L

STRIP PROBLEM

Full + free BC shear parameter free ?

extra free BC

$$M_{nn} = -K\left(\frac{\partial^2 w}{\partial n^2} + \nu \frac{\partial^2 w}{\partial t^2}\right) = 0$$

$$V_{nn} = -K\left(\frac{\partial^3 w}{\partial n^3} + (2-\nu)\frac{\partial}{\partial n}\left(\frac{\partial^2 w}{\partial t^2}\right)\right) = 0$$

$$K = \frac{Eh^3}{12(1-\nu^2)}$$

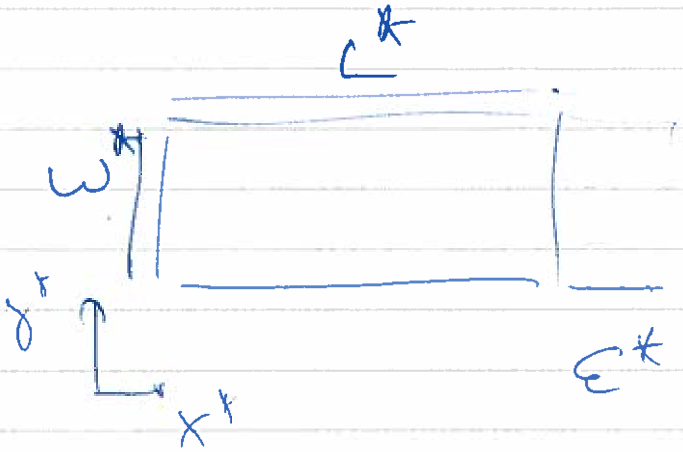
As long as homogeneous no added parameters from BC

In plane:

$$\sigma^*_{\alpha\beta} n_\beta = t_\alpha = 0$$

Also no extra params.

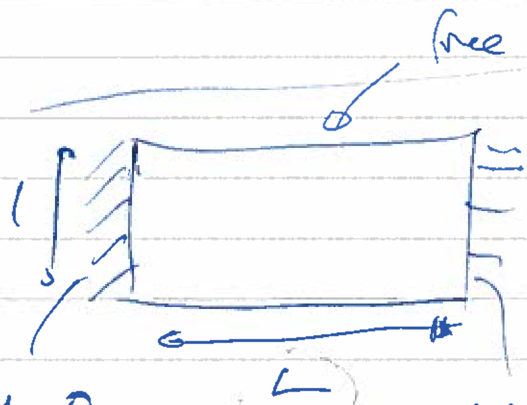
Strip



Dim

$\mathcal{L} = W^*$

Nondim



$u = 0$

$w = \frac{\partial w}{\partial n} = 0$

$u = \mathcal{E}\, e_x$

$w = \frac{\partial w}{\partial n} = 0$

$$\mathcal{E} = \frac{E^* \mathcal{L}}{h^{*2}} = \frac{E^* W^*}{h^{*2}}$$

$u^* = h^* \frac{h^*}{W^*} u$

$w^* = h^* w$

$L = \frac{L^*}{W^*}$

$h = \frac{h^*}{W^*}$

Keep strain, material etc fixed.

⇒ everything is only fct of $\mathcal{E}$.

⇒ $\boxed{\text{unique } \mathcal{E}_{wrinkle} \text{ \& } N_{wrinkle}}$ "pattern"

LSA

$$\mathcal{E}_{\text{wrinkle}} = \text{const} = \frac{\mathcal{E}^*_{\text{wrinkle}}(h^*)\, w^*}{h^{*2}}$$

$$\Rightarrow \boxed{\mathcal{E}^*_{\text{wrinkle}} = \mathcal{E}_{\text{wrinkle}} \frac{h^{*2}}{w^*}} \quad (\text{CSA})$$

Large displ. problem governed by same eqns

$$\Rightarrow w(x_\alpha) = w(x_\alpha, \mathcal{E}) = \frac{w^*\left(\frac{x^*_\alpha}{w^*}, \frac{\mathcal{E}^* w^*}{h^{*2}}\right)}{h^*}$$

$$u^* = h^* \frac{h^*}{w^*} u$$

$$\mathcal{E}^* = \frac{h^{*2}}{w^*} \mathcal{E}$$

$$\Rightarrow w^* = h^* w(x_\alpha; \mathcal{E})$$

"shape" universal in terms of $\mathcal{E}$

$$w^* = h^* w\left(x_\alpha; \frac{\mathcal{E}^* w^*}{h^{*2}}\right)$$

amplitude only

shape; i.e. wave ff depends on

$$N = N\left(\frac{\mathcal{E}^* w^*}{h^{*2}}\right)$$

Mohodevans:

$$k = \frac{2\pi n}{\omega} \quad ; \lambda = \frac{2\pi}{k} = \frac{2\pi\,\omega}{2\pi n} = \frac{\omega}{n}$$

(obv)

$$\lambda = \frac{\omega}{n} = \frac{(2\pi L t)^{1/2}}{\left(3(1-\nu^2)\mu\right)^{1/4}}$$

$$n = \frac{\omega\,\left(3(1-\nu^2)\mu\right)^{1/4}}{(2\pi L t)^{1/2}}$$

into my notation

$$\mu = \frac{\varepsilon^*}{\omega} \quad ; \quad t = h \quad ; \quad n = N$$

$$N = \frac{3(1-\nu^2)^{1/4}}{\underbrace{(2\pi)^{1/2}}_{C}}\;\frac{\omega^{1/4}\,\omega^{3/4}\;\varepsilon^{*1/4}}{L^{1/4}\,L^{1/2}\,h^{1/2}}$$

$$= C\left(\frac{\omega}{L}\right)^{3/4}\frac{\omega^{1/4}\varepsilon^{*1/4}}{h^{1/4}\,h^{1/4}}$$

$$N = C\left(\frac{\omega}{L}\right)^{3/4}\underbrace{\left(\frac{\omega^*\varepsilon^*}{h^{*2}}\right)^{1/4}}$$

~~same~~

"Shape" - dep. on ε^* consistent with our analysis

$\Rightarrow$ FvK does/ can

capture dep. of wave #

on ε^* (stretch)

Note also that

Mahadevan predicts (& therefore that does too?)

that amplitude of wrinkles goes like

$$A \sim (\ell L t)^{1/2} \left(\frac{16\,\mu}{3\pi^2 (1-\nu^2)} \right)^{1/4}$$

μ = stretch so amplitude does increase unboundedly (as observed by Healey)

[TODO: Q. check Healey data for dep. on μ.]

$\sim \mu^{1/4}$?

<u>A:</u> Doesn't look like it: (log/log plot of max|w| vs. ε^*)