Reversal Interest Rate and Macroprudential Policy*

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Abstract

Could a monetary policy loosening in a low interest rate environment have unintended recessionary effects? Using a non-linear macroeconomic model fitted to the euro area economy, we show that the effectiveness of monetary policy can decline in negative territory until it reaches a turning point, where monetary policy becomes contractionary. The risk of hitting the reversal interest rate depends on the capitalization of the banking sector, which gives rise to a novel motive for macroprudential policy. Building up macroprudential buffers in good times mitigates the probability of encountering the reversal rate and can increase the effectiveness of negative interest rate policies.

Keywords: Reversal Interest Rate, Negative Interest Rates, Macroprudential Policy, Monetary Policy, ZLB

JEL Codes: E32, E44, E52, E58, G21

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1 Introduction

The prolonged period of ultra-low interest rates in the euro area and other advanced economies has raised concerns that further monetary policy accommodation could have the opposite effect than what is intended. Specifically, there is a risk that a further loosening of monetary policy can become contractionary and reduce lending for very negative policy rates. The policy rate enters a "reversal interest rate" territory, to use the terminology of Brunnermeier and Koby (2018), in which the usual monetary transmission mechanism through the banking sector breaks down.

In this paper, we analyze the connection between monetary policy and macroprudential policy in a low interest rate environment. We develop a non-linear macroeconomic model with a banking sector fitted to the euro area economy that features asymmetric monetary policy transmission and captures the reversal rate mechanism. The framework demonstrates that a less well-capitalized banking sector amplifies the likelihood of encountering the reversal interest rate and impairs negative interest rate policies. This gives rise to a new motive for macroprudential policy. Building up macroprudential policy space in good times to support the bank lending channel of monetary policy, for instance in the form of a countercyclical capital buffer (CCyB), mitigates the risk of monetary policy hitting reversal rate territory and increases the effectiveness of negative interest rate policies.

A key component of an analysis that focuses on the reversal rate is to account for the transmission of policy rates to other interest rates. Specifically, there is growing evidence that the pass-through of policy rates to banks' deposit rates is increasingly imperfect for negative rates because banks are reluctant to cut rates below zero (e.g. Heider, Saidi and Schepens, 2019). Figure 1 highlights this fact for the euro area economy. The co-movement of the ECB deposit facility rate, which determines the interest received from reserves, and the average deposit rate paid to households decouples after approaching a low interest rate territory. Additionally, the distribution of deposit rates across individual euro area banks shows that, initially, no bank charged sub-zero deposit rates after approaching negative territory in June 2014. Even in December 2019, there is a only small but increasing

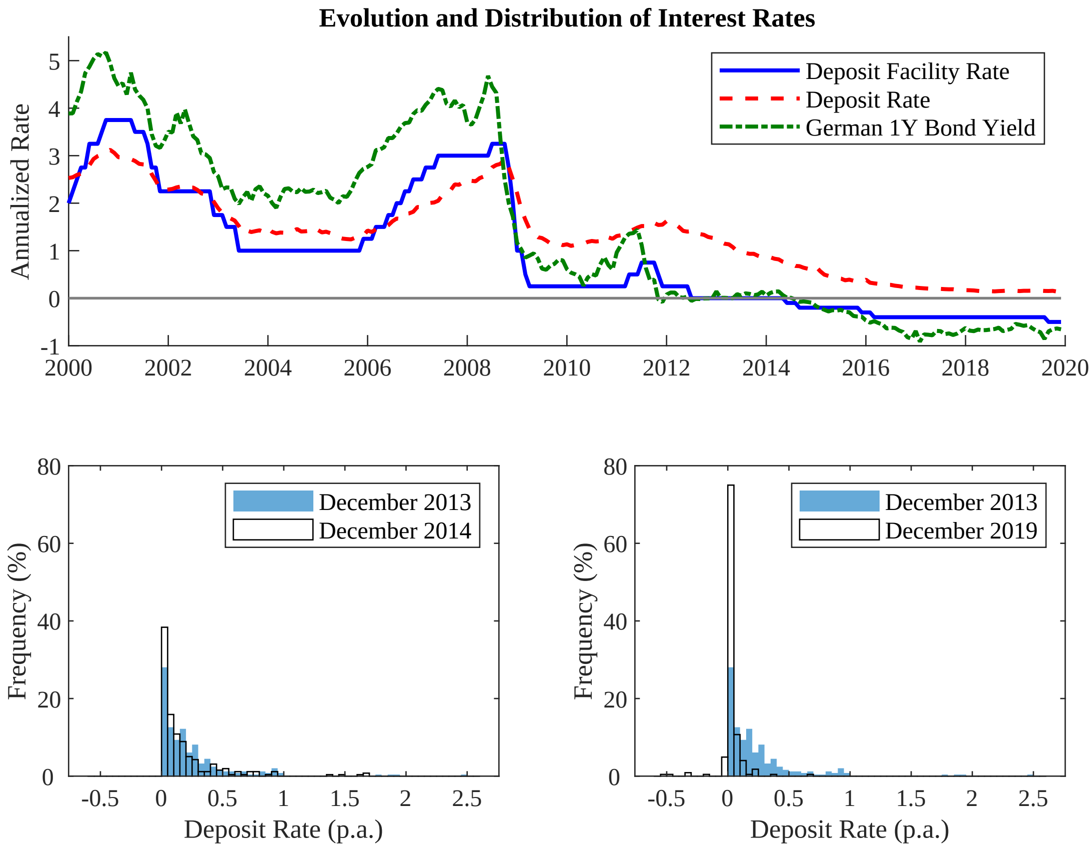

Figure 1: The upper panel shows the ECB deposit facility rate, average household deposit rate in the euro area and the German 1Y bond yield. The lower panel shows the distribution of overnight household deposit rates across banks. Details can be found in Appendix B.

fraction of banks that charged sub-zero deposit rates. At the same time, a policy rate cut directly lowers the return of liquid assets of banks such as reserves and government assets, as can be seen for the German one year bond yield in Figure 1. The diminished return on liquid assets deteriorates bank profitability, which then contracts bank lending.

We incorporate these facts in a novel macroeconomic framework by extending a New Keynesian model with a capital-constrained banking sector along two dimensions: i) by introducing an imperfect pass-through of policy rates to deposit rates for low interest rates and ii) by adding a liquidity requirement for banks to hold liquid assets. The imperfect pass-through captures the depletion of banks' market power for low interest rates as in Brunnermeier and Koby (2018).[1] The requirement for liquid assets reflects both

[1]The relevance of banks' market power in deposit markets is theoretically pointed out e.g. by Klein (1971), while empirical evidence is provided, for instance, in Sharpe (1997). Hainz, Marjenko and Wildgruber (2017) show that banks market power is declining with low interest rates as the switching costs of banks are falling. Drechsler, Savov and Schnabl (2017) document that market power in the deposit market affects monetary policy transmission.

monetary policy and regulatory considerations.[2] The key implication of this framework is that monetary policy can have contractionary effects in negative territory due to a deterioration of the banking sector's profitability.

The framework suggests that, for the euro area, the reversal interest rate is located at around -1% p.a. and that the policy rate enters this territory with a probability of less than three percent. To establish this result, we fit the model to salient features of the euro area economy and solve the model using global methods that can capture non-linear dynamics. The bank lending channel is state-dependent and the transmission of shocks is asymmetric. In particular, a lowering of the policy rate has only a modest impact on credit supply and aggregate demand due to the imperfect pass-through in a low interest rate environment. At the same time, a reduction of the policy rate lowers the return on banks' government asset holdings and reduces their net worth. If the latter channel is the dominant one, monetary policy reaches a turning point (the reversal rate), from which on a monetary policy loosening reduces bank lending and contracts output. The main insight is that a lower policy rate requires a larger interest rate cut in order to have the same expansionary impact. However, this is conditional on there being enough space left before approaching the reversal interest rate.

The threat of the reversal rate gives rise to a new motive for macroprudential policy as it can help to strengthen the bank lending channel in a "lower for longer" interest rate environment.[3] The reason is that the capitalization of the banking sector plays a key role for the transmission of monetary policy in low or negative interest rate environment. This opens up the possibility of using macroprudential policy to alleviate the diminishing effectiveness of monetary policy. In particular, building up macroprudential policy space in good times can mitigate the risk of monetary policy entering a reversal rate territory. The additional space can be released during downturns to increase the resilience of the

[2] In relation to monetary policy, banks are required to hold minimum reserves with the central bank. The minimum reserve requirements aim at stabilizing money market rates and creating (or enlarging) a structural liquidity shortage, but may also reflect the need to maintain a certain amount of eligible securities to be able to participate in open market operations. On the regulatory side, liquid asset holdings are needed to comply with minimum liquidity requirements (e.g. the Liquidity Coverage Ratio).

[3] The strategic interaction between macroprudential policy and the reversal rate is a potential positive side effect of macroprudential policy, which aims to safeguard financial stability.

banking sector. To emphasize this motive, we incorporate macroprudential policy in the form of a countercyclical capital buffer that can impose additional capital requirements. The buffer is created during a phase of credit expansion and can then be released during a recession following the features of the Basel III framework. Therefore, the buffer is asymmetric and restricted to be non-negative, which we capture with an occasionally binding rule.

We demonstrate that macroprudential policy lowers the probability of hitting the reversal interest rate and increases the effectiveness of negative interest rate policies. The welfare-optimizing capital buffer rule reduces the probability of being at or below the reversal rate by around 23%. The banking sector builds up additional equity in good times, which can then subsequently be released during a recession. Having accumulated additional capital buffers during good times, the negative impact of monetary policy loosening on bank balance sheets is then dampened in a low interest rate environment. Consequently, monetary policy becomes more effective during economic downturns and the reversal interest rate is less likely to materialize, which improves overall welfare.[4] We thereby provide evidence of important strategic complementarities between monetary policy and macroprudential policies.

Literature Review The paper adds to the growing literature about negative interest rates and the reversal interest rate, which is summarized for instance in Brandao-Marques et al. (2021) or Heider, Saidi and Schepens (2021). Our paper builds on the seminal contribution by Brunnermeier and Koby (2018), where the reversal interest rate is endogenously determined in a framework with an imperfect pass-through. Eggertsson et al. (2019) show the importance of reserve holdings for the bank lending channel with negative interest rates. Ulate (2019) emphasizes the trade-off between increasing demand and reducing bank profitability for negative interest rates. Sims and Wu (2021) connect the size of the central bank's balance sheet to the impact of negative interest rates.[5] With

[4]The impact of macroprudential policy on the effectiveness of monetary policy could also affect the optimal inflation target because the optimal inflation target depends on the probability of encountering constrained monetary policy as discussed in Coibion, Gorodnichenko and Wieland (2012).

[5]Further connected papers are De Groot and Haas (2020) and Balloch and Koby (2019), among others.

respect to the existing literature, we incorporate macroprudential policy and assess its interaction with negative interest rate policies and the reversal rate. Importantly, the location of the reversal rate is endogenous in our framework so that the negative interest rate policies can be initially expansionary, albeit with diminishing effectiveness, before becoming contractionary at the reversal rate. Unlike the studies, we use global solution methods to fully capture the non-linearities associated with the reversal rate.

This paper is also related to the large body of literature on the interaction between monetary policy and macroprudential policies.[6] Farhi and Werning (2016) and Korinek and Simsek (2016) show the importance of macroprudential policy in an environment with a binding zero lower bound. Lewis and Villa (2016) demonstrate that a countercyclical capital requirement can mitigate the output contractions in the presence of a zero lower bound. We assess macroprudential policy in a negative interest rate environment, where the intended effect of monetary policy can endogenously reverse. This creates a new motive for macroprudential policy and emphasizes the strategic complementarity with monetary policy. We also contribute to the modeling of the countercyclical capital buffer, which is one of the main macroprudential instruments considered in the literature. As a new feature, we incorporate the asymmetric design of the CCyB with an occasionally binding policy rule.[7]

The paper is also connected to the fast-growing empirical literature on negative policy rates. Jackson (2015) and Bech and Malkhozov (2016) analyze the early experiences with negative policy rates and find that a negative policy rate has a limited pass-through.[8] Heider, Saidi and Schepens (2019) document that negative policy rates impact bank lending in the euro area. Banks are reluctant to pass through the policy rates to their depositors, which results in less lending for banks that depend heavily on deposit funding. Hainz, Marjenko and Wildgruber (2017) provide empirical evidence that this is related to a fall in market power. Firms that are exposed to negative rates are likely to switch their

[6]See e.g. Darracq-Pariès, Kok and Rodriguez-Palenzuela (2011), Angelini, Neri and Panetta (2014), Rubio and Carrasco-Gallego (2014), Benes and Kumhof, 2015, Collard et al. (2017), De Paoli and Paustian (2017), Gelain and Ilbas (2017), Bluwstein et al. (2020), among many others.

[7]Van der Ghote (2018) computes a constrained optimal macroprudential policy.

[8]Other studies are e.g. Ampudia and Van den Heuvel (2018), Basten and Mariathasan (2018), Altavilla et al. (2019), Eisenschmidt and Smets (2019), Mendicino, Puglisi and Supera (2021), among others.

banks and take other measures to alleviate the costs of negative rates. Borio, Gambacorta and Hofmann (2017) and Claessens, Coleman and Donnelly (2018) show that banks' profitability deteriorated for low interest rates. Additionally, Fuster, Schelling and Towbin (2021) point out that a reduction in the reserve requirement increases the profitability of banks. We incorporate this evidence about an imperfect pass-through and reduced bank profitability in a non-linear macroeconomic model to assess monetary policy effectiveness, the reversal rate and the interaction with macroprudential policy.

Outline The paper is organized as follows. In Section 2, the non-linear macroeconomic model is introduced. We calibrate the model and parametrize the imperfect deposit rate pass-through in Section 3. In Section 4, we study the reversal rate and derive the effective lower bound on monetary policy. In Section 5, we incorporate macroprudential policy to study its interaction with the reversal rate. We conclude in Section 6.

2 The Model

The setup is a New Keynesian framework with a capital-constrained banking sector giving rise to financial accelerator effects as in Gertler and Karadi (2011). We embed two further financial frictions in this model that enable the possibility of a reversal interest rate: i) an imperfect pass-through of monetary policy to deposit rates as in Brunnermeier and Koby (2018) and ii) a reserve and liquidity requirement for the banking sector which generates substantial government asset holdings as in Eggertsson et al. (2019).

The imperfect pass-through captures that banks' market power in deposit markets depletes with low interest rates, which affects monetary policy transmission when the economy approaches negative interest rate territory. Once the pass-through becomes increasingly imperfect, the funding costs of the banks decrease less as well as the aggregate demand stimulus via households is weaker. This channel creates a substantially diminished effectiveness of negative interest rate policies.

However, the combination with a reserve requirement for monetary policy purposes and regulatory liquidity constraints is necessary to observe a reversal rate.[9] This second friction forces banks to hold liquid government bonds, where the return on these bonds comoves with the policy rate, for a fraction of their deposits. Even though the government bonds provide a stable profit in normal times, they can cause losses during periods of low rates. When the policy rate is reduced to a sufficient low level, the spread between the policy rate and deposit rate diminishes and can even turn negative due to the imperfect deposit rate pass-through. As a consequence, bank profitability deteriorates, which then reduces credit supply and enables the reversal rate.

For this reason, the effectiveness of monetary policy diminishes step by step in low rate environment until it reaches a turning point, where the bank lending channel of monetary policy breaks down and reverses. To capture these state-dependencies and asymmetries, we solve the model with global methods in its non-linear specification.

2.1 Model Description

Households The representative household is a family with perfect consumption insurance for the different members. The family consists of workers and bankers with constant fractions. The workers elastically supply labor to the non-financial firms, while the bankers manage a bank that transfers its proceedings to the household. Additionally, the household also owns the non-financial firms and receives the profits.

The household can hold deposits at the bank for which it earns the predetermined nominal rate R_t^D. In addition to this, the return also depends exogenously on the risk premium shock η_t, which follows an AR(1) process and is based on Smets and Wouters (2007). This shock is shown to be empirically very important in explaining the Great Recession and zero lower bound episodes in estimated DSGE models.[10] This shock creates a wedge that distorts the choice of deposits as it affects the decision between consumption

[9] With regard to monetary policy operations, the reserve holdings relate to the standard central bank minimum reserve requirements. On the regulatory side, liquid asset holdings are needed to comply with minimum liquidity requirements such as the Liquidity Coverage Ratio and the Net Stable Funding Ratio. Given the typically low risk weights on such liquid instruments, banks may also have an incentive to hoard them on the balance sheet in order to retain a solid capital ratio.

[10] For instance Barsky, Justiniano and Melosi (2014) and Christiano, Eichenbaum and Trabandt (2015) show this using linearized medium-sized DSGE models, among others. Gust et al. (2017) estimate a non-linear model featuring this shock.

and saving. At the same time, the risk premium shock impacts the refinancing costs of the banking sector as it alters the payments on the deposits to the households. Its structural interpretation is further outlined in Appendix C.

The nominal budget constraint reads as follows:

$$P_tC_t = P_tW_tL_t + P_{t-1}D_{t-1}R^D_{t-1}\eta_{t-1} - P_tD_t + P_t\Pi^P_t - P_t\tau_t \tag{1}$$

where P_t is an aggregate price index, C_t is consumption, W_t is the wage, L_t is labor supply, D_t are the deposits and Π^P_t are the real profits from the capital good producers, retailers and transfers with the banks and τ_t is the lump sum tax.

The household maximizes its utility that depends on consumption and leisure:

$$E_t\sum_{t=0}^{\infty}\beta^t\left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi\frac{L_t^{1+\varphi}}{1+\varphi}\right] \tag{2}$$

The first-order conditions are given as:

$$\beta R^D_t\eta_t E_t\frac{\Lambda_{t,t+1}}{\Pi_{t+1}} = 1$$

$$\chi L_t^{\varphi} = C_t^{-\sigma}W_t$$

where $\Lambda_{t-1,t} = C_t^{-\sigma}/C_{t-1}^{-\sigma}$ and Π_t is gross inflation. The risk premium shock creates a wedge in the Euler equation. An exogenous increase in the risk premium leads to a higher return on deposits. This induces the households to increase their deposit holdings and to postpone consumption, which lowers aggregate demand.

Banking Sector The banks' role is to intermediate funds between the households and non-financial firms. They hold net worth n_t and collect deposits d_t from households to buy securities s_t from the intermediate good producers at the real price Q_t and reserve assets a_t from the government. The flow of fund constraint in nominal terms is

$$Q_tP_ts_t + P_ta_t = P_tn_t + P_td_t \tag{3}$$

where the lowercase letters indicate an individual banker's variable, and the uppercase letters denote the aggregate variable. The banker earns the stochastic return R_{t+1}^{K} on the securities and pays the nominal interest R_t^D as well as a risk premium for the deposits. The reserve assets earn the nominal gross return R_t^A, which is the policy rate. Leverage is defined as securities over assets:

$$\phi_t = \frac{Q_t s_t}{n_t}$$

To accrue net worth, the earnings are retained:

$$P_{t+1} n_{t+1} = R_{t+1}^{K} Q_t P_t s_t + R_t^A P_t a_t - R_t^D \eta_t P_t d_t \tag{4}$$

which can be written in real terms as

$$n_{t+1} = \frac{R_{t+1}^{K} Q_t s_t + R_t^A a_t - R_t^D \eta_t d_t}{\Pi_{t+1}} \tag{5}$$

The banker closes its bank with an exogenous probability of $1-\theta$ and transfers the accumulated net worth to households in case of exit. Therefore, the banker maximizes its net worth:

$$v_t(n_t) = \max_{s_t, d_t, a_t} (1-\theta)\beta E_t \Lambda_{t,t+1} \Big((1-\theta) n_{t+1} + \theta v_{t+1}(n_{t+1})\Big) \tag{6}$$

The banker is subject to an agency problem, which imposes a constraint on the leverage decision. The banker can divert a fraction λ of the bank's assets as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). Since this fraction cannot be recovered by the households, funds are only supplied if the banker's net worth exceeds the fraction λ of bank assets. Furthermore, the banker faces a requirement to hold a certain amount of government assets that cover at least a fraction δ^B of the deposits. This requirement is meant to capture both regulatory liquidity constraints and the reserve requirements for monetary policy purposes.[11] The two constraints can be summed up as:

$$v_t(n_t) \geq \lambda(Q_t s_t + a_t) \tag{7}$$

$$a_t \geq \delta^B d_t \tag{8}$$

The banker's problem is given as:

$$\psi_t = \max_{\phi_t} \mu_t \phi_t + \nu_t \tag{9}$$

$$\text{s.t.} \quad \mu_t \phi_t + \nu_t \geq \lambda\Big(\frac{1}{1-\delta^B}\phi_t - \frac{\delta^B}{1-\delta^B}\Big) \tag{10}$$

where we define $\psi_t = \frac{v_t(n_t)}{n_t}$ and assume that the reserve ratio $a_t = \delta^B d_t$ is binding and discussed later. μ_t is the expected discounted marginal gain of expanding securities for constant net worth, ν_t is the expected discounted marginal gain of expanding net worth for constant assets and R_t is the deposit rate adjusted for the holding of reserve assets:

$$\mu_t = \beta E_t \Lambda_{t,t+1} \left(1 - \theta + \theta\psi_t\right) \frac{R^K_{t+1} - R_t}{\Pi_{t+1}} \tag{11}$$

$$\nu_t = \beta E_t \Lambda_{t,t+1} \left(1 - \theta + \theta\psi_t\right) \frac{R_t}{\Pi_{t+1}} \tag{12}$$

$$R_t = (\eta_t R^D_t)\frac{1}{1-\delta^B} - R^A_t \frac{\delta^B}{1-\delta^B} \tag{13}$$

The banker's leverage maximization results in an optimality condition:

$$\xi_t = \frac{\lambda/(1-\delta^B) - \mu_t}{\mu_t} \tag{14}$$

where ξ_t is the multiplier on the market-based leverage constraint in the banker's problem. This constraint is binding if $0 < \mu_t < \lambda/(1-\delta^B)$, which requires the return on the security to be larger than the combined interest rate adjusted for inflation $E_t(R^K_{t+1} - R_t)/\Pi_{t+1} \geq 0$. The reserve asset ratio is binding as long as the expected return of the security is larger than the policy rate adjusted for inflation $E_t(R^k_{t+1} - R^A_t)/\Pi_{t+1} \geq 0$. Both constraints are binding at the relevant state space, which we verify numerically.

[11] Curdia and Woodford (2011) and Eggertsson et al. (2019) use a function in which reserves lower the intermediation costs of the banks. The regulatory liquidity requirement is not explicitly modeled but provides an additional motivation for banks to hold substantial amounts of liquid government bonds and other assets on their balance sheets.

The individual leverage ϕ_t does not depend on bank-specific components so that it can be summed up over the individual bankers, that is:[12]

$$Q_t S_t = \phi_t N_t \tag{15}$$

The aggregate evolution of net worth N_t is the sum of the net worth of surviving bankers N_t^S and newly entering banks that N_t^N that receive a transfer from the households:

$$N_t = N_t^S + N_t^N \tag{16}$$

$$N_t^S = \theta N_{t-1} \frac{R_t^K - R_{t-1}\phi_{t-1} + R_{t-1}^D}{\Pi_t} \tag{17}$$

$$N_t^N = \omega^N \frac{S_{t-1}}{\Pi_t} \tag{18}$$

Non-financial Firms The non-financial firms are the intermediate good producers, retailers subject to Rotemberg pricing and capital good producers.

Intermediate good producers produce output using labor and capital:

$$Y_t = A^P K_{t-1}^{\alpha} L_t^{1-\alpha} \tag{19}$$

where A^P is the productivity. It sells the output at price P_t^M to the retailers. It pays the labor at wage W_t. The firm purchases capital at market price Q_{t-1} in period $t-1$, which is financed with a loan from the bank. It pays the state-contingent interest rate R_t^K to the banks. Thus, the maximization problem of the firm can be written as

$$\max_{K_{t-1},L_t} \sum_{i=0}^{\infty} \beta \Lambda_{t,t+1} \left[P_t P_t^M Y_t + P_t Q_t (1-\delta) K_{t-1} - R_t^K P_{t-1} Q_{t-1} K_{t-1} - P_t W_t L_t \right] \tag{20}$$

This gives the nominal rate of return on capital:

$$R_t^k = \frac{(P_t^m \alpha Y_t / K_{t-1} + (1-\delta) Q_t)}{Q_{t-1}} \Pi_t$$

The final good retailers, which are subject to Rotemberg pricing, buy the intermediate

[12]Similarly, the leverage ratio associated with reserve assets does not depend on bank specific components.

goods and bundle them to the final good using a CES production function:

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}} \tag{21}$$

where $Y_t(f)$ is the demand of output from intermediate good producer j. Cost minimization implies the following intermediate good demand:

$$Y_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\epsilon} \tag{22}$$

where the price index P_t of the bundled good reads as follows

$$P_t = \left[\int_0^1 P_t(f)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \tag{23}$$

The retailer then maximizes its profits

$$E_t \left\{ \sum_{t=0}^{\infty} \left[\left(\frac{P_t(f)}{P_t} - MC_t \right) Y_t(f) - \frac{\rho^r}{2} Y_t \left(\frac{P_t(f)}{P_{t-1}(f)\Pi} - 1 \right)^2 \right] \right\} \tag{24}$$

where $MC_t = P_t^M$ and Π is the inflation target of the central bank. This gives us the New Keynesian Phillips curve:

$$\left(\frac{\Pi_t}{\Pi} - 1 \right) \frac{\Pi_t}{\Pi} = \frac{\epsilon}{\rho^r} \left(P_t^m - \frac{\epsilon - 1}{\epsilon} \right) + \beta E_t \Lambda_{t,t+1} \frac{Y_{t+1}}{Y_t} \left(\frac{Pi_{t+1}}{\Pi_t} - 1 \right) \frac{\Pi_{t+1}}{\Pi}$$

Capital good producers have access to the function $\Gamma(I_t, K_{t-1})$ which they can use to create capital out of an investment I_t. The capital is then sold so that the maximization problem reads as follows:

$$\max_{I_t} Q_t \Gamma(I_t, K_{t-1}) K_{t-1} - I_t \tag{25}$$

The real price of capital is then given as

$$Q_t = [\Gamma'(I_t, K_{t-1}) K_{t-1}]^{-1}$$

The stock of capital evolves then as:

$$K_t = (1-\delta)K_{t-1} + \Gamma(I_t, K_{t-1})K_{t-1} \tag{26}$$

Monetary Policy and Imperfect Deposit Rate Pass-Through The central bank sets the nominal interest rate for the reserve asset.[13] It responds to inflation and output deviations, while it faces an iid monetary policy shock ζ_t.[14] Furthermore, the central bank can set a lower bound $\tilde{R}^A$ that restricts the level of the interest rate. The policy rule reads as follows:

$$R_t^A = \max\left[R^A\left(\frac{\Pi_t}{\Pi}\right)^{\theta_\Pi}\left(\frac{Y_t}{Y}\right)^{\theta_Y}, \tilde{R}^A\right]\zeta_t \tag{27}$$

The lower bound gives the central bank the opportunity to endogenously restrain itself from lowering the policy rate below a specific rate as the model features a potential reversal interest rate. This level could be a negative or positive net interest rate as we will later determine based on welfare considerations. In contrast to this, a zero lower bound exogenously restricts the central bank from setting a negative net interest rate.

Importantly, there is an imperfect pass-through of the policy instrument to retail deposit rates for a low rate environment. The reason is that banks' market power in deposit markets depletes for low policy rates due to a reduction in the costs of switching banks for depositors. This implies that the markdown for the deposit rate varies with the level of the policy rate R_t^A, which can be written as:

$$R_t^D = \omega(R_t^A), \tag{28}$$

where $\omega(R_t^A)$ is a function. We use a flexible functional form to capture this mapping that is fitted to the declining pass-through in the data.[15] The functional form and the

[13]The central banker has no access to other policy tools such as quantitative easing that could be used as substitutes for conventional monetary policy as shown in Sims and Wu (2020).

[14]The advantage of an iid monetary policy shock is that it prevents the monetary policy shock from being used as a device to keep interest rates low for a prolonged period and influencing the economy via future expectations. De Groot and Haas (2020) discuss such a signaling channel in a negative interest rate environment.

[15]An alternative approach would be to use an asymmetric cost function with higher downward than upward interest rate rigidity similar to Levieuge and Sahuc (2021).

parameterization strategy with a non-linear least squares approach is described in Section 3. A microfoundation for this function that is based on a depletion of banks' market power in deposit markets as in Brunnermeier and Koby (2018) can be found in Appendix D.[16] The outlined microfoundation also shows that this modeling approach is robust to the Lucas critique so that policy experiments such as variations in the effective lower bound and macroprudential policy can be assessed with this framework.

Government and Resource Constraint The government has a balanced budget constraint. It holds the reserve assets and taxes the households with a lump sum tax:

$$P_t \tau_t + P_t A_t = R_{t-1}^A P_{t-1} A_{t-1} \tag{29}$$

The resource constraint is:

$$Y_t = C_t + I_t + \frac{\rho^r}{2} \left(\frac{\Pi_t}{\Pi} - 1 \right)^2 Y_t \tag{30}$$

2.2 Competitive Equilibrium

The competitive equilibrium is defined as a sequence of quantities $\{C_t, Y_t, K_t, L_t, I_t, D_t, S_t, \Pi_t^P, N_t, N_t^E, N_t^N\}_{t=0}^{\infty}$, prices $\{R_t, R_t^D, R_t^A, R_t^K, Q_t, \Pi_t, \Lambda_{t,t+1}, W_t, P_t^M\}_{t=0}^{\infty}$, bank variables $\{\psi_t, \nu_t, \mu_t, \phi_t\}$, and exogenous variable $\{\eta_t\}_{t=0}^{\infty}$ given the initial conditions $\{K_{-1}, R_{-1}D_{-1}, \eta_{-1}\}$ and a sequence of shocks $\{e_t^{\eta}, \zeta_t\}_{t=0}^{\infty}$ that satisfies the non-linear equilibrium system of this economy provided in Appendix A.

2.3 Global Solution Method

The model is solved in its non-linear specification with global methods. This approach is necessary to capture the state-dependency of the monetary policy pass-through. In particular, this setting allows monetary policy to have a different quantitative as well as qualitative impact depending on the state of the economy. Another advantage of the non-linear approach is that agents take future uncertainty into account, which is particularly

[16] Driscoll and Judson (2013) show that a menu-cost model can feature different stickiness for downward and upward deposit rate changes.

relevant due to the highly non-linear region of low and negative interest rates. The solution method is time iteration with piecewise linear policy functions based on Richter, Throckmorton and Walker (2014). The algorithm description is in Appendix F.

3 Calibration

The model is calibrated to the euro area economy with a particular emphasis on the current low interest rate environment. The considered horizon begins in 2000Q1 and ends in 2019Q4. The data to parametrize the model is mostly based on the ECB's statistical data warehouse and the AWM database, which is built for the ECB's large-scale DSGE model (New Area-Wide Model II).[17] Appendix B contains the details regarding the data sources and construction.

Table 1 summarizes the calibration. The discount factor is set to 0.9975, which corresponds to a risk-free rate of 1% per annum. This is in line with the average estimate of 1.27 for the euro area from Holston, Laubach and Williams (2017).[18] The inflation target is set to 1.9% to match the ECB's inflation target of below, but close to, 2%. The inverse Frisch Labor Elasticity φ equals 1.5 to be in line with the evidence provided in Chetty et al. (2011). The disutility of labor aims that agents work 1/3 of their time. The parameter α is set to 0.33 in line with the capital share of production. The depreciation rate is 0.025 to match an annualized depreciation rate of 10%. The elasticity of the asset price is parameterized to 0.25 as in Bernanke, Gertler and Gilchrist (1999). We target a mark-up of 10% so that $\epsilon = 11$. The Rotemberg parameter $\rho^r = 1000$ implies a 1% slope of the New Keynesian Phillips curve. The inflation and output response are set to 2.5 and 0.125, which are standard values in the literature. The monetary authority does not lower the systemic component of the policy rate below -2% per annum, which gives $\tilde{R}^A = 0.995$ for the endogenous lower bound.

Deposit Rate Pass-Through The pass-through is parameterized using data of bank retail deposit rates and the policy rate for the euro area. We use a weighted measure

[17]The AMW database provides data only until 2017Q4.

[18]Even though our value is slightly lower, this accounts for the trend of falling real interest rates.

Table 1: Calibration

Parameters	Sign	Value	Target
a) Preferences, technology and monetary policy			
Discount factor	β	0.9975	Risk-free rate = 1% p.a.
Risk aversion	σ	1	Risk aversion = 1
Disutility of labor	χ	12.38	SS labor supply = 1/3
Inverse Frisch labor elasticity	φ	1.5	Chetty et al. (2011)
Capital production share	α	0.33	Capital income share = 33%
Capital depreciation rate	δ	0.025	Annual depreciation rate = 10%
Elasticity of asset price	η_i	0.25	Bernanke, Gertler and Gilchrist (1999)
Investment parameter 1	a_i	0.5302	$Q = 1$
Investment parameter 2	b_i	-0.0083	$\Gamma(I/K) = I$
Elasticity of substitution	ϵ	11	Market power of 10%
Rotemberg adjustment costs	ρ^r	1000	1% slope of NK Phillips curve
Inflation	Π	1.0047	Inflation Target = 1.9% p.a.
Inflation response	κ_π	2.5	Standard
Output response	κ_Y	0.125	Standard
Endogenous lower bound	$\tilde{R}^A$	0.995	Lower bound of -2% p.a.
b) Deposit rate pass-through			
Pass-through parameter 1	ω_1	-0.0008	Perfect pass-through at SS
Pass-through parameter 2	ω_2	0.0027	Markdown $R^A = \bar{R}^A = 0.56\%$ p.a.
Pass-through parameter 3	ω_3	124.73	Imperfect pass-through if $R^A < \bar{R}^A$
Banks' Market Power	ς	0.001	Markdown if $R^A > \bar{R}^A = 0.56\%$ p.a.
c) Financial Sector			
Reserve asset requirement	δ^B	0.2545	Government asset share = 23% if $R^A < 1$
Survival probability	θ	0.9	$R_K - R_D = 2\%$ p.a.
Diversion banker	λ	0.1540	Leverage = 8
Proportional transfer to new banks	ω^N	0.00523	Uniquely determined from θ and λ
d) Shocks			
Persistence risk premium shock	ρ^η	0.75	Probability of negative policy rate
Std. dev. risk premium shock	σ^η	0.125%	Standard deviation of detrended output = 0.021
Std. dev. monetary policy shock	σ^ζ	0.0001	Small value to avoid distortion

of different deposit rates to take into account the different maturities in the data. The policy rate is defined as the deposit facility rate. The evolution of both series can be seen in the upper panel of Figure 1. The imperfect deposit rate pass-through in the model is captured in the equation $R_t^D = \omega(R_t^A)$. For this mapping, we follow the functional form in Brunnermeier and Koby (2018). This function separates the connection between the two rates in a region with an imperfect pass-through ($R_t^A < \bar{R}^A$) and a region with a perfect pass-through ($R_t^A \geq \bar{R}^A$), where the threshold parameter $\bar{R}^A$ is the deterministic steady state of the policy rate. The functional form is given as

$$R_t^D = \omega(R_t^A) = \begin{cases} \omega^1 + \omega^2 \exp(\omega^3(R_t^A - 1)) + 1 & \text{if } R_t^A < \bar{R}^A \\ R_t^A - \varsigma & \text{else} \end{cases} \tag{31}$$

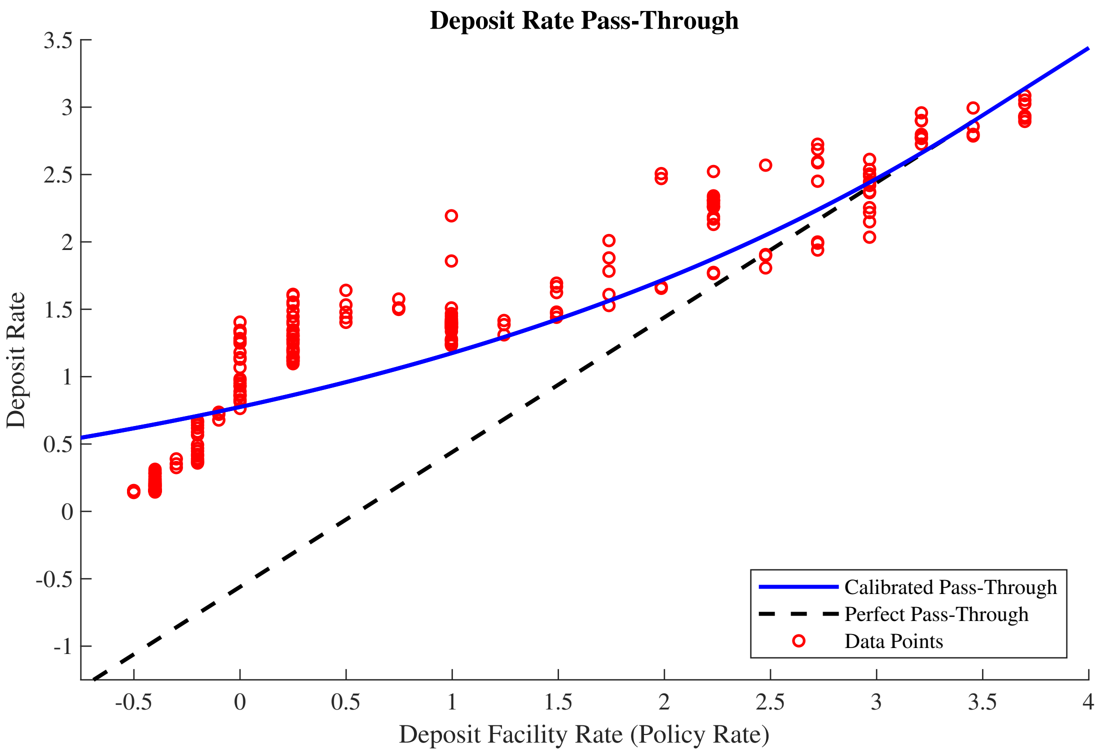

Figure 2: Figure shows the deposit rate pass-through estimated with a non-linear least squares approach. The blue line is the imperfect pass-through, the black dashed line is a scenario with a perfect pass-through and the red dots refer to the data points.

where ω^1, ω^2 and ω^3 determines the shape of the imperfect deposit pass-through and ς is related to banks market power.

We use a non-linear least squares approach to parameterize this functional form to the varying deposit rate pass-through in the euro area economy, as can be seen in Figure 2. Specifically, we calibrate the shape parameters to minimize the distance between the connection of the policy and deposit rate. This approach uses the observations that are below the threshold $\bar{R}^A$. Furthermore, we impose two restrictions on this minimization. First, there is a perfect deposit rate pass-through at the steady state.[19] Second, the markdown at the steady state is 0.56% in annualized terms. For the markdown, we use the measured average spread between the deposit rate and the deposit rate facility conditional on being at or above the steady state. This also gives the markdown for the region with perfect pass-through $\varsigma = 0.0014$. We then fit the curve using a non-linear least square approach that incorporates the described constraint. The fitted values of ω^1, ω^2 and ω^3 are -0.0008, 0.0027 and 124.73. The details of the non-linear least squares

[19] This implies that the derivative of the function at the steady state equals 1.

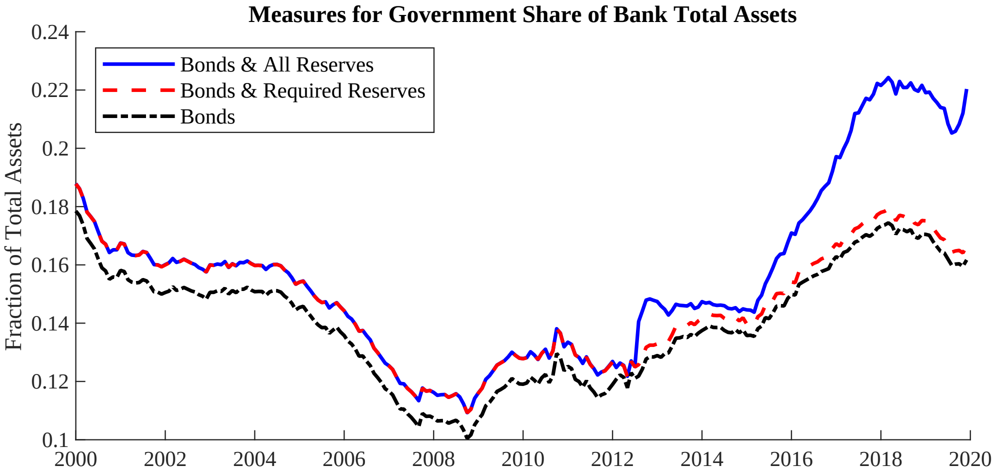

Figure 3: Figure shows different measures of the share of government assets in the bank's balance sheet.

approach are outlined in Appendix B.2.

Banking Sector We calibrate the financial friction parameter λ to match a leverage ratio of 8. The banks have to hold at least a fraction δ^B of their deposits as government assets. Different measures of government asset shares in the banks' balance sheet can be compared in the lower panel of Figure 3. The different shares are government bonds only, government bonds plus required reserve assets, and government bonds plus reserve assets. We match the model to the broadest measure as our requirement captures government bonds as well as reserve assets. According to this measure, since the introduction of negative interest rates in the euro area in 2014, the share of government assets to total banking sector assets has edged up to almost 25%. In line with this, we target that banks have a government asset share of 23% during periods of negative interest rates. The corresponding value for the fraction of deposits is then 0.2545. The banker's survival rate θ is set to 0.9 to obtain an average spread between the return on capital and deposit rate of 2% p.a. at the steady state similar to the New Area-Wide Model II. The average spread between the lending rate and deposit rate is around 2.5% p.a. in the data. However, there is a maturity mismatch in the data as loans have longer maturities on average. Moreover, the survival probability θ and the financial friction parameter λ uniquely determine the endowment to new bankers ω^N.

Shocks The risk premium shock is parameterized to match the fluctuations in output and the frequency of a negative interest rate environment. We set the standard deviation σ^η to 0.125% and the persistence to 0.75. The model predicts a standard deviation of 2.2% for output in line with the data.[20] The policy rate falls below -1% with a 2.7% probability. A negative policy rate occurs with a probability of 5% in the model. A caveat is that the model underestimates the materialization of a negative policy rate compared to the recent experience in the euro, where the policy rate entered negative territory for the first time in June 11 in 2014 and is still below zero in the last quarter of 2019. Substantially increasing the episodes with negative interest rates poses a problem for a model featuring monetary policy ineffectiveness, as shown in Bianchi, Melosi and Rottner (2019) and Fernández-Villaverde et al. (2015) for the zero lower bound. The reason is that overly prolonged episodes in which monetary policy is not effective affect the stability of the model and can result in deflationary spirals.[21] The standard deviation of the monetary policy shock is set to a negligible value. This ensures that this shock does not affect the moments of the model.

4 Reversal Interest Rate and Effective Lower Bound

This section outlines the transmission of shocks in a low or negative interest rate environment and the conditions that give rise to the reversal interest rate. In particular, we demonstrate that the transmission of shocks is asymmetric and state-dependent due to the non-linear features of the model. The model predicts that negative interest rate policies can be effective, even though the effectiveness diminishes until monetary policy approaches the reversal interest rate. At this turning point, further monetary policy accommodation becomes contractionary. This threat of a reversal creates an effective lower

[20]The standard deviation of detrended real GDP is 2.1%. As the model does not have a trend, we detrend the logarithm of real output linearly.

[21]Bianchi, Melosi and Rottner (2019) show that a high frequency of being at the zero lower bound can result in deflationary spirals so that an equilibrium does not exist anymore. The probability of a constrained monetary policy leads to a vicious circle of low inflation and rising real interest rates, which in turn leads to lower inflation. Fernández-Villaverde et al. (2015) show that, for instance, a tax that affects the Euler equation can help to match the duration and frequency of a zero lower bound episode. Dordal-i-Carreras et al. (2016) suggest to use a regime switching process instead of an AR(1) shock to better capture that periods at the zero lower bound are rare but long-lived.

bound on the monetary policy rule, which limits how negative the policy rate can go. We use our framework to locate the effective lower bound.

4.1 Impulse Response Functions and Non-Linearities

We first want to assess the potential non-linearities in the transmission of shocks that results from approaching low or negative interest rate territory.

Risk Premium Shock We begin with an impulse response analysis of the preference (demand) shock, which is shown in Figure 4. To detect asymmetries in the transmission of the shock over the business cycle, we consider expansionary and contractionary shocks with varying magnitudes. The starting point of the economy is the risky steady state, which is the point to which the economy would converge if future shocks are expected and the realizations turn out to be zero (Coeurdacier, Rey and Winant, 2011).

To begin with, the model has the standard financial accelerator which amplifies the impact of financial shocks. An increase in the risk premium, which is a contractionary shock, affects the consumption and saving decisions of the households as well as the refinancing costs of the banks. The households postpone consumption so that output drops. This affects banks as their return on assets is lower and asset prices fall. In addition, the funding costs of the banks increase. Both effects reduce the net worth and weaken the balance sheet of the banks which amplifies the shock via the financial accelerator mechanism. In response, the central bank lowers the interest rate to mitigate the bust. However, the impact of such a policy is non-linear due to the imperfect deposit rate pass-through and the reserve requirement.

The stronger relative impact of a contractionary risk premium shock compared to an expansionary one demonstrates that monetary policy can lose its effectiveness. As can be seen in Figure 4, this asymmetry is visible from the reaction of output, the policy rates, bank net worth and leverage, all of which have a more pronounced response for a risk premium increase. Monetary policy is less effective in stabilizing the economy in a downturn as deposit rates move less than one-to-one due to the imperfect pass-through.

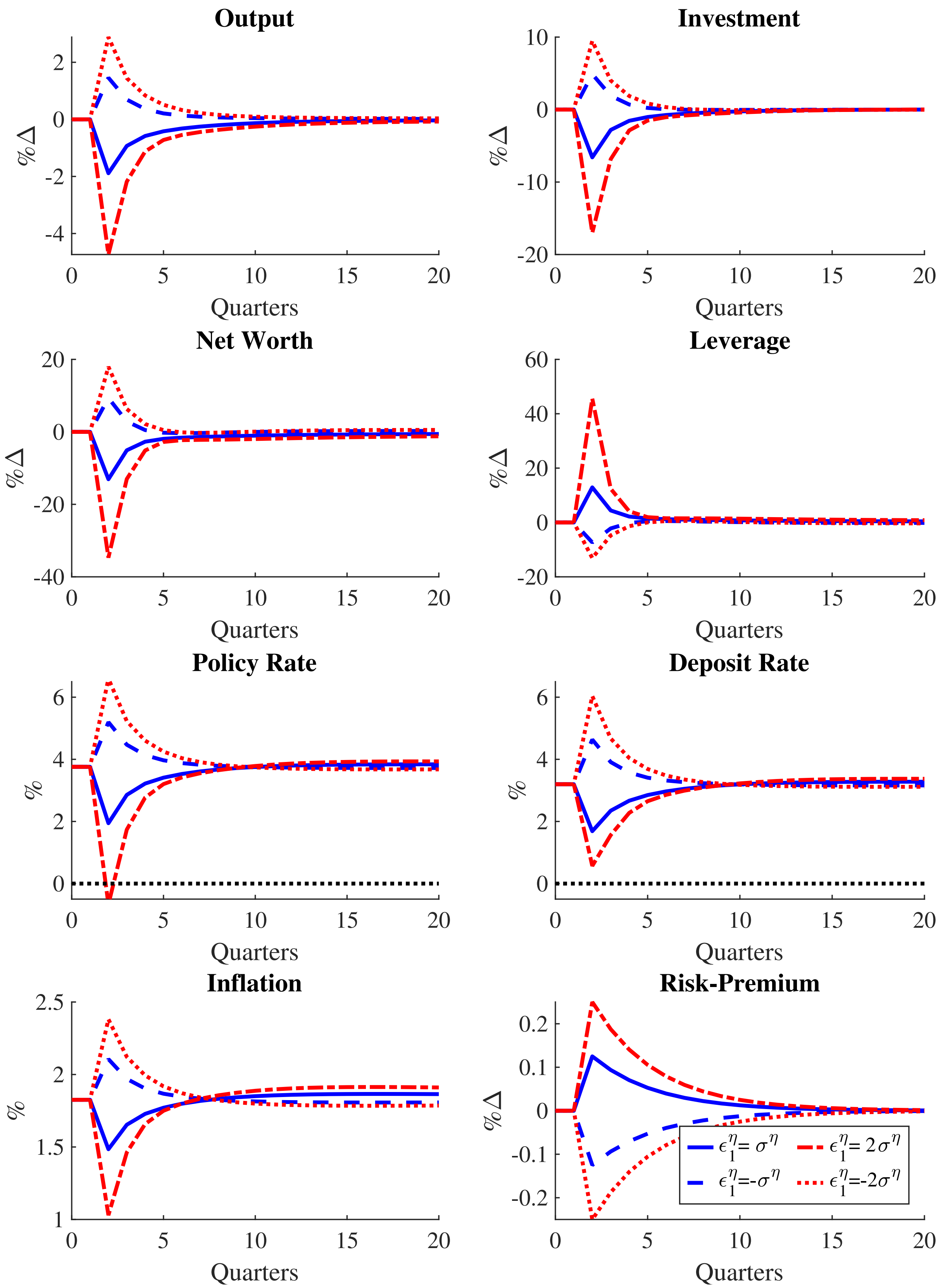

Figure 4: Impulse response functions of the risk premium shock that differ in the size and sign of the innovation. A one standard deviation increase (blue solid line) and decrease (blue dashed line) as well as a two standard deviation increase (red dash-dotted line) and decrease (red dotted line) for the innovation ϵ_t^η is shown. The black dotted line is the zero lower bound. The scales are either percentage deviations from the risky steady state (%Δ) or annualized net rate (%).

This stems from two different channels that operate via the households and banks. First, the deposit interest rates offset less of the increase in the wedge in the household's Euler equation. This results in a stronger drop in consumption. Second, the funding costs of the banking sector do not decrease by much, as the deposit rates are decoupled from the policy rate. At the same time, the spread of the reserve assets also diminishes. This together implies that the banks' net worth losses are comparatively more severe so that there is a strong contraction of lending and output. Importantly, the financial accelerator increases such effects.

Furthermore, another non-linear feature can be discerned from the fact that the size of the contractionary shock matters for how forcefully it is transmitted to the economy. The economy responds considerably more than twice as strongly to a two standard deviation than a one standard deviation shock increase. The reason is that the deposit rate pass-through becomes more sluggish the deeper the recession. This effect is reinforced through the government asset requirement. This shows that monetary policy becomes less and less effective around negative interest rates. In contrast to this, the size of a decrease in the risk premium has less of an effect if the economy is initially at the steady state. There is a perfect pass-through in this part of the state space, meaning that the size of the shock does not matter.

Monetary Policy Shock An exogenous lowering of the monetary policy rate boosts the economy if the economy starts at the risky steady state. Reducing the policy rate affects the deposit rate, which induces households to consume more and reduces the refinancing costs of banks. This leads to a rise in aggregate demand and increases the credit supply. Around the risky steady is almost perfect deposit rate pass-through, so that monetary policy is very effective and the non-linearities are very small. The transmission of varying monetary policy shocks can be seen in Figure 12 in Appendix G. However, we will show now that this result depends on the state of the business cycle.

4.2 Reversal Interest Rate

The previous simulation suggests at first glance that accommodative monetary policy is effective and there is no reversal interest rate. This is due to the fact that the starting point for the simulations is the risky steady state, which implies that the economy is in a region with normal interest rates and close to perfect deposit rate pass-through. However, the impact of monetary policy depends on the interest rate environment. For instance, negative interest rate policies could be much less effective or even contractionary. Therefore, combining the monetary policy shock with simultaneously occurring risk-premium shocks allows us to assess the monetary policy shock at different points of the business cycle.

Figure 5 shows the impulse responses of a negative one standard deviation monetary policy shock at different states of the business cycle. To approximate the business cycle, we use different risk premium innovations ϵ_1^η. A larger risk premium shock contracts the economy more severely. The starting point is still the steady state, but the risk premium shock contracts the economy. The displayed paths show the percentage deviations between a path with and without the monetary policy shock for varying risk premium innovations. Therefore, the combination of the shocks allows to analyse the state-dependent response to a monetary policy shock.

Depending on the size of the contractionary risk premium shock, the monetary policy shock becomes less powerful. The expansionary impact of monetary policy shock decreases with the strength of the risk premium shock as can be seen in the responses of output and net worth. In other words, monetary policy is less effective during a severe recession. In fact, its impact even reverses for a scenario with ϵ_t^η close to $3\sigma_t^\eta$, which approximates a severe recession. In this case, monetary policy, which is intended to be accommodative, actually reduces banks' profitability and output. The reason is that the nominal interest rate is so low when the risk premium shock occurs that monetary policy not only becomes less effective, but is even harmful to the economy. It turns out that an increase in the nominal rate would actually be beneficial in such a state. The reduction in the interest rate hurts the net worth of the banks sufficiently strongly due to their substantial

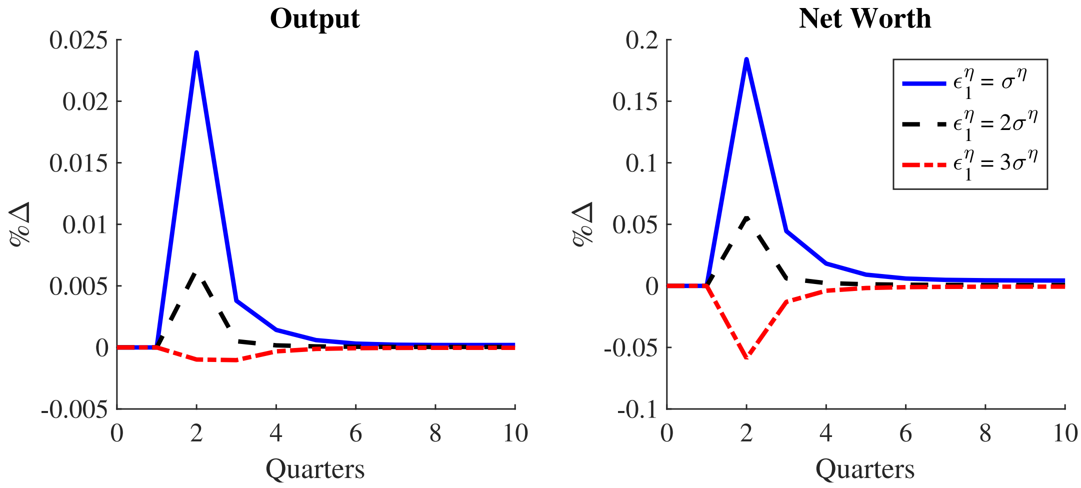

Figure 5: Impulse response analysis to show the state-dependent impact of monetary policy shocks. To generate the state dependency, the monetary policy shock is combined with different sized risk premium shocks. The blue solid line captures the combination with a one std. dev. contractionary risk premium shock, while the black dashed and red dash-dotted ones capture a two and three std. dev. contractionary risk premium shock, respectively. Each line displays the difference between a path with a negative one std. dev. monetary policy shock in period 1 relative to a path without a monetary policy innovation for a given risk premium shock. The deviations are in percent.

government asset holdings. At the same time, the refinancing costs and aggregate demand of households are mostly unaffected as the deposit rate is very sticky in this state of the economy.

For a more detailed analysis about the reversal of monetary policy, we assess the impact of an interest rate over the business cycle. Figure 6 shows the first period impact of an exogenous one standard deviation monetary policy shock for varying risk premium shocks, which are used to proxy the business cycle. If the risk premium shock is negative or around zero, which can be interpreted as an expansion or as tranquil times, monetary policy is very effective. Importantly, the policy rate is high and efficiently passed through.

In contrast to this, monetary policy is considerably less powerful in recessions than in booms, as can be seen by the impact on output. Monetary policy is initially to some extent still effective once the economy approaches negative territory, which is marked as blue shaded area. However, a more deep recession triggers a reversal of the impact of negative interest rate policies. Specifically, the turning point is reached around a risk premium shock of ϵ^η close to 3 standard deviations. From this point onwards, a policy rate cut triggers a fall in output and inflation. This is explained by the sluggish deposit rate pass through and the strong drop in bank net worth in this state of the economy.

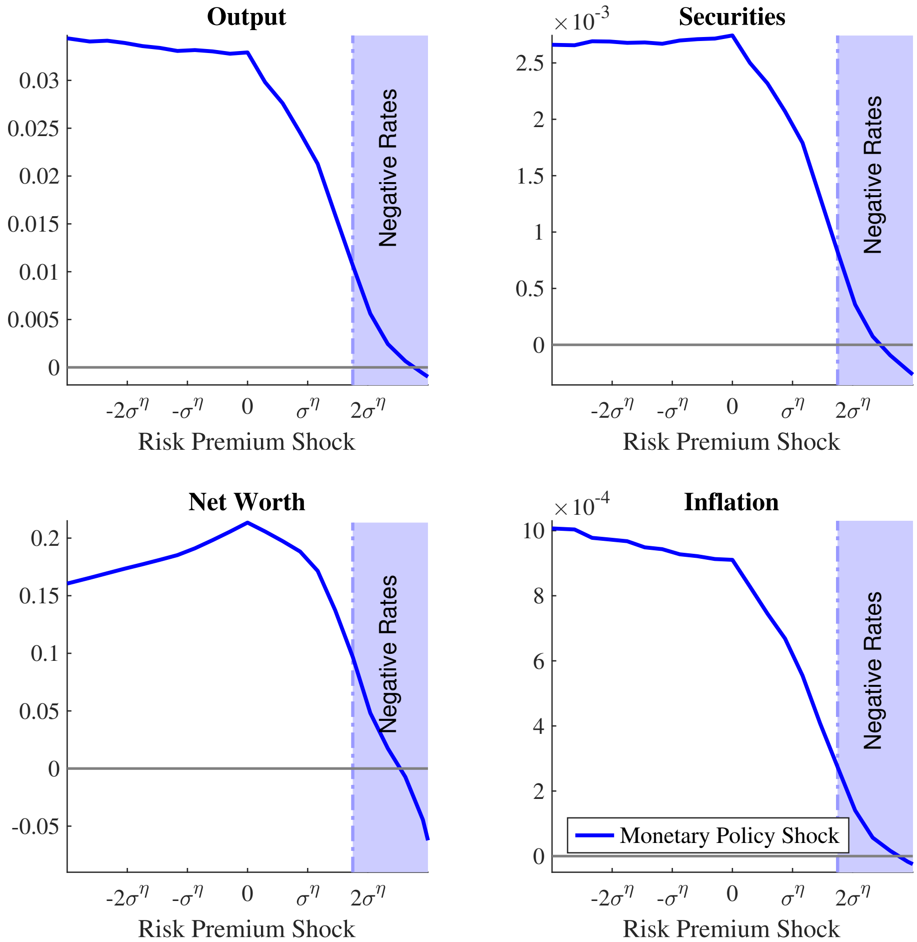

Figure 6: First period of an impulse response function to illustrate the state-dependent impact of monetary policy shocks. To generate the state dependency, the monetary policy shock is combined with different sized risk premium shocks, which are displayed on the horizontal axis. The vertical axis displays the state-dependent difference for the period $t = 1$ impulse response between a shocked path, which faces additionally a negative one std. dev. monetary policy shock, and a path, in which the monetary policy innovation does not occur. The deviations are in percent. The blue shaded area indicates the territory, where the risk premium shock pushes the economy in negative interest rate territory.

In summary, the effectiveness of monetary policy is diminishing in a low interest rate territory. For sufficient negative interest rates, the impact reverses and a policy rate cut is then contractionary.

Deposit Rate Pass-Through and Government Asset Holdings The deposit rate pass-through and the banking sector's government asset holdings are the key factors that generate state-dependent monetary policy and the reversal rate in our framework. To analyze their impact, the frictions are relaxed one at a time.

First, a model featuring perfect deposit rate pass-through is considered. Accordingly, the deposit rate equals the policy rate adjusted for the mark down:

$$R_t^D = R_t^A - \varsigma \tag{32}$$

As a consequence, the pass-through is not state-dependent. Consequently, monetary policy transmission is equally effective in an expansion as well as in a recession. The central bank can also stimulate demand and lower the refinancing costs for the banking sector during a downturn. Simultaneously, the negative effects via the government bonds are shut down as the government spread is fixed, that is $R_t^A - R_t^D = \varsigma$. There are almost no state-dependencies any more and the monetary policy shock has almost the same impact over the same cycle. Consequently, monetary policy is always effective and this specification does not feature a reversal interest rate. This highlights the importance of the imperfect deposit rate pass-through of monetary policy. This property is also illustrated in Figure 13 in Appendix G.

The second experiment is to alter the amount of reserve assets, while keeping an imperfect deposit rate pass-through. In particular, we consider a calibration in which the banks only hold half the share of government assets than assumed in the benchmark model calibration. Monetary policy is still state dependent and less powerful in recessions due to the imperfect deposit rate pass-through. However, a reversal rate does not materialize in this setting because monetary policy does not result in net worth losses of bankers. While monetary policy becomes less effective for low interest rates, it does not become contractionary. In fact, monetary policy stabilizes the banking sector even in a severe recession. Figure 13 in Appendix G shows this experiment as well.

4.3 Effective Lower Bound on Monetary Policy

The model can generate a reversal interest rate, in which an exogenous lowering of the interest rate contracts the economy. Importantly, the same mechanism holds for the lower bound of monetary policy. A very loose lower bound can have adverse effects. The endogenous lower bound R^A can avoid such adverse effects. At the same time, setting too conservative a bound would restrict monetary policy unnecessarily.

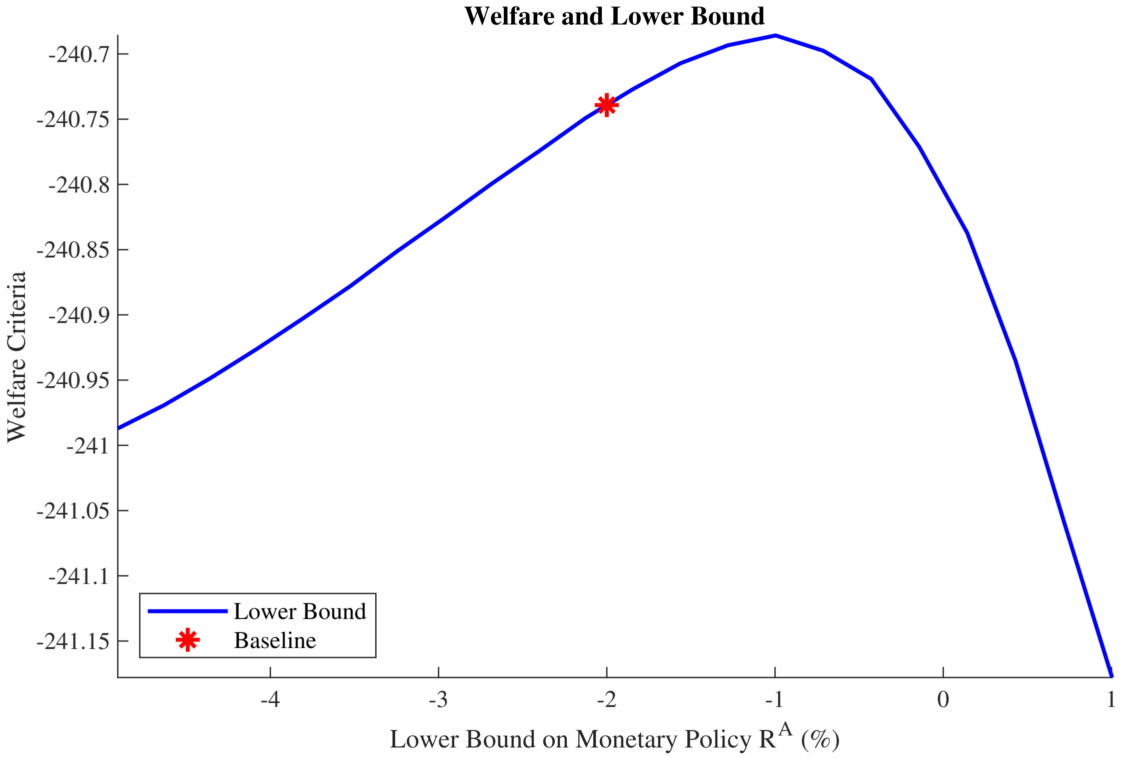

Figure 7: Welfare for different lower bounds of the policy rule R^A (measured as annualized net rate). The x-axis shows the interest rate in percent per annum. The star marks the baseline of a lower bound at -2%.

We evaluate the effective lower bound in our model using the welfare of the households, which is given by:

$$W_0 = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \right] \tag{33}$$

In addition to this, we consider the impact of the lower bound on selected moments.

Figure 7 shows the shape of welfare depending on the variation in the lower bound. The effective lower bound on monetary policy is around -1% per annum. At this rate, the trade-off between lowering the interest rate with diminishing deposit rate pass-through and lowering banks' income on their government asset holdings is optimally balanced. This is the endogenously determined reversal interest rate in our model. It should be noted that an overly restrictive lower bound such as keeping the policy rate at positive levels lowers welfare as the central bank forgoes potentially beneficial monetary accommodation. This highlights the problem with monetary policy accommodation when approaching reversal interest rate territory. Monetary policy needs to balance inflation stabilization with the stability of the banking sector.

We can compare the impact of the lower bound on the moments of the model. Table 2 shows the different selected moments for a very negative lower bound at -5% , the baseline case with -2% and a rather large and positive lower bound at 1% using a simulation of 200000 periods (after a burn-in period). The differences between a very negative lower bound and the baseline case are rather small. In particular, we can see that output and leverage is slightly larger in the economy with a lower bound at -2%. The banking sector is allowed to be more highly leveraged as the banks do not face potential losses through the reversal interest rate. The strongest difference is in the behavior of inflation, where a very low lower bound leads to increased inflation. In addition to this, leverage is much more volatile for a lower bound with $R_t^A = -5\%$. Nevertheless, the differences are rather small because interest rates are rarely so negative. If the economy were to experience more often such a severe recession that can trigger very low rates, the differences in the moments would be stronger. At the same time, we see a stronger response of the moments if the lower bound is set very tight. A lower bound of 1% results in considerably lower average output. We also see much more deflation as the central bank does not respond to deflationary pressure sufficiently. In addition to this, the economy is also much more volatile as monetary policy intervenes less.

The observation that the differences are larger for a high lower bound compared to a very low stems from the fact that the economy only infrequently encounters very low interest rates where the reversal rate affects the economy. Therefore, an overly restricted monetary policy does not stabilize the economy for macroeconomic outcomes that occur frequently, while the occurrence of the reversal interest rate hurts the economy. However, this is more of a tail event. This suggests that setting the effective lower bound involves a trade-off between financial stability and inflation stabilization for low interest rates.

5 Macroprudential Policy

Macroprudential policy is an important tool that can help to restore the efficiency of monetary policy in a "lower for longer" interest rate environment. The reason is that the capitalization of the banking sector plays a decisive role in the transmission of mon-

Table 2: Selected Moments for Varying Monetary Policy Lower Bound R^A

Moment	Model I: $R^A = -5$	Model II: $R^A = -2$	Model III: $R^A = 1$
a) Mean			
$\overline{Y}$	1.0040	1.0042	1.0015
$\overline{N}$	1.1477	1.1465	1.1517
$\overline{\phi}$	8.1943	8.2076	8.2282
$\overline{\pi}$	2.0157	1.9835	1.9727
b) Standard Deviation			
$\sigma(Y)$	0.0219	0.0223	0.0246
$\sigma(N)$	0.1675	0.1712	0.1907
$\sigma(\phi)$	5.1945	4.2370	6.2787
$\sigma(\pi)$	0.4057	0.4152	0.4564

etary policy. This gives rise to a new motive for macroprudential policy because it can strengthen the bank lending channel of monetary policy.

The macroprudential regulator can impose restrictions on the bank capital ratio, which is defined as the inverse of leverage $1/\phi$. In particular, the regulator can require the banks to build up additional capital buffers and release them subsequently. This policy instrument is based on the countercyclical capital buffer (CCyB) that was introduced as part of the Basel III requirements. The CCyB is built up during an expansion and can then be subsequently released, even though it can never fall below 0%, during a downturn.

We incorporate this asymmetry using an occasionally binding macroprudential rule. The policy cannot reduce the capital requirements below the market-based capital demands. Although the regulator could theoretically set capital ratios below the market ones, the market-based constraint would be the binding constraint for the banks. In that regard, the market enforces a lower bound on regulatory capital requirements. This restriction diminishes the welfare gains of macroprudential policy as the scope of policy interventions during a downturn is limited to the previously created buffers.[22] This emphasizes the importance of building up buffers in good times in order to create sufficient macroprudential space that can be employed to relax capital requirements in bad times and, thus ensuring macroprudential policy efficiency.

[22] The usual approach in the DSGE literature is based on unrestricted rules without a lower bound in assessing countercyclical capital requirements. An exception is, for instance, Van der Ghote (2018), where the market-based leverage constraint restricts optimal macroprudential regulation.

5.1 Macroprudential Policy Rule

The macroprudential regulator can set a time-varying capital buffer τ_t that imposes additional capital requirements. We use the following rule:

$$\tau_t = \max\left\{(\phi^{MPP} - \phi_t^M)\tau^{MPP}, 0\right\} \tag{34}$$

where τ^{MPP} is the responsiveness and ϕ^{MPP} is the anchor value of the buffer. The rule responds to deviations of the market-based leverage ϕ_t^M from the anchor value ϕ^{MPP}. The asymmetry of the buffer depends directly on ϕ^{MPP}.[23] The max operator ensures that the buffer can only have non-negative values, which creates an asymmetry in the buffer in line with the Basel III requirements.

The market-based capital constraint stems from the agency problem of the banker (see equation (10)) and is repeated for convenience:

$$\phi_t^M = \frac{\nu_t + \frac{\delta^B}{1-\delta^B}}{\frac{\lambda}{1-\delta^B} - \mu_t} \tag{35}$$

This implicitly ensures that the buffer is countercyclical in our model if $\tau^{MPP} > 0$ since market-based bank leverage is countercyclical in the model. As the buffer is additional to the market-based equity requirements, the banks' capital ratio reads as follows

$$\frac{1}{\phi_t} = \frac{1}{\phi_t^M} + \tau_t \tag{36}$$

Due to the non-negativity restrictions of the buffer, the policy instrument only occasionally affects leverage. If the buffer is at zero, leverage is determined directly from ϕ_t^M. Therefore, the regulatory capital buffer affects asymmetrically the capitalization of the banking sector because it imposes additional capital requirements during periods of banks' balance sheet expansion.

Importantly, the capital buffer affects the transmission of shocks and dampens economic downturns. As the buffer is released after contractionary shocks, banks can better

[23] We consider different potential anchor values. The alternative approach would be to impose the non-negativity at a pre-imposed point such as the steady state. However, this would unnecessarily restrict how macroprudential policy space is built up and released.

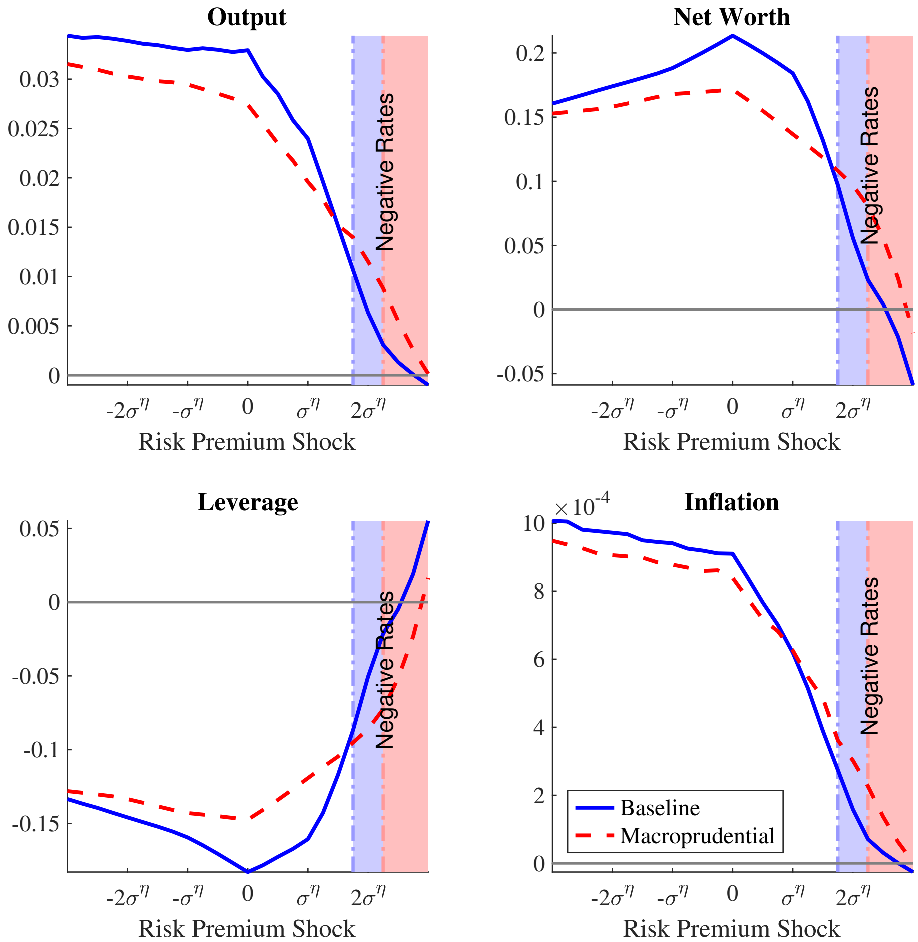

Figure 8: Comparison of state dependency of monetary policy shocks for economy without (baseline) and with macroprudential policy, where first period of an impulse response function of monetary policy shocks is shown. To generate the state dependency, the monetary policy shock is combined with different sized risk premium shocks, which are displayed on the horizontal axis. The vertical axis displays the state-dependent difference for the period $t = 1$ impulse response between a shocked path, which faces additionally a negative one std. dev. monetary policy shock, and a path without a monetary policy shock. The deviations are in percent. The blue and red shaded area indicates the territory, where the risk premium shock pushes the economy without and with macroprudential policy in negative interest rate territory.

absorb their losses. This stabilizes the economy and reduces economic losses during a downturn. Furthermore, the central bank needs to respond less strongly, which gives additional space before approaching low interest rate territory as well as the reversal rate. This highlights that macroprudential policy has the potential to impact the probability of encountering negative interest rates and the reversal interest rate. It thereby also reduces the asymmetric response to expansionary and contractionary shocks. In Appendix G, these properties are illustrated in Figure 14 with an impulse response functions analysis .

5.2 Macroprudential Policy and Reversal Interest Rate

We have shown and highlighted the importance of the reversal interest rate for economic outcomes. As the impact of monetary policy on banking sector leverage is key for the possibility of entering reversal rate territory, a better-capitalized banking sector can compensate for losses and reduce the asymmetry of monetary policy shocks. To illustrate the beneficial role of macroprudential policy, we compare the impact of the capital buffer rule on the reversal interest rate.

Figure 8 shows the initial impact of a negative one standard deviation monetary policy shock over the business cycle, where varying risk premium shocks approximate the state of the economy. We compare the welfare-optimizing macroprudential policy, which is derived in the next subsection, to the benchmark economy without a buffer. The shaded areas indicate from which point onwards the respective economy enters negative interest rate territory. This clearly illustrates that macroprudential policy reduces the probability of encountering negative interest rates and the reversal rate, as can be seen in the difference of the shaded areas and the location of the reversal of output. As the buffer dampens contractionary shocks, the economy encounters less severe recessions and fewer interest rate reductions. Therefore, monetary policy retains more of its efficiency for large ϵ_t^η and is less likely to enter the region, where a monetary policy rate cut would deteriorate output and credit supply.[24] In addition to this, macroprudential policy also helps to stabilize inflation.

While the countercyclical capital buffer rule helps to restore the monetary policy transmission mechanism in case of large contractionary shocks, it also affects it in normal times. As the banking sector is better capitalized, monetary policy is less powerful during an expansion. For instance, the increase in output or net worth is smaller in an economy with an active macroprudential policy.

[24] In addition to this channel, macroprudential policy could have an additional positive impact on systemic risk in a low interest rate environment (Van der Ghote, 2020).

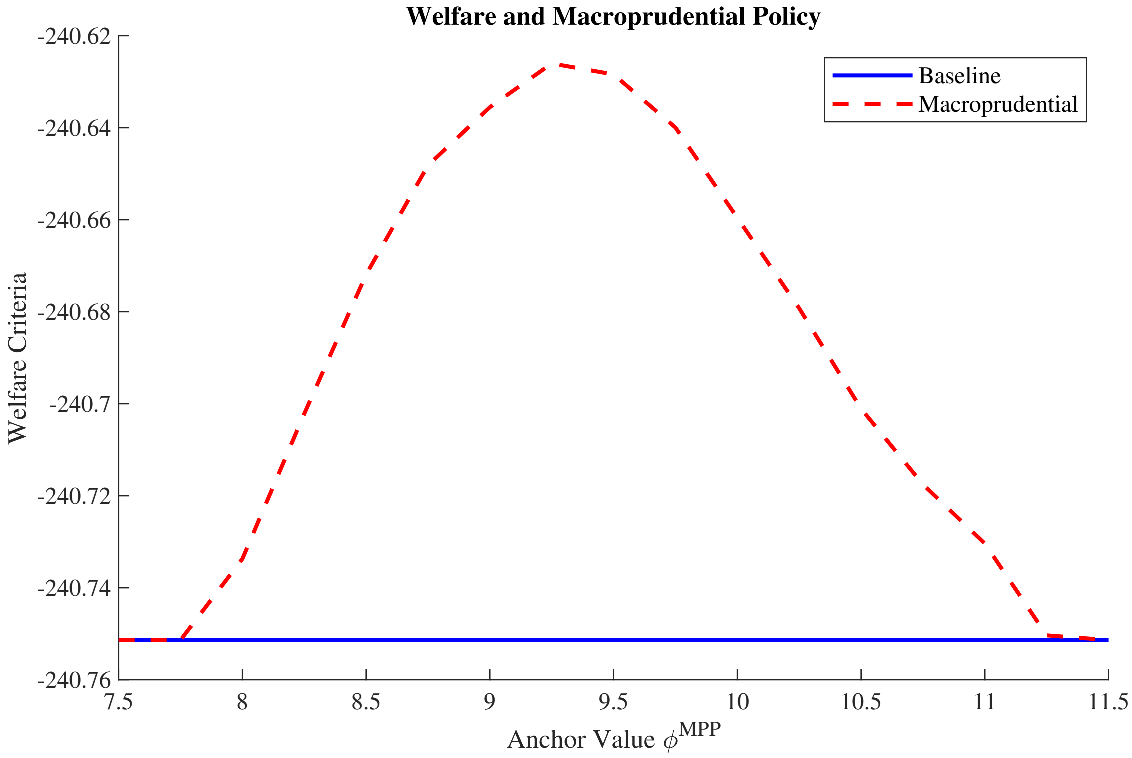

Figure 9: Welfare for different anchor values ϕ^{MPP}, which are varied on the horizontal axis. The response to deviations τ^{MPP} is set optimally to maximize welfare for each value of ϕ^{MPP}.

5.3 Optimal Macroprudential Policy

Active macroprudential policy can reduce the threat of the reversal interest rate. This notwithstanding, an excessively large buffer could also depress the economy. In other words, the macroprudential regulator could face a trade-off between building up macroprudential space to support monetary policy and the potential costs of creating this space in good times. We evaluate this trade-off using the same welfare criteria, as specified in equation (33). To determine the optimal macroprudential policy rule, we vary the anchor value ϕ^M and calculate for each anchor value the corresponding responsiveness τ^{MPP}. Appendix E contains more details about the interactions between the parameters ϕ^{MPP} and τ^{MPP}.

The outlined trade-off can be seen by the hump-shaped welfare curve in Figure 9. A large buffer helps the banking system to absorb losses and reduces the threat of the reversal rate during a severe downturn. At the same time, the build-up of the buffer is costly, which limits the optimal macroprudential space. It should be noted that the positive impact of this rule is due to the introduction of the imperfect deposit rate pass-through and reserve and liquidity requirement. For instance, in an economy with a perfect pass-through, the proposed macroprudential policy rules would result in a welfare loss. In fact, it would be optimal to not have the capital rule (or to set $\tau^{MPP} = 0$) as the costs of building up the buffers outweigh the benefits in such an economy without a reversal rate.

The optimal macroprudential policy rule that strikes the balance between building up sufficient, but not excessive, macroprudential space has the anchor value $\phi^M = 9.25$ with $\tau^{MPP} = 0.019\%$. This policy reduces the risk of large output contraction and lowers the standard deviation of output by 11%. The likelihood to encounter negative interest rates and the reversal rate fall by around 22% and 23%, respectively.

5.4 Macroprudential Policy and Effective Lower Bound

The previous analysis has highlighted that macroprudential and monetary policy are strategic complements. Therefore, it is important to understand the interaction between macroprudential policy and the effective lower bound on monetary policy. To address this question, we compare the different lower bounds for an economy without and with macroprudential policy, which is displayed in Figure 10. For each lower bound, we choose the optimal macroprudential policy to calculate welfare.[25]

First of all, macroprudential increases welfare independent for all considered lower bounds because it helps to prevent the economy from approaching reversal rate territory. As it stabilizes the banking sector, the recession and the threat of ultra low interest rates is less severe. Via this channel, the welfare-optimizing capital rules improve welfare regardless of the specific lower bound. This again highlights how the reversal interest rate creates this novel motive for macroprudential policy.

We can also see that the capital buffer does not directly affect the choice of the effective lower bound. The reason is that the macroprudential policy space is already released once the policy rate is lowered to such a negative territory. If the macroprudential policy space were also to affect the capital holdings in a negative region of -1%, the lower bound would

[25]This implies that we maximize ϕ^{MPP} and τ^{MPP} for each value of τ^{MPP}.

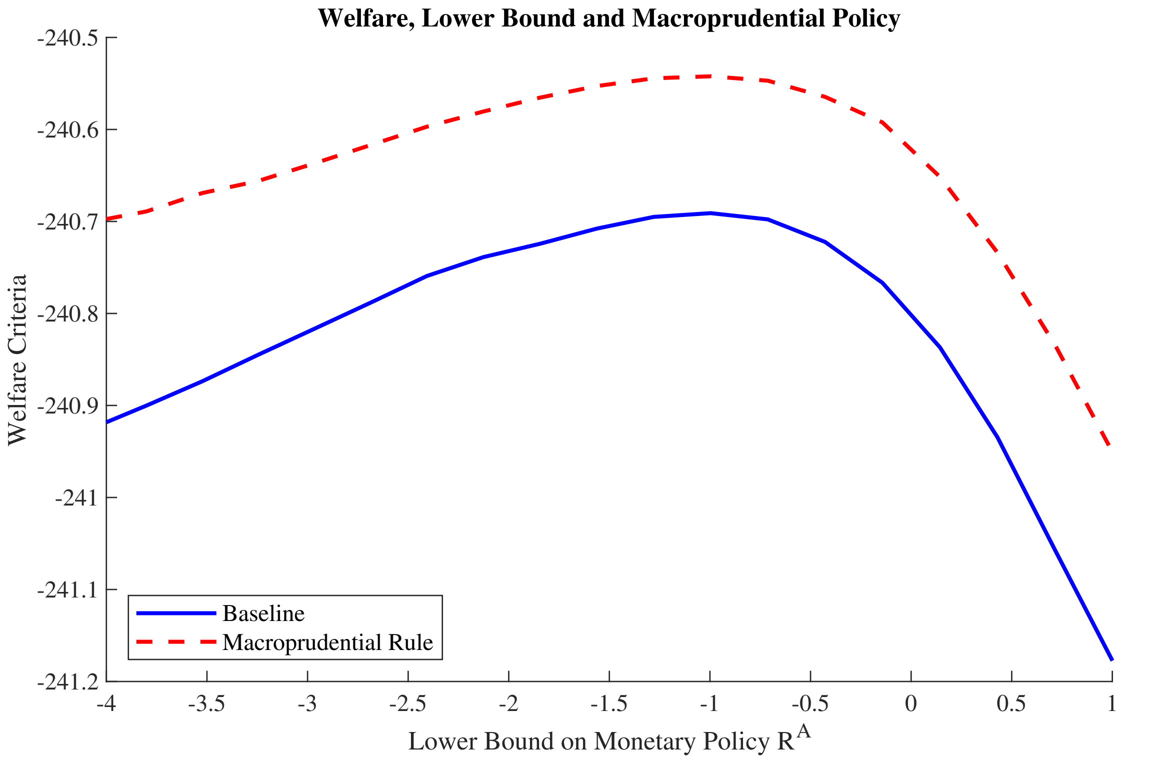

Figure 10: Welfare with and without macroprudential policy for different lower bounds on monetary policy (measured as annualized net rate). The macroprudential policy rule parameters ϕ^{MPP} and τ^{MPP} are optimized separately for each lower bound.

adjust. This could be the case if the central bank were to require very large buffer holdings or increase the general level of capital requirements. However, as discussed before, this would be not optimal as the costs in normal times would outweigh the benefits.

In addition to the increase in welfare, the macroprudential policy results in a flatter curve. The capital buffer rule smoothes the fluctuations and the economy is less often in such a low interest rate area. A suboptimal lower bound, which is either unnecessary high or low, has less of an impact. In other words, macroprudential policy mitigates the danger of either too loose or too restrictive monetary policy. This connection further contributes to the strategic complementarity between macroprudential and monetary policy in a low interest rate environment.

6 Concluding Remarks

In this paper, we investigate monetary policy effectiveness and macroprudential policy in a negative interest rate environment. Using a novel macroeconomic model fitted to the euro area, we illustrate how a "lower for longer" interest rate environment gives rise to important asymmetries and non-linearities. Our model predicts the possibility of a reversal rate where a lowering of the policy rate may give rise to an unintended contraction of output. To avoid a reversal of monetary policy, the framework suggests that the effective lower bound on monetary policy is located at -1% per annum.

We document a new motive for macroprudential policy that exploits the link between banks' capitalization, the effectiveness of negative interest rate policies and the reversal rate. The build-up of macroprudential space in good times supports the bank lending channel of monetary policy and reduces the risk of approaching reversal interest rate territory. Importantly, such a macroprudential policy also increases the effectiveness of negative interest rate policies and mitigates the costs of an either too loose or too restrictive monetary policy in a low interest rate environment. This also implies that macroprudential policy could impact the optimal inflation target since the optimal level depends on the effectiveness of monetary policy as shown in Coibion, Gorodnichenko and Wieland (2012).

The analysis has at least two important policy implications. First, macroprudential policy using a countercyclical capital buffer approach has the potential to alleviate and mitigate the risks of entering a reversal rate territory. Second, there are important strategic complementarities between monetary policy and a countercyclical capital-based macroprudential policy in the sense that the latter can help facilitate the effectiveness of monetary policy, in periods of low, or even negative, interest rates. Overall, the findings in this paper provide important insights into the relevance of financial stability considerations in monetary policy strategy discussions.

The developed non-linear framework, which captures asymmetric monetary policy transmission and a contains an endogenous reversal interest rate, could be extended to evaluate other central banks' tools. In particular, the impact of quantitative easing on the reversal interest rate and the effectiveness of negative interest rate policies seems to be an important question in the current interest rate environment.

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A Non-Linear Equilibrium Equations

Households

$$C_t = W_t L_t + D_{t-1}\frac{R^D_{t-1}}{\Pi_t}\eta_{t-1} - D_t + \Pi^P_t - \tau_t$$

$$\beta R^D_t \eta_t E_t \frac{\Lambda_{t,t+1}}{\Pi_{t+1}} = 1$$

$$\chi L^{\varphi}_t = C^{-\sigma}_t W_t$$

Banks

$$\mu_t\phi_t + \nu_t \geq \lambda\Big(\frac{1}{1-\delta^B}\phi_t - \frac{\delta^B}{1-\delta^B}\Big)$$

$$\psi_t = \mu_t\phi_t + \nu_t$$

$$\mu_t = \beta E_t \Lambda_{t,t+1}\left(1-\theta+\theta\psi_t\right)\frac{R^K_{t+1} - R_t}{\Pi_{t+1}}$$

$$\nu_t = \beta E_t \Lambda_{t,t+1}\left(1-\theta+\theta\psi_t\right)\frac{R_t}{\Pi_{t+1}}$$

$$Q_t S_t = \phi_t N_t$$

$$R_t = (\eta_t R^D_t)\frac{1}{1-\delta^B} - R^A_t\frac{\delta^B}{1-\delta^B}$$

$$N_t = N^S_t + N^N_t$$

$$N^S_t = \theta N_{t-1}\frac{R^K_t - R_{t-1}\phi_{t-1} + R_{t-1}}{\Pi_t}$$

$$N^N_t = \omega^N \frac{S_{t-1}}{\Pi_t}$$

Production, Investment and New Keynesian Phillips Curve

$$Y_t = A^P K^{\alpha}_{t-1} L^{1-\alpha}_t$$

$$W_t = P^m_t(1-\alpha)Y_t/L_t$$

$$R^k_t = \frac{(P^m_t \alpha Y_t/K_{t-1} + (1-\delta)Q_t)}{Q_{t-1}}\Pi_t$$

$$Q_t = \frac{1}{(1-\eta_i)a_i}\Big(\frac{I_t}{K_{t-1}}\Big)^{\eta_i}$$

$$K_t = (1-\delta)K_{t-1} + (a_i(I_t/K_{t-1})^{(1-\eta_i)} + b_i)K_{t-1}$$

$$\left(\frac{\Pi_t}{\Pi}-1\right)\frac{\Pi_t}{\Pi}=\frac{\epsilon}{\rho^r}\left(P_t^m-\frac{\epsilon-1}{\epsilon}\right)+\beta E_t\Lambda_{t,t+1}\frac{Y_{t+1}}{Y_t}\left(\frac{Pi_{t+1}}{\Pi_t}-1\right)\frac{\Pi_{t+1}}{\Pi}$$

Policy Rule, Interest Rates, Government Budget Constraint and Aggregate Resource Constraint

$$R_t^A=\max\left[R^A\left(\frac{\Pi_t}{\Pi}\right)^{\theta_\Pi}\left(\frac{Y_t}{Y}\right)^{\theta_Y},\tilde{R}^A\right]\zeta_t$$

$$R_t^D=R_t^A-\omega(R_t^A)$$

$$R_t^D=\mathbf{1}_{R_t^A\geq R^{A\,SS}}\left[R_t^A-\varsigma\right]+(1-\mathbf{1}_{R_t^A\geq R^{A\,SS}})\left[\omega^1+\omega^2\exp(\omega^3(R_t^A-1))+1\right]$$

$$\tau_t+A_t=\frac{R_{t-1}^A}{\Pi_t}A_{t-1}$$

$$Y_t=C_t+I_t+\frac{\rho^r}{2}\left(\frac{\Pi_t}{\Pi}-1\right)^2Y_t$$

A.1 Occasionally Binding Regulatory Constraint

The non-negative capital buffer is

$$\tau_t=\min\left\{\tau^{MPP}(\phi^{MPP}-\phi_t^M),0\right\} \tag{37}$$

The market imposed leverage constraint is given from the run-away constraint

$$\phi_t^M=\frac{\nu_t+\frac{\delta^B}{1-\delta^B}}{\frac{\lambda}{1-\delta^B}-\mu_t}$$

Banks leverage is then given as

$$\phi_t=\left(\frac{1}{\phi_t^M}+\tau_t\right)^{-1} \tag{38}$$

B Data and Calibration

B.1 Data Sources and Construction

This section describes the data sources and construction. Table 3 shows all used series and their source. We use euro area data from 2002Q1 until 2019Q4.[26]

[26] The data from the euro area have a changing composition.

Deposit Rate The deposit rate weights the different lending rates for varying maturities, where the rates are from ECB SDW MIR data and the volume is based on the ECB SDW - BSI data. The used rates are the overnight deposit rate, deposit rate up to 1 year for new business, deposit rate over 1 and up to 2 years for new business and the deposit rate over 2 years for new business. Their contribution is weighted with their relative outstanding amount in the balance sheet. All different rates and outstanding amounts are for deposits from households. The constructed deposit rate R_t^D reads then as follows:

$$R_t^D = \frac{DS0_t \times RD0_t + DS1_t \times RD1_t + DS2_t \times RD2_t + DS3_t \times RD3_t}{DS0_t + DS1_t + DS2_t + DS3_t} \tag{39}$$

Lending Rate The lending rate uses data from the ECB SDW - MIR data and the volume to weight is based on BSI data. For the lending rate, we use up to 1 year, over 1 year and below 5 years, and over 5 years to non-financial corporates and outstanding amounts. The volume data has the same maturity and is the outstanding amount to all non-financial corporations. The constructed lending rate R_t^K is the weighted index of the different rates:

$$R_t^K = \frac{LR1_t \times LS1_t + LR2_t \times LS2_t + LR3_t \times LS3_t}{LS1_t + LS2_t + LS3_t} \tag{40}$$

Policy Rate The main policy rate is the ECB's deposit facility rate. Euribor 3-month and the Eonia rate are the typical alternatives in the New Keynesian literature for the euro area.

Government Assets The share of government assets uses data from the ECB SDW - BSI data. We use loans to euro area government held by monetary financial institutions (MFIs), euro area government debt securities held by MFIs, required reserves held by credit institutions and excess reserves held by credit institutions.[27] This is compared to the total assets held by the MFIs. The consolidated balance sheet of the euro area MFIs

[27]There are two important regulatory changes for the reserve requirement. Initially, the reserve requirement was 2% of the deposit base, which was lowered to 1% from 18 January 2012. Furthermore, a two-tier system takes effect rom 30 October 2019. This system exempts credit institutions from remunerating part of their excessive holdings.

is used for each series. The different measures include to a different extent the reserves:

$$\frac{A_t^1}{S_t + A_t^1} = \frac{LG + LS}{TA} \tag{41}$$

$$\frac{A_t^2}{S_t + A_t^2} = \frac{LG + LS + RR}{TA} \tag{42}$$

$$\frac{A_t^3}{S_t + A_t^3} = \frac{LG + LS + RR + ER}{TA} \tag{43}$$

The different series can be seen in the lower panel of Figure 3 in the main text.

Bank Level Deposit Rates The deposit rates for different banks are based on the ECB IMIR data.

Government Bond Yield The government bond yield is shown for the German one year bond, with the data is being extracted from Datastream.

B.2 Non-Linear Least Squares

The model function that relates the deposit rate data dd_i and the policy rate data pd_i (conditional on being below the threshold) is given as

$$dd_i = (\omega_1 + \omega_2 \exp(\omega_3 pd_i)$$

We impose two restrictions, which allow us to express ω_1 and ω_2 in terms of ω_3. First, the markdown at the threshold value corresponds to ς. Second, the pass-through at the threshold value is 1, which implies perfect pass-through. Thus, the shape parameters ω_1 and ω_2 can be written as:

$$\omega_1 = i^{SS} - \varsigma - \frac{1}{\omega_3}$$

$$\omega_2 = \frac{1}{\omega_3 \exp(\omega_3 i^{SS})}$$

where i^{SS} is the threshold parameter.

The non-linear least squares now finds the parameter ω_3 that minimizes the squared

Table 3: Data Sources

Data	Name	Source
a) Deposit Rate		
Overnight Deposit Rate, Households (HH)	RD0	ECB SDW - MIR
Deposit rate, maturity up to 1 year, HH, New Business	RD1	ECB SDW - MIR
Deposit rate, maturity over 1 and up to 2 years, HH, New Business	RD2	ECB SDW - MIR
Deposit rate, maturity over 2 years, HH, New Business	RD3	ECB SDW - MIR
Overnight deposits, Total, HH	DS0	ECB SDW - BSI
Deposits, maturity up to 1 year, HH, Outstanding	DS1	ECB SDW - BSI
Deposits, maturity over 1 and up to 2 years, HH,Outstanding	DS2	ECB SDW - BSI
Deposits, maturity over 2 years, HH, Outstanding	DS2	ECB SDW - BSI
b) Lending Rate		
Lending rate, maturity up to 1 year, NF-Corp., Outstanding (Out)	LR1	ECB SDW - MIR
Lending rate, maturity over 1 and up to 5 years, NF-Corp., (Out)	LR2	ECB SDW - MIR
Lending rate, maturity over 5 years, NF-Corp., Outstanding	LR3	ECB SDW - MIR
Loans, maturity up to 1 year, NF-Corp., Outstanding	LS1	ECB SDW - BSI
Loans, maturity over 1 and up to 5 years, NF-Corp., Outstanding	LS2	ECB SDW - BSI
Loans, maturity over 5 years, NF-Corp., Outstanding	LS3	ECB SDW - BSI
c) Policy Rate		
ECB Deposit facility rate	PR1	ECB SDW - FM
Euribor 3-month	PR2	ECB SDW - FM
Eonia rate	PR3	ECB SDW - FM
d) Government Asset		
Loans to government, MFI, Stock	LG	ECB SDW - BSI
Government debt securities, MFI, Stock	LS	ECB SDW - BSI
Reserve Maintenance Required Reserves, Credit Inst.	RR	ECB SDW - BSI
Reserve Maintenance Excess Reserves, Credit Inst.	ER	ECB SDW - BSI
Total Assets, MFI	TA	ECB SDW - BSI
e) Bank Level Data		
Overnight Deposit Rate, Households	RDi	ECB SWD - IMIR
f) Government bond yield		
German government 1 year bond yield	G1Y	Datastream

residuals r_i from the model function:

$$r_i = dd_i - \left(i^{SS} - \varsigma - \frac{1}{\omega_3} + \frac{\exp(\omega_3 pd_i)}{\omega_3 \exp(\omega_3 i^{SS})} \right)$$

C Structural Interpretation of the Risk Premium Shock

The risk premium shock of Smets and Wouters (2007) is empirically very important in structural DSGE models, and can explain the zero lower bound episodes. However, its structural interpretation as a risk premium shock is heavily criticized in Chari, Kehoe and McGrattan (2009). They argue that it is best interpreted as a flight to quality shock that affects the demand for a safe and liquid asset such as government debt. Fisher

(2015) microfounds this argument and indeed shows that this shock can be interpreted as a preference shock for treasury bills.

We show that the risk premium shock in our model can be interpreted as a flight to quality shock in government bonds in line with the argument above. For this reason, we incorporate government debt as an additional asset that earns the one period ahead nominal gross interest rate R_t^G. Following Fisher (2015), the government bond enters the household utility function as additive term and is subject to an exogenous preference shock Ω_t so that the household problem is given as:

$$\max_{C_t, L_t, D_t, B_t} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} + \Omega_t U(B_t) \right]$$

$$\text{s.t.} \quad P_t C_t = P_t W_t L_t + P_{t-1} D_{t-1} R_{t-1}^D \eta_{t-1} + P_{t-1} B_{t-1} R_{t-1}^B - P_t D_t - P_t B_t + P_t \Pi_t^P - P_t \tau_t$$

where $U(\cdot)$ is positive, increasing and concave. η_t is not an exogenous innovation in the model in this setup. Instead, the nominal gross interest is now artificially divided as $R_{t-1}^D \eta_{t-1}$ to better illustrate the mapping between the flight to quality shock and the risk premium shock. The first-order conditions with respect to deposits and government bonds are

$$\beta R_t^D \eta_t E_t \frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} = C_t^{-\sigma}$$

$$\beta R_t^G E_t \frac{C_{t+1}^{-\sigma}}{\Pi_{t+1}} = C_t^{-\sigma} - \Omega_t U'(B_t)$$

which can be combined to:

$$R_t^D \eta_t = R_t^G \frac{1}{1 - \Omega_t U'(B_t)}$$

This equation suggests that η_t captures changes in the preference for the safe asset Ω_t. In particular, an exogenous increase in the demand for the government bond would require either the nominal deposit rate to increase or the return on government bonds to fall. If R_t^G does not respond to wholly offset the impact of the shock, then there is a direct mapping from the flight to quality preference shock to our risk premium shock. η_t accounts for the rise in the nominal interest rate shock that resulted from a change in the risk premium.

The rise in the nominal interest rate resulting from the preference shock can be accounted for by an adjustment in η_t ,which we can then use as the risk premium shock. To avoid any impact on the household's budget constraint, the government bond can be in zero net supply. [28]

Regarding the bankers, their maximization problem is not directly affected from the flight to quality preference shock. The only impact on them is on the change in the nominal interest rates on deposits exactly as in the model. However, the increased funding costs for the banks via deposits are taken into account.

To conclude, there is a direct mapping of our version of the risk premium shock to the interpretation in Chari, Kehoe and McGrattan (2009) and Fisher (2015). An increase in the risk premium of deposits captures an increased demand in government bonds via a substitution effect.

Flight to Quality and Deposits Since our original model abstracts from government bonds for simplicity, an alternative approach would be to introduce a preference for holding deposits instead of government bonds in the utility function. The exogenous shock ω_t now targets the preference for deposits:

$$\max_{C_t,L_t,D_t} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} + \omega_t U(D_t) \right]$$
$$\text{s.t.} \quad P_t C_t = P_t W_t L_t + P_{t-1} D_{t-1} R_{t-1}^D \eta_{t-1} - P_t D_t + P_t \Pi_t^P - P_t \tau_t$$

where η_t is not an exogenous innovation in this setup, but part of the interest rate as before. The first-order condition can be written as

$$\beta R_t^D \eta_t E_t \frac{\Lambda_{t+1}}{\Pi_{t+1}} = 1 + \omega_t^\star U(D_t)$$

where the shock is normalized with respect to marginal utility of consumption $\Omega_t^\star = \omega_t / C_t^{-\sigma}$. Thus, the shock can be interpreted as a preference shifter of deposits: $\eta_t = 1 + \omega_t U(D_t)$. To capture the idea of a flight to safety to government bonds that increases

[28] One other potential caveat could be that this shock could actually also capture potential heterogeneities in the pass-through of deposits and governments. Nevertheless, the shock would still capture the impact of flight to quality, only adjusted for the different pass-through.

the nominal interest rate of deposits, it is important to realize that the shocks Ω_t and ω_t are inversely related. A flight to safety scenario implies an increase in Ω_t and a reduction in ω_t so that eta_t increases. As before, this setup is consistent with our modeling of the banking sector

Bank Default Finally, an alternative could be that the wedge accounts for the probability of default of the banks as our model abstracts from idiosyncratic default and bank runs. If the default probability of deposits is p_t, then the budget optimization problem would be:

$$\max_{C_t,L_t,D_t} E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{L_t^{1+\varphi}}{1+\varphi} \right] \tag{44}$$

$$\text{s.t.} \quad P_t C_t = P_t W_t L_t + P_{t-1} D_{t-1} R_{t-1}^D \eta_{t-1}(1-p_t) - P_t D_t + P_t \Pi_t^P - P_t \tau_t \tag{45}$$

where η_t should again be interpreted as part of the nominal interest rate. The Euler equations reads as:

$$\beta R_t^D \eta_t E_t (1-p_{t+1}) \frac{\Lambda_{t,t+1}}{\Pi_{t+1}} = 1$$

Therefore, our risk premium shock would be a proxy for the impact of the probability of default of the bank. It is important to note that the difference in timing between the risk shock and the probability of default. While η_t is known in period t, the probability of default is uncertainty and we have $E_t p_{t+1}$. This approach requires that the problem of the bank side is adjusted behind the increase in nominal rates. Rational bankers would take the probability of (idiosyncratic) default into account in their maximization framework. Thus, the model could be extended to include banking default.

D Microfoundation of the Imperfect Deposit Rate Pass-Through of Monetary Policy

In this section, we provide a microfoundation for the imperfect deposit rate pass-through of monetary policy, which follows Brunnermeier and Koby (2018).[29] The microfoundation captures that banks' market power in setting deposit rates depletes with low policy rates due to a fall in the costs of switching banks for depositors. This mechanism is in line with the empirical evidence of Hainz, Marjenko and Wildgruber (2017).

Microfoundation In the setup, there exists a continuum of banks that compete on prices. Each bank is associated with a continuum of households, which hold deposits at the bank. There exists to some degree deposit stickiness due to switching costs for the households. The households start to search for a new bank if the spread between the the policy rate R^A and the deposit rate t^D is sufficiently large, which can be summarized as the activation level $\tilde{\omega}(R^A)$. As a consequence, the households start to search if

$$R^A - R^D > \tilde{\omega}(R^A) \tag{46}$$

The necessary spread $R^A - R^D$ that triggers households to search for alternatives depends on the level of the interest rates. In particular, the activation level $\omega(R_t^A)$ decreases in the level of the interest rates.

Therefore, the households associated with bank j stay if the spread between the two rates is smaller or equal than the activation level or the offered deposit rates at all competitor banks are lower. Formally, the share of households Υ that keeps having deposits with their associated bank j is given by

$$\Upsilon(R_j^D; \mathbf{R}_{-j}^D, R^A) \equiv \mathbf{1}_{R^A - R^D \geq \tilde{\omega}(R^A) \ \vee \ R_j^D > \max \mathbf{R}_{-j}^D} \tag{47}$$

where $\mathbf{R}_{-j}^D$ is the vector of prices set by the remaining banks. This gives the extensive margin of the deposit rate choice of the banks. The intensive margin is that the amount

[29] An alternative approach could be to use a menu cost model as in Driscoll and Judson (2013).

of deposits provided by each households also varies with the deposit rate. We assume that the intensive margin is inelastic and therefore, focus only on the extensive margin.

The demand of bank j can then be written as

$$D_j(R_j^D; \mathbf{R}_{-j}^D, R^A) = \Upsilon(R_j^D; \mathbf{R}_{-j}^D, R^A) \tag{48}$$

As the banks want to keep their depositors, the deposit rate is given as

$$R_t^D = R_t^A - \tilde{\omega}(R^A) \tag{49}$$

Defining $\omega(R^A) \equiv R_t^A - \tilde{\omega}(R^A)$, this can be mapped back into the equation 28 that maps the level of the policy rate to the deposit rate:

$$R^D = \omega(R^A) \tag{50}$$

Thus, the markdown on the deposit rate R_t^D varies with the level of the policy rate R_t^A.

Functional Form We use a functional form $\omega(R_t^A)$ that separates the connection between the two rates in a region with an increasingly imperfect pass-through and a region with a perfect pass-through

$$R_t^D = \omega(R_t^A) = \begin{cases} \omega^1 + \omega^2 \exp(\omega^3(R_t^A - 1)) + 1 & \text{if } R_t^A < \bar{R}^A \\ R_t^A - \varsigma & \text{else} \end{cases} \tag{51}$$

where ω^1, ω^2 and ω^3 determines the shape of the imperfect deposit pass-through and ς is related to banks market power. Figure 2 shows the parameterized pass-through after fitting the model to the data with a non-linear least squares approach.

Policy Invariance to the Lucas Critique The microfoundation creates a structural mapping from the policy rate to the deposit rate, which is captured with the structural parameters ω^1, ω^2, ω^3 and ς. The relation does not depend on other characteristics of the economy or the banking sector. Therefore, the framework can be used for the policy experiments conducted in this paper (e.g. the effective lower bound on monetary policy and the macroprudential policy rule). In other words, the Lucas critique does not apply here even though we use a functional form for the mapping between the deposit rate and the policy rate.[30]

E Macroprudential Policy Rule Parameters

The rule consists of two parameters that interact with each other. Figure 11 shows the impact on welfare for different combinations of ϕ^{MPP} and τ^{MPP}. The optimal rule has a rather large anchor value with a small response parameter. This ensures the build-up of a small buffer that can then be released during a crisis. If the anchor value is too large, the economy has on average too many buffers that it never releases.

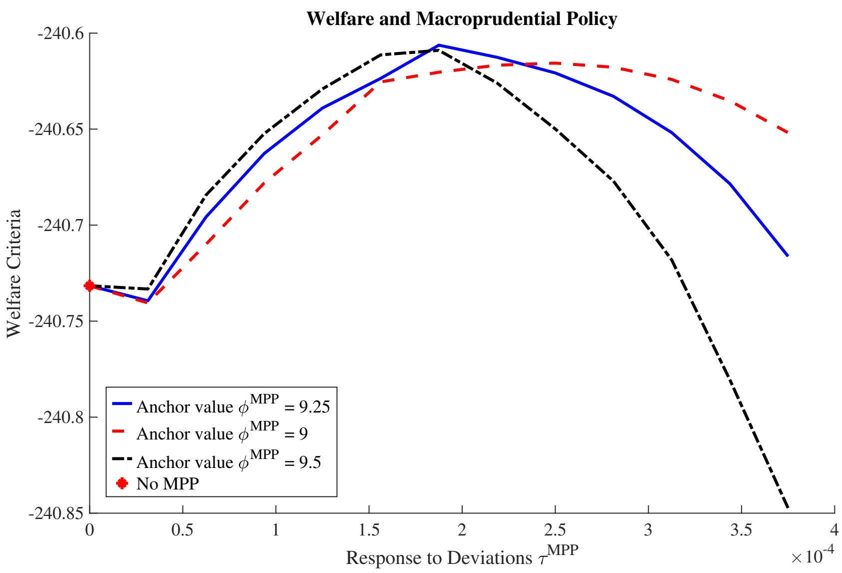

Figure 11: Welfare for response to deviations τ^{MPP} and anchor values ϕ^{MPP}. τ^{MPP} is varied on the horizontal axis. Welfare is on the horizontal axis

F Solution Method

The non-linear model is solved with policy function iterations. In particular, we use time iteration (Coleman, 1990) and linear interpolation of the policy functions as in Richter,

[30]The condition for a valid policy experiment is that it does not affect the activation level function $\tilde{\omega}$. A hypothetical policy experiment that would violate this condition would be for instance the introduction of a policy that facilitates the comparison of interest rate between different banks and would thereby alter $\tilde{\omega}$.

Throckmorton and Walker (2014). We solve for the policy functions and law of motions. We rewrite the model to use net worth N_t as a state variable instead of $D_{t-1}R_{t-1}$ to ease the computation.

The algorithm has the following steps:

1. Define the state space and discretize the shock with the Rouwenhorst method.

2. Use an initial guess for the policy functions.

3. Solve for all the time t variables for a given state vector and a law of motion of net worth. Given the state vector $K_{t-1}, N_t, \eta_t, \zeta_t$, the policy variables Q_t, C_t, ψ_t, Π_t and the law of motion of the net worth, we can solve for the following variables in period t:

$$
\begin{aligned}
I_t &= (Q_t(1-\eta_i)a_i)^{\frac{1}{\eta_i}} K_{t-1} \\
Y_t &= \frac{C_t + I_t}{\left(1 - \frac{\rho^r}{2}\left(\frac{\Pi_t}{\Pi} - 1\right)^2\right)} \\
L_t &= \left(\frac{Y_t}{K_{t-1}^{\alpha}}\right)^{\frac{1}{1-\alpha}} \\
W_t &= \chi L^{\varphi} C^{\sigma} \\
MC_t &= \frac{W_t}{1-\alpha}\frac{L}{Y} \\
R_t^A &= R^A\left(\frac{\Pi_t}{\Pi}\right)^{\kappa_\Pi}\left(\frac{Y_t}{Y}\right)^{\kappa_Y} \\
R_t^D &= \mathbf{1}_{R_t^A \geq R^{A^{SS}}}\left[R_t^A - \varsigma\right] + (1 - \mathbf{1}_{R_t^A \geq R^{A^{SS}}})\left[\omega^1 + \omega^2 \exp(\omega^3(R_t^A - 1)) + 1\right]
\end{aligned}
$$

The endogenous state variables are capital and net worth, which are given from the law of motion of capital and the guess for the law of motion of net worth

$$
\begin{aligned}
K_t &= (1-\delta)K_t + \left(a_i\left(\frac{I_t}{K_t}\right)^{1-\eta_i} + b_i\right)K_{t-1} \\
N_{t+1} &= \mathcal{T}(K_{t-1}, N_t, \zeta_t, \eta, \zeta_{t+1}, \eta_{t+1})
\end{aligned}
$$

Note that capital is predetermined, while net worth depends on the shocks. Therefore, we have a net wroth at each integration node for the shocks. At each node

i, we then now the policy function $Q^i_{t+1}, C^i_{t+1}, \psi^i_{t+1}, \Pi^i_{t+1}$. At this step, we linearly interpolate the policy functions

$$
\begin{aligned}
I^i_{t+1} &= \left(Q^i_{t+1}(1-\eta_i)a_i\right)^{\frac{1}{\eta_i}} K_t \\
Y^i_{t+1} &= \frac{C^i_{t+1} + I^i_{t+1}}{\left(1 - \frac{\rho^r}{2}\left(\frac{\Pi^i_{t+1}}{\Pi} - 1\right)^2\right)} \\
L^i_{t+1} &= \left(\frac{Y^i_{t+1}}{K^\alpha_t}\right)^{\frac{1}{1-\alpha}} \\
W^i_{t+1} &= \chi \left(L^i_{t+1}\right)^{\varphi} \left(C^i_{t+1}\right)^{\sigma} \\
MC^i_{t+1} &= \frac{W^i_{t+1}}{1-\alpha} \frac{L^i_{t+1}}{Y^i_{t+1}} \\
R^{k,i}_{t+1} &= \frac{MC^i_{t+1} \alpha Y^i_{t+1}/K_t + Q^i_{t+1}(1-\delta)}{Q_t} \Pi^i_{t+1}
\end{aligned}
$$

We can now calculate the following items:

$$
\begin{aligned}
\phi_t &= \frac{Q_t K_t}{N_t} \\
R_t &= R^D_t \eta_t \frac{1}{1-\delta^B} - R^A_t \frac{\delta^B}{1-\delta^B} \\
\mu_t &= \beta E_t \left(\frac{C_t}{C_{t+1}}\right)^{\sigma} (1 - \theta + \theta\psi_t) \left(\frac{R^K_{t+1} - R_t}{\Pi_{t+1}}\right) \\
\nu_t &= \beta E_t \left(\frac{C_t}{C_{t+1}}\right)^{\sigma} (1 - \theta + \theta\psi_t) \left(\frac{R_t}{\Pi_{t+1}}\right)
\end{aligned}
$$

where the expectations are based on the weighting of the different integration nodes. The Rouwenhorst method discretizes the shocks and gives the weighting matrix. Finally, we can calculate the errors for the four remaining equations

$$
\begin{aligned}
err_1 &= \left(\frac{\Pi_t}{\Pi} - 1\right)\frac{\Pi_t}{\Pi} - \left(\frac{\epsilon}{\rho^r}\left(MC_t - \frac{\epsilon - 1}{\epsilon}\right) + \beta E_t \left(\frac{C_t}{C_{t+1}}\right)^{-\sigma} \frac{Y_{t+1}}{Y_t}\left(\frac{\Pi_{t+1}}{\Pi_t} - 1\right)\frac{\Pi_{t+1}}{\Pi}\right) \\
err_2 &= \beta R^D_t \eta_t E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{1}{\Pi_{t+1}} \\
err_3 &= \psi_t - (\mu_t \phi_t + \nu_t) \\
err_4 &= \psi_t - \left(\lambda\Big(\frac{1}{1-\delta^B}\phi_t - \frac{\delta^B}{1-\delta^B}\Big)\right)
\end{aligned}
$$

We minimize the errors using a root solver the policy functions in period t. The policy functions for period $t+1$ are taken from the previous iteration.

4. This step is only relevant for the extension with the countercyclical capital rule. Otherwise, it can be skipped. Check if the occasionally binding constraint is binding. If we introduce the capital requirement, it is occasionally binding. Therefore, we have to check if

$$\phi^R > \phi^M$$

where ϕ^M is the market based leverage that we calculated as ϕ in the previous step. If this is the case, the capital constraint is binding. We now replace two equations from before, namely we impose directly

$$\phi = \phi^R$$

Furthermore, one of the remaining equations is now adjusted as the market based leverage constraint is not binding anymore. Therefore, we remove $\phi_t = \frac{Q_t K_t}{N_t}$ from the calculations and actually minimize the error:

$$err_4 = \phi_t - \frac{Q_t K_t}{N_t}$$

Note that we do not need $\psi_t \geq \left(\lambda\left(\frac{1}{1-\delta^B}\phi_t - \frac{\delta^B}{1-\delta^B}\right)\right)$ from the previous step as it is not binding.

5. Update the law of motion for net worth. We have assumed that we know the actual law of motions. Using the policy functions, we improve our guess of the policy function. Using the result from the previous steps (depending on the binding of the constraint), we update it as follows

$$N^i_{t+1} = \theta\left(\left(R^{k,i}_{t+1} - R_t\right)\phi_t - R_t\right) + \omega K_t$$

We have to update the law of motion for each possible shock realization in the next period.

6. Check convergence for the policy functions and the law of motion of net worth for a predefined criterion.

G Additional Figures

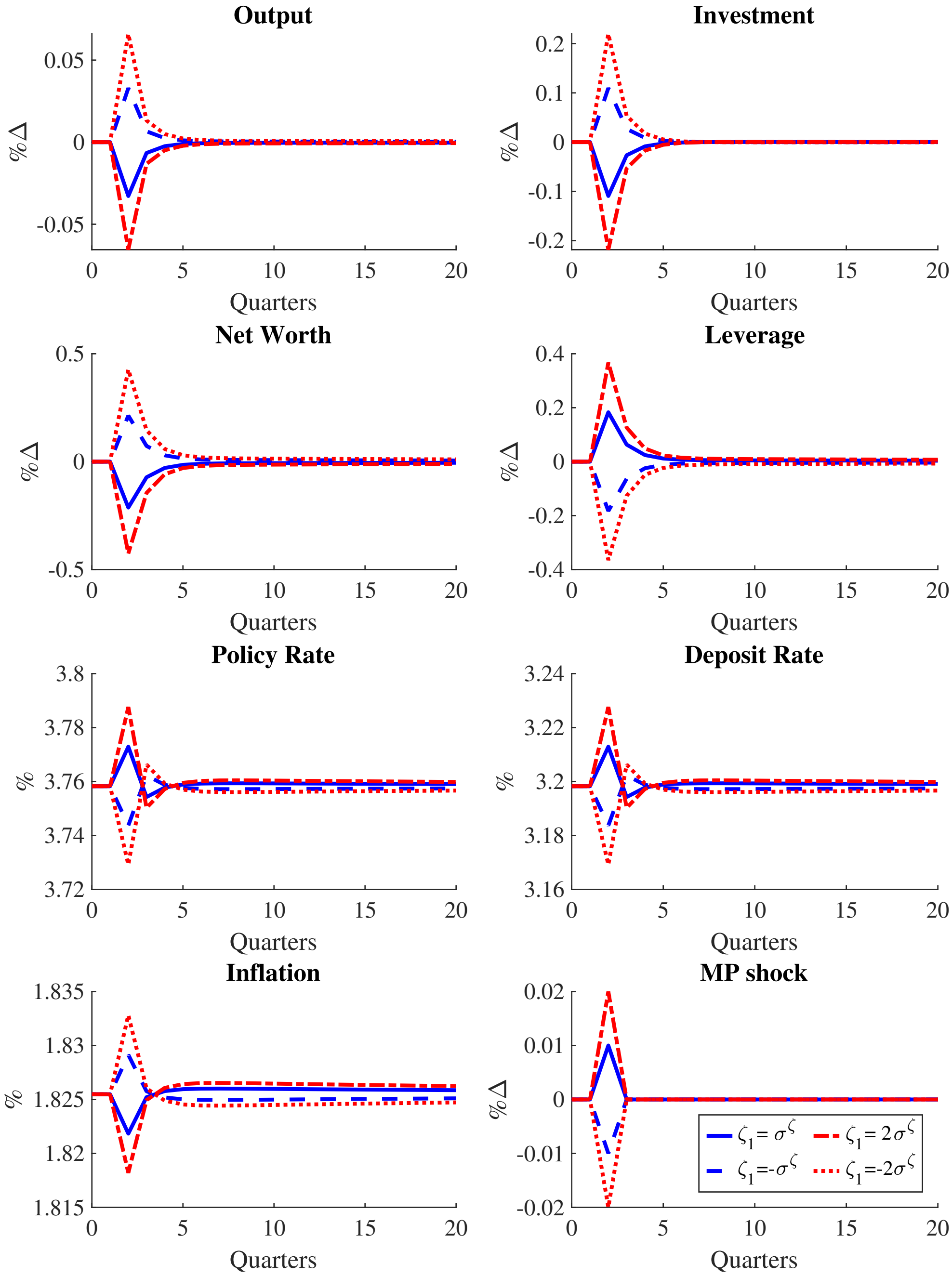

Figure 12: Impulse response functions of the monetary policy shock that differ in the size and sign of the innovation. A one standard deviation increase (blue solid) and decrease (blue dashed) as well as a two standard deviation increase (red dash-dotted) and decrease (red dotted) for the innovation ζ is shown. The responses are displayed in percentage deviations from the risky steady state, which is the initial point of the economy.

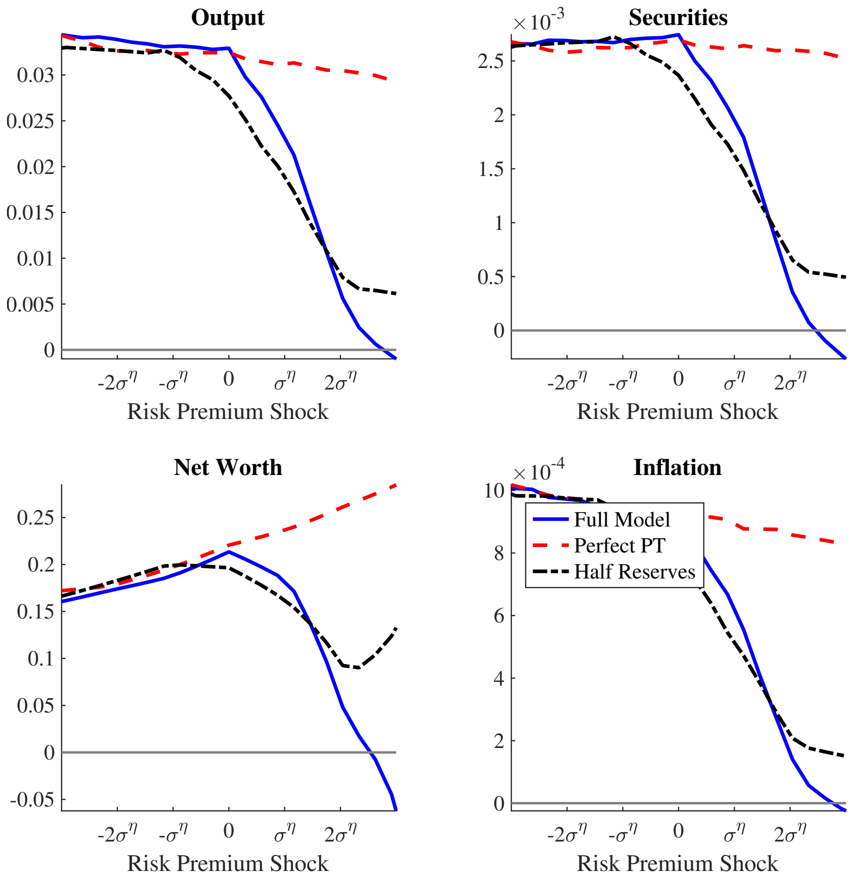

Figure 13: First period response to a monetary policy shock combined with different sized risk premium shocks. The vertical axis displays the state-dependent difference for the period $t = 1$ response between a shocked path, which introduces a negative one standard deviation innovation for the monetary policy shock $\zeta_1 = \sigma^\zeta$, and a path, in which the monetary policy innovation does not occur. The state-dependence results from the different sized risk premium shock that occurs simultaneously in the first period, which is displayed on the horizontal axis.

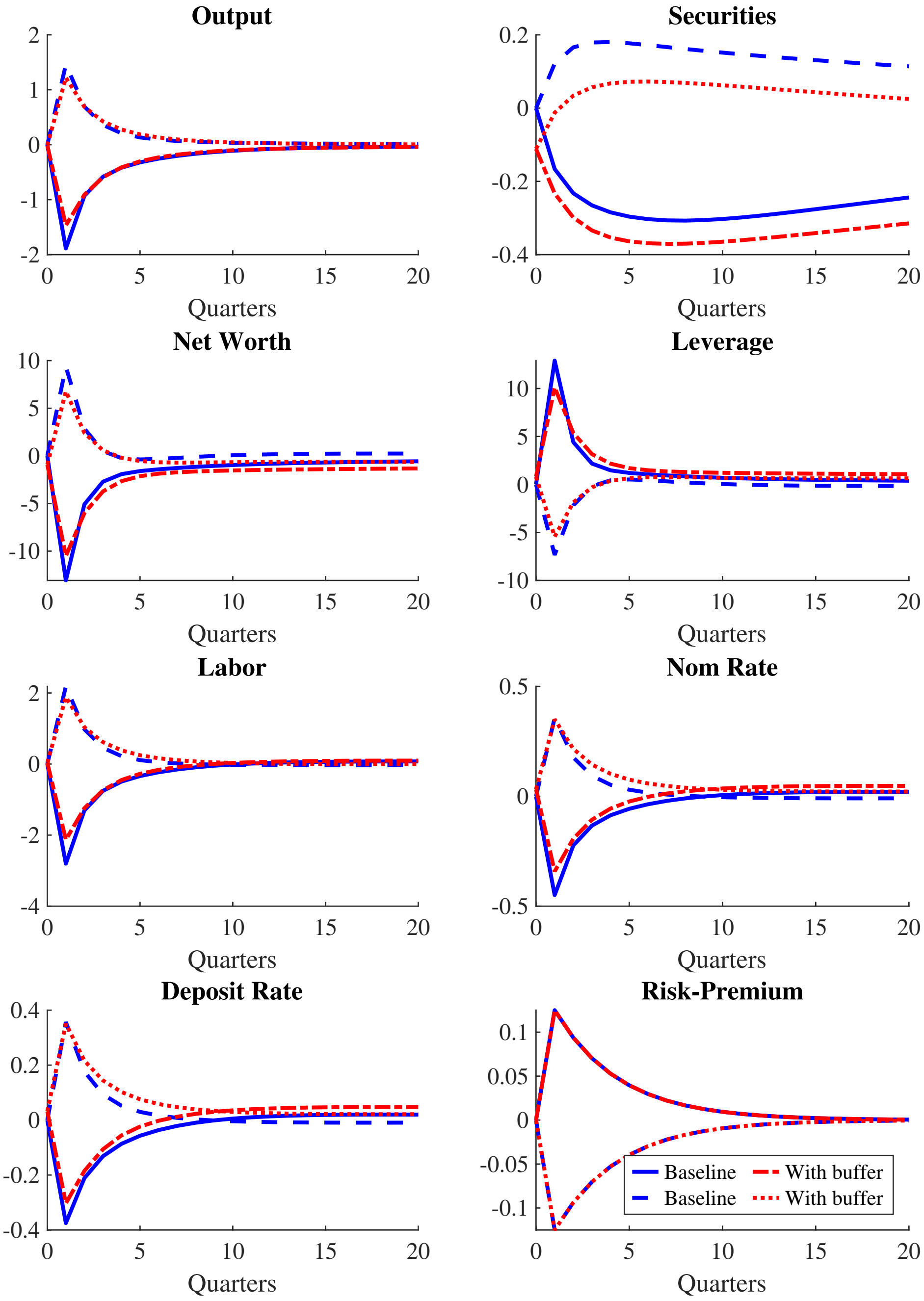

Figure 14: Impulse response functions of different risk premiums shock depending on the capital buffer are shown. A one standard deviation increase and decrease is shown for the baseline model without buffer (blue solid dotted and blue dashed line, respectively) and an economy with a buffer $\tau^{MPP} = 0.016\%$ and $\phi^{MPP} = 9.75$ (red dash-dotted and red dotted line, respectively). Starting point is the risky steady state of each economy. Deviations are in percent relative to the risk steady state of the economy without a capital buffer rule.