## **Chapter 1**

## Demo problem: Large-amplitude shock-wave propagation in a circular disk

Detailed documentation to be written. Here's the already fairly well documented driver code...

```
//LIC// This file forms part of oomph-lib, the object-oriented,
//LIC// multi-physics finite-element library, available
//LIC// at http://www.oomph-lib.org.
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//LIC// The authors may be contacted at oomph-lib@maths.man.ac.uk.
//LIC//
// Driver for large-displacement elasto-dynamic test problem:
// Circular disk impulsively loaded by compressive load.
#include <iostream>
#include <fstream>
#include <cmath>
//My own includes
#include "generic.h"
#include "solid.h"
//Need to instantiate templated mesh
#include "meshes/quarter_circle_sector_mesh.h"
using namespace std;
using namespace oomph;
namespace Global_Physical_Variables
 /// Pointer to constitutive law
 ConstitutiveLaw* Constitutive_law_pt;
 /// Elastic modulus
 double E=1.0;
 /// Poisson's ratio
 double Nu=0.3;
```

```
/// Uniform pressure
double P = 0.00;
 /// Constant pressure load
 void constant_pressure(const Vector<double> &xi,const Vector<double> &x,
                       const Vector<double> &n, Vector<double> &traction)
  unsigned dim = traction.size();
  for(unsigned i=0;i<dim;i++)</pre>
   traction[i] = -P*n[i];
}
/// Elastic quarter circle sector mesh with functionality to
/// attach traction elements to the curved surface. We "upgrade"
/// the RefineableQuarterCircleSectorMesh to become an
\ensuremath{///} SolidMesh and equate the Eulerian and Lagrangian coordinates,
/// thus making the domain represented by the mesh the stress-free
/// configuration.
/// The member function \c make_traction_element_mesh() creates
/// a separate mesh of \dot{\text{SolidTractionElements}} that are attached to the
/// mesh's curved boundary (boundary 1).
template <class ELEMENT>
class ElasticRefineableQuarterCircleSectorMesh :
 public virtual RefineableQuarterCircleSectorMesh<ELEMENT>,
public virtual SolidMesh
public:
 /// Constructor: Build mesh and copy Eulerian coords to Lagrangian
 /// ones so that the initial configuration is the stress-free one.
ElasticRefineableQuarterCircleSectorMesh<ELEMENT>(GeomObject* wall_pt,
                                       const double& xi_lo,
                                        const double& fract_mid,
                                        const double& xi_hi,
                                        TimeStepper* time_stepper_pt=
                                        &Mesh::Default_TimeStepper) :
 RefineableQuarterCircleSectorMesh<ELEMENT>(wall_pt,xi_lo,fract_mid,xi_hi,
                                           time_stepper_pt)
#ifdef PARANOID
   /// Check that the element type is derived from the SolidFiniteElement
   SolidFiniteElement* el_pt=dynamic_cast<SolidFiniteElement*>
    (finite_element_pt(0));
   if (el_pt==0)
    throw OomphLibError(
     "Element needs to be derived from SolidFiniteElement\n",
     OOMPH_CURRENT_FUNCTION,
     OOMPH_EXCEPTION_LOCATION);
#endif
  // Make the current configuration the undeformed one by
  // setting the nodal Lagrangian coordinates to their current
   // Eulerian ones
  set_lagrangian_nodal_coordinates();
 /// Function to create mesh made of traction elements
 void make_traction_element_mesh(SolidMesh*& traction_mesh_pt)
  {
   // Make new mesh
  traction_mesh_pt=new SolidMesh;
   // Loop over all elements on boundary 1:
  unsigned b=1;
  unsigned n_element = this->nboundary_element(b);
   for (unsigned e=0;e<n_element;e++)</pre>
   {
     // The element itself:
    FiniteElement* fe_pt = this->boundary_element_pt(b,e);
    // Find the index of the face of element e along boundary b
    int face_index = this->face_index_at_boundary(b,e);
     // Create new element
    traction_mesh_pt->add_element_pt(new SolidTractionElement<ELEMENT>
                                     (fe_pt, face_index));
    }
```

```
}
 /// Function to wipe and re-create mesh made of traction elements
 \verb|void remake_traction_element_mesh| (SolidMesh*\& traction_mesh_pt)|
   // Wipe existing mesh (but don't call it's destructor as this
   // would wipe all the nodes too!)
   traction_mesh_pt->flush_element_and_node_storage();
   // Loop over all elements on boundary 1:
   unsigned b=1;
   unsigned n_element = this->nboundary_element(b);
   for (unsigned e=0;e<n_element;e++)</pre>
     // The element itself:
    FiniteElement* fe_pt = this->boundary_element_pt(b,e);
    // Find the index of the face of element e along boundary b
    int face_index = this->face_index_at_boundary(b,e);
     // Create new element
    traction_mesh_pt->add_element_pt(new SolidTractionElement<ELEMENT>
                                    (fe_pt,face_index));
};
/// "Shock" wave propagates through an impulsively loaded
/// circular disk.
template<class ELEMENT, class TIMESTEPPER>
class DiskShockWaveProblem : public Problem
public:
 /// Constructor:
 DiskShockWaveProblem();
 /// Run the problem; specify case_number to label output
 /// directory
 void run(const unsigned& case_number);
 /// Access function for the solid mesh
 ElasticRefineableQuarterCircleSectorMesh<ELEMENT>*& solid_mesh_pt()
   return Solid mesh pt:
 /// Access function for the mesh of surface traction elements
 SolidMesh*& traction_mesh_pt()
   return Traction_mesh_pt;
 /// Doc the solution
 void doc_solution();
 /// Update function (empty)
 void actions_after_newton_solve() {}
 /// Update function (empty)
 void actions_before_newton_solve() {}
 /// Actions after adaption: Kill and then re-build the traction
 /// elements on boundary 1 and re-assign the equation numbers
 void actions_after_adapt();
 /// Doc displacement and velocity: label file with before and after
 void doc_displ_and_veloc(const int& stage=0);
 /// Dump problem-specific parameters values, then dump
 /// generic problem data.
 void dump_it(ofstream& dump_file);
 /// Read problem-specific parameter values, then recover
 /// generic problem data.
 void restart (ifstream& restart file);
private:
 // Output
 DocInfo Doc_info;
 /// Trace file
 ofstream Trace_file;
```

```
/// Vector of pointers to nodes whose position we're tracing Vector<Node*> Trace_node_pt;
 /// Pointer to solid mesh
ElasticRefineableQuarterCircleSectorMesh<ELEMENT>* Solid_mesh_pt;
 /// Pointer to mesh of traction elements
SolidMesh* Traction_mesh_pt;
/// Constructor
template<class ELEMENT, class TIMESTEPPER>
DiskShockWaveProblem<ELEMENT, TIMESTEPPER>::DiskShockWaveProblem()
 //Allocate the timestepper
add_time_stepper_pt(new TIMESTEPPER);
 // Set coordinates and radius for the circle that defines
 // the outer curvilinear boundary of the domain
double x_c=0.0;
double y_c=0.0;
double r=1.0;
 // Build geometric object that specifies the fish back in the
 // undeformed configuration (basically a deep copy of the previous one)
GeomObject* curved_boundary_pt=new Circle(x_c,y_c,r,time_stepper_pt());
 // The curved boundary of the mesh is defined by the geometric object
 // What follows are the start and end coordinates on the geometric object:
double xi lo=0.0;
double xi_hi=2.0*atan(1.0);
 // Fraction along geometric object at which the radial dividing line
 // is placed
 double fract_mid=0.5;
 //Now create the mesh
 solid_mesh_pt() = new ElasticRefineableQuarterCircleSectorMesh<ELEMENT>(
 curved_boundary_pt,xi_lo,fract_mid,xi_hi,time_stepper_pt());
 // Set up trace nodes as the nodes on boundary 1 (=curved boundary) in
 // the original mesh (they exist under any refinement!)
unsigned nnod0=solid_mesh_pt()->nboundary_node(0);
 unsigned nnod1=solid_mesh_pt()->nboundary_node(1);
 Trace_node_pt.resize(nnod0+nnod1);
 for (unsigned j=0; j<nnod0; j++)</pre>
  Trace_node_pt[j]=solid_mesh_pt()->boundary_node_pt(0,j);
 for (unsigned j=0; j<nnod1; j++)</pre>
  Trace_node_pt[j+nnod0]=solid_mesh_pt()->boundary_node_pt(1,j);
 // Build traction element mesh
 solid_mesh_pt()->make_traction_element_mesh(traction_mesh_pt());
 // Solid mesh is first sub-mesh
add_sub_mesh(solid_mesh_pt());
 // Traction mesh is first sub-mesh
 add_sub_mesh(traction_mesh_pt());
 // Build combined "global" mesh
build_global_mesh();
 // Create/set error estimator
solid_mesh_pt()->spatial_error_estimator_pt()=new Z2ErrorEstimator;
 // Fiddle with adaptivity targets and doc
 solid_mesh_pt()->max_permitted_error()=0.006; //0.03;
 solid_mesh_pt()->min_permitted_error()=0.0006;// 0.0006; //0.003;
 solid_mesh_pt()->doc_adaptivity_targets(cout);
 // Pin the bottom in the vertical direction
unsigned n_bottom = solid_mesh_pt()->nboundary_node(0);
 //Loop over the node
 for (unsigned i=0;i<n_bottom;i++)</pre>
  solid_mesh_pt()->boundary_node_pt(0,i)->pin_position(1);
 // Pin the left edge in the horizontal direction
 unsigned n_side = solid_mesh_pt()->nboundary_node(2);
 //Loop over the node
 for (unsigned i=0;i<n_side;i++)</pre>
  solid mesh pt()->boundary node pt(2,i)->pin position(0);
 //Find number of elements in solid mesh
 unsigned n_element =solid_mesh_pt()->nelement();
 //Set parameters and function pointers for solid elements
 for(unsigned i=0;i<n_element;i++)</pre>
```

```
//Cast to a solid element
   ELEMENT *el_pt = dynamic_cast<ELEMENT*>(mesh_pt()->element_pt(i));
   //Set the constitutive law
   el_pt->constitutive_law_pt() =
   Global_Physical_Variables::Constitutive_law_pt;
   // Switch on inertia
  el_pt->enable_inertia();
 }
// Pin the redundant solid pressures
PVDEquationsBase<2>::pin_redundant_nodal_solid_pressures(
 solid_mesh_pt()->element_pt());
//Find number of elements in traction mesh
n_element=traction_mesh_pt()->nelement();
 //Set function pointers for traction elements
 for (unsigned i=0;i<n_element;i++)</pre>
 {
   //Cast to a solid traction element
  SolidTractionElement<ELEMENT> *el_pt =
    dynamic_cast<SolidTractionElement<ELEMENT>*>
    (traction_mesh_pt()->element_pt(i));
   //Set the traction function
  el_pt->traction_fct_pt() = Global_Physical_Variables::constant_pressure;
 //Attach the boundary conditions to the mesh
 cout « assign_eqn_numbers() « std::endl;
 // Refine uniformly
 refine_uniformly();
refine_uniformly();
refine uniformly();
 // Now the non-pinned positions of the SolidNodes will have been
 // determined by interpolation. This is appropriate for uniform
 // refinements once the code is up and running since we can't place
 \ensuremath{//} new SolidNodes at the positions determined by the MacroElement.
 // However, here we want to update the nodes to fit the exact
// intitial configuration.
 // Update all solid nodes based on the Mesh's Domain/MacroElement
 // representation
bool update_all_solid_nodes=true;
 solid_mesh_pt()->node_update(update_all_solid_nodes);
 // Now set the Eulerian equal to the Lagrangian coordinates
 solid_mesh_pt()->set_lagrangian_nodal_coordinates();
//-----
/// Kill and then re-build the traction elements on boundary 1,
/// pin redundant pressure dofs and re-assign the equation numbers.
                   -----
template<class ELEMENT, class TIMESTEPPER>
void DiskShockWaveProblem<ELEMENT,TIMESTEPPER>::actions_after_adapt()
 // Wipe and re-build traction element mesh
solid_mesh_pt()->remake_traction_element_mesh(traction_mesh_pt());
 // Re-build combined "global" mesh
rebuild_global_mesh();
 //Find number of elements in traction mesh
unsigned n_element=traction_mesh_pt()->nelement();
 //Loop over the elements in the traction element mesh
 for (unsigned i=0;i<n_element;i++)</pre>
   //Cast to a solid traction element
  SolidTractionElement<ELEMENT> *el_pt =
   dynamic_cast<SolidTractionElement<ELEMENT>*>
    (traction_mesh_pt()->element_pt(i));
   //Set the traction function
  el_pt->traction_fct_pt() = Global_Physical_Variables::constant_pressure;
 }
// Pin the redundant solid pressures
PVDEquationsBase<2>::pin_redundant_nodal_solid_pressures(
 solid_mesh_pt()->element_pt());
 //Do equation numbering
cout « assign_eqn_numbers() « std::endl;
/// Doc the solution
template<class ELEMENT, class TIMESTEPPER>
void DiskShockWaveProblem<ELEMENT,TIMESTEPPER>::doc_solution()
 // Number of plot points
unsigned npts;
npts=5;
 // Output shape of deformed body
```

```
ofstream some_file;
char filename[100];
sprintf(filename, "%s/soln%i.dat", Doc_info.directory().c_str(),
        Doc_info.number());
some_file.open(filename);
solid_mesh_pt()->output(some_file,npts);
some_file.close();
// Output traction
unsigned nel=traction_mesh_pt()->nelement();
sprintf(filename, "%s/traction%i.dat", Doc_info.directory().c_str(),
        Doc_info.number());
some_file.open(filename);
Vector<double> unit_normal(2);
Vector<double> traction(2);
Vector<double> x_dummy(2);
Vector<double> s_dummy(1);
for (unsigned e=0; e<nel; e++)
  some_file « "ZONE " « std::endl;
  for (unsigned i=0;i<npts;i++)</pre>
    s_dummy[0]=-1.0+2.0*double(i)/double(npts-1);
    SolidTractionElement<ELEMENT>* el_pt=
    dynamic_cast<SolidTractionElement<ELEMENT>*>(
      traction_mesh_pt()->finite_element_pt(e));
    el_pt->outer_unit_normal(s_dummy,unit_normal);
    el_pt->traction(s_dummy,traction);
    « std::endl;
   }
some_file.close();
// Doc displacement and velocity
doc_displ_and_veloc();
// Get displacement as a function of the radial coordinate
// along boundary 0
 // Number of elements along boundary 0:
 unsigned nelem=solid_mesh_pt()->nboundary_element(0);
 // Open files
 sprintf(filename, "%s/displ%i.dat", Doc_info.directory().c_str(),
         Doc_info.number());
 some_file.open(filename);
 Vector<double> s(2);
 Vector<double> x(2);
 Vector<double> dxdt(2);
 Vector<double> xi(2);
 Vector<double> r_exact(2);
 Vector<double> v_exact(2);
 for (unsigned e=0;e<nelem;e++)</pre>
   some_file « "ZONE " « std::endl;
   for (unsigned i=0;i<npts;i++)</pre>
     // Move along bottom edge of element
     s[0]=-1.0+2.0*double(i)/double(npts-1);
     s[1] = -1.0;
     // Get pointer to element
     SolidFiniteElement* el_pt=dynamic_cast<SolidFiniteElement*>
      (solid_mesh_pt()->boundary_element_pt(0,e));
     // Get Lagrangian coordinate
     el_pt->interpolated_xi(s,xi);
     // Get Eulerian coordinate
     el_pt->interpolated_x(s,x);
     // Get velocity
     el_pt->interpolated_dxdt(s,1,dxdt);
     }
 some_file.close();
// Write trace file
Trace_file « time_pt() -> time() « " ";
unsigned ntrace_node=Trace_node_pt.size();
for (unsigned j=0; j<ntrace_node; j++)</pre>
  \label{trace_file} {\tt Trace\_file} \  \, \hbox{\tt w sqrt(pow(Trace\_node\_pt[j] -> x(0),2) +} \\
                      pow(Trace_node_pt[j]->x(1),2)) « " ";
```

```
Trace_file « std::endl;
 // removed until Jacobi eigensolver is re-instated
 // // Output principal stress vectors at the centre of all elements
 // SolidHelpers::doc_2D_principal_stress<ELEMENT>(Doc_info,solid_mesh_pt());
// // Write restart file
  sprintf(filename, "%s/restart%i.dat", Doc_info.directory().c_str(),
           Doc_info.number());
// some_file.open(filename);
// dump_it(some_file);
// some_file.close();
cout « "Doced solution for step "
     « Doc_info.number()
      « std::endl « std::endl;
/// Doc displacement and veloc in displ_and_veloc*.dat.
/// The int stage defaults to 0, in which case the ' *' in the
/// filename is simply the step number specified by the Problem's
/// DocInfo object. If it's +/-1, the word "before" and "after"
/// get inserted. This allows checking of the veloc/displacment
/// interpolation during adaptive mesh refinement.
template<class ELEMENT, class TIMESTEPPER>
void DiskShockWaveProblem<ELEMENT,TIMESTEPPER>::doc_displ_and_veloc(
const int& stage)
ofstream some_file;
char filename[100];
 // Number of plot points
unsigned npts;
npts=5;
 // Open file
 if (stage==-1)
  sprintf(filename,"%s/displ_and_veloc_before%i.dat",
          Doc_info.directory().c_str(),Doc_info.number());
else if (stage==1)
  sprintf(filename,"%s/displ_and_veloc_after%i.dat",
          Doc_info.directory().c_str(),Doc_info.number());
else
  sprintf(filename, "%s/displ_and_veloc%i.dat",
          Doc_info.directory().c_str(),Doc_info.number());
 some file.open(filename);
Vector<double> s(2),x(2),dxdt(2),xi(2),displ(2);
 //Loop over solid elements
 unsigned nel=solid_mesh_pt()->nelement();
 for (unsigned e=0;e<nel;e++)</pre>
  some_file « "ZONE I=" « npts « ", J=" « npts « std::endl;
   for (unsigned i=0;i<npts;i++)</pre>
    s[0]=-1.0+2.0*double(i)/double(npts-1);
     for (unsigned j=0; j<npts; j++)</pre>
      s[1]=-1.0+2.0*double(j)/double(npts-1);
       // Cast to full element type
      ELEMENT* el_pt=dynamic_cast<ELEMENT*>(solid_mesh_pt()->
                                            finite_element_pt(e));
       // Eulerian coordinate
      el_pt->interpolated_x(s,x);
       // Lagrangian coordinate
      el_pt->interpolated_xi(s,xi);
       // Displacement
      displ[0]=x[0]-xi[0];
       displ[1] = x[1] - xi[1];
       // Velocity (1st deriv)
      « std::endl;
 some_file.close();
/// Dump the solution
template<class ELEMENT, class TIMESTEPPER>
void DiskShockWaveProblem<ELEMENT, TIMESTEPPER>::dump_it(ofstream& dump_file)
```

```
// Call generic dump()
Problem::dump(dump_file);
/// Read solution from disk
template<class ELEMENT, class TIMESTEPPER>
void DiskShockWaveProblem<ELEMENT,TIMESTEPPER>::restart(ifstream& restart_file)
 // Read generic problem data
Problem::read(restart_file);
/// Run the problem; specify case_number to label output directory
template<class ELEMENT, class TIMESTEPPER>
void DiskShockWaveProblem<ELEMENT,TIMESTEPPER>::run(
const unsigned& case_number)
{
// If there's a command line argument, run the validation (i.e. do only
 // 3 timesteps; otherwise do a few cycles
 unsigned nstep=400;
 if (CommandLineArgs::Argc!=1)
   nstep=3;
 // Define output directory
char dirname[100];
sprintf(dirname, "RESLT%i", case_number);
Doc_info.set_directory(dirname);
 // Step number
 Doc_info.number()=0;
 // Open trace file
 char filename[100];
 sprintf(filename, "%s/trace.dat", Doc_info.directory().c_str());
 Trace_file.open(filename);
 // Set up trace nodes as the nodes on boundary 1 (=curved boundary) in
 // the original mesh (they exist under any refinement!)
 unsigned nnod0=solid_mesh_pt()->nboundary_node(0);
unsigned nnodl=solid_mesh_pt()->nboundary_node(1);
Trace_file « "VARIABLES=\"time\"";
for (unsigned j=0;j<nnod0;j++)</pre>
   Trace_file « ", \"radial node " « j « "\" ";
 for (unsigned j=0; j<nnod1; j++)</pre>
   Trace_file « ", \"azimuthal node " « j « "\" ";
 Trace_file « std::endl;
  // Restart?
    // Pointer to restart file
    ifstream* restart_file_pt=0;
    // No restart
    if (CommandLineArgs::Argc==1)
..
|/
|/
|/
      cout « "No restart" « std::endl;
    // Restart
    else if (CommandLineArgs::Argc==2)
// Open restart file
      restart_file_pt=new ifstream(CommandLineArgs::Argv[1],ios_base::in);
      if (restart_file_pt!=0)
        cout « "Have opened " « CommandLineArgs::Argv[1] «
         " for restart. " « std::endl;
      else
        cout « "ERROR while trying to open " « CommandLineArgs::Argv[1] «
         " for restart." « std::endl;
      // Do the actual restart
      pause("need to do the actual restart");
      //problem.restart(*restart file pt);
    // More than one restart file specified?
    else
      cout « "Can only specify one input file " « std::endl; cout « "You specified the following command line arguments: " « std::endl;
      CommandLineArgs::output();
```

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```
//assert(false);
// Initial parameter values
Global_Physical_Variables::P = 0.1;
// Initialise time
double time0=0.0;
time_pt()->time()=time0;
// Set initial timestep
double dt=0.01;
// Impulsive start
assign_initial_values_impulsive(dt);
// Doc initial state
doc_solution();
Doc_info.number()++;
// First step without adaptivity
unsteady_newton_solve(dt);
doc_solution();
Doc_info.number()++;
//Timestepping loop for subsequent steps with adaptivity
unsigned max_adapt=1;
for(unsigned i=1;i<nstep;i++)</pre>
  unsteady_newton_solve(dt,max_adapt,false);
  doc_solution();
  Doc_info.number()++;
/// Driver for simple elastic problem
//============
int main(int argc, char* argv[])
// Store command line arguments
CommandLineArgs::setup(argc,argv);
//Initialise physical parameters
Global_Physical_Variables::E = 1.0; // ADJUST
Global_Physical_Variables::Nu = 0.3; // ADJUST
  // "Big G" Linear constitutive equations:
 Global_Physical_Variables::Constitutive_law_pt =
  new GeneralisedHookean(&Global_Physical_Variables::Nu,
                           &Global_Physical_Variables::E);
 //Set up the problem:
 unsigned case_number=0;
// Pure displacement formulation
  cout « "Running case " « case_number
        « ": Pure displacement formulation" « std::endl;
  DiskShockWaveProblem<RefineableQPVDElement<2,3>, Newmark<1> > problem;
  problem.run(case_number);
  case_number++;
// Pressure-displacement with Crouzeix Raviart-type pressure
  cout « "Running case " « case_number
        « ": Pressure/displacement with Crouzeix-Raviart pressure" « std::endl;
  DiskShockWaveProblem<RefineableQPVDElementWithPressure<2>, Newmark<1> >
   problem;
  problem.run(case number);
  case_number++;
  // Pressure-displacement with Taylor-Hood-type pressure
  cout « "Running case " « case_number
  « ": Pressure/displacement with Taylor-Hood pressure" « std::endl;
DiskShockWaveProblem<RefineableQPVDElementWithContinuousPressure<2>,
   Newmark<1> > problem;
  problem.run(case_number);
  case_number++;
// Clean up
delete Global_Physical_Variables::Constitutive_law_pt;
Global_Physical_Variables::Constitutive_law_pt=0;
```

## 1.1 PDF file

A pdf version of this document is available.