Financial Crises and Shadow Banks: A Quantitative Analysis

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Abstract

Motivated by the build-up of shadow bank leverage prior to the Great Recession, I develop a nonlinear macroeconomic model that features excessive leverage accumulation and show how this can cause a bank run. Introducing risk-shifting incentives to account for fluctuations in shadow bank leverage, I use the model to illustrate that extensive leverage makes the shadow banking system runnable, thereby raising the vulnerability of the economy to future financial crises. The model is taken to U.S. data with the objective of estimating the probability of a run in the years preceding the financial crisis of 2007-2008. According to the model, the estimated risk of a bank run was already considerable in 2004 and kept increasing due to the upsurge in leverage. I show that levying a leverage tax on shadow banks would have substantially lowered the probability of a bank run. Finally, I present reduced-form evidence that supports the tight link between leverage and the possibility of financial crises.

Keywords: Financial crises, Shadow banks, Leverage, Credit booms, Bank runs

JEL classification: E32, E44, G23

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1 Introduction

The financial crisis of 2007-2008 was, at the time, the most severe economic downturn in the US since the Great Depression. Although the origins of the financial crisis are complex and various, the financial distress in the shadow banking sector has been shown to be one of the key factors.[1] The shadow banking sector, which consists of financial intermediaries operating outside normal banking regulation, expanded considerably before the crisis. Crucially, there was an excessive build-up of leverage (asset to equity ratio) for these unregulated banks. Lehman Brothers, a major investment bank at the time, elevated its leverage by around 30% in 2007 compared to just three years earlier for instance.[2] The subsequent collapse of Lehman Brothers in September 2008 intensified a run on the short-term funding of many financial intermediaries with severe repercussions for the real economy in the fourth quarter of 2008. Figure 1 documents these stylized facts about GDP growth and shadow bank leverage using balance sheet data from Compustat and the Flow of Funds.[3]

In this paper, I build a new nonlinear quantitative macroeconomic model with shadow banks and occasional bank runs that captures these dynamics. I use the model to point out how risk-shifting incentives for shadow banks are a key factor for financial fragility to emerge. Importantly, the occurrence of a run on the shadow banking system depends on economic fundamentals and is in particular linked to leverage. Risk-shifting incentives for shadow banks resulting from limited liability may endogenously lead to excessive leverage, which then makes the shadow banking system runnable. To assess the empirical implications of the model, the framework is fitted to the salient features of the U.S. economy and especially the shadow banking sector. With the objective of estimating the probability of a run around the time of the 2007-2008 financial crisis, I use a particle filter to extract the sequence of shocks that account for output growth and shadow bank leverage. The estimated probability of a bank run increases considerably from 2004 onwards and peaks in 2008. I discuss the idea of a leverage-tax, which is a tax on the deposits of shadow banks, and quantify its impact on the probability of a bank run. Using quantile regressions, I also provide reduced form evidence for the connection between leverage and the risk of financial crisis.

The framework is a canonical quantitative New Keynesian dynamic stochastic general equilibrium model extended with a banking sector. The banks in the model should be seen and interpreted as shadow banks. The two features that are at the heart of the model are i) risk-shifting incentives, which determine leverage, and ii) bank runs, which can occur because a deposit insurance is absent. Crucially, the interaction of these two characteristics is the main mechanism that explains a financial crisis as will become clear below.

The first feature is the risk-shifting incentives of the banks, which determine how they accumulate leverage as in Adrian and Shin (2014) and Nuño and Thomas (2017). Bankers

[1] See e.g. Adrian and Shin (2010), Bernanke (2018), Brunnermeier (2009) and Gorton and Metrick (2012).

[2] The leverage ratio rose from 24 to 31 between 2004 and 2007 (e.g. Wiggins, Piontek and Metrick, 2014). Other investment banks such as Merrill Lynch and Goldman Sachs also elevated their leverage significantly.

[3] The leverage series relies on book equity, which is the difference between the value of the assets and the liabilities. The details are shown in Appendix A.

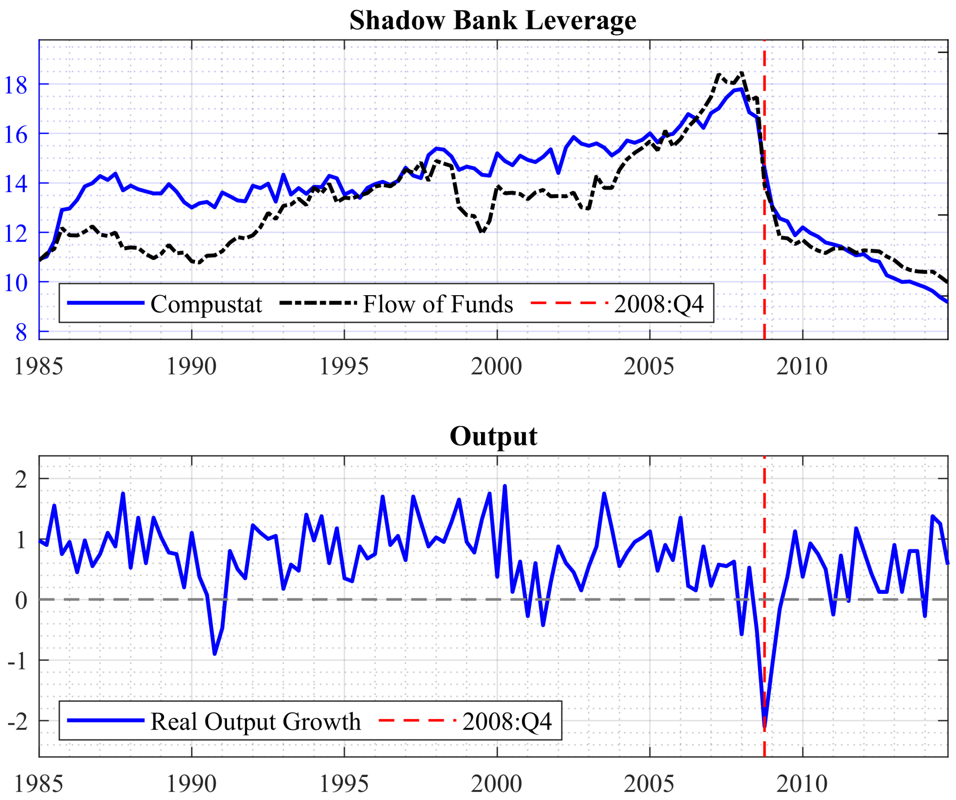

Figure 1: The upper graph shows two measures of shadow bank book leverage. The first measure is based on shadow bank balance sheet data from Compustat (left axis). The alternative one uses Flow of Funds data (right axis). Appendix A shows the details. The lower graph shows the quarter on quarter real output growth rate.

have to choose between securities with different volatilities in their returns. At the same time, the bankers are protected by limited liability, which restricts the downside risk and thus distorts the security choice.[4] To ensure an effective investment by the banks, leverage is constrained by the depositors.[5] In addition to this, the relative volatility of the securities, or simply volatility, fluctuates exogenously as in Nuño and Thomas (2017), who also provide empirical evidence for the importance of this shock. Fluctuations in volatility affect the leverage accumulation by changing the risk-shifting incentives, and can result in periods of excessive leverage.

The second feature is that banks runs are possible in the spirit of Diamond and Dybvig (1983) because there does not exist a deposit insurance. In contrast to a classical bank run, the run in my model is a self-fulfilling rollover crisis as in Gertler, Kiyotaki and Prestipino (2020).[6] If depositors expect the banking sector to default, they stop to roll over their

[4]As shown in Adrian and Shin (2014), this financial friction based on the corporate finance theory micro-founds a value-at-risk approach, which is a very common risk management approach for shadow banks.

[5]Shadow banks mainly borrow from wholesale funding markets. The depositors, who borrow to the shadow banks, are thus best thought of as institutional investors.

[6]Cole and Kehoe (2000) introduce the concept of a self-fulfilling roll-over crisis for sovereign bond markets.

deposits, and as a consequence, the banks have to sell their securities. The securities market breaks down so that asset prices drop significantly. The losses from the firesale result in bankruptcy, which validates the depositors' original belief. As the banking system is only runnable if the firesale losses wipe out the equity of the banks, the chance of such a self-fulfilling rollover crisis occuring depends on economic fundamentals, and links it directly to the balance sheet of the banks.

My main theoretical contribution is to propose risk-shifting incentives as the new underlying driver for a self-fulfilling rollover crisis. The mechanism directly relates to the build-up of leverage and is as follows. First, low volatility reduces the risk-shifting incentives, which results in elevated leverage. Subsequently, credit and output expand. At the same time, the banking sector's loss absorbing capacities are diminished as the banks hold relatively low equity buffers. A negative shock can then cause a bank run through self-fulfilling expectations. The banking panic sets off a sharp contraction in output and pushes the economy from an expansion into a severe recession. Importantly, the elevated leverage reduces loss absorbing capacities so that the banks become runnable. This demonstrates how high leverage sows the seed for a crisis.

The predictions of the model match not only the facts about leverage and output, but also other major empirical observations concerning financial crises. Schularick and Taylor (2012) use historical data for a large panel of countries to establish that a financial crisis is usually preceded by a credit boom. Credit spreads, one of the most watched financial variables, are low in the pre-crisis period and increase sharply during a financial crisis as documented empirically in Krishnamurthy and Muir (2017). The results of my model fit well with these facts about financial crises.

After calibrating the model to U.S. data, I conduct the main experiment of the model, which is a quantitative assessment of the risk of a banking panic in the run-up to the Great Recession. The horizon of the analysis is between 1985 and 2014 to include the period before the failure of Lehman Brothers in 2008. I use a particle filter to extract the sequence of structural shocks that accounts for shadow bank leverage and real output growth. The sequence allows to calculate the probability of a bank run implied by the model. The estimated probability of a financial crisis starts to increase significantly from 2004 onwards and peaks in the first quarter of 2008. The framework predicts in 2007:Q4 that the risk of a roll-over crisis in the next quarter is around 5%. For the entire year ahead, the probability of a run increases to slightly below 20%.[7] The estimation highlights the importance of low volatility in causing the rise in leverage and making the banking system prone to instability.

The model captures the strong decline in output in the fourth quarter of 2008 after the failure of Lehman Brothers. As the occurrence of a bank run is not exogenously imposed, the particle filter determines that occurrence from the data. Importantly, this assessment of the bank run clearly selects that self-fulfilling rollover crisis captures the bust. A counterfactual

[7]The emergence of the possibility of a bank run as an additional equilibrium has also been supported recently with a non-structural approach by Adrian, Boyarchenko and Giannone (2019*a*). They show the existence of multiple equilibria for the fourth quarter of 2008 conditional on data in the previous quarter using a reduced-form approach.

analysis in which deposits are rolled over suggests that GDP growth would have been close to zero in the absence of the run, instead of very negative.

The framework can be used not only to evaluate the potential trade-offs of macroprudential policies but also to quantify their potential impact on the vulnerability of the shadow banking system during the financial crisis in 2007-2008. An idea discussed in policy circles is to implement a leverage tax, which would tax the deposit holdings, for shadow banks. Specifically, the "Minnesota Plan to End too Big too Fail" from the Minneapolis Federal Reserve Bank in 2017 proposes to tax the borrowing of shadow banks. This policy would encourage banks to substitute deposits with equity, and the tax would also increase the funding costs of the shadow banks. While this could result in lower net worth and increased financial fragility, the surge in costs could also lead to a reduction in the market share of the shadow banks. I use the model to illustrate that such a tax would increase financial stability, as an annual tax of 0.25% on deposits would mitigate the emergence of financial fragility considerably. A counterfactual analysis shows that the leverage tax would have lowered the probability of a crisis by around 10% to 20% in the period prior to the financial crisis of 2007-2008.

Additionally, I provide reduced-form evidence for the link between shadow bank leverage and macroeconomic tail-risk, which is used as a proxy for a financial crisis. To study the tail-risk, it is useful to focus on the entire distribution of GDP growth instead of a single estimate such as the mean. Using conditional quantile regressions similar to the econometric approach of Adrian, Boyarchenko and Giannone (2019*b*), I study how shadow bank leverage impacts the lower tails of the GDP distribution. The reduced-form analysis associates an increasing probability of large economic contractions with elevated leverage. This finding corroborates the tight link between shadow bank leverage and financial crisis.

In conclusion, I create a model to capture the dynamics of the accumulation of leverage and show how this can endogenously result in a bank run. In the model, I show that the risk-shifting incentives of shadow banks can capture the dynamics of key financial variables like leverage, credit and credit spreads. I take the model to the data and estimate the sequence of shocks with a particle filter. This allows me to assess the underlying drivers of a financial crisis. This estimation exercise also helps in understanding the probability of a financial crisis and in evaluating the impact of macroprudential policies.

Related Literature Even though bank runs and leverage cycles have both been analysed independently, I show that the connection between these approaches is the key to explain the run on the shadow banking sector after the collapse of Lehman Brothers. Gertler, Kiyotaki and Prestipino (2020) pioneer the incorporation of self-fulfilling rollover crises into macroeconomic models and show that a bank run can account for the large drop in output observed in the fourth quarter of 2008.[8] Compared to their paper, mine emphasizes the importance of leverage and shows how elevated leverage endogenously creates the scenario of a boom

[8] Gertler and Kiyotaki (2015) and Gertler, Kiyotaki and Prestipino (2016) are preceding important contributions that integrate bank runs in standard macro models. Cooper and Corbae (2002) is an early study that features a dynamic equilibrium model with runs that can be interpreted as a rollover crisis.

going bust as empirically shown by Schularick and Taylor (2012). Introducing risk-shifting incentives as new channel allows to account for the build-up of leverage prior to the financial crisis and connect it to the run on the shadow banking system.[9] The other major difference is that I take the model to the data and estimate with a particle filter the probability of a run on the shadow banking sector in the years preceding the financial crisis. My work is also connected to other papers that incorporate bank runs in quantitative macroeconomic frameworks such as Faria-e Castro (2019), Ferrante (2018), Mikkelsen and Poeschl (2019), Paul (2019) and Poeschl (2020).

The paper also contributes to the literature about financial crises and its connection to excessive leverage and credit booms.[10] Adrian and Shin (2010), Brunnermeier and Pedersen (2009), Gorton and Ordonez (2014) and Geanakoplos (2010) stress the importance of leverage, or of leverage cycles, for the emergence of financial crises. In contrast to the literature, I show how the excessive accumulation of leverage can cause a self-fulfilling bank run that then results in a financial crisis. More generally, Lorenzoni (2008) and Bianchi (2011) point out that excessive borrowing can result in systemic risk because of a pecuniary externality that is not internalised by agents. Several approaches can capture credit booms, including asymmetric information (e.g. Boissay, Collard and Smets, 2016), optimistic beliefs (e.g. Bordalo, Gennaioli and Shleifer, 2018) and learning (e.g. Boz and Mendoza, 2014; Moreira and Savov, 2017), among others. Additionally, (time-varying) rare disasters (Barro, 2006; Wachter, 2013) can also capture the disruptive effects of a financial crisis.

This paper also builds on contributions about financial frictions in macroeconomic models. One strand of this literature points out the importance of volatility shocks, sometimes alternatively labeled as uncertainty or risk shocks, in connection with financial frictions for the business cycle (e.g. Christiano, Motto and Rostagno, 2014; Gilchrist, Sim and Zakrajšek, 2014). Adrian and Shin (2014) and Nuño and Thomas (2017) outline the importance of volatility in financial markets in relation to risk-shifting incentives and emphasize the relevance of volatility fluctuations for the dynamics of procyclical leverage. Brunnermeier and Sannikov (2014) show that lower exogenous risk can result in financial vulnerability, what is called the volatility paradox. I add to this literature that low volatility increases leverage and the associated upsurge in leverage makes the shadow banking system runnable.

The quantitative analysis adds to an evolving literature that empirically assesses nonlinear models with multiple equilibria. To capture the nonlinearities of such models, a particle filter, as advocated in Fernández-Villaverde and Rubio-Ramírez (2007), is needed. I take inspiration from the approach of Borağan Aruoba, Cuba-Borda and Schorfheide (2018) who examine the probability of different inflation regimes. In the sovereign default literature, Bocola and Dovis

[9]The risk-shifting incentives have a very different impact on leverage compared to that of a run-away constraint, where a banker can divert a fraction of assets that cannot be reclaimed, as used in Gertler, Kiyotaki and Prestipino (2020). Risk-shifting incentives combined with the volatility shock generate procyclical leverage, while leverage is normally countercyclical because of the run-away constraint. To reconcile the run-away constraint with the evidence for credit booms that generate busts, they rely on misbeliefs. Households and bankers have different beliefs, where bankers are overly optimistic about future news.

[10]The paper is about the distress in the banking sector. Other studies such as Justiniano, Primiceri and Tambalotti (2015) and Guerrieri and Lorenzoni (2017) also emphasize the role of housing booms.

(2019) use a particle filter to estimate the likelihood of a government default. Like my work, Faria-e Castro (2019) also obtains model-implied probabilities for a bank run. However, there are important differences in the work of this paper. My model incorporates the idea of a credit boom that turns to a bust, which allows for a discussion how financial fragility emerges during good times. In addition to this, the question asked here is very different, because Faria-e Castro (2019) models a run on the commercial banking sector and analyses the potential scope of capital requirements, but I focus on a self-fulfilling rollover crisis in the shadow banking sector. Other non-structural approaches that identify the multiplicity of equilibria resulting from a financial crisis are based on estimated (multimodal) distributions (Adrian, Boyarchenko and Giannone, 2019*a*) or Markov-Switching VARs (e.g. Bianchi, 2020). Finally, the link between financial conditions and macroeconomic downside risk has also been studied recently (e.g. Giglio, Kelly and Pruitt, 2016; Adrian, Boyarchenko and Giannone, 2019*b*).

Layout The rest of this paper is organized as follows. Section II outlines the dynamic stochastic general equilibrium model, while the conditions for a bank run and its connection to leverage are discussed in Section III. I present the quantitative properties including the nonlinear solution method and calibration in Section IV. Afterwards, I introduce the particle filter with the objective of estimating the probability of a bank run prior to the recent financial crisis in Section V. The policy tool of a leverage tax is analysed in Section VI. In Section VII provides the reduced-form evidence based on quantile regressions. The last section concludes.

2 Model

The setup is a New Keynesian dynamic stochastic general equilibrium model with a banking sector. The banks in the model correspond to shadow banks as they are unregulated and not protected by deposit insurance.The main features are the endogenous bank leverage constraint and the occurrence of bank runs.

Bankers have risk-shifting incentives based on Adrian and Shin (2014) and Nuño and Thomas (2017). They have to choose between two securities that face idiosyncratic shocks to their return. Importantly, the two assets differ in the mean and standard deviations of the idiosyncratic shock. Limited liability, which protects the losses of the bankers, distorts the decision between the two securities as it limits the downside losses. This determines leverage endogenously. The other key element is that the banking sector occasionally faces system-wide bank runs similar to Gertler, Kiyotaki and Prestipino (2020). The occurrence of the bank run depends on fundamentals and in particular on the leverage of the banking sector. During a run, households stop to roll over their deposits.[11] This forces the banks to

[11] There is no explicit distinction between households and typical lenders on the wholesale market such as commercial banks in the model. Poeschl (2020) discusses this assumption and shows that adding commercial banks separately can result in amplification under some conditions.

sell their assets. The asset price drops significantly as all banks sell at the same time, which justifies the run in the first place.

The rest of the economy follows a canonical New Keynesian model.[12] There exist intermediate goods firms, retailers and capital good producers. The retailers face nominal rigidities via Rotemberg pricing, and the capital good producers face investment adjustment costs. Monetary policy follows a Taylor rule.

2.1 Household

There is a large number of identical households. The representative household consists of workers and bankers that have perfect consumption insurance. Workers supply labor L_t and earn the wage W_t. Bankers die with a probability of $1-\theta$ and return their net worth to the household to avoid self-financing. Simultaneously, new bankers enter each period and receive a transfer from the household. The household owns the non-financial firms, from where it receives the profits. The variable Ξ_t captures all transfers between households, banks and non-financial firms.

The household is a net saver and has access to two different assets that are also actively used. The first option is to make one-period deposits D_t into shadow banks that promise to pay a predetermined gross interest rate $\bar{R}_t$. However, the occurrence of a bank run in the following periods alters the bank's ability to honor its commitment. In this scenario the household receives only a fraction x_t, which is the recovery ratio, of the promised return. The gross rate R_t is thus state-dependent:

$$R_t = \begin{cases} \bar{R}_{t-1} & \text{if no bank run takes place in period } t \\ x_t \bar{R}_{t-1} & \text{if a bank run takes place in period } t \end{cases} \tag{1}$$

Securities are the other option. I distinguish between beginning of period securities K_t that are used to produce output and end of period securities S_t. The households' end of period securities S_t^H give them a direct ownership in the non-financial firms. The household earns the stochastic rental rate Z_t. The household can trade the securities with other households and banks at the market clearing price Q_t. The securities of households and banks, where the latter are denoted as S_t^B, are perfect substitutes. Total end of period capital holdings S_t are:

$$S_t = S_t^H + S_t^B. \tag{2}$$

The households are less efficient in managing capital holdings as in the framework of Brunnermeier and Sannikov (2014). Following the shortcut of Gertler, Kiyotaki and Prestipino (2020), capital holdings are costly in terms of utility. The costs are given as:

$$UC_t = \frac{\Theta}{2}\Big(\frac{S_t^H}{S_t} - \gamma^F\Big)^2 S_t, \tag{3}$$

[12] The banking sector is embedded in a New Keynesian setup because nominal rigidities help to replicate the large drop in asset prices during a bank run (Gertler, Kiyotaki and Prestipino, 2020).

where $\Theta > 0$ and $\gamma^F > 0$. An increase in households' capital holdings increases the utility costs, while an increase in total capital holdings decreases the utility costs:

$$\frac{\partial UC_t}{\partial S_t^H} = \Theta\Big(\frac{S_t^H}{S_t} - \gamma^F\Big) \qquad > 0, \tag{4}$$

$$\frac{\partial UC_t}{\partial S_t} = -\gamma^F \Theta\Big(\frac{S_t^H}{S_t} - \gamma^F\Big)\frac{S_t^H}{S_t} < 0. \tag{5}$$

The budget constraint reads as follows:

$$C_t = W_t L_t + D_{t-1}R_t - D_t + \Xi_t + Q_t S_t^H + (Z_t + (1-\delta)Q_t)S_{t-1}^H, \tag{6}$$

where C_t is consumption. The utility function reads as follows:

$$U_t = E_t\Big\{\sum_{\tau=t}^{\infty}\beta^{\tau-t}\Big[\frac{(C_\tau)^{1-\sigma^h}}{1-\sigma^h} - \frac{\chi L_\tau^{1+\varphi}}{1+\varphi} - \frac{\Theta}{2}\Big(\frac{S_\tau^H}{S_\tau} - \gamma^F\Big)^2 S_\tau^H\Big]\Big\}. \tag{7}$$

The first order conditions with respect to consumption, labor, deposits and household securities are:

$$\varrho_t = (C_t)^{-\sigma}, \tag{8}$$

$$\varrho_t W_t = \chi L_t^{\varphi}, \tag{9}$$

$$1 = \beta E_t \Lambda_{t,t+1} R_{t+1}, \tag{10}$$

$$1 = \beta E_t \Lambda_{t,t+1}\frac{Z_{t+1} + (1-\delta)Q_{t+1}}{Q_t + \Theta(S_t^H/S_t - \gamma^F)/\varrho_t}, \tag{11}$$

where ϱ_t is the marginal utility of consumption and $\beta E_t \Lambda_{t,t+1} = \beta E_t \varrho_{t+1}/\varrho_t$ is the stochastic discount factor. The first order conditions with respect to the two assets can be combined to:

$$E_t R_{t+1} = E_t \frac{Z_{t+1} + (1-\delta)Q_{t+1}}{Q_t + \Theta(S_{H,t}/S_t - \gamma^F)/\varrho_t}. \tag{12}$$

This shows that the household's marginal gain of the two assets must be equal in the equilibrium. There is a spread between the return on capital and deposit rates due to the utility costs.

2.2 Banks

The bankers' leverage decision depends on risk-shifting incentives and the possibility of a bank run. I first present the risk-shifting incentives abstracting from bank runs. Afterwards, the possibility of a bank run is incorporated into the decision problem.

2.2.1 Risk-shifting Incentives Moral Hazard Problem

The banks face a moral hazard problem due to risk-shifting incentives that limits their leverage as in Adrian and Shin (2014) and Nuño and Thomas (2017). The bank can invest in two different securities with distinct risk profiles. Limited liability protects the banker's losses in case of default and creates incentives to choose a strategy that is too risky from the depositors' point of view. To circumvent this issue, an incentive constraint restricts the leverage choice. To ensure that households provide deposits, the bankers also face a participation constraint. On that account, the risk-shifting problem is formulated as a standard contracting problem in line with the corporate finance theory. The bankers maximise their net worth subject to a participation and incentive constraint.

There is a continuum of bankers indexed by j that intermediate funds between households and non-financial firms. The banks hold net worth N_t and collect deposits D_t to buy securities S_t^B from the intermediate goods producers:

$$Q_t S_t^{Bj} = N_t^j + D_t^j. \tag{13}$$

Bank leverage is defined as

$$\phi_t^j = \frac{Q_t S_t^{B,j}}{N_t^j}. \tag{14}$$

After receiving funding and purchasing the securities, the banker converts the securities into efficiency units facing the idiosyncratic volatility ω_{t+1} at the end of the period similar to Christiano, Motto and Rostagno (2014). The arrival of the shock is i.i.d over time and banks. The banker has to choose between two different conversions - a good security and a substandard security - that differ in their cross-sectional idiosyncratic volatility. The good type ω and the substandard type $\tilde{\omega}$ have the following distinct distributions:

$$\log \omega_t = 1, \tag{15}$$

$$\log \tilde{\omega}_t \overset{iid}{\sim} N\left(\frac{-\sigma_t^2 - \psi}{2}, \sigma_t\right), \tag{16}$$

where $\psi < 1$ and σ_t, which affects the idiosyncratic volatility, is an exogenous driver specified below. I abstract from idiosyncratic volatility for the good security. For that reason, its distribution is a dirac delta function, where $\Delta_t(\omega)$ denotes the cumulative distribution function. The substandard one follows a log normal distribution, where $F_t(\tilde{\omega_t})$ is the cumulative distribution function.

Importantly, the good security is superior as it has a higher mean and a lower variance:

$$E(\omega) = \omega = 1 > e^{-\frac{\psi}{2}} = E(\tilde{\omega}), \tag{17}$$

$$Var(\omega) = 0 < [e^{\sigma^2} - 1]e^{-\psi} = Var(\tilde{\omega}), \tag{18}$$

because $\psi < 1$.[13] However, the substandard security has a higher upside risk. In case of a very high realization of the idiosyncratic shock, the return of the substandard security is larger. More formally, I assume that $\Delta_t(\omega)$ cuts $F_t(\tilde{\omega})$ once from below to ensure this

[13]In line with this assumption, Ang et al. (2006) shows empirically that stocks with high idiosyncratic variance have low average returns.

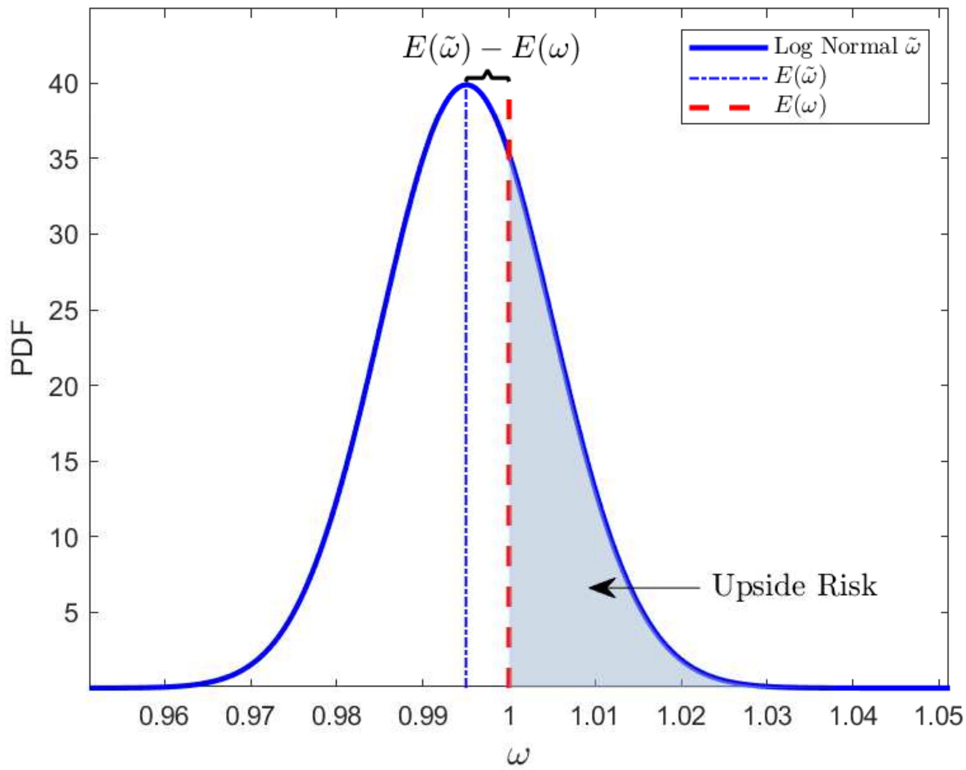

Figure 2: Trade-off between mean return and upside risk. The blue line depicts the PDF of the log normal distribution associated with the substandard security. The blue dash dotted and red dashed line are the mean return of the substandard and good security, respectively. The shaded area indicates the area associated with the upside risk.

property. This means that there is a single ω^* such that

$$(\Delta_t(\omega) - \tilde{F}_t(\omega))(\omega - \omega^*) \geq 0 \quad \forall \omega. \tag{19}$$

Figure 2 shows the distributions and highlights the difference in mean, variance and upside risk.

As specified later in detail, limited liability distorts the choice between the securities and creates risk-shifting incentives for shadow bankers. In fact, the difference in mean return and upside risk are weighted against each other. The upside risk gives an incentive to choose the substandard security despite being inefficient as limited liability protects the banker from large losses.

The shock σ_t affects the relative cross-sectional idiosyncratic volatility of the securities. In particular, it changes the upside risk, while preserving the mean spread $E(\omega) - E(\tilde{\omega})$.[14] I

[14]This result does not depend on the assumption that the good security does not contain idiosyncratic risk. For instance, the following distribution would give the same result: $\log \tilde{\omega}_t \overset{iid}{\sim} N(-0.5\eta\sigma_t^2 - \psi, \sqrt{\eta}\sigma_t)$, where $\eta < 1$. The risk shock would preserve the mean between the two distributions: $E(\omega) = 1 > e^{-\frac{\psi}{2}} = E(\tilde{\omega})$. The variance of the substandard shock would respond stronger to changes in σ_t: $Var(\omega) = [e^{\eta\sigma^2} - 1] < [e^{\sigma^2} - 1]e^{-\psi} = Var(\tilde{\omega})$ which can satisfy the assumption that the distributions cut once from below.

label the variable as volatility and assume an AR(1) process:

$$\sigma_t = (1-\rho^\sigma)\sigma + \rho^\sigma \sigma_{t-1} + \sigma^\sigma \epsilon_t^\sigma, \tag{20}$$

where $\epsilon_t^\sigma \sim N(0,1)$.

The banker earns the return $R_t^{K,j}$ on its securities that depends on the stochastic aggregate return R_t^K and the experienced idiosyncratic volatility conditional on its conversion choice:

$$R_t^{Kj} = \omega_t^j R_t^K = R_t^K \qquad \text{if good type} \tag{21}$$

$$R_t^{Kj} = \tilde{\omega}_t^j R_t^K \qquad \text{if substandard type} \tag{22}$$

The aggregate return depends on the asset price and the gross profits per unit of effective capital Z_t:

$$R_{K,t} = \frac{[(1-\delta)Q_t + Z_t]}{Q_{t-1}}. \tag{23}$$

Based on this, I can define a threshold value $\overline{\omega}_t^j$ for the idiosyncratic volatility of the substandard security where the banker can exactly cover the face value of the deposits $\bar{D}_t = \bar{R}_t^D D_t$:

$$\overline{\omega}_t^j = \frac{\bar{D}_{t-1}^j}{R_t^k Q_{t-1} S_{t-1}^{Bj}}. \tag{24}$$

The threshold is independent of the type of the security. However, the substandard security is more likely to fall below this value due to the lower mean and higher upside risk.

If the realized idiosyncratic volatility is below $\overline{\omega}_t^j$, the banker declares bankruptcy. However, limited liability protects the banker in such a scenario. The household ceases all the assets, but cannot reclaim the promised repayment. This results in two constraints for the contract between the banker and the household. First, limited liability distorts the choice between the two securities. The bank may invest in the substandard security despite its inefficiency due to lower mean return and higher upside risk. To ensure that the bankers only choose the good security, the banker faces an incentive constraint. Second, the household's expected repayment needs to be larger or equivalent to investing in the security. This constitutes the participation constraint.

I begin with the incentive constraint that deals with the risk-shifting incentives resulting from limited liability. This friction limits the losses for bankers in case of default and thereby creates thereby incentives to choose the substandard securities. The banks profit from the upside risk, while the costs of downside risk are taken by the households. This resembles a put option for the banker in the contract between the banker and households. The household receives only the return of the assets in case of defaults, or put differently, the banker has the option to sell its asset at strike price $\overline{\omega}_{t+1}^j$. Thus, the substandard security contains a put option $\tilde{\pi}_t$ that insures the bank from the downside risk and is given as:

$$\tilde{\pi}_t(\overline{\omega}_{t+1}^j) = \int^{\overline{\omega}_{t+1}^j} (\overline{\omega}_{t+1}^j - \tilde{\omega}) dF_t(\tilde{\omega}). \tag{25}$$

By these assumptions, the put option of the substandard technology is larger at a given strike price $\overline{\omega}_t^j$:

$$\tilde{\pi}_t(\overline{\omega}_{t+1}^j) > \pi_t(\overline{\omega}_{t+1}^j) = 0. \tag{26}$$

In particular, the put option of the standard security $\pi_t(\bar{\omega_{t+1}^j})$ is zero due to the absence of idiosyncratic risk. Thus, there is a trade-off between higher mean return of the good security and the higher upside risk of the substandard security. Due to this trade-off, the bank faces an incentive constraint that ensures the choice of the good security:

$$\begin{aligned} &E_t\beta\Lambda_{t,t+1}\Big\{\theta V_{t+1}^j(\omega, S_t^{Bj}, \overline{D}_t^j) + (1-\theta)[R_{t+1}^K Q_t S_t^{Bj} - \overline{D}_t^j]\Big\} \geq \\ &E_t\beta\Lambda_{t,t+1}\int_{\overline{\omega}_t^j}^{\infty}\Big\{\theta V_{t+1}(\omega, S_t^{Bj}, \overline{D}_t^j) + (1-\theta)[R_{t+1}^K Q_t S_t^{Bj}\omega_{t+1}^j - \overline{D}_t^j]\Big\} d\tilde{F}_{t+1}(\omega), \end{aligned} \tag{27}$$

where $V_{t+1}^j(\omega, S_t^{Bj}, \overline{D}_t^j)$ is the value function of a banker. The LHS is the banker's gain of the standard securities and the RHS is the gain of deviating to the substandard security. It is important to note that the banks only hold the standard security if the incentive constraint holds.

The participation constraint ensures that the households provide deposits. I can focus entirely on the good security as it is the only choice in equilibrium. Due to the absence of bank runs and idiosyncratic volatility for the good security, the banks do not default. The households receive the predetermined interest rate so that the households' expected face value of the deposits reads as follows:

$$E_t[R_{t+1}^{Dj} D_t^j] = \bar{R}_t^D D_t^j. \tag{28}$$

The households provide only funds to the banks if it is optimal to invest in deposits, which is captured in its related first order condition:

$$E_t\beta\Lambda_{t,t+1}R_{t+1}^D D_t \geq D_t. \tag{29}$$

Combining the two previous equations, the participation constraint reads as follows:

$$\beta E_t[\Lambda_{t,t+1}\bar{R}_t^D D_t^j] \geq D_t. \tag{30}$$

This condition ensures that the households hold deposits.

The problem of the banker is then to maximise the value of being a banker V_t:

$$V_t^j(N_t^j) = \max_{S_t^{Bj}, \bar{D}_t} E_t\Lambda_{t,t+1}\Big[\theta V_{t+1}^j(N_{t+1}^j) + (1-\theta)(R_t^K Q_t S_t^{Bj} - \bar{D}_t^j)\Big] \tag{31}$$

subject to Incentive Constraint [Equation (27)],

Participation Constraint [Equation (30)],

where N_t is the banker's net worth. The participation constraint and incentive constraint are both binding in equilibrium and can be written as

$$\beta E_t[\Lambda_{t,t+1}\bar{R}_t^D] \geq 1, \tag{32}$$

$$1 - e^{-\frac{\psi}{2}} = E_t\left[\tilde{\pi}_{t+1}^j\right], \tag{33}$$

where the derivation is in the Appendix C.[15]

The incentive constraint shows the trade-off between higher mean return of the good security and the put option of the substandard security. This constraint forces the banker to hold enough "skin in the game" and limits the leverage of the banker. The reason is that the value of the put option depends on ϕ_t:

$$E_t[\tilde{\pi}_{t+1}^j] = E_t\left[\overline{\omega}_{t+1}^j \Phi\left(\frac{\log(\overline{\omega}_{t+1}^j) + \frac{1}{2}(\psi + \sigma_{t+1}^2)}{\sigma_{t+1}}\right) - e^{-\psi/2}\Phi\left(\frac{\log(\overline{\omega}_{t+1}^j) + \frac{1}{2}(\psi - \sigma_{t+1}^2)}{\sigma_{t+1}}\right)\right], \tag{34}$$

$$\text{where} \quad \overline{\omega}_{t+1}^j = \frac{\bar{R}_t^D\left(\phi_t^j - 1\right)}{R_{t+1}^K \phi_t^j}. \tag{35}$$

Specifically, the value of the put option increases in leverage, that is $\partial E_t[\tilde{\pi}_{t+1}]/\partial\phi_t > 0$.

The participation and incentive constraint do not depend on bank-specific characteristics so that the optimal choice of leverage is independent of net worth. Therefore, we can sum up across individual bankers to get the aggregate values. Bankers demand for assets depends on leverage and aggregate banker net worth and is given as:

$$Q_t S_t^B = \phi_t N_t. \tag{36}$$

The net worth evolution is as follows in the absence of bank runs. Surviving bankers retain their earnings, while newly entering bankers get a transfer from households:

$$N_{S,t} = R_t^K Q_t S_{t-1}^B - R_t^D D_t, \tag{37}$$

$$N_{N,t} = (1-\theta)\zeta S_{t-1}, \tag{38}$$

where $N_{S,t}$ and $N_{N,t}$ are the net worth of surviving respectively new bankers. Aggregate net worth N_t is given as:

$$N_t = \theta N_{S,t} + N_{N,t}. \tag{39}$$

2.2.2 Bank Run Possibility and Risk-Shifting Moral Hazard Problem

A bank run is a systemic event that affects the entire banking sector. In particular, a run eradicates the net worth of all banks, that is $N_t = 0$. All bankers are thus bankrupt and stop to operate. However, all agents incorporate the possibility of a run. Their decision problem responds to the probability of a financial crisis. I discuss in the following the implications on

[15] I check numerically that the multipliers associated with the constraints are positive for the relevant state space.

the contract between banker and households. Appendix C contains a full derivation.

The banker can only continue operating or return its net worth to the household in the absence of a run. The value function depends now on the probability p_t that a bank run takes place next period:

$$V_t(N_t) = (1-p_t)E_t\left[\Lambda_{t,t+1}(\theta V_{t+1}(N_{t+1}) + (1-\theta)(R^K_t Q_t S^B_t - \bar{D}_t))\Big|\text{no run}\right], \tag{40}$$

where $E_t[\cdot|\text{no run}]$ is the expectation conditional on no run in $t+1$. For the ease of exposition, I use now a superscript if the expectations are conditioned on the occurrence of a run or not, that is $E^N_t[\cdot] = E_t[\cdot|\text{no run}]$ and $E^R_t[\cdot] = E_t[\cdot|\text{run}]$

The probability p_t is endogenous and is described in detail in the next section, where I derive the conditions for a bank run. The bank's value function decreases with the probability of a run as a run wipes out the entire net worth. The banks' commitment to repay the households is also altered. As the bankers are protected by limited liability, the households do not receive the promised repayments. Instead, households recover the gross return of bank securities $R^K_t Q_{t-1} S^B_{t-1}$. The gross rate R_t is thus state-dependent:

$$R_t = \begin{cases} \bar{R}_{t-1} & \text{if no bank run takes place in period } t \\ R^K_t Q_{t-1} S^B_{t-1}/D_{t-1} & \text{if a bank run takes place in period } t \end{cases} \tag{41}$$

The participation constraint, which is binding in equilibrium, includes the probability of a run as the banks need to compensate the households for the tail-event of a run:

$$(1-p_t)E^N_t[\beta\Lambda_{t,t+1}\bar{R}_t D_t] + p_t E^R_t[\beta\Lambda_{t,t+1}R^K_{t+1}Q_t S^B_t] \geq D_t. \tag{42}$$

The return in a run scenario is lower than the promised repayment. Consequently, the funding costs of the bank, namely the interest rate $\bar{R}_t$, increase with p_t. As the funding costs increase for the banks, they have lower expected profits. As this increases the value of the limited liability, this pushes leverage down.

Furthermore, the possibility of a run also alters the incentive constraint that is binding in equilibrium, which reads as follows:[16]

$$\begin{aligned}&(1-p_t)E^N_t[\Lambda_{t,t+1}R^K_{t+1}(\theta\lambda_{t+1} + (1-\theta))[1 - e^{\frac{-\psi}{2}} - \tilde{\pi}_{t+1}]] = \\ &p_t E^R_t[\Lambda_{t,t+1}R^K_{t+1}(e^{-\frac{\psi}{2}} - \varpi_{t+1} + \tilde{\pi}_{t+1})],\end{aligned} \tag{43}$$

where λ_t is the multiplier on the participation constraint.

$$\lambda_t = \frac{(1-p_t)E^N_t \Lambda_{t,t+1}R^K_{t+1}[\theta\lambda_{t+1} + (1-\theta)](1-\varpi_{t+1})}{1 - (1-p_t)E^N_t[\Lambda_{t,t+1}R^K_{t+1}\varpi_{t+1}] - p_t E^R_t[\Lambda_{t,t+1}R^K_{t+1}]}. \tag{44}$$

The trade-off between higher mean return and upside risk still prevails, which is displayed on

[16]Investing in substandard securities is an outside equilibrium strategy, which allows a banker to survive a run in case of a very high realization of the idiosyncratic shock. It is assumed that the surviving bankers repay their depositors fully and return their remaining net worth to the households.

the LHS in equation (43). It is now weighted with the probability to survive as the banker does not profit from a higher mean return in case of a bank run.

However, there now exists an additional channel that affects the leverage decision and the funding of the banks. If a bank run happens, then $\overline{\omega}_t > 1$ holds by construction as the bank cannot repay the deposits for the standard securities with $\omega = 1$. Thus, no bank survives a run using the standard security. In contrast to this, the substandard security offers the possibility to survive a bank run as the idiosyncratic volatility $\tilde{\omega}_t^i$ is drawn from a distribution. If $\tilde{\omega}_t^i > \overline{\omega}_t$, the bank can repay its depositors because it profits from the upside risk of the substandard security. It is important to note that this is out of equilibrium as no bank actually has substandard securities. Thus, I assume that if a bank would invest in substandard securities and survived a run, then the bank would shut down and repay its remaining net worth to the household. The RHS of equation (43) shows this channel. An increase in the run probability makes the substandard technology more attractive from the bankers' perspective and reduces leverage. It is thus a dampening force operating in the opposite direction of the moral hazard problem.

The anticipation of a potential bank run is an additional channel that affects the leverage decision and funding of the banks. An increase in the run probability makes the substandard technology more attractive from the bankers' perspective. Consequently, this pushes leverage down.

During a bank run, the net worth of the banks, who existed in the previous period is zero due to limited liability. However, new banks are entering due to transfers from households so that net worth is given as

$$N_t = (1-\theta)\zeta S_t^H \text{ if bank run happens in period t.} \tag{45}$$

Afterwards, the banking sector starts to rebuild in period $t+1$ with the same transfer from households and using retained profits in addition, that is:

$$N_{t+1} = \theta[(R_{k,t+2} - R_{t+2}\phi_{t+1}) + R_{t+2}]N_{t+1} + (1-\theta)\zeta S_{t+1}^H. \tag{46}$$

As the banking sector starts to immediately rebuild in period t, a bank run is already possible again in period $t+1$.

2.3 Production and Closing the Model

The non-financial firms sector consists of intermediate goods producers, final good producers and capital good producers. A standard Taylor rule determines the nominal interest rate.

2.3.1 Intermediate Goods Producers

There is a continuum of competitive intermediate good producers. The representative intermediate good producer produces the output Y_t with labor L_t and working capital K_t as input:

$$Y_t^j = A_t(K_{t-1}^j)^\alpha (L_t^j)^{1-\alpha}. \tag{47}$$

A_t is total factor productivity, which follows an AR(1) process. The firm pays the wage W_t to the households. The firm purchases in period $t-1$ capital S_{t-1} at the market price Q_{t-1}. The firm finances the capital with securities S_{t-1}^B from the bank and the households S_{t-1}^H, so that:

$$K_{t-1} = S_{t-1}^H + S_{t-1}^B. \tag{48}$$

This loan is frictionless and the intermediate firm pays the state-contingent interest rate $R_{K,t}$. After using the capital in period t for production, the firm sells the undepreciated capital $(1-\delta)K_t$ at market price Q_t. The intermediate output is sold at price MC_t^M, which turns out to be equal to the marginal costs. The problem can be summarized as:

$$\max_{K_{t-1},L_t} \sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i} \begin{Bmatrix} MC_{t+i}Y_{t+i} + Q_{t+i}(1-\delta)K_{t-1+i} \\ -R_{k_{t+i}}Q_{t-1+i}K_{t-1+i} - W_{t+i}L_{t+i} \end{Bmatrix}. \tag{49}$$

This is a static problem, of which the first order conditions are:

$$MC_t(1-\alpha)\frac{Y_t}{L_t} = W_t, \tag{50}$$

$$R_t^K = \frac{Z_t + Q_t(1-\delta)}{Q_{t-1}}, \tag{51}$$

$$Z_t = MC_t\alpha\frac{Y_t}{K_{t-1}}. \tag{52}$$

2.3.2 Final goods retailers

The final goods retailers buy the intermediate goods and transform it into the final good using a CES production technology:

$$Y_t = \left[\int_0^1 (Y_t^j)^{\frac{\epsilon-1}{\epsilon}} df\right]^{\frac{\epsilon}{\epsilon-1}}. \tag{53}$$

The price index and intermediate goods demand are given by:

$$P_t = \left[\int_0^1 (P_t^j)^{1-\epsilon} df\right]^{\frac{1}{1-\epsilon}}, \tag{54}$$

$$Y_t^j = \left(\frac{P_t^j}{P_t}\right)^{-\epsilon} Y_t. \tag{55}$$

The final retailers are subject to Rotemberg price adjustment costs. Their maximization problem is:

$$E_t\left\{\sum_{i=0}^{T} \Lambda_{t,t+i}\left[\left(\frac{P_{t+i}^j}{P_{t+i}} - MC_{t+i}\right)Y_{t+i}^j - \frac{\rho^r}{2}Y_{t+i}\left(\frac{P_{t+i}^j}{P_{t+i-1}^j} - \Pi\right)^2\right]\right\}, \tag{56}$$

where Π is the inflation target of the monetary authority.

I impose a symmetric equilibrium and define $\Pi_t = P_t/P_{t-1}$. The New Keynesian Phillips curve reads as follows:

$$(\Pi_t - \Pi)\Pi_t = \frac{\epsilon}{\rho^r}\Big(MC_t - \frac{\epsilon - 1}{\epsilon}\Big) + \Lambda_{t,t+1}(\Pi_{t+1} - \Pi_{SS})\Pi_{t+1}\frac{Y_{t+1}}{Y_t}. \tag{57}$$

2.3.3 Capital goods producers

Competitive capital goods producers produce new end of period capital using final goods. They create $\Gamma(I_t/S_{t-1})S_{t-1}$ new capital S_{t-1} out of an investment I_t, which they sell at market price Q_t:

$$\max_{I_t} Q_t\Gamma\Big(\frac{I_t}{S_{t-1}}\Big)S_{t-1} - I_t, \tag{58}$$

where the functional form is $\Gamma(I_t/S_{t-1}) = a_1(I_t/S_{t-1})^{1-\eta} + a_2$ as in Bernanke, Gertler and Gilchrist (1999). The FOC gives a relation for the price Q_t depending on investment and the capital stock:

$$Q_t = \left[\Gamma'\Big(\frac{I_t}{S_{t-1}}\Big)\right]^{-1}. \tag{59}$$

The law of motion for capital is:

$$S_t = (1-\delta)S_{t-1} + \Gamma\Big(\frac{I_t}{S_{t-1}}\Big)S_{t-1}. \tag{60}$$

2.3.4 Monetary Policy and Resource Constraint

The monetary authority follows a standard Taylor Rule for setting the nominal interest rate i_t:

$$i_t = \frac{1}{\beta}\left(\frac{\Pi_t}{\Pi}\right)^{\kappa_\Pi}\left(\frac{MC_t}{MC}\right)^{\kappa_y}, \tag{61}$$

where deviations of marginal costs from its deterministic steady state MC capture the output gap. To connect this rate to the household, there exists one-period bond in zero net supply that pays the riskless nominal rate i_t. The associated Euler equation reads as follows:

$$\beta\Lambda_{t,t+1}\frac{i_t}{\Pi_{t+1}} = 1. \tag{62}$$

The aggregate resource constraint is

$$Y_t = C_t + I_t + G\frac{\rho^r}{2}(\Pi_t - 1)^2 Y_t, \tag{63}$$

where G is government spending and the last term captures the adjustment costs of Rotemberg pricing.

2.4 Equilibrium

The recursive competitive equilibrium is a price system, monetary policy, policy functions for the households, the bankers, the final good producers, intermediate good producers and capital good producers, law of motion of the aggregate state and perceived law of motion of the aggregate state, such that the policy functions solve the agents' respective maximization problem, the price system clears the markets and the perceived law of motion coincides with the law of motion. The aggregate state of the economy is described by the vector of state variables $\mathcal{S}_t = (N_t, S^B_{t-1}, A_t, \sigma_t, \iota_t)$, where ι_t is a sunspot shock related to bank runs that is specified in the next chapter. The details regarding the equilibrium description and different equilibrium equations can be found in the Appendix B.

3 Multiple Equilibria, Bank Runs and Leverage

A bank run is a self-fulfilling event that depends on the state of the world. This scenario enters as an additional equilibrium to the normal one in which households roll over their deposits. At first, I discuss the existence of this equilibrium. Afterwards, the connection between the emergence of a bank run and leverage is discussed.

3.1 Bank Run and Multiple Equilibria

The model contains multiple equilibria resulting from the possibility of a bank run as in Diamond and Dybvig (1983). Importantly, the existence of the bank run equilibrium emerges endogenously similar to Gertler, Kiyotaki and Prestipino (2020). The possibility of self-fulfilling expectations about a bank run depends on the aggregate state and especially on the banks' balance sheet strength. A household only expect the additional bank run equilibrium if banks do not survive it, that is an eradication of the entire banking sector would take place ($N_t = 0$). Therefore, the multiplicity of equilibria occurs only in some states of the world similar to Cole and Kehoe (2000), who characterize self-fulfilling rollover crisis in the context of sovereign bond markets.

The multiplicity of equilibria originates from a heterogeneous asset demand of households and bankers. During normal times, that is in the absence of a bank run, households roll over their deposits. Banks and households demand capital and the market clears at price Q_t. This price can be interpreted as the fundamental price. The bank can cover the promised repayments for the fundamental price:

$$[(1-\delta)Q_t + Z_t]S^B_{t-1} > \bar{R}_{t-1}D_{t-1}. \tag{64}$$

In contrast to this, a run wipes out the banking sector. Households stop to roll over their deposits in a run so that banks liquidate their entire assets to repay the households. This eliminates their demand for securities. Households are the only remaining agents that can buy assets in a run. Subsequently, the asset price must fall to clear the market. The drop is particularly severe because it is costly for households to hold large amounts of capital. This firesale price $Q_t^\star$ depresses the potential liquidation value of banks' securities. A bank run can only take place if the banks cannot repay the household. This is the case if the firesale liquidation value is smaller than the household claims:

$$[(1-\delta)Q_t^\star + Z_t^\star]S_{t-1}^B < \bar{R}_{t-1}D_{t-1}, \tag{65}$$

where the superscript $\star$ indicates the bank run equilibrium. The recovery ratio x_t captures this relation:

$$x_t = \frac{[(1-\delta)Q_t^\star + Z_t^\star]S_{t-1}^B}{\bar{R}_{t-1}D_{t-1}}. \tag{66}$$

The numerator is the firesale liquidation value and the denominator is the promised repayments. A bank run can only take place if

$$x_t < 1, \tag{67}$$

as banks then do not have sufficient means to cover the claims of the households under the firesale price $Q_t^\star$.

Based on the recovery ratio x_t, I can partition the state space in a safe and a fragile zone. $x_t > 1$ characterizes the safe zone. The bank can cover the claims under the fundamental and firesale price. Therefore, bank runs are not possible and only the normal equilibrium exists. In the fragile zone $x_t < 1$, both equilibria exist. The banks have only sufficient means given the fundamental price. Technically, there is a third scenario. If an absolutely disastrous shock hits the economy, the bank could not even repay the households in the absence of a bank run. I neglect the scenario as the probability is infinitesimal in the quantitative model.

As the run equilibrium coexists with the normal equilibrium, I occasionally have multiple equilibria. I select among the equilibria using a sunspot shock similar to Cole and Kehoe (2000). The sunspot ι_t takes the value 1 with probability Υ and 0 with probability $1-\Upsilon$. If $\iota_t = 1$ materializes and $x_t < 1$, a bank run takes place. The condition on the recovery ratio x_t ensures that the run equilibrium is only chosen if it is optimal. Otherwise, the normal equilibrium is chosen. The probability for a run in period $t+1$ is the probability of being in the crisis zone next period and drawing the sunspot shock:

$$p_t = \text{prob}(x_{t+1} < 1)\Upsilon. \tag{68}$$

The bank run probability is time-varying as x_{t+1} depends on the macroeconomic and financial circumstances.

3.2 Bank Run and Leverage

Leverage and the volatility shock determine to a large extent the possibility of a bank run for which I derived the condition $x_t < 1$. The recovery ratio x_t can be expressed in terms of leverage ϕ_t:

$$x_t = \frac{\phi_{t-1}}{\phi_{t-1}-1}\frac{[(1-\delta)Q_t^\star + Z_t^\star]}{Q_{t-1}\bar{R}_{t-1}}. \tag{69}$$

An increase in leverage reduces the recovery ratio, that is

$$\frac{\partial x_t}{\partial \phi_{t-1}} = -\frac{1}{(\phi_{t-1}-1)^2}\frac{[(1-\delta)Q_t^\star + Z_t^\star]}{Q_{t-1}\bar{R}_{t-1}} < 0. \tag{70}$$

This shows that a highly leveraged banking sector is more likely to be subject to a run. In particular, a previous period leverage that shifts the recovery ratio below 1 enables a bank run.[17] The volatility shock affects the recovery ratio via the return on capital in the run equilibrium $R_t^{K\star} = (1-\delta)Q_t^\star + Z_t^\star$. An increase in volatility lowers the return and thereby the recovery ratio for a given level of leverage:

$$\frac{\partial x_t}{\partial \sigma_t} = \frac{\partial x_t}{\partial R_t^{K\star}}\frac{\partial R_t^{K\star}}{\partial \sigma_t} < 0. \tag{71}$$

Figure 3 illustrates how the combination of the volatility shock and leverage determines the region. The $x_t = 1$ line is downward sloping and divides the two regions. First, it can be be seen that a large level of previous period leverage and an increase in volatility pushes the economy in the fragile region as discussed previously. Second, there is a region in which only the safe zone exists for the displayed shock size. Low leverage implies that the economy is in the safe region as it would require very large shocks that are highly unlikely to push the economy into the crisis.

The pre-crisis period is critical for the build-up of financial fragility because high leverage facilitates a bank run. The model requires a credit boom that increases leverage in the first place. A reduction in the volatility shock reduces the volatility of the substandard security. For a given leverage, the value of the put option declines. The risk-shifting incentives become less severe. Therefore, banks can increase leverage such that the incentive constraint is binding. This channel generates procyclical leverage. Thus, there needs to be initially a period of decreases in volatility that increase leverage. This enables the bank run in the first place. Tranquil periods sow the seed of a crisis.

However, leverage and thus the probability of a bank run are constrained in the model. The agents in the economy are aware of the prospect of a bank run which generates an opposing force in a boom. In case of a run, their return on the standard security is zero. In contrast to this, they can profit from the substandard security. If they draw a large individual realization, they can repay the depositors and survive the run. Thus, bankers are more tempted to invest in the substandard security as the run likelihood rises. To satisfy the incentive constraint, the banking sector reduces its leverage. This channel endogenously limits the probability of multiple equilibria. A bank run is thus a tail event as leverage is bounded. This implies that banks can be highly leveraged as long as the probability of a

[17] I abstract from the potential indirect impact of ϕ_{t-1} on other variables such as the firesale price in the derivative.

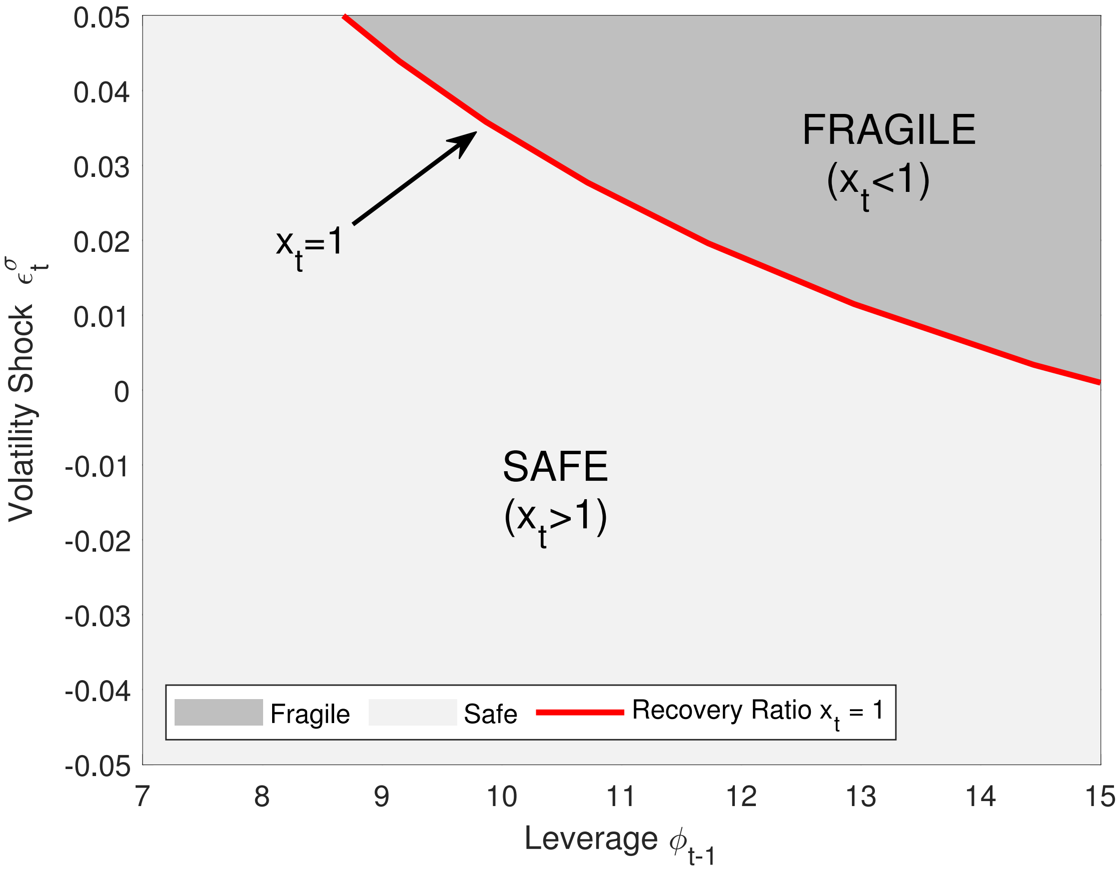

Figure 3: The dependence on the safe and fragile regions on leverage ϕ_{t-1} and the volatility shock. ϵ_t^σ

run is not too large. Despite this channel, the model contains a bank run externality. While the agents anticipate a run, they do not take into account the effect of their decisions on the probability of a run.

In the following sections, the model is taken to the data and its quantitative implications are analysed.

4 Model Evaluation

In this section, I evaluate the model and its predictions with the objective of analyzing a typical financial crisis and how the banking system becomes runnable. First, the data and the parameterization of the model are discussed. Then, the nonlinear solution method that incorporates the bank run equilibrium is explained. Afterwards, the quantitative properties of the model are shown. Specifically, the emergence and unfolding of a typical financial crisis is discussed.

4.1 Calibration

This section explains the mapping of the model to the data. The emphasis of the calibration is on the shadow banking sector and its role in the recent financial crisis in the United

States. I use quarterly data from 1985:Q1 to 2014:Q4 to adapt to the changing regulation of shadow banking activities. The starting point coincides with major changes in the contracting conventions of the repurchasement agreement (repo) market, which is an important source of funding for shadow banks, that took place after the failure of some dealers in the beginning of 1980s (Garbade, 2006).[18] In addition, this allows to focus on the period after the Great Inflation. After the financial crisis, new regulatory reforms such as Basel III and Dodd-Frank Wall Street and Consumer Protection act overhauled the financial system, which rationalizes to stop the sample in 2014.[19]

Preferences, Technologies and Monetary Policy The discount factor is set to 0.995, which corresponds to an annualized long run real interest rate of 2%. This is line with the average of the real interest rates estimates of Laubach and Williams (2003).[20] The Frisch elasticity is set to match the elasticity of 0.75 for aggregate hours as suggested in Chetty et al. (2011).[21] The risk aversion is parameterized to 1, which implies a logarithmic utility function. Output is normalized to 1, which implies a total factor productivity of $A = 0.49$. Government spending G matches 20% of total GDP in the deterministic steady state. The production parameter α matches a capital income share of 33%. The annual depreciation is chosen to be 10%, which implies $\delta = 0.025$. I set the price elasticity of demand to 10 to get a markup of 11%. I target a 1% slope of the New Keynesian Philipps curve so that the Rotemberg adjustment cost parameter is $\rho = 1000$. I match the elasticity of the asset price ρ^r of 0.25 as in Bernanke, Gertler and Gilchrist (1999). The parameters of the investment function normalize the asset price to $Q = 1$ and the investment $\Gamma(I/K) = I$ in the deterministic steady state. Monetary policy responds to deviations of output and inflation, where the target inflation rate Π is normalized to 0% per annum. I parameterize $\kappa_\pi = 1.5$ and $\kappa_y = 0.125$ in line with the literature.

Financial Sector The financial sector represents the shadow banking sector, which is defined as runnable intermediaries. In particular, I define these as entities that rely on short-term deposits that are not protected by the Federal Deposit Insurance Corporations and do not have access to the FED's discount window.[22] The share of total assets, which are held directly by the shadow banking sector, was 37.1% in 2006 and dropped to 28.3% in 2012 as

[18]There were three major changes in contracting conventions as documented in Garbade (2006). First, the Bankruptcy Amendments and Federal Judgeship Act of 1984 altered the bankruptcy regulation of repos. Second, lenders could earn interest in a repurchasement agreement. Third, a new repo contract called a tri-party-repo has been adopted.

[19]Adrian and Ashcraft (2012) discuss the impact of these two most comprehensive reforms on the financial system such as changed accounting standards. In addition to this, most of the major broker-dealers are now part of bank holding companies, which gives additional protection.

[20]The two-sided estimated long run real interest rate of Laubach and Williams (2003) is 2.16% for the considered horizon.

[21]Chetty et al. (2011) find a Frisch elasticity of 0.5 on the intensive margin and 0.25 extensive margin, which points to an elasticity of 0.75 for the representative households in this framework.

[22]This definition applies to the following entities: Money market mutual funds, government-sponsored enterprises, agency- and GSE-backed mortgage pools, private-label issuers of asset-backed securities, finance companies, real estate investment trusts, security brokers and dealers, and funding corporations.

Table 1: Calibration

Parameters	Sign	Value	Target / Source
(a) Conventional Parameters			
Discount factor	β	0.995	Risk free rate = 2% p.a.
Frisch labor elasticity	$1/\varphi$	0.75	Elasticity aggregate hours = 0.75%
Risk aversion	σ^H	1	Log Utility
TFP level	A	0.4070	Output = 1
Government spending	G	0.2	Governm. spending to output = 0.2
Capital share	α	0.33	Capital income share = 33 %
Capital depreciation	δ	0.025	Depreciation Rate = 10% p.a.
Price elasticity of demand	ϵ	10	Markup = 11%
Rotemberg adjustment costs	ρ^r	1000	Slope of NK Phillips curve = 0.1%
Elasticity of asset price	η_i	0.25	Elasticity of asset price = 25%
Investment Parameter 1	a_1	0.5302	Asset Price Q = 1
Investment Parameter 2	a_2	-.0083	$\Gamma(I/K) = I$
Target inflation	Π	1.00	Normalization
MP response to inflation	κ_π	1.5	Literature
MP response to marginal costs	κ_y	0.25	Literature
(b) Financial Sector			
Parameter asset share	γ^F	0.355	Size of shadow bank = 33%
Mean Substandard Technology	ψ	0.01	Leverage = 14.4
Parameter intermediation cost	Θ	0.1024	Annual Run probability = 2.5%
Fraction start capital	ζ	.0131	Credit spread = 2.31% p.a.
Survival Rate Banker	θ	0.92	Implied from other parameters
(c) Shocks			
Std. dev. Volatility shock	σ^σ	0.0058	Std. dev. of leverage = 1.90
Std. dev. TFP shock	σ^A	0.002	Std. dev. of output growth = 0.59
Persistence volatility shock	ρ^σ	0.92	Persistence of leverage = 0.94
Persistence TFP shock	ρ^A	0.95	Fernald (2014)
Sunspot Shock	Υ	0.5	Strong increase of volatility in run

shown by Gallin (2015) using the financial accounts of the United States.[23] In line with this, I target that the shadow banking sector holds 33% of total assets on average, setting the parameter $\gamma^F = 0.42$.[24]

The leverage ratio of the shadow banking sector is matched to balance sheet data of shadow banks from Compustat as show in Figure 1.[25] Specifically, the leverage series is

[23] Based on a broader definition of shadow banking activities, Poszar et al. (2010) suggest a larger share of the shadow banking sector around 50%.

[24] This share is also in line with the macroeconomic modeling literature (e.g. Begenau and Landvoigt, 2018; Gertler, Kiyotaki and Prestipino, 2020).

[25] In particular, I classify shadow banks as companies with SIC codes between 6141 - 6172 and 6199 - 6221. This characterization contains credit institutions, business credit institutions, finance lessors, finance services, mortgage bankers and brokers, security brokers, dealers and flotation companies, and commodity contracts brokers and dealers.. The Appendix contains more information on the construction.

calculated based on book equity, which is the difference between value of portfolio claims and liabilities of financial intermediaries. An alternative measure is the financial intermediaries' market capitalization (e.g. market valuation of financial intermediaries). Importantly, the appropriate concept in this context is book equity because the occurrence of a bank run in the model depends directly on book equity that is denoted as net worth in the model. Furthermore, the interest is on credit supply and financial intermediaries lending decision, which requires the usage of book equity as stressed for instance in Adrian and Shin (2014).[26] In the data prior to the financial crisis (from 1985:Q1 until 2007:Q3), the leverage ratio was on average around 14.4. The matching concept is the stochastic steady state of Coeurdacier, Rey and Winant (2011). It is defined as the point to which the economy converges if shocks are expected, but they do not realize. In that regard, the calibration incorporates the possibility of future financial crisis. The choice of $\psi = 0.01$ for the substandard technology firm calibrates leverage at the risky steady state.

The intermediation cost parameter Θ is set to match an annual run probability of 2.5% which corresponds to a bank run on average every 40 years. This frequency is in line with the historical macroeconomic data of Jordà, Schularick and Taylor (2017). Their database suggests that the average yearly probability of a financial crisis is 2.7% in the U.S. and 1.9% for a sample of advanced economies since the second world war.[27] With the fraction of the start capital ζ, I target an average spread of 2.25% to match the spread between the BAA bond yield and a 10 year treasury bond. The survival rate θ is implied from the other parameters of the banking sector.

Shocks I calibrate the volatility shock to match the persistence and standard deviation of the leverage measure. In addition to this, I use the TFP shock to match the standard deviation of output growth. The persistence of the shock is aligned with the estimated persistence of the TFP series in Fernald (2014). The sunspot shock is parameterized to result in a bank run with 50% in case of the multiplicity. Consequently, a bank run requires a rather large increase in the volatility shock. This captures the strong increase in different volatility measures experienced during the financial crisis. As a rather large contractionary shock is needed for a bank run, this ensures that leverage is not increasing too much in a crisis in line with the data.

4.2 Solution Method

The model is solved with global methods that incorporate the multiplicity of equilibria. A nonlinear global solution is necessary due to the incorporation of bank runs. First, a financial

[26]He, Khang and Krishnamurthy (2010) and He, Kelly and Manela (2017) emphasize the importance of market leverage. However, market capitalization is the appropriate measure related to the issuance of new shares or acquisitions decision as argued in Adrian, Colla and Shin (2013). Nuño and Thomas (2017) also provide a detailed view about the two concepts in a dynamic stochastic general equilibrium framework.

[27]The average probability of a financial crisis for is 4.4% for the U.S. and 3.8% for the advanced economies if the considered horizon starts in 1870. The Appendix contains more details regarding the construction of the probability.

panic destroys the entire banking sector so that the dynamics are highly nonlinear. Second, the agents take the likelihood of a bank run into account and respond to it.

I use a time iteration algorithm with piecewise linear policy functions based on Coleman (1990) and Richter, Throckmorton and Walker (2014). The method is adjusted to factor in the multiplicity of equilibria similarly to Gertler, Kiyotaki and Prestipino (2020). The details of the numerical solution are left to Appendix D.

4.3 Typical Financial Crisis and its Macroeconomic Impact

To understand the underlying drivers of a financial crisis, I assess the dynamics around a typical bank run. As the possibility of a bank run is endogenous and depends on economic fundamentals, the emergence of financial fragility can be studied. I simulate the economy for 500000 periods so that in total 3084 banks runs are observed. Collecting the bank run episodes, I can construct an event window centered 10 quarters before and 10 quarters after a bank run.[28] Figure 4 displays the median along with 68% and 90% confidence bands of this event analysis. The median is particularly informative about the emergence of a typical financial crisis.

A bank run is preceded by a build-up of leverage, elevated credit and low finance premium. While this implies an economic expansion, financial fragility arises simultaneously due to the rising leverage of banks. Specifically, the increase on leverage sows the seed for the crisis as the probability of a run in the quarter ahead increases. The occurrence of a bank run causes a sharp economic contraction. Output drops by 2 percent from the previous quarter in the period of run - which correspond to 8% annualized - for the median. In addition, the credit boom goes bust and the finance premium spikes. Importantly, the observed dynamics reconcile the empirical facts outlined in the introduction: i) sharp drop in output, ii) elevated leverage of shadow banks prior to collapse, iii) credit boom precedes a financial crisis and iv) credit spreads are low before and spike during crisis.[29]

Crucially, a period of low volatility is the underlying force behind the emergence of the bank run. The simulation points out this circumstance as not only the median, but also both confidence intervals of volatility are considerably below its risky steady state prior to a run. On that account, this framework features a volatility paradox in the spirit of Brunnermeier and Sannikov (2014). In contrast to this, the productivity shock is less important for the build-up. While a positive TFP level enforces the boom, the median is only slightly above the risky steady state. The mechanism behind the endogenous emergence of a financial crisis is as follows. Low volatility results in a build-up of leverage. This creates financial fragility and opens up the possibility of a bank run. The banks have relative low net worth to absorb potential losses. The realization of negative shocks pushes the economy into the fragile zone. If depositors do not rollover their deposits, or in technical jargon a sunspot shock materializes,

[28] The simulation starts after a burn-in of 100000 periods. In total, the simulation features 3177 bank run episodes, which corresponds to a bank run probability of 2.54% per year in line with the target of the calibration.

[29] Among others, see for instance Adrian and Shin (2010)), Bernanke (2018), Schularick and Taylor (2012) and Krishnamurthy and Muir (2017) for the different stylized facts.

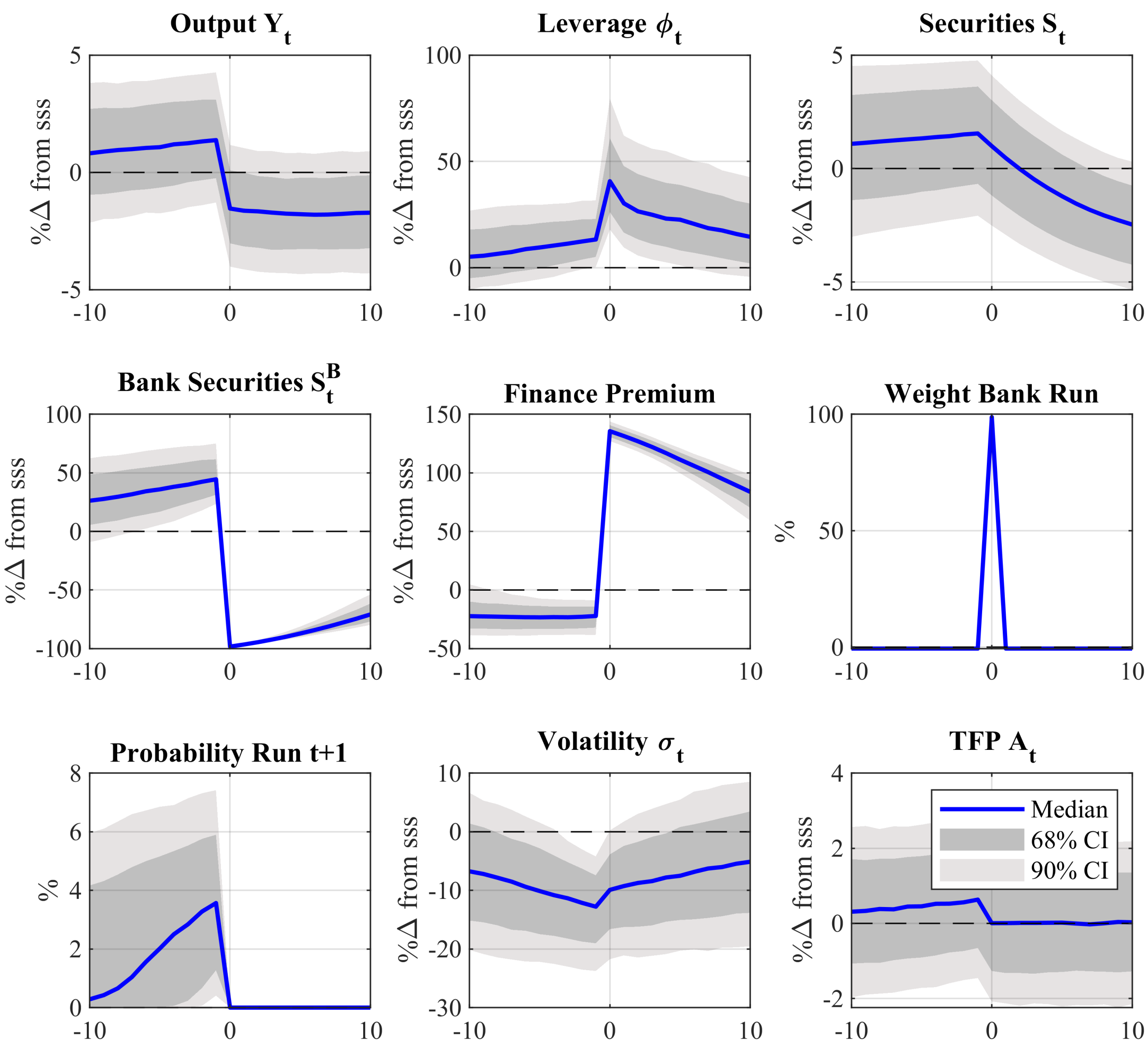

Figure 4: Event-window around bank run episodes. Based on simulation of 500000 periods, median path and 68% as well as 90% confidence intervals of all bank runs are displayed ten quarters before and after bank run that takes place in period 0. The scales are either as percentage deviations from the risky steady state (sss), basis points deviations from the rss or in levels. The weight variable weight bank run shows the probability that a bank run occured in the period.

the banks are forced to sell their assets at a firesale price. As the bankers do not have enough equity to cover the losses, a self-fulfilling run occurs.

The cyclicality of leverage is the key determinant in the model. Due to the volatility shock and the risk-shifting incentives, leverage is procyclical in the model. This means that an increase in leverage raises output and credit. This comovement generates the credit boom gone bust dynamics as shown in Schularick and Taylor (2012). To better understand the importance of procyclical leverage, I discuss now the consequences of countercyclicality for the predictions of the model. In the case of countercyclical leverage, high leverage would imply low output and low assets. A bank run would then occur in a bust and therefore could not capture the credit boom gone bust dynamics.[30] For that reason, understanding the leverage dynamics and matching the increase in leverage is key to replicate the patterns around the financial crisis.

After the bank run, output drops sharply and securities fall. All banks are failing and only the newly entering banks operate in the market. For this reason, the credit supply from the banking sector is very low. The leverage of these new banks is initially very large, which overshoots the prediction in the data. As the old banks fail, the returns for new banks are very large, which increases leverage initially. While this is a common problem in the literature, the increase in leverage is already much more in line with the data. As seen later, this model can actually track the observed behavior in leverage in the financial crisis.[31] Mikkelsen and Poeschl (2019) show that a bank run affects uncertainty endogenously. An extension along this path would be to incorporate an increase in volatility in the bank run equilibrium. In that case, the model would even better capture the dynamics of leverage due to the additional increase in volatility.

Financial Fragility and Macroeconomic Downside Risk The model shows that there is a substantial increase in financial fragility prior to the bank run. The probability of a run for the next quarter peaks in the period before the run. The median is on average 4% after it increases steadily in the periods before. At the same time, the upper bound of financial fragility is limited as it peaks around 8%. The reason is that agents are aware of the possibility of a bank run which endogenously limits the leverage of bankers due to the incentive constraint shown in equation (43). This limits that the threat of a financial crisis arises. In other words, the model excludes a scenario in which the possibility of a run next period is too large as this is conflicting with the decision of the agents. An extension of the model could relax the rational expectations assumptions. For instance, agents could cognitively discount the probability of a bank run.[32] In such an extension, the probability of a run would increase.

The other important implication of this model is that not every boom ends in a bust. Even though elevated leverage increases the likelihood, the economy can also converge back to normal times. This is the case if either no sufficient contractionary shocks arrive or no sunspot shock materializes. Importantly, this property is in line with recent empirical evidence of Gorton and Ordonez (2020). They show that not each boom results in a bust.

[30] As an example of countercyclical leverage, see for instance the baseline model of Gertler, Kiyotaki and Prestipino (2020), where a bust precedes a bank run.

[31] In fact, my model is the first that allows for new bankers operating in the same period. Other models predict increases in leverage up to 2000% after a bank run. As these newly entering banks have so high leverage, asset prices would increase so much that the bank run equilibrium does not exist anymore.

[32] Gabaix (2020) introduces the idea of cognitive discounting.

Summary of Results Taken together, the quantitative framework reconciles important stylized facts about bank runs. For this reason, the model is well-suited to assess the emergence of financial fragility and study underlying drivers around the recent financial crisis in 2007-2008 from a quantitative perspective. Therefore, I estimate the probability of a financial crisis based on a filter in the next section.

5 Quantitative Assessment: Financial Crisis of 2007 - 2009

I am now turning to the empirical implications of the model. The main goal is to examine the financial fragility around the recently experienced financial crisis. In particular, I want to estimate the likelihood of a bank run in the periods ahead in the run-up to the Great Recession. Furthermore, the structural drivers can be assessed. Finally, the estimated path of shocks can be used to conduct counterfactuals.

The strategy is to employ a filter, which retrieves the sequence of the shocks including the sunspot shock. This in turn can be used to calculate the objects of interest such as the macroeconomic tail-risk. To capture the nonlinearity of the model, I use a particle filter as suggested in Fernández-Villaverde and Rubio-Ramírez (2007).[33] I adapt the particle filter to take into account specifically the multiplicity of equilibria similar to Boragğan Aruoba, Cuba-Borda and Schorfheide (2018). Actually, I extend the approach to handle not only multiplicity of equilibria, but also that the probabilities of the equilibria are endogenously time-varying. This adjustment is necessary to take account of the endogeneity of bank runs. The considered horizon is from 1985:Q1 to 2014:Q4 in line with the calibration.

5.1 Particle Filter

The particle filter can be used to estimate the hidden states and shocks based on a set of observables. It is convenient to cast the model in a nonlinear state-space representation as starting point:

$$\mathbb{X}_t = f(\mathbb{X}_{t-1}, v_t, \iota_t), \tag{72}$$

$$\mathbb{Y}_t = g(\mathbb{X}_t) + u_t. \tag{73}$$

The first set of equations contains the transition equations that depend on the state variables $\mathbb{X}_t$, the structural shocks v_t and the sunspot shock ι_t. In particular, the state variables and shocks determine endogenously the selected equilibrium of the model, which are the normal and the run equilibrium. The transition equations are different for the different regimes. The nonlinear functions f are obtained from the global solution method. The second set of equations contains the measurement equations which connects the state variables with the observables $\mathbb{Y}_t$, which are specified in the next paragraph. It also includes an additive measurement error u_t.

[33] The particle filter uses sequential importance resampling based on Atkinson, Richter and Throckmorton (2019) and Herbst and Schorfheide (2015)

The particle filter extracts a sequence of conditional distributions for the structural shocks $v_t|\mathbb{Y}_{1:t}$ and the sunspot shock $\iota_t|\mathbb{Y}_{1:t}$, which provides the empirical implications of the model.[34] Thereby, the filter evaluates when a bank run occurs and provides the probability of a bank run in the next quarter. The algorithm and the adaptation to the multiplicity of regimes is laid out in Appendix E.

Observables The observables $\mathbb{Y}_t$ are GDP growth and shadow bank leverage, which have been used for the calibration and are shown in the Figure 1 in the introduction. GDP growth is chosen as a model with financial crisis should capture the large reduction in economic activity. GDP growth is measured as the quarter-to-quarter real GDP growth rate. I demean output growth as the trend growth is zero in the model. The model is also fitted to leverage to capture the discussed key trade-off between leverage and financial fragility. I use the shadow bank leverage measure based on shadow bank balance sheet data from Compustat as in the calibration. As stressed in the calibration, the leverage measure is based on book equity, which corresponds directly to the definition of the model.[35] Thus, the observation equation can be written as:

$$\begin{bmatrix} \text{Output Growth}_t \\ \text{Leverage}_t \end{bmatrix} = \begin{bmatrix} 100 \ln \left(\frac{Y_t}{Y_{t-1}} \right) \\ \phi_t \end{bmatrix} + u_t, \tag{74}$$

where the measurement error is given by $u_t \sim N(0, \Sigma_u)$

Measurement Error I include a measurement error in the observation equation. The particle filter requires a measurement error to avoid degeneracy of the likelihood function. Another advantage of including the measurement error is that can take into account noisy data. As it is very complicated to measure shadow bank leverage, the underlying series is potentially very noisy series. The variance of the measurement error is set as a fraction μ_u of the sample variance:

$$\Sigma_u = \mu_u \text{diag}(V(\mathbb{Y}_{1:t})), \tag{75}$$

where V is the covariance matrix of the observables. The measurement error is set to 25% variance of the sample data, which implies $\mu_u = 0.25$.[36]

5.2 Results

To analyze the implications around the recent financial crisis, Figure 5 compares the data for leverage and output with the estimated sequence based on the particle filter. The model can capture the fluctuations in the data as the filtered median and the 68% confidence interval tracks closely the observables. In line with the data, leverage increases substantially prior to

[34] The filter also evaluates the likelihood function of the nonlinear model

[35] Appendix D contains more details on the data construction and its connection with the model.

[36] A measurement error of 25% is a conventional choice and has been used e.g. in Gust et al. (2017).

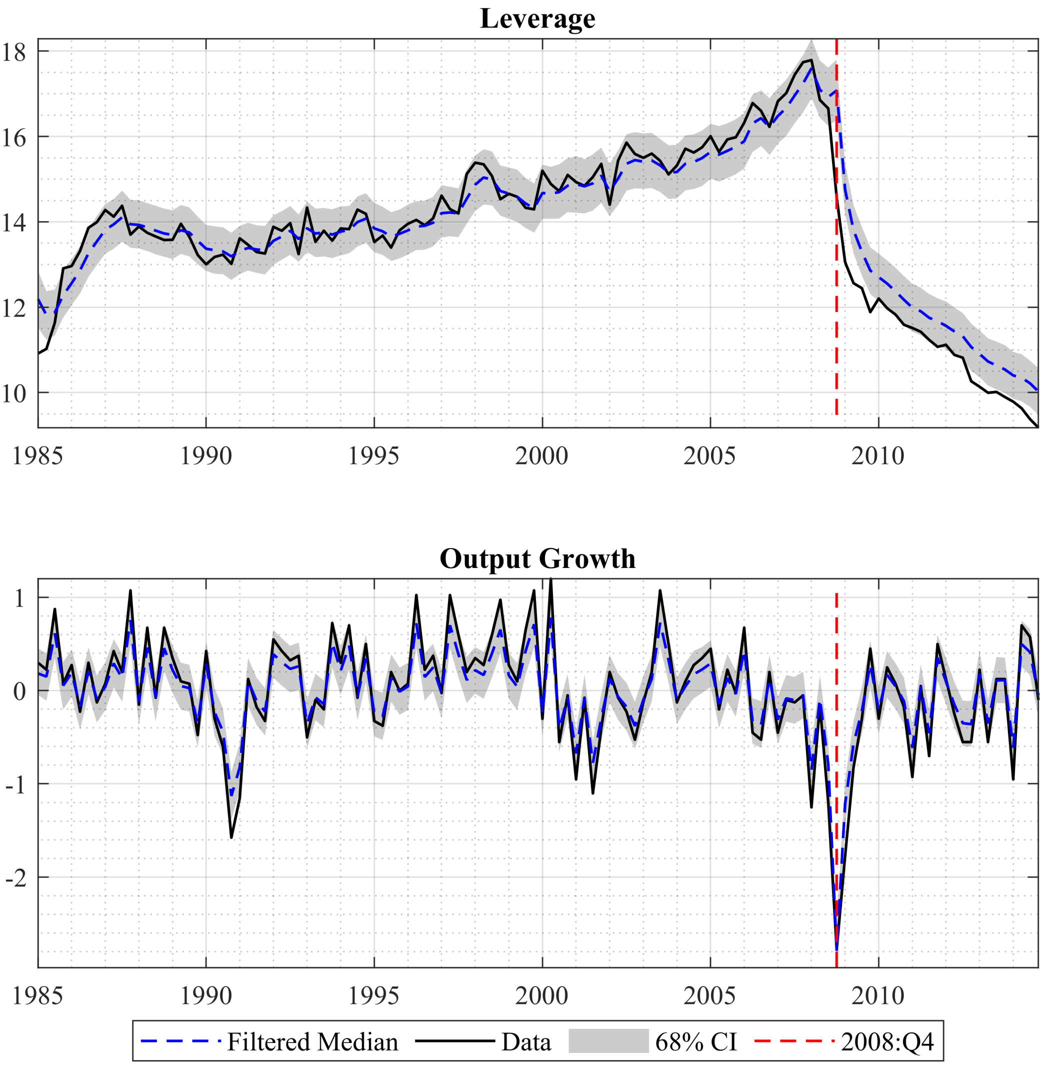

Figure 5: Filtered median of leverage and output growth is the blue line together with its 68% confidence interval. The observables are shadow bank leverage and (demeaned) real output growth. The red line corresponds to the fourth quarter of 2008:Q4.

the financial crisis. The peak is around 2008 with a leverage of close to 18. The model also takes account of the strong decrease in output in the fourth quarter of 2008.

Crucially, the model can account for this sharp drop in the fourth quarter of 2008 event via two different channels: a run on the banking sector or large contractionary shocks. As the equilibria are not exogenously imposed, the particle filter selects the regime depending on the fit with the data. This gives a real-time assessment if a bank run took place. As shown in Figure 6, the median including the confidence intervals select the run regime, which indicates a strong favor of the bank run regime based on the data.[37] Bernanke (2018) and Gorton and Metrick (2012) argue that the run on the shadow banking sector is responsible

[37]The filtered weight of run in the fourth quarter of 2008 is 97%. Otherwise, the weight of the run regime is basically 0% in all remaining periods.

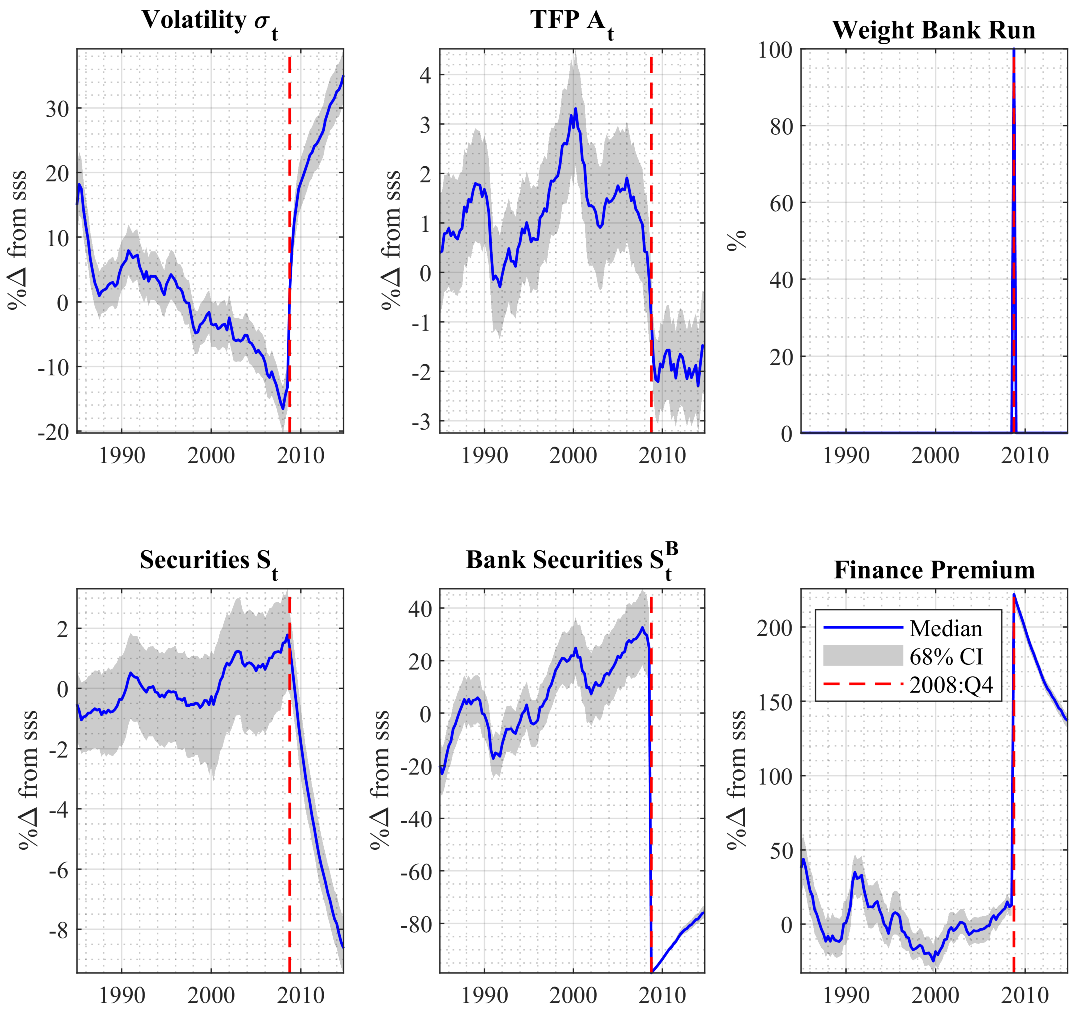

Figure 6: Filtered median of volatility and TFP with its 68% confidence interval. The third plot shows the regime selection. The red line indicates the fourth quarter of 2008.

for the sharp and large decrease in economic activity. To assess this through the lenses of the model, I construct a counterfactual analysis in which no sunspot shock materializes in the fourth quarter of 2008. This scenario without a run results in a diminished economic contraction. To be precise, the bank run results in an additional 2.2% growth reduction in quarterly terms. In that regard, the bank run contributed to 85% of the output drop, while the shocks in isolation are responsible for 15% of the contraction in 2008:Q4.

To inspect the economic drivers behind the bank run in 2008:Q4, the filtered series of volatility and total factor productivity are shown in Figure 6. There is a series of reductions in the volatility prior to the financial crisis. In line with the idea of the volatility paradox, this period sows the seed of a crisis as it increases leverage and thus financial fragility. In the fourth quarter of 2008, there are contractionary volatility and TFP shocks, which trigger the bank run. While contractionary shocks are necessary to enable the run, the extent is very

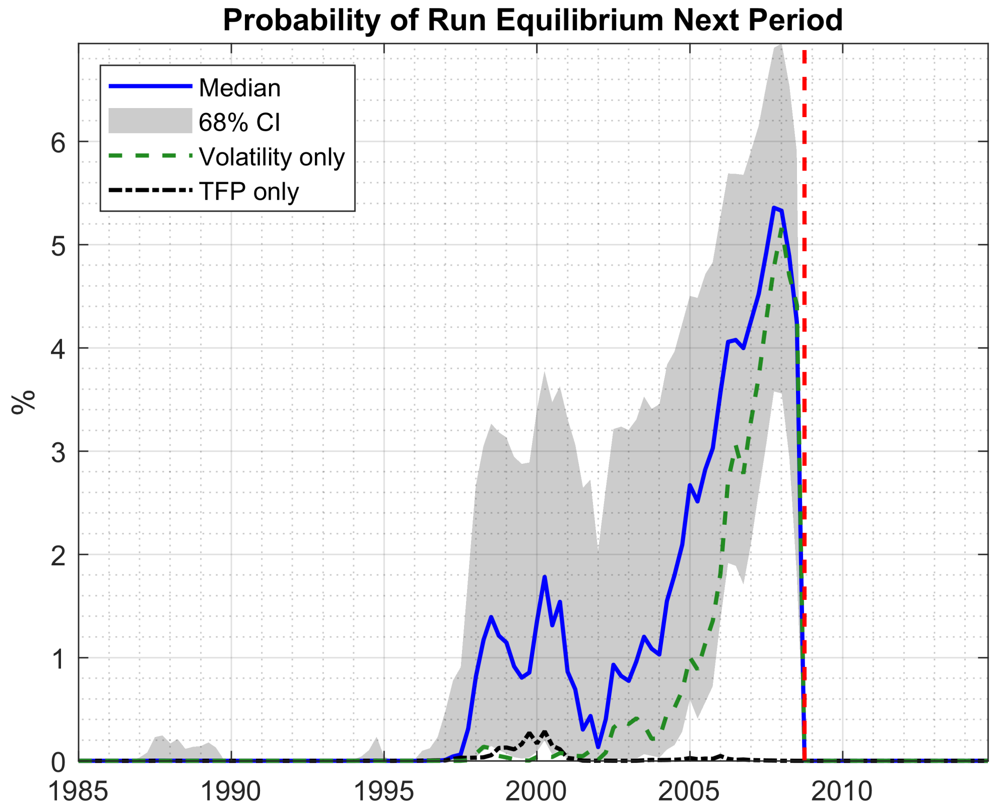

Figure 7: Filtered median probability of a bank run next quarter with its 68% confidence interval. To disentangle the impact of the structural shocks, the realizations of the volatility shock and TFP shock are set to 0 one at a time. The dashed green line is a scenario that only uses the extracted volatility shocks. The black dash-dotted line is a scenario that only uses the extracted TFP shocks. The red line indicates the fourth quarter of 2008.

large to lower leverage after the run. At the same time, related concepts such as national financial conditions index or uncertainty measures spike up in the fourth quarter in line with the predictions.

As a validity check of the empirical experiment, the filtered series of securities and finance premium can be compared to events in the financial crisis. There is a securities boom that begins in the early 2000s, which ends in a severe credit crunch. Furthermore, the finance premium is also very low prior to 2007. With the bank run, there is a large spike in the finance premium. As the filter did not target credit or a credit spread such as BAA - 10 year government spread, this corroborates the predictions of the model. Taken together, the model can reconcile important dynamics of the recent financial crisis. This is a crucial requirement as this enables to assess the emergence of financial fragility through the lenses of the model.

Financial Fragility Figure 7 shows the probability of a bank run in the period ahead, which is a measure of financial fragility. While there is a slight increase around the recession in 2001, there is a remarkable increase from 2004 onwards. In that regard, the model finds a strong increase in financial fragilities preceding the financial crisis by a few years. The rising

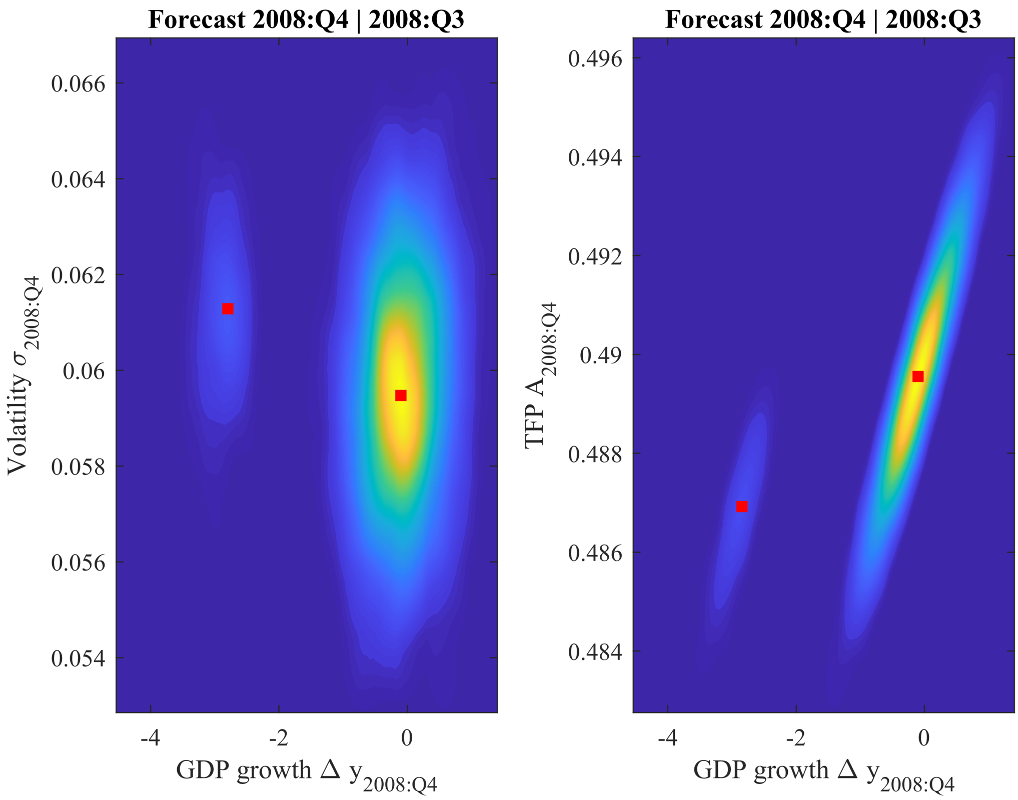

Figure 8: The Figure shows the multimodality in the one quarter ahead forecast that arises due to the bank run equilibrium using contour plot. The contour plots display the joint distribution of GDP growth and volatility respectively TFP for 2008:Q4 conditional on 2008:Q3. Yellow indicates a high density, while dark blue indicates a low density The red square shows the two modes. The forecasts are conditioned on the median realization in 2008:Q3.

leverage is the reason for this.

The probability of a bank run peaks in 2008 with a probability of around 5% for the next quarter. This estimate can be also mapped in a probability of a bank run in the next four quarters. In particular, the probability to observe a financial crisis during 2008 was close to but below 20% from a 2007:Q4 perspective, which highlights the high level of financial fragility. Through the lenses of the model, the financial crisis is a tail risk, even though a substantial one.

As a next step, a counterfactual analysis is considered to disentangle the structural sources of the financial crisis. Only the extracted sequence of shocks for one shock is used at a time, while the other shock realizations are set to 0. This not only allows to analyse the impact of each shock individually, but also the nonlinear impact of a combination. While the total factor productivity has a modest impact around the recession in 2001, it does not cause financial fragility in isolation in the run-up to the financial crisis. In contrast to this, the volatility shock is the main driver as it explains more than 90% in 2008. Nevertheless, the combination of the shocks can increase financial fragility. This can be seen in the years preceding 2007 because financial fragility is considerably higher in a scenario with both shocks than the sum of them in isolation.

Macroeconomic Tail Risk and Multimodality The increase in financial fragility causes large macroeconomic downside risk as the possibility of a bank run emerges. To assess the downside risk, I evaluate the one quarter ahead distribution of output and structural shocks. Figure 8 shows a contour plot of the one quarter ahead joint distribution of output and the structural shocks for 2008:Q4 conditional on 2008:Q3. The distribution of output is bimodal due to the possibility of a bank run. The normal equilibrium is associated with output growth centered around 0. The run equilibrium is associated with a large economic contraction of around minus 2 to 3 percent matching the actual drop in of -2.7% in 2008:Q4. This analysis also highlights the importance of contractionary shocks for the emergence of a crisis that trigger the run. The run equilibrium is associated with high volatility and low TFP.

The multimodality of equilibria has also been detected recently using a non-structural approach by Adrian, Boyarchenko and Giannone (2019*a*). They estimate conditional joint distributions of economic fundamentals and financial conditions. Similar to the predictions in the model, they find the occurrence of a second equilibrium for 2008:Q4 conditional on 2008:Q3.[38] Similar to my finding, the probability of the normal equilibrium is larger. Importantly, the possibility of a bank run causes the appearance of a multimodal distribution.

6 Leverage Tax

A discussed idea in policy circles is to implement a leverage tax - a tax on the deposit holdings - for shadow banks. Specifically, the "Minnesota Plan to End too Big too Fail" from the Minneapolis Federal Reserve Bank in 2017 proposes to tax the borrowing of shadow banks.[39] It is potentially very difficult to regulate shadow banks as these are financial intermediaries that by definition operate outside the regulatory banking umbrella. Another problem poses that a reliable measure of the leverage of shadow banks is quite challenging. A tax on the borrowing is a simple and tractable approach, which is well suited for that reason.

From a theoretical perspective, the models provides a motive for macroprudential policy for banks because of a bank run externality. Agents do not take into account the impact of their own decisions on the bank run probability. As a consequence, leverage is potentially too high, especially during good times when financial vulnerability arises endogenously. On that account, the leverage tax could be interpreted in the Pigouvian sense as this policy aims to correct undesirable leverage accumulation, which makes the banking system runnable in the first place.[40]

However, the impact of the leverage tax on output and financial stability are ambiguous.

[38] While I find the multiplicity of equilibria for a prolonged horizon, their study finds the multiplicity of equilibria only for this one period.

[39] The plan consists of several recommendations for the entire banking sector. The focus in this paper is only on the leverage tax. In line with the mentioned proposal, the leverage tax only applies to shadow banks in the model.

[40] The proposed tax is constant and does not vary with the business cycle. This could be a limitation as the bank run externality is state-dependent as the probability of a bank run depends on economic fundamentals. For instance, it is not needed to regulate the shadow banks during normal times without the threat of a bank run.

The leverage tax could increase output costs in non-crisis times, but could lower the frequency of a financial crisis. In that regard, the change in economic activity in fluctuations are unclear. Furthermore, the impact of the leverage tax on financial stability is ambiguous as it causes different opposing effects that create a potential trade-off. In fact, there is an income effect, a substitution effect and an indirect effect.

The leverage tax requires the banker to pay at the end of the period a tax τ for its borrowings from households. This income effect changes the net worth accumulation, which is now given as:

$$N_t = R_t^K Q_{t-1} S_{t-1}^B - R_{t-1}^D D_{t-1} - \tau^\phi D_{t-1}. \tag{76}$$

The tax is costly and lowers the net worth for a given leverage choice in partial equilibrium. As a consequence, the recovery ratio in case of a bank run is diminished taking all other variables as given:

$$x_t = \frac{[(1-\delta)Q_t^\star + Z_t^\star]S_{t-1}^B}{\left(\bar{R}_{t-1} + \tau^\phi\right) D_{t-1}}. \tag{77}$$

The income effect actually increases the threat of a financial crisis. The substitution effect operates over the risk-shifting incentives. Specifically, the gain from limited liability increases, which makes the substandard securities more attractive. The threshold value $\overline{\omega}_t$ for the idiosyncratic volatility of the substandard security where the banker can exactly cover the face value of the deposits is now larger:

$$\overline{\omega}_t = \frac{\bar{D}_{t-1}}{R_t^k Q_{t-1} S_{t-1}^{Bj}}. \tag{78}$$

As a larger realization of the idiosyncratic return is needed to avoid bankruptcy, the gain from limited liability $\tilde{\pi}_t$ increases. The incentive constraint as shown in equation (43) requires thus a lower leverage to ensure an investment in the good security. This substitution effect lowers leverage and thereby lowers the bank run probability.

The reduction in net worth and leverage forces the banks to hold less assets so that the relative share of assets that is intermediated via the banking sectors falls. In case of a firesale, the drop in the price is now more moderate as the households have to absorb fewer securities. This indirect effect directly lowers the systemic risk. The reduced amount of assets stabilizes the economy and lowers the exposure to bank runs.

Quantitative Evaluation of the Leverage Tax To assess the ambiguous impact on output and financial stability, a quantitative solution is required. The leverage tax is calibrated to $\tau = 0.625/100$, which implies that the banks have to pay a leverage tax of 0.25% annually on their deposits. To put the size of the tax in proportion, the leverage tax corresponds approximately to 1/10 of the external finance premium.

Table 2 shows selected moments of the economy that are obtained from simulating the economy with and and without the leverage tax for 500000 periods. The major take-away

Table 2: Selected moments of the model with and without a leverage tax

Model Specification/Moments		Run probability	$\overline{S_t^B/S_t}$	$\overline{y}$	$\sigma(y)$	$\overline{\phi}$	$\sigma(\phi)$
With Runs	Baseline model	2.54%	32.6%	0.986	0.52	14.8	2.04
	Leverage tax τ_t	1.83%	31.3%	0.982	0.505	14.7	2.00
Without Runs	Baseline model	0%	38.6%	0.994	0.47	14.5	1.59
	Leverage tax τ_t	0%	35.3%	0.987	0.47	14.5	1.67

is that the leverage tax helps to reduce the frequency of bank runs. The annual bank run probability drops to 1.8% compared to 2.5% in a world without taxes, which shows the potential stabilization impact of the leverage tax. A lower frequency of bank runs also helps to reduce the volatility of output. However, the tax lowers output in non-crisis times, so that average output in total is lower than in the baseline model without a tax. This highlights the potential trade-off between output and financial stability in the long run.

Even though leverage falls slightly, the main mechanism for the reduction in the frequency of bank runs is the drop in the market share of shadow bank assets S_t^B/S_t. The market share falls by more than 1% in the model with runs. As shown in the same table, the relative drop is even larger in an economy without the realization of bank runs. Agents take into account the possibility of a bank run, however a bank run does not materialize.[41] In fact, the relative small effect of the leverage tax on leverage is also emphasized in the no run scenario.

In addition to this, the impact of the leverage tax for the build-up of financial vulnerabilities in the financial crisis in 2007 and 2008 can be evaluated. Based on the extracted sequence of distribution of shocks in Section 5, a counterfactual scenario with a leverage tax can be studied. In other words, the filtered shocks are fed in the model to recalculate the evolution of the economy with a leverage tax.[42] Figure 9 shows the impact of the leverage tax on the vulnerability of the economy to a financial crisis between 1985 and 2014. This allows to compare the baseline prediction with its confidence interval to the median realization with a leverage tax. The leverage tax would have lowered the bank run probability between 0.5 and 1 percentage points.

7 Reduced Form Evidence: Quantile Regressions

The structural model predicts an increasing probability of a financial crisis already from 2004:Q4 onwards due to increasing leverage in the shadow banking sector. I want to compare this prediction to the results from a non-structural approach. Specifically, I am interested in studying the tail-risk of output growth conditional on shadow banking leverage. For this reason, it is useful to focus on the entire distribution of GDP growth instead of a single

[41] This is the case if the realization of the sunspot shock is always 0, that is $\iota_t = 0 \; \forall t$.
[42] In this exercise, it is assumed that the leverage tax is active for the entire period.

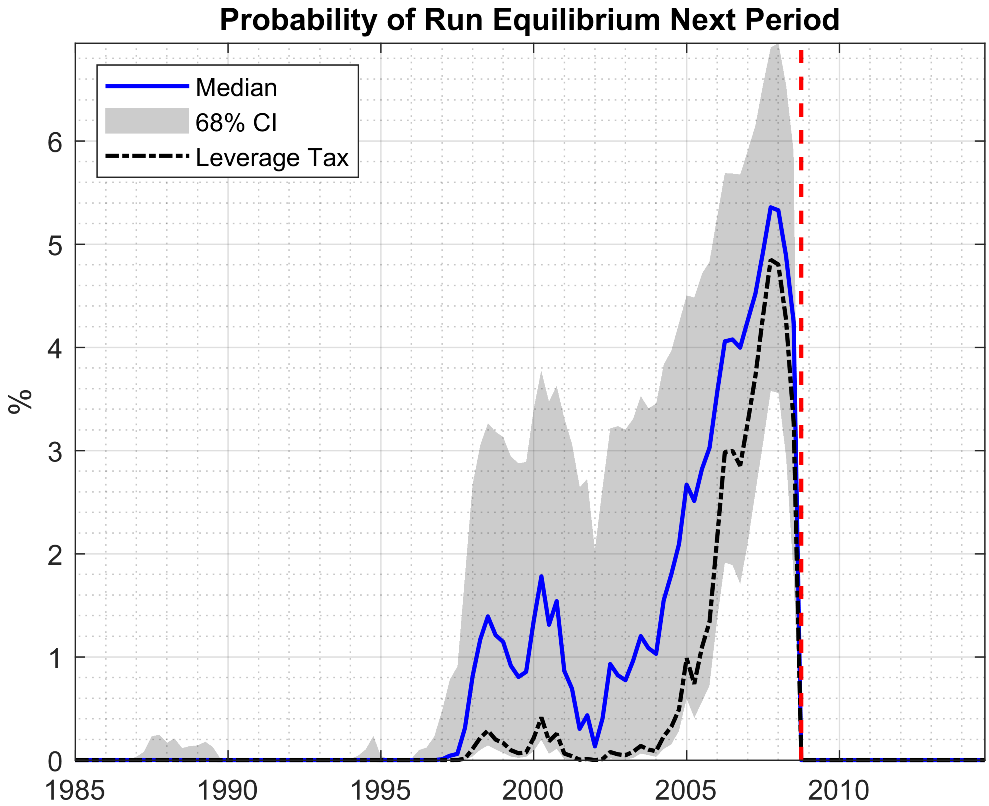

Figure 9: Filtered median probability (blue) of a bank run next quarter with its 68% confidence interval in the baseline model is compared to the median (red) of the model with the leverage tax.

estimate such as the mean forecast. Using this distribution, I can assess the behavior of the tails over time. For that purpose, I am calculating the distribution of GDP growth one year ahead based on the econometric approach of Adrian, Boyarchenko and Giannone (2019*b*), which is based on quantile regressions. The forecast can be conditioned on a different set of variables, which allows to evaluate the impact of shadow banking leverage on estimated downside risk. Even though there have been several studies focusing on different predictors, the role of shadow bank leverage has not been assessed.[43]

Econometric Specification The starting point of the econometric specifications are quantile regressions (Koenker and Bassett Jr, 1978). I estimate different quantiles of real GDP growth one year ahead, which is defined as the average growth rate in the last four quarters $\bar{y}_{t+4} = \sum_{j=1}^{4}(y_{t+j} - y_{t+j-1})/4 = \sum_{j=1}^{4}(\Delta y_{t+j})/4$. I regress the one year ahead GDP on current real GDP growth $\Delta y_t = y_t - y_{t-1}$ and current shadow banking leverage ϕ_t:

$$\hat{Q}_\tau(\bar{y}_{t+4}|x_t) = x_t\hat{\beta}_\tau, \tag{79}$$

[43]See e.g. Giglio, Kelly and Pruitt (2016), Hasenzagl, Reichlin and Ricco (2020) and Loria, Matthes and Zhang (2019), among others.

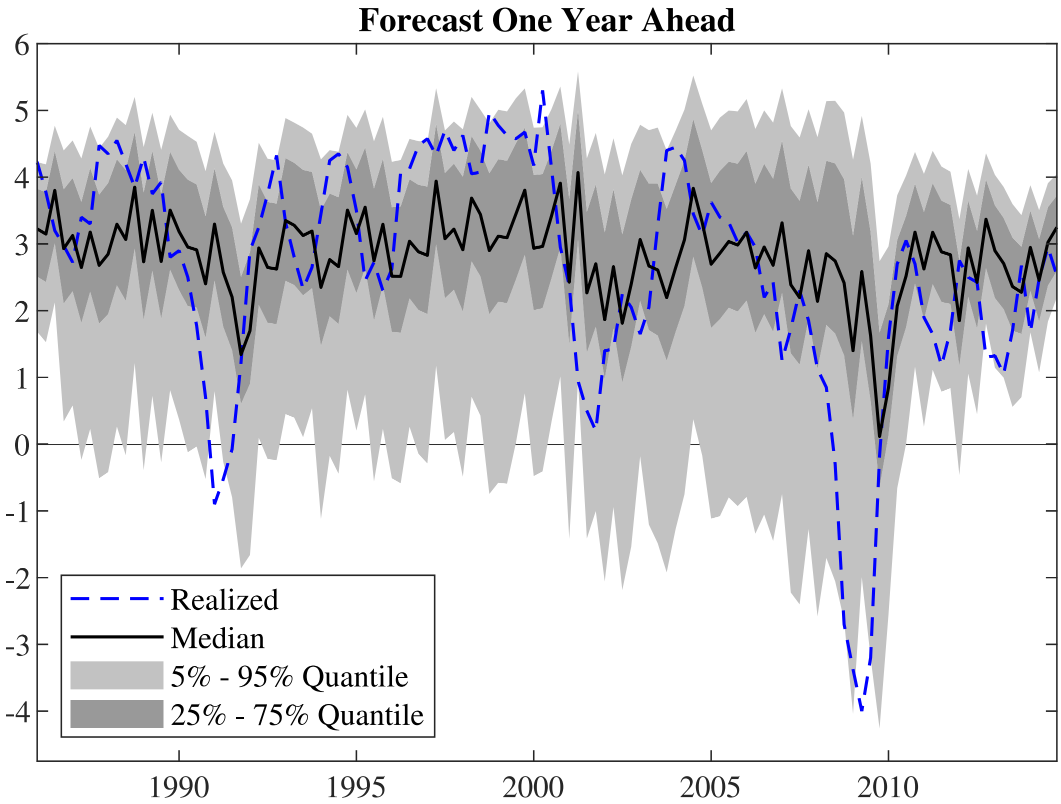

Figure 10: The one year ahead forecast of real GDP conditional on current GDP growth and shadow bank leverage using data from 1985:Q1 to 2014:Q4. Different quantiles and the realized value are displayed.

where $\hat{Q}_\tau$ is a consistent linear estimator of the quantile function, τ indicates the chosen quantile and $x_t = [\Delta y_t \ \phi_t]$. $\hat{\beta}_\tau$ minimizes the quantile weighted value of absolute errors:

$$\beta_\tau = \arg\min_{\beta} \sum_{t=1}^{T-4} \left(\tau \mathbf{1}_{y_{t+4} \geq x_t\beta_\tau} \left| y_{t+4} - x_t\beta_\tau \right| + (1-\tau)\mathbf{1}_{y_{t+4} < x_t\beta_\tau} \left| y_{t+4} - x_t\beta_\tau \right| \right), \tag{80}$$

where $\mathbf{1}$ is an indicator function.

Results I estimate the quantile regressions using data from 1985:Q1 to 2014:Q4 in line with the structural model. Figure 10 displays the one year-ahead forecast of GDP, which is also compared to the realized GDP growth. The 95% quantile, which is the upper end of the area and can be interpreted as the upside risk, is very stable over the entire horizon. In contrast to this, the 5% quantile, which is the lower end of the area and measures the macroeconomic downside risk, is much more volatile.

To begin with, the downside risk during the recessions is discussed. If GDP growth is very low, the macroeconomic downside risk increases. For this reason, the downside risk is very large in each recession (1990-1991, 2001, 2007 -2009) that is observed in the sample.

However, there is a difference between the financial crisis and the other recessions. There was increasing macroeconomic downside risk already prior to the financial crisis, which is not observed for the other recessions. The 5% quantile falls from 2004 onward considerably until the fourth quarter of 2008. This suggests the build-up of downside risk due to the financial crisis already in 2004 in line with the increase in shadow bank leverage.

In fact, rising leverage drives the increasing downside risk from 2004 onwards. To demonstrate this, I compare 5% quantile to a specification that is only conditioning on current GDP growth. While the differences in the downside risk for the different specifications are negligible for the recessions in 1990-1991 and in 2001, the difference is very large until the fourth quarter in 2008. For instance, the estimated 5% quantile for 2008:Q4 is -2.6% in the baseline model, where leverage is included. In contrast to this, the 5% quantile is only -1.0% if it is only conditioned on GDP. After the fourth quarter in 2008, the downside risk measures for both specifications are closer as current GDP falls. Taken together, shadow bank leverage seems to be connected to the emergence of financial fragility.

A related measure of macroeconomic downside risk is the probability that GDP drops below a specified level - for instance below the annualized growth rate in 2008:Q4. Using the results from the quantile regressions, this measure can be derived as shown in the Appendix F. In a nutshell, the conditional quantile estimates need to be mapped into a quantile distribution.[44] Based on the quantile distribution, the measure can then be derived. The main prediction is that a specification without conditioning on leverage predicts a considerably lower probability. To be precise, the inclusion of leverage almost doubles the probability of a large economic contraction starting from 2004 until the arrival in 2008. In line with the previous result, this suggests again a tight link between leverage and macroeconomic downside risk. This difference in the probability is unique to the period before the financial crisis. In the Appendix F, the conditional probability of a drop in output as in 2008:Q4 is shown for the entire horizon.

8 Conclusion

I investigate the financial crisis of 2007 - 2009 through the lenses of a new nonlinear macroeconomic model that is taken to the data. Understanding the dynamics that lead to a financial crisis is very important as it can help to detect financial fragilities early on in the future and allows to understand the impact of macroprudential policies.

The first contribution is to highlight the build-up of shadow bank leverage as key determinant in the emergence of a bank run. In particular, elevated leverage causes financial fragility, which sets the stage for a financial crisis. I show that the interaction of high leverage with the endogenous occurrence of a bank run can reconcile a credit boom gone bust and countercyclical credit spreads.

My second contribution is the quantitative analysis of the recent financial crisis. Fitting the model to important moments of the shadow banking sector in the U.S., I can estimate the model-implied probability of a bank run in the 2000s based on a particle filter. Crucially,

[44]In practice, I am using a skewed t-distribution to approximate the quantile function as discussed in the Appendix F.

I find that financial fragility starts to increase considerably from 2004 onwards. Conditional on 2007:Q4, the model predicts that the probability of a bank run during 2008 is close to but below 20%.

The framework is also used to evaluate macroprudential policies. Specifically, a leverage tax based on a proposal from the Minneapolis Federal Reserve Bank in 2017 would reduce the risk of a bank run. Based on a counterfactual analysis around the recent financial crisis, I show that a conservatively calibrated leverage tax could have mitigated the probability of a bank run by around 10%.

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A Data

A.1 Leverage of Shadow Banks

The leverage series in this paper uses book equity, which is the difference between the value of portfolio claims and liabilities of financial intermediaries. An alternative measure is the financial intermediaries' market capitalization (e.g. market valuation of financial intermediaries). The appropriate concept in this context is book equity because the interest is on credit supply and financial intermediaries' lending decision as stressed for instance in Adrian and Shin (2014).[45] In contrast to this, market capitalization is the appropriate measure related to the issuance of new shares or acquisition decisions (Adrian, Colla and Shin, 2013). In the context of the model, the occurrence of a bank run also depends on book equity, which rationalizes this choice. On that account, book leverage based on book equity is the appropriate concept in our context.

A related issue is that marked-to-market value of book equity, which is the difference between the market value of portfolio claims and liabilities of financial intermediaries, is conceptually very different from market capitalization. As argued in Adrian and Shin (2014), the book value of equity should be measured as marked-to-market. In such a case, the valuation of the assets is based on market values. Importantly, the valuation of assets is marked-to-market in the balance sheet of financial intermediaries that hold primarily securities (Adrian and Shin, 2014). Crucially, the concept of marked-to-market value of book equity corresponds to the approach of leverage in the model as the value of the securities depends on their market price. Therefore, the interest is marked-to-market book leverage.

A.2 Compustat

The book leverage of the shadow banking sector is constructed using balance sheet data from Compustat. I include financial firms that are classified with SIC codes between 6141 - 6172 and 6199 - 6221. This characterization contains credit institutions, business credit institutions, finance lessors, finance services, mortgage bankers and brokers, security brokers, dealers and flotation companies, and commodity contracts brokers and dealers.[46] In total, the unbalanced panel consists out of 562 firms.[47]

Equity is computed as the difference between book assets and book liabilities for each firm is:

$$Equity_{i,t} = Book\ Assets_{i,t} - Book\ Liabilities_{i,t}. \tag{A.1}$$

[45] He, Khang and Krishnamurthy (2010) and He, Kelly and Manela (2017) provide an opposing view with an emphasis on market leverage.

[46] Finance lessors and finance services with the SIC codes 6172 and 6199 are not official SIC codes, but are used by the U.S. Securities and Exchange commission.

[47] As a robustness check, I keep only firms that have balance sheet data for at least two consecutive years. While the number of firms reduces to 462, the leverage series and its moments are robust to this change.

The leverage of the shadow banking sector is then defined as

$$Leverage_t = \frac{\sum_i Book\ Assets_{i,t}}{\sum_i Book\ Equity_{i,t}}, \tag{A.2}$$

where I sum up equity and assets over the different entities.

A.3 Flow of Funds

An alternative to this leverage measure is to use data from the Flow of Funds as in Nuno and Thomas (2017). A caveat is that there is only balance sheet data for assets and liabilities available for a limited set of financial firms. In particular, I can calculate the leverage using assets and equity for finance companies as well as security brokers and dealers:[48]

$$Leverage_t = \frac{Assets\ Finance\ Companies_t + Assets\ Security\ Brokers\ and\ Dealers_t}{Equity\ Finance\ Companies_t + Equity\ Security\ Brokers\ and\ Dealers_t}. \tag{A.3}$$

A more narrow measure would rely only on the security broker and dealers as in Adrian and Shin (2010) because these are the marginal investors. While there is a shift in the level, the implications are very similar to the other two measures.

[48] I adjust for discontinuities and breaks in the data.

B Model Equations and Equilibrium

The equilibrium conditions for the normal equilibrium are shown, and afterwards the bank run equilibrium is discussed.

B.1 Normal Equilibrium

Households

$$C_t = W_t L_t + D_{t-1} R_t - D_t + \Xi_t + Q_t S_t^H + (Z_t + (1-\delta) Q_t) S_{t-1}^H, \tag{B.1}$$

$$\varrho_t = (C_t)^{-\sigma}, \tag{B.2}$$

$$\varrho_t W_t = \chi L_t^{\varphi}, \tag{B.3}$$

$$1 = \beta E_t \Lambda_{t,t+1} R_{t+1}, \tag{B.4}$$

$$1 = \beta E_t \Lambda_{t,t+1} \frac{Z_{t+1} + (1-\delta) Q_{t+1}}{Q_t + \Theta(S_t^H / S_t - \gamma^F)/\varrho_t}, \tag{B.5}$$

$$\beta E_t \Lambda_{t,t+1} = \beta E_t \varrho_{t+1} / \varrho_t. \tag{B.6}$$

Banks

$$Q_t S_t^B = \phi_t N_t, \tag{B.7}$$

$$\overline{\omega}_t = \frac{\phi_{t-1} - 1}{R_t^K \phi_{t-1}}, \tag{B.8}$$

$$(1-p_t) E_t^N [\beta \Lambda_{t,t+1} \bar{R}_t D_t] + p_t E_t^R [\beta \Lambda_{t,t+1} R_{t+1}^K Q_t S_t^B] \geq D_t, \tag{B.9}$$

$$(1-p_t) E_t^N [\Lambda_{t,t+1} R_{t+1}^K (\theta \lambda_{t+1} + (1-\theta))[1 - e^{\frac{-\psi}{2}} - \tilde{\pi}_{t+1}]] = p_t E_t^R [\Lambda_{t,t+1} R_{t+1}^K (e^{-\frac{\psi}{2}} - \overline{\omega}_{t+1} + \tilde{\pi}_{t+1})], \tag{B.10}$$

$$\lambda_t = \frac{(1-p_t) E_t^N \Lambda_{t,t+1} R_{t+1}^K [\theta \lambda_{t+1} + (1-\theta)](1 - \overline{\omega}_{t+1})}{1 - (1-p_t) E_t^N [\Lambda_{t,t+1} R_{t+1}^K \overline{\omega}_{t+1}] - p_t E_t^R [\Lambda_{t,t+1} R_{t+1}^K]}, \tag{B.11}$$

$$\kappa_t = \frac{\beta (1-p_t) E_t^N \Lambda_{t,t+1} \left[\lambda_t - (\theta \lambda_{t+1} + 1 - \theta)\right]}{(1-p_t) E_t^N \Lambda_{t,t+1} \left[(\theta \lambda_{t+1} + 1 - \theta) \tilde{F}_{t+1}(\overline{\omega}_{t+1})\right] + p_t E_t^R \Lambda_{t,t+1} \left[(\theta \lambda_{t+1} + 1 - \theta) \left(1 - \tilde{F}_{t+1}(\overline{\omega}_{t+1})\right)\right]}, \tag{B.12}$$

$$N_{S,t} = R_t^K Q_{t-1} S_{t-1}^B - \overline{R}_{t-1} D_{t-1}, \tag{B.13}$$

$$N_{N,t} = (1-\theta) \zeta S_{t-1}, \tag{B.14}$$

$$N_t = \theta N_{S,t} + N_{N,t}. \tag{B.15}$$

Non-Financial Firms

$$Y_t = A_t (K_{t-1})^{\alpha} (L_t)^{1-\alpha}, \tag{B.16}$$

$$K_t = S_t, \tag{B.17}$$

$$MC_t (1-\alpha) \frac{Y_t}{L_t} = W_t, \tag{B.18}$$

$$R_t^K = \frac{Z_t + Q_t(1-\delta)}{Q_{t-1}}, \tag{B.19}$$

$$Z_t = MC_t \alpha \frac{Y_t}{K_{t-1}}, \tag{B.20}$$

$$(\Pi_t - \Pi_{SS})\Pi_t = \frac{\epsilon}{\rho^r}\Big(MC_t - \frac{\epsilon - 1}{\epsilon}\Big) + \Lambda_{t,t+1}, (\Pi_{t+1} - \Pi_{SS})\Pi_{t+1}\frac{Y_{t+1}}{Y_t}, \tag{B.21}$$

$$\Gamma\Big(\frac{I_t}{K_t}\Big) = a_1\Big(\frac{I_t}{K_t}\Big)^{(1-\eta)} + a_2, \tag{B.22}$$

$$Q_t = \left[\Gamma'\Big(\frac{I_t}{S_{t-1}}\Big)\right]^{-1}, \tag{B.23}$$

$$S_t = (1-\delta)S_{t-1} + \Gamma\Big(\frac{I_t}{S_{t-1}}\Big)S_{t-1}. \tag{B.24}$$

Monetary Policy and Market Clearing

$$i_t = \frac{1}{\beta}\Pi_t^{\kappa_\Pi}(Y_t/Y_{SS})^{\kappa_y}, \tag{B.25}$$

$$\beta\Lambda_{t,t+1}\frac{i_t}{\Pi_{t+1}} = 1, \tag{B.26}$$

$$Y_t = C_t + I_t + G + \frac{\rho^r}{2}(\Pi_t - 1)^2 Y_t, \tag{B.27}$$

$$S_t = S_t^H + S_t^B. \tag{B.28}$$

Shocks

$$\sigma_t = (1-\rho^\sigma)\sigma + \rho^\sigma\sigma_{t-1} + \sigma^\sigma\epsilon_t^\sigma, \tag{B.29}$$

$$A_t = (1-\rho^A)A + \rho^A A_{t-1} + \sigma^A\epsilon_t^A, \tag{B.30}$$

$$\iota_t = \begin{cases} 1 & \text{with probability}\Upsilon \\ 0 & \text{with probability}1-\Upsilon \end{cases}. \tag{B.31}$$

Bank Run Specific Variables

$$p_t = \text{prob}(x_{t+1} < 1)\Upsilon, \tag{B.32}$$

$$R_t = \begin{cases} \bar{R}_{t-1} & \text{if no bank run takes place in period } t \\ x_t\bar{R}_{t-1} & \text{if a bank run takes place in period } t \end{cases}, \tag{B.33}$$

$$x_t = \frac{[(1-\delta)Q_t^\star + Z_t^\star]S_{t-1}^B}{\bar{R}_{t-1}D_{t-1}}, \tag{B.34}$$

$$\tag{B.35}$$

where $\star$ indicates the variables conditional on the run equilibrium

B.2 Bank Run Equilibrium

The bank run equilibrium has almost the same equations as the normal equilibrium, which are not repeated for convenience. In the bank run equilibrium, all banks from the previous period are bankrupt due to $x_t < 1$. Thus, the net worth of surviving banks is zero:

$$N_{S,t} = 0. \tag{B.36}$$

Furthermore, the return on deposits is lower than the promised one and is given as:

$$R_t = x_t \bar{R}_{t-1} \tag{B.37}$$

As the banking sector starts to rebuild in the same period, the other banking equations are unchanged.

C Derivation of Banker's Problem

In the following, I derive the banker's problem for two cases: (i) absence of bank runs and (ii) anticipation of bank runs.

C.1 Absence of bank runs

The banker maximises the value of its bank V_t subject to a participation and incentive constraint, which reads as follows:[49]

$$
\begin{aligned}
V_t^j(N_t^j) &= \max_{S_t^{Bj}, \bar{D}_t} \beta E_t \Lambda_{t,t+1} \Big[\theta V_{t+1}^j(N_{t+1}^j) + (1-\theta)(R_{t+1}^K Q_t S_t^{Bj} - \bar{D}_t^j) \Big], && \text{(C.1)} \\
\text{subject to} \quad & \beta E_t[\Lambda_{t,t+1} \bar{R}_t^D D_t^j] \geq D_t^j, && \text{(C.2)} \\
& \beta E_t \Lambda_{t,t+1} \Big\{ \theta V_{t+1}^j(S_t^{Bj}, \overline{D}_t^j) + (1-\theta)[R_{t+1}^K Q_t S_t^{Bj} - \overline{D}_t^j] \Big\} \geq && \text{(C.3)} \\
& \beta E_t \Lambda_{t,t+1} \int_{\overline{\omega}_{t+1}^j}^{\infty} \Big\{ \theta V_{t+1}(\omega, S_t^{Bj}, \overline{D}_t^j) + (1-\theta)[R_{t+1}^K Q_t S_t^{Bj} \omega_{t+1}^j - \overline{D}_t^j] \Big\} d\tilde{F}_{t+1}(\omega).
\end{aligned}
$$

The banker's problem can be written as the following Bellman equation:

$$
\begin{aligned}
V_t(N_t^j) = \max_{\{S_t^{Bj}, \overline{b}_t^j\}} & \beta E_t \Lambda_{t,t+1} \Bigg[\theta V_{t+1} \bigg(\Big(1 - \frac{\overline{b}_t^j}{R_{t+1}^K}\Big) R_{t+1}^K Q_t S_t^{Bj} \bigg) + (1-\theta)\big(1 - \frac{\overline{b}_t^j}{R_{t+1}^K}\big) R_{t+1}^K Q_t S_t^{Bj} \Bigg] \\
& + \lambda_t^j \Big[\beta E_t \Lambda_{t,t+1} Q_t S_t^{Bj} \overline{b}_t^j - (Q_t S_t^{Bj} - N_t^j) \Big] \\
& + \kappa_t^j \beta E_t \Lambda_{t,t+1} \Bigg\{ \Bigg[\theta V_{t+1} \bigg(\Big(1 - \frac{\overline{b}_t^j}{R_{t+1}^K}\Big) R_{t+1}^K Q_t S_t^{Bj} \bigg) + (1-\theta)\Big(1 - \frac{\overline{b}_t^j}{R_{t+1}^K}\Big) R_{t+1}^K Q_t S_t^{Bj} \Bigg] \\
& - \int_{\frac{\overline{b}_t^j}{R_{t+1}^K}}^{\infty} \Bigg[\theta V_{t+1} \bigg(\Big(\omega - \frac{\overline{b}_t^j}{R_{t+1}^K}\Big) R_{t+1}^K Q_t S_t^{Bj} \bigg) + (1-\theta)\Big(\omega - \frac{\overline{b}_t^j}{R_{t+1}^K}\Big) R_{t+1}^K Q_t S_t^{Bj} \Bigg] d\tilde{F}_{t+1}(\omega) \Bigg\}
\end{aligned}
$$

where I defined $\overline{b}_t^j = \overline{D}_t^j / \left(Q_t S_t^B\right)$ and used that

$$
N_t^j = \begin{cases} \Big(1 - \frac{\overline{b}_{t-1}^j}{R_t^K}\Big) R_t^K Q_{t-1} S_{t-1}^{Bj} & \text{if standard security} \\ \Big(\omega - \frac{\overline{b}_{t-1}^j}{R_t^K}\Big) R_t^K Q_{t-1} S_{t-1}^{Bj} & \text{if substandard security} \end{cases} \tag{C.4}
$$

λ_t^j and κ_t^j are the Lagrange multiplier of the participation and incentive constraint. The first order conditions are

$$
\begin{aligned}
0 =& \beta E_t \Lambda_{t,t+1} R_{t+1}^K [\theta V_{t+1}^{\prime,j} + (1-\theta)](1 - \overline{\omega}_{t+1}^j) + \lambda_t^j E_t[\beta \Lambda_{t,t+1} R_{t+1}^K \overline{\omega}_{t+1}^j - 1] \\
& + \kappa_t^j \beta E_t \Lambda_{t,t+1} R_{t+1}^K \Bigg\{ [\theta V_{t+1}^{\prime,j} + (1-\theta)](1 - \overline{\omega}_{t+1}^j) - \int_{\overline{\omega}_{t+1}^j}^{\infty} \Big[[\theta V_{t+1}^{\prime,j} + (1-\theta)](\omega - \overline{\omega}_{t+1}^j) \Big] d\tilde{F}_{t+1}(\omega) \Bigg\}
\end{aligned}
$$

[49] The derivation is based on Nuño and Thomas (2017).

and

$$
\begin{aligned}
0 =& -\beta E_t \Lambda_{t,t+1}[\theta V'^j_{t+1} + (1-\theta)] + \lambda^j_t \beta E_t \Lambda_{t,t+1} \\
& - \kappa^j_t \beta E_t \Lambda_{t,t+1} \Big\{ [\theta V'^j_{t+1} + (1-\theta)] - \int_{\overline{\omega}^j_{t+1}}^{\infty} \Big[\theta V'^j_{t+1} + (1-\theta)\Big] d\tilde{F}_{t+1}(\omega) - \theta \frac{V_{t+1}(0)}{R^K_{t+1} Q_t S^{Bj}_t} \tilde{f}_t(\overline{\omega}^j_{t+1}) \Big\}
\end{aligned}
$$

where I used $\overline{\omega}^j_{t+1} = \overline{b}^j_t / R^K_{t+1}$. The envelope condition is given as:

$$
V'^j_t = \lambda^j_t \tag{C.5}
$$

The first order conditions can be written as:

$$
\begin{aligned}
0 =& \beta E_t \Lambda_{t,t+1} R^K_{t+1}[\theta \lambda^j_{t+1} + (1-\theta)](1 - \overline{\omega}^j_{t+1}) + \lambda^j_t E_t[\Lambda_{t,t+1} R^K_{t+1} \overline{\omega}^j_{t+1} - 1] \\
& + \kappa^j_t E_t R^K_{t+1}[\theta \lambda^j_{t+1} + (1-\theta)] \Big\{ (1 - \overline{\omega}^j_{t+1}) - \int_{\overline{\omega}^j_{t+1}}^{\infty} \Big[(\omega - \overline{\omega}^j_{t+1})\Big] d\tilde{F}_{t+1}(\omega) \Big\} \\
0 =& -\beta E_t \Lambda_{t,t+1}[\theta \lambda^j_{t+1} + (1-\theta)] + \lambda^j_t \beta E_t \Lambda_{t,t+1} \\
& - \kappa^j_t \beta E_t \Lambda_{t,t+1} \Big\{ [\theta \lambda^j_{t+1} + (1-\theta)] - \int_{\overline{\omega}^j_{t+1}}^{\infty} \Big[\theta \lambda^j_{t+1} + (1-\theta)\Big] d\tilde{F}_{t+1}(\omega) - \theta \frac{V_{t+1}(0)}{R_{{t+1}^K Q_t S^{Bj}_t}} \tilde{f}_t(\overline{\omega}^j_{t+1}) \Big\}
\end{aligned}
$$

To continue solving the problem, I use a guess and verify approach. I guess that the value function is linear in net worth, so that the value function reads as follows:

$$
V_t = \lambda^j_t N^j_t \tag{C.6}
$$

Furthermore, I guess the multipliers are equal across banks, that is $\lambda^j_t = \lambda_t$ and $\kappa^j_t = \kappa_t \ \forall j$. Using the guess, the incentive constraint can be written as:

$$
\beta E_t \Lambda_{t,t+1} \left\{ \begin{array}{l} \Big[\theta \lambda_{t+1}(1 - \overline{\omega}^j_{t+1}) R^K_{t+1} Q_t S^B_t + (1-\theta)(1 - \overline{\omega}^j_{t+1}) R^K_{t+1} Q_t S^B_t\Big] - \\ \int_{\overline{\omega}^j_{t+1}}^{\infty} \Big[\theta \lambda_{t+1}(\omega_t - \overline{\omega}^j_{t+1}) R^K_{t+1} Q_t S^B_t + (1-\theta)(\omega_t - \overline{\omega}^j_{t+1}) R^K_{t+1} Q_t S^B_t\Big] d\tilde{F}_{t+1}(\omega) \end{array} \right\} \geq 0
$$

and reformulated to:

$$
\beta E_t \Lambda_{t,t+1}(\theta \lambda_{t+1} + (1-\theta)) \Big\{ (1 - \overline{\omega}^j_{t+1}) - \int_{\overline{\omega}^j_{t+1}}^{\infty} (\omega_t - \overline{\omega}^j_{t+1}) d\tilde{F}_{t+1}(\omega) \Big\} \geq 0 \tag{C.7}
$$

The next step is to simplify the first order conditions. I use that if either the incentive constraint binds or if not then $\lambda_t = 0$ (Kuhn Tucker conditions) to simplify the participation constraint and use that the guess for the value function evaluated at 0 so that the first order conditions are given as:

$$
0 = E_t \Lambda_{t,t+1} R^K_{t+1}[\theta \lambda_{t+1} + (1-\theta)](1 - \overline{\omega}_{t+1}) + \lambda_t E_t[\Lambda_{t,t+1} R^K_{t+1} \overline{\omega}^j_{t+1} - 1] \tag{C.8}
$$

$$
0 = -\beta E_t \Lambda_{t,t+1}[\theta \lambda_{t+1} + (1-\theta)] + \lambda_t \beta E_t \Lambda_{t,t+1} - \kappa_t \beta E_t \Lambda_{t,t+1}(\theta \lambda_{t+1} + (1-\theta)) \tilde{F}_{t+1}(\overline{\omega}^j_{t+1}) \tag{C.9}
$$

I can now get the following expression for the multipliers:

$$\lambda_t = \frac{\beta E_t \Lambda_{t,t+1} R^K_{t+1} [\theta \lambda_{t+1} + (1-\theta)](1-\overline{\omega}^j_{t+1})}{1 - \beta E_t \Lambda_{t,t+1} R^K_{t+1} \overline{\omega}^j_{t+1}} \tag{C.10}$$

$$\kappa_t = \frac{\beta E_t \Lambda_{t,t+1}(\lambda_t - [\theta \lambda_{t+1} + (1-\theta)])}{\beta E_t \Lambda_{t,t+1}(\theta \lambda_{t+1} + (1-\theta)) \tilde{F}_{t+1}(\overline{\omega}^j_{t+1})} \tag{C.11}$$

I now want to show that the multipliers are symmetric across banks. Assuming that equation (C.7), which is the incentive constraint, is binding, I can get $\omega^j_t = \omega_t$. Due to $b^j_t = \overline{\omega}^j_{t+1} R^K_t$, $b^j_t = b_t$ can be obtained. At the same time, we have $\overline{\omega}^j_{t+1} = \overline{\omega}_{t+1}$ and $\overline{b}^j_t = \overline{b}_t$. Then, equation (C.10) implies that $\lambda^j_t = \lambda_t$ and equation (C.11) shows $\kappa^j_t = \kappa_t$. This verifies our guess that the multipliers are equalized. I check numerically that the participation and incentive constraint are binding, that is $\lambda_t > 0$ and $\kappa_t = 0$.

To show that the leverage ratio is symmetric, I use the participation constraint and assume that it is binding:

$$E_t \Lambda_{t,t+1} Q_t S^{Bj}_t \overline{b}^j_t - (Q_t S^{Bj}_t - N^j_t) = 0. \tag{C.12}$$

The leverage ratio is then given as:

$$\phi^j_t = \frac{1}{1 - E_t \Lambda_{t,t+1} R^K_{t+1} \overline{\omega}_{t+1}}. \tag{C.13}$$

As the leverage ratio does not depend on j, this implies that $\phi_t = \phi^j_t$.

The final step is to show that our guess $V_t = \lambda_t N^j_t$ is correct. The starting point is again the value function:

$$V_t(N^j_t) = \beta E_t \big[(\theta \lambda_{t+1} N_{t+1} + (1-\theta)\big(1-\overline{\omega}_{t+1}\big) R^K_{t+1} Q_t S^{Bj}_t) \big],$$

where I used $N^j_{t+1} = (1-\overline{\omega}_{t+1}) R^K_{t+1} Q_t S^{Bj}_t$. I insert the guess to obtain:

$$\lambda_t N^j_t = \phi_t N^j_t \beta E_t \Lambda_{t,t+1} \big[\theta \lambda_{t+1} + (1-\theta)\big] \big(1-\overline{\omega}_{t+1}\big) R^K_{t+1}. \tag{C.14}$$

and reformulate it to

$$\lambda_t = \phi_t E_t \Lambda_{t,t+1} \big[\theta \lambda_{t+1} + (1-\theta)\big] \big(1-\overline{\omega}_{t+1}\big) R^K_{t+1} \tag{C.15}$$

This gives us again a condition for λ_t:

$$\lambda_t = E_t \big[(\theta \lambda_{t+1} N_{t+1} + (1-\theta)\big(1-\overline{\omega}_{t+1}\big) R^K_{t+1} Q_t S^{Bj}_t) \big] \tag{C.16}$$

$$= \phi_t \beta E_t \Lambda_{t,t+1} \big[\theta \lambda_{t+1} + (1-\theta)\big] \big(1-\overline{\omega}_{t+1}\big) R^K_{t+1}. \tag{C.17}$$

Inserting (C.13), the condition for λ_t becomes:

$$\lambda_t = \frac{\beta E_t \Lambda_{t,t+1} \big[\theta \lambda_{t+1} + (1-\theta)\big] \big(1-\overline{\omega}_{t+1}\big) R^K_{t+1}}{1 - \beta E_t \Lambda_{t,t+1} R^K_{t+1} \overline{\omega}_{t+1}}. \tag{C.18}$$

This coincides with the equation (C.10). This verifies the guess.

C.2 With Bank Runs

In this section, the possibility of bank runs is included. The banker maximises V_t subject to a participation and incentive constraint, which reads as follows:

$$V_t^j(N_t^j) = \max_{S_t^{Bj},\bar{D}_t} (1-p_t^j)\beta E_t^N \Lambda_{t,t+1}\left[\theta V_{t+1}^j\left(N_{t+1}^j\right) + (1-\theta)(R_{t+1}^K Q_t S_t^{Bj} - \bar{D}_t^j)\right] \quad \text{(C.19)}$$

$$\text{s.t.} \quad (1-p_t^j)\beta E_t^N[\Lambda_{t,t+1}Q_t S_t^{Bj}\bar{b}_t^j] + p_t^j\beta E_t^R[R_{t+1}^K Q_t S_t^{Bj}] \geq (Q_t S_t^{Bj} - N_t^j) \quad \text{(C.20)}$$

$$(1-p_t^j)E_t^N\left[\Lambda_{t,t+1}\theta V_{t+1}\left(N_{t+1}^j\right) + (1-\theta)\Big(1-\frac{\bar{b}_t^j}{R_{t+1}^K}\Big)R_{t+1}^K Q_t S_t^{Bj}\right] \geq \quad \text{(C.21)}$$

$$\beta\Lambda_{t,t+1}E_t\left[\Lambda_{t,t+1}\int_{\frac{\bar{b}_t^j}{R_{t+1}^K}}^{\infty}\theta V_{t+1}\left(N_{t+1}^j\right) + (1-\theta)\Big(\omega-\frac{\bar{b}_t^j}{R_{t+1}^K}\Big)R_{t+1}^K Q_t S_t^{Bj}\,d\tilde{F}_{t+1}(\omega)\right]$$

The banker's specific can be written as Bellman equation:

$$\begin{aligned}
V_t(N_t^j) = &\max_{\{\phi_t^j,\bar{b}_t^j\}} (1-p_t^j)\beta E_t^N\Lambda_{t,t+1}\left[\theta V_{t+1}\left(\Big(1-\frac{\bar{b}_t^j}{R_{t+1}^K}\Big)R_{t+1}^K\phi_t^j N_t^j\right) + (1-\theta)\big(1-\frac{\bar{b}_t^j}{R_{t+1}^K}\big)R_{t+1}^K\phi_t^j N_t^j\right] \\
&+ \lambda_t^j\left[(1-p_t^j)\beta E_t^N[\Lambda_{t,t+1}\phi_t^j N_t^j\bar{b}_t^j] + p_t^j\beta E_t^R[R_{t+1}^K\phi_t^j N_t^j] - (\phi_t^j N_t^j - N_t^j)\right] \\
&+ \kappa_t^j\beta\Bigg\{\left[(1-p_t^j)E_t^N\Lambda_{t,t+1}\left[\Lambda_{t,t+1}\theta V_{t+1}\left(\Big(1-\frac{\bar{b}_t^j}{R_{t+1}^K}\Big)R_{t+1}^K\phi_t^j N_t^j\right) + (1-\theta)\Big(1-\frac{\bar{b}_t^j}{R_{t+1}^K}\Big)R_{t+1}^K\phi_t^j N_t^j\right]\right] \\
&- \beta E_t\left[\Lambda_{t,t+1}\int_{\frac{\bar{b}_t^j}{R_{t+1}^K}}^{\infty}\theta V_{t+1}\left(\Big(1-\frac{\bar{b}_t^j}{R_{t+1}^K}\Big)R_{t+1}^K\phi_t^j N_t^j\right) + (1-\theta)\Big(\omega-\frac{\bar{b}_t^j}{R_{t+1}^K}\Big)R_{t+1}^K\phi_t^j N_t^j\,d\tilde{F}_{t+1}(\omega)\right]\Bigg\}
\end{aligned}$$

The first order conditions with respect to ϕ_t^j can be written as

$$\begin{aligned}
0 =&(1-p_t^j)E_t^N\Lambda_{t,t+1}R_{t+1}^K[\theta V_{t+1}'^j + (1-\theta)](1-\varpi_{t+1}^j) \\
&+ \lambda_t^j((1-p_t^j)E_t^N[\Lambda_{t,t+1}R_{t+1}^K\varpi_{t+1}^j] + p_t E_t^R[\Lambda_{t,t+1}R_{t+1}^K] - 1) \\
&+ \kappa_t^j((1-p_t^j)\beta E_t^N\Lambda_{t,t+1}R_{t+1}^K[\theta V_{t+1}'^j + (1-\theta)](1-\varpi_{t+1}^j) \\
&- \kappa_t^j\beta E_t\Lambda_{t,t+1}\int_{\varpi_{t+1}^j}^{\infty}\left[R_{t+1}^K[\theta V_{t+1}'^j + (1-\theta)](\omega-\varpi_{t+1}^j)\right]d\tilde{F}_{t+1}(\omega) \\
&- \frac{\partial p_t^j}{\phi_t^j}E_t^N\Lambda_{t,t+1}R_{t+1}^K[\theta V_{t+1}'^j + (1-\theta)](1-\varpi_{t+1}^j)\left(1+\kappa_t^j\right) && \text{(C.22)} \\
&- \frac{\partial p_t^j}{\phi_t^j}E_t^N\left(R_{t+1}^K\varpi_{t+1}^j - R_{t+1}^K\right) && \text{(C.23)}
\end{aligned}$$

where I applied $\varpi_{t+1}^j = \bar{b}_t^j/R_{t+1}^K$. Gertler, Kiyotaki and Prestipino (2020) show that the even though the optimization of leverage ϕ^j affect the default probability p_t, this indirect effect on on the firm value V_t and the promised return R_t^D is zero. The reason is that at the cutoff value of default, net worth is zero, which implies $V_{t+1} = 0$. Similarly, the promised return is unchanged. The cutoff values of default is defined as:

$$\xi^D_{t+1}(\phi^j_t) = \left\{ (\sigma_{t+1}, A_{t+1}, \iota_{t+1}) : R^K_{t+1} \frac{\phi^j_t - 1}{\phi^j_t} \overline{R}^D_t \right\}. \tag{C.24}$$

At the cutoff points, the banker can exactly cover the face value of the deposits, which implies

$$\overline{\omega}^j_t = 1. \tag{C.25}$$

Based on the derivation in Gertler, Kiyotaki and Prestipino (2020), the property $\overline{\omega}^j_t = 1$ implies that

$$- \frac{\partial p_t}{S^{Bj}_t} E^N_t \Lambda_{t,t+1} R^K_{t+1} [\theta V'^j_{t+1} + (1-\theta)](1 - \overline{\omega}^j_{t+1}) \left(1 + \kappa^j_t\right) = 0, \tag{C.26}$$

$$- \frac{\partial p_t}{S^{Bj}_t} E^N_t \left(R^K_{t+1} \overline{\omega}^j_{t+1} - R^K_{t+1} \right) = 0, \tag{C.27}$$

The first order condition with respect to ϕ^B_t becomes then

$$\begin{aligned}
0 =& (1 - p^j_t) E^N_t \Lambda_{t,t+1} R^K_{t+1} [\theta V'^j_{t+1} + (1-\theta)](1 - \overline{\omega}^j_{t+1}) \\
&+ \lambda^j_t ((1 - p^j_t) E^N_t [\Lambda_{t,t+1} R^K_{t+1} \overline{\omega}^j_{t+1}] + p_t E^R_t [\Lambda_{t,t+1} R^K_{t+1}] - 1) \\
&+ \kappa^j_t ((1 - p^j_t) \beta E^N_t \Lambda_{t,t+1} R^K_{t+1} [\theta V'^j_{t+1} + (1-\theta)](1 - \overline{\omega}^j_{t+1}) \\
&- \kappa^j_t \beta E_t \Lambda_{t,t+1} \int_{\overline{\omega}^j_{t+1}}^{\infty} \left[R^K_{t+1} [\theta V'^j_{t+1} + (1-\theta)](\omega - \overline{\omega}^j_{t+1}) \right] d\tilde{F}_{t+1}(\omega)
\end{aligned}$$

The first order condition with respect to $\overline{b}^j_t$ is given as

$$\begin{aligned}
0 =& - \beta (1 - p^j_t) E^N_t \Lambda_{t,t+1} [\theta V'^j_{t+1} + (1-\theta)] \\
&+ \lambda^j_t \beta (1 - p^j_t) E^N_t \Lambda_{t,t+1} \\
&- \kappa^j_t \beta (1 - p^j_t) E^N_t \Lambda_{t,t+1} \left\{ [\theta V'^j_{t+1} + (1-\theta)] \right\} \\
&+ \kappa^j_t \beta (1 - p^j_t) E_t \Lambda_{t,t+1} \int_{\overline{\omega}^j_{t+1}}^{\infty} \left[\theta V'^j_{t+1} + (1-\theta) \right] d\tilde{F}_{t+1}(\omega) - \theta \frac{V_{t+1}(0)}{R^K_{t+1} Q_t S^{Bj}_t} \tilde{f}_t(\overline{\omega}^j_{t+1})
\end{aligned} \tag{C.28}$$

where I applied $\overline{\omega}^j_{t+1} = \overline{b}^j_t / R^K_{t+1}$

Similar to before, I use the following guess for the value function

$$V_t = \lambda^j_t N^j_t \tag{C.29}$$

and also that the multipliers are equal across banks, that is $\lambda^j_t = \lambda_t$ and $\kappa^j_t = \kappa_t \forall j$. In addition, I also guess now that the probability of a bank run does not depend on individual characteristics, that is $p^j_t = p_t$.

The incentive constraint can then then be written as

$$\beta (1 - p^j_t) E^N_t \left[\Lambda_{t,t+1} (\theta \lambda_{t+1} + (1-\theta))(1 - \overline{\omega}^j_{t+1}) R^K_{t+1} \right] \geq \tag{C.30}$$

$$\beta E_t \left[\Lambda_{t,t+1} \int_{\frac{\bar{b}_t^j}{R_{t+1}^K}}^{\infty} (\theta\lambda_{t+1} + (1-\theta)) \left(\omega - \overline{\omega}_{t+1}^j\right) R_{t+1}^K d\tilde{F}_{t+1}(\omega) \right]$$

The two first order conditions can then be adjusted similar to section C.1 and be written as

$$\begin{aligned}
0 =&(1-p_t)E_t^N \Lambda_{t,t+1} R_{t+1}^K [\theta\lambda_{t+1} + (1-\theta)](1-\overline{\omega}_{t+1}^j) + \\
&\lambda_t((1-p_t)E_t^N[\Lambda_{t,t+1}R_{t+1}^K \overline{\omega}_{t+1}] + p_t E_t^R[\Lambda_{t,t+1}R_{t+1}^K] - 1) \qquad \text{(C.31)} \\
0 =& -\beta(1-p_t)E_t^N \Lambda_{t,t+1}[\theta\lambda_{t+1} + (1-\theta)] + \lambda_t \beta (1-p_t) E_t^N \Lambda_{t,t+1} \\
&- \kappa_t \beta \Big\{ (1-p_t) E_t^N \Lambda_{t,t+1} \left[(\theta\lambda_{t+1} + 1 - \theta) \tilde{F}_{t+1}(\overline{\omega}_{t+1}^j) \right] \\
&\quad + p_t E_t^R \Lambda_{t,t+1} \left[(\theta\lambda_{t+1} + 1 - \theta) \left(1 - \tilde{F}_{t+1}(\overline{\omega}_{t+1}^j)\right) \right] \Big\} \qquad \text{(C.32)}
\end{aligned}$$

Using the same strategy as in C.1, the guess about the equalized multipliers can be verified. Similarly, it can be shown that leverage is the same across banks. This then verifies that the guess of the bank run probability $p_t^j = p_t$ is verified as the cutoff value is the same across banks as shown in equation (C.24). I additionally assume that in case of a run on the entire banking sector, a bank that survives shuts down and returns their net worth. This implies that $E_t^R \lambda_{t+1} = 1$. The participation constraint is given as:

$$(1-p_t)E_t^N[\beta\Lambda_{t,t+1}\bar{R}_t D_t] + p_t E_t^R[\beta\Lambda_{t,t+1}R_{t+1}^K Q_t S_t^B] = D_t. \qquad \text{(C.33)}$$

The incentive constraint is given as:

$$\begin{aligned}
&(1-p_t)E_t^N[\Lambda_{t,t+1}R_{t+1}^K(\theta\lambda_{t+1} + (1-\theta))[1 - e^{\frac{-\psi}{2}} - \tilde{\pi}_{t+1}]] = \qquad \text{(C.34)} \\
&p_t E_t^R[\Lambda_{t,t+1}R_{t+1}^K(e^{-\frac{\psi}{2}} - \overline{\omega}_{t+1} + \tilde{\pi}_{t+1})],
\end{aligned}$$

λ_t and κ_t are derived from the first order conditions in equations (C.31) and (C.32) are given as:

$$\lambda_t = \frac{(1-p_t)E_t^N \Lambda_{t,t+1} R_{t+1}^K [\theta\lambda_{t+1} + (1-\theta)](1-\overline{\omega}_{t+1})}{1 - (1-p_t)E_t^N[\Lambda_{t,t+1}R_{t+1}^K\overline{\omega}_{t+1}] - p_t E_t^R[\Lambda_{t,t+1}R_{t+1}^K]} \qquad \text{(C.35)}$$

$$\kappa_t = \frac{\beta(1-p_t)E_t^N \Lambda_{t,t+1}\left[\lambda_t - (\theta\lambda_{t+1} + 1 - \theta)\right]}{(1-p_t)E_t^N \Lambda_{t,t+1}\left[(\theta\lambda_{t+1} + 1 - \theta)\tilde{F}_{t+1}(\overline{\omega}_{t+1})\right] + p_t E_t^R \Lambda_{t,t+1}\left[(\theta\lambda_{t+1} + 1 - \theta)\left(1 - \tilde{F}_{t+1}(\overline{\omega}_{t+1})\right)\right]} \qquad \text{(C.36)}$$

It is numerically checked that $\lambda_t > 0$ and $\kappa_t > 0$ so that the participation and incentive constraint are binding.

D Global Solution Method

The Algorithm uses time iteration with piecewise linear policy functions based on Richter, Throckmorton and Walker (2014). The approach is adjusted to take into account the multiplicity of equilibria due to possibility of a bank run also that the probability of the bank run equilibrium is time-varying. The state variables are $\{S_{t-1}, N_t, \sigma_t, A_t, \iota_t\}$, where I used N_t as state variable instead of $\overline{D}_{t-1}$ for computational reasons. The policy variables are $Q_t, C_t, \bar{b}_t, \Pi_t, \lambda_t$. I solve for the following policy functions $\boldsymbol{Q(X), C(X), \bar{b}(X), \Pi(X), \lambda(X)}$, the law of motion of net worth $\boldsymbol{N'(X, \varepsilon_{t+1})}$ and the probability of a bank run next period $\boldsymbol{P(X)}$. The expectations are evaluated using Gauss-Hermite quadrature, where the matrix of nodes is denoted as $\boldsymbol{\varepsilon}$. The Algorithm is summarized below:

1. Define a state grid $\boldsymbol{X} \in [\underline{S}_{t-1}, \overline{S}_{t-1}] \times [\underline{N}_t, \overline{N}_t] \times [\underline{\sigma}_t, \overline{\sigma}_t] \times [\underline{A}_t, \overline{A}_t]$ and integration nodes $\boldsymbol{\epsilon} \in [\underline{\epsilon^\sigma}_{t+1}, \overline{\epsilon^\sigma}_{t+1}] \times [\underline{\epsilon^A}_{t+1}, \overline{\epsilon^A}_{t+1}]$ to evaluate expectations based on Gauss-Hermiture quadrature

2. Guess the piecewise linear policy functions to initialize the algorithm[50]

   (a) the "classical" policy functions $\boldsymbol{Q(X), C(X), \bar{b}(X), \Pi(X), \lambda(X)}$

   (b) a function $\boldsymbol{N'(X, \varepsilon_{t+1})}$ at each point from the nodes of next period shocks based on Gauss-Hermite Quadrature

   (c) the probability $\boldsymbol{P(X)}$ that a bank run occurs next period

3. Solve for all time t variables for a given state vector. Take from the previous iteration j the law of motion $\boldsymbol{N'_j}(\boldsymbol{X}, \varepsilon_{t+1})$ and the probability of a bank run as given $\boldsymbol{P_j(X)}$ and calculate time $t+1$ variables using the guess j policy functions with $\boldsymbol{X'}$ as state variables. The expectations are calculated using numerical integration based on Gauss-Hermiture Quadrature. A numerical root finder with the time t policy functions as input minimises the error in the following five equations:

$$\begin{aligned} \text{err}_1 &= (\Pi_t - \Pi_{SS})\Pi_t \\ &\quad - \left(\frac{\epsilon}{\rho^r} \left(MC_t - \frac{\epsilon - 1}{\epsilon} \right) + \Lambda_{t,t+1}(\Pi_{t+1} - \Pi_{SS})\Pi_{t+1} \frac{Y_{t+1}}{Y_t} \right), \end{aligned} \tag{D.1}$$

$$\text{err}_2 = 1 - \beta\Lambda_{t,t+1} \frac{i_t}{\Pi_{t+1}} 1, \tag{D.2}$$

$$\text{err}_3 = (1 - p_t)E_t^N \left[\beta\Lambda_{t,t+1}\bar{R}_t D_t \right] + p_t E_t^R \left[\beta\Lambda_{t,t+1} R_{t+1}^K Q_t S_t^B \right] - D_t, \tag{D.3}$$

$$\begin{aligned} \text{err}_4 &= (1 - p_t)E_t^N \left[\Lambda_{t,t+1} R_{t+1}^K (\theta\lambda_{t+1} + (1 - \theta))(1 - e^{\frac{-\psi}{2}} \tilde{\pi}_{t+1}) \right] \\ &\quad - p_t E_t^R \left[\Lambda_{t,t+1} R_{t+1}^K (e^{-\frac{\psi}{2}} - \overline{\omega}_{t+1} + \tilde{\pi}_{t+1}) \right], \end{aligned} \tag{D.4}$$

$$\text{err}_5 = \lambda_t - \frac{(1 - p_t)E_t^N \Lambda_{t,t+1} R_{t+1}^K [\theta\lambda_{t+1} + (1 - \theta)](1 - \overline{\omega}_{t+1})}{1 - (1 - p_t)E_t^N [\Lambda_{t,t+1} R_{t+1}^K \overline{\omega}_{t+1}] - p_t E_t^R [\Lambda_{t,t+1} R_{t+1}^K]}. \tag{D.5}$$

[50] In practice, it can be helpful to solve first for the economy with only one shock, for instance the volatility shock, and solve this model in isolation. The resulting policy functions can then be used as starting point for the full model with two shocks.

4. Take the iteration j policy functions , $\boldsymbol{N'_j}(\boldsymbol{X}, \boldsymbol{\varepsilon}_{t+1})$ and $\boldsymbol{P_j(X)}$ as given and and solve the whole system of time t and $(t+1)$ variables. Calculate then N_{t+1} using the "law of motion" for net worth

$$N_{t+1} = \max\left[R^K_{t+1} Q_t S^B_t - \overline{R}_t D_t, 0\right] + (1-\theta)\zeta S_t \tag{D.6}$$

   A bank run occurs at a specific point if

$$R^K_{t+1} Q_t S^B_t - \overline{R}_t D_t \leq 0 \tag{D.7}$$

   In such a future state, the weight of a bank run is 1. In other state, the weight of a bank run.[51] This can be now used to evaluate the probability of a bank run next period based on Gauss-Hermiture quadrature so that we know p_t.

5. Update the policy policy functions slowly $\boldsymbol{Q}(\boldsymbol{X}), \boldsymbol{C}(\boldsymbol{X}), \boldsymbol{\psi}(\boldsymbol{X}), \boldsymbol{\pi}(\boldsymbol{X})$. For instance for consumption policy function, this could be written as:

$$\boldsymbol{C_{j+1}}(\boldsymbol{X}) = \alpha^{U1} \boldsymbol{C_j}(\boldsymbol{X}) + (1-\alpha^{U1}) \boldsymbol{C_{sol}}(\boldsymbol{X}) \tag{D.8}$$

   where the subscript *sol* denotes the solution for this iteration and α^{U1} determines the weight of the previous iteration. Furthermore, $\boldsymbol{N'}(\boldsymbol{X}, \boldsymbol{\varepsilon}_{t+1})$ and $\boldsymbol{P(X)}$ are updated using the results from step 4:

$$\boldsymbol{N'_{j+1}}(\boldsymbol{X}, \boldsymbol{\varepsilon}_{t+1}) = \alpha^{U2} \boldsymbol{N'_j}(\boldsymbol{X}, \boldsymbol{\varepsilon}_{t+1}) + (1-\alpha^{U2}) \boldsymbol{N'_{sol}}(\boldsymbol{X}, \boldsymbol{\varepsilon}_{t+1}) \tag{D.9}$$

$$\boldsymbol{P_{j+1}}(\boldsymbol{X}) = \alpha^{U3} \boldsymbol{P_j}(\boldsymbol{X}) + (1-\alpha^{U3}) \boldsymbol{P_{sol}}(\boldsymbol{X}) \tag{D.10}$$

6. Repeat steps 3,4 and 5 until the errors of all functions, which are the classical policy functions $\boldsymbol{Q(X), C(X), \overline{b}(X), \Pi(X), \lambda(X)}$ together with the law of motion of net worth $\boldsymbol{N'(X, \varepsilon_{t+1})}$ and the probability of a bank $\boldsymbol{P}(\boldsymbol{X})$, at each point of the discretized state are sufficiently small.

[51] This procedure would imply a zero and one indicator, which is very unsmooth. For this reason, we use the following functional forms based on exponential function: $\frac{\exp(\zeta_1(1-D_{t+1}))}{1+\exp(\zeta_1*(1-D_{t+1}))}$ where $D_{t+1} = \frac{R^k_{t+1}}{R^D_t}\frac{\phi}{\phi-1}$ at each calculated N_{t+1}. ζ_1 a large value of zeta ensures sufficient steepness so that the approximation is close to an indicator function of 0 and 1.

E Particle Filter

I use particle filter with sequential importance resampling based on Atkinson, Richter and Throckmorton (2019) and Herbst and Schorfheide (2015). The algorithm is adapted to incorporate sunspot shocks and endogenous equilibria similar to Borağan Aruoba, Cuba-Borda and Schorfheide (2018), who have a model with sunspot shocks that directly determine the equilibria. Furthermore, I extend this approach to include that the probability of equilibria is endogenously time-varying. The total number of particles M is set to 10000 as in Borağan Aruoba, Cuba-Borda and Schorfheide (2018).

1. **Initialization** Use the risky steady state of the model as starting point and draw $\{v_{t,m}\}_{t=-24}^{0}$ for all particles $m \in \{0, ..., M\}$. I set $\{\iota_{t,m} = 0\}_{t=-24}^{0}$, which excludes a bank run in the initialization. The simulation of these shocks provides the start values for the state variables $\mathbb{X}_{0,m}$.

2. **Recursion** Filter the nonlinear model for periods $t = 1, ..., T$

   (a) Draw the sunspot shock $\iota_{t,m}$ and the structural shocks $v_{t,m}$ for each particle $m = \{1, ..., M\}$. The sunspot shock is drawn from a binomial distribution with realizations $0, 1$:

   $$\iota_{t,m} \sim \mathcal{B}(1, \Upsilon) \tag{E.1}$$

   where 1 indicates the number of trails and Υ is the probability of $\iota = 1$.[52] The structural shocks are drawn from a proposal distribution that distinguishes between the realizations of the sunspot shock :

   $$v_{t,m} \sim N(\overline{v_t}^{\iota=0}, I) \qquad \text{if} \quad \iota_{t,m} = 0 \tag{E.2}$$
   $$v_{t,m} \sim N(\overline{v_t}^{\iota=1}, I) \qquad \text{if} \quad \iota_{t,m} = 1 \tag{E.3}$$

   As the regime selection is endogenous in the model, the proposal distribution can be the same for the two realizations of the sunspot shock. This is the case if the model does not suggest the realization of a bank run. The difference in using the proposal distribution is that instead of drawing directly from a distribution, I draw from an adapted distribution. I derive the proposal distribution by maximizing the fit of the shock for the average state vector $\overline{\mathbb{X}}_{t-1} = \frac{1}{M}\sum_{m=1}^{M} \mathbb{X}_{t-1,m}$

      i. Calculate a state vector $\overline{\mathbb{X}}_t$ from $\overline{\mathbb{X}}_{t-1}$ and a guess of $\overline{v}_t$ for the possible realizations of the sunspot shock:

      $$\mathbb{X}_t^{\iota=0} = f(\overline{\mathbb{X}}_{t-1}, \overline{v}_t^{\iota=0}, \iota_t = 0) \tag{E.4}$$
      $$\mathbb{X}_t^{\iota=1} = f(\overline{\mathbb{X}}_{t-1}, \overline{v}_t^{\iota=1}, \iota_t = 1) \tag{E.5}$$

[52] In practice, I draw from a uniform distribution bounded between 0 and 1 and categorize the sunspot accordingly.

ii. Calculate the measurement error from the observation equation for the two cases

$$u_t^{\iota=0} = \mathbb{Y}_t - g(\mathbb{X}_t^{\iota=0}) \tag{E.6}$$

$$u_t^{\iota=0} = \mathbb{Y}_t - g(\mathbb{X}_t^{\iota=1}) \tag{E.7}$$

The measurement error follows a multivariate normal distribution, so that the probabilities of observing the measurement error for the different sunspot shocks are given by

$$p(u_t^{\iota=0}|\mathbb{X}_t^{\iota=0}) = (2\pi)^{-n/2}|\Sigma_u|^{-0.5} \exp\left(-0.5(u_t^{\iota=0})'\Sigma_u^{-1}(u_t^{\iota=0})\right) \tag{E.8}$$

$$p(u_t^{\iota=1}|\mathbb{X}_t^{\iota=1}) = (2\pi)^{-n/2}|\Sigma_u|^{-0.5} \exp\left(-0.5(u_t^{\iota=1})'\Sigma_u^{-1}(u_t^{\iota=1})\right) \tag{E.9}$$

where Σ_u is the variance of the measurement error and n is the number of observables, which is 2 in this setup.

iii. Calculate the probability of observing $\mathbb{X}_t^{\iota=0}$ respectively $\mathbb{X}_t^{\iota=1}$ conditional on the average state vector from the previous period

$$p(\mathbb{X}_t^{\iota=0}|\overline{\mathbb{X}}_{t-1}) = (2\pi)^{-n/2} \exp\left(-0.5(\overline{v_t}^{\iota=0})'(\overline{v_t}^{\iota=0})\right) \tag{E.10}$$

$$p(\mathbb{X}_t^{\iota=1}|\overline{\mathbb{X}}_{t-1}) = (2\pi)^{-n/2} \exp\left(-0.5(\overline{v_t}^{\iota=1})'(\overline{v_t}^{\iota=1})\right) \tag{E.11}$$

iv. To find the proposal distribution, maximise the following objects with respect $\overline{v_t}^{\iota=0}$ respectively $\overline{v_t}^{\iota=1}$:

$$p(\mathbb{X}_t^{\iota=0}|\overline{\mathbb{X}}_{t-1})p(u_t^{\iota=0}|\mathbb{X}_t^{\iota=0}) \tag{E.12}$$

$$p(\mathbb{X}_t^{\iota=0}|\overline{\mathbb{X}}_{t-1})p(u_t^{\iota=0}|\mathbb{X}_t^{\iota=0}) \tag{E.13}$$

This provides the proposal distributions $N(\overline{v_t}^{\iota=0}, I)$ and $N(\overline{v_t}^{\iota=1}, I)$

(b) Propagate the state variables $\mathbb{X}_{t,m}$ by iterating the state-transition equation forward given $\mathbb{X}_{t-1,m}$, $v_{t,m}$ and $\iota_{t,m}$:

$$\mathbb{X}_{t,m} = f(\mathbb{X}_{t-1,m}, v_{t,m}, \iota_t) \tag{E.14}$$

(c) Calculate the measurement error

$$u_{tm} = \mathbb{Y}_t - g(\mathbb{X}_{t,m}) \tag{E.15}$$

The incremental weights of the particle m can be written as

$$w_{t,m} = \frac{p(u_{t,m}|\mathbb{X}_{t,m})p(\mathbb{X}_{t,m}|\mathbb{X}_{t-1,m})}{f(\mathbb{X}_{t,m}|\mathbb{X}_{t-1,m}, \mathbb{Y}_t, \iota_{t,m})} \tag{E.16}$$

$$= \begin{cases} \frac{(2\pi)^{-n/2}|\Sigma_u|^{-0.5}\exp\left(-0.5u'_{t,m}\Sigma_u^{-1}u_{t,m}\right)\exp\left(-0.5v'_{t,m}v_{t,m}\right)}{\exp\left(-0.5(v_{t,m}-\overline{v}_t^{\iota=0})'(v_{t,m}-\overline{v}_t^{\iota=0})\right)} & \text{if } \iota_{t,m} = 0 \\ \frac{(2\pi)^{-n/2}|\Sigma_u|^{-0.5}\exp\left(-0.5u'_{t,m}\Sigma_u^{-1}u_{t,m}\right)\exp\left(-0.5v'_{t,m}v_{t,m}\right)}{\exp\left(-0.5(v_{t,m}-\overline{v}_t^{\iota=1})'(v_{t,m}-\overline{v}_t^{\iota=1})\right)} & \text{if } \iota_{t,m} = 1 \end{cases} \tag{E.17}$$

where the density $f(\cdot)$ depends on the realization of the sunspot shock. The incremental weights determine the log-likelihood contribution in period t:

$$\ln(l_t) = \ln\left(\frac{1}{M}\sum_{m=1}^{M} w_{t,m}\right) \tag{E.18}$$

(d) Resample the particles based on the weights of the particles. First, the normalized weights $W_{t,m}$ are given by:

$$W_{t,m} = \frac{w_{t,m}}{\sum_{m=1}^{M} w_{t,m}} \tag{E.19}$$

Second, the deterministic algorithm of Kitagawa (2016) resamples the particles by drawing from the current set of particles adjusted for their relative weights. This gives a resampled distribution of state variables $\mathbb{X}_{t,m}$.

3. **Likelihood Approximation** Determine the approximated log-likelihood function of the model as

$$\ln(\mathcal{L}_t) = \sum_{t=1}^{T} \ln(l_t) \tag{E.20}$$

The results are robust to using a particle filter without drawing from a proposal distribution. In this scenario, I do not maximise the fit of structural shocks to find $\overline{v_t}^{\iota=0}, \overline{v_t}^{\iota=1}$. Instead, I draw directly draw from a standard normal distribution, which implies $\overline{v_t}^{\iota=0} = \overline{v_t}^{\iota=1} = 0$. The rest of the algorithm remains the same.

F Reduced-Form Evidence

This section discusses the derivation of the probability measure of a severe crisis. To begin with, the quantile estimates need to mapped into a quantile distribution. Based on this distribution, the measure can then be derived.

F.1 Distribution of GDP Growth

So far, the different quantiles have been analysed. I am now interested in extending this approach to assess the entire distribution and shed more light on the role of leverage. While the estimated quantiles $\hat{Q}_\tau(\bar{y}_{t+4}|x_t)$ can be mapped into the quantile function $F^{-1}_{\bar{y}_{t+4}}(\tau|x_t)$, it is in practice difficult to obtain the quantile function as argued in Adrian, Boyarchenko and Giannone (2019*b*). Following their approach, I fit THE estimated quantiles to a skewed-t-distribution of Azzalini and Capitanio (2003), which then gives us with the following probability density function:

$$f(\bar{y}_{t+h}; x_t, \mu_t, \sigma_t, \alpha_t, \nu_t) = \frac{2}{\sigma_t} t\left(\bar{z}_{t+h}; \nu_t\right) T\left(\alpha \bar{z}_{t+h}\sqrt{\frac{\nu_t+1}{\nu_t+\bar{z}^2_{t+h}}}; \nu_t+1\right) \tag{F.1}$$

where $\bar{z}_{t+h} = (\bar{y}_{t+h} - \mu_t)/\sigma_t$. $t(\cdot)$ and $T(\cdot)$ are the probability density function and cumulative density function of the student t-distribution. The four parameters determine the location ($\mu_t \in \mathbb{R}$), the scale ($\sigma_t \in \mathbb{R}^+$), skewness ($\alpha_t \in \mathbb{R}$) and kurtosis ($\nu_t \in \mathbb{Z}$) of the distribution.

Following Adrian, Boyarchenko and Giannone (2019*b*), I minimize the squared distance between the estimated quantile function $\hat{Q}_\tau(\bar{y}_{t+4}|x_t)$ and the quantile function of the skewed t-distribution $F\bar{y}^{-1}_{t+4}(\tau; x_t, \mu_t, \sigma_t, \alpha_t, \nu_t)$ to match the 5%,25%, 75% and 95% quantiles:

$$\{\hat{\mu}_t, \hat{\sigma}_t, \hat{\alpha}_t, \hat{\nu}_t\} = \arg\min_{\mu_t,\sigma_t,\alpha_t,\nu_t} \sum_\tau \left[\hat{Q}_\tau(\bar{y}_{t+4}|x_t) - F^{-1}_{\bar{y}_{t+4}}(\tau; x_t, \mu_t, \sigma_t, \alpha_t, \nu_t)\right]^2 \tag{F.2}$$

where the four parameters are exactly identified. I can now compute the skewed t-distribution at each point in time for our sample.

Figure 11 shows the forecasted probability density function of GDP growth in 2008:Q4, which is conditioned on 2007:Q4. The fourth quarter of 2008 is a key date as this coincides with the run on the shadow banking sector after Lehman Brother's bankruptcy in September 2008 and the largest GDP reduction quarter-on-quarter during the financial crisis. I can see that the distribution has very fat tails. A common measure of the downside tail risk is the GDP growth associated with the 5% quantile, which is denoted as the blue shaded area. The predicted value is close to -3%, which is very close to the actual realized GDP growth in 2008:Q4.

To focus on the impact of leverage, I compare our baseline specification to a setup where I disregard shadow banking leverage. In this setup, I only regress future GDP growth on current GDP growth, that is $x_t = [\Delta y_t \ \phi_t]$. This specification has much thinner tails. The prediction of the 5% quantile is now around -1%, which is far from the actual realized value. This shows that the high leverage in 2007:Q4 indicates an increasing tail-risk in line with the

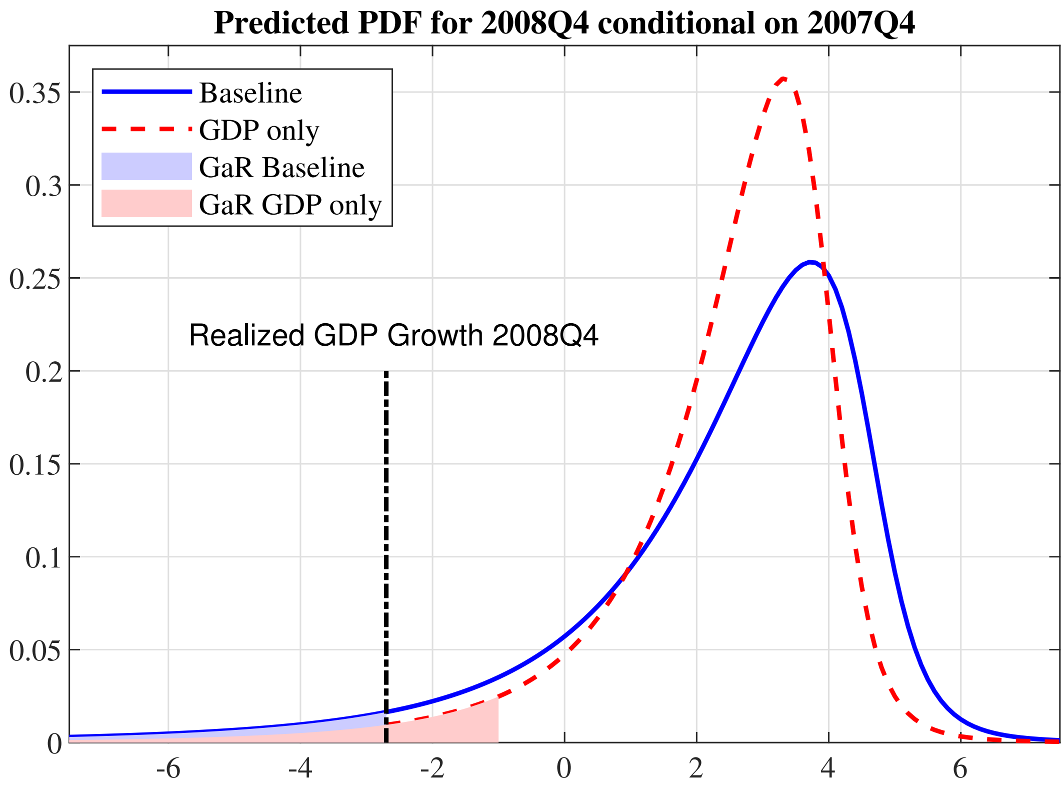

Figure 11: The predicted probability distribution function for 2008Q4 conditional on 2007Q4. The blue line shows the PDF that is conditioned on the baseline scenario with current GDP growth and leverage. The red line displays the PDF that is only conditioned on current GDP growth. The blue and red shaded area indicate the area below the 5% quantile for both specifications.

structural model.

F.2 Probability Measure of a Severe Crisis

Another important prediction of the model is that the probability of a financial crisis already increased substantially prior to 2007 and thus prior to the onset of the financial crisis. For this reason, I want to assess the downside risk that is associated with a financial crisis over the entire horizon. As a measure of downside risk, I use the probability that GDP drops below a specified level $y^{\star}$.[53] This can be written as

$$Prob_t(\bar{y}_{t+4} < y^{\star}|x_t) = \int_{-\infty}^{y^{\star}} f(\bar{y}_{t+h}; x_t, \mu_t, \sigma_t, \alpha_t, \nu_t) \tag{F.3}$$

where I condition on the explanatory variables x_t. I choose the threshold value as the fall in GDP as in 2008:Q4, which is

$$y^{\star} = \bar{y}_{2008:Q4} \tag{F.4}$$

[53] López-Salido and Loria (2020) use this conditional probability approach in the context of inflation.

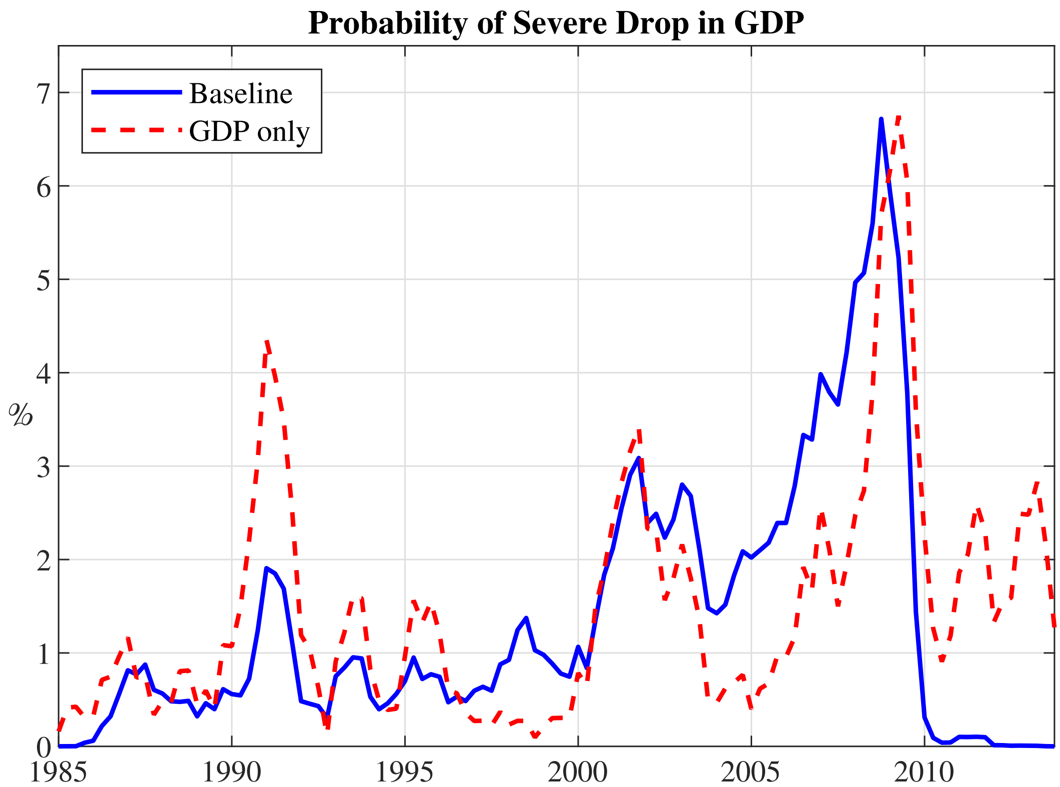

Figure 12: The conditional probability for a fall in outout below a certain threshold four quarters later is displayed from 1985:Q1 onward. The blue line shows the probability that is conditioned on the baseline scenario with current GDP growth and leverage. The red line displays the probability that is only conditioned on current GDP growth.

The measure $Prob_t(\bar{y}_{t+4} < \bar{y}_{2008:Q4}|x_t)$ gives us the conditional probability that GDP growth is below the realized value of 2008:Q4. The purpose is to relate this measure to the probability of a financial crisis as in 2008. Compared to the structural model, I cannot use the realization of a bank run. Instead, I rely on the connection between a severe drop in output that is associated with a financial crisis.

Figure 12 shows the probability $Prob_t(\bar{y}_{t+4} < \bar{y}_{2008:Q4}|x_t)$ from 1985:Q1 onwards. The interpretation is as follows. For instance, the conditional probability in 1985:Q1 for a severe output drop one year ahead, that is in 1986:Q1, below $y^\star$ is basically 0%. The measure increases around the three recessions (1990-91,2001,2007-2009) in our sample. One important difference for the financial crisis compared to the other recessions is that the tail-risk increases already substantially before. In particular, I can see a steady increase from 2004 onwards in this measure in line with the predictions of the structural model. The conditional probability rises up to 5% in 2007:Q4, which is the prediction related to 2007:Q4 .

I am interested in assessing the importance of leverage. For that purpose, I compare our baseline version in which I condition on current GDP growth and leverage to a scenario in which the probability is only conditioned on GDP growth. First, our baseline case reports a much higher risk of a large reduction from 2004 onwards until the onset of the crisis. The probability in the baseline version is around 2 percentage points higher during this period. Therefore, the model without leverage has a significantly lower tail-risk prior to the financial crisis. Taken together, the non-structural model highlights also the two most important observations. First, shadow bank leverage is important to capture an increase in tail-risk. Second, the tail-risk increases already in 2004, considerably prior to the outburst of the financial crisis.