Reversal Interest Rate and Macroprudential Policy*

Matthieu Darracq Pariès†
European Central Bank

Christoffer Kok‡
European Central Bank

Matthias Rottner§
Deutsche Bundesbank

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Abstract

Could a monetary policy loosening in a low interest rate environment have unintended recessionary effects? Using a non-linear macroeconomic model fitted to the euro area economy, we show that the effectiveness of monetary policy can decline in negative territory until it reaches an endogenous turning point, where monetary policy becomes contractionary. The framework demonstrates that the risk of hitting the rate at which the effect reverses depends on the capitalization of the banking sector. The possibility of the reversal interest rate gives rise to a novel motive for macroprudential policy. We show that macroprudential policy in the form of a countercyclical capital buffer, which prescribes the build-up of buffers in good times, substantially mitigates the probability of encountering the reversal rate and increases the effectiveness of negative interest rate policies. This new motive emphasizes the strategic complementarities between monetary policy and macroprudential policy.

Keywords: Reversal Interest Rate, Negative Interest Rates, Macroprudential Policy, Monetary Policy, ZLB

JEL Codes: E32, E44, E52, E58, G21

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†matthieu.darracq_paries@ecb.europa.eu

‡christoffer.kok@ecb.europa.eu

§matthias.rottner@bundesbank.de

1 Introduction

The prolonged period of ultra-low interest rates in the euro area and other advanced economies has raised concerns that further monetary policy accommodation could have the opposite effect than what is intended. Specifically, there is a risk that a further loosening of monetary policy can become contractionary and reduce lending for very negative policy rates. The policy rate enters a "reversal interest rate" territory, to use the terminology of Abadi, Brunnermeier and Koby (2022), in which the usual monetary transmission mechanism through the banking sector breaks down.

In this paper, we analyze the connection between monetary policy and macroprudential policy in a low interest rate environment. We develop a non-linear macroeconomic model with a banking sector fitted to the euro area economy that features asymmetric monetary policy transmission and captures the reversal rate mechanism. The framework demonstrates that a less well-capitalized banking sector amplifies the likelihood of encountering the reversal interest rate and impairs negative interest rate policies. This gives rise to a new motive for macroprudential policy. Building up macroprudential policy space in good times to support the bank lending channel of monetary policy, for instance in the form of a countercyclical capital buffer (CCyB), mitigates the risk of monetary policy hitting reversal rate territory and increases the effectiveness of negative interest rate policies. Introducing a reserve tiering system as a policy experiment, we discuss how measures that are aimed to support banks in a low interest rate environment can affect the reversal rate, negative interest rate policies and the role of macroprudential policy.

A key component of an analysis that focuses on the reversal rate is to account for the transmission of policy rates to other interest rates. Specifically, there is growing evidence that the pass-through of policy rates to banks' deposit rates is increasingly imperfect for negative rates because banks are reluctant to cut rates below zero (e.g. Heider, Saidi and Schepens, 2019). Figure 1 highlights this fact for the euro area economy. The co-movement of the ECB deposit facility rate, which determines the interest received from reserves, and the average deposit rate paid to households decouples after approaching a low interest rate territory. Additionally, the distribution of deposit rates across individual euro area banks shows that, initially, no bank charged sub-zero deposit rates after approaching negative territory in June 2014. Even in December 2019, there is a only small but increasing fraction of banks that charged sub-zero deposit rates. At the same time, a policy rate cut directly lowers the return of liquid assets of banks such as reserves and government assets, as can be seen for the German one year bond yield in Figure 1. The diminished return on liquid assets deteriorates bank profitability, which then contracts bank lending.

We incorporate these facts in a novel macroeconomic framework by extending a New Keynesian model with a capital-constrained banking sector along two dimensions: i) by introducing an imperfect pass-through of policy rates to deposit rates for low interest rates

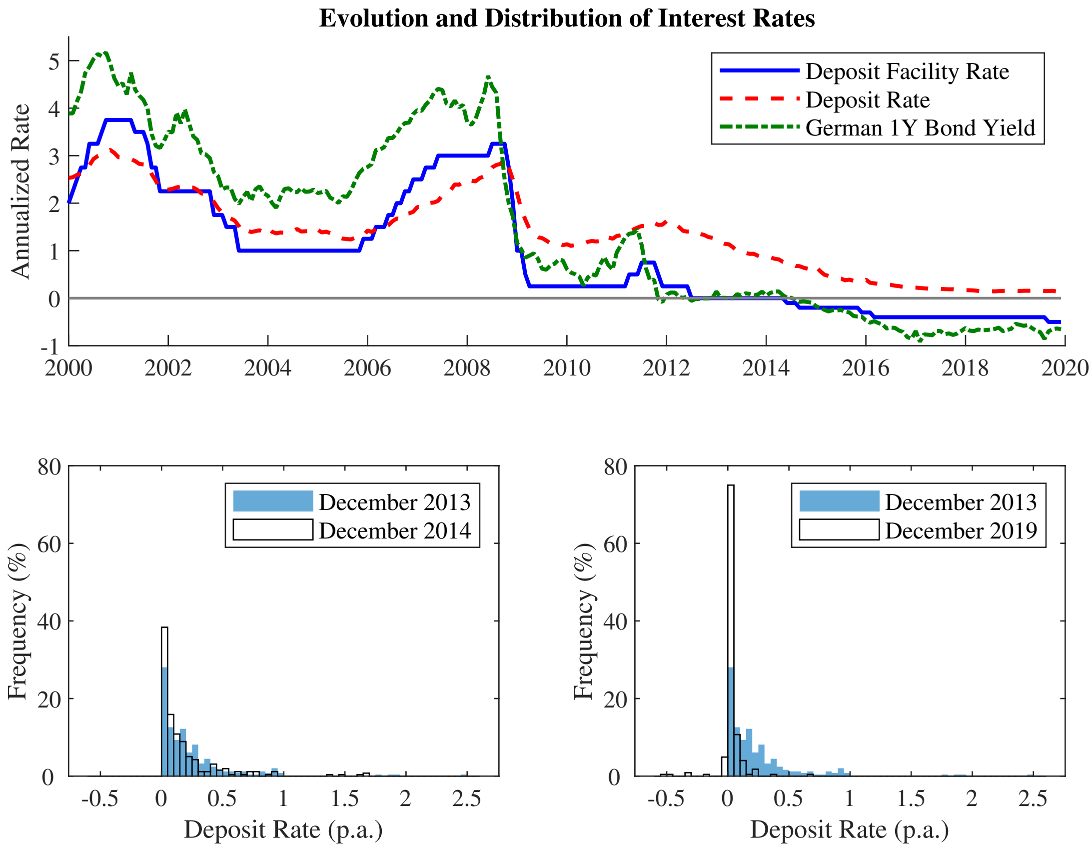

Figure 1: The upper panel shows the ECB deposit facility rate, average household deposit rate in the euro area and the German 1Y bond yield. The lower panel shows the distribution of overnight household deposit rates across banks. Details can be found in Appendix B.

and ii) by adding a liquidity requirement for banks to hold liquid assets. The imperfect deposit rate pass-through results from a depletion of banks' market power for low interest rates.[1] We formalize this relation by introducing a search model with switching costs in the deposit market based on Brunnermeier and Koby (2018). The requirement for liquid assets reflects both monetary policy and regulatory considerations.[2] The key implication of this framework is that monetary policy can have contractionary effects in negative territory due to a deterioration of the banking sector's profitability.

The framework suggests that, for the euro area, the reversal interest rate is located at around -1% p.a. and that the policy rate enters this territory with a probability of less than three percent. To establish this result, we fit the model to salient features of the euro area economy and solve the model using global methods that can capture non-linear

[1]The relevance of banks' market power in deposit markets is theoretically pointed out e.g. by Klein (1971), while empirical evidence is provided, for instance, in Sharpe (1997). Hainz, Marjenko and Wildgruber (2017) show that banks market power is declining with low interest rates as the switching costs of banks are falling. Drechsler, Savov and Schnabl (2017) document that market power in the deposit market affects monetary policy transmission.

[2]In relation to monetary policy, banks are required to hold minimum reserves with the central bank. The minimum reserve requirements aim at stabilizing money market rates and creating (or enlarging) a structural liquidity shortage, but may also reflect the need to maintain a certain amount of eligible securities to be able to participate in open market operations. On the regulatory side, liquid asset holdings are needed to comply with minimum liquidity requirements (e.g. the Liquidity Coverage Ratio).

dynamics. The bank lending channel is state-dependent and the transmission of shocks is asymmetric. In particular, a lowering of the policy rate has only a modest impact on credit supply and aggregate demand due to the imperfect pass-through in a low interest rate environment. At the same time, a reduction of the policy rate lowers the return on banks' government asset holdings and reduces their net worth. If the latter channel is the dominant one, monetary policy reaches a turning point (the reversal rate), from which on a monetary policy loosening reduces bank lending and contracts output. The main insight is that a lower policy rate requires a larger interest rate cut in order to have the same expansionary impact. However, this is conditional on there being enough space left before approaching the reversal interest rate. The declining effectiveness of monetary policy also implies that a monetary policy tightening is substantially more powerful than a loosening in a low or negative interest rate territory. As a consequence, the efficacy of monetary policy is successively restored once the economy starts to move away from very low interest rates.

The threat of the reversal rate and the declining effectiveness of negative interest rate policies give rise to a new motive for macroprudential policy as it can help to strengthen the bank lending channel in a "lower for longer" interest rate environment. The reason is that the capitalization of the banking sector plays a key role for the transmission of monetary policy in a low or negative interest rate environment. This opens up the possibility of using macroprudential policy to alleviate the diminishing effectiveness of monetary policy. In particular, building up macroprudential policy space in good times can mitigate the risk of monetary policy entering a reversal rate territory. The additional space can be released during downturns to increase the resilience of the banking sector. This motive suggests to have a stronger countercyclical capital response if the economy can face a low interest rate environment. To emphasize this motive, we incorporate macroprudential policy in the form of a countercyclical capital buffer that can impose additional capital requirements. The buffer is created during a phase of credit expansion and can then be released during a recession following the features of the Basel III framework. Therefore, the buffer is asymmetric and restricted to be non-negative, which we capture with an occasionally binding rule.

We demonstrate that macroprudential policy rule lowers the probability of hitting the reversal interest rate and increases the effectiveness of negative interest rate policies. The welfare-optimizing capital buffer rule reduces the probability of being at or below the reversal rate by around 23%. The banking sector builds up additional equity in good times, which can then subsequently be released during a recession. Having accumulated additional capital buffers during good times, the negative impact of monetary policy loosening on bank balance sheets is then dampened in a low interest rate environment. Consequently, monetary policy becomes more effective during economic downturns and the reversal interest rate is less likely to materialize, which improves overall welfare.

So far, we have analyzed the role of macroprudential policy without providing a historical perspective. We now move a step further and take the nonlinear model to the data using a particle filter. We then conduct a counterfactual analysis showing that the welfare-optimal macroprudential policy would have stabilized the credit supply in the low rate environment of the 2010s. Additionally, this approach provides to some extent an empirical validation of the model.

We extend our model to analyse the impact of a tiering system on the reversal rate and the role of macroprudential policy. Such a tiering system aims to lessen the burden of negative interest rates on banks' balance sheet. Tiering exempts part of the banks' excess reserves from negative renumeration, which then reduces the losses from holding liquid assets in a negative interest rate period. The ECB introduced a so-called two-tier system for reserves on October 30, 2019.[3] Incorporating the ECB's two tier-system in our framework, we find that such a policy helps to stabilize banks' profitability as well as the effectiveness of monetary policy in a low rate environment. Our framework also suggests that this kind of tiering system shifts the reversal rate to a level of around -1.5% and lowers the probability of encountering it. Despite the introduction of a tiering system, the profitability of banks still deteriorates and can reverse with sufficiently low interest rates. As a consequence, the strategic complementarity between monetary and macroprudential policy still prevails. We find that macroprudential policy provides similar welfare gains, even if the two-tier system is in place. We also show that implementing the two-tier system together with macroprudential policy provides the largest welfare gains.

Literature Review The paper adds to the growing literature about negative interest rates and the reversal interest rate, which is summarized for instance in Brandão-Marques et al. (2021), Heider, Saidi and Schepens (2021), and Balloch, Koby and Ulate (2022). Our paper builds on the seminal contribution by Abadi, Brunnermeier and Koby (2022), where the reversal interest rate is endogenously determined in a framework with an imperfect pass-through. Eggertsson et al. (2019) show the importance of reserve holdings for the bank lending channel with negative interest rates. Ulate (2021*b*) emphasizes the trade-off between increasing demand and reducing bank profitability for negative interest rates. Sims and Wu (2021) connect the size of the central bank's balance sheet to the impact of negative interest rates. In addition to these studies, De Groot and Haas (2020) show that negative interest rates can be used as a signal about future monetary policy.[4] With respect to the existing literature, we incorporate macroprudential policy and assess its interaction with negative interest rate policies and the reversal rate. Importantly, the location of the reversal rate is endogenous in our framework so that the negative interest

[3]Bank of Japan, Danmarks Nationalbank, Sveriges Riksbank and the Swiss National Bank also use a reserve tiering system.

[4]Further connected studies are Balloch and Koby (2019), Ulate (2021*a*) and Koenig and Schliephake (2022), among others.

rate policies can be initially expansionary, albeit with diminishing effectiveness, before becoming contractionary at the reversal rate. Unlike these studies, we use global solution methods to fully capture the non-linearities associated with the reversal rate.

This paper is also related to the large body of literature on the interaction between monetary policy and macroprudential policies.[5] Farhi and Werning (2016) and Korinek and Simsek (2016) show the importance of macroprudential policy in an environment with a binding zero lower bound. Lewis and Villa (2016) demonstrate that a countercyclical capital requirement can mitigate the output contractions in the presence of a zero lower bound. We assess macroprudential policy in a negative interest rate environment, where the intended effect of monetary policy can endogenously reverse. This creates a new motive for macroprudential policy and emphasizes the strategic complementarity with monetary policy. We also contribute to the modeling of the countercyclical capital buffer, which is one of the main macroprudential instruments considered in the literature. As a new feature, we incorporate the asymmetric design of the CCyB with an occasionally binding policy rule.[6]

The paper is also connected to the fast-growing empirical literature on negative policy rates. Jackson (2015) and Bech and Malkhozov (2016) analyze the early experiences with negative policy rates and find that a negative policy rate has a limited pass-through.[7] Heider, Saidi and Schepens (2019) document that negative policy rates impact bank lending in the euro area. Banks are reluctant to pass through the policy rates to their depositors, which results in less lending for banks that depend heavily on deposit funding. Hainz, Marjenko and Wildgruber (2017) provide empirical evidence that this is related to a fall in market power. Firms that are exposed to negative rates are likely to switch their banks and take other measures to alleviate the costs of negative rates. Borio, Gambacorta and Hofmann (2017) and Claessens, Coleman and Donnelly (2018) show that banks' profitability deteriorated for low interest rates. Additionally, Fuster, Schelling and Towbin (2021) point out that implementing a tiering system increases the profitability of banks. We incorporate this evidence about an imperfect pass-through and reduced bank profitability in a non-linear macroeconomic model to assess monetary policy effectiveness, the reversal rate and the interaction with macroprudential policy.

Outline The paper is organized as follows. In Section 2, the non-linear macroeconomic model is introduced. We calibrate the model and parametrize the imperfect deposit rate pass-through in Section 3. In Section 4, we study the reversal rate and derive the effective

[5]See e.g. Darracq-Pariès, Kok and Rodriguez-Palenzuela (2011), Angelini, Neri and Panetta (2014), Rubio and Carrasco-Gallego (2014), Benes and Kumhof, 2015, Collard et al. (2017), De Paoli and Paustian (2017), Gelain and Ilbas (2017), Bluwstein et al. (2020), among many others.

[6]Van der Ghote (2021) computes a constrained optimal macroprudential policy.

[7]Other studies are e.g. Ampudia and Van den Heuvel (2018), Basten and Mariathasan (2018), Altavilla et al. (2021), Eisenschmidt and Smets (2019), Mendicino, Puglisi and Supera (2021), among others.

lower bound on monetary policy. In Section 5, we incorporate macroprudential policy to study its interaction with the reversal rate. We conclude in Section 6.

2 The Model

The setup is a New Keynesian framework with a capital-constrained banking sector giving rise to financial accelerator effects as in Gertler and Karadi (2011). We embed two further financial frictions in this model that enable the possibility of a reversal interest rate: i) a search model with switching costs in the deposit market, which microfounds an imperfect pass-through of policy rates to deposit rates for low interest rates, as in Brunnermeier and Koby (2018) and ii) a reserve and liquidity requirement for the banking sector which generates substantial government asset holdings as in Eggertsson et al. (2019).

The imperfect pass-through results from a depletion of banks' market power in deposit markets with low interest rates, which affects monetary policy transmission when the economy approaches negative interest rate territory. Once the pass-through becomes increasingly imperfect, the funding costs of the banks decrease less as well as the aggregate demand stimulus via households is weaker. This channel creates a substantially diminished effectiveness of negative interest rate policies.

However, the combination with a reserve requirement for monetary policy purposes and regulatory liquidity constraints is necessary to observe a reversal rate.[8] This second friction forces banks to hold liquid government bonds, where the return on these bonds comoves with the policy rate, for a fraction of their deposits. Even though the government bonds provide a stable profit in normal times, they can cause losses during periods of low rates. When the policy rate is reduced to a sufficiently low level, the spread between the policy rate and deposit rate diminishes and can even turn negative due to the imperfect deposit rate pass-through. As a consequence, bank profitability deteriorates, which then reduces credit supply and enables the reversal rate.

For this reason, the effectiveness of monetary policy diminishes step by step in low rate environment until it reaches a turning point, where the bank lending channel of monetary policy breaks down and reverses. To capture these state-dependencies and asymmetries, we solve the model with global methods in its non-linear specification.

[8]With regard to monetary policy operations, the reserve holdings relate to the standard central bank minimum reserve requirements. On the regulatory side, liquid asset holdings are needed to comply with minimum liquidity requirements such as the Liquidity Coverage Ratio and the Net Stable Funding Ratio. Given the typically low risk weights on such liquid instruments, banks may also have an incentive to hoard them on the balance sheet in order to retain a solid capital ratio.

2.1 Model Description

Households The representative household is a family with perfect consumption insurance for the different members. The family consists of workers and bankers with constant fractions. The workers elastically supply labor to the non-financial firms, while the bankers manage a bank that transfers its proceedings to the household. Additionally, the household also owns the non-financial firms and receives the profits.

The household can hold deposits D_t for which it earns a predetermined interest rate. Importantly, the household holds the deposits only at a single bank j to which the household is matched at the beginning of the period. The bank j pays the predetermined nominal rate R^D_{jt} on the deposits. To deposit its fund at another bank, the household needs to pay switching costs, which implies market power for banks. We specify this search model in more detail in the bank problem. As shown later, it turns out that all banks set the same deposit rate in equilibrium. Therefore, we can drop the subscript j for the deposit rate in what follows. This also implies that households do not switch their bank so that we can refrain from adding these costs in the budget constraint or utility function in what follows.

In addition to this, the return on the deposits also depends exogenously on the risk premium shock η_t, which follows an AR(1) process and is based on Smets and Wouters (2007). This shock is shown to be empirically very important in explaining the Great Recession and zero lower bound episodes in estimated DSGE models.[9] This shock creates a wedge that distorts the choice of deposits as it affects the decision between consumption and saving. At the same time, the risk premium shock impacts the refinancing costs of the banking sector as it alters the payments on the deposits to the households. Its structural interpretation is further outlined in Appendix C.

The nominal budget constraint reads as follows:

$$P_t C_t = P_t W_t L_t + P_{t-1} D_{t-1} R^D_{t-1} \eta_{t-1} - P_t D_t + P_t \Pi^P_t - P_t \tau_t \tag{1}$$

where P_t is an aggregate price index, C_t is consumption, W_t is the wage, L_t is labor supply, D_t are the deposits and Π^P_t are the real profits from the capital good producers, retailers and transfers with the banks and τ_t is the lump sum tax.

The household maximizes its utility that depends on consumption and leisure:

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \right] \tag{2}$$

[9] For instance Barsky, Justiniano and Melosi (2014) and Christiano, Eichenbaum and Trabandt (2015) show this using linearized medium-sized DSGE models, among others. Gust et al. (2017) estimate a non-linear model featuring this shock.

The first-order conditions are given as:

$$\beta R_t^D \eta_t E_t \frac{\Lambda_{t,t+1}}{\Pi_{t+1}} = 1$$
$$\chi L_t^{\varphi} = C_t^{-\sigma} W_t$$

where $\Lambda_{t-1,t} = C_t^{-\sigma}/C_{t-1}^{-\sigma}$ and Π_t is gross inflation. The risk premium shock creates a wedge in the Euler equation. An exogenous increase in the risk premium leads to a higher return on deposits. This induces the households to increase their deposit holdings and to postpone consumption, which lowers aggregate demand.

Banking Sector The banks' role is to intermediate funds between the households and non-financial firms. The bank can be thought as consisting of a main branch that governs most of its activity and a separate branch that sets the deposit rate. The main branch collects deposits from households to buy securities with the aim to maximize the net worth of the bank. The separate branch is responsible for setting the deposit rate and has varying market power.

We first derive the problem of the main branch of the banks. The main branch holds net worth n_t and collects deposits d_t from households to buy securities s_t from the intermediate good producers at the real price Q_t and reserve assets a_t from the government. The flow of fund constraint in nominal terms is

$$Q_t P_t s_t + P_t a_t = P_t n_t + P_t d_t \tag{3}$$

where the lowercase letters indicate an individual banker's variable, and the uppercase letters denote the aggregate variable. The banker earns the stochastic return R_{t+1}^K on the securities and pays the nominal interest R_t^D as well as a risk premium for the deposits. The main branch takes the level of the deposit rate as given as it is set by a separate branch. The reserve assets earn the nominal gross return R_t^A, which is the policy rate. Leverage is defined as securities over assets:

$$\phi_t = \frac{Q_t s_t}{n_t}$$

To accrue net worth, the earnings are retained:

$$P_{t+1} n_{t+1} = R_{t+1}^K Q_t P_t s_t + R_t^A P_t a_t - R_t^D \eta_t P_t d_t \tag{4}$$

which can be written in real terms as

$$n_{t+1} = \frac{R_{t+1}^K Q_t s_t + R_t^A a_t - R_t^D \eta_t d_t}{\Pi_{t+1}} \tag{5}$$

The banker closes its bank with an exogenous probability of $1 - \theta$ and transfers the accumulated net worth to households in case of exit. Therefore, the banker maximizes its net worth:

$$v_t(n_t) = \max_{s_t, d_t, a_t} (1-\theta)\beta E_t \Lambda_{t,t+1} \Big((1-\theta) n_{t+1} + \theta v_{t+1}(n_{t+1}) \Big) \tag{6}$$

The banker is subject to an agency problem, which imposes a constraint on the leverage decision. The banker can divert a fraction λ of the bank's assets as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). Since this fraction cannot be recovered by the households, funds are only supplied if the banker's net worth exceeds the fraction λ of bank assets. Furthermore, the banker faces a requirement to hold a certain amount of government assets that cover at least a fraction δ^B of the deposits. This requirement is meant to capture both regulatory liquidity constraints and the reserve requirements for monetary policy purposes.[10] The two constraints can be summed up as:

$$v_t(n_t) \geq \lambda(Q_t s_t + a_t) \tag{7}$$

$$a_t \geq \delta^B d_t \tag{8}$$

The banker's problem is given as:

$$\psi_t = \max_{\phi_t} \mu_t \phi_t + \nu_t \tag{9}$$

$$\text{s.t.} \quad \mu_t \phi_t + \nu_t \geq \lambda \Big(\frac{1}{1-\delta^B} \phi_t - \frac{\delta^B}{1-\delta^B} \Big) \tag{10}$$

where we define $\psi_t = \frac{v_t(n_t)}{n_t}$ and assume that the reserve ratio $a_t = \delta^B d_t$ is binding and discussed later. μ_t is the expected discounted marginal gain of expanding securities for constant net worth, ν_t is the expected discounted marginal gain of expanding net worth for constant assets and R_t is the deposit rate adjusted for the holding of reserve assets:

$$\mu_t = \beta E_t \Lambda_{t,t+1} \left(1-\theta+\theta\psi_t\right) \frac{R^K_{t+1} - R_t}{\Pi_{t+1}} \tag{11}$$

$$\nu_t = \beta E_t \Lambda_{t,t+1} \left(1-\theta+\theta\psi_t\right) \frac{R_t}{\Pi_{t+1}} \tag{12}$$

$$R_t = (\eta_t R^D_t) \frac{1}{1-\delta^B} - R^A_t \frac{\delta^B}{1-\delta^B} \tag{13}$$

The banker's leverage maximization results in an optimality condition:

$$\xi_t = \frac{\lambda/(1-\delta^B) - \mu_t}{\mu_t} \tag{14}$$

where ξ_t is the multiplier on the market-based leverage constraint in the banker's problem. This constraint is binding if $0 < \mu_t < \lambda/(1-\delta^B)$, which requires the return on the security

[10] Curdia and Woodford (2011) and Eggertsson et al. (2019) use a function in which reserves lower the intermediation costs of the banks. The regulatory liquidity requirement is not explicitly modeled but provides an additional motivation for banks to hold substantial amounts of liquid government bonds and other assets on their balance sheets.

to be larger than the combined interest rate adjusted for inflation $E_t(R^K_{t+1} - R_t)/\Pi_{t+1} \geq 0$. The reserve asset ratio is binding as long as the expected return of the security is larger than the policy rate adjusted for inflation $E_t(R^k_{t+1} - R^A_t)/\Pi_{t+1} \geq 0$. Both constraints are binding at the relevant state space, which we verify numerically.

The individual leverage ϕ_t does not depend on bank-specific components so that it can be summed up over the individual bankers, that is:[11]

$$Q_t S_t = \phi_t N_t \tag{15}$$

The aggregate evolution of net worth N_t is the sum of the net worth of surviving bankers N^S_t and newly entering banks that N^N_t that receive a transfer from the households:

$$N_t = N^S_t + N^N_t \tag{16}$$

$$N^S_t = \theta N_{t-1} \frac{(R^K_t - R_{t-1})\phi_{t-1} + R_{t-1}}{\Pi_t} \tag{17}$$

$$N^N_t = \omega^N \frac{S_{t-1}}{\Pi_t} \tag{18}$$

Deposit Rates and the Imperfect Pass-Through of the Policy Rate The model features an imperfect pass-through of the policy rate, which we microfound using a search model with switching costs for deposits based on Brunnermeier and Koby (2018).[12] Each household can hold its deposits D^i_t only at single bank j to which the household is matched at the beginning of the period. The bank j pays the return R^D_{jt} on the deposit. To deposit its fund at another bank, the household needs to pay a switching cost. This can be thought of an activation level $\tilde{\omega}(R^A)$, which depends on the policy rate R^A_t. The household is willing switch if the difference between the interest rate set by the central bank R^A and the bank specific deposit rate R^D_{jt} is larger than the activation level:

$$R^A_t - R^D_{jt} + \omega^C > \tilde{\omega}(R^A_t) \tag{19}$$

where the spread is adjusted for a constant ω^C. The constant enables to have a scenario with switching costs $\tilde{\omega}(R^A_t) > 0$ and $R^A_t < R^D_{jt}$ simultaneously.

The activation level (and thus the cost of switching) is state-dependent in the level of the policy rate. Specifically, the activation level falls in a low interest rate environment. In other words, everything else equal, households are more likely to switch banks if the policy rate is low. This assumption is e.g. in line with the empirical evidence of Hainz, Marjenko and Wildgruber (2017), who provide firm survey evidence of increased switching of banks in a negative interest rate rate environment. Furthermore, we impose that this

[11] Similarly, the leverage ratio associated with reserve assets does not depend on bank specific components.

[12] We integrate the search model with state-dependent switching costs for the deposit market from Brunnermeier and Koby (2018) in a general equilibrium framework.

decrease for the activation level is accelerating with a lower level of the policy rate so that households become more and more willing to switch with low rates. Formally, the activation level satisfies the following conditions for the first and second order derivative:

$$\frac{\partial \tilde{\omega}(R_t^A)}{\partial R_t^A} > 0 \quad \text{and} \quad \frac{\partial^2 \tilde{\omega}(R_t^A)}{\partial {R_t^A}^2} < 0 \quad \text{if } R_t^A < \bar{R}^A \tag{20}$$

These relations are conditioned on R^A being below some threshold value $\bar{R}^A$ because the focus is on a low rate environment. The implication is that households are more likely to switch banks for a given spread between the policy rate and their bank's deposit rate in a low rate environment.

We connect this now to the banks' branch that sets the deposit rate. The branch of bank j is matched with a continuum of households that provide deposits.[13] The households stay with the matched bank j if the spread between R_t^A and R_{jt}^D (adjusted for the constant ω^C) is smaller than the switching costs or the offered deposit rates at all competitor banks are lower. Formally, the share of households Υ that keeps having their deposits with their associated bank j is given by

$$\Upsilon(R_{jt}^D; R_{-jt}^D, R_t^A) \equiv \mathbf{1}_{R_t^A - R_t^D + \omega^C \geq \tilde{\omega}(R^A) \ \vee \ R_j^D > \max R_{-j}^D} \tag{21}$$

where $\mathbf{1}$ is an indicator function and R_{-j}^D is the set of prices chosen by the remaining banks. This gives the extensive margin of the deposit rate choice of the banks. The intensive margin is the amount of deposits that is provided by each households for the posted interest rate R_j^D, which is denoted as D_t^j, The supply of deposits to bank j can then be written as

$$\bar{D}_{jt}(R_j^D; R_{-j}^D, R^A) = \Upsilon(R_j^D; R_{-j}^D, R^A) D_t^j \tag{22}$$

We use a guess and verify approach to solve this problem. In equilibrium, each branch sets the deposit rate to the lowest possible level that avoids switching, that is:

$$R_t^D = R_t^A - \tilde{\omega}(R^A) + \omega^C \tag{23}$$

To verify the guess, we need to show that no bank would like to deviate from this symmetric equilibrium. If the branch of bank j would set the rate lower, then they would not receive any deposits. The matched households would pay the switching costs and move their deposits to another banks. If the bank would set a higher interest rate, it would reduce its profit. The reason is that the branch, which sets the deposit rate, takes the demand of deposits from the main branch as given.[14] guess is verified. Furthermore, we

[13] The banks can have a different size of the balance sheet. The relative share of households that is matched to each bank is proportional to the banks' net worth.

[14] Note that another potential set of equilibria is one, in which all banks jointly set the same deposit rate below the activation level. In such an equilibrium, each matched household would switch banks. One way to avoid this equilibrium would to impose a infinitesimal costs for banks if their customers switch.

can drop the subscript j for the deposit rate because each bank sets the same deposit rate. Therefore, equation (23) summarizes the equilibrium relation between the deposit and policy rate.

To sum up, there is an imperfect pass-through of the policy instrument to retail deposit rates for a low rate environment. The reason is that banks' market power in deposit markets depletes for low policy rates due to a reduction in the costs of switching banks for depositors. This implies that the markdown for the deposit rate varies with the level of the policy rate R_t^A.

Non-financial Firms The non-financial firms are the intermediate good producers, retailers subject to Rotemberg pricing and capital good producers.

Intermediate good producers produce output using labor and capital:

$$Y_t = A^P K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{24}$$

where A^P is the productivity. It sells the output at price P_t^M to the retailers. It pays the labor at wage W_t. The firm purchases capital at market price Q_{t-1} in period $t-1$, which is financed with a loan from the bank. It pays the state-contingent interest rate R_t^K to the banks. Thus, the maximization problem of the firm can be written as

$$\max_{K_{t-1},L_t} \sum_{i=0}^{\infty} \beta \Lambda_{t,t+1} \left[P_t P_t^M Y_t + P_t Q_t (1-\delta) K_{t-1} - R_t^K P_{t-1} Q_{t-1} K_{t-1} - P_t W_t L_t \right] \tag{25}$$

This gives the nominal rate of return on capital:

$$R_t^k = \frac{(P_t^m \alpha Y_t / K_{t-1} + (1-\delta) Q_t)}{Q_{t-1}} \Pi_t$$

The final good retailers, which are subject to Rotemberg pricing, buy the intermediate goods and bundle them to the final good using a CES production function:

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \tag{26}$$

where $Y_t(f)$ is the demand of output from intermediate good producer j. Cost minimization implies the following intermediate good demand:

$$Y_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\epsilon} \tag{27}$$

Then each bank would like to set slightly higher interest rates than its competitors to avoid paying the switching cost. Thus, this set of equilibria would not exist anymore.

where the price index P_t of the bundled good reads as follows

$$P_t = \left[\int_0^1 P_t(f)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \tag{28}$$

The retailer then maximizes its profits

$$E_t \left\{ \sum_{t=0}^{\infty} \left[\left(\frac{P_t(f)}{P_t} - MC_t \right) Y_t(f) - \frac{\rho^r}{2} Y_t \left(\frac{P_t(f)}{P_{t-1}(f)\Pi} - 1 \right)^2 \right] \right\} \tag{29}$$

where $MC_t = P_t^M$ and Π is the inflation target of the central bank. This gives us the New Keynesian Phillips curve:

$$\left(\frac{\Pi_t}{\Pi} - 1 \right) \frac{\Pi_t}{\Pi} = \frac{\epsilon}{\rho^r} \left(P_t^m - \frac{\epsilon - 1}{\epsilon} \right) + \beta E_t \Lambda_{t,t+1} \frac{Y_{t+1}}{Y_t} \left(\frac{Pi_{t+1}}{\Pi_t} - 1 \right) \frac{\Pi_{t+1}}{\Pi}$$

Capital good producers have access to the function $\Gamma(I_t, K_{t-1})$ which they can use to create capital out of an investment I_t. The capital is then sold so that the maximization problem reads as follows:

$$\max_{I_t} Q_t \Gamma(I_t, K_{t-1}) K_{t-1} - I_t \tag{30}$$

The real price of capital is then given as

$$Q_t = \left[\Gamma'(I_t, K_{t-1}) K_{t-1} \right]^{-1}$$

The stock of capital evolves then as:

$$K_t = (1 - \delta) K_{t-1} + \Gamma(I_t, K_{t-1}) K_{t-1} \tag{31}$$

Monetary Policy The central bank sets the nominal interest rate for the reserve asset.[15] It responds to inflation and output deviations, while it faces an iid monetary policy shock ζ_t. Furthermore, the central bank can set a lower bound $\tilde{R}^A$ that restricts the level of the interest rate. The policy rule reads as follows:

$$R_t^A = \max \left[R^A \left(\frac{\Pi_t}{\Pi} \right)^{\theta_\Pi} \left(\frac{Y_t}{Y} \right)^{\theta_Y}, \tilde{R}^A \right] \zeta_t \tag{32}$$

The lower bound gives the central bank the opportunity to endogenously restrain itself from lowering the policy rate below a specific rate as the model features a potential reversal interest rate. This level could be a negative or positive net interest rate as we will later determine based on welfare considerations. In contrast to this, a zero lower bound exogenously restricts the central bank from setting a negative net interest rate.

[15]The central banker has no access to other policy tools such as quantitative easing that could be used as substitutes for conventional monetary policy as shown in Sims and Wu (2020).

Government and Resource Constraint The government has a balanced budget constraint. It holds the reserve assets and taxes the households with a lump sum tax:

$$P_t \tau_t + P_t A_t = R^A_{t-1} P_{t-1} A_{t-1} \tag{33}$$

The resource constraint is:

$$Y_t = C_t + I_t + \frac{\rho^r}{2}\left(\frac{\Pi_t}{\Pi} - 1\right)^2 Y_t \tag{34}$$

2.2 Competitive Equilibrium

The competitive equilibrium is defined as a sequence of quantities $\{C_t, Y_t, K_t, L_t, I_t, D_t, S_t, \Pi^P_t, N_t, N^E_t, N^N_t\}_{t=0}^{\infty}$, prices $\{R_t, R^D_t, R^A_t, R^K_t, Q_t, \Pi_t, \Lambda_{t,t+1}, W_t, P^M_t\}_{t=0}^{\infty}$, bank variables $\{\psi_t, \nu_t, \mu_t, \phi_t\}$, and exogenous variable $\{\eta_t\}_{t=0}^{\infty}$ given the initial conditions $\{K_{-1}, R_{-1}D_{-1}, \eta_{-1}\}$ and a sequence of shocks $\{e^{\eta}_t, \zeta_t\}_{t=0}^{\infty}$ that satisfies the non-linear equilibrium system of this economy provided in Appendix A.

2.3 Global Solution Method

The model is solved in its non-linear specification with global methods. This approach is necessary to capture the state-dependency of the monetary policy pass-through. In particular, this setting allows monetary policy to have a different quantitative as well as qualitative impact depending on the state of the economy. Another advantage of the non-linear approach is that agents take future uncertainty into account, which is particularly relevant due to the highly non-linear region of low and negative interest rates. The solution method is time iteration with piecewise linear policy functions based on Richter, Throckmorton and Walker (2014). The algorithm description is in Appendix G.

3 Calibration

The model is calibrated to the euro area economy with a particular emphasis on the current low interest rate environment. The considered horizon begins in 2000Q1 and ends in 2019Q4. The data to parametrize the model is mostly based on the ECB's statistical data warehouse and the Statistical Office of the European Communities. Appendix B contains the details regarding the data sources and construction.

Table 1 summarizes the calibration. The discount factor is set to 0.9975, which corresponds to a risk-free rate of 1% per annum. This is in line with the average estimate of 1.27 for the euro area from Holston, Laubach and Williams (2017).[16] The inflation target is set to 1.9% to match the ECB's inflation target at the time of below, but close to, 2%.

[16]Even though our value is slightly lower, this accounts for the trend of falling real interest rates.

Table 1: Calibration

Parameters	Sign	Value	Target
a) Preferences, technology and monetary policy			
Discount factor	β	0.9975	Risk-free rate = 1% p.a.
Risk aversion	σ	1	Risk aversion = 1
Disutility of labor	χ	12.38	SS labor supply = 1/3
Inverse Frisch labor elasticity	φ	1.5	Chetty et al. (2011)
Capital production share	α	0.33	Capital income share = 33%
Capital depreciation rate	δ	0.025	Annual depreciation rate = 10%
Elasticity of asset price	η_i	0.25	Bernanke, Gertler and Gilchrist (1999)
Investment parameter 1	a_i	0.5302	$Q = 1$
Investment parameter 2	b_i	-0.0083	$\Gamma(I/K) = I$
Elasticity of substitution	ϵ	11	Market power of 10%
Rotemberg adjustment costs	ρ^r	1000	1% slope of NK Phillips curve
Inflation	Π	1.0047	Inflation Target = 1.9% p.a.
Inflation response	κ_π	2.5	Standard
Output response	κ_Y	0.125	Standard
Endogenous lower bound	$\tilde{R}^A$	0.995	Lower bound of -2% p.a.
b) Deposit rate pass-through			
Switching cost parameter 1	ω_1	-0.0008	Perfect pass-through at SS
Switching cost parameter 2	ω_2	0.0027	Markdown $R^A = \bar{R}^A = 0.56\%$ p.a.
Switching cost parameter 3	ω_3	124.73	Imperfect pass-through if $R^A < \bar{R}^A$
Switching cost parameter 4	ς	0.001	Markdown if $R^A > \bar{R}^A = 0.56\%$ p.a.
c) Financial Sector			
Reserve asset requirement	δ^B	0.2545	Government asset share = 23% if $R^A < 1$
Survival probability	θ	0.9	$R_K - R_D = 2\%$ p.a.
Diversion banker	λ	0.1540	Leverage = 8
Proportional transfer to new banks	ω^N	0.00523	Uniquely determined from θ and λ
d) Shocks			
Persistence risk premium shock	ρ^η	0.75	Probability of negative policy rate
Std. dev. risk premium shock	σ^η	0.125%	Standard deviation of detrended output = 0.021
Std. dev. monetary policy shock	σ^ζ	$6.25e^{-4}$	Normalization to 25 basis points

The inverse Frisch Labor Elasticity φ equals 1.5 to be in line with the evidence provided in Chetty et al. (2011). The disutility of labor aims that agents work 1/3 of their time. The parameter α is set to 0.33 in line with the capital share of production. The depreciation rate is 0.025 to match an annualized depreciation rate of 10%. The elasticity of the asset price is parameterized to 0.25 as in Bernanke, Gertler and Gilchrist (1999). We target a mark-up of 10% so that $\epsilon = 11$. The Rotemberg parameter $\rho^r = 1000$ implies a 1% slope of the New Keynesian Phillips curve. The inflation and output response are set to 2.5 and 0.125, which are standard values in the literature. The monetary authority does not lower the systemic component of the policy rate below -2% per annum, which gives $\tilde{R}^A = 0.995$ for the endogenous lower bound in the baseline version. We choose this value to allow for a potential contractionary impact of interest rate cuts in our experiments. We later derive the welfare optimal lower bound and also discuss the implications of the lower bound on selected key moments.

Deposit Rate Pass-Through The switching costs are fitted based on the declining deposit rate pass-through in the data. Specifically, we use a flexible functional form that allows for state-dependent switching costs $\tilde{\omega}(R_t^A) = -\omega(R_t^A) + R_t^A + \omega^C$ where the function $\omega(R_t^A)$ will be defined below. Using equation (23), the deposit rate is then given as $R_t^D = \omega(R_t^A)$. We choose a flexible function $\omega(R_t^A)$ in line with Brunnermeier and Koby (2018) so that the we obtain the following mapping between the deposit rate and policy rate:

$$R_t^D = \omega(R_t^A) = \begin{cases} \omega^1 + \omega^2 \exp(\omega^3(R_t^A - 1)) + 1 & \text{if } R_t^A < \bar{R}^A \\ R_t^A - \varsigma & \text{else} \end{cases} \tag{35}$$

This setup separates the connection between the two rates in a region with an imperfect pass-through ($R_t^A < \bar{R}^A$) and a region with a perfect pass-through ($R_t^A \geq \bar{R}^A$), where the threshold parameter $\bar{R}^A$ is the deterministic steady state of the policy rate. The parameters ω^1, ω^2, ω^3, ς determine the switching costs and thus the banks' state-dependent market power in the deposit market. As the equation shows, this results in an imperfect deposit rate pass-through.

We target the connection between bank retail deposit rates and the policy rate in the euro area. We use a weighted measure of different deposit rates to take into account the different maturities in the data. The policy rate is defined as the deposit facility rate. The evolution of both series can be seen in the upper panel of Figure 1.

To calibrate the parameters and capture the varying deposit rate pass-through in the euro area economy, we use a non-linear least squares approach, as can be seen in Figure 2. Specifically, we calibrate the shape parameters to minimize the distance between the connection of the policy and deposit rate. This approach uses the observations that are below the threshold $\bar{R}^A$. Furthermore, we impose two restrictions on this minimization. First, there is a perfect deposit rate pass-through at the steady state.[17] Second, the markdown at the steady state is 0.56% in annualized terms. For the markdown, we use the measured average spread between the deposit rate and the deposit rate facility conditional on being at or above the steady state. This also gives the markdown for the region with perfect pass-through $\varsigma = 0.0014$. We then fit the curve using a non-linear least square approach that incorporates the described constraint. The fitted values of ω^1, ω^2 and ω^3 are -0.0008, 0.0027 and 124.73. Note that the parameter ω^C does not need to be specified as long as the level is sufficiently high that switching is costly for households in a low rate environment. Importantly, the used functional form for the switching costs satisfies the outlined conditions in the model section for the first and second order derivative for the chosen calibration, that is $\partial\tilde{\omega}(R_t^A)/\partial R_t^A > 0, \forall R_t^A < \bar{R}^A$ and $\partial^2\tilde{\omega}(R_t^A)/\partial {R_t^A}^2 < 0$. The details of the non-linear least squares approach are outlined in Appendix B.2.

[17]This implies that the derivative of the function at the steady state equals 1.

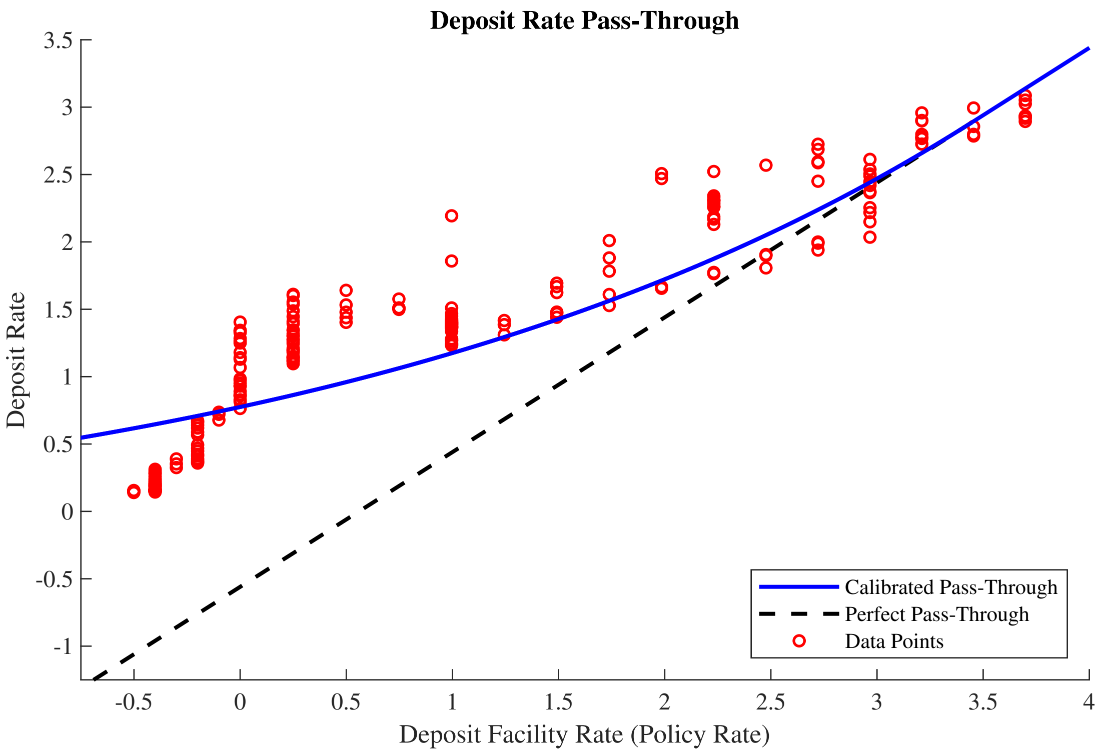

Figure 2: Figure shows the deposit rate pass-through estimated with a non-linear least squares approach. The blue line is the imperfect pass-through, the black dashed line is a scenario with a perfect pass-through and the red dots refer to the data points.

Banking Sector We calibrate the financial friction parameter λ to match a leverage ratio of 8. A leverage of 8 implies a capital requirement of 12.5%, which is very much in line with actual capital ratios based on Common Equity Tier 1 (CET1) relative to risk-weighted assets. For instance, the CET1 ratio of institutions directly supervised by the ECB stood at 14.9% in December 2019. The banks have to hold at least a fraction δ^B of their deposits as government assets. Different measures of government asset shares in the banks' balance sheet can be compared in Figure 3. The different shares are government bonds only, government bonds plus required reserve assets, and government bonds plus reserve assets. We match the model to the broadest measure as our requirement captures government bonds as well as reserve assets. According to this measure, since the introduction of negative interest rates in the euro area in 2014, the share of government assets to total banking sector assets has edged up to almost 25%. In line with this, we target that banks have a government asset share of 23% during periods of negative interest rates. The corresponding value for the fraction of deposits is then 0.2545. The banker's survival rate θ is set to 0.9 to obtain an average spread between the return on capital and deposit rate of 2% p.a. at the steady state similar to the New Area-Wide Model II.[18]

[18]The parameter can also be interpreted with regard to aggregate dividend payments, which are the returned net worth minus the transfer for new households. It is convenient to express the dividend as a ratio relative to market capitalization and book value of equity, respectively. At the deterministic steady state, the model implies a quarterly rate of 2.7% for the dividend payments to market capitalization ratio (Div_t) and of 3.3% for the dividend payments to book value of equity ratio. The values are broadly in

The average spread between the lending rate and deposit rate is around 2.5% p.a. in the data. However, there is a maturity mismatch in the data as loans have longer maturities on average. Moreover, the survival probability θ and the financial friction parameter λ uniquely determine the endowment to new bankers ω^N.

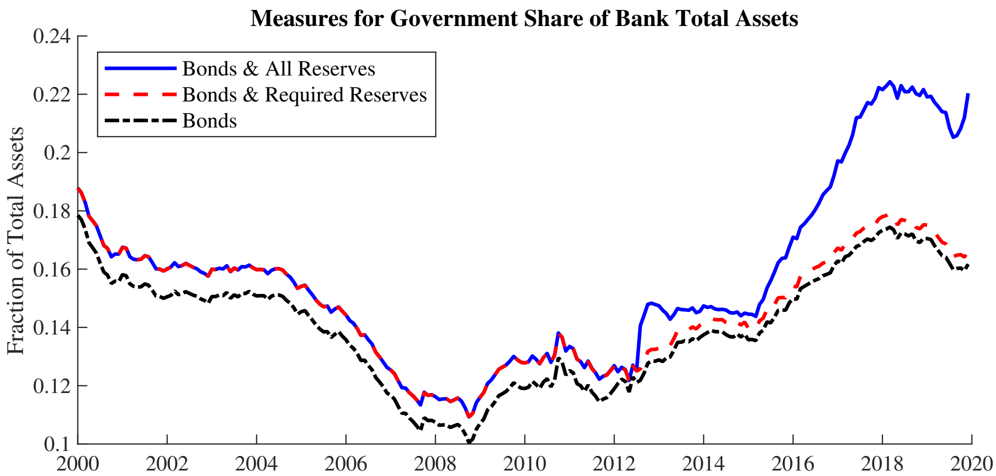

Figure 3: Figure shows different measures of the share of government assets in the bank's balance sheet.

Shocks The risk premium shock is parameterized to match the fluctuations in output and the frequency of a negative interest rate environment. We set the standard deviation σ^{η} to 0.125% and the persistence to 0.75. The model predicts a standard deviation of 2.2% for output in line with the data.[19] The policy rate falls below -1% with a 2.7% probability. A negative policy rate occurs with a probability of 5% in the model. A caveat is that the model underestimates the materialization of a negative policy rate compared to the recent experience in the euro, where the policy rate entered negative territory for the first time in June 11 in 2014 and is still below zero in the last quarter of 2019. Substantially increasing the episodes with negative interest rates poses a problem for a model featuring monetary policy ineffectiveness, as shown in Bianchi, Melosi and Rottner (2021) and Fernández-Villaverde et al. (2015) for the zero lower bound. The reason is that overly prolonged episodes in which monetary policy is not effective affect the stability of the model and can result in deflationary spirals.[20] The standard deviation of the monetary policy shock is set to 6.25e-4. This implies that a positive one standard

line (even though clearly on the upper side) with the return on equity of banks in the euro area.

[19]The standard deviation of detrended real GDP is 2.1%. As the model does not have a trend, we detrend the logarithm of real output linearly.

[20]Bianchi, Melosi and Rottner (2021) show that a high frequency of being at the zero lower bound can result in deflationary spirals so that an equilibrium does not exist anymore. The probability of a constrained monetary policy leads to a vicious circle of low inflation and rising real interest rates, which in turn leads to lower inflation. Fernández-Villaverde et al. (2015) show that, for instance, a tax that affects the Euler equation can help to match the duration and frequency of a zero lower bound episode. Dordal-i-Carreras et al. (2016) suggest to use a regime switching process instead of an AR(1) shock to better capture that periods at the zero lower bound are rare but long-lived.

deviation monetary policy shock would result in an interest rate raise of 25bps in a partial equilibrium setup.

4 Reversal Interest Rate and Effective Lower Bound

This section outlines the transmission of shocks in a low or negative interest rate environment and the conditions that give rise to the reversal interest rate. In particular, we demonstrate that the transmission of shocks and effectiveness of monetary policy is asymmetric and state-dependent due to the non-linear features of the model. The model predicts that negative interest rate policies can be effective, even though the effectiveness diminishes until monetary policy approaches the reversal interest rate. At this turning point, further monetary policy accommodation becomes contractionary. This threat of a reversal creates an effective lower bound on the monetary policy rule, which limits how negative the policy rate can go. We use our framework to locate the effective lower bound.

4.1 Non-Linearities and Asymmetric Monetary Policy

We first want to assess the potential non-linearities in the transmission of shocks that results from approaching low or negative interest rate territory.

Risk Premium Shock We begin with an impulse response analysis of the risk premium (demand) shock, which is shown in Figure 4. To detect asymmetries in the transmission of the shock over the business cycle, we consider expansionary and contractionary shocks with varying magnitudes. The starting point of the economy is the risky steady state, which is the point to which the economy would converge if future shocks are expected and the realizations turn out to be zero (Coeurdacier, Rey and Winant, 2011).

To begin with, the model has the standard financial accelerator which amplifies the impact of financial shocks. An increase in the risk premium, which is a contractionary shock, affects the consumption and saving decisions of the households as well as the refinancing costs of the banks. The households postpone consumption so that output drops. This affects banks as their return on assets is lower and asset prices fall. In addition, the funding costs of the banks increase. Both effects reduce the net worth and weaken the balance sheet of the banks which amplifies the shock via the financial accelerator mechanism. In response, the central bank lowers the interest rate to mitigate the bust. However, the impact of such a policy is non-linear due to the imperfect deposit rate pass-through and the reserve requirement.[21]

[21] The nonlinear mechanism creates a downside risk for output that is connected to the financial sector. While some approaches focus on the impact of financial crisis on economic downside risk (e.g. Gertler, Kiyotaki and Prestipino, 2020; Rottner, 2021), the mechanism here is different. It outlines the impact of low and negative interest rates on banks' profitability and its associated macroeconomic tail risk.

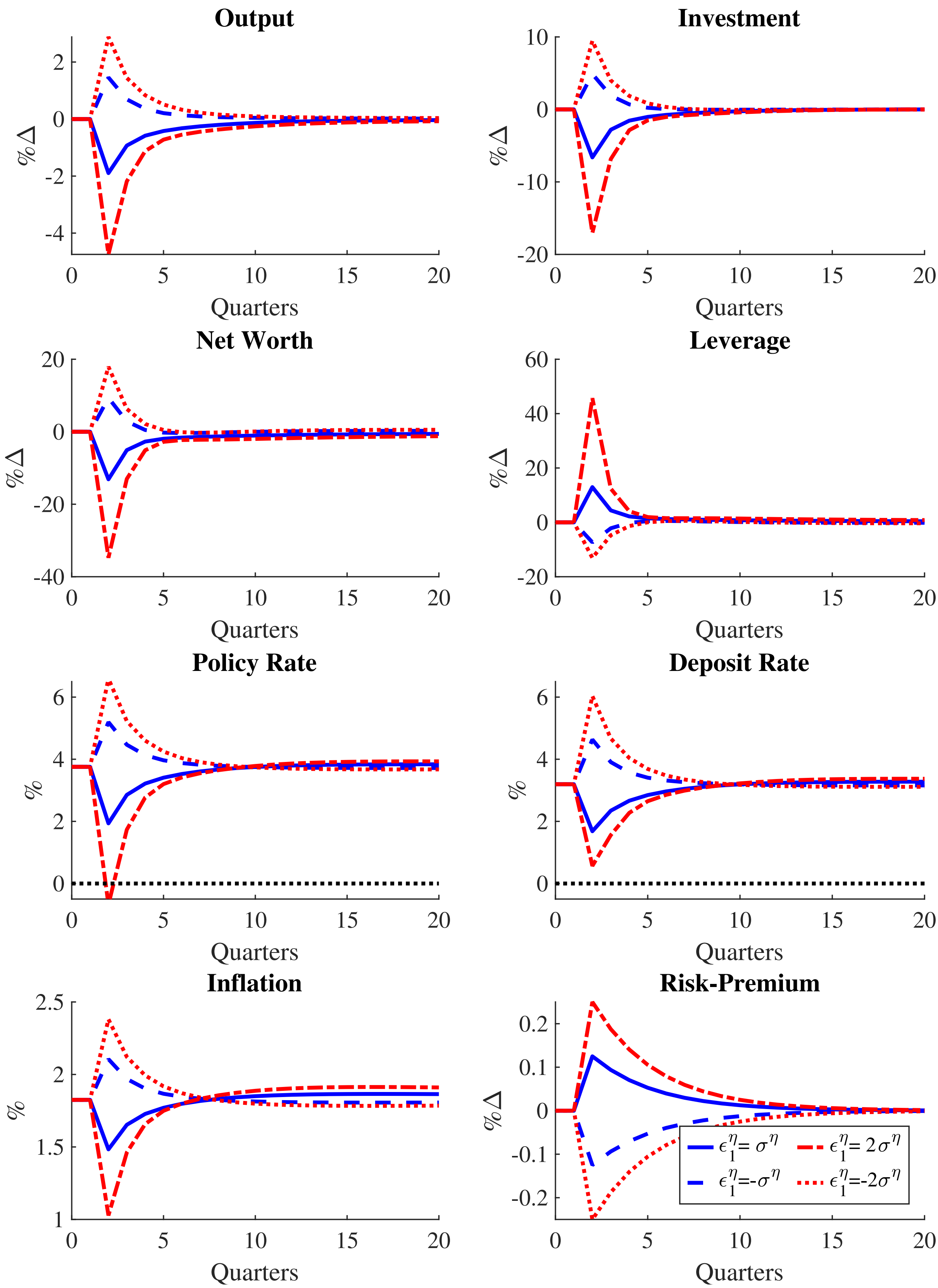

Figure 4: Impulse response functions of the risk premium shock that differ in the size and sign of the innovation. A one standard deviation increase (blue solid line) and decrease (blue dashed line) as well as a two standard deviation increase (red dash-dotted line) and decrease (red dotted line) for the innovation ϵ_t^η is shown. The black dotted line is the zero lower bound. The scales are either percentage deviations from the risky steady state (%Δ) or annualized net rate (%).

The stronger relative impact of a contractionary risk premium shock compared to an expansionary one demonstrates that monetary policy can lose its effectiveness. As can be seen in Figure 4, this asymmetry is visible from the reaction of output, the policy rates, bank net worth and leverage, all of which have a more pronounced response for a risk premium increase. Monetary policy is less effective in stabilizing the economy in a downturn as deposit rates move less than one-to-one due to the imperfect pass-through. This stems from two different channels that operate via the households and banks. First, the deposit interest rates offset less of the increase in the wedge in the household's Euler equation. This results in a stronger drop in consumption. Second, the funding costs of the banking sector do not decrease by much, as the deposit rates are decoupled from the policy rate. At the same time, the spread of the reserve assets also diminishes. This together implies that the banks' net worth losses are comparatively more severe so that there is a strong contraction of lending and output. Importantly, the financial accelerator increases such effects.

Furthermore, another non-linear feature can be discerned from the fact that the size of the contractionary shock matters for how forcefully it is transmitted to the economy. The economy responds considerably more than twice as strongly to a two standard deviation than a one standard deviation shock increase.[22] The reason is that the deposit rate pass-through becomes more sluggish the deeper the recession. This effect is reinforced through the government asset requirement. This shows that monetary policy becomes less and less effective around negative interest rates. In contrast to this, the size of a decrease in the risk premium has less of an effect if the economy is initially at the steady state. There is a perfect pass-through in this part of the state space, meaning that the size of the shock does not matter.

Monetary Policy Shock An exogenous lowering of the monetary policy rate boosts the economy if the economy starts at the risky steady state. Reducing the policy rate affects the deposit rate, which induces households to consume more and reduces the refinancing costs of banks. This leads to a rise in aggregate demand and an increased credit supply. Around the risky steady state the deposit rate pass-through is almost perfect, so that monetary policy is very effective and the non-linearities are very small. Furthermore, a monetary tightening or loosening has the same relative impact. The transmission of varying monetary policy shocks can be seen in Figure 18 in Appendix I. However, we will show now that this result depends on the state of the business cycle.

[22] As a more general point, it should be noted that the used model class features only some slight non-linearities if they are not combined with extra elements. Therefore, the imperfect deposit pass-through is very important to generate strong non-linearities.

4.2 Reversal Interest Rate and Asymmetric Monetary Policy

The previous simulation suggests at first glance that accommodative monetary policy is effective and there is no reversal interest rate. This is due to the fact that the starting point for the simulations is the risky steady state, which implies that the economy is in a region with normal interest rates and close to perfect deposit rate pass-through. However, the impact of monetary policy depends on the interest rate environment. For instance, negative interest rate policies could be much less effective or even contractionary. Therefore, combining the monetary policy shock with simultaneously occurring risk-premium shocks allows us to assess the monetary policy shock at different points of the business cycle.

Figure 5 shows the impulse responses of a negative one standard deviation monetary policy shock at different states of the business cycle. To approximate the business cycle, we use different risk premium innovations ϵ_1^η. A larger risk premium shock contracts the economy more severely. The starting point is still the steady state, but the risk premium shock contracts the economy. The displayed paths show the percentage deviations between a path with and without the monetary policy shock for varying risk premium innovations. Therefore, the combination of the shocks allows to analyse the state-dependent response to a monetary policy shock.

Depending on the size of the contractionary risk premium shock, the monetary policy shock becomes less powerful. The expansionary impact of monetary policy shock decreases with the strength of the risk premium shock as can be seen in the responses of output and net worth. In other words, monetary policy is less effective during a severe recession. In fact, its impact even reverses for a scenario with ϵ_t^η close to $3\sigma_t^\eta$, which approximates a severe recession. In this case, monetary policy, which is intended to be accommodative, actually reduces banks' profitability. As a consequence, the banks curtial their security holdings, which is also very persistent.

The reason is that the nominal interest rate is so low when the risk premium shock occurs that monetary policy not only becomes less effective, but is even harmful to the economy. While the economy is still in a positive interest rate area of around 2% for the first scenario (σ_t^η), the economy is already in a negative territory around -0.5 in the second scenario $2\sigma_t^\eta$. In the last scenario $3\sigma_t^\eta$, the interest rate is at the lower bound of -2.0%. It turns out that then an increase in the nominal rate would actually be beneficial in such a state of low rates. The reduction in the interest rate hurts the net worth of the banks sufficiently strongly due to their substantial government asset holdings. At the same time, the refinancing costs and aggregate demand of households are mostly unaffected as the deposit rate is very sticky in this state of the economy. As an additional point, the figure shows that the monetary policy shock has more impact on the policy rate in a low rate territory due to endogenous component of the Taylor rule. However, the deposit rate

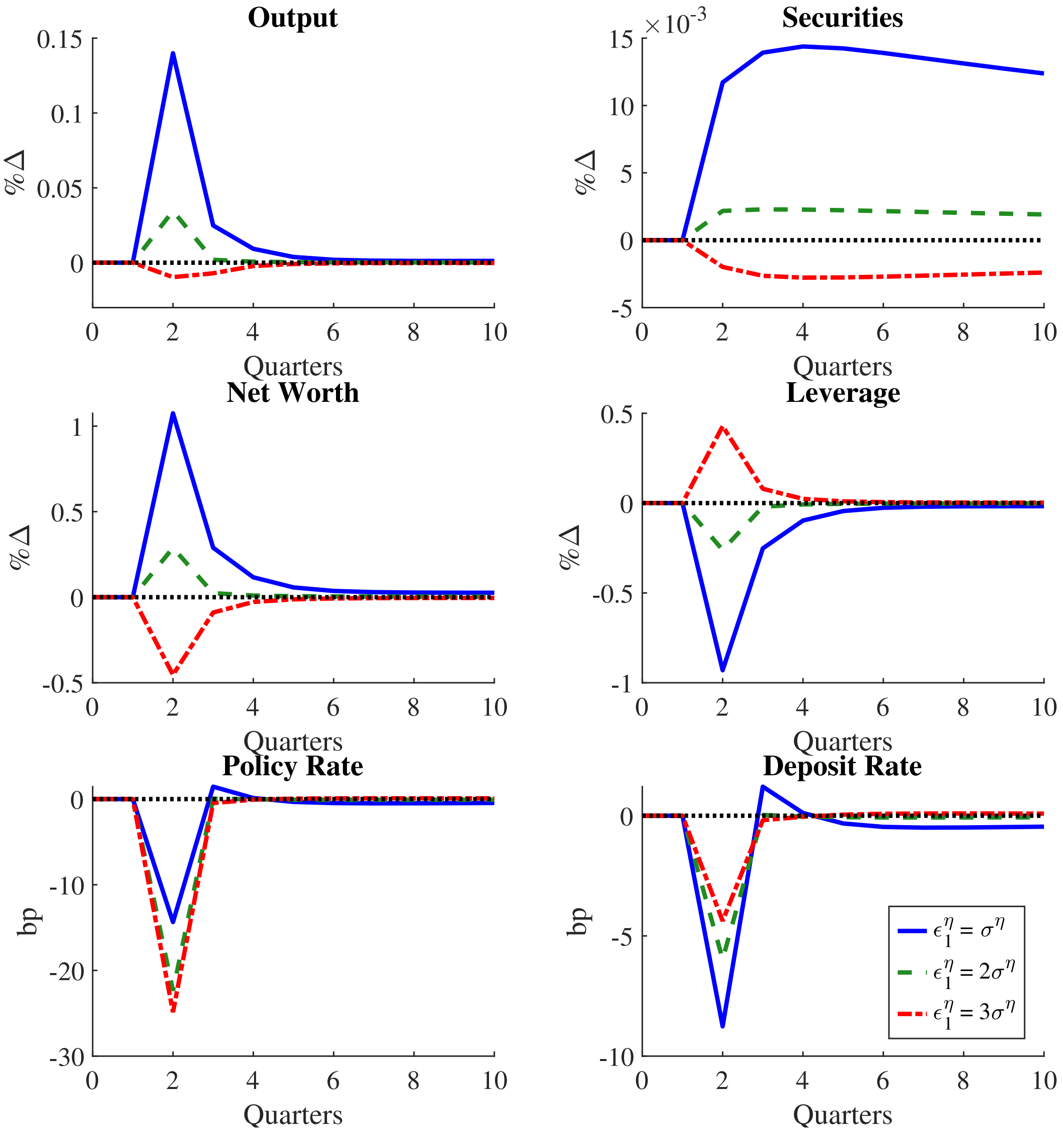

Figure 5: Impulse response analysis to show the state-dependent impact of monetary policy shocks. To generate the state dependency, the monetary policy shock is combined with different sized risk premium shocks. The blue solid line captures the combination with a one std. dev. contractionary risk premium shock, while the black dashed and red dash-dotted ones capture a two and three std. dev. contractionary risk premium shock, respectively. Each line displays the difference between a path with a negative one std. dev. monetary policy shock in period 1 relative to a path without a monetary policy innovation for a given risk premium shock. The deviations are in percent and basis points.

responds significantly less due to the imperfect pass-through.

For a more detailed analysis about the reversal of monetary policy, we assess the impact of an interest rate cut over the business cycle. Figure 6 shows the first period impact of an exogenous one standard deviation negative monetary policy shock for varying risk premium shocks, which are used to proxy the business cycle. We want to emphasize the connection between Figure 5 and Figure 6. While Figure 5 shows for three shocks the time-path of the impulse responses, 6 displays for a continuum of shocks the first period impact of the impulse responses.

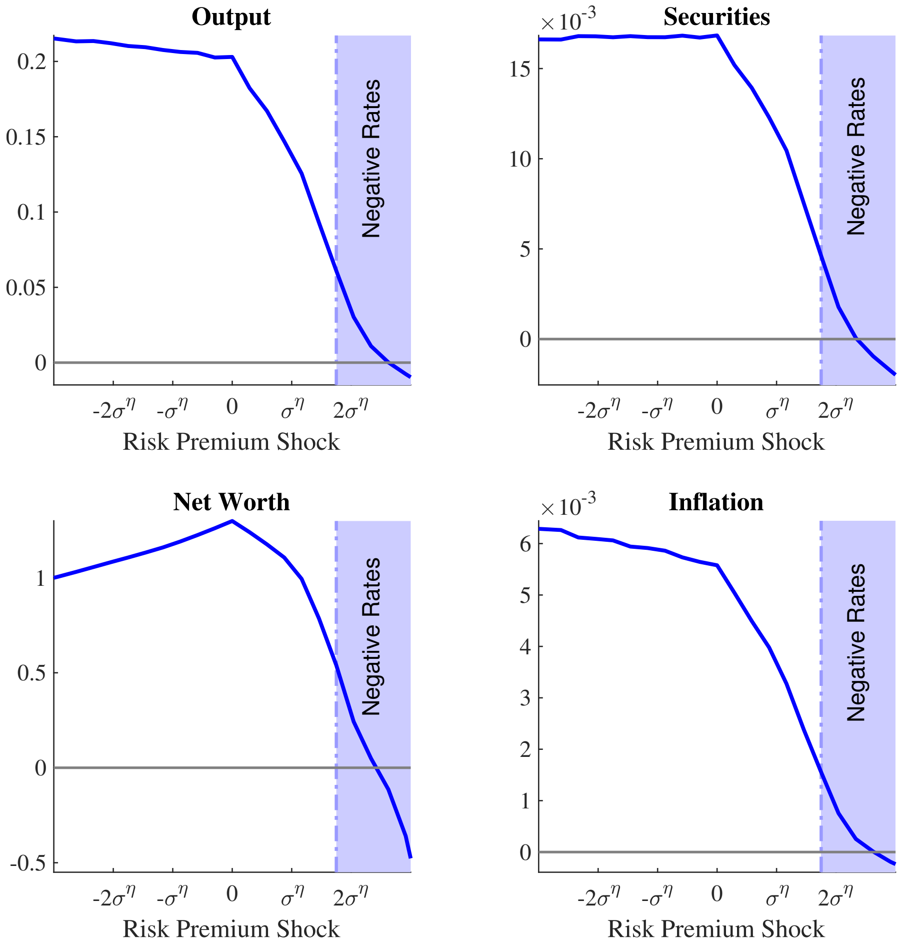

Figure 6: First period of an impulse response function to illustrate the state-dependent impact of monetary policy shocks. To generate the state dependency, the monetary policy shock is combined with different sized risk premium shocks, which are displayed on the horizontal axis. The vertical axis displays the state-dependent difference for the period $t = 1$ impulse response between a shocked path, which faces additionally a negative one std. dev. monetary policy shock, and a path, in which the monetary policy innovation does not occur. The deviations are in percent. The blue shaded area indicates the territory, where the risk premium shock pushes the economy in negative interest rate territory.

If the risk premium shock is negative or around zero, which can be interpreted as an expansion or as tranquil times, monetary policy is very effective. Importantly, the policy rate is high and efficiently passed through. In contrast to this, monetary policy is considerably less powerful in recessions than in booms, as can be seen by the impact on output. Monetary policy is initially to some extent still effective once the economy approaches negative territory, which is marked as blue shaded area. However, a more deep recession triggers a reversal of the impact of negative interest rate policies. Specifically, the turning point is reached around a risk premium shock of ϵ^η close to 3 standard deviations. From this point onwards, a policy rate cut triggers a fall in output and inflation. This is explained by the sluggish deposit rate pass through and the strong drop in bank net worth in this state of the economy.[23] Figure 19 in the Appendix I provides an alternative

[23] This mechanism is also highlighted by the dynamics of the policy rate and deposit rate, which are

display of the same chart, where the first period impact is connected to the interest rate level.[24] The figure also underlines that the effectiveness of monetary policy is diminishing in a low interest rate territory. For sufficient negative interest rates, the impact reverses and a policy rate cut is then contractionary.

Asymmetric Impact of Monetary Policy Tightening and Loosening Another implication is that the impact of a monetary tightening and loosening is asymmetric in a low rate environment. As monetary policy becomes less effective when lowering the rate further, a rate hike is relatively more powerful. Figure 7 shows the propagation of a 50 basis points increase or decrease when the economy starts in negative territory of -0.25%.[25] This highlights the asymmetric impact of monetary policy effectiveness. An interest rate hike has almost twice the impact on output and inflation relative to a cut. As a consequence, the effectiveness of monetary policy increases successively once the economy starts to move away from a low rate environment. Simultaneously, the potential negative effects of interest hikes on banks' net worth are alleviated. The return on government asset holdings increases and helps to stabilize the income of the banks.

Deposit Rate Pass-Through and Government Asset Holdings The deposit rate pass-through and the banking sector's government asset holdings are the key factors that generate state-dependent monetary policy and the reversal rate in our framework. To analyze their impact, the frictions are relaxed one at a time.

First, a model featuring perfect deposit rate pass-through is considered. Accordingly, the deposit rate equals the policy rate adjusted for the mark down:

$$R_t^D = R_t^A - \varsigma \tag{36}$$

As a consequence, the pass-through is not state-dependent. Consequently, monetary policy transmission is equally effective in an expansion as well as in a recession. The central bank can also stimulate demand and lower the refinancing costs for the banking sector during a downturn. Simultaneously, the negative effects via the government bonds are shut down as the government spread is fixed, that is $R_t^A - R_t^D = \varsigma$. There are almost no state-dependencies any more and the monetary policy shock has almost the same

displayed in Figure 20 in Appendix I. An increasing severe recession implies that a monetary policy shock has a stronger impact on the policy rate, while its impact on the deposit rate is more muted. In adidtion to this, the lower bound on the policy rate starts to bind if the recession is very severe. The binding of the lower bound can be seen in the chart from the constant lines of the policy and deposit rate once the risk premium shock is around 2.5 standard deviations.

[24] The lower bound on the policy rate is set to -5% for this Figure as the baseline calibration of -2% is already binding for sufficient large shocks. Lowering the lower bound on the interest rate can result in a stronger reversal, as can be seen in the graph in the Appendix.

[25] In the simulation, a 1.84 standard deviation risk shock creates a recession and pushes the interest rate to -0.25%. This path is combined with a monetary tightening/loosening shock that results in a 50bps increase/drop of the interest rate.

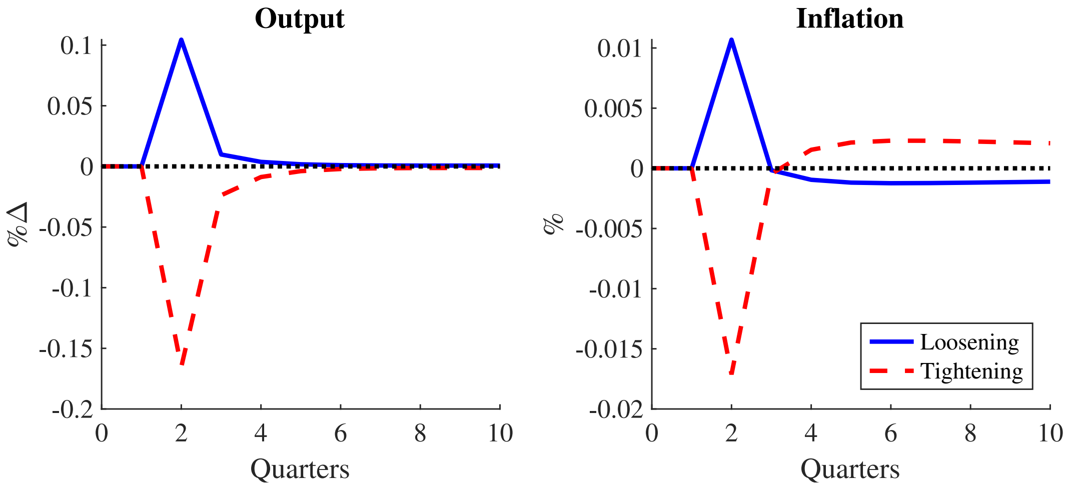

Figure 7: Impulse response analysis to highlight the asymmetric impact of monetary tightening and loosening shocks in a low rate territory. A 1.84 standard deviation risk shock creates a recession, in which the interest rate drops to -0.25%. This path is then combined with a monetary tightening/loosening shock that results in a 50bps increase/drop of the interest rate. The difference between a path with such a monetary shock (tightening/loosening) relative to a scenario without a monetary policy shock is displayed.

impact over the same cycle. Consequently, monetary policy is always effective and this specification does not feature a reversal interest rate. This highlights the importance of the imperfect deposit rate pass-through of monetary policy. This property is also illustrated in Figure 20 in Appendix I.

The second experiment is to alter the amount of reserve assets, while keeping an imperfect deposit rate pass-through. In particular, we consider a calibration in which the banks only hold half the share of government assets than assumed in the benchmark model calibration. Monetary policy is still state dependent and less powerful in recessions due to the imperfect deposit rate pass-through. However, a reversal rate does not materialize in this setting because monetary policy does not result in net worth losses of bankers. While monetary policy becomes less effective for low interest rates, it does not become contractionary. In fact, monetary policy stabilizes the banking sector even in a severe recession. Figure 20 in Appendix I shows this experiment as well.

Predictions and Empirical Support One prediction of our model is that banks' profitability decreases in a low rate environment, which is in line with the empirical studies of Borio, Gambacorta and Hofmann (2017) and Claessens, Coleman and Donnelly (2018). The overall consensus is that the the low interest rates had negative effects on the net interest margin in Europe (Abadi, Brunnermeier and Koby, 2022). One empirical observation that we cannot match by assumption is that there has been a strong increase in excess reserves (as can be seen in Figure 3) because we assume (basically) constant reserves.[26] Nevertheless, the assumption of constant reserve requirements should have

[26]Such state-dependent liquid asset holdings could, for instance, be microfounded by incorporating endogenous time-varying risk of a financial crisis. One approach could be to allow for endogenous runs

only a limited impact on the conclusions of the paper. The reason is that the liquid assets only have a negative impact on banks' net worth in a low interest rate environment. For this reason, we focus in the calibration on matching the asset holdings in a low rate territory, which we calibrate to 23% in line with the data for government bonds plus reserve assets since the introduction of negative rates in 2014.

4.3 Effective Lower Bound on Monetary Policy

The model can generate a reversal interest rate, in which an exogenous lowering of the interest rate contracts the economy. Importantly, the same mechanism holds for the lower bound of monetary policy. A very loose lower bound can have adverse effects. The endogenous lower bound R^A can avoid such adverse effects. At the same time, setting too conservative a bound would restrict monetary policy unnecessarily.

We evaluate the effective lower bound in our model using the welfare of the households, which is given by:

$$W_0 = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \right] \tag{37}$$

In addition to this, we consider the impact of the lower bound on selected moments.[27]

Figure 8 shows the shape of welfare depending on the variation in the lower bound. The effective lower bound on monetary policy is around -1% per annum. At this rate, the trade-off between lowering the interest rate with diminishing deposit rate pass-through and lowering banks' income on their government asset holdings is optimally balanced. This is the endogenously determined reversal interest rate in our model. It should be noted that an overly restrictive lower bound such as keeping the policy rate at positive levels lowers welfare as the central bank forgoes potentially beneficial monetary accommodation. This highlights the problem with monetary policy accommodation when approaching reversal interest rate territory. Monetary policy needs to balance inflation stabilization with the stability of the banking sector.

We can compare the impact of the lower bound on the moments of the model. Table 2 shows the different selected moments for a very negative lower bound at -5% , the baseline case with -2% and a rather large and positive lower bound at 1% using a simulation of 200000 periods (after a burn-in period). The differences between a very negative lower bound and the baseline case are rather small. In particular, we can see that output and leverage is slightly larger in the economy with a lower bound at -2%. The banking sector is allowed to be more highly leveraged as the banks do not face potential losses through the reversal interest rate. The strongest difference is in the behavior of inflation, where a very low lower bound leads to increased inflation. In addition to this, leverage is much

on the financial sector, e.g. as in Gertler, Kiyotaki and Prestipino (2020).

[27]The monetary policy shock is set to a very low value to avoid distorting the optimal lower bound.

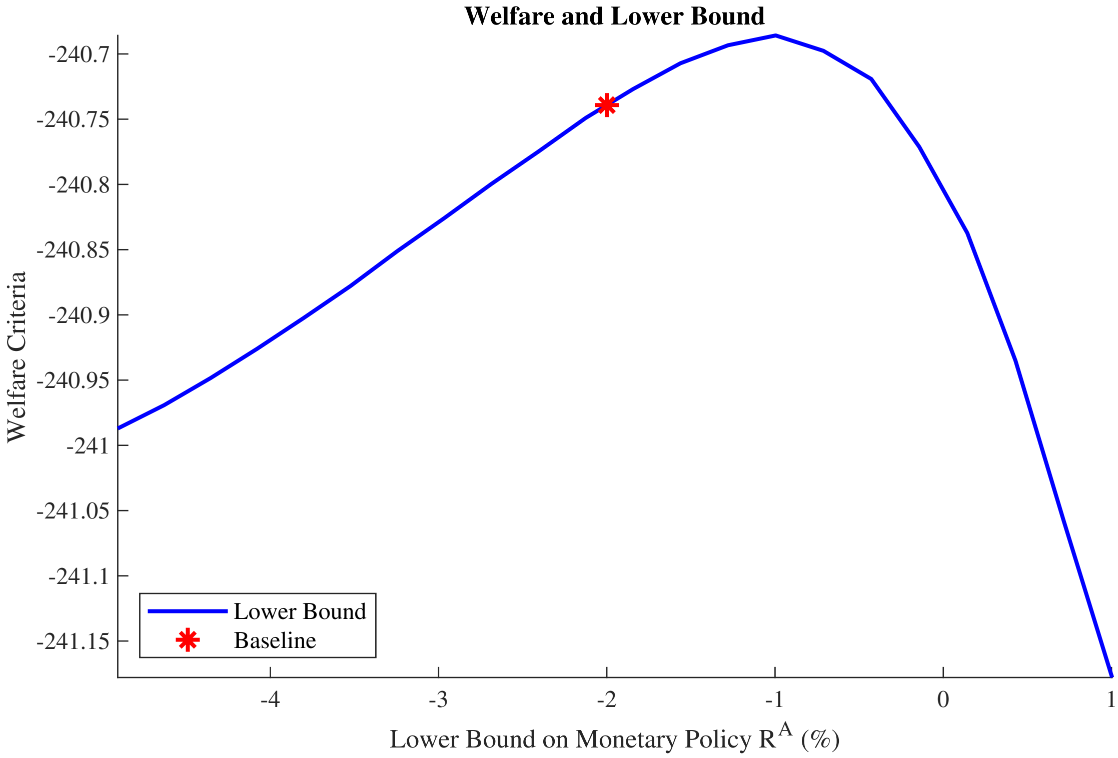

Figure 8: Welfare for different lower bounds of the policy rule R^A (measured as annualized net rate). The x-axis shows the interest rate in percent per annum. The star marks the baseline of a lower bound at -2%.

more volatile for a lower bound with $R_t^A = -5\%$. Nevertheless, the differences are rather small because interest rates are rarely so negative. The reason is that the economy needs to be in a very severe recession to even reach a lower bound of -2%, which happens only very rarely. Therefore, the instances when this differences matters are very rare as the economy does not face often a sequence of shocks that would push it in this area. However, there would be significant differences if a specific scenario of very contractionary shocks would be compared (instead of moments). At the same time, we see a stronger response of the moments if the lower bound is set very tight. A lower bound of 1% results in considerably lower average output. We also see much more deflation as the central bank does not respond to deflationary pressure sufficiently. In addition to this, the economy is also much more volatile as monetary policy intervenes less.

The observation that the differences are larger for a high lower bound compared to a very low stems from the fact that the economy only infrequently encounters very low interest rates where the reversal rate affects the economy. Therefore, an overly restricted monetary policy does not stabilize the economy for macroeconomic outcomes that occur frequently, while the occurrence of the reversal interest rate hurts the economy. However, this is more of a tail event. This suggests that setting the effective lower bound involves a trade-off between financial stability and inflation stabilization for low interest rates.

Table 2: Selected Moments for Varying Monetary Policy Lower Bound R^A

Moment	Model I: $R^A = -5$	Model II: $R^A = -2$	Model III: $R^A = 1$
a) Mean			
$\overline{Y}$	1.004	1.004	1.002
$\overline{N}$	1.148	1.146	1.152
$\overline{\phi}$	8.20	8.21	8.23
$\overline{\pi}$	2.02	1.98	1.97
b) Standard Deviation			
$\sigma(Y)$	0.022	0.022	0.025
$\sigma(N)$	0.17	0.17	0.19
$\sigma(\phi)$	5.22	4.28	6.30
$\sigma(\pi)$	0.41	0.42	0.46

5 Macroprudential Policy

Macroprudential policy is an important tool that can help to restore the efficiency of monetary policy in a "lower for longer" interest rate environment. The reason is that the capitalization of the banking sector plays a decisive role in the transmission of monetary policy. This gives rise to a new motive for macroprudential policy because it can strengthen the bank lending channel of monetary policy.

The macroprudential regulator can impose restrictions on the bank capital ratio, which is defined as the inverse of leverage $1/\phi$. In particular, the regulator can require the banks to build up additional capital buffers and release them subsequently. This policy instrument is based on the countercyclical capital buffer (CCyB) that was introduced as part of the Basel III requirements. The CCyB is built up during an expansion and can then be subsequently released, even though it can never fall below 0%, during a downturn.

We incorporate this asymmetry using an occasionally binding macroprudential rule. The policy cannot reduce the capital requirements below the market-based capital demands. Although the regulator could theoretically set capital ratios below the market ones, the market-based constraint would be the binding constraint for the banks. In that regard, the market enforces a lower bound on regulatory capital requirements. This restriction diminishes the welfare gains of macroprudential policy as the scope of policy interventions during a downturn is limited to the previously created buffers.[28] This emphasizes the importance of building up buffers in good times in order to create sufficient macroprudential space that can be employed to relax capital requirements in bad times and, thus ensuring macroprudential policy efficiency.

[28]The usual approach in the DSGE literature is based on unrestricted rules without a lower bound in assessing countercyclical capital requirements. An exception is, for instance, Van der Ghote (2021), where the market-based leverage constraint restricts optimal macroprudential regulation.

5.1 Macroprudential Policy Rule

The macroprudential regulator can set a time-varying capital buffer τ_t that imposes additional capital requirements. We use the following rule that is conditioned on the net worth of the banking sector:

$$\tau_t = \max\left\{\left(N_t/N^{MPP}\right)^{\tau^{MPP}} - 1, 0\right\} \tag{38}$$

where the parameter τ^{MPP} determines how much the macroprudential regulator responds to the deviations from target. N^{MPP} is the value to which the level of the net worth is compared to and determines the relevant deviation for the macroprudential rule. Note that it is often assumed that this corresponds to the steady state value, however our nonlinear specifications allows to easily treat this as a free parameter that can be set by the regulator. The macroprudential policy can only build up buffers if the regulatory-imposed requirements are above the market one, which we capture with the max operator in the rule. Thus, the additional buffer needs to build up in good times as the buffer cannot fall below 0.[29] This results in a natural asymmetric rule design, which is also in line with the CCyB of Basel III. By varying N^{MPP} the regulator controls at which state of the business cycle banks need to build up additional buffers.

As the buffer is additional to the market-based capital ratio requirements, the banks' capital ratio reads as follows

$$\frac{1}{\phi_t} = \frac{1}{\phi_t^M} + \tau_t \tag{39}$$

where the market-based capital constraint $1/\phi_t^M$ stems directly from the agency problem of the banker (see equation (10)). Due to the non-negativity restrictions of the buffer, the policy instrument only occasionally affects leverage. If the buffer is at zero, leverage is determined directly from ϕ_t^M. Therefore, the regulatory capital buffer affects asymmetrically the capitalization of the banking sector because it imposes additional capital requirements during periods of banks' balance sheet expansion.

Importantly, the capital buffer affects the transmission of shocks and dampens economic downturns. As the buffer is released after contractionary shocks, banks can better absorb their losses. This stabilizes the economy and reduces economic losses during a downturn. Furthermore, the central bank needs to respond less strongly, which gives additional space before approaching low interest rate territory as well as the reversal rate. This highlights that macroprudential policy has the potential to impact the probability of encountering negative interest rates and the reversal interest rate. It thereby also reduces the asymmetric response to expansionary and contractionary shocks. In Appendix I, these properties are illustrated in Figure 21 with an impulse response functions analysis.

[29]The macroprudential regulator could set a negative buffer. However, it would then have no impact as the market-imposed capital requirement would be the binding one.

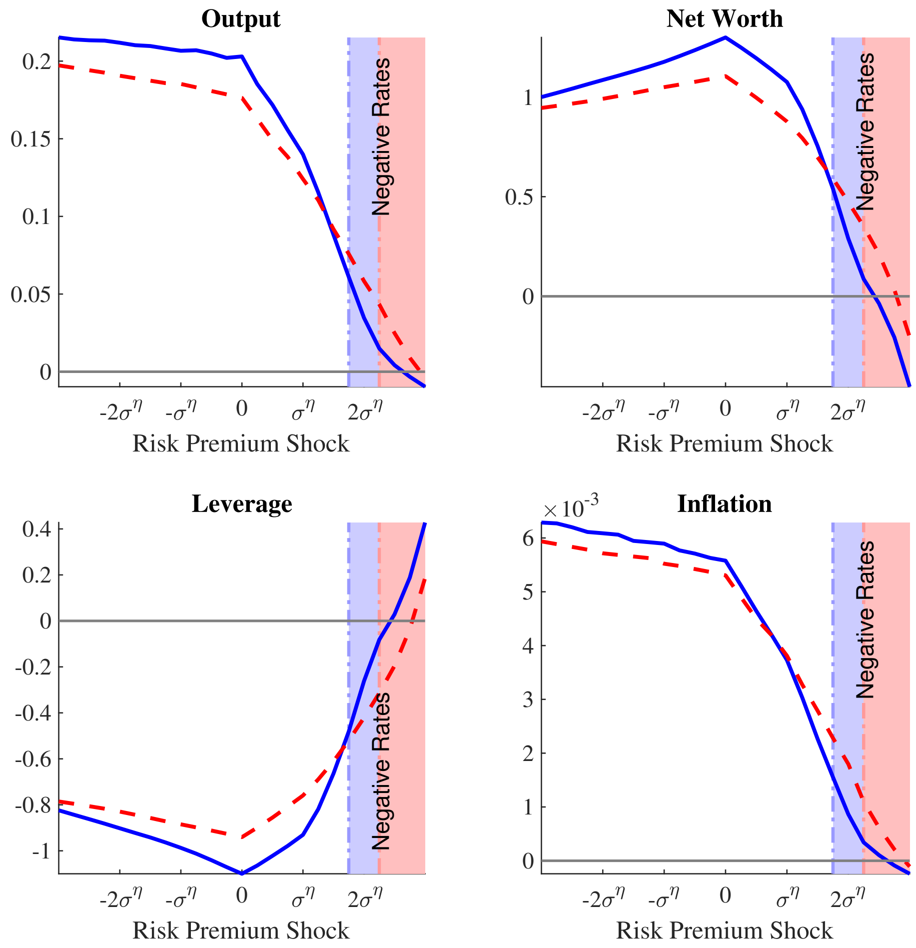

Figure 9: Comparison of state dependency of monetary policy shocks for economy without (baseline) and with macroprudential policy, where first period of an impulse response function of monetary policy shocks is shown. To generate the state dependency, the monetary policy shock is combined with different sized risk premium shocks, which are displayed on the horizontal axis. The vertical axis displays the state-dependent difference for the period $t = 1$ impulse response between a shocked path, which faces additionally a negative one std. dev. monetary policy shock, and a path without a monetary policy shock. The deviations are in percent. The blue and red shaded area indicates the territory, where the risk premium shock pushes the economy without and with macroprudential policy in negative interest rate territory.

The rule could also be conditioned on other variables. A popular alternative could be banks' asset holdings. We delegate the description of this rule to Appendix E, in which we compare it to our baseline specification. We focus on the baseline rule based on net worth because it turns out that this rule is superior from a welfare perspective. The details of the alternative rule based on asset holdings are in the appendix, while we discuss the welfare implications of the baseline rule later in this section.

5.2 Macroprudential Policy and Reversal Interest Rate

We have shown and highlighted the importance of the reversal interest rate for economic outcomes. As the impact of monetary policy on banking sector leverage is key for the possibility of entering reversal rate territory, a better-capitalized banking sector can compensate for losses and reduce the asymmetry of monetary policy shocks. To illustrate the beneficial role of macroprudential policy, we compare the impact of the capital buffer rule on the reversal interest rate.

Figure 9 shows the initial impact of a negative one standard deviation monetary policy shock over the business cycle, where varying risk premium shocks approximate the state of the economy. We compare the welfare-optimizing macroprudential policy, which is derived in the next subsection, to the benchmark economy without a buffer. The shaded areas indicate from which point onwards the respective economy enters negative interest rate territory. This clearly illustrates that macroprudential policy reduces the probability of encountering negative interest rates and the reversal rate, as can be seen in the difference of the shaded areas and the location of the reversal of output. As the buffer dampens contractionary shocks, the economy encounters less severe recessions and fewer interest rate reductions. Therefore, monetary policy retains more of its efficiency for large ϵ_t^η and is less likely to enter the region, where a monetary policy rate cut would deteriorate output and credit supply.[30] In addition to this, macroprudential policy also helps to stabilize inflation.

While the countercyclical capital buffer rule helps to restore the monetary policy transmission mechanism in case of large contractionary shocks, it also affects it in normal times. As the banking sector is better capitalized, monetary policy is less powerful during an expansion. For instance, the increase in output or net worth is smaller in an economy with an active macroprudential policy.

5.3 Optimal Macroprudential Policy

Active macroprudential policy can reduce the threat of the reversal interest rate. This notwithstanding, an excessively large buffer could also depress the economy. In other words, the macroprudential regulator could face a trade-off between building up macroprudential space to support monetary policy and the potential costs of creating this space in good times. We evaluate this trade-off using the same welfare criteria, as specified in equation (37).

To determine the optimal macroprudential policy rule, we jointly optimize over both parameters of the rule (N^{MPP} and τ^{MPP}). The outlined trade-off can be seen by the hump-shaped welfare curve in Figure 10. For the presentation of the optimal rule, we display only the variation of over N^{MPP}. For each N^{MPP}, we use the corresponding optimal τ^{MPP}. Appendix D contains more details about the interactions between the parameters N^{MPP} and τ^{MPP}.

The figure shows that a large buffer helps the banking system to absorb losses and reduces the threat of the reversal rate during a severe downturn. At the same time, the

[30] In addition to this channel, macroprudential policy could have an additional positive impact on systemic risk in a low interest rate environment (Van der Ghote, 2020).

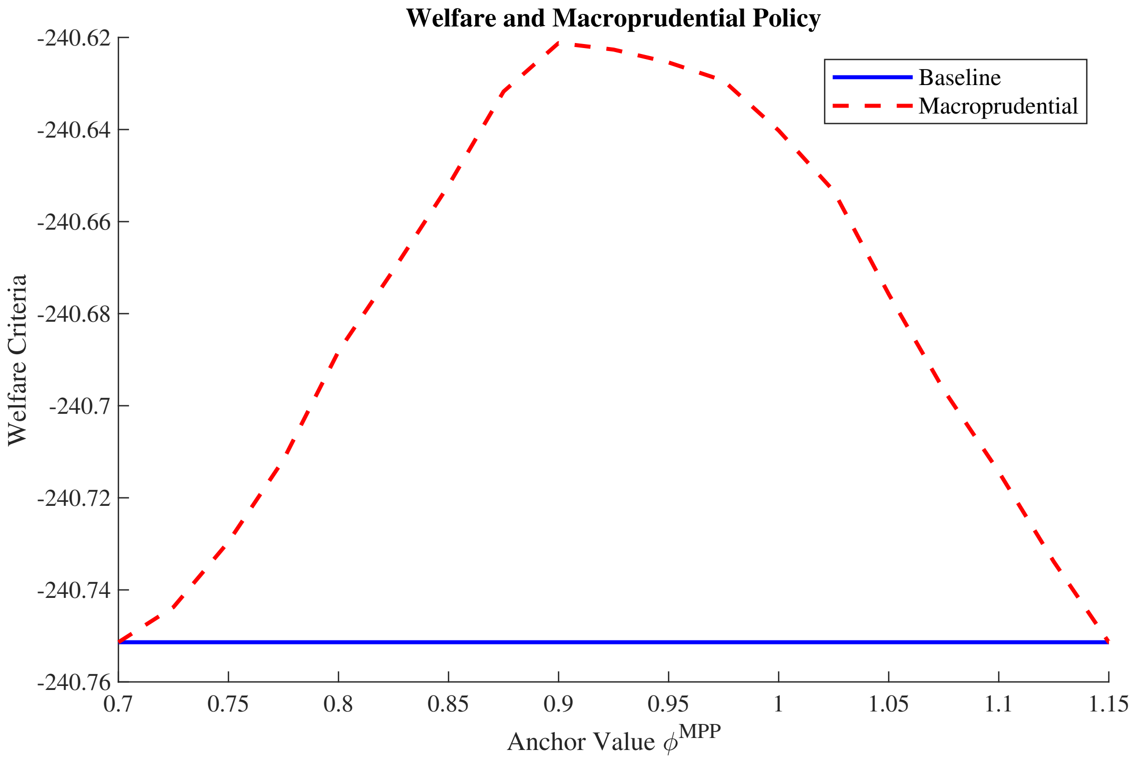

Figure 10: Welfare for different parameter values N^{MPP}, which are varied on the horizontal axis. The response to deviations τ^{MPP} is set optimally to maximize welfare for each value of N^{MPP}.

build-up of the buffer in good times is costly. Therefore, the optimal macroprudential policy space that is created during a boom is limited.[31] It should be noted that the positive impact of this rule is due to the introduction of the imperfect deposit rate pass-through and reserve and liquidity requirement. For instance, in an economy with a perfect pass-through, the proposed macroprudential policy rules would result in a welfare loss. In fact, it would be optimal to not have the capital rule (or to set $\tau^{MPP} = 0$) as the costs of building up the buffers outweigh the benefits in such an economy without a reversal rate. To better understand how macroprudential policy helps to improve welfare, Appendix D provides an additional assessment of some key moments.

The optimal macroprudential policy rule that strikes the balance between building up sufficient, but not excessive, macroprudential space has the following parameter values: $N^{MPP} = 0.9$ and $\tau^{MPP} = 0.11\%$. This policy reduces the risk of large output contraction and lowers the standard deviation of output by 11%. The likelihood to encounter negative interest rates and the reversal rate fall by around 23% and 25%, respectively. Furthermore, the optimal value of N^{MPP} is below the stochastic steady state. Therefore, the macroprudential regulator builds up a buffer in normal and good times, which can be

[31]The setup does not allow to derive the optimal additional buffer level on top of an otherwise constant capital requirement. This would be necessary if the results should be directly mapped to a specific percentage value for the countercyclical capital requirement and other related regulatory buffers. The reason is that we build on the seminal financial friction of Gertler and Karadi (2011), which results in market-based time-varying leverage. We assume that the macroprudential regulation is added on top of the market-based requirements.

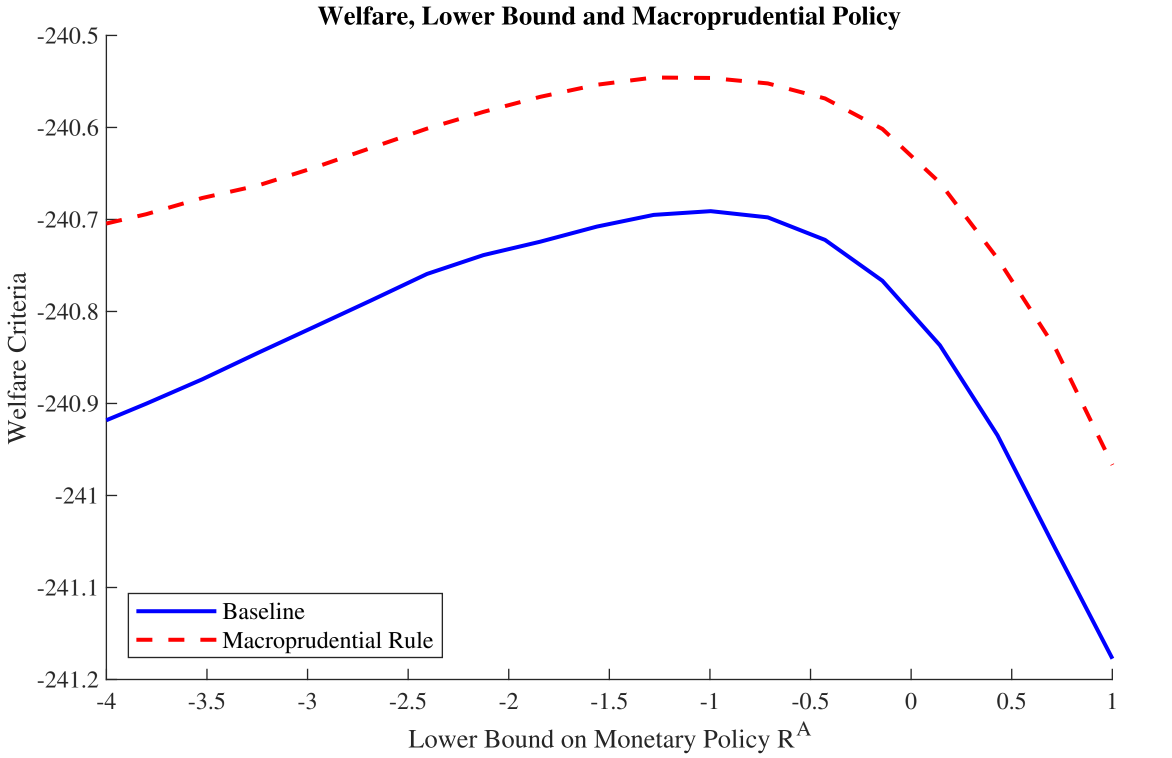

Figure 11: Welfare with and without macroprudential policy for different lower bounds on monetary policy (measured as annualized net rate). The macroprudential policy rule parameters N^{MPP} and τ^{MPP} are jointly optimized for each lower bound.

then released once contractionary shocks materialize. However, there are costs associated with building up the buffer, which limits how low the optimal value of N^{MPP} is set. As a consequence, there are severe recessions, in which the entire macroprudential space is used.

5.4 Macroprudential Policy and Effective Lower Bound

The previous analysis has highlighted that macroprudential and monetary policy are strategic complements. Therefore, it is important to understand the interaction between macroprudential policy and the effective lower bound on monetary policy. To address this question, we compare the different lower bounds for an economy without and with macroprudential policy, which is displayed in Figure 11. For each lower bound, we choose the optimal macroprudential policy to calculate welfare.[32]

First of all, macroprudential policy increases welfare independent for all considered lower bounds because it helps to prevent the economy from approaching reversal rate territory. As it stabilizes the banking sector, the recession and the threat of ultra low interest rates is less severe. Via this channel, the welfare-optimizing capital rules improve welfare regardless of the specific lower bound. This again highlights how the reversal interest rate creates this novel motive for macroprudential policy.

[32]This implies that we maximize N^{MPP} and τ^{MPP} for each value of τ^{MPP}.

We can also see that the optimal capital buffer does not directly affect the choice of the effective lower bound. The reason is that the macroprudential policy space is already released once the policy rate is lowered to such a negative territory. However, macroprudential policy can affect the location of the reversal rate. If the central bank still has the option to lower the capital requirements once the economy approaches very low territory, the location can change. As an example, we choose a suboptimal macroprudential rule, which builds up buffer already at a lower level ($N^{MPP} = 0.6$). The location of the lower bound moves in this specific example from around -1.0% to -1.3%. Appendix F contains the simulation and more details.

In addition to the increase in welfare, the shown welfare curve in Figure 11 is flatter with an active macroprudential policy. The capital buffer rule smooths the fluctuations and the economy is less often in such a low interest rate area. A suboptimal lower bound, which is either unnecessary high or low, has less of an impact. In other words, macroprudential policy mitigates the danger of either too loose or too restrictive monetary policy. This connection further contributes to the strategic complementarity between macroprudential and monetary policy in a low interest rate environment.

5.5 Counterfactual Analysis of Macroprudential Policy

So far, we have analyzed the role of macroprudential policy without providing a historical perspective. We now move a step further and evaluate how optimal macroprudential policy would have affected credit supply and economic growth since 2000. Specifically, we take the nonlinear model to the data for a counterfactual policy scenario, in which we analyze how the welfare-optimal macroprudential policy would have affected the dynamics of output and asset holdings between 2000:Q1 and 2019:Q4.

To take the nonlinear model to the data, we follow a two step-procedure.[33] In the first step, we use the particle filter (Fernández-Villaverde and Rubio-Ramírez, 2007) to estimate the structural (risk premium and monetary) shocks that explain the observed time series of detrended GDP, the deposit rate and deposit facility rate set by the ECB from 2000:Q1 until 2019:Q4 using the calibrated model without macroprudential policy.[34] The used observation equation that connects the model with the data is given as:

$$\begin{bmatrix} \text{Detrended Log GDP} \\ \text{Deposit Rate} \\ \text{Deposit Facility Rate} \end{bmatrix} = \begin{bmatrix} Y_t - Y \\ 400\left(R_t^D - 1\right) \\ 400\left(R_t^A - 1\right) \end{bmatrix} + \nu_t \tag{40}$$

The variance of the measurement error ν_t is set to 25% of the sample variance of the

[33]Rottner (2021) and Bianchi, Melosi and Rottner (2021) also use a similar two step procedure to conduct policy counterfactuals.

[34]We calibrate the volatility of the monetary policy shock for this quantitative analysis such that one standard deviation shock corresponds to 100bps.

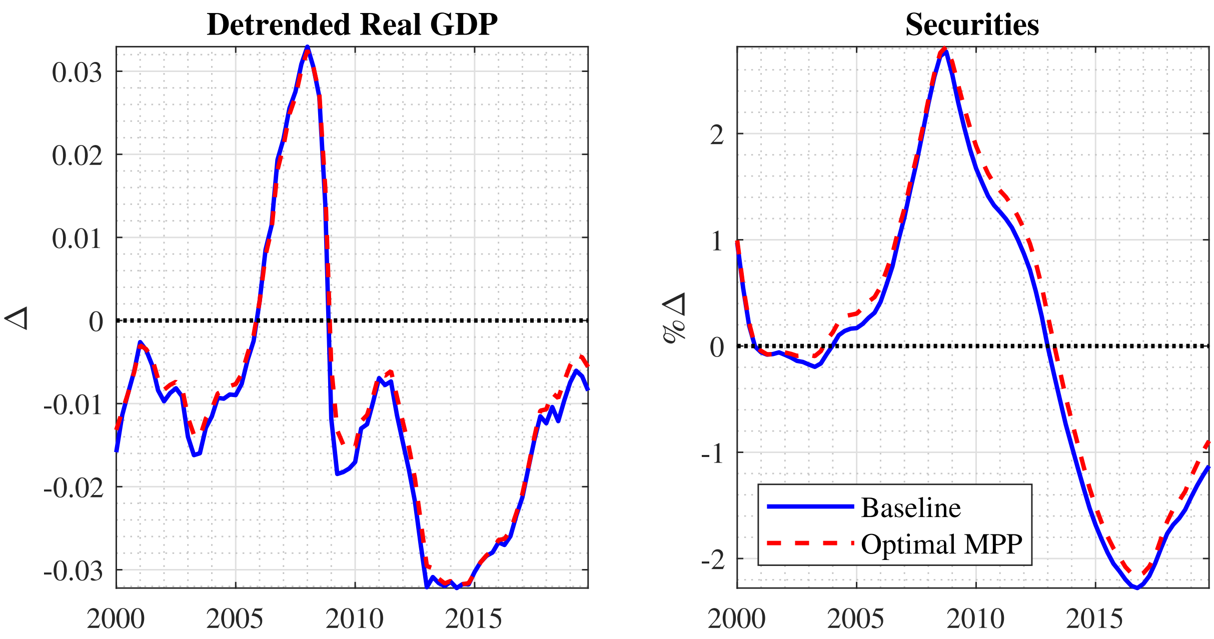

Figure 12: Counterfactual dynamics with the welfare-optimal macroprudential policy rule. The filtered median path of output and securities under the baseline economy without The blue line shows the filtered median path. The detrended real GDP is shown as deviations, while the securities are shown as percentage deviations from the mean between 2000:Q1 and 2019:Q4.

data.[35] In the second step, we use these estimated shocks from Step 1 to calculate the counterfactual scenario of the economy assuming the welfare-optimal macroprudential policy would have been in place. We provide more details on the particle filter in the Appendix H. We also show how the results from the particle filter exercise can be used to some extent as an external validation of the model.

Figure 12 displays how the welfare-optimal macroprudential policy would have affected credit supply and economic growth. The left plot shows the counterfactual path of output if macroprudential policy would have been active. During normal times, output is the same or slightly lower under the macroprudential policy regime. This is related to the prescribed build-up of buffers in non-crisis times. However, the subsequent release of the buffers allows to stabilize the fall in output, which can be seen very well for instance around 2010. Furthermore, the results indicate that the deviation from the mean in 2019:Q4 for output would be around 33% lower if the welfare-optimal macroprudential policy would have been active. The level of output would be only 0.56% below its mean instead of 0.84%. The reason is that macroprudential policy helps to stabilize the level of the securities as shown in the right plot. While the level of the security holdings is the same during the peak in 2008, the supply of credit is then more stable throughout the period of low rates in the 2010s. This exemplifies the working of the macroprudential policy. The banking sector needs to build-up additional buffers in good times so that they can be released during times of downturns, in which its net worth gets squeezed.

[35]The particle filter requires a measurement error to avoid a degeneracy of the likelihood function.

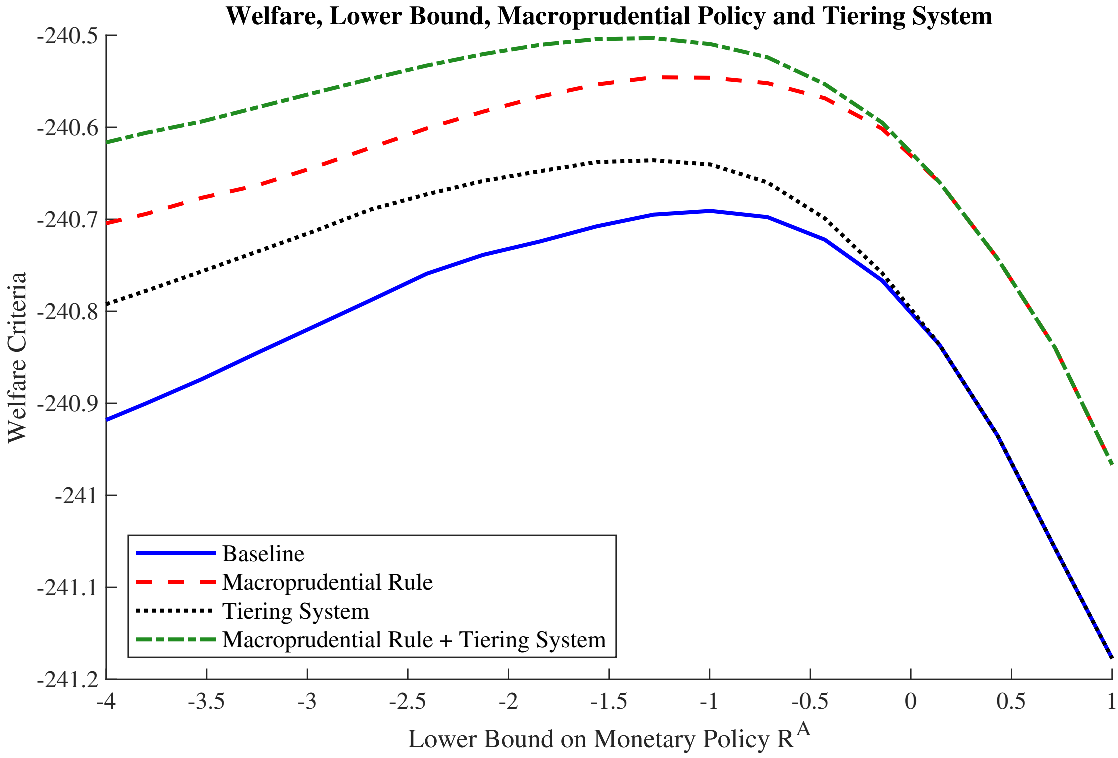

Figure 13: The impact of the tiering system on welfare, macroprudential policy and the lower bound on monetary policy (measured as annualized net rate). The macroprudential policy rule parameters N^{MPP} and τ^{MPP} are optimized separately for an economy with and without a tiering system as well as each lower bound.

5.6 Interaction with Other Policies: Reserve Tiering System

We now extend our analysis to investigate how macroprudential policy interacts with other measures that aim to support the bank lending channel in a low rate environment. Our focus is on a tiering system for reserves that has been introduced by the ECB and some other central banks. Such a tiering system exempts parts of the banks' excess reserves from negative renumeration. The purpose is to lessen the burden of negative interest rates on the profitability of banks.

We incorporate the ECB's two-tier system in our framework. The two-tier exempts part of the reserve holdings from negative rates. As a consequence, a fraction of the government asset holdings (which includes reserves and government bonds jointly in our model) is renumerated at 0% if the policy rate is negative. In other words, a zero lower bound protects the return of a fraction of the government asset. The return on the reserve asset R_t^T can then be expressed as

$$R_t^T = (1-\delta^T)R_t^A + \delta^T \max\left[1, R_t^A\right], \tag{41}$$

where the scenario $\delta^T = 0$ nests our baseline framework without a tiering system. Following the design of the ECB, the minimum reserves plus a multiple of the minimum reserves are exempted from a negative renumeration. The minimum reserve ratio is 1% and the multiple is set at 6 since the introduction of the tiering system in 2019. This corresponds to setting $\delta^T = 0.079$.

We find that the tiering system increases the effectiveness of negative interest rate policies as it lowers the costs of liquid asset holdings.[36] Nevertheless, it only exempts a part of the government asset from negative renumeration. Thus, the banks' profitability still deteriorates with low rates, even though the process is now more slow. As a consequence, the impact of negative interest rate policies still declines and the economy encounters the reversal rate occasionally, albeit with a smaller probability.

The welfare consequences can be analyzed in Figure 13, where we calculate the welfare in an economy with a tiering system for different lower bounds. As monetary policy is now more effective in negative territory, the two-tiering system improves welfare. The relative gain of using a tiering system of course depends to what extent a central bank is willing to use negative interest rate policies. The welfare gain of the tiering system starts once the lower bound is below 0 and is then increasing if the central bank sets the lower bound in more negative territory. Importantly, the tiering system alters the location of the reversal rate. The optimal endogenous lower bound is now around -1.5% relative to -1%, which was the optimal location in a scenario without a tiering system.

Nevertheless, the strategic complementarity between monetary and macroprudential policy still prevails because low interest rates still deteriorate banks' capitalization. In fact, an active usage of macroprudential policy provides similar welfare gains, even if the tiering system is in place. One reason is that some parts of the liquid assets are still costly in a low rate territory. Therefore, macroprudential policy can help to absorb the losses. Another reason is that macroprudential policy can already support monetary policy before the economy reaches negative territory. As both policies support monetary policy, the combination of macroprudential policy and the tiering system provides the largest welfare gains. Therefore, the outlined new motive for macroprudential policy is still very important, even if the central bank introduces a tiering system.

5.7 Extensions

Our objective was to study the nonlinear environment associated with the reversal rate. For this reason, we solve the model using global solution methods. While this approach allows to capture the nonlinear elements and explore the quantitative implications of the reversal rate and macroprudential policy, solving such models is at the current stage very difficult. With that in mind, we discuss three extensions that could be studied by adapting our model.

[36] Sims and Wu (2021) also find that tiering improves the efficiency of negative interest rate policies.

Maturity Mismatch We assume that the bonds in the model are 1-period and abstract from maturity mismatch. Long-term bonds are another channel that influence the reversal rate, which Abadi, Brunnermeier and Koby (2022) denote as the capital gain channel. In response to an interest rate cute, banks make gains from the cut due to a maturity mismatch for their long-term bonds. As this effect has a positive impact on the net worth of banks, the reversal rate would then be lower than in our model. Therefore, the 1-period bond assumption could potentially inflate the severity of the negative impact of low rates on banks' profitability. While adding long-term bonds would potentially lower the reversal rate, as long as these effects are sufficiently small, it would not turn the result of impaired bank profitability in a low rate environment around.[37] For instance, Abadi, Brunnermeier and Koby (2022) present a model which features a reversal rate despite maturity mismatch. Therefore, we conclude that low rates would still impair banks profitability and monetary policy becomes less effective in a low rate environment until it reverse.

Endogenous Financial Crises An important element could be to allow for endogenous financial crisis. One way could be to integrate the possibility for runs on the financial sector, e.g. as in Gertler, Kiyotaki and Prestipino (2020) and Rottner (2021). This model class predicts that a low capitalization of the banking sector is key for the occurrence of a run, which highlights the importance of macroprudential policy. An important finding for our specific context is that the introduction of the zero lower bound exacerbates the frequency and severity of a financial crisis (Rottner, 2021). Our model would predict something similar in spirit as negative interest rate policies are less powerful and the reversal rate would even hurt additionally. Endogenous financial crisis would be an additional channel that points to the importance of a sufficient macroprudential space. Therefore, our results on the potential stabilization impact of macroprudential policy in the context of the reversal rate can be seen as a lower bound because we abstract from endogenous financial crises.

Model Complexity and Estimation Our model features certain limitations, including a limited number of shocks and a lack of typical backward-looking element, which enables us to solve it in its nonlinear specification with the used global methods. These limitations should be considered when interpreting the results. Furthermore, we do not conduct a full blown estimation, but instead we calibrate the model carefully (and also provide a historical perspective using a particle filter approach). Future research that employs novel solution and estimation techniques, such as the neural networks approach in Kase, Melosi and Rottner (2022), could use more elaborated and estimated models to

[37] The role of the maturity mismatch is also particularly important in the context of quantitative easing in a low interest rate environment.

study the reversal rate and macroprudential policy from an even more quantitative angle.

6 Concluding Remarks

In this paper, we investigate monetary policy effectiveness and macroprudential policy in a negative interest rate environment. Using a novel macroeconomic model fitted to the euro area, we illustrate how a "lower for longer" interest rate environment gives rise to important asymmetries and non-linearities. Our model predicts the possibility of a reversal rate where a lowering of the policy rate may give rise to an unintended contraction of output. To avoid a reversal of monetary policy, the framework suggests that the effective lower bound on monetary policy is located at -1% per annum.

We document a new motive for macroprudential policy that exploits the link between banks' capitalization, the effectiveness of negative interest rate policies and the reversal rate. The build-up of macroprudential space in good times supports the bank lending channel of monetary policy and reduces the risk of approaching reversal interest rate territory. Importantly, such a macroprudential policy also increases the effectiveness of negative interest rate policies and mitigates the costs of an either too loose or too restrictive monetary policy in a low interest rate environment. Using a historical perspective, we show in a counterfactual scenario that macroprudential policy could have supported credit supply and economic growth in the period of low rates during the 2010s.

The analysis has at least three important policy implications. First, macroprudential policy using a countercyclical capital buffer approach has the potential to alleviate and mitigate the risks of entering a reversal rate territory. Second, there are important strategic complementarities between monetary policy and a countercyclical capital-based macroprudential policy in the sense that the latter can help facilitate the effectiveness of monetary policy, in periods of low, or even negative, interest rates. Overall, the findings in this paper provide important insights into the relevance of financial stability considerations in monetary policy strategy discussions. Third, our framework suggests that a monetary policy tightening is substantially more powerful than a monetary loosening in a low interest rate territory. As a consequence, the effectiveness of monetary policy could increase successively once the economy starts to move away from very low interest rates.

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A Non-Linear Equilibrium Equations

Households

$$C_t = W_t L_t + D_{t-1} \frac{R^D_{t-1}}{\Pi_t} \eta_{t-1} - D_t + \Pi^P_t - \tau_t$$

$$\beta R^D_t \eta_t E_t \frac{\Lambda_{t,t+1}}{\Pi_{t+1}} = 1$$

$$\chi L^{\varphi}_t = C^{-\sigma}_t W_t$$

Banks

$$\mu_t \phi_t + \nu_t \geq \lambda \Big(\frac{1}{1-\delta^B} \phi_t - \frac{\delta^B}{1-\delta^B} \Big)$$

$$\psi_t = \mu_t \phi_t + \nu_t$$

$$\mu_t = \beta E_t \Lambda_{t,t+1} \left(1 - \theta + \theta \psi_t\right) \frac{R^K_{t+1} - R_t}{\Pi_{t+1}}$$

$$\nu_t = \beta E_t \Lambda_{t,t+1} \left(1 - \theta + \theta \psi_t\right) \frac{R_t}{\Pi_{t+1}}$$

$$Q_t S_t = \phi_t N_t$$

$$R_t = (\eta_t R^D_t) \frac{1}{1-\delta^B} - R^A_t \frac{\delta^B}{1-\delta^B}$$

$$N_t = N^S_t + N^N_t$$

$$N^S_t = \theta N_{t-1} \frac{R^K_t - R_{t-1}\phi_{t-1} + R_{t-1}}{\Pi_t}$$

$$N^N_t = \omega^N \frac{S_{t-1}}{\Pi_t}$$

Production, Investment and New Keynesian Phillips Curve

$$Y_t = A^P K^{\alpha}_{t-1} L^{1-\alpha}_t$$

$$W_t = P^m_t (1-\alpha) Y_t / L_t$$

$$R^k_t = \frac{(P^m_t \alpha Y_t / K_{t-1} + (1-\delta) Q_t)}{Q_{t-1}} \Pi_t$$

$$Q_t = \frac{1}{(1-\eta_i) a_i} \Big(\frac{I_t}{K_{t-1}} \Big)^{\eta_i}$$

$$K_t = (1-\delta) K_{t-1} + (a_i (I_t / K_{t-1})^{(1-\eta_i)} + b_i) K_{t-1}$$

$$\left(\frac{\Pi_t}{\Pi} - 1 \right) \frac{\Pi_t}{\Pi} = \frac{\epsilon}{\rho^r} \left(P^m_t - \frac{\epsilon - 1}{\epsilon} \right) + \beta E_t \Lambda_{t,t+1} \frac{Y_{t+1}}{Y_t} \left(\frac{Pi_{t+1}}{\Pi_t} - 1 \right) \frac{\Pi_{t+1}}{\Pi}$$

Policy Rule, Interest Rates, Government Budget Constraint and Aggregate Resource Constraint

$$R_t^A = \max \left[R^A \left(\frac{\Pi_t}{\Pi} \right)^{\theta_\Pi} \left(\frac{Y_t}{Y} \right)^{\theta_Y}, \tilde{R}^A \right] \zeta_t$$

$$R_t^D = R_t^A - \omega(R_t^A)$$

$$R_t^D = \mathbf{1}_{R_t^A \geq R^{A\,SS}} \left[R_t^A - \varsigma \right] + (1 - \mathbf{1}_{R_t^A \geq R^{A\,SS}}) \left[\omega^1 + \omega^2 \exp(\omega^3 (R_t^A - 1)) + 1 \right]$$

$$\tau_t + A_t = \frac{R_{t-1}^A}{\Pi_t} A_{t-1}$$

$$Y_t = C_t + I_t + \frac{\rho^r}{2} \left(\frac{\Pi_t}{\Pi} - 1 \right)^2 Y_t$$

A.1 Occasionally Binding Regulatory Constraint

The non-negative capital buffer is

$$\tau_t = \max \left\{ \left(N_t / N^{MPP} \right)^{\tau^{MPP}} - 1, 0 \right\} \tag{42}$$

The market imposed leverage constraint is given from the run-away constraint

$$\phi_t^M = \frac{\nu_t + \frac{\delta^B}{1-\delta^B}}{\frac{\lambda}{1-\delta^B} - \mu_t}$$

Banks leverage is then given as

$$\phi_t = \left(\frac{1}{\phi_t^M} + \tau_t \right)^{-1} \tag{43}$$

B Data and Calibration

B.1 Data Sources and Construction

This section describes the data sources and construction. Table 3 shows all used series and their source. We use euro area data from 2002Q1 until 2019Q4.[38]

Deposit Rate The deposit rate weights the different lending rates for varying maturities, where the rates are from ECB SDW MIR data and the volume is based on the ECB SDW - BSI data. The used rates are the overnight deposit rate, deposit rate up to 1 year for new business, deposit rate over 1 and up to 2 years for new business and the deposit rate over 2 years for new business. Their contribution is weighted with their relative outstanding amount in the balance sheet. All different rates and outstanding amounts

[38]The data from the euro area have a changing composition.

are for deposits from households. The constructed deposit rate R_t^D reads then as follows:

$$R_t^D = \frac{DS0_t \times RD0_t + DS1_t \times RD1_t + DS2_t \times RD2_t + DS3_t \times RD3_t}{DS0_t + DS1_t + DS2_t + DS3_t} \tag{44}$$

Lending Rate The lending rate uses data from the ECB SDW - MIR data and the volume to weight is based on BSI data. For the lending rate, we use up to 1 year, over 1 year and below 5 years, and over 5 years to non-financial corporates and outstanding amounts. The volume data has the same maturity and is the outstanding amount to all non-financial corporations. The constructed lending rate R_t^K is the weighted index of the different rates:

$$R_t^K = \frac{LR1_t \times LS1_t + LR2_t \times LS2_t + LR3_t \times LS3_t}{LS1_t + LS2_t + LS3_t} \tag{45}$$

Policy Rate The main policy rate is the ECB's deposit facility rate. Euribor 3-month and the Eonia rate are the typical alternatives in the New Keynesian literature for the euro area.

Government Assets The share of government assets uses data from the ECB SDW - BSI data. We use loans to euro area government held by monetary financial institutions (MFIs), euro area government debt securities held by MFIs, required reserves held by credit institutions and excess reserves held by credit institutions.[39] This is compared to the total assets held by the MFIs. The consolidated balance sheet of the euro area MFIs is used for each series. The different measures include to a different extent the reserves:

$$\frac{A_t^1}{S_t + A_t^1} = \frac{LG + LS}{TA} \tag{46}$$

$$\frac{A_t^2}{S_t + A_t^2} = \frac{LG + LS + RR}{TA} \tag{47}$$

$$\frac{A_t^3}{S_t + A_t^3} = \frac{LG + LS + RR + ER}{TA} \tag{48}$$

The different series can be seen in Figure 3 in the main text.

Bank Level Deposit Rates The deposit rates for different banks are based on the ECB IMIR data.

Government Bond Yield The government bond yield is shown for the German one year bond, with the data is being extracted from Datastream.

[39] There are two important regulatory changes for the reserve requirement. Initially, the reserve requirement was 2% of the deposit base, which was lowered to 1% from 18 January 2012. Furthermore, a two-tier system takes effect rom 30 October 2019. This system exempts credit institutions from remunerating part of their excessive holdings.

Table 3: Data Sources

Data	Name	Source
a) Deposit Rate		
Overnight Deposit Rate, Households (HH)	RD0	ECB SDW - MIR
Deposit rate, maturity up to 1 year, HH, New Business	RD1	ECB SDW - MIR
Deposit rate, maturity over 1 and up to 2 years, HH, New Business	RD2	ECB SDW - MIR
Deposit rate, maturity over 2 years, HH, New Business	RD3	ECB SDW - MIR
Overnight deposits, Total, HH	DS0	ECB SDW - BSI
Deposits, maturity up to 1 year, HH, Outstanding	DS1	ECB SDW - BSI
Deposits, maturity over 1 and up to 2 years, HH,Outstanding	DS2	ECB SDW - BSI
Deposits, maturity over 2 years, HH, Outstanding	DS2	ECB SDW - BSI
b) Lending Rate		
Lending rate, maturity up to 1 year, NF-Corp., Outstanding (Out)	LR1	ECB SDW - MIR
Lending rate, maturity over 1 and up to 5 years, NF-Corp., (Out)	LR2	ECB SDW - MIR
Lending rate, maturity over 5 years, NF-Corp., Outstanding	LR3	ECB SDW - MIR
Loans, maturity up to 1 year, NF-Corp., Outstanding	LS1	ECB SDW - BSI
Loans, maturity over 1 and up to 5 years, NF-Corp., Outstanding	LS2	ECB SDW - BSI
Loans, maturity over 5 years, NF-Corp., Outstanding	LS3	ECB SDW - BSI
c) Policy Rate		
ECB Deposit facility rate	PR1	ECB SDW - FM
Euribor 3-month	PR2	ECB SDW - FM
Eonia rate	PR3	ECB SDW - FM
d) Government Asset		
Loans to government, MFI, Stock	LG	ECB SDW - BSI
Government debt securities, MFI, Stock	LS	ECB SDW - BSI
Reserve Maintenance Required Reserves, Credit Inst.	RR	ECB SDW - BSI
Reserve Maintenance Excess Reserves, Credit Inst.	ER	ECB SDW - BSI
Total Assets, MFI	TA	ECB SDW - BSI
e) Bank Level Data		
Overnight Deposit Rate, Households	RDi	ECB SWD - IMIR
f) Government bond yield		
German government 1 year bond yield	G1Y	Datastream

B.2 Non-Linear Least Squares

The model function that relates the deposit rate data dd_i and the policy rate data pd_i (conditional on being below the threshold) is given as

$$dd_i = (\omega_1 + \omega_2 \exp(\omega_3 pd_i)$$

We impose two restrictions, which allow us to express ω_1 and ω_2 in terms of ω_3. First, the markdown at the threshold value corresponds to ς. Second, the pass-through at the threshold value is 1, which implies perfect pass-through. Thus, the shape parameters ω_1 and ω_2 can be written as:

$$\omega_1 = i^{SS} - \varsigma - \frac{1}{\omega_3}$$

$$\omega_2 = \frac{1}{\omega_3 \exp(\omega_3 i^{SS})}$$

where i^{SS} is the threshold parameter.

The non-linear least squares now finds the parameter ω_3 that minimizes the squared residuals r_i from the model function:

$$r_i = dd_i - \left(i^{SS} - \varsigma - \frac{1}{\omega_3} + \frac{\exp(\omega_3 pd_i)}{\omega_3 \exp(\omega_3 i^{SS})} \right)$$

C Structural Interpretation of the Risk Premium Shock

The risk premium shock of Smets and Wouters (2007) is empirically very important in structural DSGE models, and can explain the zero lower bound episodes. However, its structural interpretation as a risk premium shock is heavily criticized in Chari, Kehoe and McGrattan (2009). They argue that it is best interpreted as a flight to quality shock that affects the demand for a safe and liquid asset such as government debt. Fisher (2015) microfounds this argument and indeed shows that this shock can be interpreted as a preference shock for treasury bills.

We show that the risk premium shock in our model can be interpreted as a flight to quality shock in government bonds in line with the argument above. For this reason, we incorporate government debt as an additional asset that earns the one period ahead nominal gross interest rate R_t^G. Following Fisher (2015), the government bond enters the household utility function as additive term and is subject to an exogenous preference shock Ω_t so that the household problem is given as:

$$\max_{C_t, L_t, D_t, B_t} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} + \Omega_t U(B_t) \right]$$
$$\text{s.t.} \quad P_t C_t = P_t W_t L_t + P_{t-1} D_{t-1} R_{t-1}^D \eta_{t-1} + P_{t-1} B_{t-1} R_{t-1}^B - P_t D_t - P_t B_t + P_t \Pi_t^P - P_t \tau_t$$

where $U(\cdot)$ is positive, increasing and concave. η_t is not an exogenous innovation in the model in this setup. Instead, the nominal gross interest is now artificially divided as $R_{t-1}^D \eta_{t-1}$ to better illustrate the mapping between the flight to quality shock and the risk premium shock. The first-order conditions with respect to deposits and government bonds are

$$\beta R_t^D \eta_t E_t \frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} = C_t^{-\sigma}$$
$$\beta R_t^G E_t \frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} = C_t^{-\sigma} - \Omega_t U'(B_t)$$

which can be combined to:

$$R_t^D \eta_t = R_t^G \frac{1}{1 - \Omega_t U'(B_t)}$$

This equation suggests that η_t captures changes in the preference for the safe asset Ω_t. In particular, an exogenous increase in the demand for the government bond would require either the nominal deposit rate to increase or the return on government bonds to fall. If R_t^G does not respond to wholly offset the impact of the shock, then there is a direct mapping from the flight to quality preference shock to our risk premium shock. η_t accounts for the rise in the nominal interest rate shock that resulted from a change in the risk premium. The rise in the nominal interest rate resulting from the preference shock can be accounted for by an adjustment in η_t ,which we can then use as the risk premium shock. To avoid any impact on the household's budget constraint, the government bond can be in zero net supply. [40]

Regarding the bankers, their maximization problem is not directly affected from the flight to quality preference shock. The only impact on them is on the change in the nominal interest rates on deposits exactly as in the model. However, the increased funding costs for the banks via deposits are taken into account.

To conclude, there is a direct mapping of our version of the risk premium shock to the interpretation in Chari, Kehoe and McGrattan (2009) and Fisher (2015). An increase in the risk premium of deposits captures an increased demand in government bonds via a substitution effect.

Flight to Quality and Deposits Since our original model abstracts from government bonds for simplicity, an alternative approach would be to introduce a preference for holding deposits instead of government bonds in the utility function. The exogenous shock ω_t now targets the preference for deposits:

$$\max_{C_t,L_t,D_t} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} + \omega_t U(D_t) \right]$$
$$\text{s.t.} \quad P_t C_t = P_t W_t L_t + P_{t-1} D_{t-1} R_{t-1}^D \eta_{t-1} - P_t D_t + P_t \Pi_t^P - P_t \tau_t$$

where η_t is not an exogenous innovation in this setup, but part of the interest rate as before. The first-order condition can be written as

$$\beta R_t^D \eta_t E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} = 1 + \omega_t^\star U(D_t)$$

where the shock is normalized with respect to marginal utility of consumption $\Omega_t^\star = \omega_t / C_t^{-\sigma}$. Thus, the shock can be interpreted as a preference shifter of deposits: $\eta_t = 1 + \omega_t U(D_t)$. To capture the idea of a flight to safety to government bonds that increases the nominal interest rate of deposits, it is important to realize that the shocks Ω_t and ω_t are inversely related. A flight to safety scenario implies an increase in Ω_t and a reduction in ω_t so that eta_t increases. As before, this setup is consistent with our modeling of the

[40]One other potential caveat could be that this shock could actually also capture potential heterogeneities in the pass-through of deposits and governments. Nevertheless, the shock would still capture the impact of flight to quality, only adjusted for the different pass-through.

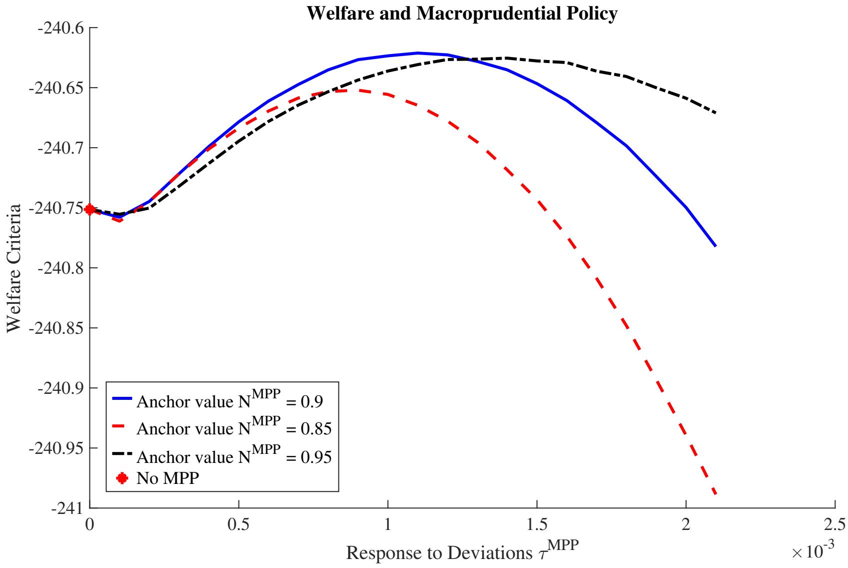

Figure 14: Welfare for response to deviations τ^{MPP} and parameter values N^{MPP}. τ^{MPP} is varied on the horizontal axis. Welfare is on the horizontal axis

banking sector

Bank Default Finally, an alternative could be that the wedge accounts for the probability of default of the banks as our model abstracts from idiosyncratic default and bank runs. If the default probability of deposits is p_t, then the budget optimization problem would be:

$$\max_{C_t,L_t,D_t} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \right] \tag{49}$$

$$\text{s.t.} \quad P_t C_t = P_t W_t L_t + P_{t-1} D_{t-1} R_{t-1}^D \eta_{t-1}(1-p_t) - P_t D_t + P_t \Pi_t^P - P_t \tau_t \tag{50}$$

where η_t should again be interpreted as part of the nominal interest rate. The Euler equations reads as:

$$\beta R_t^D \eta_t E_t (1-p_{t+1}) \frac{\Lambda_{t,t+1}}{\Pi_{t+1}} = 1$$

In this case, the risk premium shock would be a proxy for the impact of the probability of default of the bank. One potential shortcoming of this interpretation is that the modeling of the banking sector should then also explicitly incorporate default, similar to Gertler, Kiyotaki and Prestipino (2020) for instance.

Table 4: Macroprudential Policy and the Mean of Selected Variables

Variable	$\tau^{MPP}=0$	$\tau^{MPP}=0.05\%$	$\tau^{MPP}=0.11\%$	$\tau^{MPP}=0.2\%$
Mean of selected variables				
$\overline{Y}$	1.004	1.005	1.005	1.005
$\overline{S}$	9.116	9.123	9.129	9.125
$\overline{N}$	1.146	1.148	1.151	1.156
$\overline{\phi}$	8.210	8.162	8.104	8.010
$\overline{\pi}$	1.985	1.990	1.993	2.022

D Macroprudential Policy Rule: Parameters, Welfare and Moments

The rule consists of two parameters that interact with each other. Figure 14 shows the impact on welfare for different combinations of N^{MPP} and τ^{MPP}. The optimal rule has a rather low N^{MPP} parameter value with a small value for the response parameter τ^{MPP}. This ensures the build-up of a small buffer that can then be released during a crisis.

We also assess how variations in τ^{MPP} affect key moments to better understand how the macroprudential rule affects welfare. Table 4 shows the average of output, security holdings, net worth, leverage and inflation for when the model is simulated for different macroprudential policy regimes. In particular, we vary the response strength to the deviations in net worth from target value, that is the parameter τ^{MPP}. For simplicity, we fix $N^{MPP}=0.9$ and set the monetary policy lower bound to -2% as in the calibration. Note that $\tau^{MPP}=0$ corresponds to the scenario of no macroprudential policy, while $\tau^{MPP}=0.11\%$ is the welfare optimal value. While the differences are rather small, we see that a more active macroprudential policy increases average output. An important result is that the amount of security holdings is hump shaped. While raising τ^{MPP} initially increases even the level of the security holdings, a too large buffer already reduces the level again. This shows that some build-up of the buffer is helpful to stabilize lending. But, the buffer is associated with costs if it becomes too large. Additionally, we can see that increasing τ^{MPP} raises average inflation. Taken together, this shows that macroprudential policy can help to some extent to stabilize output and credit supply.

E Alternative Macroprudential Rule

We consider as an alternative a rule that responds to the banks' asset holdings. We formulate the rule based on the mark to market value of the banks' security holdings:

$$\tilde{\tau}_t = \max\left\{\left(Q_t S_t / S^{MPP}\right)^{\tilde{\tau}^{MPP}} - 1, 0\right\} \tag{51}$$

where we use the tilde to denote the rule responding to the security holdings of banks.

We compare this rule to our baseline macroprudential policy which responds to net

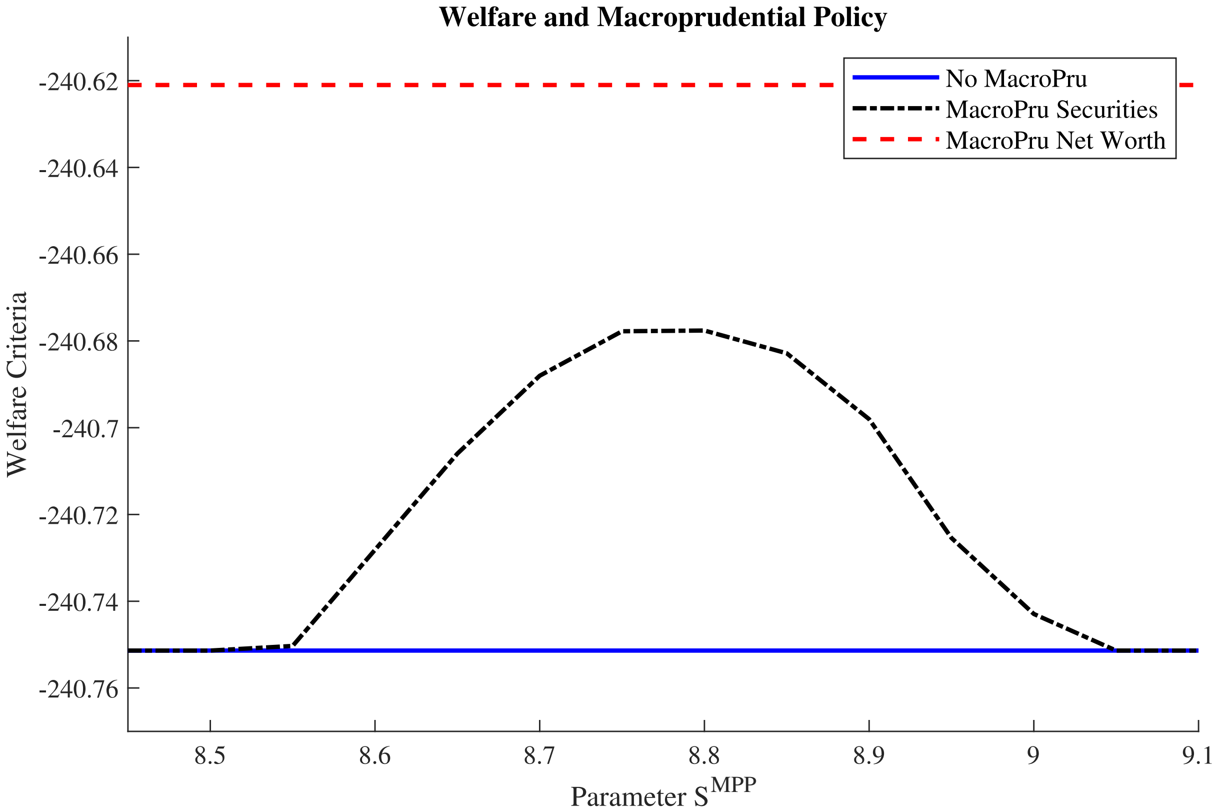

Figure 15: Welfare for the macroprudential rule based on security holdings. The parameter values S^{MPP} are varied on the horizontal axis. The response to deviations $\tilde{\tau}^{MPP}$ is set optimally to maximize welfare for each value of S^{MPP}. This is shown as dash-dotted black line. It is compared to the welfare level in an economy without macroprudential policy (blue solid line) and the maximum attainable welfare level for our main specification for the macroprudential rule that is conditioned on net worth.

worth. Specifically, Figure 15 shows the welfare level for varying parameter values S^{MPP}, where the response to deviations $\tilde{\tau}^{MPP}$ is set optimally to maximize welfare. This is shown as the dash-dotted black line. The line is compared to the welfare level in an economy without macroprudential policy (blue solid line) and the maximum attainable welfare level in our baseline macroprudential rule that is conditioned on net worth (dashed red line). First of all, both rules provide welfare gains compared to an economy without macroprudential policy. However, the baseline macroprudential policy rule that responds to net worth provides larger welfare gains compared to the rule that respond to the asset side. The reason is that net worth is more correlated with the business cycle than securities in the model. As a consequence, targeting net worth allows to better capture the financial impact on the business cycle.

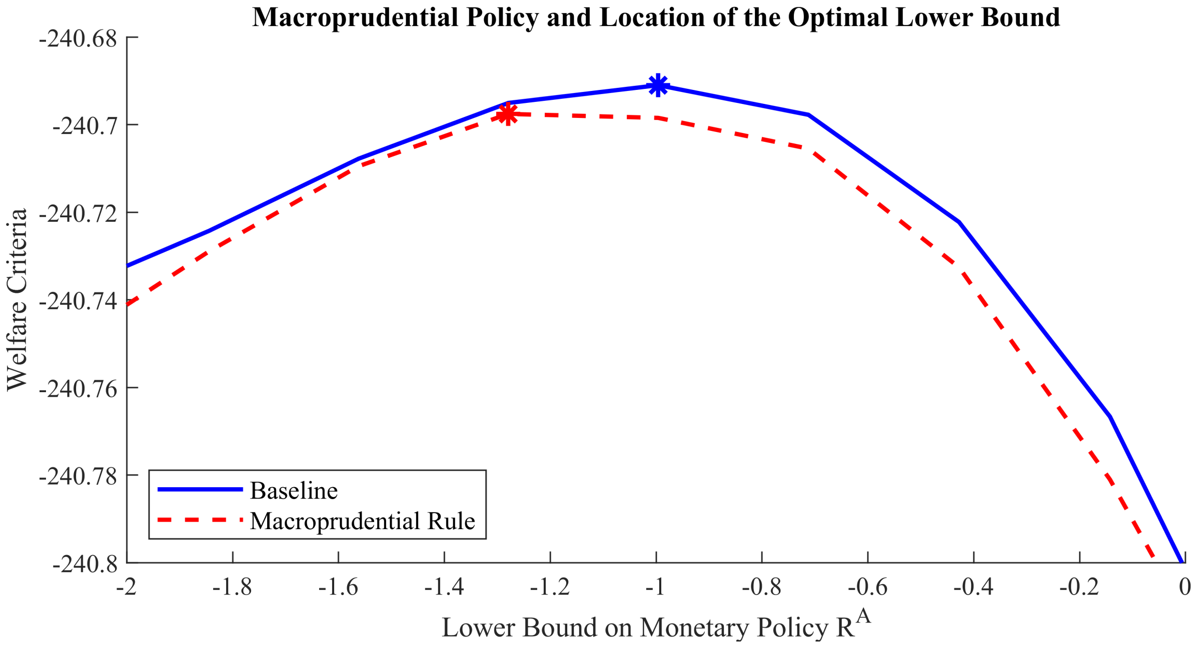

Figure 16: Change of location of the optimal lower bound due to macroprudential policy. The macroprudential policy is chosen to be suboptimal ($N^{MPP} = 0.6$ and $\tau^{MPP} = 0.0005$) to highlight that macroprudential policy can affect the location of reversal rate, which is denoted with a star.

F Macroprudential Policy and the Location of the Effective Lower Bound

Our main exercise shows that the optimal capital buffer does not directly affect the location of the optimal lower bound. The reason is that it is optimal to only build up limited macroprudential policy space. Once the economy approaches such a negative territory, the macroprudential space is already released. However, macroprudential policy can affect the location of the reversal rate. If the central bank still has the option to lower the capital requirements once the economy approaches very low territory, the location changes. As an example, we choose a suboptimal macroprudential rule, which builds up the buffer already at a lower level ($N^{MPP} = 0.6$). Figure 16 highlights that the location of the reversal rate moves from -1% in the absence of macroprudential policy to around -1.3% due to the chosen macroprudential policy.

G Solution Method

The non-linear model is solved with policy function iterations. In particular, we use time iteration (Coleman, 1990) and linear interpolation of the policy functions as in Richter, Throckmorton and Walker (2014). We solve for the policy functions and law of motions. We rewrite the model to use net worth N_t as a state variable instead of $D_{t-1}R_{t-1}$ to ease the computation.

The algorithm has the following steps:

1. Define the state space and discretize the shock with the Rouwenhorst method.
2. Use an initial guess for the policy functions.
3. Solve for all the time t variables for a given state vector and a law of motion of net worth. Given the state vector $K_{t-1}, N_t, \eta_t, \zeta_t$, the policy variables Q_t, C_t, ψ_t, Π_t and the law of motion of the net worth, we can solve for the following variables in period t:

$$
\begin{aligned}
I_t &= \left(Q_t(1-\eta_i)a_i\right)^{\frac{1}{\eta_i}} K_{t-1} \\
Y_t &= \frac{C_t + I_t}{\left(1 - \frac{\rho^r}{2}\left(\frac{\Pi_t}{\Pi} - 1\right)^2\right)} \\
L_t &= \left(\frac{Y_t}{K_{t-1}^{\alpha}}\right)^{\frac{1}{1-\alpha}} \\
W_t &= \chi L^{\varphi} C^{\sigma} \\
MC_t &= \frac{W_t}{1-\alpha}\frac{L}{Y} \\
R_t^A &= R^A\left(\frac{\Pi_t}{\Pi}\right)^{\kappa_\Pi}\left(\frac{Y_t}{Y}\right)^{\kappa_Y} \\
R_t^D &= \mathbf{1}_{R_t^A \geq R^{A\,SS}}\left[R_t^A - \varsigma\right] + (1 - \mathbf{1}_{R_t^A \geq R^{A\,SS}})\left[\omega^1 + \omega^2\exp(\omega^3(R_t^A - 1)) + 1\right]
\end{aligned}
$$

The endogenous state variables are capital and net worth, which are given from the law of motion of capital and the guess for the law of motion of net worth

$$
\begin{aligned}
K_t &= (1-\delta)K_t + \left(a_i\left(\frac{I_t}{K_t}\right)^{1-\eta_i} + b_i\right)K_{t-1} \\
N_{t+1} &= \mathcal{T}(K_{t-1}, N_t, \zeta_t, \eta, \zeta_{t+1}, \eta_{t+1})
\end{aligned}
$$

Note that capital is predetermined, while net worth depends on the shocks. Therefore, we have a net wroth at each integration node for the shocks. At each node i, we calculate now the policy function $Q_{t+1}^i, C_{t+1}^i, \psi_{t+1}^i, \Pi_{t+1}^i$. At this step, we linearly interpolate the policy functions

$$
\begin{aligned}
I_{t+1}^i &= \left(Q_{t+1}^i(1-\eta_i)a_i\right)^{\frac{1}{\eta_i}} K_t \\
Y_{t+1}^i &= \frac{C_{t+1}^i + I_{t+1}^i}{\left(1 - \frac{\rho^r}{2}\left(\frac{\Pi_{t+1}^i}{\Pi} - 1\right)^2\right)} \\
L_{t+1}^i &= \left(\frac{Y_{t+1}^i}{K_t^{\alpha}}\right)^{\frac{1}{1-\alpha}} \\
W_{t+1}^i &= \chi\left(L_{t+1}^i\right)^{\varphi}\left(C_{t+1}^i\right)^{\sigma} \\
MC_{t+1}^i &= \frac{W_{t+1}^i}{1-\alpha}\frac{L_{t+1}^i}{Y_{t+1}^i}
\end{aligned}
$$

$$R^{k,i}_{t+1} = \frac{MC^i_{t+1}\alpha Y^i_{t+1}/K_t + Q^i_{t+1}(1-\delta)}{Q_t}\Pi^i_{t+1}$$

We can now calculate the following items:

$$\phi_t = \frac{Q_t K_t}{N_t}$$

$$R_t = R^D_t \eta_t \frac{1}{1-\delta^B} - R^A_t \frac{\delta^B}{1-\delta^B}$$

$$\mu_t = \beta E_t \left(\frac{C_t}{C_{t+1}}\right)^{\sigma} (1-\theta+\theta\psi_t)\left(\frac{R^K_{t+1} - R_t}{\Pi_{t+1}}\right)$$

$$\nu_t = \beta E_t \left(\frac{C_t}{C_{t+1}}\right)^{\sigma} (1-\theta+\theta\psi_t)\left(\frac{R_t}{\Pi_{t+1}}\right)$$

where the expectations are based on the weighting of the different integration nodes. The Rouwenhorst method discretizes the shocks and gives the weighting matrix. Finally, we can calculate the errors for the four remaining equations

$$err_1 = \left(\frac{\Pi_t}{\Pi} - 1\right)\frac{\Pi_t}{\Pi} - \left(\frac{\epsilon}{\rho^r}\left(MC_t - \frac{\epsilon-1}{\epsilon}\right) + \beta E_t\left(\frac{C_t}{C_{t+1}}\right)^{-\sigma}\frac{Y_{t+1}}{Y_t}\left(\frac{\Pi_{t+1}}{\Pi_t} - 1\right)\frac{\Pi_{t+1}}{\Pi}\right)$$

$$err_2 = \beta R^D_t \eta_t E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\frac{1}{\Pi_{t+1}}$$

$$err_3 = \psi_t - (\mu_t\phi_t + \nu_t)$$

$$err_4 = \psi_t - \left(\lambda\Big(\frac{1}{1-\delta^B}\phi_t - \frac{\delta^B}{1-\delta^B}\Big)\right)$$

We minimize the errors using a root solver the policy functions in period t. The policy functions for period $t+1$ are taken from the previous iteration.

4. This step is only relevant for the extension with the countercyclical capital rule. Otherwise, it can be skipped. Check if the occasionally binding constraint is binding. If we introduce the capital requirement, it is occasionally binding. Therefore, we have to check if

   $$\phi^R > \phi^M$$

   where ϕ^M is the market based leverage that we calculated as ϕ in the previous step. If this is the case, the capital constraint is binding. We now replace two equations from before, namely we impose directly

   $$\phi = \phi^R$$

   Furthermore, one of the remaining equations is now adjusted as the market based leverage constraint is not binding anymore. Therefore, we remove $\phi_t = \frac{Q_t K_t}{N_t}$ from the calculations and actually minimize the error:

$$err_4 = \phi_t - \frac{Q_t K_t}{N_t}$$

Note that we do not need $\psi_t \geq \left(\lambda\left(\frac{1}{1-\delta^B}\phi_t - \frac{\delta^B}{1-\delta^B}\right)\right)$ from the previous step as it is not binding.

5. Update the law of motion for net worth. We have assumed that we know the actual law of motions. Using the policy functions, we improve our guess of the policy function. Using the result from the previous steps (depending on the binding of the constraint), we update it as follows

$$N_{t+1}^{i} = \theta\left(\left(R_{t+1}^{k,i} - R_t\right)\phi_t - R_t\right) + \omega K_t$$

We have to update the law of motion for each possible shock realization in the next period.

6. Check convergence for the policy functions and the law of motion of net worth for a predefined criterion.

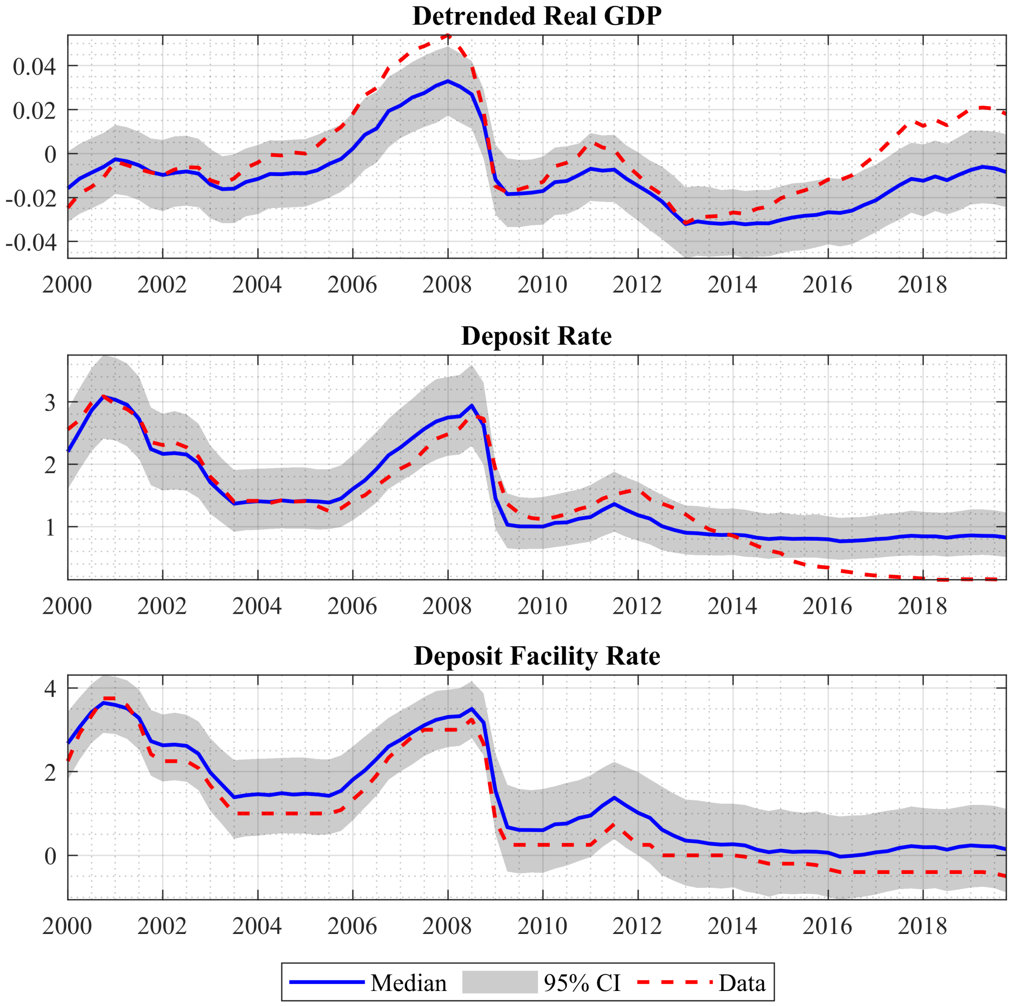

Figure 17: Comparison of the observables (linear detrended log real GDP, deposit rate and the deposit rate facility) with the model implied values. The solid blue line is the median and the shaded area is the 95% CI. The deposit rate and deposit rate facility are expressed in annualized terms.

H Particle Filter and External Validation

The particle filter algorithm follows Atkinson, Richter and Throckmorton (2020), Bianchi, Melosi and Rottner (2021), and Rottner (2021). We summarize the main idea of the particle filter and refer for a detailed description of the algorithm to these papers, particularly Bianchi, Melosi and Rottner (2021).

The particle filter provides a distribution of the structural shocks (the risk premium and monetary policy shock) to explain the data. The relative weights of the shocks are given from the fit of the data. The shocks can then be used to obtain the a historical perspective for the model's variables. In the second step, we use the filtered shocks and feed them into an alternative economy with active macroprudential policy. This allows to back out the counterfactual path of economic variables under an alternative policy regime.

External Validation We take the nonlinear model to the data to also provide an empirical validation from a historical perspective. Specifically, we include more observables than structural shocks and allow for large measurement errors, which can also account for the data. Figure 17 shows the filtered median and its 95% confidence interval and compares it to the data. The filtering results suggest that the model can capture the dynamics quite well. The filtered series captures the deposit rate facility rate very well. The deposit rate follows the path mostly very nice as well, but the model-implied value is slightly higher than in the data towards the end of the sample. The model also predicts a deeper recession in in the late 2010s than the data. One reason for the divergence between the model's predictions and the data at the end of the sample comes from the fact that our nonlinear model does not create enough persistence. We abstract from some important bells and whistles that increase the fit of the model, e.g. a backward looking term in the monetary policy rule, due to the complexity of solving the model numerically.

I Additional Figures

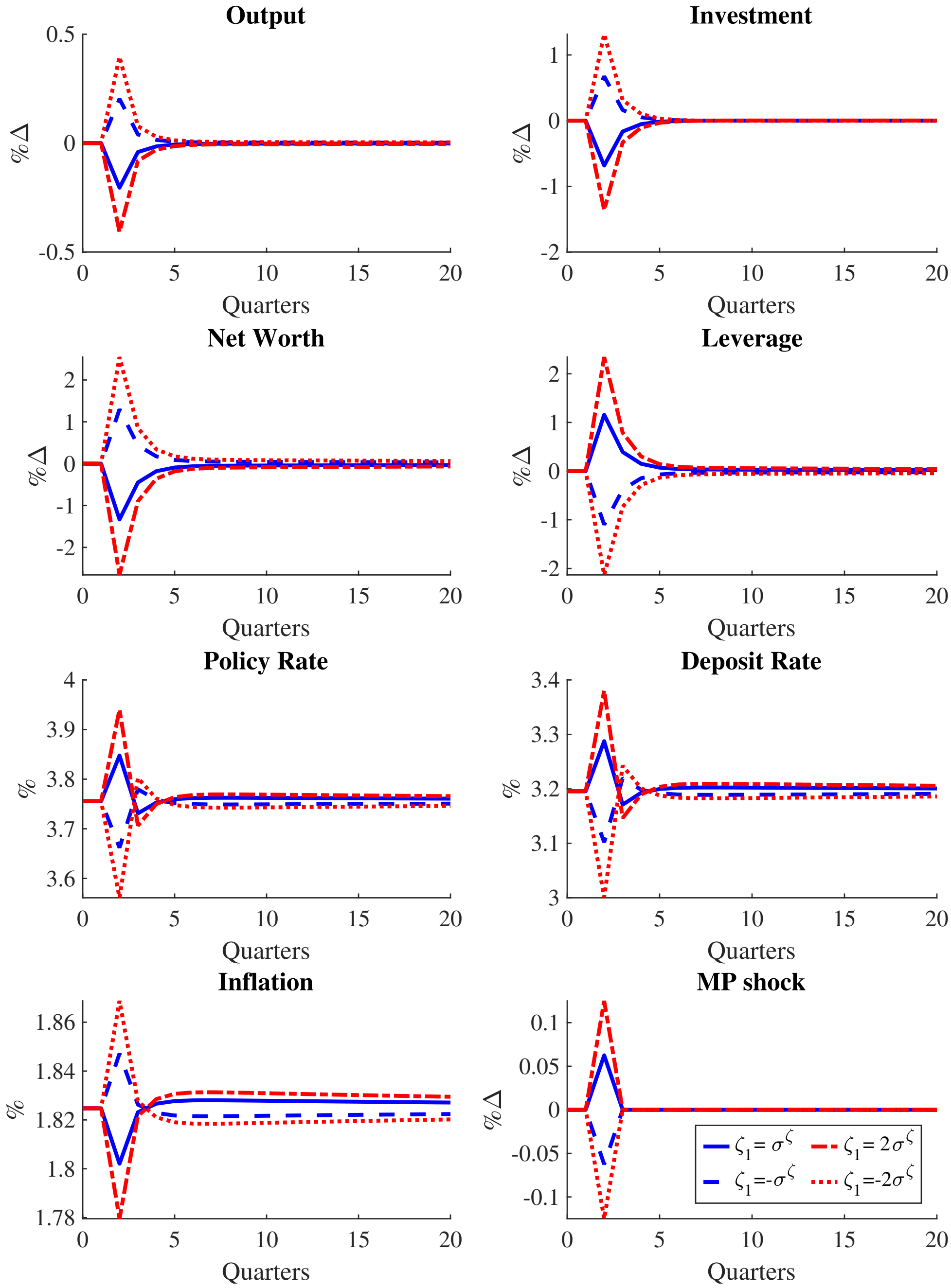

Figure 18: Impulse response functions of the monetary policy shock that differ in the size and sign of the innovation. A one standard deviation increase (blue solid) and decrease (blue dashed) as well as a two standard deviation increase (red dash-dotted) and decrease (red dotted) for the innovation ζ is shown. The responses are displayed in percentage deviations from the risky steady state, which is the initial point of the economy.

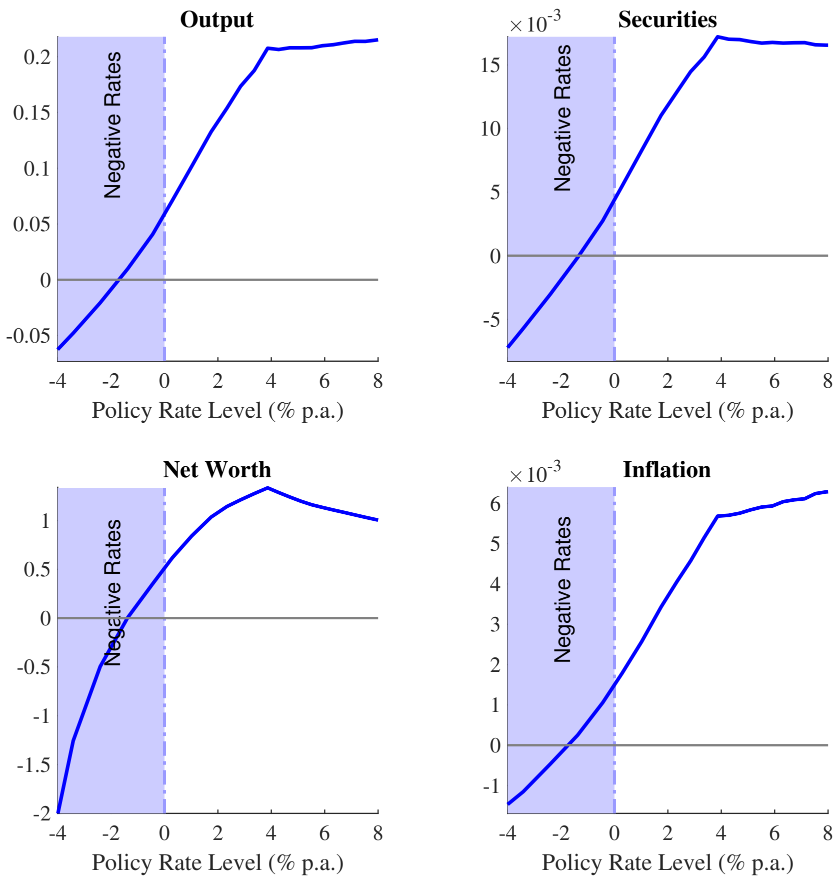

Figure 19: First period of an impulse response function to illustrate the state-dependent impact of monetary policy shocks. To generate the state dependency, the monetary policy shock is combined with different sized risk premium shocks. Instead of showing the risk-premium shock, the horizontal axis displays the associated policy rate level. The vertical axis displays the state-dependent difference for the period $t = 1$ impulse response between a shocked path, which faces additionally a negative one std. dev. monetary policy shock, and a path, in which the monetary policy innovation does not occur. The deviations are in percent. The blue shaded area indicates the territory, where the risk premium shock pushes the economy in negative interest rate territory. Note that Tte lower bound on the policy rate is set to -5% for this Figure as the baseline calibration of -2% is already binding for sufficient large shocks.

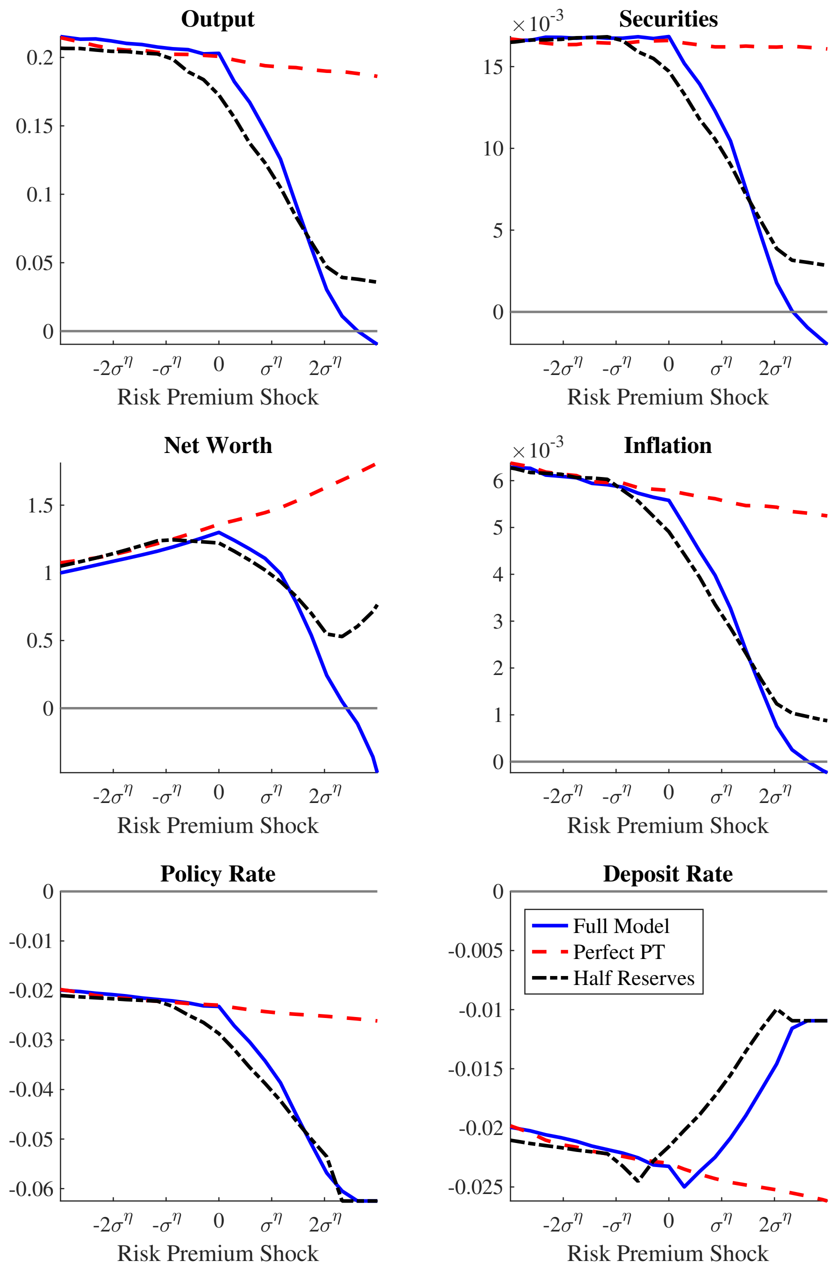

Figure 20: First period response to a monetary policy shock combined with different sized risk premium shocks. The vertical axis displays the state-dependent difference for the period $t = 1$ response between a shocked path, which introduces a negative one standard deviation innovation for the monetary policy shock $\zeta_1 = \sigma^{\zeta}$, and a path, in which the monetary policy innovation does not occur. The state-dependence results from the different sized risk premium shock that occurs simultaneously in the first period, which is displayed on the horizontal axis.

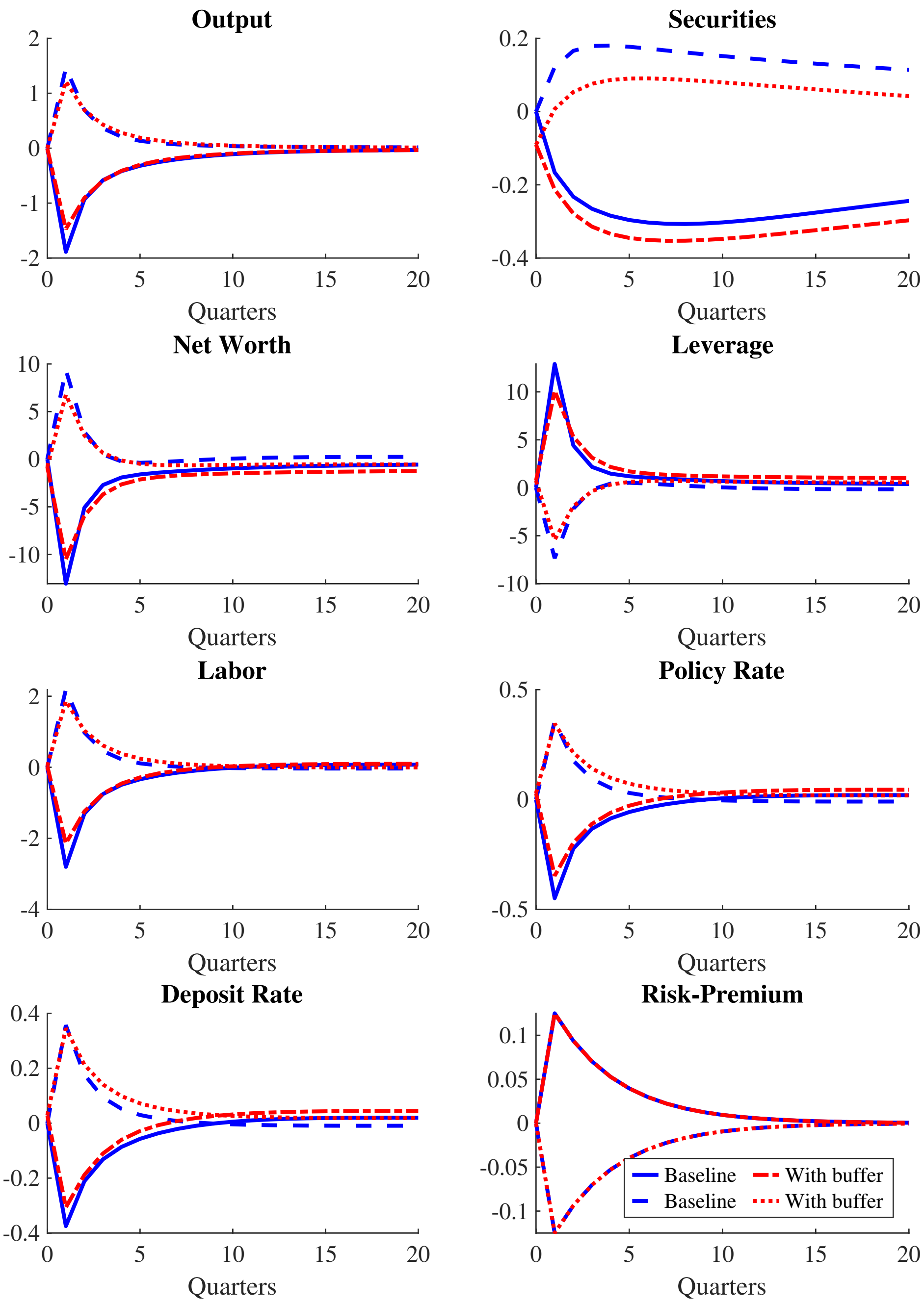

Figure 21: Impulse response functions for an economy with and without a macroprudential buffer are shown. Furthermore, different risk premium shocks are considered. A one standard deviation increase and decrease is shown for the baseline model without buffer (blue solid dotted and blue dashed line, respectively) and an economy with a buffer $\tau^{MPP} = 0.11\%$ and $N^{MPP} = 0.9$ (red dash-dotted and red dotted line, respectively). Starting point is the risky steady state of each economy. Deviations are in percent relative to the risk steady state of the economy without a capital buffer rule.