the model's nonlinearities and state-dependencies in this analysis, I build on the literature that empirically assesses nonlinear models with multiple equilibria. Following Aruoba, Cuba-Borda and Schorfheide (2018), I use a particle filter (Fernández-Villaverde and Rubio-Ramírez, 2007) that is adapted to account for the multiplicity of equilibria.[4] In the literature on sovereign default, Bocola and Dovis (2019) use a particle filter to estimate the likelihood of a government default. Faria-e-Castro (2019) applies a particle filter to conduct a counterfactual with countercyclical capital

[2] Gertler and Kiyotaki (2015) and Gertler, Kiyotaki and Prestipino (2016) are important preceding contributions that integrate bank runs into standard macro models. Cooper and Corbae (2002) is an early study that features a dynamic equilibrium model with runs that can be interpreted as a roll-over crises.

[3] The risk-shifting incentives have a very different impact on leverage compared to that of a run-away constraint, where an intermediary can divert a fraction of assets that cannot be reclaimed, as used in Gertler, Kiyotaki and Prestipino (2020*b*). Risk-shifting incentives combined with the volatility shock generate procyclical leverage, while leverage is normally countercyclical with the run-away constraint. The run-away constraint can be reconciled with the evidence for credit booms that generate busts if intermediaries are overly optimistic about future news for instance. An alternative approach to obtain procyclical leverage is to have a sticky net worth accumulation of financial intermediaries (Ikeda and Matsumoto, 2021).

[4] I adjust the filter to handle not only multiplicity of equilibria, but also the fact that the equilibrium probabilities are endogenously time-varying. This adjustment is necessary to account for and measure the probability of runs.

requirements in a model that features bank runs. What renders my work novel is the estimation of the endogenous probability of a financial crises through the lens of a nonlinear structural model. In that sense, my paper is the structural counterpart to the large growing body of empirical work on growth-at-risk (e.g. Adrian, Boyarchenko and Giannone, 2019), and also on the role of multimodality in future output growth forecasts as in Adrian, Boyarchenko and Giannone (2021), Caldara et al. (2020) and Mitchell, Poon and Zhu (2021).[5] Importantly, my model-based approach provides a new tool to appraise policy counterfactuals. It offers a way of measuring the impact of a counterfactual scenario on the estimated endogenous build-up of financial fragility and the occurrence of a financial crisis.

Layout The rest of this paper is organized as follows. Section II outlines the nonlinear macroeconomic model, conditions for a run on the financial system and the nonlinear solution method. I present the calibration and quantitative properties in Section III. I then move on, in Section IV, to analyze and estimate the build-up of financial fragility and macroeconomic downside risk. The impact of counterfactual policies on the estimated build-up of financial fragility is analyzed in Section V. The final section concludes.

2 Model

The setup is a dynamic stochastic general equilibrium model with a financial sector that faces endogenous financial crises. It is embedded in a New Keynesian setup with a zero lower bound on nominal interest rates. The main features are a leverage constraint for financial intermediaries and endogenous runs on the financial sector. The financial intermediaries in the model can be best thought of shadow banks, since they are unregulated and are not protected by deposit insurance.[6]

Financial intermediaries have risk-shifting incentives based on Adrian and Shin (2014) and Nuño and Thomas (2017). They have to choose between two securities that face idiosyncratic shocks to their return. Importantly, the two assets differ in the mean and standard deviations of the idiosyncratic shock. Limited liability, which protects the losses of the intermediaries, distorts the choice between the two securities as it limits the downside losses. This determines leverage endogenously. The other key element is that the financial sector occasionally faces system-wide runs, which are state-dependent, or in other words endogenous, similar to Gertler, Kiyotaki and Prestipino (2020*b*). The occurrence of the run depends on fundamentals and in particular on the leverage of the financial sector. During a run, households stop rolling over their deposits.[7] This forces the financial intermediaries to sell their assets. The

[5] The developed model also provides the microfoundation for more stylized macroeconomic modeling approaches that capture the macroeconomic downside risk with an ad-hoc specified vulnerability function that interacts with the model equations, as in, e.g., Adrian et al. (2020).

[6] While assuming the absence of deposit insurance is a characteristic in line with shadow banks, a run could also occur in the presence of a deposit insurance system provided that the insurance is imperfect.

[7] There is no explicit distinction in the model between households and typical lenders on the wholesale market, such as commercial banks in the model. Poeschl (2020) discusses this assumption and shows that a distinction between retail and wholesale banks separately can result in amplification under some conditions.

asset price drops significantly as all intermediaries sell at the same time, justifying the run in the first place.

The rest of the economy follows a canonical New Keynesian model with the zero lower bound. There exist intermediate goods firms, retailers and capital goods producers. The retailers face nominal rigidities via Rotemberg pricing, and the capital goods producers face investment adjustment costs. Monetary policy follows a Taylor rule subject to the zero lower bound. The model featuring endogenous multiple equilibria and a zero lower bound is solved in its nonlinear specification with global methods and later also taken to the data.

2.1 Household

There is a large number of identical households. The representative household consists of workers and financial intermediaries that have perfect insurance for their consumption C_t. Workers supply labor L_t and earn the wage W_t. Financial intermediaries die with a probability of $1-\theta$ and return their net worth to the household to avoid self-financing. Simultaneously, new intermediaries enter each period and receive a transfer from the household. The household owns the non-financial firms, from where it receives the profits. The variable Ξ_t captures all transfers between households, financial intermediaries and non-financial firms.

The household is a net saver and has access to two different assets that are also actively used. The first option is to provide one-period deposits D_t to financial intermediaries that promise to pay a predetermined gross interest rate $\bar{R}_t$. However, the occurrence of a run in the following periods alters the intermediary's ability to honor its commitment. In this scenario the household receives only a fraction $x_t^\star$, which is the recovery ratio, of the promised return. The gross rate R_t is thus state-dependent:

$$R_t = \begin{cases} \bar{R}_{t-1} & \text{if no run takes place in period } t \\ x_t^\star \bar{R}_{t-1} & \text{if a run takes place in period } t \end{cases} \tag{1}$$

Securities are the other option. I distinguish between beginning-of-period securities K_t that are used to produce output and end-of-period securities S_t. The households' end of period securities S_t^H give them a direct ownership in the non-financial firms. The household earns the stochastic rental rate Z_t. The household can trade the securities with other households and intermediaries at the market clearing price Q_t. The securities of households and financial intermediaries, where the latter are denoted as S_t^B, are perfect substitutes. Total end-of-period capital holdings S_t are:

$$S_t = S_t^H + S_t^B. \tag{2}$$

The households are less efficient in managing capital holdings, as in the framework of Brunnermeier and Sannikov (2014). Following the shortcut of Gertler, Kiyotaki and Prestipino

(2020b), capital holdings are costly in terms of utility. The costs are given as:

$$UC_t = \frac{\Theta}{2}\Big(\frac{S_t^H}{S_t} - \gamma^F\Big)^2 S_t, \tag{3}$$

where $\Theta > 0$ and $\gamma^F > 0$. An increase in households' capital holdings increases the utility costs, while an increase in total capital holdings decreases the utility costs (if the condition $S_t^H/S_t - \gamma^F > 0$ holds). The household maximizes its utility function

$$U_t = E_t\Big\{\sum_{\tau=t}^{\infty}\beta^{\tau-t}\Big[\frac{(C_\tau)^{1-\sigma^h}}{1-\sigma^h} - \frac{\chi L_\tau^{1+\varphi}}{1+\varphi} - \frac{\Theta}{2}\Big(\frac{S_\tau^H}{S_\tau} - \gamma^F\Big)^2 S_\tau^H\Big]\Big\}, \tag{4}$$

subject to the budget constraint:

$$C_t = W_t L_t + D_{t-1}R_t - D_t + \Xi_t + Q_t S_t^H + (Z_t + (1-\delta)Q_t)S_{t-1}^H. \tag{5}$$

The first order conditions with respect to the two assets can be combined to:

$$\beta E_t \Lambda_{t,t+1} R_{t+1} = \beta E_t \Lambda_{t,t+1} \frac{Z_{t+1} + (1-\delta)Q_{t+1}}{Q_t + \Theta(S_{H,t}/S_t - \gamma^F)/\varrho_t}, \tag{6}$$

where ϱ_t is the marginal utility of consumption and $\beta E_t \Lambda_{t,t+1} = \beta E_t \varrho_{t+1}/\varrho_t$ is the stochastic discount factor. This emphasizes the existence of a spread between the return on capital and deposit rates due to the utility costs.

2.2 Financial Intermediaries

The financial intermediaries' leverage decision depends on the risk-shifting incentives and the possibility of a run on the financial system. I first present the risk-shifting incentives and then incorporate the possibility of a run on the financial sector in the decision problem.

2.2.1 Risk-shifting Incentives Moral Hazard Problem

The intermediaries face a moral hazard problem due to risk-shifting incentives that limits their leverage, as in Adrian and Shin (2014) and Nuño and Thomas (2017). They can invest in two different securities with distinct risk profiles. Limited liability protects the financial intermediaries' losses in case of default and creates incentives to choose a strategy that is too risky from the depositors' point of view. To circumvent this issue, the intermediaries face an incentive and a participation constraint in their maximization problem. This formulation microfounds a value-at-risk constraint - a common risk management approach for shadow banks - and corresponds to a contracting problem familiar from corporate finance theory.[8]

There is a continuum of financial intermediaries indexed by j, who intermediate funds between households and non-financial firms. The intermediaries hold net worth N_t^j and

[8] Adrian and Shin (2010) provide evidence on the value-at-risk constraint and the leverage decision of security broker-dealers, which included, at the time, major Wall Street investment banks such as Lehman Brothers.

collect deposits D_t^j to buy securities S_t^B from the intermediate goods producers:

$$Q_t S_t^{Bj} = N_t^j + D_t^j, \tag{7}$$

and the financial intermediaries' leverage is defined as follows:

$$\phi_t^j = \frac{Q_t S_t^{B,j}}{N_t^j}. \tag{8}$$

Securities with different risk profiles After purchasing the securities, the financial intermediary converts, at the end of the period, the securities into efficiency units ω_{t+1} that are subject to idiosyncratic volatility similar to Christiano, Motto and Rostagno (2014). The arrival of the idiosyncratic shock is i.i.d over time and intermediaries. In particular, the intermediary has to choose between two different conversions - a good security and a substandard security - that differ in their cross-sectional idiosyncratic volatility. The good type ω and the substandard type $\tilde{\omega}$ have the following distinct distributions:

$$\log \omega_t = 0, \qquad \text{and} \qquad \log \tilde{\omega}_t \overset{iid}{\sim} N\left(\frac{-\sigma_t^2 - \psi}{2}, \sigma_t\right), \tag{9}$$

where $\psi < 1$ and σ_t, which affects the idiosyncratic volatility, is an exogenous driver specified below. As can be seen, I abstract from idiosyncratic volatility for the good security. This implies that its distribution is a dirac delta function, where $\Delta_t(\omega)$ denotes the cumulative distribution function. The substandard security follows a log normal distribution, where $F_t(\tilde{\omega}_t)$ is the cumulative distribution function.

The good security is superior as it has a higher mean and a lower variance due to $\psi < 1$:[9]

$$E(\omega) = \omega = 1 > e^{-\frac{\psi}{2}} = E(\tilde{\omega}), \tag{10}$$

$$Var(\omega) = 0 < [e^{\sigma^2} - 1]e^{-\psi} = Var(\tilde{\omega}). \tag{11}$$

However, the substandard security features a higher upside risk as a high realization of the idiosyncratic shock $\tilde{\omega}$ results in a large return.[10] Figure 2 shows the distributions and highlights the difference in mean, variance and upside risk.

The variable σ_t is labeled as volatility as it affects the relative cross-sectional idiosyncratic volatility of the securities. In particular, it changes the upside risk, while preserving the mean spread $E(\omega) - E(\tilde{\omega})$.[11] Volatility σ_t is exogenous and follows an AR(1) process:

$$\sigma_t = (1 - \rho^\sigma)\sigma + \rho^\sigma \sigma_{t-1} + \sigma^\sigma \epsilon_t^\sigma, \tag{12}$$

where $\epsilon_t^\sigma \sim N(0, 1)$.

[9]More formally, I assume that $\Delta_t(\omega)$ cuts $F_t(\tilde{\omega})$ once from below to ensure this property. This means that there is a single ω^*, such that $(\Delta_t(\omega) - \tilde{F}_t(\omega))(\omega - \omega^*) \geq 0 \;\; \forall \omega$.

[10]Ang et al. (2006) find empirically that stocks with high idiosyncratic variance have low average returns.

[11]This result does not depend on the assumption that the good security does not contain idiosyncratic risk. For instance, the following distribution gives the same result: $\log \tilde{\omega}_t \overset{iid}{\sim} N(-0.5(\eta\sigma_t^2 - \psi), \sqrt{\eta}\sigma_t)$, where $\eta < 1$. The mean is preserved, but the variance of the substandard security responds more strongly.

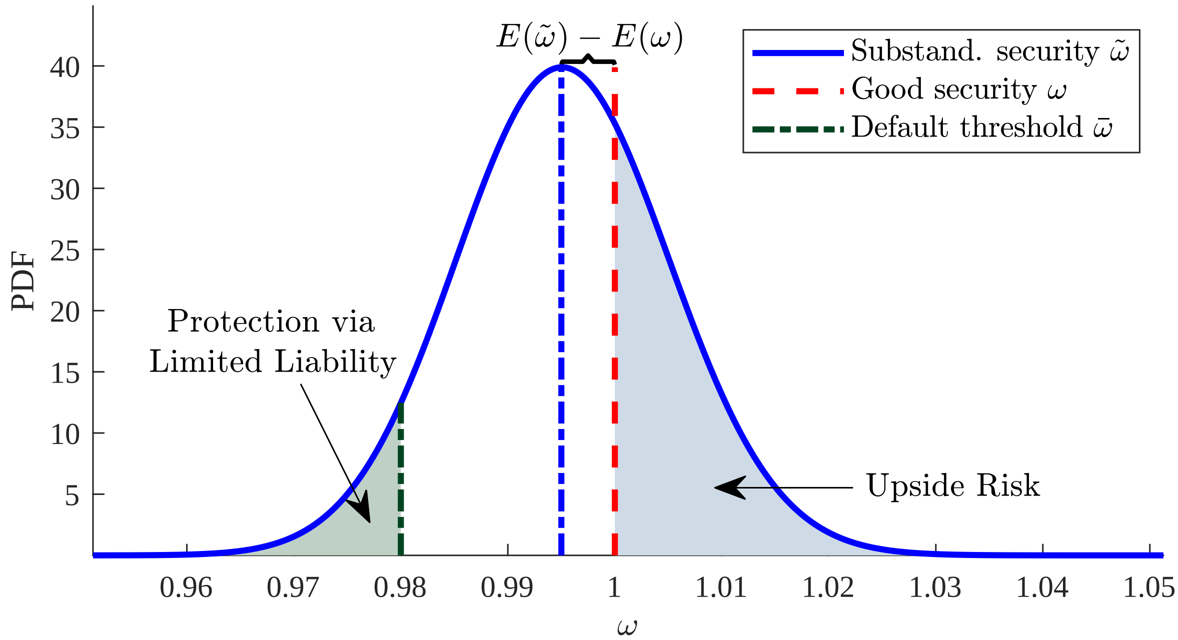

Figure 2: Trade-off between mean return, upside risk and limited liability. The blue line depicts the PDF of the substandard security (log normal distribution) and its mean (blue dashed dotted). The red dashed line is the (mean) return of the good security (dirac delta distribution). The green dash-dotted line is the default threshold value $\bar{\omega}$. The blue and green shaded areas indicate the area associated with the upside risk and protection from downside risk via limited liability, respectively.

The intermediary earns the return $R_t^{K,j}$ on its securities that depends on the stochastic aggregate return R_t^K and the experienced idiosyncratic volatility conditional on its conversion choice:

$$R_t^{Kj} = \omega_t^j R_t^K = R_t^K \qquad \text{if good type} \tag{13}$$

$$R_t^{Kj} = \tilde{\omega}_t^j R_t^K \qquad \text{if substandard type} \tag{14}$$

The aggregate return depends on the asset price and the gross profits per unit of effective capital Z_t:

$$R_{K,t} = [(1-\delta)Q_t + Z_t]/Q_{t-1}. \tag{15}$$

Based on this, a threshold value $\overline{\omega}_t^j$ for the idiosyncratic volatility defines when the intermediary can exactly cover the face value of the deposits:

$$\overline{\omega}_t^j = \frac{\bar{R}_{t-1}^D D_{t-1}^j}{R_t^k Q_{t-1} S_{t-1}^{Bj}}. \tag{16}$$

While the threshold level is independent of the type of the security, the substandard security is more likely to fall below this value due to the lower mean and higher variance.

Risk-shifting incentives and limited liability As it stands so far, the financial entities would choose to invest in the good security as it has a higher mean and lower variance.

However, the financial entities are protected by limited liability. Limited liability distorts the choice between the securities and creates risk-shifting incentives for the financial intermediaries. If the realized idiosyncratic volatility is below $\overline{\omega}_t^j$, the financial intermediary declares bankruptcy. In such a case of default, the households can seize all the assets, but they do not receive the promised repayment due to limited liability. Therefore, limited liability limits the downside risk of the substandard security, while its upside risk is unaffected. Figure 2 highlights this feature.

To avoid an investment in the substandard security, the financial intermediary faces an incentive constraint that deals with the risk-shifting incentives resulting from limited liability. The incentive constraint ensures the conversion in the good security only. The limited liability friction resembles a put option for the financial intermediary in its contract with the household. If the intermediary defaults, the household receives only the return on the assets, or put differently, the intermediary has the option to sell its asset at strike price $\overline{\omega}_{t+1}^j$. Thus, the substandard security contains a put option $\tilde{\pi}_t$ that insures the intermediary against the downside risk:

$$\tilde{\pi}_t(\overline{\omega}_{t+1}^j) = \int^{\overline{\omega}_{t+1}^j} (\overline{\omega}_{t+1}^j - \tilde{\omega}) dF_t(\tilde{\omega}). \tag{17}$$

Based on this assumption, the put option of the substandard technology is larger than the put option of good technology at a given strike price $\overline{\omega}_t^j$, so that $\tilde{\pi}_t(\overline{\omega}_{t+1}^j) > \pi_t(\overline{\omega}_{t+1}^j) = 0$. In particular, the put option of the standard security $\pi_t(\overline{\omega}_{t+1}^j)$ is zero due to the absence of idiosyncratic risk. Thus, there is a trade-off between the higher mean return of the good security and the higher upside risk of the substandard security. This results in an incentive constraint:

$$\begin{aligned} &E_t \beta \Lambda_{t,t+1} \Big\{ \theta V_{t+1}^j(\omega, S_t^{Bj}, \overline{D}_t^j) + (1-\theta)[R_{t+1}^K Q_t S_t^{Bj} - \overline{D}_t^j] \Big\} \geq \\ &E_t \beta \Lambda_{t,t+1} \int_{\overline{\omega}_t^j}^{\infty} \Big\{ \theta V_{t+1}(\omega, S_t^{Bj}, \overline{D}_t^j) + (1-\theta)[R_{t+1}^K Q_t S_t^{Bj} \omega_{t+1}^j - \overline{D}_t^j] \Big\} d\tilde{F}_{t+1}(\omega), \end{aligned} \tag{18}$$

where $V_{t+1}^j(\omega, S_t^{Bj}, \overline{D}_t^j)$ is the value function of a financial intermediary. The LHS is the intermediary's gain of the standard securities and the RHS is the gain of deviating to the substandard security. It is important to note that the good security is the only choice if the incentive constraint holds.

In addition to this, the return on deposits needs to be sufficient such that households provide deposits to intermediary j. The participation constraint can be directly derived from the households' first-order conditions (FOC) with respect to deposits:

$$\beta E_t \Lambda_{t,t+1} \bar{R}_t^D \geq 1. \tag{19}$$

The return on deposits is $\bar{R}_t^D$ is predetermined as there are no defaults for now.

Financial intermediaries' contracting problem The intermediary maximizes the value of its entity V_t subject to the incentive and participation constraint (equations 18 and 19):

$$V_t^j(N_t^j) = \max_{S_t^{Bj}, \bar{D}_t} E_t \Lambda_{t,t+1} \left[\theta V_{t+1}^j(N_{t+1}^j) + (1-\theta)(R_t^K Q_t S_t^{Bj} - \bar{D}_t^j) \right], \tag{20}$$

where N_t^j is the intermediary's net worth. The participation constraint and incentive constraints are both binding in equilibrium and can be simplified to

$$\beta E_t \left[\Lambda_{t,t+1} \bar{R}_t^D \right] = 1, \qquad \text{and} \qquad 1 - e^{-\frac{\psi}{2}} = E_t \tilde{\pi}_{t+1}^j, \tag{21}$$

where the detailed derivation is left to Appendix C.

The incentive constraint shows the trade-off between higher mean return of the good security and the put option of the substandard security. This constraint forces the intermediary to hold enough "skin in the game" and limits the leverage of the intermediary. The reason is that the value of the put option $E_t \tilde{\pi}_{t+1}^j$ increases in leverage.

The participation and incentive constraint do not depend on intermediary-specific characteristics so that the optimal choice of leverage is independent of net worth as shown in Appendix C. Therefore, I can sum up across individual intermediaries to obtain the aggregate values. Intermediaries' aggregate demand for assets depends on leverage and net worth:

$$Q_t S_t^B = \phi_t N_t. \tag{22}$$

The net worth evolution is as follows in the absence of runs. Surviving intermediaries retain their earnings, while newly entering ones receive a transfer from households:

$$N_{S,t} = R_t^K Q_t S_{t-1}^B - R_t^D D_t, \qquad \text{and} \qquad N_{N,t} = (1-\theta)\zeta S_{t-1}, \tag{23}$$

where $N_{S,t}$ and $N_{N,t}$ are the net worth of surviving and new intermediaries, respectively. Aggregate net worth N_t is given as $N_t = \theta N_{S,t} + N_{N,t}$.

2.2.2 Run on the Financial Sector and the Risk-Shifting Incentives

I now include the possibility of a run on the financial sector in the intermediaries' contracting problem. A run is a systemic event that affects all intermediaries. In particular, a run eradicates the net worth of all financial intermediaries, so that $N_t = 0$ and they stop operating. The run itself and the probability of such an event alters the decision problem of the intermediaries. The following part outlines the implications for the contracting problem, while Appendix C contains the full derivation.

The financial intermediary can only continue operating or return its net worth to the household in the absence of a run. The value function depends now on the probability p_t that a run takes place in the next period:

$$V_t(N_t) = (1-p_t) E_t \left[\Lambda_{t,t+1}(\theta V_{t+1}(N_{t+1}) + (1-\theta)(R_t^K Q_t S_t^B - \bar{D}_t)) \middle| \text{no run} \right], \tag{24}$$

where $E_t[\cdot|\text{no run}]$ is the expectation conditional on no run in $t+1$. For ease of exposition, a superscript denotes if the expectations are conditioned on the absence or occurrence of a run, that is $E_t^N[\cdot] = E_t[\cdot|\text{no run}]$ and $E_t^R[\cdot] = E_t[\cdot|\text{run}]$. The probability p_t is endogenous and state-dependent. Its derivation is described in detail in the next subsection.

The intermediaries' commitment to repay the households is also changed. Due to limited liability, households do not receive the promised repayments if a run occurs. Instead, households recover the gross return of the securities. Thus, the gross rate R_t is state-dependent:

$$R_t = \begin{cases} \bar{R}_{t-1} & \text{if no run takes place in period } t, \\ R_t^K Q_{t-1} S_{t-1}^B / D_{t-1} & \text{if a run takes place in period } t. \end{cases} \tag{25}$$

The participation constraint includes the probability of a run as the intermediaries need to compensate the households for the tail-event of a run:

$$(1-p_t)E_t^N[\beta\Lambda_{t,t+1}\bar{R}_t D_t] + p_t E_t^R[\beta\Lambda_{t,t+1}R_{t+1}^K Q_t S_t^B] = D_t. \tag{26}$$

An increase in p_t augments the funding costs as intermediaries need to compensate households for the run risk. The incentive constraint is also directly affected:

$$\begin{aligned} &(1-p_t)E_t^N \Lambda_{t,t+1} R_{t+1}^K (\theta\lambda_{t+1} + (1-\theta))[1 - e^{\frac{-\psi}{2}} - \tilde{\pi}_{t+1}] = \\ &p_t E_t^R \Lambda_{t,t+1} R_{t+1}^K (e^{-\frac{\psi}{2}} - \overline{\omega}_{t+1} + \tilde{\pi}_{t+1}), \end{aligned} \tag{27}$$

where λ_t is the multiplier on the participation constraint. The trade-off between higher mean return and upside risk still prevails, which is displayed on the LHS in equation (27). This trade-off is now weighted with the probability of surviving a run in case of investing in the substandard security, which is displayed on the RHS. The substandard security offers the possibility to survive a run as the idiosyncratic volatility $\tilde{\omega}_t^i$ is drawn from a distribution. If $\tilde{\omega}_t^i > \overline{\omega}_t$, the financial intermediary can repay its depositors because it profits from the upside risk of the substandard security.[12] This channel counteracts the risk-shifting incentives and leverage accumulation to some extent.

The net worth of the financial intermediaries, who existed in the previous period, is zero in the event of a run. However, new entities are entering due to transfers from households. The net worth of the financial sector in a run period is given as:

$$N_t = N_{N,t} = (1-\theta)\zeta S_t^H \quad \text{if a run occurs in period t.} \tag{28}$$

The financial sector continues to rebuild in period $t+1$ using retained profits and transfers from households as usual, so that $N_{t+1} = \theta N_{S,t+1} + N_{N,t+1}$. As the financial sector starts to rebuild straightaway, a run is already possible again in the next period.

[12]Investing in substandard securities is an outside equilibrium strategy, which allows a financial intermediaries to survive a run in the event of a very high realization of the idiosyncratic shock. It is assumed that the surviving intermediaries repay their depositors fully and return their remaining net worth to the households.

2.3 Endogenous Runs and Multiple Equilibria

A financial crisis is an endogenous run on the financial sector, in which depositors stop rolling over their deposits. The run is self-fulfilling element because the model features multiple equilibria in the spirit of Diamond and Dybvig (1983). The endogenous element is that the existence of the run equilibrium depends on the aggregate state and especially on the balance sheet strength of the financial intermediaries following Gertler, Kiyotaki and Prestipino (2020*b*). Therefore, the multiplicity of equilibria - a "normal" and run one - occurs only in some states of the world.

The multiplicity of equilibria originates from heterogeneous asset demand of households and intermediaries. During normal times - that is in the absence of a run - households roll over their deposits. Financial intermediaries and households demand securities and the market clears at price Q_t. This price can be interpreted as the fundamental price. The intermediary can cover the promised repayments for the fundamental price:

$$[(1-\delta)Q_t + Z_t]S^B_{t-1} > \bar{R}_{t-1}D_{t-1}. \tag{29}$$

In contrast to this, a run wipes out the entire existing financial sector, so that $N_{S,t} = 0$. Households cease to roll over their deposits in a run, causing that intermediaries need to liquidate their entire assets to repay the households. However, this eliminates their demand for securities, and households (plus the newly entering financial intermediaries) are the only remaining agents that buy securities in a run. Subsequently, the asset price falls to clear the market at a firesale price. The drop is particularly severe because it is costly for households to hold large amounts of securities. This firesale price $Q^\star_t$ depresses the potential liquidation value of intermediaries' securities. As a consequence, a run can take place if the firesale liquidation value is smaller than the households' claims:

$$[(1-\delta)Q^\star_t + Z^\star_t]S^B_{t-1} < \bar{R}_{t-1}D_{t-1}, \tag{30}$$

where the superscript $\star$ indicates the run equilibrium. Therefore, a run can occur if the intermediaries do not have sufficient means to cover the claims of the households under the firesale price $Q^\star_t$. This is the case if the recovery ratio $x^\star_t$, that is the firesale liquidation value relative to the promised repayments, is below 1:

$$x^\star_t \equiv \frac{[(1-\delta)Q^\star_t + Z^\star_t]S^B_{t-1}}{\bar{R}_{t-1}D_{t-1}} < 1. \tag{31}$$

The recovery ratio $x^\star_t$ partitions the state space into a safe and a fragile region. $x^\star_t > 1$ characterizes the safe region, where the financial intermediaries can cover the claims under the fundamental and firesale price. Therefore, runs are not possible and only the normal equilibrium exists. By contrast, both equilibria coexist in the fragile region if $x^\star_t < 1$. The financial entities have only sufficient means to repay depositors under the fundamental price. There is also a third scenario, in which the intermediaries cannot repay the depositors even under the fundamental price, which is the case if $[(1-\delta)Q_t + Z_t]S^B_{t-1} < \bar{R}_{t-1}D_{t-1}$. While

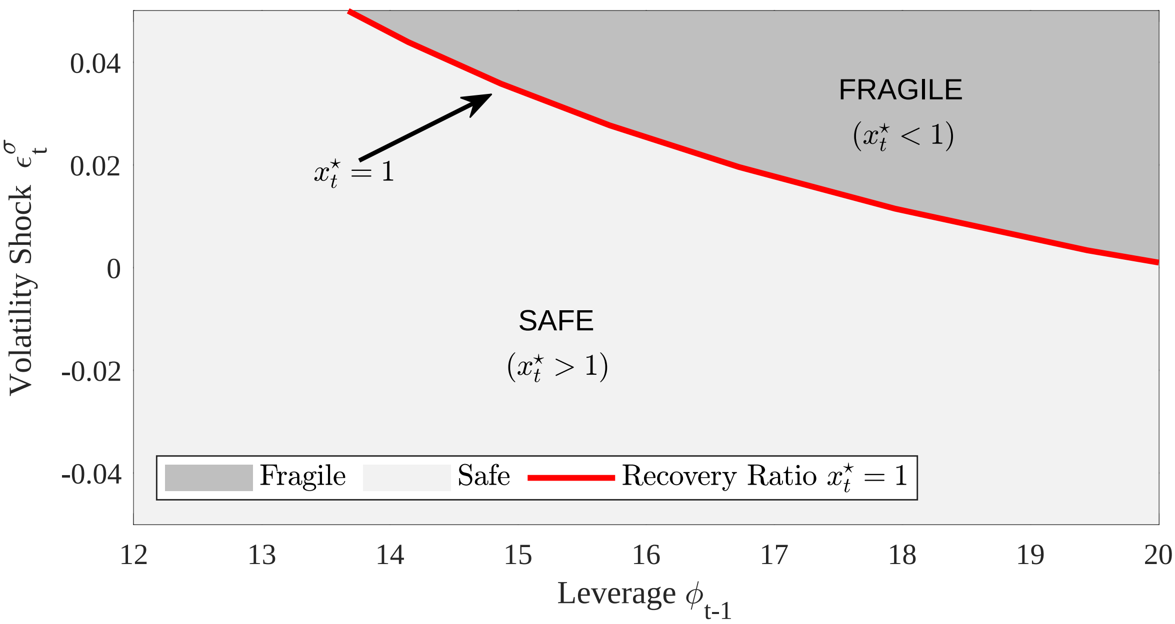

Figure 3: Illustration showing how the safe and fragile regions are dependent on leverage ϕ_{t-1} and the volatility shock ϵ_t^σ.

this third case is accounted and checked for, this scenario is neglected because the probability of it occurring is infinitesimally small in the quantitative model.

The importance of leverage can be shown by rewriting the recovery ratio $x_t^\star$:

$$x_t^\star = \frac{\phi_{t-1}}{\phi_{t-1} - 1} \frac{[(1-\delta)Q_t^\star + Z_t^\star]}{Q_{t-1}\bar{R}_{t-1}}. \tag{32}$$

Elevated leverage levels make it more likely that the run equilibrium will occur. Furthermore, contraction shock, such as an increase in volatility or a negative TFP shock, reduces the return and can thus enable a run if the leverage of the financial sector is elevated.

Figure 3 illustrates how the combination of the volatility shock and leverage determine which region an economy falls into. The $x_t^\star = 1$ line is downward sloping and divides the two regions. First, it can be seen that a high level of previous period leverage and an increase in volatility pushes the economy into the fragile region, as discussed previously. Second, low leverage is associated with the safe region. This highlights that the pre-crisis period is critical for the build-up of financial fragility. A period of low volatility reduces the risk-shifting incentives. The financial intermediaries increase their leverage and extend their credit supply. This credit boom brings with it financial fragility due to low loss absorbing capacities. In such a scenario with high leverage, a contraction shock can then cause a roll-over crisis. To put it another way, tranquil periods sow the seed of a crisis.

In some states of the world, there are now multiple equilibria, in which the normal and run equilibrium are both possible. A sunspot shock then selects between the equilibria, following Cole and Kehoe (2000).[13] The sunspot ι_t takes the value 1 with probability Υ and 0 with

[13] An alternative to the sunspot shock would be to use a global games approach to determine the equilibrium. Ikeda and Matsumoto (2021) use global games in a framework with runs. De Groot (2021) also includes global

probability $1 - \Upsilon$. If $\iota_t = 1$ materializes and $x_t^\star < 1$, a run takes place. The condition on the recovery ratio $x_t^\star$ ensures that the run equilibrium is only chosen if it is optimal. If $x_t^\star > 1$, then the sunspot shock has no impact on the equilibrium choice.

Taken together, the probability for a run in period $t+1$ depends on the probability of being in the crisis region in the next period and of drawing a sunspot shock:

$$p_t = \text{prob}(x_{t+1}^\star < 1)\Upsilon. \tag{33}$$

The run probability is time-varying and endogenous, as $x_{t+1}^\star$ depends on the macroeconomic and financial circumstances.

While all the structural elements of the two equilibria are the same, I assume that there is the possibility for an additional increase in volatility if a run occurs. The general properties of the model are unchanged by the assumption. However, it allows for a better fit with the data. Adding this features allows the model to account for the magnitude of the increase in a variety of volatility measures as observed during the financial crises. The volatility process is now:

$$\sigma_t = (1-\rho^\sigma)\sigma + \rho^\sigma \sigma_{t-1} + \mathbf{1}_{x_t^\star < 1 \wedge \iota_t = 1} \sigma^\sigma \epsilon^{\sigma\star} + \sigma^\sigma \epsilon_t^\sigma, \tag{34}$$

where $\mathbf{1}$ is an indicator function, and $\epsilon^{\sigma\star}$ is the size of the increase in case of a run.

2.4 Production, Monetary Policy and Closing the Model

The non-financial firms sector consists of intermediate goods producers, final goods producers and capital goods producers. The central bank follows a Taylor rule with a zero lower bound.

Intermediate Goods Producers There is a continuum of competitive intermediate goods producers. The representative intermediate goods producer produces the output Y_t with labor L_t and working capital K_t as input:

$$Y_t^j = A_t (K_{t-1}^j)^\alpha (L_t^j)^{1-\alpha}. \tag{35}$$

A_t is total factor productivity, which follows an AR(1) process. The firm pays the wage W_t to the households. The firm purchases in period $t-1$ capital S_{t-1} at the market price Q_{t-1}. The firm finances the capital with securities S_{t-1}^B from the financial sector and the households S_{t-1}^H, so that:

$$K_{t-1} = S_{t-1}^H + S_{t-1}^B. \tag{36}$$

This loan is frictionless and the intermediate firm pays the state-contingent interest rate $R_{K,t}$. After using the capital in period t for production, the firm sells the undepreciated capital $(1-\delta)K_t$ at market price Q_t. The intermediate output is sold at price M_t, which turns out

games in a model with a financial accelerator and partial runs on the financial sector.

to be equal to the marginal costs φ^{mc}. The problem can be summarized as:

$$\max_{K_{t-1},L_t} \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} \left(M_{t+i} Y_{t+i} + Q_{t+i}(1-\delta) K_{t-1+i} - R_{k_{t+i}} Q_{t-1+i} K_{t-1+i} - W_{t+i} L_{t+i} \right).$$

Final goods retailers The final goods retailers buy the intermediate goods and transform them into the final good using a CES production technology:

$$Y_t = \left[\int_0^1 (Y_t^j)^{\frac{\epsilon-1}{\epsilon}} df \right]^{\frac{\epsilon}{\epsilon-1}}. \tag{37}$$

The price index and intermediate goods demand are given by:

$$P_t = \left[\int_0^1 (P_t^j)^{1-\epsilon} df \right]^{\frac{1}{1-\epsilon}}, \quad \text{and} \quad Y_t^j = \left(P_t^j / P_t \right)^{-\epsilon} Y_t. \tag{38}$$

The final retailers are subject to Rotemberg price adjustment costs. Their maximization problem is:

$$E_t \left\{ \sum_{i=0}^{T} \Lambda_{t,t+i} \left[\left(\frac{P_{t+i}^j}{P_{t+i}} - \varphi_{t+i}^{mc} \right) Y_{t+i}^j - \frac{\rho^r}{2} Y_{t+i} \left(\frac{P_{t+i}^j}{\Pi P_{t+i-1}^j} - 1 \right)^2 \right] \right\}, \tag{39}$$

where Π is the inflation target of the monetary authority.

Capital goods producers Competitive capital goods producers produce new end of period capital using final goods. They create $\Gamma(I_t/S_{t-1})S_{t-1}$ new capital S_{t-1} out of an investment I_t, which they sell at market price Q_t:

$$\max_{I_t} Q_t \Gamma\left(I_t/S_{t-1}\right) S_{t-1} - I_t, \tag{40}$$

where the functional form is $\Gamma(I_t/S_{t-1}) = a_1(I_t/S_{t-1})^{1-\eta} + a_2$ as in Bernanke, Gertler and Gilchrist (1999). The FOC gives a relation for the price Q_t depending on investment and the capital stock, which is $Q_t = 1/\left[\Gamma'\left(I_t/S_{t-1}\right)\right]$. The law of motion for capital is $S_t = (1-\delta)S_{t-1} + \Gamma\left(I_t/S_{t-1}\right) S_{t-1}$.

Monetary Policy, Effective Lower Bound and Resource Constraint The monetary authority follows a standard Taylor Rule for setting the nominal interest rate R_t^I that is constrained by the zero lower bound:

$$R_t^I = \max\left[R^I \left(\frac{\Pi_t}{\Pi} \right)^{\kappa_\Pi} \left(\frac{\varphi_t^{mc}}{\varphi^{mc}} \right)^{\kappa_y}, 1 \right], \tag{41}$$

where deviations of marginal costs from its deterministic steady state φ^{mc} capture the output gap.[14] To connect this rate to the household, there exists one-period bond in zero net supply

[14] While the focus is on the zero lower bound, the model could also be extended to evaluate negative interest rate policies, e.g. along the lines of Darracq Pariès, Kok and Rottner (2020).

that pays the riskless nominal rate R_t^I. The associated Euler equation reads as follows:

$$\beta \Lambda_{t,t+1} R_t^I / \Pi_{t+1} = 1. \tag{42}$$

The aggregate resource constraint is

$$Y_t = C_t + I_t + G + \frac{\rho^r}{2} \left(\frac{\Pi_t}{\Pi} - 1 \right)^2 Y_t, \tag{43}$$

where G is government spending and the last term captures the adjustment costs of Rotemberg pricing. This constitutes a recursive competitive equilibrium, where the details of the equilibrium description can be found in Appendix B.

2.5 Global Solution Method, Occasional Runs and the Zero Lower Bound

The model is solved with global methods to account fully for the highly nonlinear features of the model such as the multiplicity of equilibria and the zero lower bound. I use a time iteration algorithm with piecewise linear policy functions based on Richter, Throckmorton and Walker (2014). The global solution method is adopted to factor in the multiplicity of equilibria. The details of the numerical solution are left to Appendix D.

3 Model Evaluation

This section focuses on the quantitative properties of the nonlinear model that is solved with global methods. I start by explaining how the model is mapped to the data, before moving on to analyse the mechanism behind the emergence and unfolding of a crisis. I also outline the role of low interest rates and the zero lower bound. The estimation of financial fragility and macroeconomic downside risk is covered in the next section.

3.1 Model Parameterization and Selected Key Moments

The emphasis of the calibration is on the recent financial crisis in the United States and the shadow banking sector. The financial sector variables and shock processes are set to match selected moments, while the conventional parameters are chosen based on the literature. The focus is mostly on quarterly data from 1985:Q1 to 2014:Q4 to accommodate the changing regulation of shadow banking activities. The starting point coincides with major changes in the contracting conventions of the repurchasement agreement (repo) market - an important source of funding for shadow banks - that took place after the failure of a number of dealers in the early 1980s (Garbade, 2006).[15] This also captures the period after the Great Inflation. After the financial crisis, new regulatory reforms such as Basel III and the Dodd-Frank Wall Street Reform and Consumer Protection Act act overhauled the financial system, meaning

[15]There were three major changes in contracting conventions as documented in Garbade (2006). First, the Bankruptcy Amendments and Federal Judgeship Act of 1984 altered the treatment of repos under bankruptcy law. Second, lenders became able to earn interest in a repurchasement agreement. Third, a new repo contract called a tri-party-repo emerged.

Table 1: Calibration

a) Conventional Parameters		Value	Target / Source
Discount factor	β	0.997	Risk free rate = 1.2% p.a.
Frisch labor elasticity	$1/\varphi$	0.75	Chetty et al. (2011)
Risk aversion	σ^H	1	Log utility for consumption
TFP level	A	0.407	Output = 1
Government spending	G	0.2	Govt. spending to output = 0.2
Capital share	α	0.33	Capital income share = 33 %
Capital depreciation	δ	0.025	Depreciation rate = 10% p.a.
Price elasticity of demand	ϵ	10	Markup = 11%
Rotemberg adjustment costs	ρ^r	178	Calvo duration of 5 quarters
Elasticity of asset price	η_i	0.25	Bernanke, Gertler and Gilchrist (1999)
Investment Parameter 1	a_1	0.530	Asset Price Q = 1
Investment Parameter 2	a_2	-.008	$\Gamma(I/K) = I$
Target inflation	Π	1.005	Inflation Target of 2%
MP response to inflation	κ_π	2.0	Standard
MP response to output	κ_y	0.125	Standard

(b) Financial Sector & Shocks		Value	Moment	Data	Model
Parameter asset share HH	γ^F	0.33	Share shadow banking sector	33%	34%
Mean Substandard Security	ψ	0.01	Mean shadow bank leverage	15.5	15.6
Intermediation cost HH	Θ	0.04	Financial crisis probability	2.2%	2.2%
Survival rate	ζ	0.88	Mean credit spread	2.3%	2.9%
Persistence volatility	ρ^σ	0.96	Persistence of leverage	0.96	0.94
Std. dev. volatility shock	σ^σ	0.0031	Std. dev. of leverage	3.0	3.0
Persistence TFP	ρ^A	0.95	Persistence TFP	0.95	0.95
Std. dev. TFP shock	σ^A	0.0026	Std. dev. of output growth	0.6	0.6
Volatility increase run	$\epsilon^{\sigma\star}$	5.50	Drop in leverage during run	24%	23%
Sunspot Shock	Υ	0.50	Output drop during run	2.8%	2.5%

that it makes sense to end the sample a few years on from 2008. Table 1 summarizes the calibration and the match with targeted moments in the data.

Conventional Parameters The discount factor is set to 0.997, which corresponds to an annualized long-run real interest rate of 1.2%. This low interest rate environment makes it possible to evaluate the connection between the zero lower bound and financial crises. The Frisch elasticity is set to match an elasticity of 0.75, as suggested in Chetty et al. (2011). Risk aversion is parameterized to 1, which implies a logarithmic utility function. Total factor productivity A normalizes output to 1 in the deterministic steady state (DSS). Government spending G is 20% of total GDP in the DSS. The production parameter α matches a capital income share of 33%. The annual depreciation is chosen to be 10%, which pins down $\delta = 0.025$. The price elasticity of demand is set to 10. The Rotemberg adjustment costs correspond to a five-quarter average duration of resetting prices in the related Calvo framework. The elasticity of the asset price ρ^r is 0.25 as in Bernanke, Gertler and Gilchrist

(1999). The parameters of the investment function normalize the asset price to $Q = 1$ and the investment $\Gamma(I/K) = I$ in the DSS. Monetary policy responds to deviations of marginal costs ($\kappa_y = 0.125$) and inflation ($\kappa_\pi = 2.0$), where the target inflation rate is normalized to 2% per annum.

Financial Sector and Shock Processes The parameters related to the financial sector and the shock processes are set to target selected moments of the shadow banking sector, the frequency of financial crises and the dynamics of output. The financial sector represents the shadow banking sector. Specifically, I define these as entities that rely on short-term deposits that are not protected by the Federal Deposit Insurance Corporations and do not have access to the FED's discount window.[16] The share of total assets held directly by the shadow banking sector was 37.1% in 2006 and dropped to 28.3% in 2012, as shown by Gallin (2015) using the financial accounts of the United States. In line with this, the parameter γ^F specifies that the shadow banking sector holds 33% of total assets on average.[17] The leverage measure combines balance sheet data from security broker dealers and finance companies using the U.S. Flow of Funds data. The leverage series itself is calculated based on book equity, which is the difference between the (market) value of the portfolio and the liabilities of financial intermediaries as discussed in detail in Appendix A.[18] The return of $\psi = 0.01$ for the substandard security is used to target a mean leverage ratio of 15.5 as in the data. The intermediation cost parameter Θ is set to match an annual run probability of 2.2%, which corresponds to a financial crisis on average every 45 years. This frequency is in line with the historical macroeconomic data of Jordà, Schularick and Taylor (2017). The average yearly probability of a financial crisis is around 2.7% for the United States and 1.9% for a sample of advanced economies since the Second World War. The survival rate θ is set such that the finance premium corresponds to an average spread of 2.3% as observed between the BAA bond yield and a 10 year Treasury bond. The fraction of the start capital ζ is implied from the other financial parameters.

The volatility shock's persistence ρ^σ and standard deviation σ^σ is set to match the persistence and standard deviation of the described shadow bank leverage measure. The standard deviation of the TFP shock σ^A targets the standard deviation of real quarter-to-quarter GDP growth. The persistence of the shock is aligned with the estimated persistence of the linear detrended TFP series of Fernald (2014). Finally, the additional exogenous increase in volatility $\epsilon^{\sigma\star}$ is parameterized to have an average drop in leverage of 24% during a run, as observed

[16]This definition applies to the following entities: Money market mutual funds, government-sponsored enterprises, agency- and GSE-backed mortgage pools, private-label issuers of asset-backed securities, finance companies, real estate investment trusts, security brokers and dealers, and funding corporations.

[17]Based on a broader definition of shadow banking activities, Pozsar et al. (2010) suggest a share of 50% for the shadow banking sector.

[18]An alternative important measure is the financial intermediaries' market capitalization (e.g. market valuation of financial intermediaries) as emphasized in He, Khang and Krishnamurthy (2010) and He, Kelly and Manela (2017). However, the appropriate concept in this context is book equity because the occurrence of a run in the model depends directly on book equity that is denoted as net worth in the model. Market capitalization would be the appropriate measure related to the issuance of new shares or acquisitions decision as argued in Adrian, Colla and Shin (2013).

in the fourth quarter of 2008. This exogenous increase in volatility is also introduced to match the substantial surge in different volatility measures observed for this period.[19] Finally, the sunspot shock materializes with a probability of 50%, which helps to match the demeaned output growth of -2.8% in 2008:Q4.

3.2 Financial Crises and its Macroeconomic Impact

The model enables us to study the endogenous vulnerability to a financial crisis because a run on the financial sector is endogenous and depends on economic fundamentals. For this reason, I can assess the typical dynamics around a run to evaluate the fluctuations of key macroeconomic as well as financial variables, the underlying drivers of a financial crisis and the build-up of financial fragility.

Specifically, I conduct an event analysis around a financial crises, which is based on a simulation over 500,000 periods with $2,767$ runs. Figure 4 displays the run dynamics using an event window approach, where the window contains the path for ten quarters before and after a run. The typical run on the financial sector, as captured by the median path, is preceded by a build-up of leverage, elevated credit supply, a low finance premium and higher output levels. The run then causes a sharp economic contraction. Output drops severely - with a quarter-to-quarter growth rate close to 2.5%, which corresponds to a 10% annualized rate. Leverage also falls due to a surge in volatility. The observed dynamics reconcile not only the path of leverage and output, but also the tendency for credit boom to precede a financial crisis (Schularick and Taylor, 2012) and for credit spreads to be low before a crisis and to spike during it (Krishnamurthy and Muir, 2017).

The underlying reason for the run is a period of low volatility. The fact that even the 5% quantile of volatility is considerably below its long-run mean highlights this. In that regard, the framework features a volatility paradox in the spirit of Brunnermeier and Sannikov (2014). In contrast to volatility, the total factor productivity (TFP) is less important for the build-up. While a positive TFP level enforces the boom, the median is only slightly above the risky steady state. Therefore, the main mechanism relates to low volatility as the underlying driver. Low volatility reduces the risk-shifting incentives so that financial intermediaries increase leverage and extend credit. This results in a credit boom and boosts output. At the same time, financial fragilities increase due to low equity holdings relative to the asset size. The realization of a contractionary shock pushes the economy into the fragile region, where a run is possible. If then a sunspot shock materializes so that depositors do not roll over their deposits, the financial intermediaries are forced to sell their assets at a firesale price. Due to the firesale, the financial intermediaries then do not have enough equity to cover their losses and a self-fulfilling run occurs.

The dynamics point out the importance of leverage because run occurs during periods

[19]While the model predicts an increase in volatility during a run independent of this component, the magnitude could not match the increase observed in the fourth quarter of 2008:Q4 throughout different related concepts such as financial uncertainty (Jurado, Ludvigson and Ng, 2015) or cross-sectional idiosyncratic uncertainty (Bloom et al., 2018).

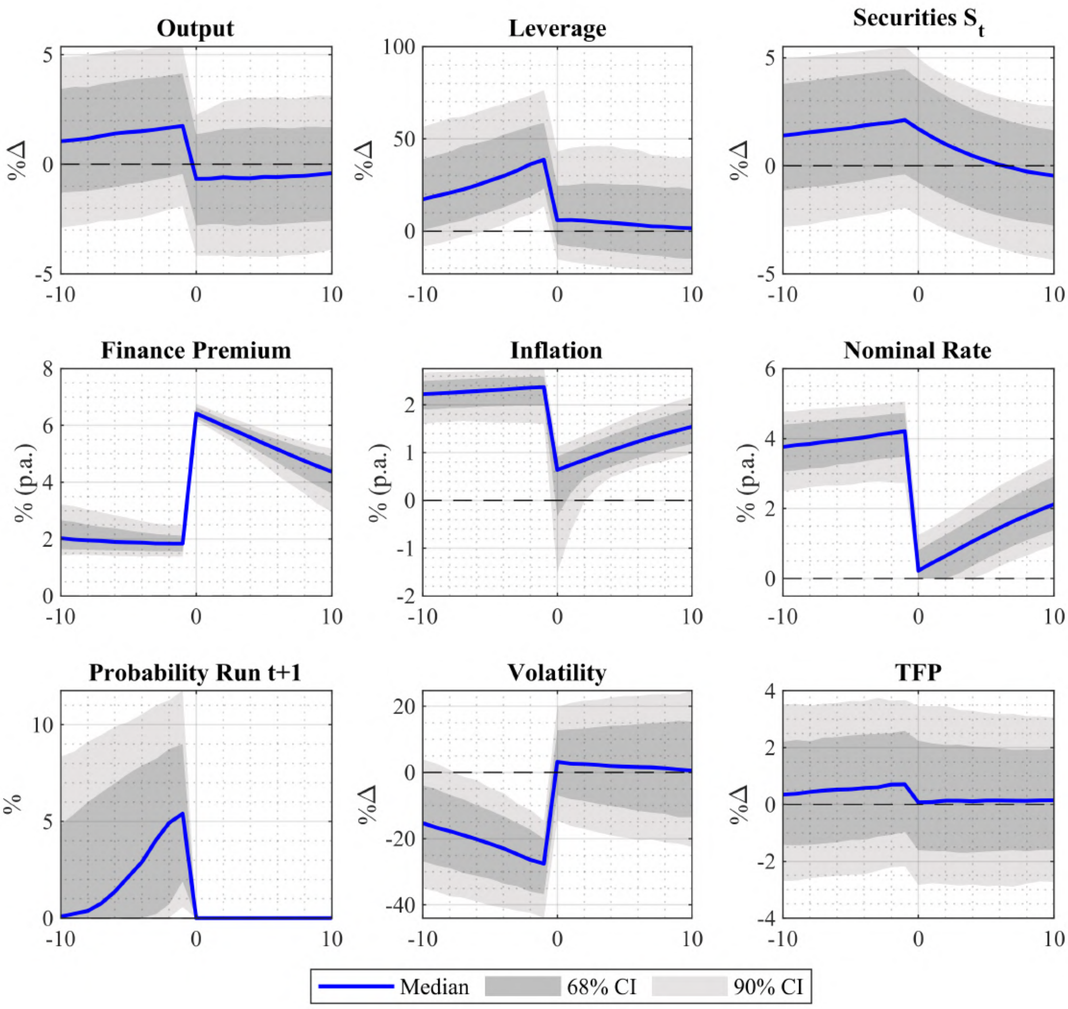

Figure 4: Event window around run episodes. Based on a simulation of 500,000 periods, the median path and the 68% as well as 90% confidence intervals of all runs are displayed ten quarters before and after a run in period 0. The scales are either percentage deviations from the simulated mean (%Δ), annualized percent or percent.

of excessive leverage accumulation. The incorporation of the risk-shifting incentives and the volatility shock result in procyclical leverage, as an increase in leverage raises output and credit. Due to procyclical leverage dynamics, the framework captures the way that a boom precedes a run. In the case of countercyclical leverage, high leverage would imply low output and low assets. A run would then occur in a bust and therefore could not capture this important empirical fact. Furthermore, the model can capture the drop in leverage during a run as observed in 2008:Q4. Even though the return on securities is very large, the risk-shifting incentives combined with the increase in volatility limits the leverage of the newly entering financial intermediaries. [20] The strong increase in volatility stems from two sources. First, contractionary shocks are needed to trigger a run, which results in enhanced volatility. Second, the model features an exogenous increase in volatility if a run occurs. Even though

[20]In fact, this model allows for new intermediaries operating in the same period. Other models predict increases in leverage up to 2000% after a run. The high level of leverage among these newly entering intermediaries would increase asset prices so much that the run equilibrium would no longer exist.

the latter element is not necessary to generate the explained boom-bust dynamics, the model would then predict a small increase in leverage after a run instead of a drop as observed in the data.

Financial Fragility and Macroeconomic Downside Risk The model shows that there is a substantial increase in financial fragility prior to the run. The probability of a run for the next quarter peaks in the period before the run. The median is 5%, corresponding to 20% as an annualized rate, after it increases steadily in the periods before. At the same time, the upper bound of financial fragility is limited as it peaks around 12%. The reason is that agents are aware of the possibility of a run which endogenously limits the leverage of the financial sector. This reduces the threat of a financial crisis arising. In other words, the model precludes a scenario in which the possibility of a run in the next period is too large as this would be in conflict with the decision of the agents. An extension of the model could relax the rational expectations assumption, which could allow for a higher probability of a run.

Another important implication is that not every boom ends in a bust. Even though elevated leverage increases the likelihood, the economy can also converge back to normal times. This is the case if either no sufficient contractionary shocks occur or no sunspot shock materializes. Importantly, this property is in line with recent empirical evidence that not each boom ends in a bust (Gorton and Ordonez, 2020).

3.3 Financial Crises in Low Interest Rate Environment

The next step is to shed light on the connection between a low interest rate environment and financial crises. The central bank faces the zero lower bound, which restricts potential interest rate cuts during a financial collapse. As a consequence, the zero lower bound is potentially an important channel that intensifies the probability and severity of a financial crisis. While the event window analysis (Figure 4) shows that the zero lower bounds can bind during a financial crisis, the potential impact cannot be directly evaluated from this exercise.

For this reason, Figure 5 compares a financial crisis in an economy with and without a zero lower bound. Each respective economy is simulated with the same series of (volatility, productivity and sunspot) shocks, which are chosen to generate boom-bust dynamics with a binding zero lower bound.[21] The analysis highlights the relevance of the zero lower bound for financial fragility, output and inflation. First of all, the zero lower bound increases the probability of a run. This is because the central bank is limited in their response, making a financial crisis is more likely. The drop in output is also more severe, especially in the periods with a binding zero lower bound. While a financial crisis is already associated with a strong downside risk in inflation - as also empirically found in López-Salido and Loria (2020) - the zero lower bound exacerbates the downside risk of inflation even further. Furthermore,

[21]The sequence is based on the event window analysis and is the mean shock realization 50 periods before and 10 periods after a run conditional on a binding zero lower bound in the simulation. The respective starting point is the stochastic steady state of each economy.

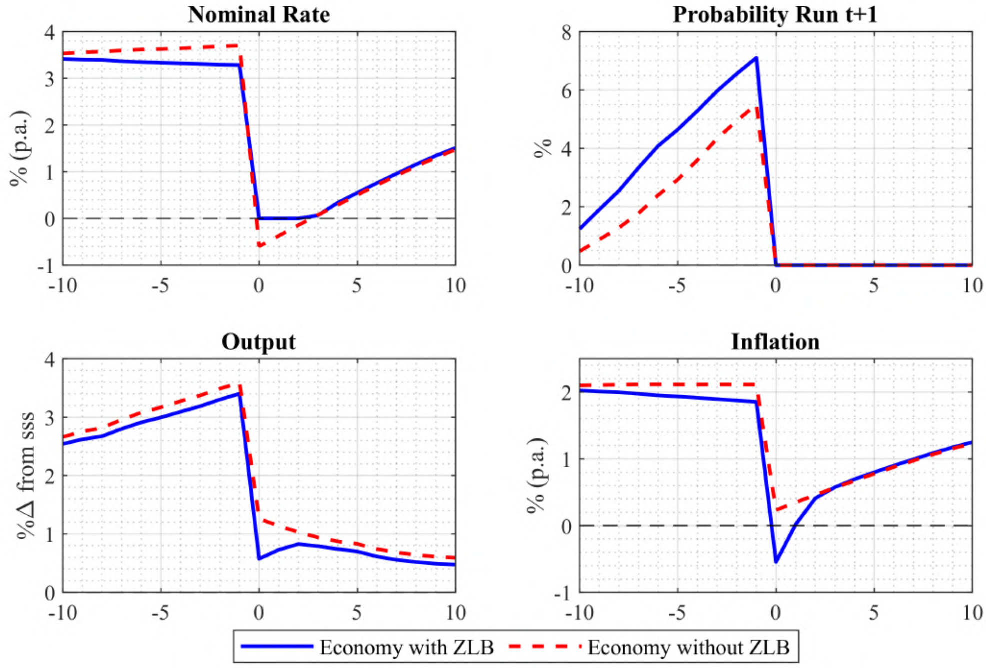

Figure 5: Comparison of crisis dynamics between an economy with a zero lower bound and an economy without such a constraint. Both economies are subject to the same shock sequence that generates boom-bust dynamics. The abbreviation sss denotes the stochastic steady state from the economy with the ZLB.

the threat of encountering the zero lower bound creates deflationary pressure in periods of high financial fragility. This constitutes a further deflationary channel of the lower bound, in addition to the one which has already been studied in the literature, as e.g. in Bianchi, Melosi and Rottner (2021). The described effects intensify even further with an increase in the anticipated time spent at the zero lower bound. Taken together, the results highlight an important interaction between monetary policy and financial crises in a low interest rate environment.[22]

4 Estimation of Financial Fragility

I estimate the build-up of financial fragility and evaluate the macroeconomic downside risk around the recently experienced financial crisis in 2008 through the lens of the quantitative model with endogenous runs. This approach delivers a new structural estimate for the probability of a financial crisis. The analysis also provides insights on the structural drivers and can be used for counterfactuals that evaluate alternative monetary and macroprudential strategies. The considered horizon stretches from 1985:Q1 to 2014:Q4, in line with the calibration.

The estimation strategy is to employ a nonlinear filter. The filter retrieves the sequence of the shocks including the sunspot shock using the parameterized model. This sequence can,

[22]The average duration at the zero lower bound after a financial crisis is rather small in the model, which is a setting required for the model to be solved. The reason is that the detrimental general equilibrium effects of the zero lower bound in combination with a financial crisis are too strong. This result is in line with work on the role of the zero lower bound on the existence of an equilibrium (Bianchi, Melosi and Rottner, 2021).

in turn, be used to obtain other objects of interest such as the estimated probability of a run. To capture the nonlinearity of the model, I use a particle filter as suggested in Fernández-Villaverde and Rubio-Ramírez (2007).[23] I adapt the particle filter to specifically take into account the multiplicity of equilibria similar to Aruoba, Cuba-Borda and Schorfheide (2018). Additionally, I extend their approach to handle not only multiplicity of equilibria, but also the endogenously time-varying nature of the equilibria probabilities. This adjustment is necessary to take account of the endogeneity of runs.

The outlined approach to estimating the endogenous financial fragility based on a structural model with financial crises is general and provides the structural equivalent to the fast-growing empirical growth-at-risk approach (Adrian, Boyarchenko and Giannone, 2019).

4.1 Particle Filter

The particle filter estimates the hidden states and shocks based on a set of observables. It is convenient to cast the model in a nonlinear state-space representation as a starting point:

$$\mathbb{X}_t = f(\mathbb{X}_{t-1}, v_t, \iota_t), \tag{44}$$

$$\mathbb{Y}_t = g(\mathbb{X}_t) + u_t. \tag{45}$$

The first set of equations contains the transition equations that depend on the state variables $\mathbb{X}_t$, the structural shocks v_t and the sunspot shock ι_t. In particular, the state variables and shocks determine endogenously the selected equilibrium of the model, either the normal equilibrium or the run equilibrium. The transition equations are different for the different regimes. The nonlinear functions f are obtained from the nonlinear model that is fitted to selected moments and solved with the global solution method. The second set of equations contains the measurement equations, which connect the state variables with the observables $\mathbb{Y}_t$. It also includes an additive measurement error u_t.[24]

The particle filter extracts a sequence of conditional distributions for the structural shocks $v_t|\mathbb{Y}_{1:t}$ and the sunspot shock $\iota_t|\mathbb{Y}_{1:t}$, which provides the empirical implications of the model. Thereby, the filter evaluates when a run occurs and provides the probability of a run in the next quarter. The algorithm and the adaptation to the multiplicity of regimes is laid out in Appendix E.

Observables The observables $\mathbb{Y}_t$ are GDP growth and shadow bank leverage. GDP growth is included as a model with financial crisis should capture the sizeable reduction in economic activity. GDP growth is measured as the quarter-to-quarter real GDP growth rate. Output growth is demeaned as the trend growth is zero in the model. The model is fitted to leverage to capture the key trade-off between leverage and financial fragility explored above. The

[23]The particle filter is based on Atkinson, Richter and Throckmorton (2019) and Herbst and Schorfheide (2015).

[24]The particle filter requires a measurement error to avoid a degeneracy of the likelihood function. Another advantage of including the measurement error is that it can take into account noisy data. Measuring shadow bank leverage is highly complex so the underlying series may be very noisy.

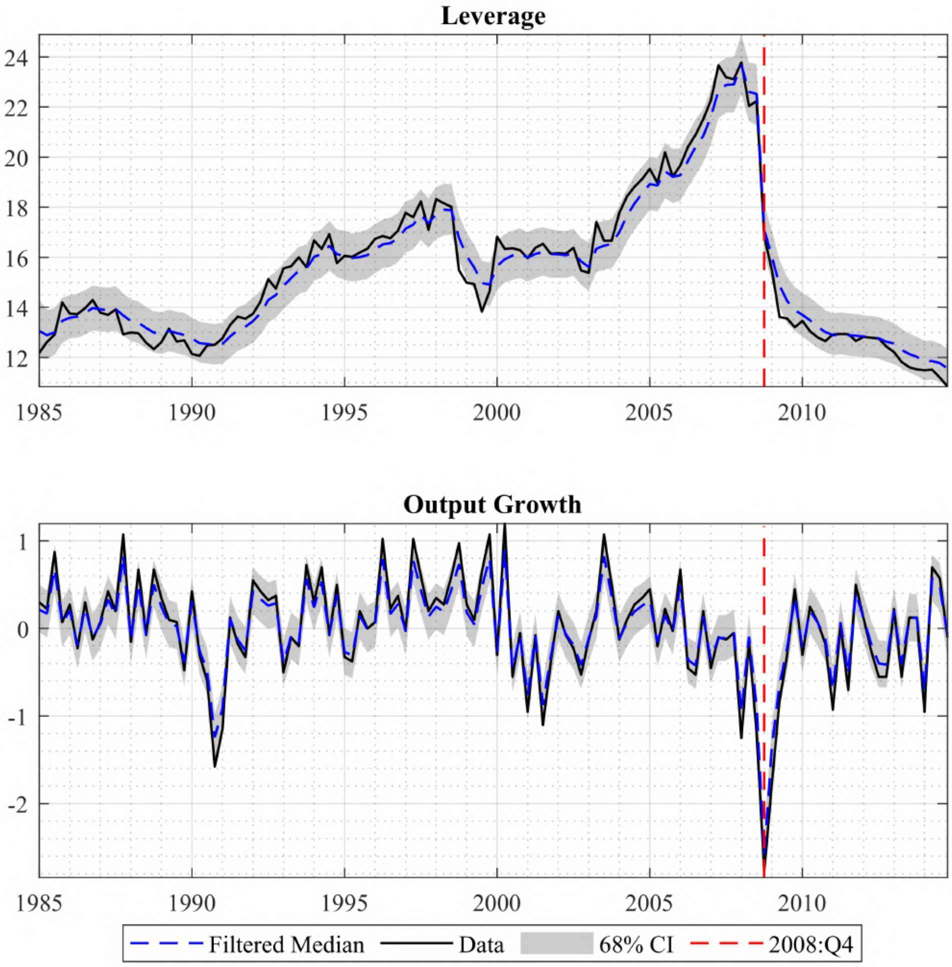

Figure 6: Filtered median of leverage and output growth is the blue line together with its 68% confidence interval. The observables are shadow bank leverage and (demeaned) real output growth. The red line corresponds to the fourth quarter of 2008:Q4.

measure relies on shadow bank leverage as discussed in the calibration. It uses balance sheet data for security broker-dealers and finance companies from the U.S. Flow of Funds, and is based on book equity as also discussed in Appendix A. The observation equation is:

$$\begin{bmatrix} \text{Output Growth}_t \\ \text{Leverage}_t \end{bmatrix} = \begin{bmatrix} 100 \ln \left(\frac{Y_t}{Y_{t-1}} \right) \\ \phi_t \end{bmatrix} + u_t, \tag{46}$$

where the measurement error is given by $u_t \sim N(0, \Sigma_u)$. The variance of the measurement error is set to 25% of the sample variance, similar to Gust et al. (2017).

4.2 Results

To establish that the filtered model captures the fluctuations in the observables, Figure 6 compares the estimated sequence for leverage and output against the data. In line with the data, leverage increases substantially prior to the financial crisis. The peak comes in around 2008, with leverage close to 24. The filtered path also takes account of the strong decrease

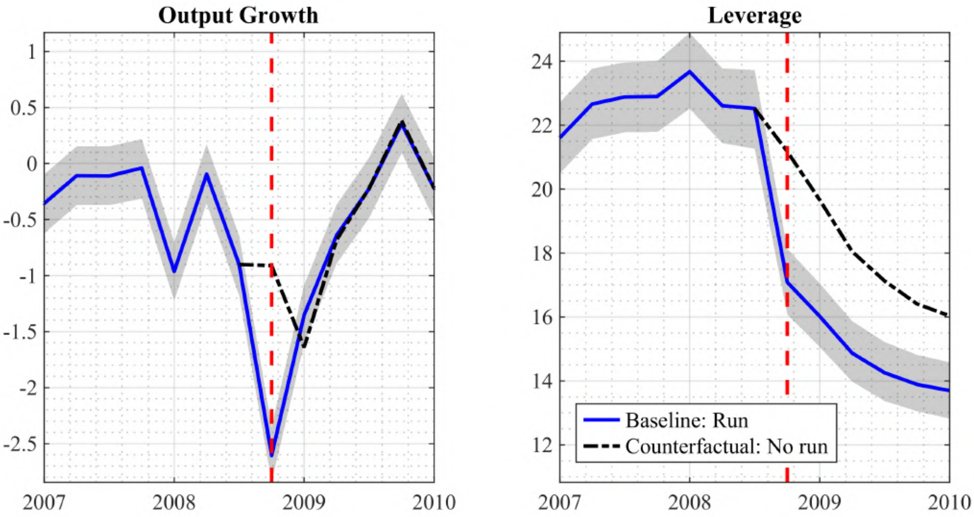

Figure 7: Comparison of baseline estimate to a counterfactual scenario without a run. The baseline median (blue line with its 68% confidence interval) is compared to the counterfactual median, where no sunspot shock materializes in 2008:Q4.

in output and leverage in the fourth quarter of 2008.

Crucially, the model can account for this sharp drop in the fourth quarter of 2008 via two different channels: a run on the financial sector or large contractionary shocks. As the equilibria are not exogenously imposed, the particle filter selects the regime depending on the fit with the data. This gives a real-time assessment if a run took place. The model clearly favors a run. The filter assigns a weight of (close to) 100% to a run to explain the 2008:Q4 period, while the weight of the run regime is basically 0% in all other periods.

Bernanke (2018) and Gorton and Metrick (2012) argue that the run on the financial sector is responsible for the sharp and large decrease in economic activity. To assess this through the lens of the model, a counterfactual compares the estimated path to a hypothetical scenario without a self-fulfilling run as no sunspot shock materializes in 2008:Q4. The crucial take-away is that the economic contraction is considerably smaller in the absence of a run, as shown in Figure 7. The endogenous amplification via a run enables to account for the drop in output. To be precise, the endogenous mechanism of a run results in an additional 1.8 percentage points growth reduction quarter to quarter. The run alone explains around 70% of the output drop. The impact of the contractionary shocks without a run would only explain 30% of the contraction in 2008:Q4. This result - on the importance of the endogenous run mechanism to account for the time series evidence from the U.S - also implies that studying the role of financial shocks using linearized models without runs is difficult.

To inspect the economic drivers behind the run in 2008:Q4, the filtered series of volatility and total factor productivity can be compared as shown in Figure F.1 in the Appendix. A series of shocks reduces volatility σ_t prior to the financial crisis. In line with the idea of the volatility paradox, this period sows the seed of a crisis as leverage and financial fragility increase. In the fourth quarter of 2008, contractionary volatility and TFP shocks in combination with a sunspot shock then trigger the run. Figure F.1 also provides a validity check of the empirical experiment, as other filtered series of key variables such as securities, inflation,

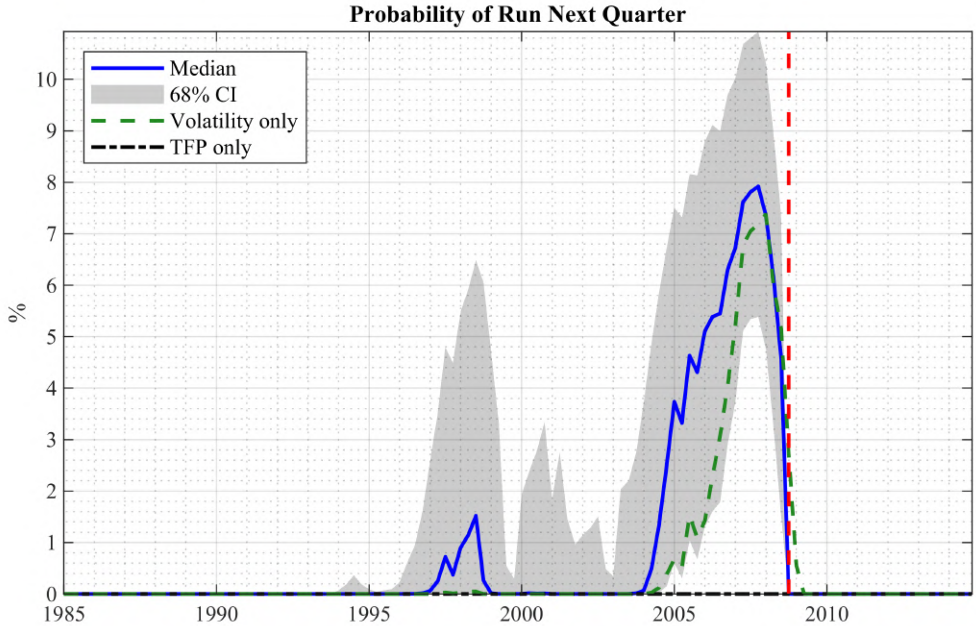

Figure 8: Filtered median probability of a run in the next quarter with its 68% confidence interval. To disentangle the impact of the structural shocks, the realizations of the volatility shock and TFP shock are set to 0 one at a time. The dashed green line is a scenario that only uses the extracted volatility shocks. The black dash-dotted line is a scenario that only uses the extracted TFP shocks. The red line indicates the fourth quarter of 2008.

finance premium can be assessed. The estimated series predicts a credit boom gone bust, a countercyclical finance premium and a period of low inflation after the run in line with the data.

Financial Fragility A crucial advantage of the developed approach is that it provides a model-implied (real-time) estimate of financial fragility. In particular, Figure 8 shows the shows the probability of a run for the period ahead. This probability fluctuates over time as it depends on the business cycle and on the vulnerability of the financial sector. While there is a slight increase around 1998, there is a remarkable surge of financial fragility from 2005 onwards. Thus, the model suggests that that there had already been a substantial build-up of financial fragilities a few years prior to the outbreak of the financial crisis. From a quantitative perspective, the median one-quarter-ahead forecast for a financial crisis in 2008:Q4 is around 5%, which would be 20% in annualized terms.

As a next step, I disentangle the structural sources of the financial crisis using a counterfactual analysis. In particular, the estimated series of the productivity and volatility shock are evaluated in isolation by setting the other shock to zero for the entire horizon. Thus, the financial fragility measure is then revisited if only volatility shocks or TFP shocks drive the economy. Furthermore, the counterfactual setup sheds lights on the question of whether a shock alone would have still resulted in a run. Total factor productivity in isolation causes (virtually) no financial fragility in the run-up to the financial crisis, and no run would have taken place. By contrast, the volatility shock is the main driver, explaining, by itself, up to 90% of total fragility in 2008. The analysis emphasizes the importance of nonlinearities as the

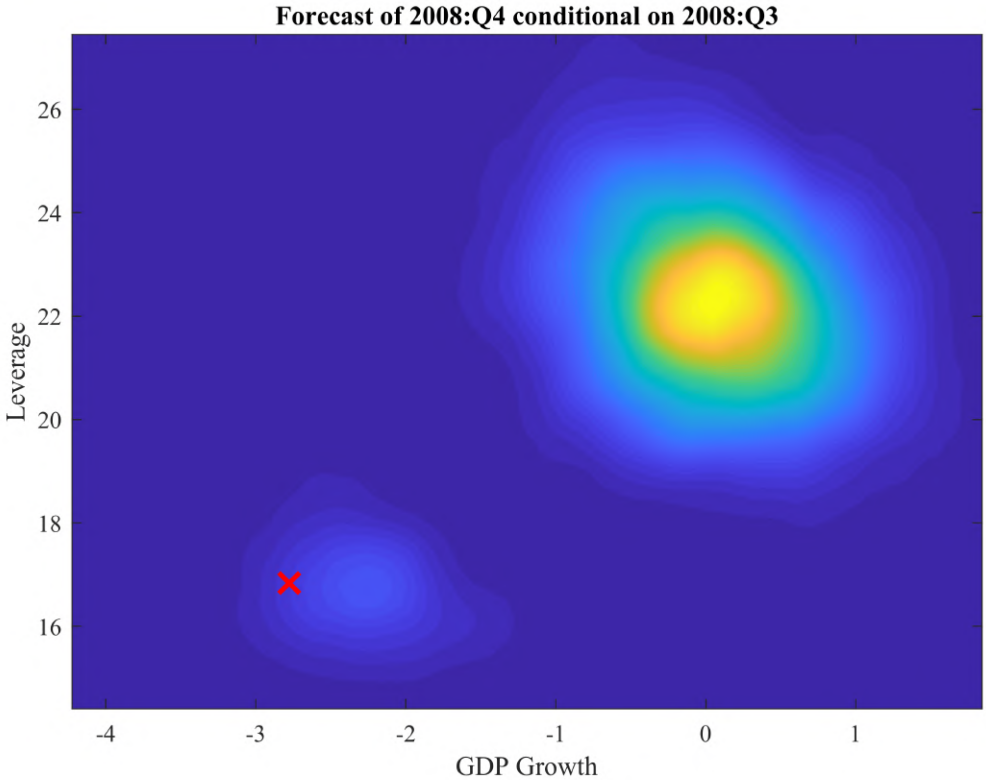

Figure 9: The contour plots displays the joint distribution of GDP growth and leverage for 2008:Q4 conditional on 2008:Q3. It shows that the multimodality arises due to the run equilibrium. Yellow indicates a high density, while dark blue indicates a low density. The red square shows the actual data realization in 2008:Q4. The forecasts are conditioned on the median realization in 2008:Q3.

combination of the shocks can increase (or decrease) financial fragility. This can be seen in the years preceding 2008: the measured financial fragility is considerably higher in a scenario with both shocks than the sum of them in isolation. Furthermore, the volatility shock alone is also not sufficient to trigger a run. The reason is that the combination of contractionary TFP and volatility shocks trigger the run in the estimated series. Taken together, the volatility shock is more important than the TFP shock to explain the financial crisis from a horse race perspective. But, there are very important interactions between the shocks.

Macroeconomic Tail Risk and Multimodality The increase in financial fragility induces substantial macroeconomic downside risk as the possibility of a financial crisis emerges. To better understand the downside risk, I evaluate the joint one-quarter-ahead distribution of output and leverage. Figure 9 shows a contour plot of the one-quarter-ahead joint distribution of output and leverage for 2008:Q4 conditional on 2008:Q3. This points out that the distribution of output and leverage is bimodal due to the possibility of a run. There is a high probability that the normal equilibrium with an output growth centered around 0 is chosen. However, there is the significant tail risk of a run on the financial sector that brings with it a large drop in GDP and leverage. This figure also emphasizes how the model can explain the data. The probability of observing such large shocks for TFP and volatility, which could account for the observed drop in output without a run, is infinitesimally small. Instead, an endogenous financial crisis can capture the data and this sharp recession. The incorporation of nonlinear features permits it to explain the data.

From a time perspective, the possibility of a financial crisis opens up in 2005 as the period of low volatility induces high leverage and financial fragility. Before 2005 and from 2009 onwards, the distribution is overwhelmingly unimodal as a run is (almost) not possible. Figure F.2 in the Appendix displays how the conditional joint distribution evolves over time.

The analysis is the structural equivalent to the large body of literature on growth-at-risk starting with Adrian, Boyarchenko and Giannone (2019), where the downside risk over time is evaluated. Recent reduced-form empirical approaches also suggest that the downside risk comes from a bimodal distribution, in line with my findings (see e.g Adrian, Boyarchenko and Giannone, 2021, Caldara et al., 2020 and Mitchell, Poon and Zhu, 2021). The work most closely related is that of Adrian, Boyarchenko and Giannone (2021), who estimate the conditional joint distributions of economic fundamentals and financial conditions. Similarly to the predictions in the model, they find the occurrence of a second equilibrium for 2008:Q4 conditional on 2008:Q3. They also find that the probability of the normal equilibrium is higher. Comparing it with the reduced form approach, the structural approach makes an assessment of the impact of policy counterfactuals on the emergence of macroeconomic downside risk possible. This will be the focus of the next section.

5 Counterfactuals for Monetary and Macroprudential Policies

This section conducts policy counterfactuals using the quantitative nonlinear model with financial crisis and the estimated endogenous financial fragility measure. In particular, I evaluate the impact of alternative monetary and macroprudential strategies on the build-up of financial fragility, economic downside risk and their potential to avoid a financial crisis in 2008. Using the filtered shocks from the particle filter, I can directly evaluate how alternative policies would have affected the estimated probability of a run and the occurrence of a run on the financial sector in 2008:Q4. While the focus is on specific monetary and macroprudential instruments in this section, the outlined approach is general and can be applied to address other policies.

5.1 Monetary Policy: “Leaning Against the Wind”

With regard to monetary policy, I evaluate the potential of “leaning against the wind” policies to lower financial fragility. “Leaning against the wind” prescribes a tighter monetary policy (e.g. higher interest rates) during a potential credit boom for financial stability purposes. The advantage of monetary policy over macroprudential policy and supervision is that it gets in all of the cracks that macroprudential policy and supervision fail to reach. This is especially relevant in my setup because the analysis focuses on the financial fragility associated with

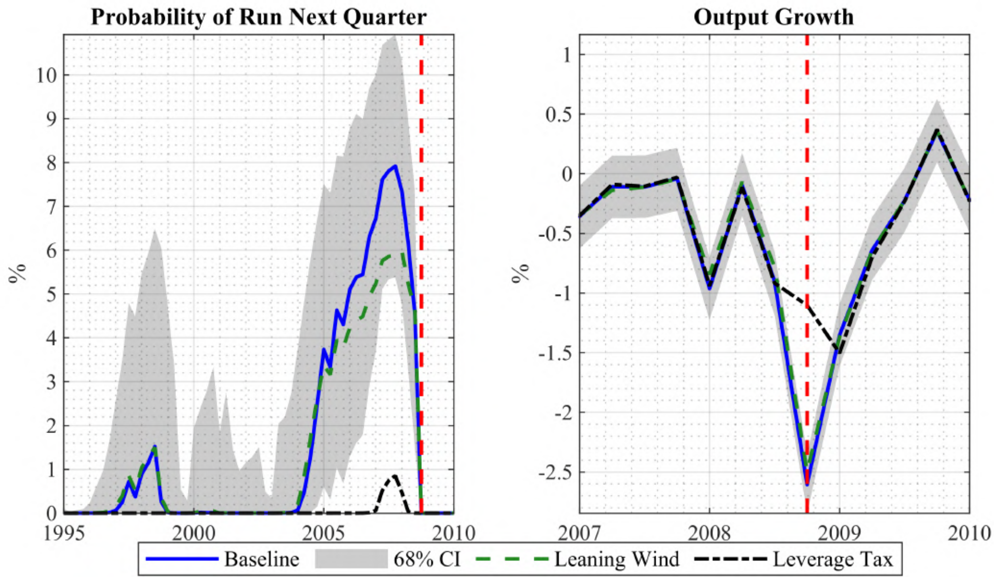

Figure 10: Counterfactual policy analysis of "leaning against the wind" and the leverage tax. The filtered median probability of a run in the next quarter and output growth (solid blue) with its 68% confidence interval is shown for the baseline scenario. Using the estimated shocks, the median for the counterfactual scenario of "leaning against the wind" (dashed green) and leverage tax (dash-dotted black) is shown. The red line indicates the fourth quarter of 2008.

the unregulated part of the financial sector. While there has been active debate about its costs and benefits, this setup can provide novel insights as it uses an estimated endogenous financial stability measure based on a nonlinear structural model.

To incorporate "leaning against the wind", the central bank raises its interest rate if the security holdings of financial intermediaries exceed a certain target value. This adds an asymmetric component to the Taylor rule that incorporates financial considerations in the monetary policy response during a boom:

$$R_t^I = \max\left\{R^I\left(\frac{\Pi_t}{\Pi}\right)^{\kappa_\Pi}\left(\frac{\varphi_t^{mc}}{\varphi^{mc}}\right)^{\kappa_y}\left[\mathbf{1}_{S_t^B>S^T}\left(\frac{S_t^B}{S^T}\right)^{\kappa_s}+(1-\mathbf{1}_{S_t^B>S^T})\right],1\right\}, \tag{47}$$

where $\mathbf{1}$ is an indicator function and κ_s is the response to deviations above a target value S^T set by the central bank. The asymmetric design implies that the sole policy gain and additional space to cut rates comes from raising rates during a credit boom, in line with the idea of "leaning against the wind".

The impact on financial stability is ambiguous as an increase in the interest rate can lead to a substitution towards more equity and less deposit-based financing. At the same time, the increased funding cost due to higher interest rates can also result in less loss absorbing capacities and thus, an increased risk of financial stability. A quantitative analysis shows that the impact of "leaning against the wind" depends on the shocks. "Leaning against the wind" helps to increase financial stability in a world with only volatility shocks. However, the probability of financial crises increases in the full model that also includes TFP shocks. This limits the potential scope of the response κ_S, which is set to 0.04. At this value, the probability of a financial crisis increases slightly to 2.3% from 2.2%, but it limits the build-up of very high leverage levels and its associated financial fragility.

The counterfactual scenario with a monetary authority that "leans against the wind" for the financial crisis of 2008 can now be evaluated. Based on the estimated sequence of distribution of shocks in the previous section, an alternative scenario with the adjusted monetary policy rule can be studied. The filtered shocks are fed into the adapted model, which allows me to calculate the counterfactual evolution of the economy. Figure 10 summarizes the the counterfactual path. The build-up of financial fragility is now slightly reduced. The probability of a run peaks now at around 6% instead of 8%. This shows that "leaning against the wind" offers some potential in terms of protecting the economy from financial crisis. However, the policy is not enough in this scenario. Although the economy is slightly less fragile, the estimated shocks still trigger a financial crisis and output drops (almost) as much as in the baseline scenario.[25]

5.2 Macroprudential Policy: Leverage Tax

I also evaluate the evolution of financial fragility under an alternative macroprudential policy, namely a leverage tax for shadow banks. This is a tax on the deposit holdings and has been the subject of recent discussions among policymaking circles. Specifically, the "Minneapolis Plan to End Too Big to Fail" drawn up by the Minneapolis Federal Reserve Bank in 2017 proposes taxing the borrowing of shadow banks.[26] Given that shadow banks are financial intermediaries which, by their very definition, operate outside of the regulatory banking umbrella, regulating them is potentially extremely difficult. In addition, finding a reliable measure of shadow banks' leverage is quite a challenging task, which poses another problem. A tax on the borrowing of shadow banks is a rather simple and tractable approach, making it well suited.

The leverage tax τ^{ϕ} requires the banker to pay at the end of the period a tax τ for its borrowings from households:

$$N_t = R_t^K Q_{t-1} S_{t-1}^B - R_{t-1}^D D_{t-1} - \tau^{\phi} D_{t-1} + \tau^L, \tag{48}$$

They also receive lump sum transfer τ^L, where the lump sum transfer is chosen so that the leverage tax is budget neutral for each intermediary. The leverage tax incentivizes the intermediaries to substitute from deposit holdings towards equity. The reduction in leverage can then increase financial stability and lower the frequency of financial crises. The tax τ^{ϕ} could be interpreted in a Pigouvian sense as the model features a run externality.[27] Agents

[25] While an even more aggressive "leaning against the wind" policy could have mitigated the build-up potentially even more, such a policy is not applicable in this model. This is because a stronger asymmetric response in the Taylor rule (e.g. increase κ_S) results in a situation, whereby the equilibrium cannot be found with global methods anymore. This is a general issue with models featuring strong nonlinearities. The nonlinear features can result in the non-existence of an equilibrium so that the model does not possess a solution anymore.

[26] The plan consists of several recommendations for the entire banking sector. This paper focuses solely on the leverage tax. In line with the proposal, the leverage tax only applies to shadow banks in the model. An alternative policy could be to alter the corporate tax rate, which also affects the cost of external financing.

[27] The proposed tax is constant and does not vary with the business or financial cycle. Thus, the proposed tax cannot be optimal because - similarly to the probability of a run - the run externality is state-dependent.

do not take into account the impact of their own decisions on the run probability so that the risk-taking and leverage of the financial intermediaries can be too high. The leverage tax aims to correct undesirable leverage accumulation and thereby to avoid the build-up of financial fragility. I set the leverage tax τ^{ϕ} to 0.015 to target a reduction in the run frequency by more than 75%. While the threat of financial crisis is muted, the leverage tax reduces financial intermediation. This lowers output and credit during non-crisis times.

The counterfactual path of the economy with a leverage tax is summarized in Figure 10. It shows how the macroprudential instrument alters the estimated build-up of financial fragility and the occurrence of a run on the financial sector in 2008. I find that the leverage tax significantly reduces the probability of a financial crisis. The peak is now around 1% and it is close to 0% in 2008:Q3. Furthermore, the leverage tax succeeds in averting a financial meltdown. The economy does not encounter a run on the financial system in 2008:Q4 due to the lowered leverage levels of the intermediaries. As a consequence, the fall in output is much more muted in 2008 in this counterfactual scenario. Taken together, a sufficient limit on leverage would have reduced the build-up of financial fragility prior to the financial crisis significantly and would have avoided the run on the financial sector in 2008:Q4 itself.

6 Conclusion

I investigate the endogenous build-up of financial fragility around financial crises with a new nonlinear macroeconomic model that accounts for key macroeconomic and financial features. The framework highlights the build-up of leverage as a decisive determinant for a run on the financial sector. It also emphasizes the importance of a low real interest rate environment and the zero lower bound when it comes to financial stability.

The quantitative model is taken to the data to estimate the build-up of financial fragility and the macroeconomic downside risk in the years preceding the financial crisis of 2007-2008. My results suggest that the estimated financial fragility and the economic downside risk increase considerably from 2005 onwards and peak in 2008. The developed approach offers a new strategy for the appraisal of alternative policies, letting me evaluate counterfactual monetary and macroprudential policies with respect to the estimated endogenous build-up of the probability of a financial crisis. The counterfactual analysis suggests that macroprudential policy limiting the leverage of shadow banks would have reduced the build-up of financial fragility and would have avoided the run on the financial sector in 2008 itself.

The outlined concept for estimating and analyzing the endogenous build-up of financial fragility and conducting policy counterfactuals through the lens of a structural nonlinear model is general and can be adapted to a variety of countries, scenarios and policies. The approach can provide a real-time structural estimate of financial fragility and can help to quantify the potential financial stability impact of different policy scenarios such as an increase in the interest rate after a prolonged period of low rates or the tapering of asset purchase programs.

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A Data: Shadow Bank Leverage

The leverage series in this paper uses book equity, which is the difference between the value of the portfolio and liabilities of financial intermediaries. An alternative measure is the financial intermediaries' market capitalization (e.g. market valuation of financial intermediaries). Book equity is the appropriate concept in this context because the interest lies with credit supply and financial intermediaries' lending decisions, as stressed for instance in Adrian and Shin (2014).[28] In contrast to this, market capitalization is the appropriate measure when considering the issuance of new shares or acquisition decisions (Adrian, Colla and Shin, 2013). In the context of the model, the occurrence of a run also depends on book equity, which rationalizes this choice. With that in mind, book leverage based on book equity is the appropriate concept for my purposes.

A related issue is that marked-to-market value of book equity, which is the difference between the market value of portfolio claims and liabilities of financial intermediaries, is conceptually very different from market capitalization. As argued in Adrian and Shin (2014), the book value of equity should be measured as marked-to-market. In such a case, the valuation of the assets is based on market values. Importantly, the valuation of assets is marked-to-market in the balance sheet of financial intermediaries that hold primarily securities (Adrian and Shin, 2014). Crucially, the concept of marked-to-market value of book equity corresponds to the approach to leverage adopted in the model as the value of the securities depends on their market price. Therefore, I am interested in marked-to-market book leverage.

U.S. Flow of Funds The leverage measure for shadow banks uses U.S. Flow of Funds balance sheet data for security brokers and dealers and finance companies similar to Nuño and Thomas (2017).[29]

Equity is computed as the difference between book assets and book liabilities for both types of financial intermediaries:

$$Equity\ Brokers\ \&\ Dealers_t = Assets\ Brokers\ \&\ Dealers_t - Liabilities\ Brokers\ \&\ Dealers_t \tag{A.1}$$

$$Equity\ Finance\ Companies_t = Assets\ Finance\ Companies_t - \ Liabilities\ Finance\ Companies_t \tag{A.2}$$

The aggregate leverage measure is then defined as:

$$Leverage_t = \frac{Assets\ Finance\ Companies_t + Assets\ Brokers\ \&\ Dealers_t}{Equity\ Finance\ Companies_t + Equity\ Security\ Brokers\ \&\ Dealers_t}. \tag{A.3}$$

Compustat An alternative measure of book leverage of the shadow banking sector can be constructed with individual balance sheet data from Compustat. I include financial firms that

[28] He, Khang and Krishnamurthy (2010) and He, Kelly and Manela (2017) provide an opposing view with an emphasis on market leverage.

[29] The time series are adjusted for discontinuities and breaks in the data.

are classified with SIC codes between 6141 - 6172 and 6199 - 6221. This set contains credit institutions, business credit institutions, finance lessors, finance services, mortgage bankers and brokers, security brokers, dealers and flotation companies, and commodity contracts brokers and dealers.[30]

Equity is computed as the difference between book assets and book liabilities for each firm:

$$Equity_{i,t} = Book\ Assets_{i,t} - Book\ Liabilities_{i,t}. \tag{A.4}$$

The leverage of the shadow banking sector is then defined as

$$Leverage_t = \frac{\sum_i Book\ Assets_{i,t}}{\sum_i Book\ Equity_{i,t}}, \tag{A.5}$$

where I sum up equity and assets over the different entities.

[30]Finance lessors and finance services with the SIC codes 6172 and 6199 are not official SIC codes, but are used by the U.S. Securities and Exchange Commission.

B Model Equations and Equilibrium

The system of equations that characterizes the economy is described below.

Households

$$C_t = W_t L_t + D_{t-1}R_t - D_t + \Xi_t + Q_t S_t^H + (Z_t + (1-\delta)Q_t)S_{t-1}^H, \tag{B.1}$$

$$\varrho_t = (C_t)^{-\sigma}, \tag{B.2}$$

$$\varrho_t W_t = \chi L_t^{\varphi}, \tag{B.3}$$

$$1 = \beta E_t \Lambda_{t,t+1} R_{t+1}, \tag{B.4}$$

$$1 = \beta E_t \Lambda_{t,t+1} \frac{Z_{t+1} + (1-\delta)Q_{t+1}}{Q_t + \Theta(S_t^H/S_t - \gamma^F)/\varrho_t}, \tag{B.5}$$

$$\beta E_t \Lambda_{t,t+1} = \beta E_t \varrho_{t+1}/\varrho_t. \tag{B.6}$$

Financial Intermediaries

$$Q_t S_t^B = \phi_t N_t, \tag{B.7}$$

$$\overline{\omega}_t = \frac{\phi_{t-1} - 1}{R_t^K \phi_{t-1}}, \tag{B.8}$$

$$(1-p_t)E_t^N[\beta\Lambda_{t,t+1}\bar{R}_t D_t] + p_t E_t^R[\beta\Lambda_{t,t+1}R_{t+1}^K Q_t S_t^B] \geq D_t, \tag{B.9}$$

$$(1-p_t)E_t^N[\Lambda_{t,t+1}R_{t+1}^K(\theta\lambda_{t+1} + (1-\theta))[1 - e^{\frac{-\psi}{2}} - \tilde{\pi}_{t+1}]] = p_t E_t^R[\Lambda_{t,t+1}R_{t+1}^K(e^{-\frac{\psi}{2}} - \overline{\omega}_{t+1} + \tilde{\pi}_{t+1})], \tag{B.10}$$

$$\lambda_t = \frac{(1-p_t)E_t^N \Lambda_{t,t+1} R_{t+1}^K[\theta\lambda_{t+1} + (1-\theta)](1-\overline{\omega}_{t+1})}{1 - (1-p_t)E_t^N[\Lambda_{t,t+1}R_{t+1}^K\overline{\omega}_{t+1}] - p_t E_t^R[\Lambda_{t,t+1}R_{t+1}^K]}, \tag{B.11}$$

$$\kappa_t = \frac{\beta(1-p_t)E_t^N\Lambda_{t,t+1}\left[\lambda_t - (\theta\lambda_{t+1} + 1 - \theta)\right]}{(1-p_t)E_t^N\Lambda_{t,t+1}\left[(\theta\lambda_{t+1} + 1 - \theta)\,\tilde{F}_{t+1}(\overline{\omega}_{t+1})\right] + p_t E_t^R \Lambda_{t,t+1}\left[(\theta\lambda_{t+1} + 1 - \theta)\left(1 - \tilde{F}_{t+1}(\overline{\omega}_{t+1})\right)\right]}, \tag{B.12}$$

$$E_t[\tilde{\pi}_{t+1}] = E_t\left[\overline{\omega}_{t+1}\Phi\left(\frac{\log(\overline{\omega}_{t+1}) + \frac{1}{2}(\psi + \sigma_{t+1}^2)}{\sigma_{t+1}}\right) - e^{-\psi/2}\Phi\left(\frac{\log(\overline{\omega}_{t+1}) + \frac{1}{2}(\psi - \sigma_{t+1}^2)}{\sigma_{t+1}}\right)\right], \tag{B.13}$$

$$N_t = \theta N_{S,t} + N_{N,t}, \tag{B.14}$$

$$N_{N,t} = (1-\theta)\zeta S_{t-1}, \tag{B.15}$$

$$N_{S,t} = \begin{cases} R_t^K Q_{t-1} S_{t-1}^B - \overline{R}_{t-1} D_{t-1} & \text{if } x_t^\star \geq= 1 \vee \iota_t = 0 \text{ (no run)} \\ 0 & \text{if } x_t^\star < 1 \wedge \iota_t = 1 \text{ (run occurs)} \end{cases}, \tag{B.16}$$

$$R_t = \begin{cases} \bar{R}_{t-1} & \text{if } x_t^\star \geq= 1 \vee \iota_t = 0 \text{ (no run)} \\ x_t^\star \bar{R}_{t-1} & \text{if } x_t^\star < 1 \wedge \iota_t = 1 \text{ (run occurs)} \end{cases}. \tag{B.17}$$

Non-Financial Firms

$$Y_t = A_t(K_{t-1})^{\alpha}(L_t)^{1-\alpha}, \tag{B.18}$$

$$K_t = S_t, \tag{B.19}$$

$$\varphi_t^{mc}(1-\alpha)\frac{Y_t}{L_t} = W_t, \tag{B.20}$$

$$R_t^K = \frac{Z_t + Q_t(1-\delta)}{Q_{t-1}}, \tag{B.21}$$

$$Z_t = \varphi_t^{mc}\alpha\frac{Y_t}{K_{t-1}}, \tag{B.22}$$

$$\left(\frac{\Pi_t}{\Pi} - 1\right)\frac{\Pi_t}{\Pi} = \frac{\epsilon}{\rho^r}\Big(\varphi_t^{mc} - \frac{\epsilon-1}{\epsilon}\Big) + \Lambda_{t,t+1}, \left(\frac{\Pi_{t+1}}{\Pi} - 1\right)\frac{\Pi_{t+1}}{\Pi}\frac{Y_{t+1}}{Y_t}, \tag{B.23}$$

$$\Gamma\Big(\frac{I_t}{K_t}\Big) = a_1\Big(\frac{I_t}{K_t}\Big)^{(1-\eta)} + a_2, \tag{B.24}$$

$$Q_t = \left[\Gamma'\left(\frac{I_t}{S_{t-1}}\right)\right]^{-1}, \tag{B.25}$$

$$S_t = (1-\delta)S_{t-1} + \Gamma\Big(\frac{I_t}{S_{t-1}}\Big)S_{t-1}. \tag{B.26}$$

Monetary Policy and Market Clearing

$$R_t^I = \max\left[1, R^I\left(\frac{\Pi_t}{\Pi}\right)^{\kappa_\Pi}\left(\frac{\varphi_t^{mc}}{\varphi^{mc}}\right)^{\kappa_y}\right], \tag{B.27}$$

$$\beta E_t\Lambda_{t,t+1}\frac{R_t^I}{\Pi_{t+1}} = 1, \tag{B.28}$$

$$Y_t = C_t + I_t + G + \frac{\rho^r}{2}\left(\frac{\Pi_t}{\Pi} - 1\right)^2 Y_t, \tag{B.29}$$

$$S_t = S_t^H + S_t^B. \tag{B.30}$$

Shocks

$$\sigma_t = (1-\rho^\sigma)\sigma + \rho^\sigma\sigma_{t-1} + \mathbf{1}_{x_t^\star<1\wedge\iota_t=1}\sigma^\sigma\epsilon^{\sigma\star} + \sigma^\sigma\epsilon_t^\sigma, \tag{B.31}$$

$$A_t = (1-\rho^A)A + \rho^A A_{t-1} + \sigma^A\epsilon_t^A, \tag{B.32}$$

$$\iota_t = \begin{cases} 1 & \text{with probability } \Upsilon \\ 0 & \text{with probability } 1-\Upsilon \end{cases}. \tag{B.33}$$

Definition

The recursive competitive equilibrium is a price system, policy functions for the households, the financial intermediaries, the final goods producers, intermediate goods producers and capital goods producers, law of motion of the aggregate state and perceived law of motion of the aggregate state, such that the policy functions solve the agents' respective maximization problem, the price system clears the markets and the perceived law of motion coincides with the law of motion. The aggregate state of the economy is described by the vector of state variables $\mathcal{S}_t = (N_t, S^B_{t-1}, A_t, \sigma_t, \iota_t)$, where ι_t is a sunspot shock.

C Derivation of Financial Intermediary's Problem

In the following, I derive the financial intermediary's problem for two cases: (i) absence of runs and (ii) anticipation of runs.

C.1 Absence of runs

The financial intermediary maximizes the value of its franchise V_t subject to a participation and incentive constraint, which reads as follows:[31]

$$
\begin{aligned}
V_t^j(N_t^j) = & \max_{S_t^{Bj}, \bar{D}_t} \beta E_t \Lambda_{t,t+1} \Big[\theta V_{t+1}^j(N_{t+1}^j) + (1-\theta)(R_{t+1}^K Q_t S_t^{Bj} - \bar{D}_t^j) \Big], && \text{(C.1)} \\
\text{subject to} \quad & \beta E_t [\Lambda_{t,t+1} \bar{R}_t^D D_t^j] \geq D_t^j, && \text{(C.2)} \\
& \beta E_t \Lambda_{t,t+1} \Big\{ \theta V_{t+1}^j (S_t^{Bj}, \overline{D}_t^j) + (1-\theta)[R_{t+1}^K Q_t S_t^{Bj} - \overline{D}_t^j] \Big\} \geq && \text{(C.3)} \\
& \beta E_t \Lambda_{t,t+1} \int_{\overline{\omega}_{t+1}^j}^{\infty} \Big\{ \theta V_{t+1}(\omega, S_t^{Bj}, \overline{D}_t^j) + (1-\theta)[R_{t+1}^K Q_t S_t^{Bj} \omega_{t+1}^j - \overline{D}_t^j] \Big\} d\tilde{F}_{t+1}(\omega).
\end{aligned}
$$

The financial intermediary's problem can be written as the following Bellman equation:

$$
\begin{aligned}
V_t(N_t^j) = & \max_{\{S_t^{Bj}, \overline{b}_t^j\}} \beta E_t \Lambda_{t,t+1} \Big[\theta V_{t+1} \Big(\Big(1 - \frac{\overline{b}_t^j}{R_{t+1}^K}\Big) R_{t+1}^K Q_t S_t^{Bj} \Big) + (1-\theta)\big(1 - \frac{\overline{b}_t^j}{R_{t+1}^K}\big) R_{t+1}^K Q_t S_t^{Bj} \Big] \\
& + \lambda_t^j \Big[\beta E_t \Lambda_{t,t+1} Q_t S_t^{Bj} \overline{b}_t^j - (Q_t S_t^{Bj} - N_t^j) \Big] \\
& + \kappa_t^j \beta E_t \Lambda_{t,t+1} \Big\{ \Big[\theta V_{t+1} \Big(\Big(1 - \frac{\overline{b}_t^j}{R_{t+1}^K}\Big) R_{t+1}^K Q_t S_t^{Bj} \Big) + (1-\theta)\Big(1 - \frac{\overline{b}_t^j}{R_{t+1}^K}\Big) R_{t+1}^K Q_t S_t^{Bj} \Big] \\
& - \int_{\frac{\overline{b}_t^j}{R_{t+1}^K}}^{\infty} \Big[\theta V_{t+1} \Big(\Big(\omega - \frac{\overline{b}_t^j}{R_{t+1}^K}\Big) R_{t+1}^K Q_t S_t^{Bj} \Big) + (1-\theta)\Big(\omega - \frac{\overline{b}_t^j}{R_{t+1}^K}\Big) R_{t+1}^K Q_t S_t^{Bj} \Big] d\tilde{F}_{t+1}(\omega) \Big\}
\end{aligned}
$$

where I defined $\overline{b}_t^j = \overline{D}_t^j / \left(Q_t S_t^B\right)$ and used that

$$
N_t^j = \begin{cases} \Big(1 - \frac{\overline{b}_{t-1}^j}{R_t^K}\Big) R_t^K Q_{t-1} S_{t-1}^{Bj} & \text{if standard security} \\ \Big(\omega - \frac{\overline{b}_{t-1}^j}{R_t^K}\Big) R_t^K Q_{t-1} S_{t-1}^{Bj} & \text{if substandard security} \end{cases} \qquad \text{(C.4)}
$$

λ_t^j and κ_t^j are the Lagrange multipliers of the participation and incentive constraint. The first order conditions are

$$
\begin{aligned}
0 = & \beta E_t \Lambda_{t,t+1} R_{t+1}^K [\theta V_{t+1}^{\prime,j} + (1-\theta)](1 - \overline{\omega}_{t+1}^j) + \lambda_t^j E_t [\beta \Lambda_{t,t+1} R_{t+1}^K \overline{\omega}_{t+1}^j - 1] \\
& + \kappa_t^j \beta E_t \Lambda_{t,t+1} R_{t+1}^K \Big\{ [\theta V_{t+1}^{\prime,j} + (1-\theta)](1 - \overline{\omega}_{t+1}^j) - \int_{\overline{\omega}_{t+1}^j}^{\infty} \Big[[\theta V_{t+1}^{\prime,j} + (1-\theta)](\omega - \overline{\omega}_{t+1}^j) \Big] d\tilde{F}_{t+1}(\omega) \Big\}
\end{aligned}
$$

[31] The derivation is based on Nuño and Thomas (2017).

and

$$0 = -\beta E_t \Lambda_{t,t+1}[\theta V'^j_{t+1} + (1-\theta)] + \lambda^j_t \beta E_t \Lambda_{t,t+1}$$
$$- \kappa^j_t \beta E_t \Lambda_{t,t+1} \Big\{ [\theta V'^j_{t+1} + (1-\theta)] - \int_{\overline{\omega}^j_{t+1}}^{\infty} \Big[\theta V'^j_{t+1} + (1-\theta) \Big] d\tilde{F}_{t+1}(\omega) - \theta \frac{V_{t+1}(0)}{R^K_{t+1} Q_t S^{Bj}_t} \tilde{f}_t(\overline{\omega}^j_{t+1}) \Big\}$$

where I used $\overline{\omega}^j_{t+1} = \overline{b}^j_t / R^K_{t+1}$. The envelope condition is given as:

$$V'^j_t = \lambda^j_t \tag{C.5}$$

The first order conditions can be written as:

$$0 = \beta E_t \Lambda_{t,t+1} R^K_{t+1} [\theta \lambda^j_{t+1} + (1-\theta)](1 - \overline{\omega}^j_{t+1}) + \lambda^j_t E_t[\Lambda_{t,t+1} R^K_{t+1} \overline{\omega}^j_{t+1} - 1]$$
$$+ \kappa^j_t E_t R^K_{t+1} [\theta \lambda^j_{t+1} + (1-\theta)] \Big\{ (1 - \overline{\omega}^j_{t+1}) - \int_{\overline{\omega}^j_{t+1}}^{\infty} \Big[(\omega - \overline{\omega}^j_{t+1}) \Big] d\tilde{F}_{t+1}(\omega) \Big\}$$
$$0 = -\beta E_t \Lambda_{t,t+1}[\theta \lambda^j_{t+1} + (1-\theta)] + \lambda^j_t \beta E_t \Lambda_{t,t+1}$$
$$- \kappa^j_t \beta E_t \Lambda_{t,t+1} \Big\{ [\theta \lambda^j_{t+1} + (1-\theta)] - \int_{\overline{\omega}^j_{t+1}}^{\infty} \Big[\theta \lambda^j_{t+1} + (1-\theta) \Big] d\tilde{F}_{t+1}(\omega) - \theta \frac{V_{t+1}(0)}{R_{t+1^K Q_t S^{Bj}_t}} \tilde{f}_t(\overline{\omega}^j_{t+1}) \Big\}$$

To continue solving the problem, I use a guess and verify approach. I guess that the value function is linear in net worth, so that the value function reads as follows:

$$V_t = \lambda^j_t N^j_t \tag{C.6}$$

Furthermore, I guess the multipliers are equal across intermediaries, that is $\lambda^j_t = \lambda_t$ and $\kappa^j_t = \kappa_t \ \forall j$. Using the guess, the incentive constraint can be written as:

$$\beta E_t \Lambda_{t,t+1} \left\{ \begin{array}{l} \Big[\theta \lambda_{t+1} (1 - \overline{\omega}^j_{t+1}) R^K_{t+1} Q_t S^B_t + (1-\theta)(1 - \overline{\omega}^j_{t+1}) R^K_{t+1} Q_t S^B_t \Big] - \\ \int_{\overline{\omega}^j_{t+1}}^{\infty} \Big[\theta \lambda_{t+1} (\omega_t - \overline{\omega}^j_{t+1}) R^K_{t+1} Q_t S^B_t + (1-\theta)(\omega_t - \overline{\omega}^j_{t+1}) R^K_{t+1} Q_t S^B_t \Big] d\tilde{F}_{t+1}(\omega) \end{array} \right\} \geq 0$$

and reformulated to:

$$\beta E_t \Lambda_{t,t+1} (\theta \lambda_{t+1} + (1-\theta)) \Big\{ (1 - \overline{\omega}^j_{t+1}) - \int_{\overline{\omega}^j_{t+1}}^{\infty} (\omega_t - \overline{\omega}^j_{t+1}) d\tilde{F}_{t+1}(\omega) \Big\} \geq 0 \tag{C.7}$$

The next step is to simplify the first order conditions. I use that if either the incentive constraint binds or if not then $\lambda_t = 0$ (Kuhn Tucker conditions) to simplify the participation constraint and use that the guess for the value function evaluated at 0 so that the first order conditions are given as:

$$0 = E_t \Lambda_{t,t+1} R^K_{t+1} [\theta \lambda_{t+1} + (1-\theta)](1 - \overline{\omega}_{t+1}) + \lambda_t E_t[\Lambda_{t,t+1} R^K_{t+1} \overline{\omega}^j_{t+1} - 1] \tag{C.8}$$

$$0 = -\beta E_t \Lambda_{t,t+1}[\theta \lambda_{t+1} + (1-\theta)] + \lambda_t \beta E_t \Lambda_{t,t+1} - \kappa_t \beta E_t \Lambda_{t,t+1} (\theta \lambda_{t+1} + (1-\theta)) \tilde{F}_{t+1}(\overline{\omega}^j_{t+1}) \tag{C.9}$$

I can now get the following expression for the multipliers:

$$\lambda_t = \frac{\beta E_t \Lambda_{t,t+1} R^K_{t+1}[\theta\lambda_{t+1} + (1-\theta)](1-\overline{\omega}^j_{t+1})}{1 - \beta E_t \Lambda_{t,t+1} R^K_{t+1}\overline{\omega}^j_{t+1}} \tag{C.10}$$

$$\kappa_t = \frac{\beta E_t \Lambda_{t,t+1}(\lambda_t - [\theta\lambda_{t+1} + (1-\theta)])}{\beta E_t \Lambda_{t,t+1}(\theta\lambda_{t+1} + (1-\theta))\tilde{F}_{t+1}(\overline{\omega}^j_{t+1})} \tag{C.11}$$

I now want to show that the multipliers are symmetric across intermediaries. Assuming that equation (C.7), which is the incentive constraint, is binding, I can get $\omega^j_t = \omega_t$. Due to $b^j_t = \overline{\omega}^j_{t+1} R^K_t$, $b^j_t = b_t$ can be obtained. At the same time, I have $\overline{\omega}^j_{t+1} = \overline{\omega}_{t+1}$ and $\overline{b}^j_t = \overline{b}_t$. Then, equation (C.10) implies that $\lambda^j_t = \lambda_t$ and equation (C.11) shows $\kappa^j_t = \kappa_t$. This verifies my guess that the multipliers are equalized. I check numerically that the participation and incentive constraint are binding, that is $\lambda_t > 0$ and $\kappa_t = 0$.

To show that the leverage ratio is symmetric, I use the participation constraint and assume that it is binding:

$$E_t \Lambda_{t,t+1} Q_t S^{Bj}_t \overline{b}^j_t - (Q_t S^{Bj}_t - N^j_t) = 0. \tag{C.12}$$

The leverage ratio is then given as:

$$\phi^j_t = \frac{1}{1 - E_t \Lambda_{t,t+1} R^K_{t+1}\overline{\omega}_{t+1}}. \tag{C.13}$$

As the leverage ratio does not depend on j, this implies that $\phi_t = \phi^j_t$.

The final step is to show that my guess $V_t = \lambda_t N^j_t$ is correct. The starting point is again the value function:

$$V_t(N^j_t) = \beta E_t\big[(\theta\lambda_{t+1} N_{t+1} + (1-\theta)\big(1-\overline{\omega}_{t+1}\big) R^K_{t+1} Q_t S^{Bj}_t)\big],$$

where I used $N^j_{t+1} = (1-\overline{\omega}_{t+1}) R^K_{t+1} Q_t S^{Bj}_t$. I insert the guess to obtain:

$$\lambda_t N^j_t = \phi_t N^j_t \beta E_t \Lambda_{t,t+1}\big[\theta\lambda_{t+1} + (1-\theta)\big]\big(1-\overline{\omega}_{t+1}\big) R^K_{t+1}. \tag{C.14}$$

and reformulate it to

$$\lambda_t = \phi_t E_t \Lambda_{t,t+1}\big[\theta\lambda_{t+1} + (1-\theta)\big]\big(1-\overline{\omega}_{t+1}\big) R^K_{t+1} \tag{C.15}$$

This gives us again a condition for λ_t:

$$\lambda_t = E_t\big[(\theta\lambda_{t+1} N_{t+1} + (1-\theta)\big(1-\overline{\omega}_{t+1}\big) R^K_{t+1} Q_t S^{Bj}_t)\big] \tag{C.16}$$

$$= \phi_t \beta E_t \Lambda_{t,t+1}\big[\theta\lambda_{t+1} + (1-\theta)\big]\big(1-\overline{\omega}_{t+1}\big) R^K_{t+1}. \tag{C.17}$$

Inserting (C.13), the condition for λ_t becomes:

$$\lambda_t = \frac{\beta E_t \Lambda_{t,t+1}\big[\theta\lambda_{t+1} + (1-\theta)\big]\big(1-\overline{\omega}_{t+1}\big) R^K_{t+1}}{1 - \beta E_t \Lambda_{t,t+1} R^K_{t+1}\overline{\omega}_{t+1}}. \tag{C.18}$$

This coincides with the equation (C.10). This verifies the guess.

C.2 With Runs on the Financial Sector

In this section, the possibility of runs is included. The financial intermediary maximizes V_t subject to a participation and incentive constraint, which reads as follows:

$$V_t^j(N_t^j) = \max_{S_t^{Bj},\bar{D}_t} (1-p_t^j)\beta E_t^N \Lambda_{t,t+1}\left[\theta V_{t+1}^j\left(N_{t+1}^j\right) + (1-\theta)(R_{t+1}^K Q_t S_t^{Bj} - \bar{D}_t^j)\right] \quad \text{(C.19)}$$

$$\text{s.t.} \quad (1-p_t^j)\beta E_t^N[\Lambda_{t,t+1} Q_t S_t^{Bj}\bar{b}_t^j] + p_t^j \beta E_t^R[R_{t+1}^K Q_t S_t^{Bj}] \geq (Q_t S_t^{Bj} - N_t^j) \quad \text{(C.20)}$$

$$(1-p_t^j)E_t^N\left[\Lambda_{t,t+1}\theta V_{t+1}\left(N_{t+1}^j\right) + (1-\theta)\Big(1-\frac{\bar{b}_t^j}{R_{t+1}^K}\Big)R_{t+1}^K Q_t S_t^{Bj}\right] \geq \quad \text{(C.21)}$$

$$\beta\Lambda_{t,t+1}E_t\left[\Lambda_{t,t+1}\int_{\frac{\bar{b}_t^j}{R_{t+1}^K}}^{\infty}\theta V_{t+1}\left(N_{t+1}^j\right) + (1-\theta)\Big(\omega - \frac{\bar{b}_t^j}{R_{t+1}^K}\Big)R_{t+1}^K Q_t S_t^{Bj} d\tilde{F}_{t+1}(\omega)\right]$$

The financial intermediary's specific can be written as Bellman equation:

$$\begin{aligned}
V_t(N_t^j) = &\max_{\{\phi_t^j,\bar{b}_t^j\}} (1-p_t^j)\beta E_t^N\Lambda_{t,t+1}\left[\theta V_{t+1}\left(\Big(1-\frac{\bar{b}_t^j}{R_{t+1}^K}\Big)R_{t+1}^K\phi_t^j N_t^j\right) + (1-\theta)\big(1-\frac{\bar{b}_t^j}{R_{t+1}^K}\big)R_{t+1}^K\phi_t^j N_t^j\right] \\
&+ \lambda_t^j\left[(1-p_t^j)\beta E_t^N[\Lambda_{t,t+1}\phi_t^j N_t^j \bar{b}_t^j] + p_t^j\beta E_t^R[R_{t+1}^K\phi_t^j N_t^j] - (\phi_t^j N_t^j - N_t^j)\right] \\
&+ \kappa_t^j\beta\Bigg\{\left[(1-p_t^j)E_t^N\Lambda_{t,t+1}\left[\Lambda_{t,t+1}\theta V_{t+1}\left(\Big(1-\frac{\bar{b}_t^j}{R_{t+1}^K}\Big)R_{t+1}^K\phi_t^j N_t^j\right) + (1-\theta)\Big(1-\frac{\bar{b}_t^j}{R_{t+1}^K}\Big)R_{t+1}^K\phi_t^j N_t^j\right]\right] \\
&- \beta E_t\left[\Lambda_{t,t+1}\int_{\frac{\bar{b}_t^j}{R_{t+1}^K}}^{\infty}\theta V_{t+1}\left(\Big(1-\frac{\bar{b}_t^j}{R_{t+1}^K}\Big)R_{t+1}^K\phi_t^j N_t^j\right) + (1-\theta)\Big(\omega-\frac{\bar{b}_t^j}{R_{t+1}^K}\Big)R_{t+1}^K\phi_t^j N_t^j d\tilde{F}_{t+1}(\omega)\right]\Bigg\}
\end{aligned}$$

The first order conditions with respect to ϕ_t^j can be written as

$$\begin{aligned}
0 =&(1-p_t^j)E_t^N\Lambda_{t,t+1}R_{t+1}^K[\theta V_{t+1}'^j + (1-\theta)](1-\varpi_{t+1}^j) \\
&+ \lambda_t^j((1-p_t^j)E_t^N[\Lambda_{t,t+1}R_{t+1}^K\varpi_{t+1}^j] + p_t E_t^R[\Lambda_{t,t+1}R_{t+1}^K] - 1) \\
&+ \kappa_t^j((1-p_t^j)\beta E_t^N\Lambda_{t,t+1}R_{t+1}^K[\theta V_{t+1}'^j + (1-\theta)](1-\varpi_{t+1}^j) \\
&- \kappa_t^j\beta E_t\Lambda_{t,t+1}\int_{\varpi_{t+1}^j}^{\infty}\left[R_{t+1}^K[\theta V_{t+1}'^j + (1-\theta)](\omega-\varpi_{t+1}^j)\right]d\tilde{F}_{t+1}(\omega) \\
&- \frac{\partial p_t^j}{\phi_t^j}E_t^N\Lambda_{t,t+1}R_{t+1}^K[\theta V_{t+1}'^j + (1-\theta)](1-\varpi_{t+1}^j)\left(1+\kappa_t^j\right) && \text{(C.22)} \\
&- \frac{\partial p_t^j}{\phi_t^j}E_t^N\left(R_{t+1}^K\varpi_{t+1}^j - R_{t+1}^K\right) && \text{(C.23)}
\end{aligned}$$

where I applied $\varpi_{t+1}^j = \bar{b}_t^j/R_{t+1}^K$. Gertler, Kiyotaki and Prestipino (2020*b*) show that the even though the optimization of leverage ϕ^j affect the default probability p_t, this indirect effect on on the firm value V_t and the promised return R_t^D is zero. The reason is that at the cutoff value of default, net worth is zero, which implies $V_{t+1} = 0$. Similarly, the promised return is unchanged. The cutoff values of default is defined as:

$$\xi^D_{t+1}(\phi^j_t) = \left\{ (\sigma_{t+1}, A_{t+1}, \iota_{t+1}) : R^K_{t+1}\frac{\phi^j_t - 1}{\phi^j_t}\overline{R}^D_t \right\}. \tag{C.24}$$

At the cutoff points, the intermediary can exactly cover the face value of the deposits, which implies

$$\overline{\omega}^j_t = 1. \tag{C.25}$$

Based on the derivation in Gertler, Kiyotaki and Prestipino (2020*b*), the property $\overline{\omega}^j_t = 1$ implies that

$$-\frac{\partial p_t}{S^{Bj}_t}E^N_t\Lambda_{t,t+1}R^K_{t+1}[\theta V'^j_{t+1} + (1-\theta)](1-\overline{\omega}^j_{t+1})\left(1+\kappa^j_t\right) = 0, \tag{C.26}$$

$$-\frac{\partial p_t}{S^{Bj}_t}E^N_t\left(R^K_{t+1}\overline{\omega}^j_{t+1} - R^K_{t+1}\right) = 0, \tag{C.27}$$

The first order condition with respect to ϕ^B_t becomes then

$$\begin{aligned}
0 =&(1-p^j_t)E^N_t\Lambda_{t,t+1}R^K_{t+1}[\theta V'^j_{t+1} + (1-\theta)](1-\overline{\omega}^j_{t+1}) \\
&+ \lambda^j_t((1-p^j_t)E^N_t[\Lambda_{t,t+1}R^K_{t+1}\overline{\omega}^j_{t+1}] + p_tE^R_t[\Lambda_{t,t+1}R^K_{t+1}] - 1) \\
&+ \kappa^j_t((1-p^j_t)\beta E^N_t\Lambda_{t,t+1}R^K_{t+1}[\theta V'^j_{t+1} + (1-\theta)](1-\overline{\omega}^j_{t+1}) \\
&- \kappa^j_t\beta E_t\Lambda_{t,t+1}\int_{\overline{\omega}^j_{t+1}}^{\infty}\left[R^K_{t+1}[\theta V'^j_{t+1} + (1-\theta)](\omega - \overline{\omega}^j_{t+1})\right]d\tilde{F}_{t+1}(\omega)
\end{aligned}$$

The first order condition with respect to $\overline{b}^j_t$ is given as

$$\begin{aligned}
0 =& -\beta(1-p^j_t)E^N_t\Lambda_{t,t+1}[\theta V'^j_{t+1} + (1-\theta)] \\
&+ \lambda^j_t\beta(1-p^j_t)E^N_t\Lambda_{t,t+1} \\
&- \kappa^j_t\beta(1-p^j_t)E^N_t\Lambda_{t,t+1}\left\{[\theta V'^j_{t+1} + (1-\theta)]\right\} \\
&+ \kappa^j_t\beta(1-p^j_t)E_t\Lambda_{t,t+1}\int_{\overline{\omega}^j_{t+1}}^{\infty}\left[\theta V'^j_{t+1} + (1-\theta)\right]d\tilde{F}_{t+1}(\omega) - \theta\frac{V_{t+1}(0)}{R^K_{t+1}Q_tS^{Bj}_t}\tilde{f}_t(\overline{\omega}^j_{t+1})
\end{aligned} \tag{C.28}$$

where I applied $\overline{\omega}^j_{t+1} = \overline{b}^j_t/R^K_{t+1}$

Similar to before, I use the following guess for the value function

$$V_t = \lambda^j_tN^j_t \tag{C.29}$$

and also the fact that the multipliers are equal across intermediaries, that is $\lambda^j_t = \lambda_t$ and $\kappa^j_t = \kappa_t \forall j$. In addition, I also guess now that the probability of a run does not depend on individual characteristics, that is $p^j_t = p_t$.

The incentive constraint can then then be written as

$$\beta(1-p_t^j)E_t^N\left[\Lambda_{t,t+1}(\theta\lambda_{t+1}+(1-\theta))(1-\overline{\omega}_{t+1}^j)R_{t+1}^K\right] \geq \tag{C.30}$$
$$\beta E_t\left[\Lambda_{t,t+1}\int_{\frac{\bar{b}_t^j}{R_{t+1}^K}}^{\infty}(\theta\lambda_{t+1}+(1-\theta))\Big(\omega-\overline{\omega}_{t+1}^j\Big)R_{t+1}^K d\tilde{F}_{t+1}(\omega)\right]$$

The two first order conditions can then be adjusted similar to section C.1 and be written as

$$\begin{aligned}
0 =&(1-p_t)E_t^N\Lambda_{t,t+1}R_{t+1}^K[\theta\lambda_{t+1}+(1-\theta)](1-\overline{\omega}_{t+1}^j)+\\
&\lambda_t((1-p_t)E_t^N[\Lambda_{t,t+1}R_{t+1}^K\overline{\omega}_{t+1}]+p_tE_t^R[\Lambda_{t,t+1}R_{t+1}^K]-1) \qquad (C.31)\\
0 =&-\beta(1-p_t)E_t^N\Lambda_{t,t+1}[\theta\lambda_{t+1}+(1-\theta)]+\lambda_t\beta(1-p_t)E_t^N\Lambda_{t,t+1}\\
&-\kappa_t\beta\Big\{(1-p_t)E_t^N\Lambda_{t,t+1}\left[(\theta\lambda_{t+1}+1-\theta)\,\tilde{F}_{t+1}(\overline{\omega}_{t+1}^j)\right]\\
&\qquad+p_tE_t^R\Lambda_{t,t+1}\left[(\theta\lambda_{t+1}+1-\theta)\left(1-\tilde{F}_{t+1}(\overline{\omega}_{t+1}^j)\right)\right]\Big\} \qquad (C.32)
\end{aligned}$$

Using the same strategy as in C.1, the guess about the equalized multipliers can be verified. Similarly, it can be shown that leverage is the same across intermediaries. This then verifies that the guess of the run probability $p_t^j = p_t$ is verified as the cutoff value is the same across intermediaries as shown in equation (C.24). I additionally assume that in case of a run on the entire financial sector, a intermediary that survives shuts down and returns their net worth. This implies that $E_t^R\lambda_{t+1} = 1$. The participation constraint is given as:

$$(1-p_t)E_t^N[\beta\Lambda_{t,t+1}\bar{R}_tD_t]+p_tE_t^R[\beta\Lambda_{t,t+1}R_{t+1}^KQ_tS_t^B]=D_t. \tag{C.33}$$

The incentive constraint is given as:

$$(1-p_t)E_t^N[\Lambda_{t,t+1}R_{t+1}^K(\theta\lambda_{t+1}+(1-\theta))[1-e^{\frac{-\psi}{2}}-\tilde{\pi}_{t+1}]] = \tag{C.34}$$
$$p_tE_t^R[\Lambda_{t,t+1}R_{t+1}^K(e^{-\frac{\psi}{2}}-\overline{\omega}_{t+1}+\tilde{\pi}_{t+1})],$$

λ_t and κ_t are derived from the first order conditions in equations (C.31) and (C.32) are given as:

$$\lambda_t = \frac{(1-p_t)E_t^N\Lambda_{t,t+1}R_{t+1}^K[\theta\lambda_{t+1}+(1-\theta)](1-\overline{\omega}_{t+1})}{1-(1-p_t)E_t^N[\Lambda_{t,t+1}R_{t+1}^K\overline{\omega}_{t+1}]-p_tE_t^R[\Lambda_{t,t+1}R_{t+1}^K]} \tag{C.35}$$

$$\kappa_t = \frac{\beta(1-p_t)E_t^N\Lambda_{t,t+1}\left[\lambda_t-(\theta\lambda_{t+1}+1-\theta)\right]}{(1-p_t)E_t^N\Lambda_{t,t+1}\left[(\theta\lambda_{t+1}+1-\theta)\,\tilde{F}_{t+1}(\overline{\omega}_{t+1})\right]+p_tE_t^R\Lambda_{t,t+1}\left[(\theta\lambda_{t+1}+1-\theta)\left(1-\tilde{F}_{t+1}(\overline{\omega}_{t+1})\right)\right]} \tag{C.36}$$

If $\lambda_t > 0$ and $\kappa_t > 0$, the participation and incentive constraint are binding.

D Global Solution Method

The Algorithm uses time iteration with piecewise linear policy functions based on Richter, Throckmorton and Walker (2014). The approach is adjusted to take into account the multiplicity of equilibria due to possibility of a run also that the probability of the run equilibrium is time-varying. The state variables are $\{S_{t-1}, N_t, \sigma_t, A_t, \iota_t\}$, where I used N_t as state variable instead of $\overline{D}_{t-1}$ for computational reasons. The policy variables are $Q_t, C_t, \bar{b}_t, \Pi_t, \lambda_t$. I solve for the following policy functions $\boldsymbol{Q(X), C(X), \bar{b}(X), \Pi(X), \lambda(X)}$, the law of motion of net worth $\boldsymbol{N'(X, \varepsilon_{t+1})}$ and the probability of a run next period $\boldsymbol{P(X)}$. The expectations are evaluated using Gauss-Hermite quadrature, where the matrix of nodes is denoted as $\boldsymbol{\varepsilon}$. The Algorithm is summarized below:

1. Define a state grid $\boldsymbol{X} \in [\underline{S}_{t-1}, \overline{S}_{t-1}] \times [\underline{N}_t, \overline{N}_t] \times [\underline{\sigma}_t, \overline{\sigma}_t] \times [\underline{A}_t, \overline{A}_t]$ and integration nodes $\boldsymbol{\epsilon} \in [\underline{\epsilon^{\sigma}}_{t+1}, \overline{\epsilon^{\sigma}}_{t+1}] \times [\underline{\epsilon^{A}}_{t+1}, \overline{\epsilon^{A}}_{t+1}]$ to evaluate expectations based on Gauss-Hermite quadrature

2. Guess the piecewise linear policy functions to initialize the algorithm[32]

   (a) the "classical" policy functions $\boldsymbol{Q(X), C(X), \bar{b}(X), \Pi(X), \lambda(X)}$

   (b) a function $\boldsymbol{N'(X, \varepsilon_{t+1})}$ at each point from the nodes of next period shocks based on Gauss-Hermite quadrature

   (c) the probability $\boldsymbol{P(X)}$ that a run occurs next period

3. Solve for all time t variables for a given state vector. Take from the previous iteration j the law of motion $\boldsymbol{N'_j}(\boldsymbol{X}, \boldsymbol{\varepsilon}_{t+1})$ and the probability of a run as given $\boldsymbol{P_j(X)}$ and calculate time $t{+}1$ variables using the guess j policy functions with $\boldsymbol{X'}$ as state variables. The expectations are calculated using numerical integration based on Gauss-Hermite quadrature. A numerical root finder with the time t policy functions as input minimizes the error in the following five equations:

$$\begin{aligned}
\text{err}_1 &= (\Pi_t - \Pi_{SS})\Pi_t \\
&\qquad - \left(\frac{\epsilon}{\rho^r}\left(\varphi_t^{mc} - \frac{\epsilon-1}{\epsilon}\right) + \Lambda_{t,t+1}(\Pi_{t+1} - \Pi_{SS})\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right), && \text{(D.1)} \\
\text{err}_2 &= 1 - \beta\Lambda_{t,t+1}\frac{i_t}{\Pi_{t+1}}1, && \text{(D.2)} \\
\text{err}_3 &= (1-p_t)E_t^N\left[\beta\Lambda_{t,t+1}\bar{R}_t D_t\right] + p_t E_t^R\left[\beta\Lambda_{t,t+1}R_{t+1}^K Q_t S_t^B\right] - D_t, && \text{(D.3)} \\
\text{err}_4 &= (1-p_t)E_t^N\left[\Lambda_{t,t+1}R_{t+1}^K(\theta\lambda_{t+1} + (1-\theta))(1 - e^{\frac{-\psi}{2}}\tilde{\pi}_{t+1})\right] && \text{(D.4)} \\
&\qquad - p_t E_t^R\left[\Lambda_{t,t+1}R_{t+1}^K(e^{-\frac{\psi}{2}} - \overline{\omega}_{t+1} + \tilde{\pi}_{t+1})\right], \\
\text{err}_5 &= \lambda_t - \frac{(1-p_t)E_t^N\Lambda_{t,t+1}R_{t+1}^K[\theta\lambda_{t+1} + (1-\theta)](1-\overline{\omega}_{t+1})}{1 - (1-p_t)E_t^N[\Lambda_{t,t+1}R_{t+1}^K\overline{\omega}_{t+1}] - p_t E_t^R[\Lambda_{t,t+1}R_{t+1}^K]}. && \text{(D.5)}
\end{aligned}$$

[32]In practice, it can be helpful to solve first for the economy with only one shock, for instance the volatility shock, and solve this model in isolation. The resulting policy functions can then be used as starting point for the full model with two shocks.

4. Take the iteration j policy functions , $\boldsymbol{N'_j}(\boldsymbol{X}, \varepsilon_{t+1})$ and $\boldsymbol{P_j(X)}$ as given and solve the whole system of time t and $(t+1)$ variables. Calculate then N_{t+1} using the "law of motion" for net worth

$$N_{t+1} = \max\left[R^K_{t+1} Q_t S^B_t - \overline{R}_t D_t, 0\right] + (1-\theta)\zeta S_t. \tag{D.6}$$

A run occurs at a specific point if

$$R^K_{t+1} Q_t S^B_t - \overline{R}_t D_t \leq 0. \tag{D.7}$$

In such a future state, the weight of a run is 1. In the other state, the weight of a run 0.[33] This can be now used to evaluate the probability of a run next period based on Gauss-Hermite quadrature so that I know p_t.

5. Update the policy policy functions slowly $\boldsymbol{Q}(\boldsymbol{X}), \boldsymbol{C}(\boldsymbol{X}), \boldsymbol{\psi}(\boldsymbol{X}), \boldsymbol{\pi}(\boldsymbol{X})$. For instance for consumption policy function, this could be written as:

$$\boldsymbol{C_{j+1}}(\boldsymbol{X}) = \alpha^{U1} \boldsymbol{C_j}(\boldsymbol{X}) + (1-\alpha^{U1}) \boldsymbol{C_{sol}}(\boldsymbol{X}), \tag{D.8}$$

where the subscript *sol* denotes the solution for this iteration and α^{U1} determines the weight of the previous iteration. Furthermore, $\boldsymbol{N'}(\boldsymbol{X}, \varepsilon_{t+1})$ and $\boldsymbol{P(X)}$ are updated using the results from step 4:

$$\boldsymbol{N'_{j+1}}(\boldsymbol{X}, \varepsilon_{t+1}) = \alpha^{U2} \boldsymbol{N'_j}(\boldsymbol{X}, \varepsilon_{t+1}) + (1-\alpha^{U2}) \boldsymbol{N'_{sol}}(\boldsymbol{X}, \varepsilon_{t+1}), \tag{D.9}$$

$$\boldsymbol{P_{j+1}}(\boldsymbol{X}) = \alpha^{U3} \boldsymbol{P_j}(\boldsymbol{X}) + (1-\alpha^{U3}) \boldsymbol{P_{sol}}(\boldsymbol{X}). \tag{D.10}$$

6. Repeat steps 3,4 and 5 until the errors of all functions, which are the classical policy functions $\boldsymbol{Q(X), C(X), \overline{b}(X), \Pi(X), \lambda(X)}$ together with the law of motion of net worth $\boldsymbol{N'(X, \varepsilon_{t+1})}$ and the probability of a run $\boldsymbol{P}(\boldsymbol{X})$, at each point of the discretized state are sufficiently small.

[33] This procedure would imply a zero and one indicator, which is very unsmooth. For this reason, I use the following functional forms based on exponential function: $\frac{\exp(\zeta_1(1-D_{t+1}))}{1+\exp(\zeta_1 * (1-D_{t+1}))}$ where $D_{t+1} = \frac{R^k_{t+1}}{R^D_t} \frac{\phi}{\phi-1}$ at each calculated N_{t+1}. ζ_1 is set to 2500. This large value of ζ ensures sufficient steepness so that the approximation is close to an indicator function of 0 and 1.

E Particle Filter

I use a particle filter with sequential importance resampling based on Atkinson, Richter and Throckmorton (2019) and Herbst and Schorfheide (2015). The algorithm is adapted to incorporate sunspot shocks and endogenous equilibria similar to Aruoba, Cuba-Borda and Schorfheide (2018), who have a model with sunspot shocks that directly determine the equilibria. I extend this approach to include the circumstance that the probability of equilibria is endogenously time-varying. The total number of particles M is set to 10000 as in Aruoba, Cuba-Borda and Schorfheide (2018).

1. **Initialization** Use the risky steady state of the model as a starting point and draw $\{v_{t,m}\}_{t=-24}^{0}$ for all particles $m \in \{0, ..., M\}$. I set $\{\iota_{t,m} = 0\}_{t=-24}^{0}$, which excludes a run in the initialization. The simulation of these shocks provides the start values for the state variables $\mathbb{X}_{0,m}$.

2. **Recursion** Filter the nonlinear model for periods $t = 1, ..., T$

   (a) Draw the sunspot shock $\iota_{t,m}$ and the structural shocks $v_{t,m}$ for each particle $m = \{1, ..., M\}$. The sunspot shock is drawn from a binomial distribution with realizations $0, 1$:

   $$\iota_{t,m} \sim \mathcal{B}\,(1, \Upsilon)\,, \tag{E.1}$$

   where 1 indicates the number of trails and Υ is the probability of $\iota = 1$.[34] The structural shocks are drawn from a proposal distribution that distinguishes between the realizations of the sunspot shock :

   $$v_{t,m} \sim N(\overline{v_t}^{\iota=0}, I) \qquad \text{if} \quad \iota_{t,m} = 0, \tag{E.2}$$

   $$v_{t,m} \sim N(\overline{v_t}^{\iota=1}, I) \qquad \text{if} \quad \iota_{t,m} = 1. \tag{E.3}$$

   As the regime selection is endogenous in the model, the proposal distribution can be the same for the two realizations of the sunspot shock. This is the case if the model does not suggest the realization of a run. The difference in using the proposal distribution is that instead of drawing directly from a distribution, I draw from an adapted distribution. I derive the proposal distribution by maximizing the fit of the shock for the average state vector $\overline{\mathbb{X}}_{t-1} = \frac{1}{M}\sum_{m=1}^{M} \mathbb{X}_{t-1,m}$

   i. Calculate a state vector $\overline{\mathbb{X}}_t$ from $\overline{\mathbb{X}}_{t-1}$ and a guess of $\overline{v}_t$ for the possible realizations of the sunspot shock:

   $$\mathbb{X}_t^{\iota=0} = f(\overline{\mathbb{X}}_{t-1}, \overline{v}_t^{\iota=0}, \iota_t = 0) \tag{E.4}$$

   $$\mathbb{X}_t^{\iota=1} = f(\overline{\mathbb{X}}_{t-1}, \overline{v}_t^{\iota=1}, \iota_t = 1) \tag{E.5}$$

[34] In practice, I draw from a uniform distribution bounded between 0 and 1 and categorize the sunspot accordingly.

ii. Calculate the measurement error from the observation equation for the two cases

$$u_t^{\iota=0} = \mathbb{Y}_t - g(\mathbb{X}_t^{\iota=0}), \tag{E.6}$$
$$u_t^{\iota=0} = \mathbb{Y}_t - g(\mathbb{X}_t^{\iota=1}). \tag{E.7}$$

The measurement error follows a multivariate normal distribution, so that the probabilities of observing the measurement error for the different sunspot shocks are given by

$$p(u_t^{\iota=0}|\mathbb{X}_t^{\iota=0}) = (2\pi)^{-n/2}|\Sigma_u|^{-0.5}\exp\left(-0.5(u_t^{\iota=0})'\Sigma_u^{-1}(u_t^{\iota=0})\right), \tag{E.8}$$
$$p(u_t^{\iota=1}|\mathbb{X}_t^{\iota=1}) = (2\pi)^{-n/2}|\Sigma_u|^{-0.5}\exp\left(-0.5(u_t^{\iota=1})'\Sigma_u^{-1}(u_t^{\iota=1})\right), \tag{E.9}$$

where Σ_u is the variance of the measurement error and n is the number of observables, which is 2 in this setup.

iii. Calculate the probability of observing $\mathbb{X}_t^{\iota=0}$ respectively $\mathbb{X}_t^{\iota=1}$ conditional on the average state vector from the previous period

$$p(\mathbb{X}_t^{\iota=0}|\overline{\mathbb{X}}_{t-1}) = (2\pi)^{-n/2}\exp\left(-0.5(\overline{v_t}^{\iota=0})'(\overline{v_t}^{\iota=0})\right), \tag{E.10}$$
$$p(\mathbb{X}_t^{\iota=1}|\overline{\mathbb{X}}_{t-1}) = (2\pi)^{-n/2}\exp\left(-0.5(\overline{v_t}^{\iota=1})'(\overline{v_t}^{\iota=1})\right). \tag{E.11}$$

iv. To find the proposal distribution, maximize the following objects with respect $\overline{v_t}^{\iota=0}$ respectively $\overline{v_t}^{\iota=1}$:

$$p(\mathbb{X}_t^{\iota=0}|\overline{\mathbb{X}}_{t-1})p(u_t^{\iota=0}|\mathbb{X}_t^{\iota=0}), \tag{E.12}$$
$$p(\mathbb{X}_t^{\iota=1}|\overline{\mathbb{X}}_{t-1})p(u_t^{\iota=1}|\mathbb{X}_t^{\iota=1}). \tag{E.13}$$

This provides the proposal distributions $N(\overline{v_t}^{\iota=0}, I)$ and $N(\overline{v_t}^{\iota=1}, I)$.

(b) Propagate the state variables $\mathbb{X}_{t,m}$ by iterating the state-transition equation forward given $\mathbb{X}_{t-1,m}$, $v_{t,m}$ and $\iota_{t,m}$:

$$\mathbb{X}_{t,m} = f(\mathbb{X}_{t-1,m}, v_{t,m}, \iota_t). \tag{E.14}$$

(c) Calculate the measurement error

$$u_{tm} = \mathbb{Y}_t - g(\mathbb{X}_{t,m}). \tag{E.15}$$

The incremental weights of the particle m can be written as

$$w_{t,m} = \frac{p(u_{t,m}|\mathbb{X}_{t,m})p(\mathbb{X}_{t,m}|\mathbb{X}_{t-1,m})}{f(\mathbb{X}_{t,m}|\mathbb{X}_{t-1,m}, \mathbb{Y}_t, \iota_{t,m})} \tag{E.16}$$

$$= \begin{cases} \frac{(2\pi)^{-n/2}|\Sigma_u|^{-0.5}\exp\left(-0.5u_{t,m}'\Sigma_u^{-1}u_{t,m}\right)\exp\left(-0.5v_{t,m}'v_{t,m}\right)}{\exp\left(-0.5(v_{t,m}-\overline{v}_t^{\iota=0})'(v_{t,m}-\overline{v}_t^{\iota=0})\right)} & \text{if } \iota_{t,m} = 0 \\ \frac{(2\pi)^{-n/2}|\Sigma_u|^{-0.5}\exp\left(-0.5u_{t,m}'\Sigma_u^{-1}u_{t,m}\right)\exp\left(-0.5v_{t,m}'v_{t,m}\right)}{\exp\left(-0.5(v_{t,m}-\overline{v}_t^{\iota=1})'(v_{t,m}-\overline{v}_t^{\iota=1})\right)} & \text{if } \iota_{t,m} = 1 \end{cases} \tag{E.17}$$

where the density $f(\cdot)$ depends on the realization of the sunspot shock. The incremental weights determine the log-likelihood contribution in period t:

$$\ln(l_t) = \ln\left(\frac{1}{M}\sum_{m=1}^{M} w_{t,m}\right). \tag{E.18}$$

(d) Resample the particles based on the weights of the particles. First, the normalized weights $W_{t,m}$ are given by:

$$W_{t,m} = \frac{w_{t,m}}{\sum_{m=1}^{M} w_{t,m}}. \tag{E.19}$$

Second, the deterministic algorithm of Kitagawa (1996) resamples the particles by drawing from the current set of particles adjusted for their relative weights. This gives a resampled distribution of state variables $\mathbb{X}_{t,m}$.

3. **Likelihood Approximation** Determine the approximated log-likelihood function of the model as

$$\ln(\mathcal{L}_t) = \sum_{t=1}^{T} \ln(l_t). \tag{E.20}$$

F Further results on the Estimation of Financial Fragility

This section contains shows additional results for Section 4, in which a quantitative analysis is conducted and the build-up of financial fragility is estimated.

Estimated Time Series Figure F.1 shows further results based on the particle filter. The filtered median with its 68% confidence interval is shown for selected variables. It shows the weight of a run over time, the shock processes (volatility and TFP), and important macroeconomic and financial variables. The implications are in line with the data. The exercise predicts a credit boom, which ends in a severe credit crunch, countercyclical finance premium and a period of low inflation after the run.

Forecast of joint distribution over time Figure F.2 shows a contour plot of the one-quarter-ahead joint distribution of output and leverage over time. It expands the analysis of the forecast of 2008:Q4 conditional on 2008:Q3, which can be seen in Figure 9.

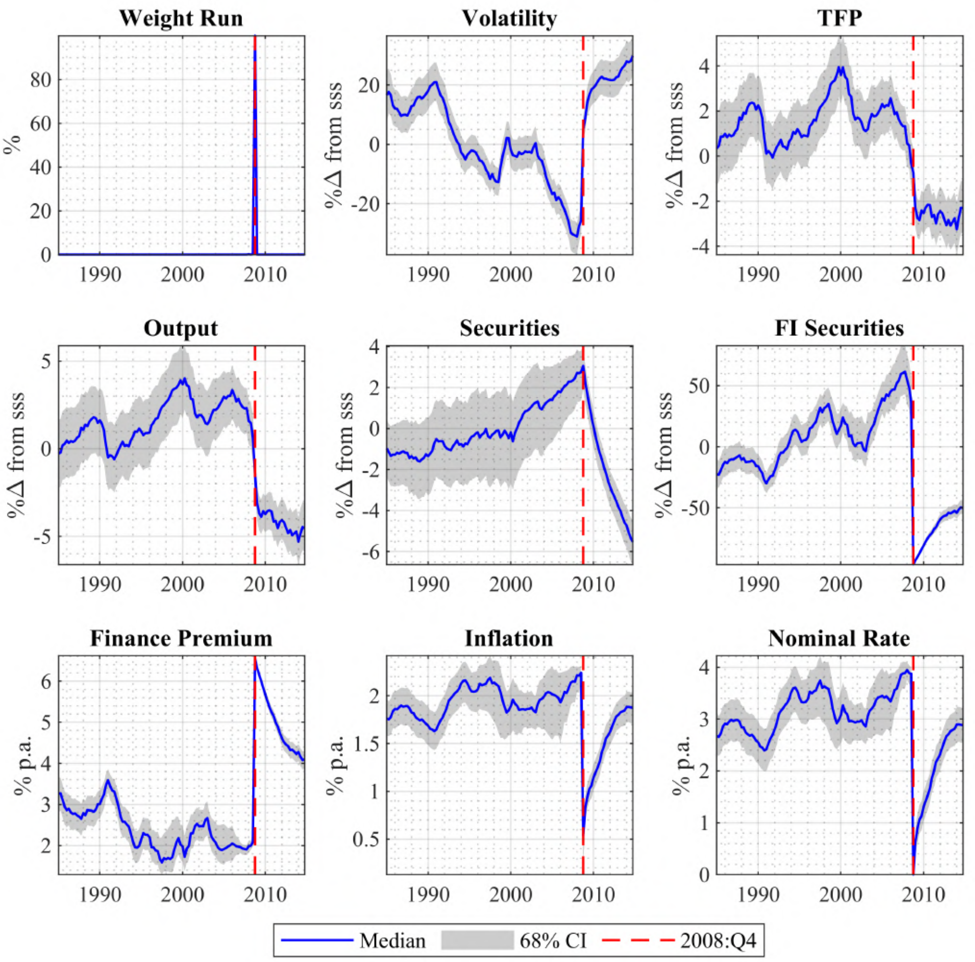

Figure F.1: Filtered median with its 68% confidence interval for important variables. The first plot shows the regime selection. The second and third plot show the exogenous drivers volatility and TFP. The remaining plots show other key variables. The red line indicates the fourth quarter of 2008.

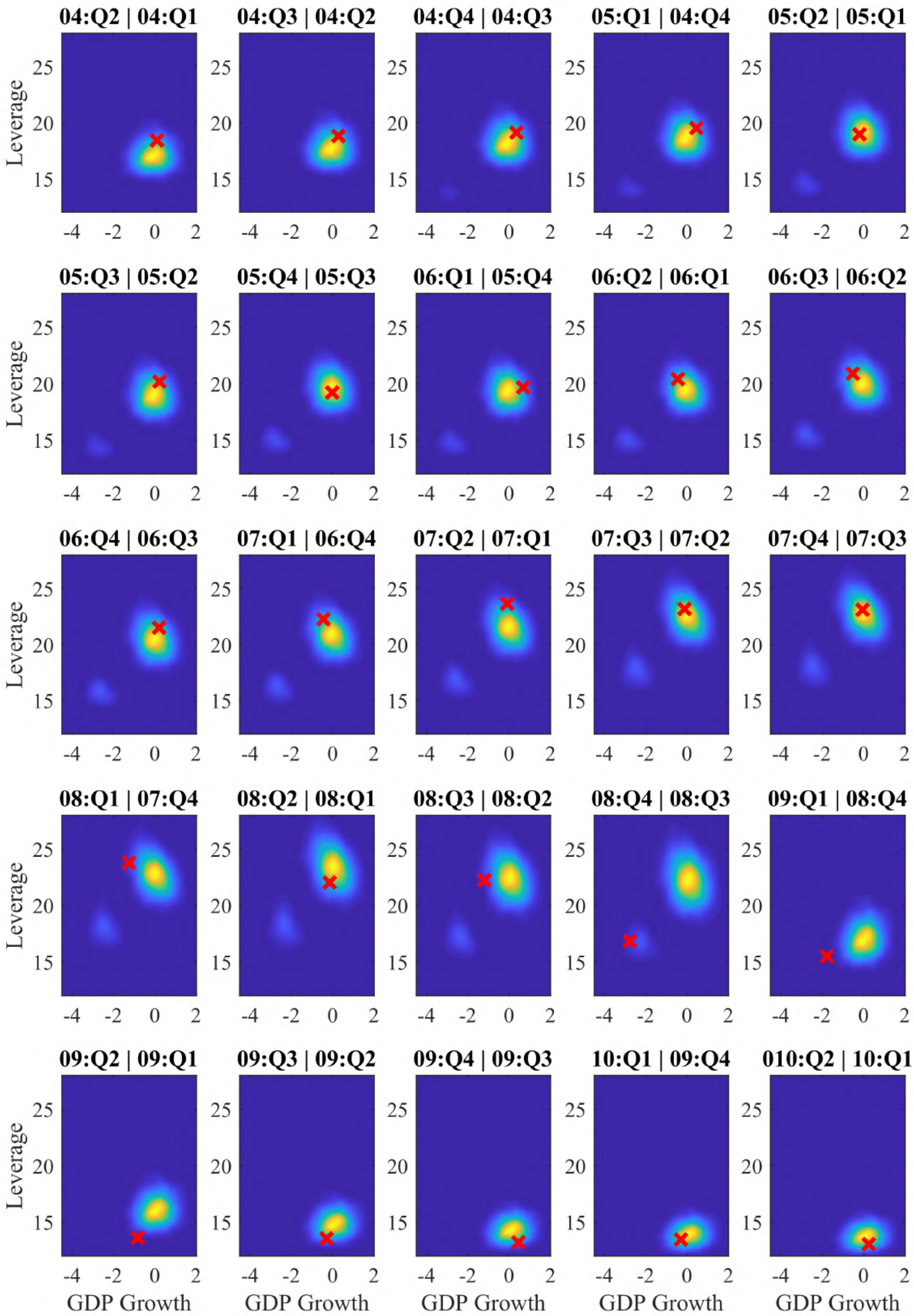

Figure F.2: The contour plots displays the one-quarter-ahead joint distribution of GDP growth and leverage over time. Leverage is on the horizontal axis, while GDP growth is on the vertical axis. Yellow indicates a high density, while dark blue indicates a low density The red square shows the actual data realization in the forecasted period. The forecasts are conditioned on the median realization.