Monetary Policy, Firm Heterogeneity, and the Distribution of Investment Rates*

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Abstract

We document three pieces of evidence about the investment channel of monetary policy. First, an interest rate cut *reshapes* the distribution of investment rates as it leads to fewer small or zero investment rates and more large investment rates. Second, the change in the distribution is more pronounced among young than old firms. We emphasize the relevance of the extensive margin——firms deciding whether to invest or not—in explaining these findings. Third, a decomposition exercise indicates that the extensive margin accounts for around 50% of the effect of monetary policy on the *average* investment rate and more than 50% of the heterogeneous effect on young firms. To interpret these empirical findings, we build a heterogeneous-firm model with fixed adjustment costs and firm life-cycle dynamics. In the model, young (small) firms—often standing in for financially constrained firms—are more sensitive to monetary policy even without a financial accelerator mechanism.

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1 Introduction

Understanding the investment channel of monetary policy is important for policy-makers because investment is a sizable and the most volatile component of aggregate GDP. To this end, the literature has extensively studied the effect of monetary policy on the *average* investment rate.[1] However, this estimated effect on the average investment rate can reflect a *parallel shifting* of the entire distribution or a change in the *shape* of the distribution. How does monetary policy affect the distribution of investment rates? Which part of the distribution is responsible for the change in the average investment rate? Moreover, a growing academic literature studies the heterogeneous effects of monetary policy on the investment behavior of different groups of firms, see, e.g., Gertler and Gilchrist (1994), Ottonello and Winberry (2020), Jeenas (2019), and Cloyne et al. (2020).[2] Which part of the distribution drives these heterogeneous effects on *average* investment rates? The answers to these questions are important to understand the transmission of monetary policy. In particular, they can help shed light on the frictions that matter for the (heterogeneous) effects of monetary policy on firm investment decisions.

We provide three pieces of evidence that address the raised questions. First, monetary policy affects the *shape* of the distribution of investment rates. Specifically, an expansionary monetary policy shock leads to fewer small or zero investment rates and more large investment rates. Second, the change in the shape of the investment rate distribution is more pronounced among young (small) firms than among old (large) firms. This paper emphasizes the relevance of the extensive margin—firms deciding whether to invest or not—in explaining these findings. Third, a decomposition exercise indicates that the extensive margin accounts for around 50% of the effect of monetary policy on the *average* investment rate and for more than 50% of the *heterogeneous effect* on firms of different age groups.

Theoretically, we provide a heterogeneous-firm model that combines capital adjustment costs, firm entry and exit, and nominal rigidities. The presence of fixed adjustment costs gives rise to an investment channel of monetary policy along the extensive margin. That is, an interest rate cut induces some firms to switch from not investing to making a large investment. Therefore, monetary policy reshapes the distribution of investment rates. In addition, the calibrated quantitative model generates

[1]See, for example, Ottonello and Winberry (2020), Jeenas (2019), or Cloyne et al. (2020).

[2]Cloyne et al. (2020) document that investment rates of young firms are on average more sensitive to monetary policy than those of old firms. Gertler and Gilchrist (1994) show a similar result for small and large firms. Clearly, these findings are connected, as age and size are strongly correlated in the data. In this paper, we focus on age but emphasize and show that our results are similar when comparing small and large firms.

sizable heterogeneous effects of monetary policy on average investment rates of firms of different age groups. The reason is that young firms are more easily induced to make an investment than old firms. The model attributes more than 50% of the heterogeneous effect across age groups to the extensive margin, as in the data.

In more detail, we address the empirical questions using quarterly Compustat data combined with identified monetary policy shocks as in Ottonello and Winberry (2020) and Cloyne et al. (2020). In contrast to the existing studies, we estimate the effects of monetary policy on different quantiles of the investment rate distribution rather than solely focusing on the first moment of the distribution—the average investment rate. We uncover that the upper quantiles respond substantially more to a monetary policy shock than the lower quantiles do. This finding suggests that monetary policy reshapes the distribution of investment rates by reducing the mass of firms making a small or no investment and increasing the mass of firms making a sizeable investment.

To visualize the change in the distribution, we approximate the shape of the entire distribution of investment rates before and after a monetary shock. We do so by fitting a flexible skewed t-distribution, using quantiles of the distribution and their estimated impulse response functions as inputs.[3] Comparing the approximated skewed t-distributions before and after an expansionary monetary policy shock, we illustrate that fewer firms make a small or no investment and more firms make a large investment—**Fact 1**. This novel evidence suggests the presence of a quantitatively relevant investment channel of monetary policy along the extensive margin.

When the same exercise is conducted for young and old firms separately, we uncover that the effect of monetary policy on the shape of the distribution of investment rates is more pronounced among young firms than among old firms—**Fact 2**. This result implies that the extensive margin investment channel is particularly important for young firms. The following exercises further support this conclusion. We estimate the effects of monetary policy on the *spike rate*, defined as the fraction of firms whose quarterly investment rate exceeds 10%, and on the *inaction rate*, defined as the fraction of firms whose quarterly investment rate is smaller than 0.5% in absolute value.[4] The spike rate rises and the inaction rate drops more strongly for young firms than for old

[3]This approach has recently been applied by Adrian et al. (2019) to transform conditional quantiles into the conditional distribution of GDP growth.

[4]In annual data, an investment spike is typically defined as an investment rate above 20%, so about twice the average investment rate, which, in most representative datasets, ranges between 10% and 12% (Zwick and Mahon, 2017). Since we do not use annual, but quarterly data and Compustat features higher average investment rates, as discussed in Online Appendix A.3, we define an investment spike to be a quarterly investment rate exceeding 10%. This too is an investment rate roughly twice the average investment rate. Inaction is typically defined as an annual investment rate less than 1% in absolute value. For the same reasons as above, we define inaction as a quarterly investment rate smaller than 0.5% in absolute value.

firms. Both differences are statistically significant.

The empirical literature has documented that young firms' *average* investment rates are more sensitive to monetary policy than old firms', see, e.g., Cloyne et al. (2020). Gertler and Gilchrist (1994) show a similar result for small and large firms. Conventional wisdom views these findings as supporting the financial accelerator mechanism, based on the narrative that young firms are financially constrained[5] and monetary policy affects financial conditions. The novel empirical evidence presented in this paper suggests that besides financial acceleration, the extensive margin investment decision, arising from fixed adjustment costs, is important for the heterogeneous sensitivity of young and old firms. The final empirical exercise quantifies the relative importance of the extensive margin.

We decompose the effect of monetary policy on the average investment rate into contributions arising from the extensive and intensive margin, respectively. We use the change in the spike rate to proxy for the extensive margin. Our decomposition suggests that the extensive margin accounts for 50% of the total effect of a monetary policy shock on the average investment rate. In addition, more than 50% of the heterogeneous sensitivity of young and old firms' average investment rates to monetary policy is due to the extensive margin.

The second part of the paper interprets the empirical findings through the lens of a quantitative model. Fixed capital adjustment costs give rise to an investment channel of monetary policy along the extensive margin, consistent with the empirical findings. As a result, monetary policy affects the distribution of investment rates.

Moreover, the model highlights that the presence of an *extensive margin* investment decision creates *heterogeneous* effects on *average* investment rates of young and old firms. Entry and exit give rise to endogenous firm life cycles and an age distribution. The age-group-specific average investment rate is the fraction of investing firms (hazard rate) times the investment rate conditional on investing. The heterogeneous effect on different age groups along the extensive margin arises from two channels. First, monetary policy has a heterogeneous effect on hazard rates. More specifically, an interest rate cut induces more young than old firms to switch from inaction to making an investment. The reason is that young firms are on average further away from their optimal level of capital and therefore more easily induced to make an investment. Second, even without a heterogeneous effect on hazard rates, we would observe a higher

[5] Rauh (2006), Fee et al. (2009), Hadlock and Pierce (2010), and more recently Cloyne et al. (2020) argue that young firms are more likely financially constrained than old firms. Gertler and Gilchrist (1994) rely on the narrative that "*...the costs of external finance apply mainly to younger firms, firms with a high degree of idiosyncratic risk, and firms that are not well collateralized. These are, on average, smaller firms...*" to motivate the use of firm size as a proxy for financial frictions.

average sensitivity of young firms. This is because young firms have a higher investment rate conditional on adjusting, again, because they are on average further away from their optimal level of capital. Overall, the model predicts that monetary policy affects the distribution of investment rates due to the extensive margin and that these effects are more pronounced among young firms, in line with the empirical evidence.

After illustrating the mechanisms in a simple model, we quantify them in a general equilibrium heterogeneous-firm model calibrated to match moments of the cross-sectional investment rate distribution and firm life-cycle patterns. According to the quantitative model, young firms are almost twice as sensitive to a monetary policy shock as old firms, explaining a large portion of the observed heterogeneous sensitivity in the data. A decomposition exercise demonstrates that the extensive margin is quantitatively dominant.

Our findings have important implications for both academic research and the conduct of monetary policy. We present a mechanism that makes firms that are typically classified as financially constrained more sensitive to monetary policy *even in the absence* of financial acceleration.[6] Thus, there is an issue of observational equivalence: The observed heterogeneous sensitivity of young (small) firms can arise not only due to a financial accelerator mechanism but also due to the presence of fixed adjustment costs as explained above. In addition, to the extent that age is correlated with popular proxies of financial frictions, as documented in Cloyne et al. (2020), the issue of observational equivalence extends beyond the comparison of firms by age or size. However, one should not interpret our results as rejecting the financial accelerator mechanism. It is likely that both the financial frictions *and* the non-financial mechanisms that we emphasize in this paper are relevant for the heterogeneous sensitivity observed in the data. Our findings highlight that there remain unresolved challenges when it comes to identifying the financial accelerator mechanism in the data.

Understanding the frictions underlying firms' (heterogeneous) investment decisions is important for guiding macroeconomic policy in recessions. Financial conditions are typically tighter in recessions, which is associated with a stronger financial accelerator mechanism. Therefore, if financial frictions are more important for firms' decisions, one would expect monetary and fiscal policy to be more effective in recessions. Conversely, if lumpy investment behavior is more important, recessions are typically associated with fewer firms at the margin of adjusting. Therefore, macroeco-

[6]Even though the capital adjustment costs that we impose can in principle be interpreted as stand-ins for financial frictions, our model does not feature a financial accelerator mechanism. The idea of the financial accelerator mechanism is that monetary policy changes the tightness of financial constraints. By construction, the capital adjustment costs are not affected by aggregate shocks, including monetary policy shocks, and therefore, there is no financial accelerator mechanism.

nomic policies are less effective in times of economic slack: see, e.g., Winberry (2021). Our paper supports the quantitative relevance of lumpy investment behavior and argues that the heterogeneous sensitivity of young firms is not sufficient evidence for financial acceleration. The relevance of the lumpy investment is consistent with separate evidence uncovered in the empirical literature: monetary and fiscal policy interventions are less potent in recessions (Tenreyro and Thwaites, 2016; Ramey and Zubairy, 2018).

Literature Review The evidence presented in this paper contributes to the empirical literature that studies the investment channel of monetary policy: see, e.g., Christiano et al. (2005) using aggregate data and Gertler and Gilchrist (1994), Ottonello and Winberry (2020), Jeenas (2019), and Cloyne et al. (2020) using firm-level data. So far, this literature has focused on the effects on average investment rates or on aggregate investment. Our contribution is to show that monetary policy reshapes the distribution of investment rates and that this effect is more pronounced among young and small firms.

Our paper also contributes to the literature emphasizing the extensive margin of firm investment, i.e. the relevance of fixed adjustment costs. A long debate has focused on whether lumpy firm-level investment behavior matters for aggregate investment and in particular for its responsiveness to shocks over the business cycle. Important contributions include Caballero et al. (1995), Caballero and Engel (1999), Thomas (2002), Khan and Thomas (2003), Khan and Thomas (2008), Bachmann et al. (2013), House (2014), Koby and Wolf (2020), Winberry (2021), and Baley and Blanco (2021). Monetary policy shocks in models with fixed adjustment costs have been analyzed in Reiter et al. (2013), Reiter et al. (2020), and Fang (2022). We develop a heterogeneous-firm model with three features: fixed adjustment costs, firm life-cycle dynamics, and a New Keynesian sticky-price setup. We contribute to the theoretical strand of this literature by demonstrating the importance of the extensive margin for the heterogeneous sensitivity of firm-level investment rates across firm groups.

The empirical strand of the literature on lumpy investment has mainly produced two types of evidence. First, the *unconditional* distribution of firm-level investment rates is in line with the presence of fixed adjustment costs, see, e.g., Caballero et al. (1995), Cooper et al. (1999), and Cooper and Haltiwanger (2006). Second, the behavior of *aggregate* investment in response to macroeconomic shocks is in line with the presence of fixed adjustment costs, see, e.g. Caballero and Engel (1999), Bachmann et al. (2013) and Fang (2022). We contribute to the empirical strand by documenting the response of the *entire distribution* of investment rates to monetary policy shocks.

The evidence supports the quantitative relevance of the investment channel of monetary policy along the extensive margin.[7] Furthermore, we emphasize that the heterogeneous effects along the extensive margin across age groups are consistent with a model with fixed adjustment costs and *endogenous firm life cycles*.

The issue of observational equivalence that we raise in this paper contributes to the literature that aims to document the financial accelerator mechanism in firm-level data. Several recent papers compare the investment behavior of groups of "likely financially constrained" and "likely financially unconstrained" firms after monetary policy shocks. To group firms, some observable proxy variable, which plausibly correlates with the severity of financial constraints, is used. For example, Gertler and Gilchrist (1994) use size to group firms,[8] Ottonello and Winberry (2020) use leverage and distance to default, and Jeenas (2019) uses liquidity. Cloyne et al. (2020) advocate the use of firm age as a proxy for financial constraints, because it correlates with most other proxy variables while being exogenous to firm decisions. A higher sensitivity to monetary policy shocks of "likely financially constrained" firms is taken as evidence supporting the financial accelerator mechanism, based on the logic that there is a heterogeneous effect on the marginal *cost* of investing. We show that two common proxies for financial constraints—firm age and firm size—predict a higher sensitivity to monetary policy shocks even in the absence of financial frictions. This is because fixed adjustment costs create a heterogeneous effect on the (marginal) *benefit* of investing. This finding does not speak against age and size being correlated with financial constraints. However, it illustrates that age and size, and therefore all proxy variables which correlate with them, also correlate with the relevance of non-financial constraints which make firms sensitive to monetary policy. In this sense, our paper relates to Crouzet and Mehrotra (2020), who argue that large firms are less cyclical than small firms because they are better diversified across industries, but not because of financial frictions.[9]

[7] Gourio and Kashyap (2007) emphasize the cyclicality of the spike rate of firms' investments—another statistic in addition to the average investment rate. In contemporaneous work, Lee (2022) estimates the effect of monetary policy shocks on the spike rates of small and large firms. We investigate the effect on the entire distribution of investment rates in addition to spike rates. Lee (2022) uses the estimates by firm size to *calibrate* a real business cycle model with *size-dependent* fixed adjustment costs and aggregate TFP shocks. We *rationalize* our findings in a New Keynesian sticky-price model. Importantly, firm entry and exit give rise to endogenous firm life cycles in our setting that explain the heterogeneous effects along the extensive margin by firm age.

[8] See also Singh et al. (2021) for the heterogeneous effects of monetary policy on labor market outcomes across firms of different sizes.

[9] In addition, our argument aligns well with Farre-Mensa and Ljungqvist (2016) who show that supposedly financially constrained firms do not behave as if they were constrained and also differ systematically from supposedly less constrained firms along other important dimensions. Along the same lines, Dinlersoz et al. (2018) argue that only private firms, but not public ones—i.e. Compustat firms which are the ones examined in the above-mentioned papers—appear financially constrained.

The remainder of this paper is organized as follows. Section 2 presents our empirical findings. Section 3 outlines the simple model and explains its key mechanisms. Section 4 presents the full New Keynesian heterogeneous-firm model. Section 5 calibrates the model and analyzes the effects of a monetary policy shock. Section 6 concludes.

2 Empirical Evidence

We present three pieces of evidence that are important to understand the investment channel of monetary policy. Section 2.1 introduces the data used throughout this paper. Section 2.2 describes the local projection model used to estimate impulse response functions (IRFs). Section 2.3 documents the effects of monetary policy on the distribution of investment rates. Section 2.4 presents the heterogeneous effects of monetary policy by firm age. Section 2.5 decomposes the (heterogeneous) effects of monetary policy into contributions arising from the extensive and intensive margins, respectively.

2.1 Firm-Level Data

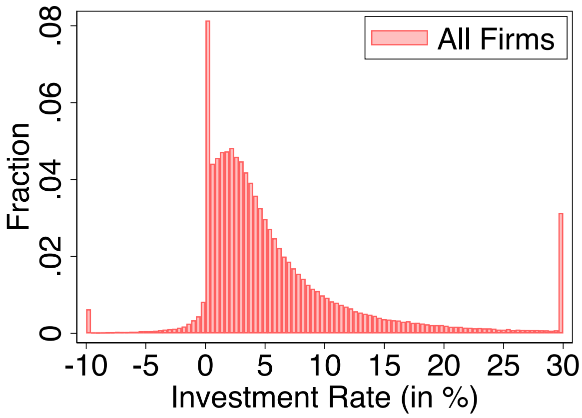
We use quarterly firm-level data from Compustat. Our sample begins with 1986Q1 and ends with 2018Q4. We exclude firms with incomplete or questionable information (e.g. negative reported sales) and those not suitable for our analysis (e.g. financial firms) from the sample. Details on the sample selection are relegated to Online Appendix A.1. Since information on firm age in Compustat is scarce, we merge age information from WorldScope and Jay Ritter's database, as explained in Online Appendix A.2.

Capital stocks reported in Compustat are *accounting* capital stocks and do not perfectly reflect *economic* capital stocks.[10] To address this issue, we use a Perpetual Inventory Method (PIM) to compute real economic capital stocks, building on Bachmann and Bayer (2014). Details of this procedure are explained in Online Appendix A.3. Our baseline measure of the investment rate is $i_{jt} = \frac{CAPX_{jt} - SPPE_{jt}}{INVDEF_t \times k_{jt-1}}$, thus, real capital expenditures (CAPX) net of sales of capital (SPPE) divided by the lagged real economic capital stock (k). More details on the construction of variables are given in Online Appendix A.4.

[10] On the one hand, accounting depreciation is driven by tax incentives and usually exceeds economic depreciation. On the other hand, accounting capital stocks are reported at historical prices, not current prices. With positive inflation, both issues make economic capital stocks exceed accounting capital stocks.

For parts of the subsequent analysis, we aggregate the firm-level data to quarterly investment rate distributions and moments thereof.[11] The distribution of investment rates, shown in Figure 1, depicts some well-known features of investment rate distributions. That is, the distribution has a positive skewness, a long right tail, substantial mass at 0, and very few negative observations.

Figure 1: Distribution of Investment Rates



Notes: This figure plots the distribution of quarterly investment rates of firms in Compustat. The investment rate is real capital expenditures (CAPX) net of sales of capital (SPPE) divided by the lagged real economic capital stock. Sample: 1986Q1 - 2018Q4.

2.2 Local Projection: Method to Construct the IRFs

To estimate the effects of monetary policy shocks, we estimate the following simple local projection (LP) models:

$$y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h} \tag{1}$$

where y_t indicates the outcome variable, ϵ_t^{MP} is the monetary policy shock, q_t is the calendar quarter, and $\mathbb{1}\{q_{t+h} = j\}$ are quarter dummies that are included to address seasonality. We use the monetary policy shocks implied by the Proxy SVAR in Gertler and Karadi (2015). These are extracted after updating the time series data used in the VAR as well as the high-frequency instruments. Details are relegated to Online Appendix A.5. The shocks are scaled to reduce the 1-year Treasury rate on impact

[11]Moments which are sensitive to outliers, such as the mean, are winsorized. Quite importantly, winsorizing is done by group and quarter. This ensures that the process does not systematically bias our sample.

by 25 basis points. Throughout, we use Newey-West standard errors to account for heteroskedasticity and autocorrelation.

Before turning to our novel findings, we verify that the monetary policy shocks have plausible effects on aggregate variables. We show in Online Appendix A.6 that an expansionary shock leads to hump-shaped increases in both investment and real GDP. The peak effects are 1.4% (investment) and 0.35% (real GDP), respectively.

2.3 Fact 1: Shape of the Distribution of Investment Rates

The literature has extensively studied the effect of monetary policy on the *average* investment rate.[12] However, this estimated effect on the average investment rate can reflect a *parallel shifting* of the entire distribution or a change in the *shape* of the distribution. We now investigate how monetary policy affects the distribution of firm-level investment rates by estimating the effects on different quantiles of the investment rate distribution. This is done by using the time series of the respective quantiles of the distribution as outcome variables in the empirical model (1).[13] If the increase in the average investment rate reflects a parallel shifting of the distribution, the effect on all quantiles must be identical.

Figure 2 shows the effect of monetary policy shocks on selected quantiles of the investment rate distribution. Panel (a) plots the responses of the 25th (in blue) and the 75th (in red) percentiles. It is evident that the right tail (the 75th percentile) responds more strongly than the left tail (the 25th percentile) of the investment rate distribution. At the peak, the 75th percentile of the investment rate distribution rises by 40 basis points, while the 25th percentile rises by only 10 basis points. This difference is statistically significant, as illustrated by the IRF of the corresponding interquantile range (Panel b). These findings are robust to the alternative choices of quantiles: see Panels (c) to (f).[14] The disproportionate change in the right tail compared to the left tail indicates that monetary policy affects the shape of the investment rate distribution. The following exercise formalizes the mapping between the heterogeneous effect on different quantiles and changes in the shape of the investment rate distribution.

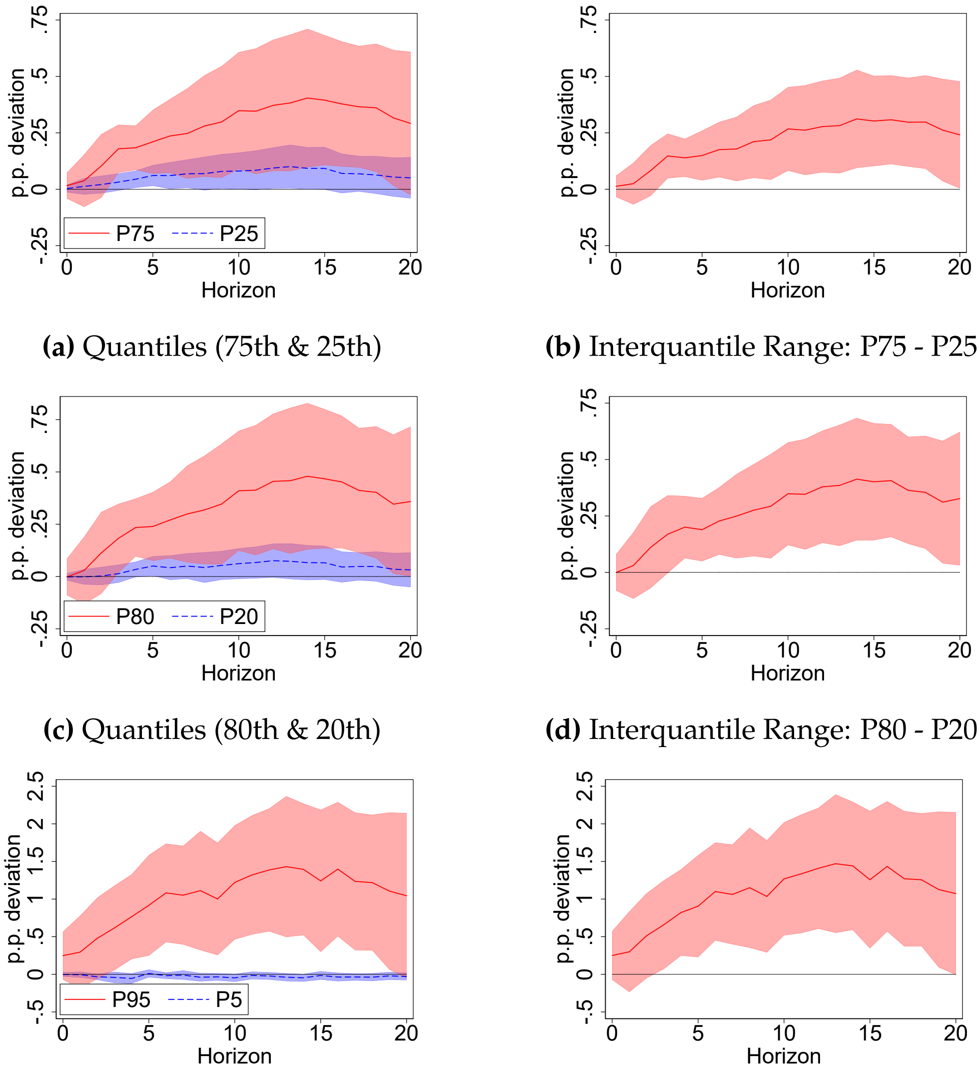
From Quantiles to Distribution To visualize the effect of monetary policy on the distribution of investment rates, we use parametric approximations of the investment

[12] We show the effect of monetary policy on the average investment rate in Panel (a) of Figure 7.

[13] Loria et al. (2022) have recently applied a similar two-step quantile local projection approach to estimate the effects of macroeconomic shocks on the conditional quantiles of GDP growth. Using identified exogenous monetary policy shocks ensures that the estimated IRFs of quantiles are not subject to endogeneity issues.

[14] The impulse response functions of additional quantiles are reported in Figure A.1.

Figure 2: Effects of Monetary Policy Shock on Different Quantiles of Investment Rates



(a) Quantiles (75th & 25th)

(b) Interquantile Range: P75 - P25

(c) Quantiles (80th & 20th)

(d) Interquantile Range: P80 - P20

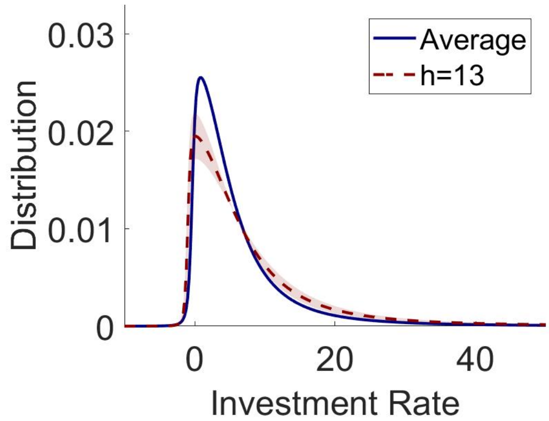
(e) Quantiles (95th & 5th)

(f) Interquantile Range: P95 - P5

Notes: This figure plots the effect of a monetary policy shock on statistics of the investment rate distribution. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$, using monetary policy shocks ϵ_t^{MP} implied by the Proxy SVAR in Gertler and Karadi (2015). The shocks are scaled to reduce the 1-year Treasury rate by 25 basis points. The shaded areas indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

rate distribution. The approximations use quantiles of the distribution and their impulse response functions as inputs. Since the investment rate distribution is skewed with a fat right tail, we choose a flexible skewed-*t* distribution (Azzalini and Capitanio,

Figure 3: Effect of Monetary Policy Shock on Distribution of Investment Rates



Notes: This figure plots the approximated average distribution of investment rates (blue line) as well as the approximated distribution at horizon 13 (peak effect) after a monetary policy shock (red dashed line). The red shaded areas depict the 90% confidence bands constructed using the corresponding confidence bands of the responses of quantiles. The monetary policy shock is scaled by a factor of 10 to make differences in the distribution better visible.

2003) to match it.[15] This approach has recently been applied by Adrian et al. (2019) to transform conditional quantiles into the conditional distribution of GDP growth.

To transform quantiles into a distribution, we estimate the parameters of the skewed-t distribution to match nine quantiles of the investment rate distribution.[16] The blue line in Figure 3 plots the average distribution using quantiles of the time-averaged distribution of firm-level investment rates. To fit the distribution of investment rates after a monetary policy shock, we repeat the estimation with the same quantiles but adding the impulse responses for a given horizon h.

The red line in Figure 3 plots the distribution at the horizon at which the effect of the monetary policy shock peaks.[17] There is a clearly visible change in the distribution of investment rates after an expansionary monetary policy shock. In particular, there are fewer small investment rates and more large investment rates. This suggests that the *average* effect of monetary policy on firm investment rates is driven to a

[15]The skewed-t distribution for a variable y features the following density function:

$$f(y|\mu,\sigma,\alpha,\nu) = \frac{2}{\sigma} t\left(\frac{y-\mu}{\sigma};\nu\right) T\left(\alpha\frac{y-\mu}{\sigma}\sqrt{\frac{\nu+1}{\nu+\left(\frac{y-\mu}{\sigma}\right)^2}};\nu+1\right), \tag{2}$$

where t and T denote the density and cumulative distribution function of the t-distribution, respectively. μ determines the location of the distribution, σ is a scale parameter, ν controls the fatness of the tails, and α governs the skewness as it controls how much the standard t-distribution is skewed.

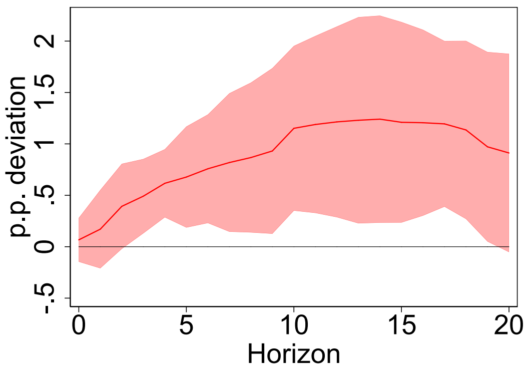
[16]Specifically, we match the 5th, 10th, 15th, 20th, 25th, 35th, 50th, 75th, and 95th percentiles. Our findings are robust to alternative choices of quantiles.

[17]Horizon 13 is when the peak effect on the average investment rate is reached.

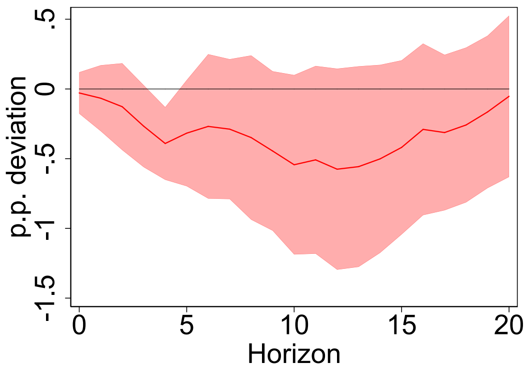
sizeable degree by the extensive margin, i.e., firms switch from making a small or no investment to making a large investment. This aligns well with the evidence that *unconditional* fluctuations in aggregate investment are driven primarily by the extensive margin (Gourio and Kashyap, 2007).

Effect on the Spike and the Inaction Rate To further investigate the hypothesis that the extensive margin is important for the effect of monetary policy on firm investment behavior, we look at two additional statistics of the investment rate distribution, namely, the *spike rate*, defined as the fraction of firms whose quarterly investment rate exceeds 10%, and the *inaction rate*, defined as the fraction of firms whose quarterly investment rate is smaller than 0.5% in absolute value.[18] Indeed, we find that following an expansionary monetary policy shock, the inaction rate falls and the spike rate rises substantially, as shown in Figure 4.

Figure 4: Effects of Monetary Policy Shock on Spike and Inaction Rates



(a) Spike Rate



(b) Inaction Rate

Notes: This figure plots the effect of a monetary policy shock on the spike rate and the inaction rate of all firms. A spike is an investment rate exceeding 10%, inaction is an investment rate less than 0.5% in absolute value. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^4 \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$, using monetary policy shocks ϵ_t^{MP} implied by the Proxy SVAR in Gertler and Karadi (2015). The shocks are scaled to reduce the 1-year Treasury rate by 25 basis points. The shaded areas indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

2.4 Fact 2: Heterogeneous Effects across Age Groups

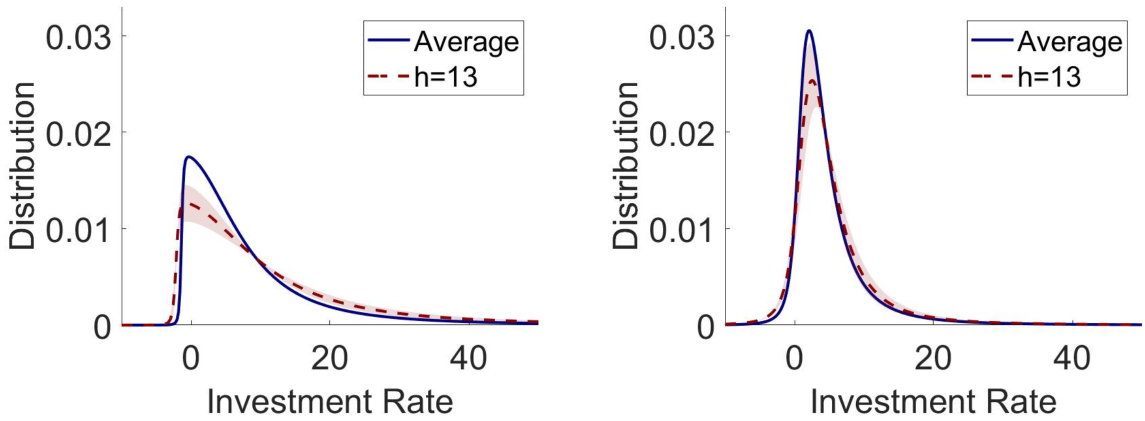
In addition to documenting the effect of monetary policy on the overall distribution of investment rates, we investigate the effect on *group-specific* investment rate distri-

[18]The choice of cutoffs reflects that the investment data is quarterly and features relatively high average investment rates. See footnote 4 for more details.

butions. In particular, we look at age-specific distributions. Cloyne et al. (2020) have documented that after an expansionary monetary shock, young firms increase their investment rates *on average* by much more than old firms. We replicate this finding in Figure A.2. Yet, this difference in averages is only to a limited extent informative about the effect of monetary policy on the age-specific distributions.

Heterogeneous Effects on Quantiles of the Investment Rate Distributions Figures A.3 and A.4 show that the disproportionate effects of monetary policy on the right tail of the investment rate distribution, documented for all firms in Figure 2, are present among both the group of young firms and the group of old firms. Quantitatively, these effects are much more pronounced among young firms, however. Next, we visualize the effect of monetary policy on the age-specific investment rate distributions by means of parametric approximations, using the IRFs of the quantiles.

Figure 5: Effect of Monetary Policy on Age-Specific Distributions of Investment Rates



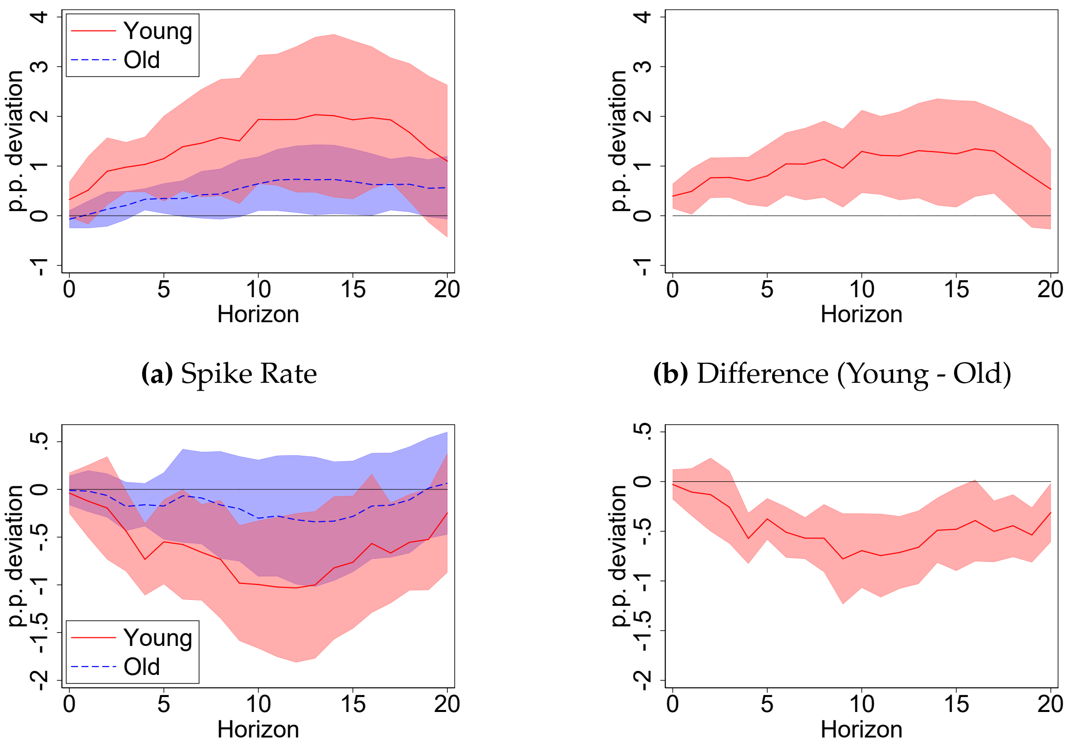
(a) Young Firms **(b)** Old Firms

Notes: This figure plots the approximated average distributions of investment rates (blue lines) as well as the approximated distributions at horizon 13 (peak effect) after a monetary policy shock (red dashed line) for young (panel a) and old (panel b) firms. Young (old) firms are less (more) than 15 years old. The red shaded areas depict the 90% confidence bands constructed using the corresponding confidence bands of the responses of quantiles. The monetary policy shock is scaled by a factor of 10 to make differences in the distribution better visible.

Heterogeneous Effects on Distributions of Investment Rates Figure 5 compares the average distribution of investment rates of young (left panel) and old (right panel) firms with the distribution after a monetary policy shock. We find that the shape of the distribution changes more visibly for young firms. In particular, the decrease in small investment rates and increase in large investment rates is more pronounced.

This suggests that the extensive margin is not only important for the *average* effect of monetary policy on investment rates, but also for the *heterogeneous* effect across age groups.

Figure 6: Effect of Monetary Policy Shock on Age-Specific Spike & Inaction Rates



(a) Spike Rate

(b) Difference (Young - Old)

(c) Inaction Rate

(d) Difference (Young - Old)

Notes: This figure plots the effect of a monetary policy shock on the spike rate and the inaction rate of young and old firms. Young (old) firms are less (more) than 15 years old. A spike is an investment rate exceeding 10%, inaction is an investment rate less than 0.5% in absolute value. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$, using monetary policy shocks ϵ_t^{MP} implied by the Proxy SVAR in Gertler and Karadi (2015). The shocks are scaled to reduce the 1-year Treasury rate by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

Heterogeneous Effects on the Spike and the Inaction Rates To lend further support to the hypothesis that the extensive margin is important for the heterogeneous sensitivity of young and old firms, we look at two additional statistics of the investment rate distribution, namely, the *spike rate* and the *inaction rate*. Figure 6 shows that the spike rate rises and the inaction rate drops more strongly for young firms. Both differences are statistically significant.

2.5 The Relative Importance of the Extensive Margin

Finally, we perform a simple decomposition exercise to gain some insights about the relative importance of the intensive and extensive margin. For this purpose, we classify investment rate observations into "spikes" ($i_{j,t} > 10\%$, as before) and "normal" investments ($i_{j,t} \leq 10\%$). It follows that the average (potentially group-specific) investment rate in period t is

$$\overline{i_t} = \psi^s_t i^s_t + (1 - \psi^s_t) i^n_t \tag{3}$$

where ψ^s_t is the fraction of firms undertaking a "spike" in period t, i^s_t and i^n_t are the average investment rates conditional on "spike" and "normal", respectively. Then, the effect of a monetary policy shock on the average investment rate can be decomposed as follows:[19]

$$\frac{\partial \mathbb{E}(\overline{i_t})}{\partial \epsilon^{MP}} \approx \underbrace{\frac{\partial \mathbb{E}(\psi^s_t)}{\partial \epsilon^{MP}} \left(\mathbb{E}(i^s_t) - \mathbb{E}(i^n_t)\right)}_{\text{Extensive Margin}} + \underbrace{\mathbb{E}(\psi^s_t) \frac{\partial \mathbb{E}(i^s_t)}{\partial \epsilon^{MP}} + (1 - \mathbb{E}(\psi^s_t)) \frac{\partial \mathbb{E}(i^n_t)}{\partial \epsilon^{MP}}}_{\text{Intensive Margin}} \tag{4}$$

Intuitively, the extensive margin reflects the change in the average investment rate that results *only* from changes in the spike rate, while the conditional investment rates are held fixed. Vice versa, the intensive margin reflects the change in the average investment rate that results *only* from changes in the conditional investment rates, while the spike rate is held fixed.

To implement this decomposition, we construct hypothetical average investment rates that would prevail if there were no changes in the extensive margin ($\overline{i_t}^{int}$) or the intensive margin ($\overline{i_t}^{ext}$):

$$\overline{i_t}^{int} = \overline{\psi^s} i^s_t + (1 - \overline{\psi^s}) i^n_t, \tag{5}$$

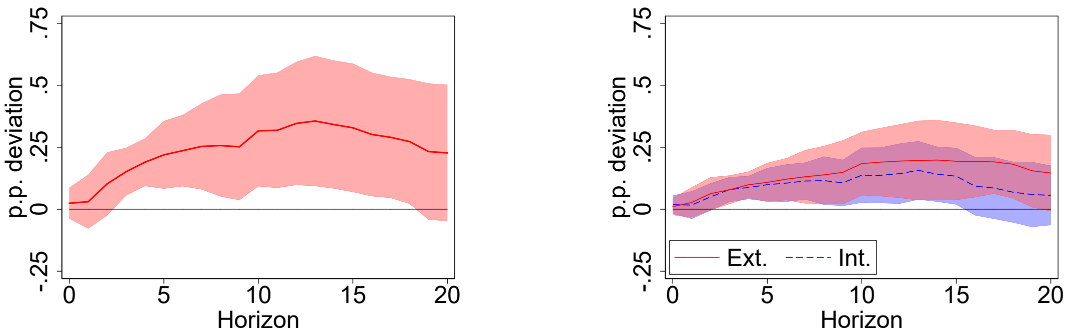
$$\overline{i_t}^{ext} = \psi^s_t \overline{i^s} + (1 - \psi^s_t) \overline{i^n}. \tag{6}$$

$\overline{i_t}^{int}$ captures fluctuations in the average investment rate arising only from the intensive margin, because the spike rate, $\overline{\psi^s}$, equals its average over time. Vice versa, $\overline{i_t}^{ext}$ captures fluctuations in the average investment rate arising only from the extensive margin, because the conditional investment rates, $\overline{i^n}$ and $\overline{i^s}$, equal their respective av-

[19]Note that this decomposition ignores two covariance terms ($Cov(\psi^s_t, i^s_t)$, $Cov(\psi^s_t, i^n_t)$), which can also be affected by the monetary shock. In the data, their contribution to the total effect on the average investment rate is very small, however. Moreover, unlike in the model, we cannot perfectly identify "spikes" in the data. Choosing a particular threshold (e.g., 10%) has the drawback that intensive margin adjustments *across* this threshold are captured as extensive margin adjustments. We, therefore, use the results of this decomposition to get a sense of the quantitative relevance of the extensive margin while acknowledging that the point estimates may be imprecise.

erages over time.

Figure 7: Decomposition of the Effect of Monetary Policy on the Avg. Inv. Rate



(a) Effect on Avg. Inv. Rate

(b) Decomposition: Ext. vs. Int. Margin

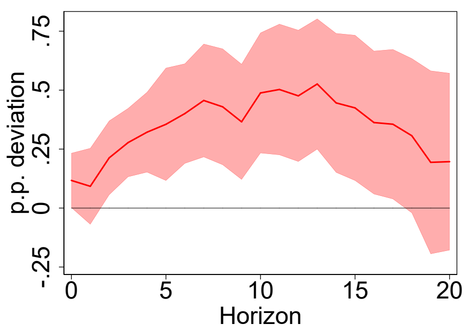
Notes: Panel (a) of this figure shows the effect of a monetary policy shock on the average investment rate ($\bar{i}_t$). Panel (b) decomposes this effect into an intensive ($\bar{i}_t^{\,int}$) and an extensive margin ($\bar{i}_t^{\,ext}$) contribution, using equation (4). The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$, using monetary policy shocks ϵ_t^{MP} implied by the Proxy SVAR in Gertler and Karadi (2015). The shocks are scaled to reduce the 1-year Treasury rate by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

Decomposition of the Effect of a Monetary Policy Shock According to Equation (4), the IRF of the average investment rate ($\bar{i}_t$) is approximately equal to the sum of the IRFs of the two hypothetical investment rates ($\bar{i}_t^{\,int}$ and $\bar{i}_t^{\,ext}$). Figure 7a plots the total effect on the average investment rate and Figure 7b presents the decomposition. It is evident that both margins contribute about 50% to the effect of monetary policy on the *average* investment rate.

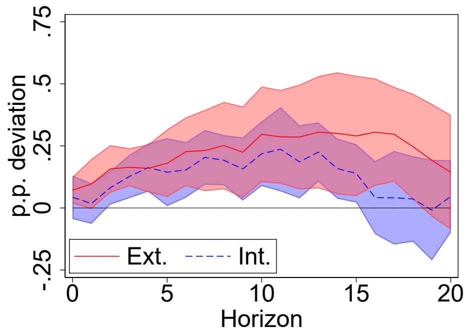
Decomposition of the Heterogeneous Effect of a Monetary Policy Shock Figure 8a plots the estimated impulse response function of the difference between the average investment rates of young and old firms to an expansionary monetary policy shock: i.e., $\frac{\partial \mathbb{E}(\bar{i}_{Y,t+h} - \bar{i}_{O,t+h})}{\partial \epsilon_t^{MP}}$. The average investment rate of young firms responds more to a monetary policy shock than that of old firms. This confirms the findings of Cloyne et al. (2020).

Figure 8b decomposes the heterogeneous effect into the contributions arising from the extensive margin ($\frac{\partial \mathbb{E}(\bar{i}_{Y,t+h}^{\,ext} - \bar{i}_{O,t+h}^{\,ext})}{\partial \epsilon_t^{MP}}$) and the intensive margin ($\frac{\partial \mathbb{E}(\bar{i}_{Y,t+h}^{\,int} - \bar{i}_{O,t+h}^{\,int})}{\partial \epsilon_t^{MP}}$). It shows that the extensive margin explains more than 50% of the heterogeneous sensitivity of young and old firms.

Figure 8: Decomposition of the Heterogeneous Effect of a Monetary Policy Shock



(a) Heterogeneous Effect (Young - Old)



(b) Decomposition: Ext. vs. Int. Margin

Notes: Panel (a) of this figure shows the heterogeneous effect of a monetary policy shock on the average investment rate of young firms as opposed to old firms. Panel (b) decomposes this heterogeneous effect into an intensive and an extensive margin contribution, using equation (4). Young (old) firms are less (more) than 15 years old. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$, using monetary policy shocks ϵ_t^{MP} implied by the Proxy SVAR in Gertler and Karadi (2015). The shocks are scaled to reduce the 1-year Treasury rate by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

Summary of Empirical Evidence We have documented three empirical findings. First, monetary policy reshapes the distribution of investment rates. Specifically, an interest rate cut leads to fewer small or zero investment rates and more large investment rates. Second, the change in the distribution is more pronounced among young firms than among old firms. Third, the extensive margin accounts for around 50% of the effect of monetary policy on the *average* investment rate and for more than 50% of the heterogeneous effect on young and old firms.

Appendix B shows that similar findings emerge when we compare small and large firms, instead of young and old ones. The second part of the paper presents a theoretical model to interpret these empirical findings.

3 A Simple Model

In Section 4, we build a heterogeneous-firm life-cycle model with capital adjustment costs and nominal rigidities. The purpose is to explain the observed effects of interest rate changes on the distribution of investment rates and why these effects are stronger among young firms. In the current section, we illustrate the mechanisms at work through the lens of a simple two-period model. Most importantly, the model features fixed capital adjustment costs which create an extensive margin investment decision.

In this simple model, we compare small and large firms. Since age and size are strongly correlated both in the data and in the quantitative model, all intuitions we provide in the simple model hold true when comparing young and old firms in the quantitative model. In Appendix B, we compare the heterogeneous sensitivity by age and by size in the data and in the quantitative model.

The simple model consists of two periods. In period 0, firms are endowed with k_0 units of capital and choose the next period's capital k_1. The price of one unit of capital relative to the price of the consumption good is q. In period 1, firms transform capital into the consumption good (y) using the decreasing returns to scale production technology $y = k_1^\theta$ with $\theta < 1$. Sales are discounted at the real interest rate r, and capital depreciates fully during production.

In the absence of adjustment costs, the firms' profit-maximization problem is

$$\max_{k_1} \frac{1}{1+r} k_1^\theta - q(k_1 - k_0). \tag{7}$$

From the first-order condition for k_1, we obtain the optimal amount of capital that the firm chooses for period 1

$$k_1^* = \left(\frac{\theta}{(1+r)q}\right)^{\frac{1}{1-\theta}} \tag{8}$$

and the optimal (gross) investment rate as a function of firm size $i^*(k_0) = \frac{k_1^*}{k_0}$.

We now introduce some features from the quantitative model. First, there is a unit mass of firms within each size category k_0 and firms are indexed by j. Second, adjusting the stock of capital is subject to a fixed adjustment cost $\xi_j \in [0, \overline{\xi}]$, which is drawn from a uniform distribution. Moreover, we assume that the economy is populated by firms whose initial capital stocks are below the desired level, i.e., $k_{j,0} < k_1^*$, $\forall k_0$.[20]

The optimization problem of a firm j with an initial stock of capital k_0 has changed to:

$$\max_{k_{1,j}} \frac{1}{1+r} k_{1,j}^\theta - q(k_{1,j} - k_0) - \xi_j \mathbb{1}\{k_{1,j} \neq k_0\}, \tag{9}$$

where $\mathbb{1}\{k_{1,j} \neq k_0\}$ is an indicator variable that equals 1 if $k_{1,j} \neq k_0$ and 0 otherwise. To solve this problem, let $VA(k_0)$ denote the value added of adjusting capital while ignoring the fixed adjustment cost:

$$VA(k_0) = \frac{1}{1+r} k_1^{*\theta} - q(k_1^* - k_0) - \frac{1}{1+r} k_0^\theta, \tag{10}$$

[20] In the steady state of the quantitative model, there are also some firms with capital stocks above their desired level. However, quantitatively, these firms play a minor role.

where k_1^* is the optimal amount of capital that firms will acquire conditional on adjusting as defined by equation (8).

Considering the adjustment cost, a firm j adjusts capital if and only if the value added exceeds the costs, i.e., $VA(k_0) > \xi_j$. The threshold value of ξ_j, which makes a firm indifferent between adjusting or not, is defined by $\xi^T(k_0) \equiv VA(k_0)$. This implies a cutoff rule, i.e., a firm j will adjust its capital stock if and only if $\xi_j < \xi^T(k_0)$. From equation (10), it is evident that this cutoff value not only depends on the initial size of the firm but also on the interest rate r and the other parameters of the model.

The average investment rate among firms of size k_0 is:

$$\bar{i}(k_0) = \lambda(k_0) \times i^*(k_0) \tag{11}$$

where $\lambda(k_0) = \frac{\xi^T(k_0)}{\bar{\xi}} \in [0,1]$ denotes the share of firms of size k_0 that choose to invest, i.e. the hazard rate. Conditional on investing, firms choose the optimal investment rate $i^*(k_0)$ as defined above.

The group-specific interest rate sensitivity of the investment rate is:

$$\frac{\partial \bar{i}(k_0)}{\partial r} = \underbrace{\frac{\partial \lambda(k_0)}{\partial r} i^*(k_0)}_{\text{Extensive Margin}} + \underbrace{\lambda(k_0)\frac{i^*(k_0)}{\partial r}}_{\text{Intensive Margin}}, \tag{12}$$

which features two components. There is an intensive margin effect, $\lambda(k_0)\frac{i^*(k_0)}{\partial r}$, because firms that would be adjusting anyways choose a different investment rate. Moreover, there is an extensive margin effect, $\frac{\partial \lambda(k_0)}{\partial r} i^*(k_0)$, because more or less firms choose to invest at all. Motivated by our empirical findings, this paper emphasizes the extensive margin effect.

Proposition 1 provides the main theoretical findings of this paper, which regard the effect of interest rate changes on the hazard rate ($\frac{\partial \lambda(k_0)}{\partial r}$) as well as how the sensitivity of the average investment rate due to the extensive margin changes with firm size.

Proposition 1. *In an economy populated by heterogeneous firms that face fixed adjustment costs as described above, it holds that*

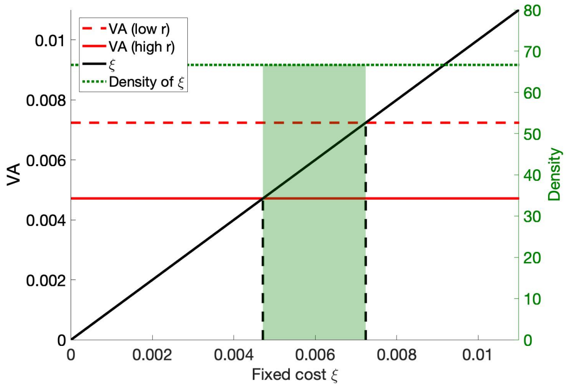
1. *An interest rate cut increases the hazard rate:* $\frac{\partial \lambda(k_0)}{\partial r} < 0$
2. *The sensitivity of the average investment rate to interest rate changes via the extensive margin is decreasing (in absolute terms) in firm size:* $\frac{\partial\left(\frac{\partial \lambda(k_0)}{\partial r} i^*(k_0)\right)}{\partial k_0} > 0$

Proof. See Online Appendix B. □

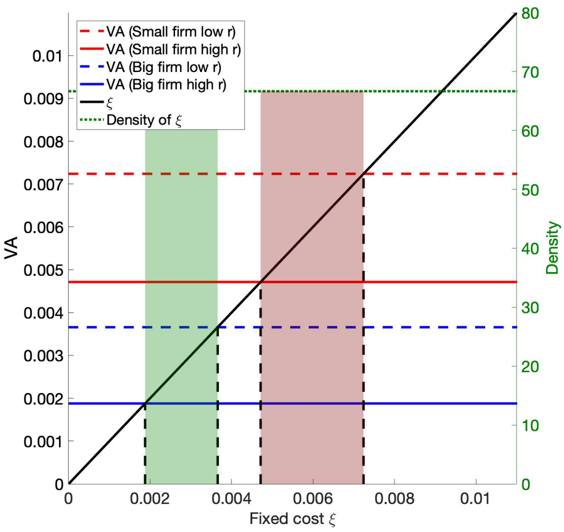
The first part of Proposition 1 establishes that an interest rate cut increases the hazard rate in line with the empirical evidence shown in Figure 6. The costs of investing (cost of additional capital, adjustment cost) are paid in period 0, whereas the benefits materialize in period 1. When the interest rate falls, the discounted benefit of investing rises. Hence, the value added of adjusting and thus the hazard rate rise.[21]

Figure 9a provides visual intuition by plotting the value added for a given k_0, $VA(k_0)$, against the random fixed cost ξ. The black upward-sloping line is the 45° line indicating the points where VA equals ξ. The intercept of the two curves pins down the cutoff value ξ^T. The green dotted line plots the density function of ξ (uniform distribution). The area under the density function to the left of the cutoff value ξ^T is the mass of adjusting firms. An interest rate cut shifts the VA curve upwards. As a result, the cutoff value ξ^T increases and so does the mass of adjusting firms as indicated by the green shaded area.

Figure 9: Intuition for Proposition 1



(a) r-Sensitivity of Hazard Rate



(b) r-Sensitivity by Firm Size

Notes: This figure plots the value added of investing (VA) of a firm against the random fixed cost ξ. The black upward-sloping line is the 45° line indicating the points where VA equals ξ. The intercept of the two curves pins down the threshold value of ξ^T. The green dotted line plots the density function of ξ (uniform distribution). The area under the density function to the left of the threshold value ξ^T is the hazard rate. The shaded area in Panel (a) plots the difference in the hazard rate after an interest rate change. Panel (b) plots the difference in the hazard rate for a small and a big firm.

The second part of Proposition 1 establishes that the effect of an interest rate cut on the group-specific average investment rate via the extensive margin is larger among

[21] In the quantitative model, there are of course additional effects, but the main intuition – an interest rate cut raising the value added of investing – remains the same.

small firms. To understand this result, it is useful to compare the extensive margin effect for groups of small (S) and large (L) firms:

$$
\begin{aligned}
HetExt_{S-L} &= \underbrace{\frac{\partial\lambda(k_{0,S})}{\partial r} i^*(k_{0,S})}_{\text{Small Firms}} - \underbrace{\frac{\partial\lambda(k_{0,L})}{\partial r} i^*(k_{0,L})}_{\text{Large Firms}} \\
&= \underbrace{\frac{\partial\lambda(k_{0,S})}{\partial r} \left(i^*(k_{0,S}) - i^*(k_{0,L})\right)}_{\text{Heterogeneous Size Effect}} + \underbrace{\left(\frac{\partial\lambda(k_{0,S})}{\partial r} - \frac{\partial\lambda(k_{0,L})}{\partial r}\right) i^*(k_{0,L})}_{\text{Heterogeneous Hazard Rate Increase}} \qquad (13)
\end{aligned}
$$

This decomposition shows that there are two mechanisms. First, there is the *heterogeneous size effect*, due to which even if an interest rate cut had the same effect on hazard rates of small and large firms, there would be a differential effect on average investment rates. This is because among the *new* adjusters, small firms have higher investment rates conditional on adjusting ($i^*(k_{0,S}) - i^*(k_{0,L}) > 0$). This follows from the observation that in this simple model, conditional on investing, all firms choose k_1^* and the investment rate is defined by $i^* = \frac{k_1^*}{k_0}$. In the absence of an extensive margin investment decision, this effect would disappear because $\frac{\partial\lambda(k_0)}{\partial r} = 0$.

Second, interestingly, an interest rate cut increases the hazard rate of small firms by more than the hazard rate of large firms. This result aligns well with the empirical evidence that the spike rate of small (young) firms reacts more strongly to a monetary shock than the spike rate of large (old) firms (see Figure B.2 for size and Figure 6 for age). As discussed above, the hazard rate rises, because the value added of investing rises, which happens because the discounted benefit of investing rises. This increase in the discounted benefit of investing is larger for small firms. The reason for this is that small firms have a higher marginal product of capital because of decreasing returns to scale. Hence, the interest rate cut has a larger effect on the hazard rate of small firms.

Figure 9b provides visual intuition for the heterogeneous effect of an interest rate cut on hazard rates. The cut in the interest rate shifts the VA of small firms (red lines) up by more than the VA of big firms (blue lines). As a result, the change in the hazard rate is more pronounced for small firms (red-shaded area) than for big firms (green-shaded area).

To sum up, we have highlighted two effects in this simple model. First, an interest rate cut increases the hazard rate, i.e. the fraction of firms deciding to make an investment. Therefore, a change in the interest rate changes the distribution of investment rates. Second, the average investment rate of small firms responds more strongly along the extensive margin to interest rate changes than the average investment rate of large firms.

Regarding the second effect, it is worth pointing out that small firms are more sensitive to interest rate changes *in the absence* of a financial accelerator mechanism. The basic idea of the financial accelerator mechanism is that interest rate changes affect financing conditions and small firms are more exposed to financing conditions than large firms. Then, interest rate changes have a heterogeneous effect on investment because there is a heterogeneous effect on the *cost* of investing, as e.g. in Ottonello and Winberry (2020). In contrast, in this model, there is a heterogeneous effect of interest rate changes on investment because of a heterogeneous effect on the *benefit* of investing.[22] This is because small firms have a higher marginal product of capital.

In the next section, we quantify the mechanisms highlighted in this section in a general equilibrium model.

4 A Quantitative Model

We build a New Keynesian model with heterogeneous firms subject to convex and fixed capital adjustment costs and entry and exit. These features have been studied separately; see, e.g., Khan and Thomas (2008), Clementi and Palazzo (2016), Ottonello and Winberry (2020), and Koby and Wolf (2020). The novelty of our model is to combine all these ingredients that are relevant for the understanding of the effect of monetary policy on age-specific investment rate distributions.

4.1 Investment Block

There exists a continuum of production firms[23] in the economy. Each firm j produces a quantity y_{jt} of the intermediate good using the production function

$$y_{jt} = z_{jt} k_{jt}^{\theta} n_{jt}^{\nu} \qquad \text{with } \theta, \nu > 0 \text{ and } \theta + \nu < 1 \tag{14}$$

where z_{jt} is total factor productivity (TFP), k_{jt} is the capital stock, and n_{jt} is the labor input. Productivity z_{jt} is subject to idiosyncratic shocks and follows an AR(1) process in logs

$$\log z_{jt} = \rho_z \log z_{jt-1} + \sigma_z \epsilon_{jt}^z \qquad \text{with } \epsilon_{jt}^z \sim \mathcal{N}(0,1) \tag{15}$$

[22] Even though the capital adjustment costs that we impose can in principle be interpreted as stand-ins for financial frictions, the model does not feature a financial accelerator mechanism. This is because by construction, the capital adjustment costs are themselves not affected by aggregate shocks, including monetary policy shocks.

[23] We normalize the mass of firms to 1. Since entry and exit is exogenous, the mass of firms does not vary in response to aggregate shocks. While our model also features retailers, a final good producer, and a capital good producer, we only refer to intermediate good producers as firms.

Labor n_{jt} can be adjusted frictionlessly in every period. Capital k_{jt} is accumulated according to

$$k_{jt+1} = (1-\delta)k_{jt} + i_{jt} \tag{16}$$

where i_{jt} is investment and δ the depreciation rate. The relative price of capital (in terms of the final good) is q_t.

Following Bachmann et al. (2013), we include *maintenance investment*. That is, a fraction χ of the depreciation δk_{jt} that occurs during the production process needs to be replaced immediately. At the end of the period, firms have $(1-\delta(1-\chi))k_{jt}$ units of capital and decide how much to invest voluntarily. To this voluntary investment, i^v_{jt}, there are capital adjustment costs, which need to be paid if $i^v_{jt} \neq 0$.[24] Total adjustment costs consist of a random fixed adjustment cost $w_t\xi_{jt}$, where ξ_{jt} is distributed uniformly between 0 and $\bar{\xi}$, and a convex adjustment cost $\frac{\phi}{2}\frac{{i^v_{jt}}^2}{k_{jt}}$:

$$AC(k_{jt}, k_{jt+1}, \xi_{jt}) = w_t\xi_{jt}\mathbb{1}\{k_{jt+1} \neq (1-\delta(1-\chi))k_{jt}\} + \frac{\phi}{2}\frac{(k_{jt+1}-(1-\delta(1-\chi))k_{jt})^2}{k_{jt}} \tag{17}$$

where w_t is the real wage. Total investment is the sum of voluntary investment and maintenance investment.

Entry & Exit Firms face independent and identically distributed (i.i.d.) exit shocks ϵ^{exit}_{jt} and are forced to exit the economy at the end of the period with probability π^{exit}. Each period, a fixed mass of newborn firms enters the economy. These entrants are endowed with k_0 units of capital and draw their initial productivity level from the distribution $\mu^{ent} \sim \mathcal{N}(0, \frac{\sigma_z^2}{1-\rho_z^2})$, which is the ergodic distribution of (15).

Timing Within any period, the timing is as follows. At stage one, idiosyncratic TFP shocks to incumbent firms realize. At stage two, a fixed mass of firms enters the economy. Entrants draw their initial productivity from μ^{ent} and are endowed with k_0 units of capital from the household. Henceforth, they are indistinguishable from incumbent firms. At stage three, firms hire labor and production takes place. Firms conduct maintenance investment. At stage four, exit shocks realize and random fixed adjustment costs are drawn. Exiting firms sell their capital stock and leave the economy. Continuing firms decide whether to adjust their capital stock or remain inactive.

[24]Matching the empirical distribution of investment rates requires a rich adjustment cost specification, as discussed in Cooper and Haltiwanger (2006).

Value Functions We characterize the firm's optimization problem recursively. The individual state variables are total factor productivity z and capital k. Subscripts for individual variables are henceforth dropped for readability and primes denote next period's values. The beginning-of-period real firm value is

$$V_t(z,k) = \max_n p_t z k^\theta n^\nu - w_t n + \pi^{exit} CV_t^{exit}(z,k) + (1-\pi^{exit}) \int_0^{\bar{\xi}} CV_t(z,k,\xi) d\xi \tag{18}$$

where CV_t^{exit} and CV_t denote the continuation values of exiting and surviving firms, respectively. With probability π^{exit}, a firm is forced to exit after the production stage. Exiting firms have the liquidation value

$$CV_t^{exit}(z,k) = (1-\delta) q_t k. \tag{19}$$

Note that exiting firms do not need to pay capital adjustment costs. Therefore, maintenance investment does not affect the liquidation value.

The continuation value of a surviving firm is

$$CV_t(z,k,\xi) = \max\left\{CV_t^a(z,k,\xi), CV_t^n(z,k)\right\}, \tag{20}$$

which reflects that surviving firms can decide whether to adjust their capital stock (CV_t^a) or not (CV_t^n). The continuation value of not adjusting is:

$$CV_t^n(z,k) = \mathbb{E}_t\left[\Lambda_{t+1} V_{t+1}(z', (1-\delta(1-\chi))k)\right] - q_t \chi \delta k, \tag{21}$$

while the continuation value of a firm that adjusts its capital stock is:

$$CV_t^a(z,k,\xi) = \max_{k'} \mathbb{E}_t\left[\Lambda_{t+1} V_{t+1}(z',k')\right] - q_t\left(k' - (1-\delta)k\right) - AC(k,k',\xi). \tag{22}$$

Policy Functions The labor decision in equation (18) is static and independent of the capital decision

$$n_t^*(z,k) = \left(\frac{p_t \nu z k^\theta}{w_t}\right)^{\frac{1}{1-\nu}}. \tag{23}$$

Thus, earnings net of labor costs are

$$\pi_t(z,k) \equiv p_t z k^\theta (n_t^*)^\nu - w_t n^*. \tag{24}$$

The optimal capital decision is computed as follows. First of all, the solution to the maximization problem in equation (22) is the policy function $k_t^a(z,k)$, which is independent of ξ. This policy function allows us to compute $CV_t^a(z,k,\xi)$. Since, $CV_t^a(z,k,\xi)$ depends on ξ linearly, we can formulate a cutoff rule for the maximization problem in equation (20). Firms choose to adjust capital if and only if their fixed adjustment cost draw ξ is smaller or equal $\xi_t^T(z,k)$:

$$k_t^*(z,k,\xi) = \begin{cases} k_t^a(z,k) & \text{if } \xi \leq \xi_t^T(z,k) \\ (1-\delta(1-\chi))k & \text{if } \xi > \xi_t^T(z,k) \end{cases} \tag{25}$$

where $\xi_t^T(z,k) = \frac{CV_t^a(z,k,\xi=0) - CV_t^n(z,k)}{w_t}$.

As in the simple model, we define the hazard rate $\lambda_t(z,k)$ as:

$$\lambda_t(z,k) = \begin{cases} 0 & \text{if } \xi_t^T(z,k) \leq 0 \\ \frac{\xi_t^T(z,k)}{\bar{\xi}} & \text{if } 0 < \xi_t^T(z,k) \leq \bar{\xi}\,. \\ 1 & \text{if } \bar{\xi} < \xi_t^T(z,k) \end{cases} \tag{26}$$

4.2 New Keynesian Block

We separate nominal rigidities from the investment block of the model. A fixed mass of retailers $i \in [0,1]$ produces differentiated varieties $\widetilde{y}_{it}$ from the undifferentiated intermediate goods produced by the production firms. There is a one-to-one production technology $\widetilde{y}_{it} = y_{it}$, where y_{it} is the amount of the intermediate good retailer i purchases. Retailers face Rotemberg quadratic price adjustment costs $\frac{\varphi}{2}\left(\frac{\widetilde{p}_{it}}{\widetilde{p}_{it-1}} - 1\right)^2 Y_t$, where $\widetilde{p}_{it}$ is the relative price of variety i.

A representative final good producer aggregates the differentiated varieties optimally into the final good according to

$$Y_t = \left(\int \widetilde{y}_{it}^{\frac{\gamma-1}{\gamma}}\, di\right)^{\frac{\gamma}{\gamma-1}} \tag{27}$$

The resulting demand function for retail good $\widetilde{y}_{it}$ is:

$$\widetilde{y}_{it} = \left(\frac{\widetilde{p}_{it}}{P_t}\right)^{-\gamma} Y_t, \tag{28}$$

where $P_t = \left(\int_0^1 \widetilde{p}_{it}^{1-\gamma} di\right)^{\frac{1}{1-\gamma}}$ is the price of the final good.

The optimization problem of a monopolistically competitive retailer i is:

$$\max_{\{\widetilde{p}_{it}\}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \Lambda_t \left\{ (\widetilde{p}_{it} - p_t)\widetilde{y}_{it} - \frac{\varphi}{2} \left(\frac{\widetilde{p}_{it}}{\widetilde{p}_{it-1}} - 1 \right)^2 Y_t \right\} \right] \tag{29}$$

subject to the demand curve (28). We log-linearize the optimality condition of the retailer's problem to obtain the familiar New Keynesian Phillips Curve (NKPC):

$$\log(1 + \pi_t) = \frac{\gamma - 1}{\varphi} \log \frac{p_t}{p^*} + \beta\, \mathbb{E}_t \log(1 + \pi_{t+1}) \tag{30}$$

where $\pi_t \equiv P_t / P_{t-1} - 1$ is the inflation rate, $p^* = \frac{\gamma-1}{\gamma}$ is the relative price (in terms of the final good) of the intermediate good in steady state.

4.3 Capital Good Producer

There is a representative capital good producer operating in a perfectly competitive market. It transforms units of the final good into new capital subject to external capital adjustment costs:

$$I_t = \left[\frac{\delta^{1/\kappa}}{1 - 1/\kappa} \left(\frac{I_t^Q}{K_t} \right)^{1-1/\kappa} - \frac{\delta}{\kappa - 1} \right] K_t, \tag{31}$$

where I_t^Q represents the amount of the final good used, I_t the amount of new capital produced, and K_t is the total stock of capital in the beginning of period t. The parameter κ determines the strength of external capital adjustment costs. The static optimization problem is:

$$\max_{I_t} q_t I_t - I_t^Q. \tag{32}$$

Optimal behavior implies that the relative price of capital (q_t) has to satisfy the following condition

$$q_t = \left(\frac{I_t^Q / K_t}{\delta} \right)^{1/\kappa}. \tag{33}$$

4.4 The Central Bank

The central bank sets the nominal interest rate r_t^n according to a Taylor rule

$$\log(1 + r_t^n) = \rho_r \log(1 + r_{t-1}^n) + (1 - \rho_r) \left[\log \frac{1}{\beta} + \varphi_\pi \log(1 + \pi_t) \right] + \epsilon_t^m, \tag{34}$$

where ϵ_t^m is a monetary policy shock, ρ_r is the interest rate smoothing parameter, and φ_π is the reaction coefficient to inflation.

4.5 Household

There is a representative household, which consumes C_t^h, supplies labor N_t^h, and saves or borrows in one-period non-contingent bonds B_t^h.

The household's objective is to maximize expected lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log(C_t^h) - \psi N_t^h\right), \tag{35}$$

subject to the flow budget constraint:

$$P_t C_t^h + Q_t^B B_t^h \leq B_{t-1}^h + W_t N_t^h + \Pi_t, \tag{36}$$

where Q_t^B is the nominal one-period risk-free bond price (one unit of B_t pays one unit of currency at $t+1$), W_t is the nominal wage, and Π_t subsumes additional transfers to and from the household.[25]

Solving the household's optimization problem leads to the following optimality conditions

$$\Lambda_{t+1} \equiv \beta \mathbb{E}_t \left[\frac{C_t^h}{C_{t+1}^h}\right], \tag{37}$$

$$w_t = \psi C_t^h, \tag{38}$$

where Λ_{t+1} is the household's stochastic discount factor between period t and $t+1$, and w_t is the real wage.

Online Appendix C.1 defines an equilibrium in this economy.

5 Quantitative Results

We calibrate the model to the U.S. economy. Wherever possible, we rely on data sources which refer to the entire economy.

We fix a subset of parameters to conventional values. These parameters are listed

[25] Π_t includes dividends from intermediate good producers, retailers, and the final good producer, as well as the initial capital endowment k_0, which entering firms receive from the household. We follow Winberry (2021) and do not rebate back adjustment costs to the household in a lump-sum manner. Therefore, convex adjustment costs do exhaust the aggregate resource constraint.

in Table 1. Given these fixed parameters, we fit the remaining parameters to match the moments listed in Table 3. The fitted parameters are listed in Table 2.

Since a model period corresponds to a quarter, the discount factor is set to $\beta = 0.99$. The labor disutility parameter is set to $\psi = 0.45$.[26] Capital and labor coefficients are set to standard values, that is, $\theta = 0.21$ and $\nu = 0.64$ (Ottonello and Winberry, 2020). The depreciation rate δ generates an annual aggregate investment rate of 7.7% as reported in Zwick and Mahon (2017). We target the standard deviation of idiosyncratic TFP shocks σ_z, but fix their persistence ρ_z due to the identification problem discussed in Clementi and Palazzo (2015). We set ρ_z to 0.95 (Khan and Thomas, 2008; Bloom et al., 2018). The exogenous exit probability π^{exit} is set to 1.625% as in Koby and Wolf (2020).[27]

We choose standard values for the parameters of the New Keynesian block, i.e. $\varphi = 90$ and $\gamma = 10$ (Ottonello and Winberry, 2020). The coefficient on inflation in the Taylor rule φ_π is set to 1.5 and the interest rate smoothing parameter ρ_r is set to 0.75. External capital adjustment costs κ are set to 8 to roughly match the peak effect of a monetary policy shock on investment relative to the peak effect on output documented in Section 2.

Table 1: Fixed Parameters

Parameter	Description	Value
Household		
β	Discount factor	0.99
ψ	Labor Disutility	0.45
Investment Block		
θ	Capital Coefficient	0.21
ν	Labor Coefficient	0.64
δ	Depreciation Rate	1.93%
ρ_z	Persistence of TFP Shock	0.95
π^{exit}	Exogenous Exit Probability	1.63%
New Keynesian Block		
φ	Price Adjustment Cost	90
γ	Elasticity of Substitution over Intermediate Goods	10
φ_π	Taylor Rule Coefficient on Inflation	1.5
ρ_r	Interest Rate Smoothing	0.75
κ	External Capital Adjustment Costs	8

The five parameters listed in Table 2 are chosen to match five targeted moments listed in Table 3. Even though all parameters are calibrated jointly, we briefly explain

[26] This value follows from normalizing the steady-state real wage w to 1.

[27] This exit probability brings the age distribution as close to the data as possible without using age-specific exit probabilities.

Table 2: Fitted Parameters

Parameter	Description	Value
σ_z	Volatility of TFP Shock	0.060
k_0	Initial Capital of Entrants	4.025
$\bar{\xi}$	Upper Bound on Fixed Adjustment Cost	0.350
ϕ	Convex Adjustment Cost	0.750
χ	Maintenance Investment Parameter	0.375

Table 3: Empirical & Simulated Moments

Moment	Data	Model
Standard Deviation of Investment Rates	0.200	0.203
Average Investment Rate	0.119	0.136
Autocorrelation of Investment Rates	0.380	0.377
Relative Size of Entrants	0.285	0.297
Relative Spike Rate of Old Firms	0.400	0.380

Notes: Data moments related to investment rates are taken from Zwick and Mahon (2017) (Appendix, Table B.1, Unbalanced Sample). The relative spike rate of old firms is computed from Compustat data. Corresponding model moments are computed from a simulation of a large panel of firms. The relative size of entrants is taken from Business Dynamics Statistics (BDS). In the model, this moment can be computed from the steady-state distribution.

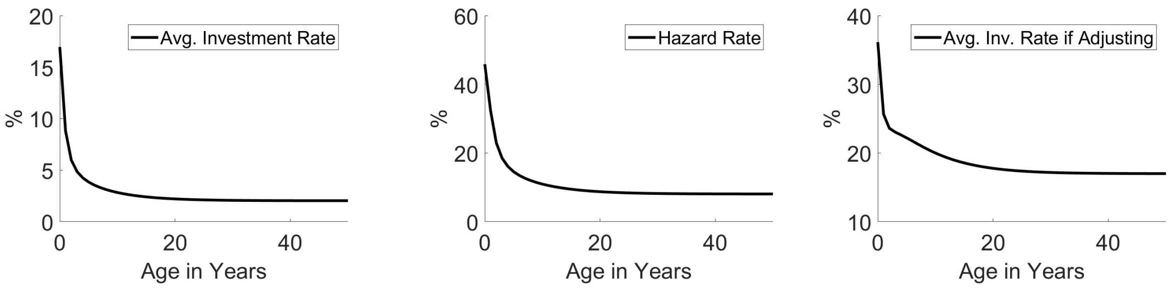
which moments are particularly informative about which parameters.

First, we target the standard deviation of investment rates, because it is informative about the volatility of idiosyncratic TFP shocks. Second, we target the average investment rate as it is informative about both adjustment cost parameters. Increasing either adjustment cost dampens investment rates in particular of young firms and therefore the average investment rate. Third, we target the autocorrelation of investment rates, because it is informative about the relative importance of fixed and convex adjustment costs. Convex adjustment costs generate a positive autocorrelation, whereas fixed adjustment costs generate a negative or zero autocorrelation. For these three moments, we use the statistics reported in Zwick and Mahon (2017). Fourth, we target the relative size of entrants, which is informative about the initial capital of entrants. This moment is computed from Business Dynamics Statistics (BDS) data. Fifth, we target the spike rate of old firms relative to the spike rate of young firms, which is informative about the maintenance investment parameter. The more depreciation is undone by maintenance investment, the less frequently do old firms need to make an extensive margin investment. Thus, a higher maintenance parameter leads to a lower spike rate among old firms. This moment needs to be computed from Compustat data, since it is the only data source which includes both investment rates and firm age.

5.1 Firm Life-Cycles and the Aggregate Effects of Monetary Policy

Before moving to the key findings of the paper, we show that the model is capable of reproducing well-known facts regarding (i) firms' life-cycles and (ii) the aggregate effects of monetary policy shocks.

Figure 10: Life-Cycle Profiles



(a) Avg. Investment Rate **(b)** Hazard Rate **(c)** Avg. Cond. Inv. Rate

Notes: Investment rates and the hazard rate refer to a quarter. Averages are computed from the steady state distribution.

Firm Life-Cycle Profiles Figure 10 shows that the model matches several untargeted investment life-cycle profiles. The empirical counterparts are shown in Figure A.5. Panel (a) shows that the average investment rate is higher for young firms and falls monotonically in age. Panels (b) and (c) decompose this average investment rate into the average probability to invest ("hazard rate") and the average investment rate conditional on investing. Evidently, the observation that young firms have higher average investment rates is driven in part by higher hazard rates and in part by a higher investment rate conditional on investing.

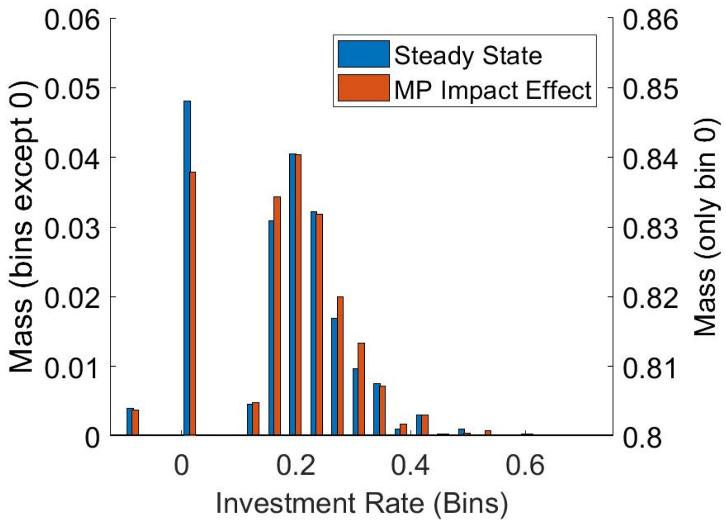
The Aggregate Effects of Monetary Policy Shocks We study the effects of an unexpected expansionary monetary policy shock followed by a perfect foresight transition back to steady state.[28] Figure A.6 plots the impulse response functions of aggregates and prices, which confirm that our model produces the typical New Keynesian effects to a monetary policy shock.

[28]This approach to constructing impulse response functions to aggregate shocks follows Boppart et al. (2018). The size of the monetary shock is chosen to roughly match the peak effects on output and investment seen in the data.

5.2 Monetary Policy and the Distribution of Investment Rates

Turning to the main focus of this paper, Figure 11 plots the effect of a monetary policy shock on the distribution of investment rates. Specifically, it plots the distribution of investment rates in steady state (blue bars) and in the period when an expansionary monetary policy shock has hit the economy (red bars). It is apparent that monetary policy affects some firms' extensive margin investment decision and therefore the distribution of investment rates: after an interest rate cut, there are fewer inactive firms and more firms choosing to make an investment. This observation corresponds to **Fact 1** documented in Section 2.3. Figure 12 shows that also the impulse response functions of spike and inaction rates as well as the relative movements of the quantiles of the investment rate distribution match the empirical evidence (see Figures 2 and 4).

Figure 11: Effect of Monetary Policy on the Distribution of Investment Rates

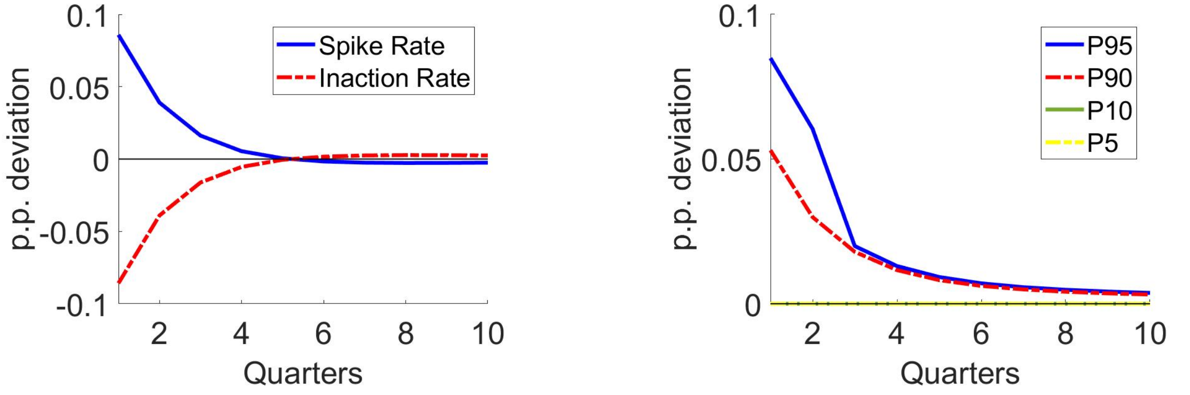


Notes: This figure plots the distribution of investment rates in steady state (blue bars) and after a monetary policy shock (red bars). The monetary policy shock is scaled by a factor of 10 to make differences in the distribution better visible.

As in the data, monetary policy affects the average investment rate not only via the extensive margin, but also via the intensive margin. To assess the relative importance of both margins, we decompose the effect on the average investment rate into contributions of the extensive and intensive margin, similar to the empirical exercise presented in Figure 7.[29] Figure 13 presents the findings. Panel (a) plots the effect of an expansionary monetary policy shock on the average investment rate. Panel (b) plots the decomposition. Evidently, the model attributes a significant portion of the change

[29]This decomposition is computed by holding either hazard rates at steady-state levels (intensive margin contribution) or investment rates conditional on investing at steady-state levels (extensive margin contribution), see also Equation (4).

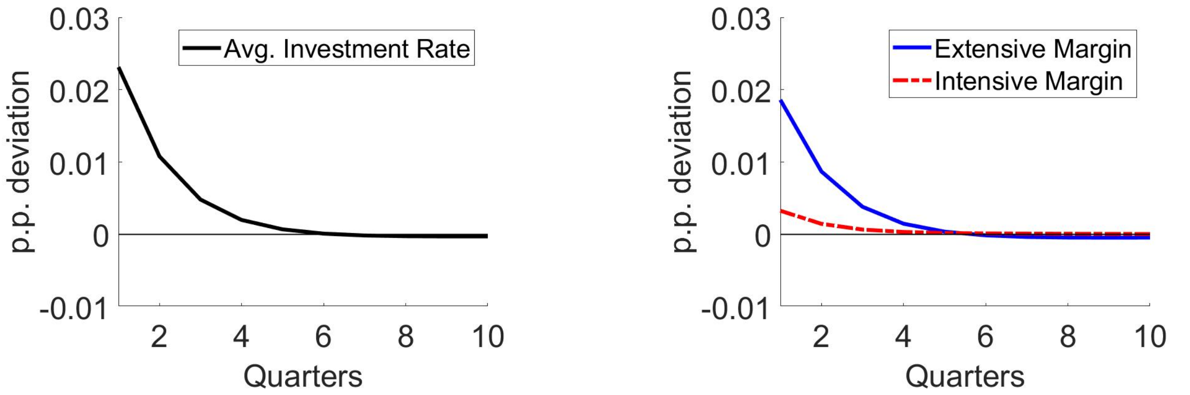
Figure 12: Effect of Monetary Policy on Spike Rate, Inaction Rate, Quantiles



(a) Spike & Inaction Rate **(b)** Quantiles of Distribution

Notes: Panel (a) of this figure plots the effect of a monetary policy shock on the spike and inaction rate of all firms. Panel (b) plots the IRFs of certain quantiles of the investment rate distribution.

Figure 13: Effects of Monetary Policy: Extensive & Intensive Margin



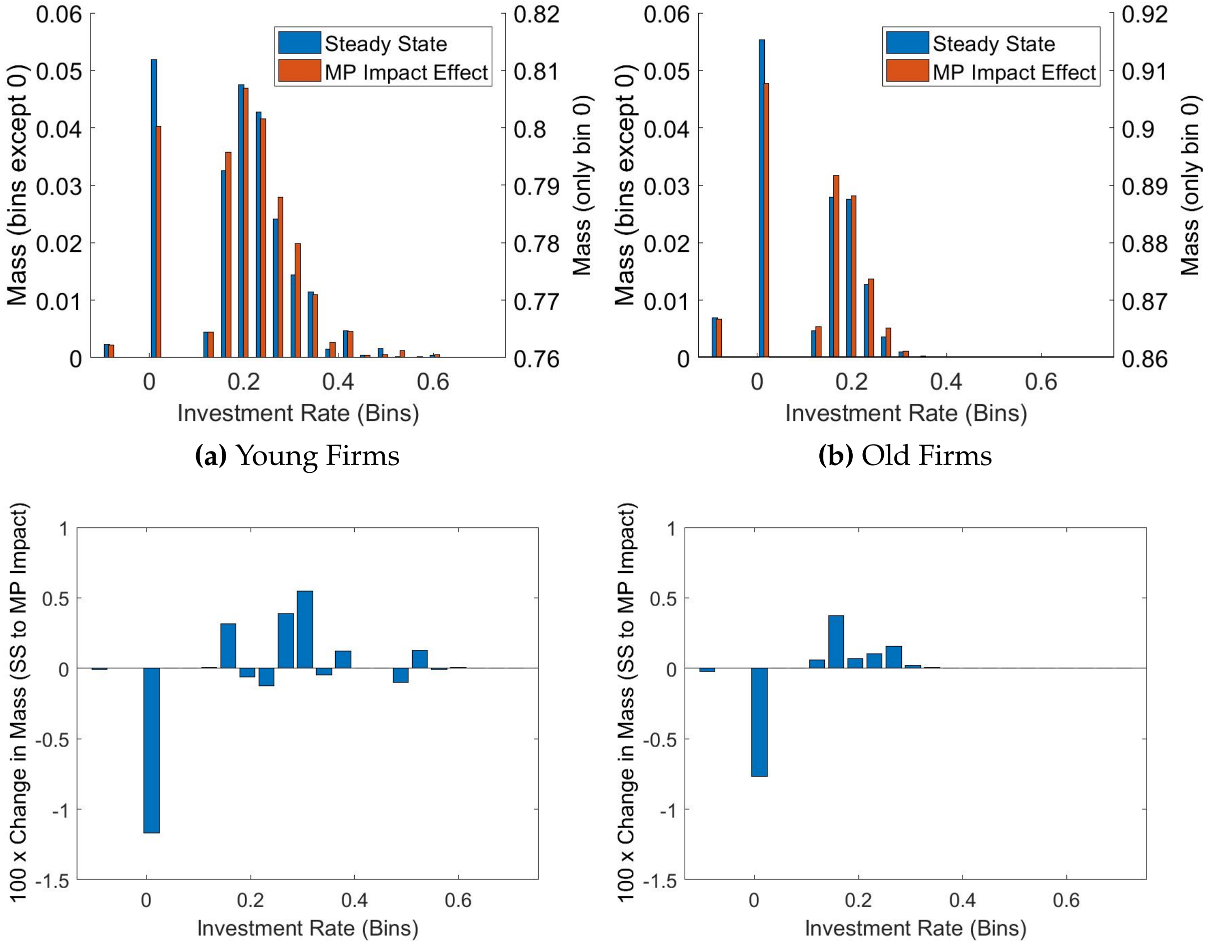
(a) Avg. Investment Rate **(b)** Decomposition

Notes: Panel (a) of this figure plots the effect of a monetary policy shock on the average investment rate of all firms. Panel (b) decomposes the IRF in panel (a) into an extensive margin contribution and an intensive margin contribution.

in the average investment rate to the extensive margin.

Heterogeneous Sensitivity of Young Firms In addition, the model reproduces the empirical finding that the effect of monetary policy on the distribution of investment rates is heterogeneous across different age groups, as shown in Figure 14. This corresponds to **Fact 2**. Panels (a) and (b) plot the distribution of investment rates before and after an expansionary monetary policy shock of young and old firms, respectively. The bottom panels plot the changes in the distribution. This shows that after an interest

Figure 14: Effect of Monetary Policy on the Distribution of Inv. Rates (by Age Group)



(a) Young Firms

(b) Old Firms

(c) Young Firms (Difference)

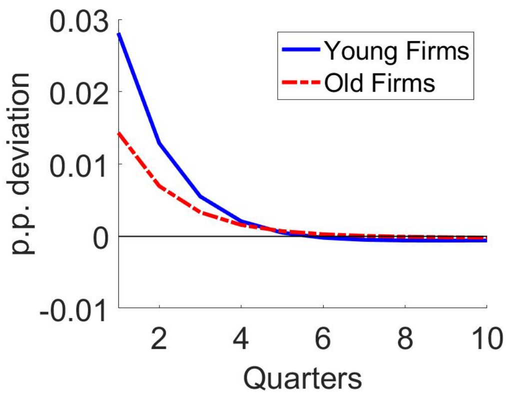
(d) Old Firms (Difference)

Notes: Panels (a) and (b) of this figure plot the distribution of investment rates of young (old) firms in steady state and after a monetary policy shock. Panels (c) and (d) plot the difference of the two distributions for young (old) firms. The monetary policy shock is scaled by a factor of 10 to make differences in the distribution better visible.

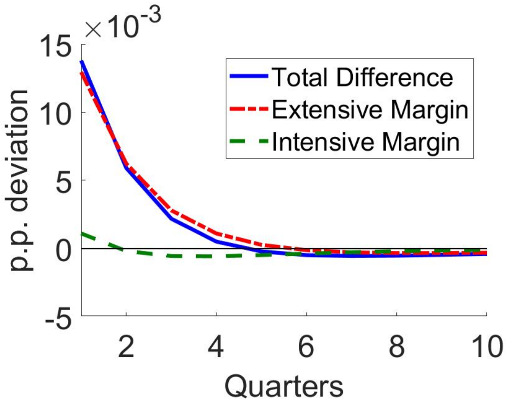
rate cut, there are more young firms than old firms switching from being inactive to making a large investment. That is, the effect of monetary policy along the extensive margin is more pronounced among young firms.

Due to the heterogeneous effect along the extensive margin, monetary policy affects *average* investment rates differently across age groups. Figure 15 plots the effect of an expansionary monetary policy shock on average investment rates by age group. Panel (a) shows that young firms increase their investment rates on average more strongly than old firms. Panel (b) decomposes this heterogeneous sensitivity into extensive and intensive margin contributions, similar to the empirical exercise shown in Figure 8b, and demonstrates that the total difference is driven by the exten-

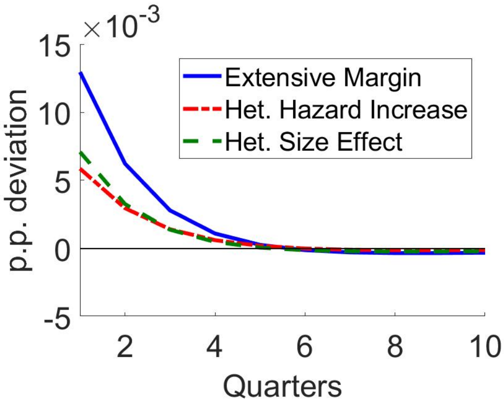
Figure 15: Heterogeneous Effect of Monetary Policy (by Age Group)



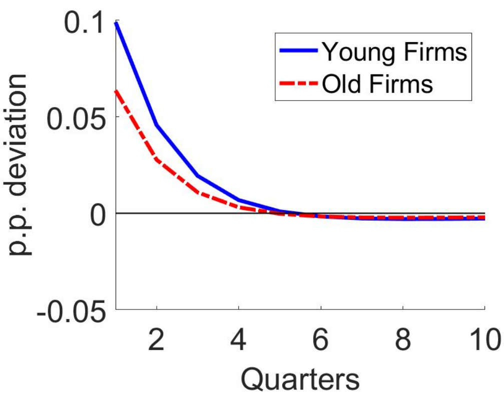
(a) Avg. Investment Rate



(b) Extensive vs. Intensive Margin



(c) Extensive Margin Decomposition



(d) Spike Rate

Notes: Panel (a) of this figure plots the effect of a monetary policy shock on the average investment rates of young and old firms. Panel (b) decomposes the differences of the two IRFs in panel (a) into an extensive margin contribution and an intensive margin contribution. Panel (c) further decomposes the IRF of the extensive margin contribution in panel (b) into the heterogeneous hazard rate increase and the heterogeneous size effect. Panel (d) plots the effect of a monetary policy shock on the spike rates of young and old firms.

sive margin. Panel (c) further decomposes the extensive margin into the two mechanisms identified in the simple model (Equation 13). On the one hand, the hazard rate rises more strongly among young firms (heterogeneous hazard rate increase), which is separately shown in panel (d). On the other hand, new young adjusters on average have a higher investment rate than new old adjusters (heterogeneous size effect). Quantitatively, the heterogeneous size effect is slightly more important.

To summarize, these results confirm that the two effects identified in the simple model in Section 3 hold and are quantitatively relevant in a calibrated general equilibrium heterogeneous-firm model. First, there is an important investment channel of monetary policy along the extensive margin. Second, this effect does not affect all firms homogeneously: young (small) firms' average investment rates are more sensitive to monetary policy even in the absence of a financial accelerator mechanism.

6 Conclusion

In this paper, we highlight two features of the investment channel of monetary policy. First, there is a quantitatively relevant investment channel along the extensive margin. That is, an interest rate cut induces some firms to switch from making a small or no investment to making a sizeable one. Second, along the extensive margin, young firms respond more strongly to interest rate changes than old firms. Therefore, young firms are more sensitive to monetary policy even in the absence of a financial accelerator mechanism.

We present three pieces of evidence in line with these effects. First, monetary policy affects the shape of the distribution of investment rates. Specifically, an interest rate cut leads to fewer small or zero investment rates and more large investment rates. Second, this change in the distribution is more pronounced among young firms than among old firms. Third, a decomposition exercise indicates that the extensive margin accounts for around 50% of the effect of monetary policy on the average investment rate and for more than 50% of the heterogeneous effect on firms of different age groups.

We build a heterogeneous-firm model that combines fixed adjustment costs, firm life-cycle dynamics, and a New Keynesian sticky-price setup to interpret these novel empirical findings. In the model, monetary policy affects firms' investment decisions along the intensive and, importantly, along the extensive margin. The extensive margin investment channel arises due to fixed capital adjustment costs. Quantitatively, the extensive margin explains a large chunk of the effect of monetary policy on the average investment rate as well as of the heterogeneous sensitivity of young firms—as in the data.

Our findings have important implications for both academic research and the conduct of monetary policy. First, the paper raises the issue of observational equivalence: firms typically classified as financially constrained (young/small) are more sensitive to monetary policy even in the absence of a financial accelerator mechanism. Second, understanding the frictions underlying firms' (heterogeneous) investment decisions is important for guiding macroeconomic policies in recessions. The financial accelerator mechanism suggests that macroeconomic policies are more effective in downturns. In contrast, the presence of an extensive margin investment decision—which we highlight in this paper—makes monetary and fiscal policy interventions less potent in recessions.

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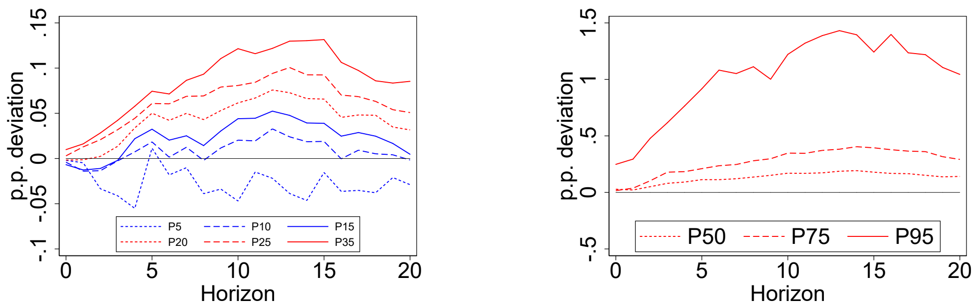
A Additional Figures

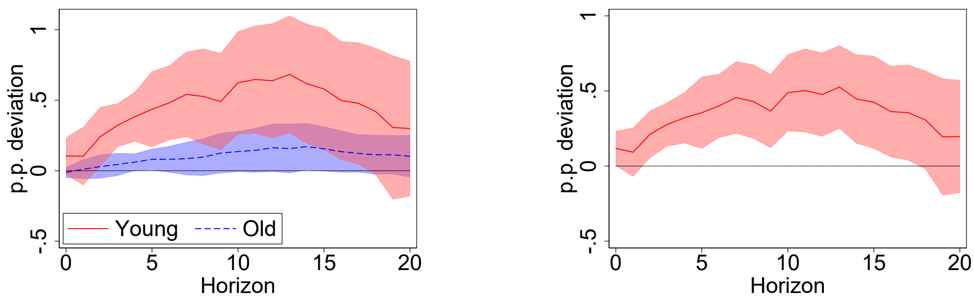
Figure A.1: Effect of Monetary Policy on Quantiles of the Inv. Rate Distribution



(a) P5 - P35 **(b)** P50, P75 & P90

Notes: See Figure A.2

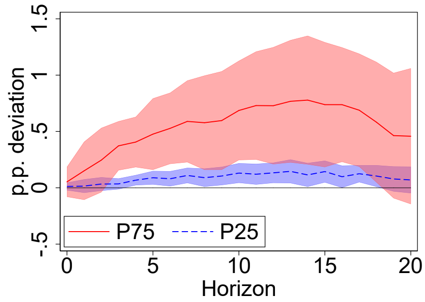
Figure A.2: Effect of Monetary Policy on Age-Group-Specific Avg. Inv. Rates



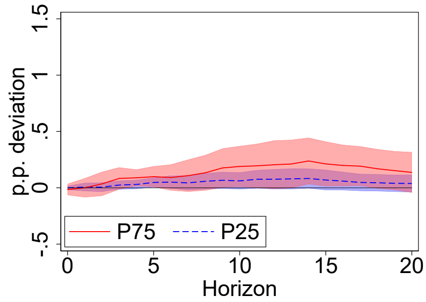
(a) Average Investment Rates **(b)** Difference (Young - Old)

Notes: Young (old) firms are less (more) than 15 years old. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$, using monetary policy shocks ϵ_t^{MP} implied by the Proxy SVAR in Gertler and Karadi (2015). The shocks are scaled to reduce the 1-year Treasury rate by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

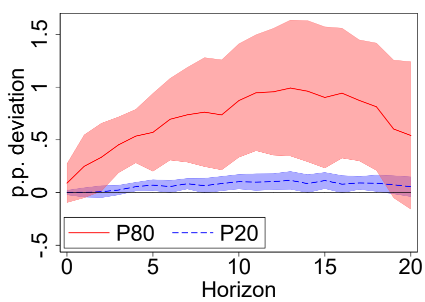
Figure A.3: Effect on Quantiles of Age-Group-Specific Inv. Rate Distributions



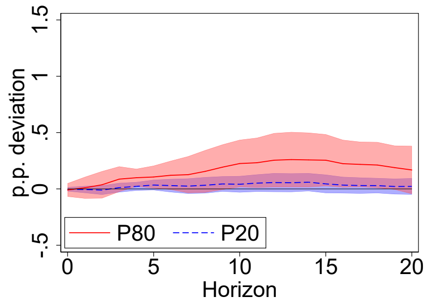
(a) 75th & 25th (Young Firms)



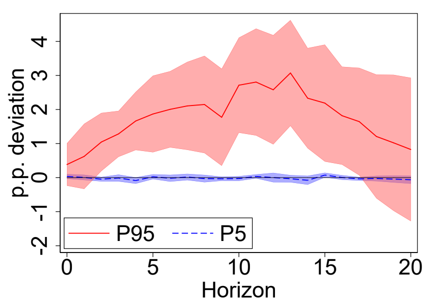
(b) 75th & 25th (Old Firms)



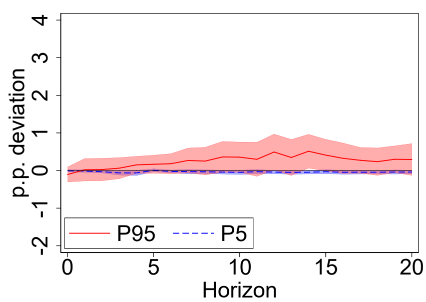
(c) 80th & 20th (Young Firms)



(d) 80th & 20th (Old Firms)



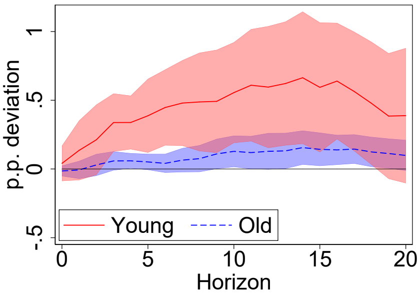
(e) 95th & 5th (Young Firms)



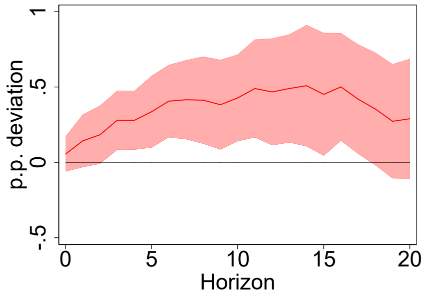
(f) 95th & 5th (Old Firms)

Notes: This figure plots the effect of a monetary policy shock on quantiles of the age-specific investment rate distributions. Young (old) firms are firms less (more) than 15 years old. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$, using monetary policy shocks ϵ_t^{MP} implied by the Proxy SVAR in Gertler and Karadi (2015). The shocks are scaled to reduce the 1-year Treasury rate by 25 basis points. The shaded areas indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

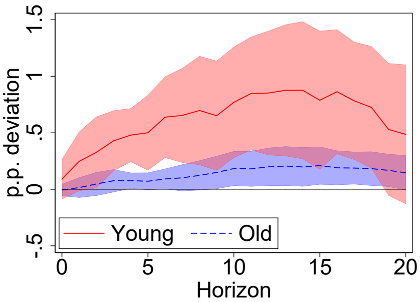
Figure A.4: Effect on Interquantile Ranges of Age-Group-Specific Inv. Rate Distributions



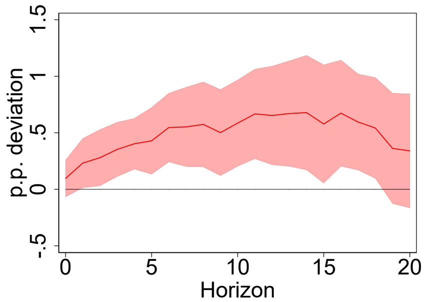
(a) Interquantile Range: P75 - P25



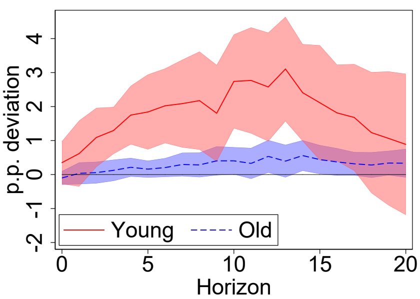
(b) Difference (Young - Old)



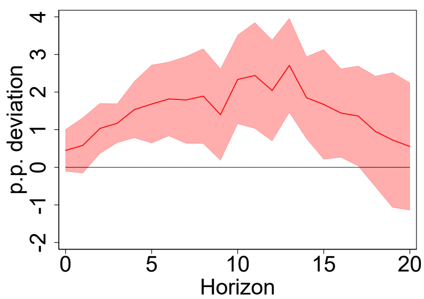
(c) Interquantile Range: P80 - P20



(d) Difference (Young - Old)



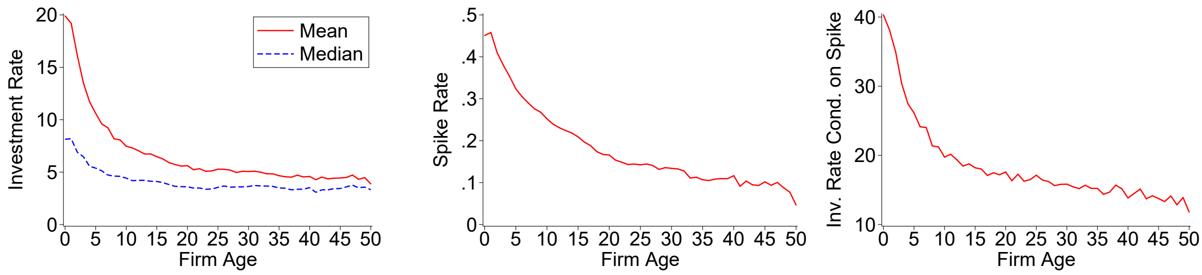
(e) Interquantile Range: P95 - P5



(f) Difference (Young - Old)

Notes: This figure plots the effect of a monetary policy shock on statistics of the age-specific investment rate distributions. Young (old) firms are firms less (more) than 15 years old. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$, using monetary policy shocks ϵ_t^{MP} implied by the Proxy SVAR in Gertler and Karadi (2015). The shocks are scaled to reduce the 1-year Treasury rate by 25 basis points. The shaded areas indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

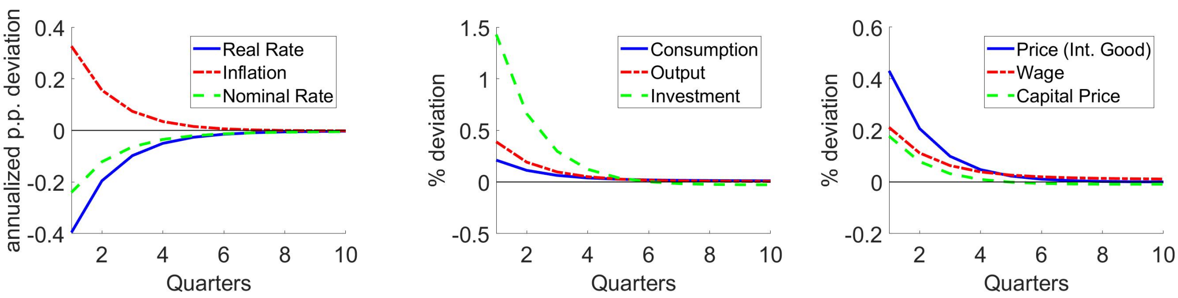
Figure A.5: Empirical Life-Cycle Profiles



(a) Investment Rate **(b)** Spike Rate **(c)** Avg. Cond. Inv. Rate

Notes: Investment rates and the spike rate refer to a quarter. A spike is defined as an investment rate $\geq 10\%$. The average conditional investment rate (panel c) is the average investment rate among all firms with an investment rate $\geq 10\%$.

Figure A.6: Aggregate Effects of an Expansionary Monetary Policy Shock



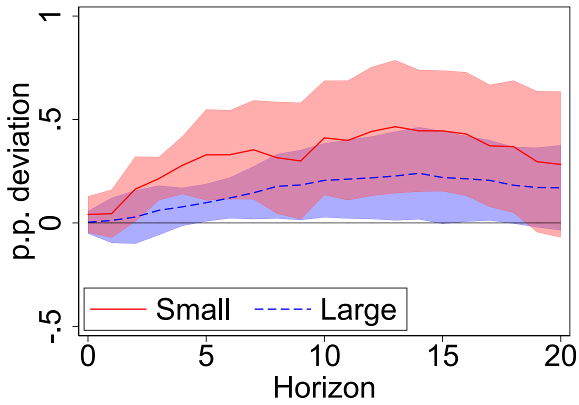
(a) Interest Rates & Inflation **(b)** Cons., Output & Investm. **(c)** Prices

Notes: This figure plots the effects of a monetary policy shock on interest rates, inflation, aggregates, and prices.

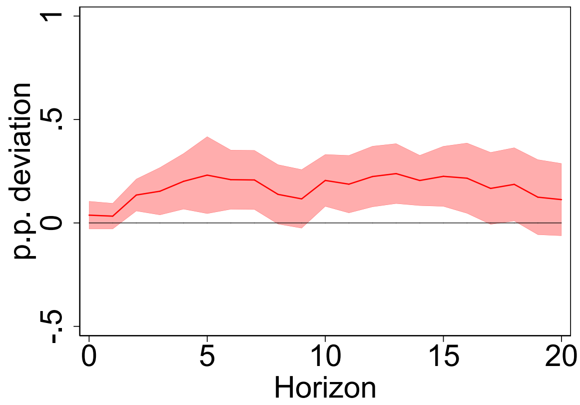
B Heterogeneous Sensitivity by Firm Size

Empirical Evidence Cloyne et al. (2020) have shown that being young is a better predictor of a firm's sensitivity to monetary policy shocks than being small. We replicate this finding in Figure B.1. Firms that are smaller than the median are at the peak on average 24 basis points more sensitive than firms which are larger than the median. In comparison, young firms are at the peak on average 53 basis points more sensitive than old firms, as shown in Figure A.2. This weaker heterogeneous sensitivity goes along with a weaker heterogeneous sensitivity of the extensive margin, as shown in Figure B.2, which replicates Figure 6 while grouping firms by size instead of age. In addition, the change in the distribution differs somewhat less across size groups than across age groups, as can be seen from comparing Figure A.3 with Figure D.1 (in the Online Appendix).

Figure B.1: Effect of Monetary Policy Shock on Average Investment Rates by Size



(a) Average Investment Rates

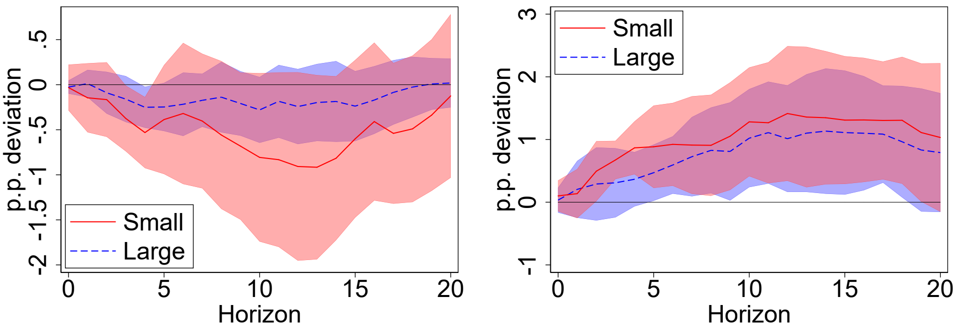


(b) Difference (Small - Large)

Notes: Small (large) firms are firms smaller (larger) than the median in a given quarter. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$, using monetary policy shocks ϵ_t^{MP} implied by the Proxy SVAR in Gertler and Karadi (2015). The shocks are scaled to reduce the 1-year Treasury rate by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

Model Predictions Our model is able to replicate the finding that young age is a better predictor of firms' sensitivity to monetary policy shocks than small size. This is evident from Figure B.3, which replicates Figure 15, panel (a), while grouping firms by size instead of age. Firms that are smaller than the median are on impact more sensitive than firms larger than the median, but the difference is by about 50% smaller than the gap between young and old firms. Intuitively, age is the better predictor of sensitivity, because young firms are more likely to be "close to making a large investment". This is because young firms are born small and will almost certainly grow in the future. In contrast, small firms may or may not be "close to making a large investment". This is because some firms are small because they are very unproductive, such that the low level of capital is their desired level of capital. In a nutshell, size correlates positively with productivity, while age is uncorrelated with productivity.

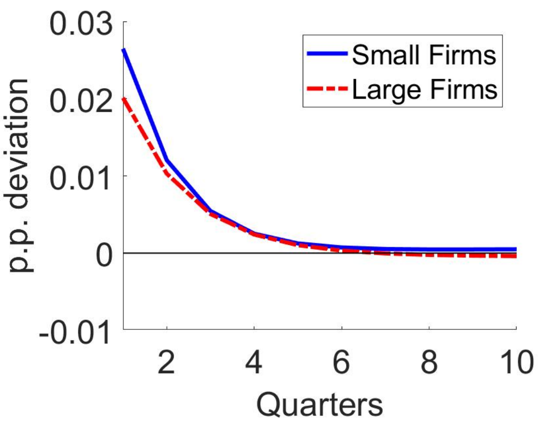
Figure B.2: Effect on Group-Specific Spike & Inaction Rates (by Size)



(a) Spike Rate (by Size) **(b)** Inaction Rate (by Size)

Notes: This figure plots the effect of a monetary policy shock on the spike rate and the inaction rate of small and large firms. Small (large) firms are firms smaller (larger) than the median in a given quarter. A spike rate is an investment rate exceeding 10%, an inaction rate is an investment rate less than 0.5% in absolute value. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$, using monetary policy shocks ϵ_t^{MP} implied by the Proxy SVAR in Gertler and Karadi (2015). The shocks are scaled to reduce the 1-year Treasury rate by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.

Figure B.3: Heterogeneous Effect (by Size Group) of an Exp. Monetary Policy Shock



(a) Avg. Investment Rate

Notes: This figure plots the effect of a monetary policy shock on the average investment rates of small and large firms. Small (large) firms are firms smaller (larger) than the median in a given quarter.

Online Appendix to "Monetary Policy, Firm Heterogeneity, and the Distribution of Investment Rates"

Matthias Gnewuch, Donghai Zhang

A Data Appendix

A.1 Sample Selection

We use the Compustat North America Fundamentals Quarterly database. Observations are uniquely identified by GVKEY & DATADATE. In line with the literature, we exclude observations which fall under the following criteria

1. not incorporated in the United States (based on FIC)
2. native currency not U.S. Dollar (based on CURNCDQ)
3. fiscal quarter does not match calendar quarter (based on FYR)
4. specific sectors
   - Utilities (SIC 4900-4999)
   - Financial Industry (SIC 6000-6999)
   - Non-operating Establishments (SIC 9995)
   - Industrial Conglomerates (SIC 9997)
   - Non-classifiable (NAICS > 999900)
5. missing industry information (SIC or NAICS code)
6. missing capital expenditures (based on CAPX)
7. missing or non-positive total assets (AT) or net capital (PPENT)
8. negative sales (SALEQ)
9. acquisitions (based on AQCY) exceed 5% of total assets (in absolute terms)
10. missing or implausible age information (see Online Appendix A.2)
11. outlier in the Perpetual Inventory Method (see Online Appendix A.3)

Our sample begins with 1986Q1 and ends with 2018Q4. In a final step, we exclude firm which we observe for less than 20 quarters, unless they are still in the sample in the final period. This ensures that we do not mechanically exclude all firms incorporated in the last five years of our sample.

A.2 Firm Age

We use data on firm age from WorldScope and Jay Ritter's database[1]. WorldScope provides the date of incorporation (Variable: INCORPDATE), while Jay Ritter's database provides the founding date. Both are merged with Compustat based on CUSIP. We define as the firm entry quarter the minimum of both dates if both are available. We do not use information on the initial public offering (IPO) of a firm to determine its age, since the time between incorporation and IPO can vary substantially. However, we use the IPO date to detect implausible age information. We exclude firms for which the IPO date reported in Compustat (IPODATE) precedes the firm entry quarter by more than four quarters. In similar fashion, we exclude firms which appear in Compustat more than four quarters before the firm entry quarter.[2] Finally, we merge information on the beginning of trading from CRSP (Variable: BEGDAT) based on CUSIP and likewise exclude firms with trading more than four quarters before the firm entry quarter.

A.3 Perpetual Inventory Method

Accounting capital stocks $k^a_{j,t}$ as reported in Compustat deviate from *economic* capital stocks for at least two reasons. First, accounting depreciation is driven by tax incentives and usually exceeds economic depreciation. Second, accounting capital stocks are reported at historical prices, not current prices. With positive inflation, both issues make the economic capital stock exceed the accounting capital stock. Therefore, we use a Perpetual Inventory Method (PIM) to compute real economic capital stocks, building on Bachmann and Bayer (2014).

Investment. In principle, there are two options to measure net nominal quarterly investment. First, investment can be measured directly ($I^{dir}_{j,t}$) from the Statement of Cash Flows as capital expenditures (CAPX) less the sale of PPE (SPPE)[3]. Second, investment can be backed out ($I^{indir}_{j,t}$) from the change in PPE (D.PPENT) plus depreci-

[1] https://site.warrington.ufl.edu/ritter/

[2] We do not construct firm age from the first appearance in Compustat. An inspection of the data reveals that this would result in wrongly classifying a number of old and established firms as young. Cloyne et al. (2020) do exactly this. However, they show in an earlier working paper version that results are unchanged if only age information from WorldScope is used.

[3] We follow Belo et al. (2014) and set missing values of SPPE to zero.

ation (DPQ), using Balance Sheet and Income Statement information. Either measure needs to be deflated to obtain real investment. We use INVDEF from FRED, which has the advantage of being quality-adjusted. We prefer the direct investment measure, since the indirect measure basically captures any change to PPE, including changes due to acquisitions. Nevertheless, we want to exclude observations where both investment measures differ strongly. To this end, we compute investment rates using lagged net accounting capital (L.PPENT), compute the absolute difference between both and discard the top 1% of that distribution.

Depreciation Rates. We obtain economic depreciation rates from the Bureau of Economic Analysis' (BEA) Fixed Asset Accounts. Specifically, we retrieve current-cost net stock and depreciation of private fixed assets by year and industry.[4] We calculate annual depreciation rates by industry and assume a constant depreciation rate within the calendar year to calculate quarterly depreciation rates.

Real Economic Capital Stocks. We initialize a firm's capital stock with the net (real) accounting capital stock $k^a_{j,1}$ (PPENT / INVDEF) whenever this variable is first observed. We iterate forward using deflated investment and the economic depreciation rate.

$$k^{(1)}_{j,1} = k^a_{j,1} \tag{39}$$

$$k^{(1)}_{j,t+1} = (1-\delta^e_t)k^{(1)}_{j,t} + \frac{p^I_t}{p_{2009,t}} I^{dir}_{j,t} \tag{40}$$

Comparing $k^{(1)}_{j,t}$ and $k^a_{j,t}$ shows non-negligible discrepancies. On average, the economic capital stock is larger, confirming the hypothesis that accounting capital stocks are understated. This makes it problematic to use the accounting capital stock as a starting value in the PIM. As a remedy, we again follow Bachmann and Bayer (2014) and use an iterative procedure to re-scale the starting value. We compute a time-invariant scaling factor ϕ at the sector-level and use it to re-scale the starting value as follows. We iterate until ϕ converges. The procedure is initialized with $k^{(0)}_{j,t} = k^a_{j,t}$ and $\phi^{(0)} = 1$.

$$\phi^{(n)} = \frac{1}{NT}\sum_{j,t} \frac{k^{(n)}_{j,t}}{k^{(n-1)}_{j,t}} \quad \text{[and not in top or bottom 1\%]} \tag{41}$$

$$k^{(n+1)}_{j,1} = \phi^{(n)} k^{(n)}_{j,1} \tag{42}$$

[4] The Fixed Asset Accounts also provide depreciation rates by asset type (Equipment, Structures, Intellectual Property Products), which we do not use since the firm-level data does not include information on capital stocks or capital expenditure by asset type.

Outliers. We exclude firms for which the economic capital stock becomes negative at any point in time. This can arise if there is a sale of capital, which exceeds current economic capital. Further, we compute the deviation between (real) accounting and economic capital stocks and discard the top 1% of that distribution. Finally, we discard firms for which we have less than 20 observations, unless they are still in the sample in the final quarter.

Evaluation. Our estimated real economic capital stock is still highly correlated with the real accounting capital stock. A simple regression has an R^2 of above 0.96 and shows that the economic capital stock is on average slightly higher (by about 4%), as expected. The investment rate (net real investment over lagged real economic capital) is highly correlated ($\rho > 0.98$) with the accounting investment rate used in Cloyne et al. (2020). A simple regression shows that on average, the economic investment rate is lower (by about 13%) than the accounting investment rate, also as expected due to the underreporting of accounting capital stocks.

A.4 Variable Construction

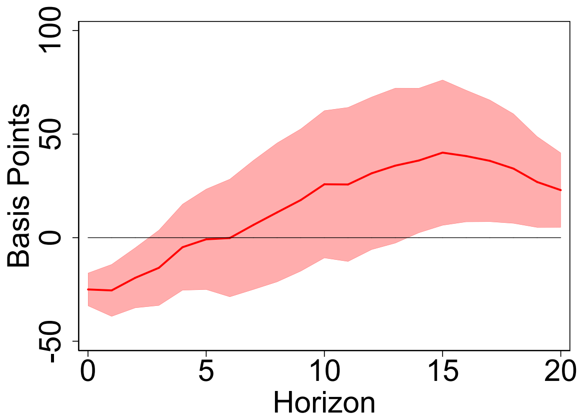
Most of our variables follow the definitions in the literature. Our baseline measure of the investment rate is $i_{jt} = \frac{CAPX_{jt} - SPPE_{jt}}{INVDEF_t \times k_{jt-1}}$, thus, real capital expenditures (CAPX) net of sales of capital (SPPE) divided by the lagged real economic capital stock, computed as described previously. To measure size, we use the log of total assets (AT).

A.5 Identification of Monetary Policy Shocks

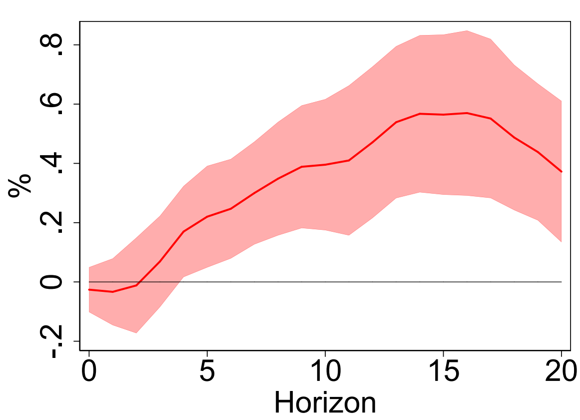
We use the monetary policy shocks implied by the proxy SVAR used in Gertler and Karadi (2015). We calculate them according to the following procedure. First, we update the data used in the Gertler and Karadi (2015) baseline SVAR. They use monthly data from 1979M7 to 2012M6. We update all time series to 2019M12. The SVAR includes (the log of) industrial production (FRED: INDPRO), (the log of) the consumer price index (FRED: CPIAUCSL), the one-year government bond rate (FRED: GS1), and the excess bond premium (Source: https://www.federalreserve.gov/econresdata/notes/feds-notes/2016/files/ebp_csv.csv, retrieved in February 2020). Moreover, we update the instrument (cumulative high-frequency FF4 surprises) to 2015M10. Then, we run the SVAR and compute the implied structural monetary policy shocks. See the appendix of Mertens and Ravn (2013) for details. Importantly, even though the instrument is only available until 2015M10, we can compute the structural monetary policy shock until 2019M12.

A.6 Effects of Monetary Policy using Aggregate Data

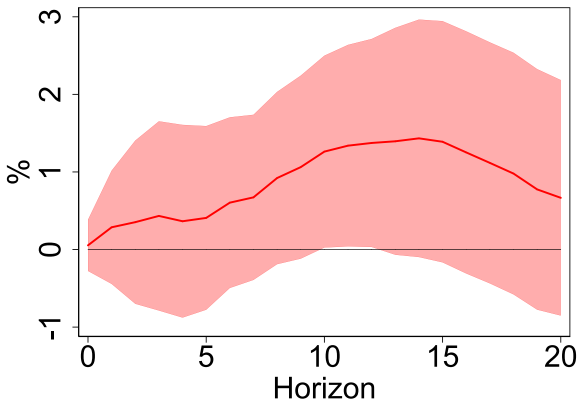
Figure A.1: Aggregate Effects of a Monetary Policy Shock



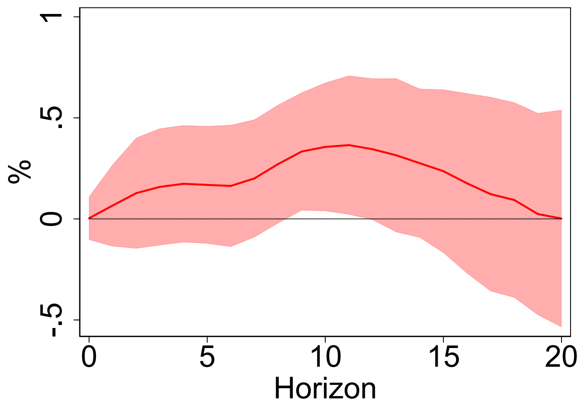
(a) 1-Year Treasury Rate



(b) Relative Price of Investment Goods



(c) Investment



(d) GDP

Notes: The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$, using monetary policy shocks ϵ_t^{MP} implied by the Proxy SVAR in Gertler and Karadi (2015). The shocks are scaled to reduce the 1-year Treasury rate by 25 basis points. The shaded areas are the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4. All variables except for the 1-year Treasury rate are in logs.

Using time series data from FRED, we document the aggregate effects of the monetary policy shocks we utilize. Qualitatively, these are quite similar to Gertler and Karadi (2015). Panel (a) of Figure A.1 shows that a monetary policy shock decreases the 1-year Treasury rate (FRED: GS1) for roughly 4 quarters. Thereafter, it overshoots, as observed in Gertler and Karadi (2015). Panels (b) and (c) show that (real) investment (FRED: PNFI) and the relative price of capital goods (FRED: PIRIC) increase strongly. The peak effect on investment is roughly 1.4%. As we will show in the model, the endogenous response of the relative price of capital generates a heterogeneous effect on young and old firms. Panel (d) shows that real GDP (FRED: GDPC1) also increases following an expansionary shock. The peak effect is about 0.35%.

B Proofs

Proposition 1. *In an economy populated by heterogeneous firms that face fixed adjustment costs as described above, it holds that*

1. *An interest rate cut increases the hazard rate:* $\frac{\partial \lambda(k_0)}{\partial r} < 0$
2. *The sensitivity of the average investment rate to interest rate changes via the extensive margin is decreasing (in absolute terms) in firm size:* $\frac{\partial\left(\frac{\partial \lambda(k_0)}{\partial r} i^*(k_0)\right)}{\partial k_0} > 0$

Proof. Rearranging equation (10), the value added of adjusting capital while ignoring the fixed adjustment cost is:

$$VA(k_0) = \frac{1}{1+r}\left(k_1^{*\theta} - k_0^{\theta}\right) - q(k_1^* - k_0) \tag{43}$$

where k_1^* was defined in equation (8). Using the definition of the cutoff $\xi^T(k_0)$ and the hazard rate $\lambda(k_0)$ from the main text, we have

$$\lambda(k_0) = \frac{1}{\bar{\xi}} VA(k_0). \tag{44}$$

Taking the derivative w.r.t. the real interest rate, we get

$$\frac{\partial \lambda(k_0)}{\partial r} = -\frac{1}{\bar{\xi}} \frac{1}{(1+r)^2}\left(k_1^{*\theta} - k_0^{\theta}\right) < 0, \tag{45}$$

which proves the first part of the proposition. Note that $k_0 < k_1^*$ by assumption.

The second part of the proposition requires

$$\frac{\partial\left(\frac{\partial \lambda(k_0)}{\partial r} i^*(k_0)\right)}{\partial k_0} = \frac{\partial^2 \lambda(k_0)}{\partial r \partial k_0} i^*(k_0) + \frac{\partial \lambda(k_0)}{\partial r}\frac{\partial i^*(k_0)}{\partial k_0} > 0. \tag{46}$$

The first term is positive, because

$$\frac{\partial^2 \lambda(k_0)}{\partial r \partial k_0} = \frac{1}{\bar{\xi}} \frac{1}{(1+r)^2} \theta k_0^{\theta-1} > 0 \tag{47}$$

and $i^*(k_0) > 0$ because $k_0, k_1 > 0$. The second term is positive because

$$\frac{\partial i^*(k_0)}{\partial k_0} = -k_1^* k_0^{-2} < 0 \tag{48}$$

and $\frac{\partial \lambda(k_0)}{\partial r} < 0$ as shown in equation (45). Thus, the inequality in equation (46) holds which completes the proof.

□

C Analysis of the Calibrated Model

C.1 Equilibrium Definition

A recursive competitive equilibrium in this model is a set of value functions $\{V_t(z,k)$, $CV_t^{exit}(z,k)$, $CV_t^a(z,k,\xi)$, $CV_t^n(z,k)\}$, policy functions $\{n_t^*(z,k)$, $k_t^*(z,k,\xi)$, $\xi_t^T(z,k)\}$, quantities $\{C_t,\ Y_t,\ I_t^Q,\ K_t,\ N_t\}$, prices $\{p_t,\ w_t,\ \pi_t,\ \Lambda_{t+1},\ q_t\}$, and distributions $\{\mu_t(z,k)\}$ such that all agents in the economy behave optimally, the distribution of firms is consistent with decision rules, and all markets clear:

1. Investment Block: Taking all prices as given, $V_t(z,k)$, $CV_t^{exit}(z,k)$, $CV_t^a(z,k,\xi)$, and $CV_t^n(z,k)$ solve the Bellman equation with associated decision rules $n_t^*(z,k)$, $k_t^*(z,k,\xi)$, and $\xi_t^T(z,k)$.

2. Household Block: Taking prices as given, C_t and C_{t+1} satisfy the household's optimality conditions (37) and (38).

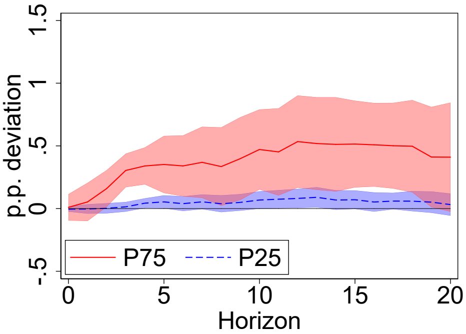
3. New Keynesian Block: The New Keynesian Phillips Curve holds. The Taylor rule holds. Taking prices a given, I_t^Q satisfies (33).

4. All markets (final good, capital, labor) clear.

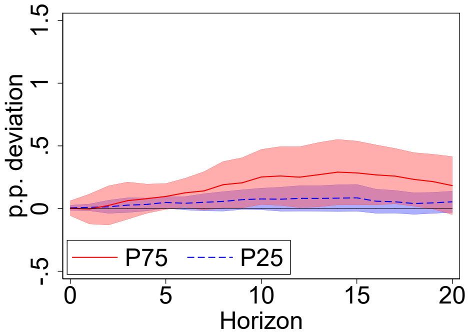
5. The distribution of firms, $\mu_t(z,k)$, evolves as implied by the decision rules $k^*(z,k,\xi)$ and $\xi_t^T(z,k)$, the exogenous process for firm-level productivity, and considering exogenous exits and entrants with capital k_0 and productivity from μ^{ent}.

D Additional Figures

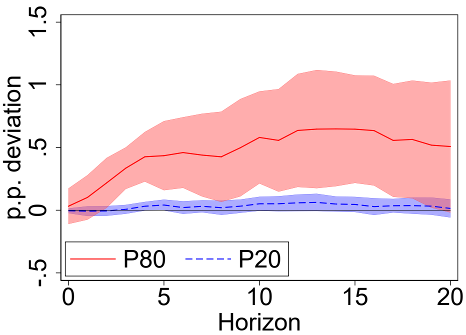
Figure D.1: Effect on Quantiles of Size-Group-Specific Inv. Rate Distributions



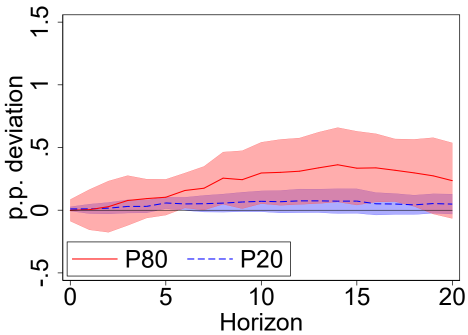
(a) 75th & 25th (Small Firms)



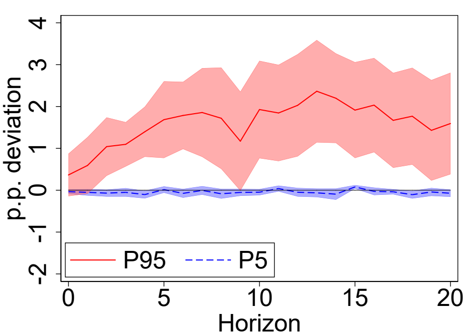
(b) 75th & 25th (Large Firms)



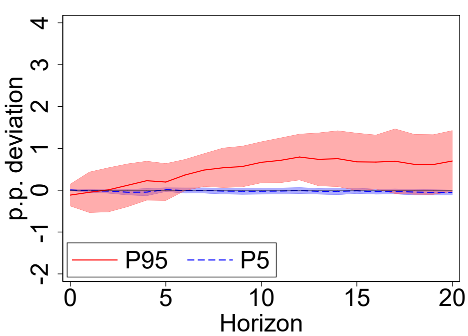
(c) 80th & 20th (Small Firms)



(d) 80th & 20th (Large Firms)



(e) 95th & 5th (Small Firms)



(f) 95th & 5th (Large Firms)

Notes: This figure plots the effect of a monetary policy shock on quantiles of the size-specific investment rate distributions. Small (large) firms are firms smaller (larger) than the median in a given quarter. The lines represent the estimated $\hat{\beta}^h$ from separate regressions: $y_{t+h} - y_{t-1} = \alpha^h + \beta^h \epsilon_t^{MP} + \sum_{j=2}^{4} \gamma^j \mathbb{1}\{q_{t+h} = j\} + e_{t+h}$, using monetary policy shocks ϵ_t^{MP} implied by the Proxy SVAR in Gertler and Karadi (2015). The shocks are scaled to reduce the 1-year Treasury rate by 25 basis points. The shaded areas indicate the 90% confidence intervals constructed using standard errors that are robust to heteroskedasticity and autocorrelation. Sample: 1986Q1 - 2018Q4.