Midterm

October 24, 2022

1 STA4364 Midterm Exam

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1.1 Import Libraries

[1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import statsmodels.api as sm
import statsmodels.formula.api as smf
from statsmodels.tools.tools import maybe_unwrap_results
from statsmodels.graphics.gofplots import ProbPlot
from statsmodels.stats.outliers_influence import variance_inflation_factor
import statsmodels
from typing import Type
import missingno as msno
from sklearn.model_selection import train_test_split
from sklearn.preprocessing import PolynomialFeatures
from sklearn_pandas import DataFrameMapper
from sklearn.metrics import mean_squared_error
```

1.2 Regression Diagnostic Class

[2]:
```
class Linear_Reg_Diagnostic():
    """
    Diagnostic plots to identify potential problems in a linear regression fit.
    Mainly,
        a. non-linearity of data
        b. Correlation of error terms
        c. non-constant variance
        d. outliers
        e. high-leverage points
        f. collinearity

"""
```

```python
    def __init__(self,
                 results: Type[statsmodels.regression.linear_model.
↪RegressionResultsWrapper]) -> None:
        """
        For a linear regression model, generates following diagnostic plots:

        a. residual
        b. qq
        c. scale location and
        d. leverage

        and a table

        e. vif

        Args:
            results (Type[statsmodels.regression.linear_model.
↪RegressionResultsWrapper]):
                must be instance of statsmodels.regression.linear_model object

        Raises:
            TypeError: if instance does not belong to above object

        Example:
        >>> import numpy as np
        >>> import pandas as pd
        >>> import statsmodels.formula.api as smf
        >>> x = np.linspace(-np.pi, np.pi, 100)
        >>> y = 3*x + 8 + np.random.normal(0,1, 100)
        >>> df = pd.DataFrame({'x':x, 'y':y})
        >>> res = smf.ols(formula= "y ~ x", data=df).fit()
        >>> cls = Linear_Reg_Diagnostic(res)
        >>> cls(plot_context="seaborn-paper")

        In case you do not need all plots you can also independently make an␣
↪individual plot/table
        in following ways

        >>> cls = Linear_Reg_Diagnostic(res)
        >>> cls.residual_plot()
        >>> cls.qq_plot()
        >>> cls.scale_location_plot()
        >>> cls.leverage_plot()
        >>> cls.vif_table()
        """
```

```
        if isinstance(results, statsmodels.regression.linear_model.
↪RegressionResultsWrapper) is False:
            raise TypeError("result must be instance of statsmodels.regression.
↪linear_model.RegressionResultsWrapper object")

        self.results = maybe_unwrap_results(results)

        self.y_true = self.results.model.endog
        self.y_predict = self.results.fittedvalues
        self.xvar = self.results.model.exog
        self.xvar_names = self.results.model.exog_names

        self.residual = np.array(self.results.resid)
        influence = self.results.get_influence()
        self.residual_norm = influence.resid_studentized_internal
        self.leverage = influence.hat_matrix_diag
        self.cooks_distance = influence.cooks_distance[0]
        self.nparams = len(self.results.params)

    def __call__(self, plot_context='seaborn-paper'):
        # print(plt.style.available)
        with plt.style.context(plot_context):
            fig, ax = plt.subplots(nrows=2, ncols=2, figsize=(10,10))
            self.residual_plot(ax=ax[0,0])
            self.qq_plot(ax=ax[0,1])
            self.scale_location_plot(ax=ax[1,0])
            self.leverage_plot(ax=ax[1,1])
            plt.show()

        self.vif_table()
        return fig, ax


    def residual_plot(self, ax=None):
        """
        Residual vs Fitted Plot

        Graphical tool to identify non-linearity.
        (Roughly) Horizontal red line is an indicator that the residual has a␣
↪linear pattern
        """
        if ax is None:
            fig, ax = plt.subplots()

        sns.residplot(
            x=self.y_predict,
            y=self.residual,
```

```python
            lowess=True,
            scatter_kws={'alpha': 0.5},
            line_kws={'color': 'red', 'lw': 1, 'alpha': 0.8},
            ax=ax)

        # annotations
        residual_abs = np.abs(self.residual)
        abs_resid = np.flip(np.sort(residual_abs))
        abs_resid_top_3 = abs_resid[:3]
        for i, _ in enumerate(abs_resid_top_3):
            ax.annotate(
                i,
                xy=(self.y_predict[i], self.residual[i]),
                color='C3')

        ax.set_title('Residuals vs Fitted', fontweight="bold")
        ax.set_xlabel('Fitted values')
        ax.set_ylabel('Residuals')
        return ax

    def qq_plot(self, ax=None):
        """
        Standarized Residual vs Theoretical Quantile plot

        Used to visually check if residuals are normally distributed.
        Points spread along the diagonal line will suggest so.
        """
        if ax is None:
            fig, ax = plt.subplots()

        QQ = ProbPlot(self.residual_norm)
        QQ.qqplot(line='45', alpha=0.5, lw=1, ax=ax)

        # annotations
        abs_norm_resid = np.flip(np.argsort(np.abs(self.residual_norm)), 0)
        abs_norm_resid_top_3 = abs_norm_resid[:3]
        for r, i in enumerate(abs_norm_resid_top_3):
            ax.annotate(
                i,
                xy=(np.flip(QQ.theoretical_quantiles, 0)[r], self.
↪residual_norm[i]),
                ha='right', color='C3')

        ax.set_title('Normal Q-Q', fontweight="bold")
        ax.set_xlabel('Theoretical Quantiles')
        ax.set_ylabel('Standardized Residuals')
        return ax
```

```python
    def scale_location_plot(self, ax=None):
        """
        Sqrt(Standarized Residual) vs Fitted values plot

        Used to check homoscedasticity of the residuals.
        Horizontal line will suggest so.
        """
        if ax is None:
            fig, ax = plt.subplots()

        residual_norm_abs_sqrt = np.sqrt(np.abs(self.residual_norm))

        ax.scatter(self.y_predict, residual_norm_abs_sqrt, alpha=0.5);
        sns.regplot(
            x=self.y_predict,
            y=residual_norm_abs_sqrt,
            scatter=False, ci=False,
            lowess=True,
            line_kws={'color': 'red', 'lw': 1, 'alpha': 0.8},
            ax=ax)

        # annotations
        abs_sq_norm_resid = np.flip(np.argsort(residual_norm_abs_sqrt), 0)
        abs_sq_norm_resid_top_3 = abs_sq_norm_resid[:3]
        for i in abs_sq_norm_resid_top_3:
            ax.annotate(
                i,
                xy=(self.y_predict[i], residual_norm_abs_sqrt[i]),
                color='C3')
        ax.set_title('Scale-Location', fontweight="bold")
        ax.set_xlabel('Fitted values')
        ax.set_ylabel(r'$\sqrt{|\mathrm{Standardized\ Residuals}|}$');
        return ax

    def leverage_plot(self, ax=None):
        """
        Residual vs Leverage plot

        Points falling outside Cook's distance curves are considered␣
↪observation that can sway the fit
        aka are influential.
        Good to have none outside the curves.
        """
        if ax is None:
            fig, ax = plt.subplots()
```

```
        ax.scatter(
            self.leverage,
            self.residual_norm,
            alpha=0.5);

        sns.regplot(
            x=self.leverage,
            y=self.residual_norm,
            scatter=False,
            ci=False,
            lowess=True,
            line_kws={'color': 'red', 'lw': 1, 'alpha': 0.8},
            ax=ax)

        # annotations
        leverage_top_3 = np.flip(np.argsort(self.cooks_distance), 0)[:3]
        for i in leverage_top_3:
            ax.annotate(
                i,
                xy=(self.leverage[i], self.residual_norm[i]),
                color = 'C3')

        xtemp, ytemp = self.__cooks_dist_line(0.5) # 0.5 line
        ax.plot(xtemp, ytemp, label="Cook's distance", lw=1, ls='--',␣
↪color='red')
        xtemp, ytemp = self.__cooks_dist_line(1) # 1 line
        ax.plot(xtemp, ytemp, lw=1, ls='--', color='red')

        ax.set_xlim(0, max(self.leverage)+0.01)
        ax.set_title('Residuals vs Leverage', fontweight="bold")
        ax.set_xlabel('Leverage')
        ax.set_ylabel('Standardized Residuals')
        ax.legend(loc='upper right')
        return ax

    def vif_table(self):
        """
        VIF table

        VIF, the variance inflation factor, is a measure of multicollinearity.
        VIF > 5 for a variable indicates that it is highly collinear with the
        other input variables.
        """
        vif_df = pd.DataFrame()
        vif_df["Features"] = self.xvar_names
        vif_df["VIF Factor"] = [variance_inflation_factor(self.xvar, i) for i␣
↪in range(self.xvar.shape[1])]
```

```
        print(vif_df
                .sort_values("VIF Factor")
                .round(2))


    def __cooks_dist_line(self, factor):
        """
        Helper function for plotting Cook's distance curves
        """
        p = self.nparams
        formula = lambda x: np.sqrt((factor * p * (1 - x)) / x)
        x = np.linspace(0.001, max(self.leverage), 50)
        y = formula(x)
        return x, y
```

1.3 Import and Clean Data

[3]:
```
df = pd.read_csv("midterm_data_1.csv", index_col = "row")
df = df.dropna()
df['feat.c'] = df['feat.c'].astype(int)
df = df.rename(columns={"feat.a": "A", "feat.b": "B", "feat.c":"C", "feat.d":
↪"D", "feat.e":"E", "feat.f":"F", "feat.g":"G", "feat.h":"H", "feat.i":"I"})
df.head()
```

[3]:
```
      response         A         B  C         D         E          F  G  \
row
1     1.658814 -0.879361 -2.297552  2 -2.052926 -1.458801   6.463630  2
2    10.691572  1.550930 -2.332102  4  1.110204 -1.744876 -11.834376  1
3    32.508862 -1.506886 -5.306166  1 -1.814569 -3.747318 -18.031607  1
4    37.747154  5.785842 -3.683903  2  6.453664 -3.645221 -10.534296  3
5    18.072661  1.988523 -3.895907  2  1.600221 -1.874998 -22.835629  1

            H         I
row
1   -0.438952 -1.307249
2    3.157077 -1.750373
3    0.065876 -3.795091
4   -0.293807 -3.495105
5    2.482003 -1.888469
```

[4]:
```
df = pd.get_dummies(df, columns = ['C','G'])
df.head()
```

[4]:
```
      response         A         B         D         E          F         H  \
row
1     1.658814 -0.879361 -2.297552 -2.052926 -1.458801   6.463630 -0.438952
```

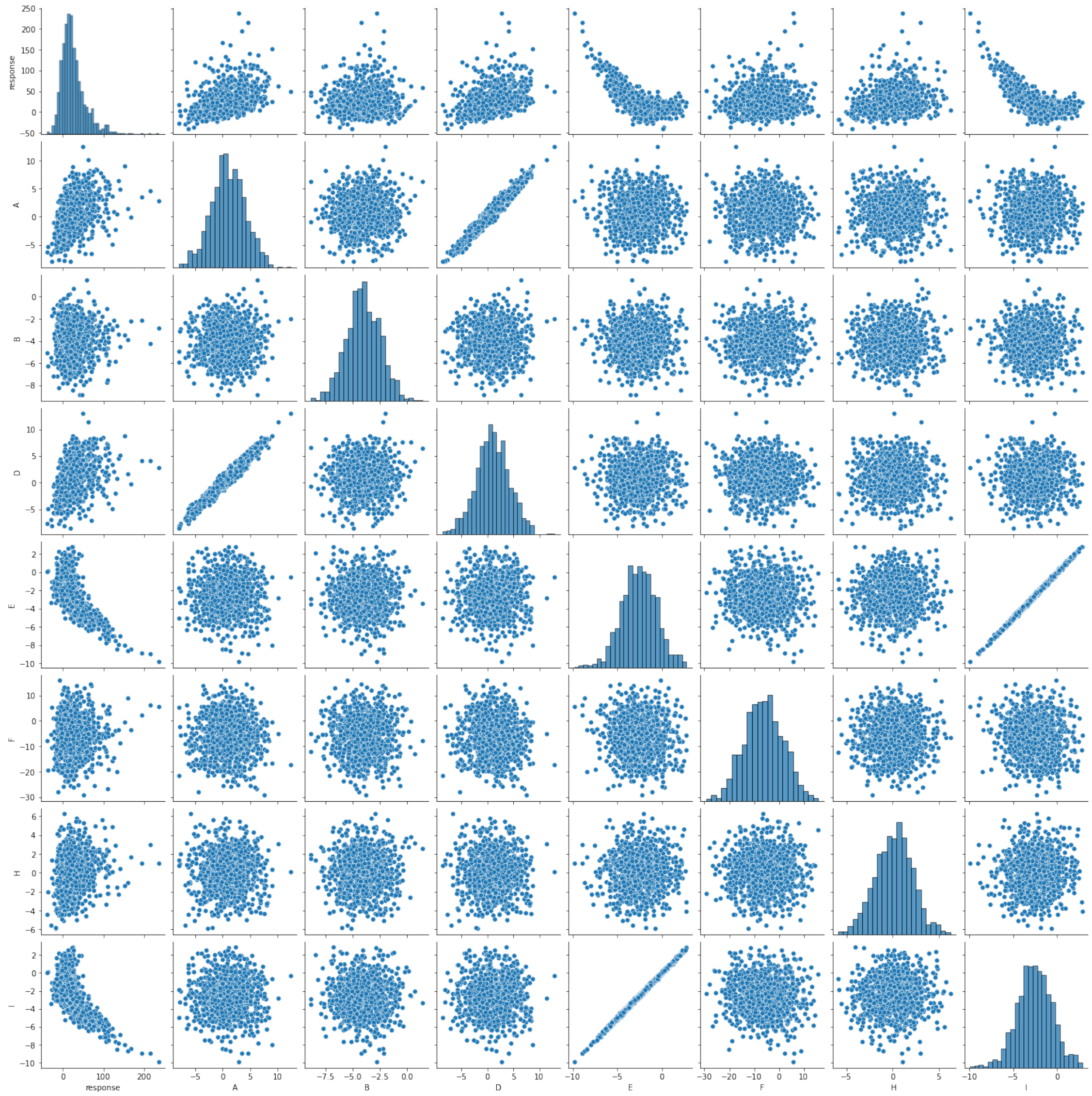
```
2    10.691572  1.550930 -2.332102  1.110204 -1.744876 -11.834376  3.157077
3    32.508862 -1.506886 -5.306166 -1.814569 -3.747318 -18.031607  0.065876
4    37.747154  5.785842 -3.683903  6.453664 -3.645221 -10.534296 -0.293807
5    18.072661  1.988523 -3.895907  1.600221 -1.874998 -22.835629  2.482003

            I  C_1  C_2  C_3  C_4  G_1  G_2  G_3
row
1   -1.307249    0    1    0    0    0    1    0
2   -1.750373    0    0    0    1    1    0    0
3   -3.795091    1    0    0    0    1    0    0
4   -3.495105    0    1    0    0    0    0    1
5   -1.888469    0    1    0    0    1    0    0
```

1.4 Plots

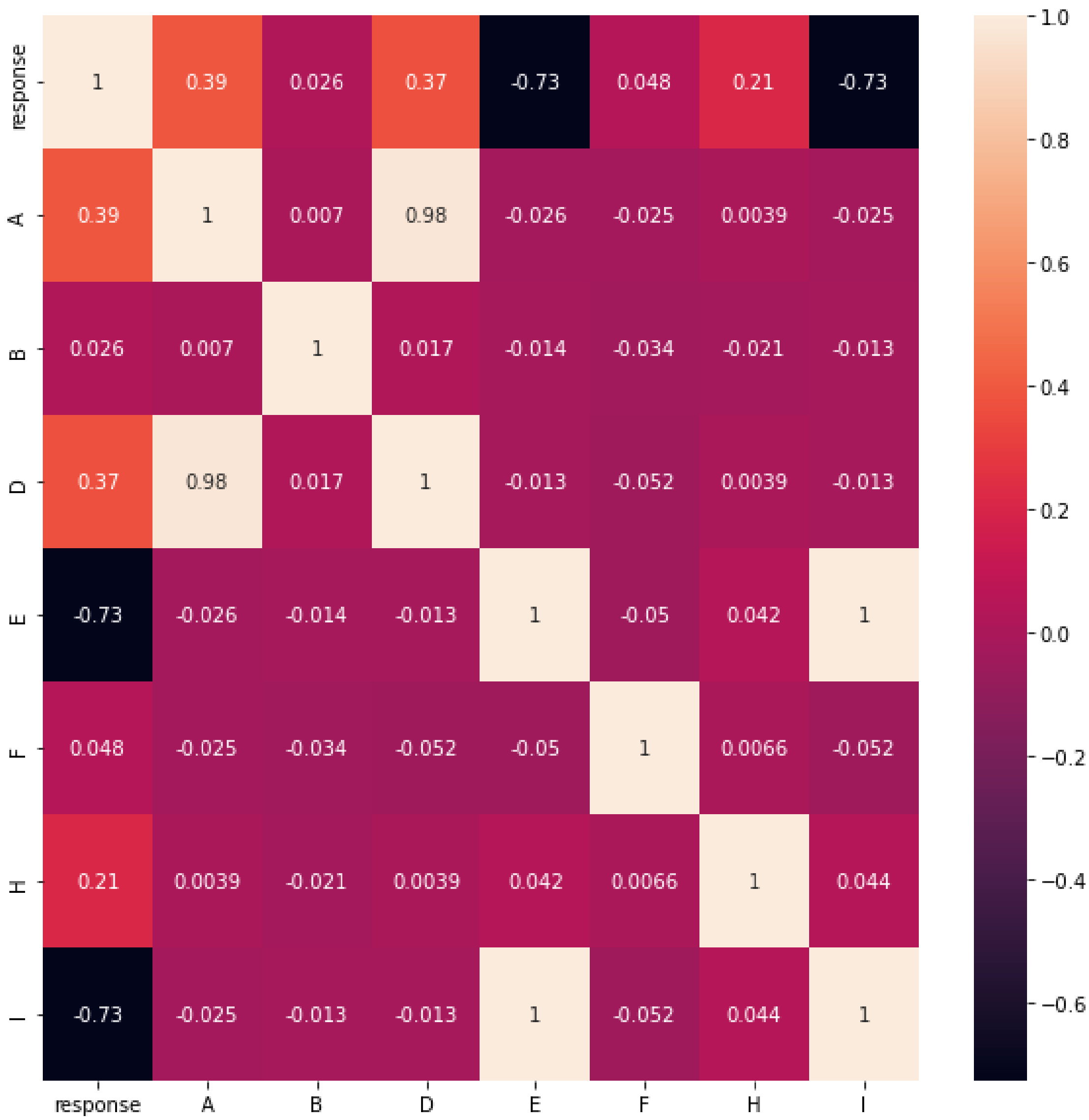
[5]:
```
sns.pairplot(df.iloc[:,:8])
```

[5]:
```
<seaborn.axisgrid.PairGrid at 0x2b09958d250>
```



[6]:
```
plt.figure(figsize = (10,10))
sns.heatmap(df.iloc[:,:8].corr(),annot = True)
```

[6]: <AxesSubplot:>



1.5 Train Test Split

[7]:
```
X = df.iloc[:,1:]
y = df.iloc[:,:1]
```

[8]:
```
X_train, X_test, y_train, y_test = train_test_split(X,y, test_size=0.25,␣
↪random_state=101)
```

1.6 First Order Model

[9]:
```
X_train_c = sm.add_constant(X_train)
model = sm.OLS(y_train,X_train_c)
results = model.fit()
results.summary()
```

```
C:\Users\matth\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142:
FutureWarning: In a future version of pandas all arguments of concat except for
the argument 'objs' will be keyword-only
  x = pd.concat(x[::order], 1)
```

[9]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:               response   R-squared:                       0.750
Model:                            OLS   Adj. R-squared:                  0.746
Method:                 Least Squares   F-statistic:                     183.7
Date:                Mon, 24 Oct 2022   Prob (F-statistic):         1.10e-211
Time:                        18:52:09   Log-Likelihood:                -3106.6
No. Observations:                 747   AIC:                             6239.
Df Residuals:                     734   BIC:                             6299.
Df Model:                          12
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -3.8786      1.167     -3.323      0.001      -6.170      -1.587
A              3.7415      0.895      4.179      0.000       1.984       5.499
B             -0.0530      0.362     -0.146      0.884      -0.763       0.657
D             -0.0715      0.875     -0.082      0.935      -1.789       1.646
E             -5.2308      5.757     -0.909      0.364     -16.532       6.071
F              0.1578      0.074      2.122      0.034       0.012       0.304
H              3.7632      0.286     13.148      0.000       3.201       4.325
I             -6.2058      5.744     -1.080      0.280     -17.483       5.071
C_1            1.1466      1.080      1.061      0.289      -0.975       3.268
C_2           -5.2634      0.999     -5.266      0.000      -7.225      -3.301
C_3            3.0249      1.051      2.879      0.004       0.962       5.087
C_4           -2.7867      1.035     -2.693      0.007      -4.819      -0.755
G_1           -1.1730      0.905     -1.297      0.195      -2.949       0.603
G_2           -0.0701      0.920     -0.076      0.939      -1.876       1.736
G_3           -2.6355      0.889     -2.965      0.003      -4.380      -0.891
==============================================================================
Omnibus:                      312.758   Durbin-Watson:                   1.951
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1907.076
Skew:                           1.784   Prob(JB):                         0.00
Kurtosis:                       9.967   Cond. No.                     1.02e+17
```

```
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The smallest eigenvalue is 8.11e-30. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.
"""
```

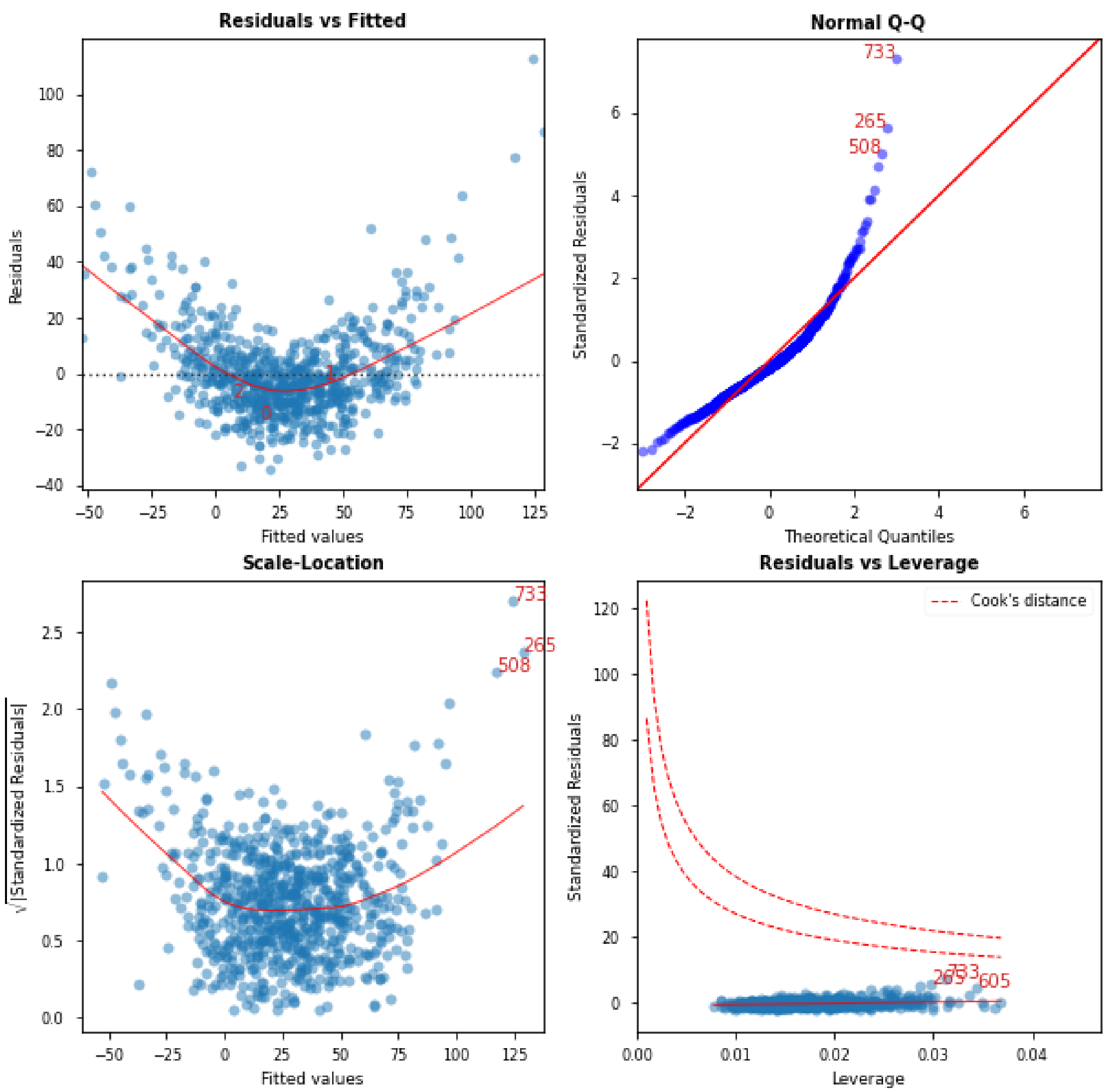
[10]:
```python
np.sqrt(results.scale)
```

[10]:
```
15.621821010599554
```

The coefficients of Feature C mean what the predicted change in the response is when the feature equals that value. If Feature C = 1, response is expected to change by -0.52, if it equals 2, response changes by -6.93, and so on. The R squared of 0.75 means that 75% of the variance in the response is explained by the model. The RSE shows that the model predicts the response with an average error of 15.62. This number can be interpreded when compared to different model with same units in order to compare performance. The F statistic shows that the model with all the variables fits better than the model without variables (intercept-only)

[11]:
```python
cls = Linear_Reg_Diagnostic(results)
fig, ax = cls()
```

```
C:\Users\matth\anaconda3\lib\site-packages\statsmodels\graphics\gofplots.py:993:
UserWarning: marker is redundantly defined by the 'marker' keyword argument and
the fmt string "bo" (-> marker='o'). The keyword argument will take precedence.
  ax.plot(x, y, fmt, **plot_style)
```



	Features	VIF Factor
0	const	0.00
2	B	1.01
6	H	1.03
5	F	1.04
1	A	24.09
3	D	24.18
4	E	399.93
7	I	399.93
8	C_1	inf
9	C_2	inf
10	C_3	inf
11	C_4	inf
12	G_1	inf

```
13      G_2          inf
14      G_3          inf
```

```
C:\Users\matth\anaconda3\lib\site-
packages\statsmodels\regression\linear_model.py:1715: RuntimeWarning: divide by
zero encountered in double_scalars
  return 1 - self.ssr/self.centered_tss
C:\Users\matth\anaconda3\lib\site-
packages\statsmodels\stats\outliers_influence.py:193: RuntimeWarning: divide by
zero encountered in double_scalars
  vif = 1. / (1. - r_squared_i)
```

The Linear Regression Diagnostic Plots show that there is a quadratic pattern in the residuals. This fails the Linearity and constant variance assumptions.

1.7 Make Data for 2nd Order Model

[12]:
```
poly = PolynomialFeatures(2, interaction_only=True)
X_train_p = pd.DataFrame(poly.fit_transform(X_train))
X_train_p.columns = poly.get_feature_names(X_train.columns)

X_test_p = pd.DataFrame(poly.fit_transform(X_test))
X_test_p.columns = poly.get_feature_names(X_test.columns)
```

[13]:
```
X_train_p['A^2'] = X_train_p['A'].pow(2)
X_test_p['A^2'] = X_test_p['A'].pow(2)

X_train_p['B^2'] = X_train_p['B'].pow(2)
X_test_p['B^2'] = X_test_p['B'].pow(2)

X_train_p['D^2'] = X_train_p['D'].pow(2)
X_test_p['D^2'] = X_test_p['D'].pow(2)

X_train_p['E^2'] = X_train_p['E'].pow(2)
X_test_p['E^2'] = X_test_p['E'].pow(2)

X_train_p['F^2'] = X_train_p['F'].pow(2)
X_test_p['F^2'] = X_test_p['F'].pow(2)

X_train_p['H^2'] = X_train_p['H'].pow(2)
X_test_p['H^2'] = X_test_p['H'].pow(2)

X_train_p['I^2'] = X_train_p['I'].pow(2)
X_test_p['I^2'] = X_test_p['I'].pow(2)
```

[14]:
```
X_train_p = X_train_p.iloc[:,1:]
X_test_p = X_test_p.iloc[:,1:]
```

[15]:
```
y_train = y_train.reset_index().drop("row", axis = 1)
y_test = y_test.reset_index().drop("row", axis = 1)
```

[16]:
```
for column in X_train_p.columns:
    if column.count('^') == 2:
        X_train_p.drop(column,axis=1, inplace=True)
    elif column.count('A') == 2:
        X_train_p.drop(column,axis=1, inplace=True)
    elif column.count('B') == 2:
        X_train_p.drop(column,axis=1, inplace=True)
    elif column.count('C') == 2:
        X_train_p.drop(column,axis=1, inplace=True)
    elif column.count('D') == 2:
        X_train_p.drop(column,axis=1, inplace=True)
    elif column.count('E') == 2:
        X_train_p.drop(column,axis=1, inplace=True)
    elif column.count('F') == 2:
        X_train_p.drop(column,axis=1, inplace=True)
    elif column.count('G') == 2:
        X_train_p.drop(column,axis=1, inplace=True)
    elif column.count('H') == 2:
        X_train_p.drop(column,axis=1, inplace=True)
    elif column.count('I') == 2:
        X_train_p.drop(column,axis=1, inplace=True)
```

[17]:
```
for column in X_test_p.columns:
    if column.count('^') == 2:
        X_test_p.drop(column,axis=1, inplace=True)
    elif column.count('A') == 2:
        X_test_p.drop(column,axis=1, inplace=True)
    elif column.count('B') == 2:
        X_test_p.drop(column,axis=1, inplace=True)
    elif column.count('C') == 2:
        X_test_p.drop(column,axis=1, inplace=True)
    elif column.count('D') == 2:
        X_test_p.drop(column,axis=1, inplace=True)
    elif column.count('E') == 2:
        X_test_p.drop(column,axis=1, inplace=True)
    elif column.count('F') == 2:
        X_test_p.drop(column,axis=1, inplace=True)
    elif column.count('G') == 2:
        X_test_p.drop(column,axis=1, inplace=True)
    elif column.count('H') == 2:
        X_test_p.drop(column,axis=1, inplace=True)
    elif column.count('I') == 2:
        X_test_p.drop(column,axis=1, inplace=True)
```

1.8 Full Model

[18]:
```
X_train_p_c = sm.add_constant(X_train_p)
model_full = sm.OLS(y_train, X_train_p_c)
results_full = model_full.fit()
results_full.summary()
```

```
C:\Users\matth\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142:
FutureWarning: In a future version of pandas all arguments of concat except for
the argument 'objs' will be keyword-only
  x = pd.concat(x[::order], 1)
```

[18]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:               response   R-squared:                       0.942
Model:                            OLS   Adj. R-squared:                  0.935
Method:                 Least Squares   F-statistic:                     134.3
Date:                Mon, 24 Oct 2022   Prob (F-statistic):               0.00
Time:                        18:52:10   Log-Likelihood:                -2558.8
No. Observations:                 747   AIC:                             5282.
Df Residuals:                     665   BIC:                             5660.
Df Model:                          81
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          2.9099      1.227      2.371      0.018       0.500       5.320
A              3.8567      1.013      3.808      0.000       1.868       5.845
B              0.7810      0.496      1.574      0.116      -0.194       1.756
D             -1.8559      0.988     -1.879      0.061      -3.795       0.083
E             -7.3724      6.642     -1.110      0.267     -20.414       5.670
F             -0.0182      0.084     -0.216      0.829      -0.183       0.147
H              0.2626      0.323      0.813      0.416      -0.371       0.897
I              8.2915      6.578      1.260      0.208      -4.625      21.208
C_1            0.1144      1.320      0.087      0.931      -2.478       2.707
C_2           -1.7044      1.284     -1.328      0.185      -4.225       0.816
C_3            5.0610      1.392      3.637      0.000       2.328       7.794
C_4           -0.5611      1.259     -0.446      0.656      -3.033       1.911
G_1           -0.9852      1.175     -0.838      0.402      -3.292       1.322
G_2            3.2642      1.206      2.707      0.007       0.896       5.632
G_3            0.6309      1.173      0.538      0.591      -1.672       2.934
A B            0.4109      0.313      1.312      0.190      -0.204       1.026
A D           -1.2014      1.069     -1.124      0.261      -3.300       0.897
A E            9.5224      5.168      1.843      0.066      -0.624      19.669
A F           -0.0601      0.062     -0.964      0.335      -0.183       0.062
A H           -0.0429      0.242     -0.177      0.860      -0.518       0.432
```

A I	-9.0693	5.180	-1.751	0.080	-19.240	1.101
A C_1	1.6114	0.906	1.778	0.076	-0.168	3.391
A C_2	0.5045	0.873	0.578	0.563	-1.209	2.218
A C_3	1.0040	0.845	1.188	0.235	-0.656	2.664
A C_4	0.7369	0.852	0.865	0.387	-0.935	2.409
A G_1	1.0407	0.763	1.365	0.173	-0.457	2.538
A G_2	1.0505	0.751	1.399	0.162	-0.424	2.525
A G_3	1.7655	0.763	2.315	0.021	0.268	3.263
B D	-0.4541	0.308	-1.477	0.140	-1.058	0.150
B E	-3.0456	2.075	-1.468	0.143	-7.120	1.029
B F	-0.0035	0.026	-0.135	0.892	-0.054	0.047
B H	-0.2219	0.099	-2.232	0.026	-0.417	-0.027
B I	3.0914	2.064	1.497	0.135	-0.962	7.145
B C_1	0.0023	0.368	0.006	0.995	-0.720	0.724
B C_2	0.0649	0.352	0.184	0.854	-0.626	0.756
B C_3	0.6540	0.369	1.771	0.077	-0.071	1.379
B C_4	0.0599	0.337	0.178	0.859	-0.603	0.722
B G_1	-0.1822	0.312	-0.585	0.559	-0.794	0.430
B G_2	0.5442	0.329	1.652	0.099	-0.103	1.191
B G_3	0.4190	0.310	1.350	0.177	-0.190	1.028
D E	-7.9389	5.051	-1.572	0.117	-17.857	1.979
D F	0.0525	0.060	0.875	0.382	-0.065	0.170
D H	0.0446	0.234	0.190	0.849	-0.416	0.505
D I	7.4853	5.060	1.479	0.140	-2.451	17.422
D C_1	-1.2885	0.879	-1.466	0.143	-3.014	0.437
D C_2	0.0853	0.861	0.099	0.921	-1.605	1.776
D C_3	-0.3924	0.830	-0.473	0.636	-2.022	1.237
D C_4	-0.2603	0.834	-0.312	0.755	-1.898	1.378
D G_1	-0.0925	0.743	-0.125	0.901	-1.552	1.367
D G_2	-0.4450	0.740	-0.601	0.548	-1.898	1.008
D G_3	-1.3183	0.745	-1.770	0.077	-2.781	0.144
E F	0.0974	0.405	0.240	0.810	-0.699	0.894
E H	-2.4030	1.532	-1.568	0.117	-5.411	0.606
E I	40.4695	45.775	0.884	0.377	-49.411	130.350
E C_1	-12.5087	5.936	-2.107	0.035	-24.164	-0.854
E C_2	-3.8846	5.367	-0.724	0.469	-14.423	6.654
E C_3	9.6150	5.604	1.716	0.087	-1.388	20.618
E C_4	-0.5941	5.615	-0.106	0.916	-11.620	10.432
E G_1	-2.8401	4.990	-0.569	0.569	-12.638	6.958
E G_2	3.3920	5.165	0.657	0.512	-6.750	13.535
E G_3	-7.9243	4.643	-1.707	0.088	-17.041	1.193
F H	-0.0251	0.021	-1.205	0.229	-0.066	0.016
F I	-0.0951	0.405	-0.235	0.815	-0.891	0.701
F C_1	-0.0661	0.070	-0.944	0.345	-0.204	0.071
F C_2	0.0544	0.072	0.751	0.453	-0.088	0.197
F C_3	-0.0143	0.071	-0.202	0.840	-0.153	0.124
F C_4	0.0078	0.074	0.105	0.916	-0.137	0.153

F G_1	-0.0320	0.065	-0.492	0.623	-0.160	0.096
F G_2	-0.0126	0.065	-0.195	0.845	-0.139	0.114
F G_3	0.0264	0.063	0.419	0.675	-0.097	0.150
H I	1.5519	1.527	1.016	0.310	-1.447	4.551
H C_1	0.2501	0.286	0.874	0.382	-0.312	0.812
H C_2	-0.1462	0.281	-0.520	0.603	-0.698	0.406
H C_3	-0.1179	0.287	-0.412	0.681	-0.681	0.445
H C_4	0.2767	0.260	1.063	0.288	-0.235	0.788
H G_1	-0.1986	0.246	-0.807	0.420	-0.682	0.285
H G_2	0.0040	0.253	0.016	0.987	-0.492	0.500
H G_3	0.4572	0.234	1.958	0.051	-0.001	0.916
I C_1	12.3156	5.899	2.088	0.037	0.733	23.898
I C_2	4.2197	5.368	0.786	0.432	-6.321	14.760
I C_3	-9.4487	5.603	-1.686	0.092	-20.450	1.553
I C_4	1.2049	5.580	0.216	0.829	-9.752	12.161
I G_1	3.2633	4.990	0.654	0.513	-6.535	13.062
I G_2	-2.9061	5.126	-0.567	0.571	-12.972	7.160
I G_3	7.9343	4.628	1.714	0.087	-1.153	17.021
C_1 G_1	-1.6435	0.977	-1.681	0.093	-3.563	0.276
C_1 G_2	1.6514	0.921	1.793	0.073	-0.157	3.460
C_1 G_3	0.1065	0.913	0.117	0.907	-1.686	1.899
C_2 G_1	-0.6739	0.838	-0.805	0.421	-2.318	0.971
C_2 G_2	-0.3288	0.896	-0.367	0.714	-2.089	1.431
C_2 G_3	-0.7016	0.909	-0.772	0.440	-2.486	1.083
C_3 G_1	0.9019	0.942	0.957	0.339	-0.948	2.752
C_3 G_2	1.9707	0.913	2.159	0.031	0.178	3.763
C_3 G_3	2.1884	0.922	2.374	0.018	0.378	3.999
C_4 G_1	0.4303	0.893	0.482	0.630	-1.324	2.185
C_4 G_2	-0.0291	0.979	-0.030	0.976	-1.951	1.893
C_4 G_3	-0.9623	0.815	-1.181	0.238	-2.562	0.638
A^2	0.6047	0.548	1.104	0.270	-0.471	1.680
B^2	0.1002	0.085	1.178	0.239	-0.067	0.267
D^2	0.6172	0.532	1.161	0.246	-0.427	1.661
E^2	-17.9185	22.939	-0.781	0.435	-62.960	27.123
F^2	-0.0037	0.004	-0.911	0.362	-0.012	0.004
H^2	-0.0480	0.056	-0.851	0.395	-0.159	0.063
I^2	-20.1967	22.859	-0.884	0.377	-65.081	24.687

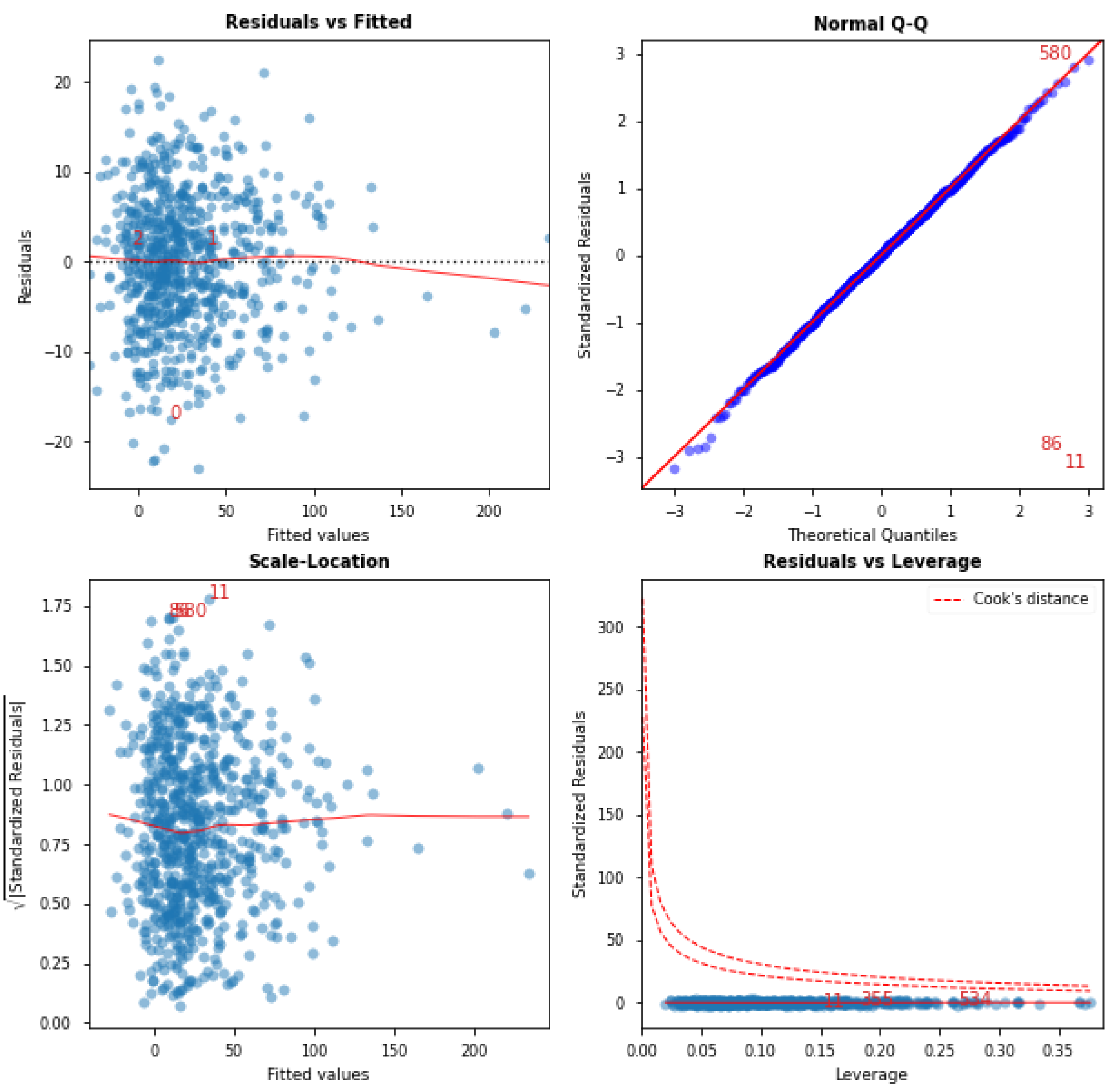
Omnibus:	0.312	Durbin-Watson:	1.920
Prob(Omnibus):	0.856	Jarque-Bera (JB):	0.321
Skew:	-0.050	Prob(JB):	0.852
Kurtosis:	2.978	Cond. No.	2.39e+16

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
[2] The smallest eigenvalue is 3.6e-26. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.
"""
```

[19]:
```
cls = Linear_Reg_Diagnostic(results_full)
fig, ax = cls()
```

```
C:\Users\matth\anaconda3\lib\site-packages\statsmodels\graphics\gofplots.py:993:
UserWarning: marker is redundantly defined by the 'marker' keyword argument and
the fmt string "bo" (-> marker='o'). The keyword argument will take precedence.
  ax.plot(x, y, fmt, **plot_style)
```




```
C:\Users\matth\anaconda3\lib\site-
packages\statsmodels\regression\linear_model.py:1715: RuntimeWarning: divide by
zero encountered in double_scalars
```

```
  return 1 - self.ssr/self.centered_tss
C:\Users\matth\anaconda3\lib\site-
packages\statsmodels\stats\outliers_influence.py:193: RuntimeWarning: divide by
zero encountered in double_scalars
  vif = 1. / (1. - r_squared_i)
```

```
    Features  VIF Factor
0      const        0.00
102      H^2        1.22
61       F H        1.95
101      F^2        3.00
31       B H        9.27
..       …          …
50     D G_3         inf
49     D G_2         inf
48     D G_1         inf
46     D C_3         inf
7          I         inf

[104 rows x 2 columns]
```

1.9 Intermediary Models

[20]:
```python
X_train_p_r = X_train_p.copy()
X_test_p_r = X_test_p.copy()
for column in X_train_p_r.columns:
    if results_full.pvalues.loc[column] > 0.38:
        X_train_p_r.drop(column,axis=1, inplace=True)
        X_test_p_r.drop(column,axis=1, inplace=True)
```

[21]:
```python
X_train_p_r_c = sm.add_constant(X_train_p_r)
model_3 = sm.OLS(y_train, X_train_p_r_c)
results_3 = model_3.fit()
results_3.summary()
```

```
C:\Users\matth\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142:
FutureWarning: In a future version of pandas all arguments of concat except for
the argument 'objs' will be keyword-only
  x = pd.concat(x[::order], 1)
```

[21]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:               response   R-squared:                       0.942
Model:                            OLS   Adj. R-squared:                  0.938
Method:                 Least Squares   F-statistic:                     220.5
Date:                Mon, 24 Oct 2022   Prob (F-statistic):               0.00
```

Time:	18:52:11	Log-Likelihood:	-2562.5
No. Observations:	747	AIC:	5229.
Df Residuals:	695	BIC:	5469.
Df Model:	51		
Covariance Type:	nonrobust		

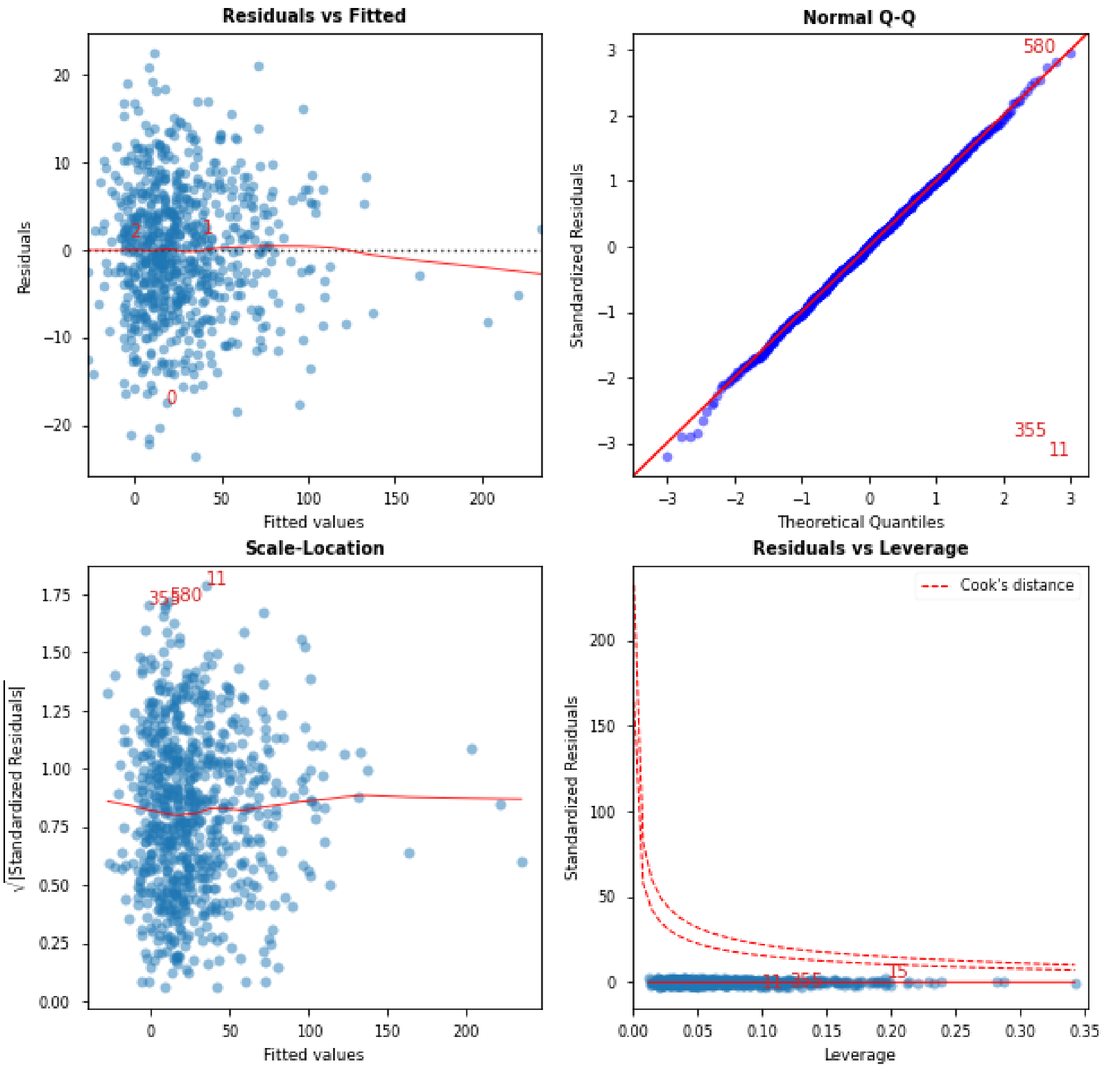
	coef	std err	t	P>|t|	[0.025	0.975]
const	2.2089	1.993	1.108	0.268	-1.704	6.122
A	4.5733	1.122	4.077	0.000	2.371	6.776
B	0.8821	0.755	1.168	0.243	-0.600	2.364
D	-2.3827	1.520	-1.567	0.118	-5.368	0.602
E	-11.2307	10.620	-1.057	0.291	-32.083	9.621
I	13.0433	10.535	1.238	0.216	-7.640	33.727
C_2	-1.7580	0.877	-2.005	0.045	-3.480	-0.036
C_3	5.5559	1.679	3.308	0.001	2.259	8.853
G_2	3.3193	1.921	1.728	0.084	-0.453	7.092
A B	0.3986	0.294	1.355	0.176	-0.179	0.976
A D	-1.1547	1.016	-1.137	0.256	-3.149	0.840
A E	9.8072	4.876	2.011	0.045	0.234	19.380
A F	-0.0068	0.013	-0.532	0.595	-0.032	0.018
A I	-9.3694	4.882	-1.919	0.055	-18.954	0.215
A C_1	0.9470	1.096	0.864	0.388	-1.206	3.100
A C_3	0.0769	0.222	0.347	0.729	-0.358	0.512
A G_1	1.3451	0.526	2.558	0.011	0.313	2.378
A G_2	1.0146	0.517	1.963	0.050	-1.92e-05	2.029
A G_3	2.2137	0.725	3.052	0.002	0.789	3.638
B D	-0.4367	0.287	-1.519	0.129	-1.001	0.128
B E	-3.2412	2.006	-1.616	0.107	-7.179	0.697
B H	-0.2361	0.055	-4.282	0.000	-0.344	-0.128
B I	3.2842	1.996	1.645	0.100	-0.635	7.203
B C_3	0.6425	0.433	1.483	0.139	-0.208	1.493
B G_2	0.5689	0.417	1.364	0.173	-0.250	1.388
B G_3	0.3314	0.268	1.236	0.217	-0.195	0.858
D E	-8.1971	4.772	-1.718	0.086	-17.567	1.172
D I	7.7594	4.776	1.625	0.105	-1.618	17.137
D C_1	-1.1211	1.065	-1.053	0.293	-3.211	0.969
D G_3	-1.3648	0.946	-1.443	0.150	-3.222	0.492
E H	-2.3694	1.447	-1.637	0.102	-5.211	0.472
E I	4.7519	1.476	3.219	0.001	1.854	7.650
E C_1	-9.4881	7.615	-1.246	0.213	-24.440	5.464
E C_3	11.6877	7.161	1.632	0.103	-2.372	25.747
E G_3	-7.7059	6.102	-1.263	0.207	-19.687	4.275
F H	-0.0289	0.018	-1.570	0.117	-0.065	0.007
F C_1	-0.0864	0.077	-1.118	0.264	-0.238	0.065
H I	1.5069	1.442	1.045	0.296	-1.325	4.339
H C_4	0.2900	0.318	0.911	0.363	-0.335	0.915

```
H G_3          0.6030      0.297      2.030      0.043      0.020      1.186
I C_1          8.7145      7.584      1.149      0.251     -6.176     23.605
I C_3        -12.0503      7.154     -1.684      0.093    -26.097      1.997
I G_3          7.1880      6.089      1.181      0.238     -4.766     19.142
C_1 G_1       -1.0870      1.527     -0.712      0.477     -4.085      1.911
C_1 G_2        2.4711      1.559      1.585      0.113     -0.590      5.532
C_3 G_1        1.2407      1.122      1.106      0.269     -0.962      3.444
C_3 G_2        2.0713      1.172      1.767      0.078     -0.230      4.373
C_3 G_3        2.2440      1.172      1.915      0.056     -0.056      4.544
C_4 G_3       -1.4971      1.339     -1.118      0.264     -4.126      1.132
A^2            0.5808      0.521      1.114      0.266     -0.443      1.604
B^2            0.1082      0.082      1.322      0.187     -0.053      0.269
D^2            0.5920      0.505      1.172      0.241     -0.399      1.584
F^2           -0.0045      0.003     -1.594      0.111     -0.010      0.001
I^2           -2.4121      1.463     -1.649      0.100     -5.284      0.460
==============================================================================
Omnibus:                        0.133   Durbin-Watson:                   1.910
Prob(Omnibus):                  0.936   Jarque-Bera (JB):                0.132
Skew:                          -0.032   Prob(JB):                        0.936
Kurtosis:                       2.987   Cond. No.                     1.37e+16
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The smallest eigenvalue is 9.96e-26. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.
"""
```

[22]:
```
cls = Linear_Reg_Diagnostic(results_3)
fig, ax = cls()
```

```
C:\Users\matth\anaconda3\lib\site-packages\statsmodels\graphics\gofplots.py:993:
UserWarning: marker is redundantly defined by the 'marker' keyword argument and
the fmt string "bo" (-> marker='o'). The keyword argument will take precedence.
  ax.plot(x, y, fmt, **plot_style)
```



	Features	VIF Factor
38	H C_4	1.35
52	F^2	1.47
35	F H	1.58
39	H G_3	1.63
36	F C_1	1.73
43	C_1 G_1	1.81
15	A C_3	1.83
6	C_2	1.96
48	C_4 G_3	2.04
44	C_1 G_2	2.06
12	A F	2.15
21	B H	2.95
25	B G_3	3.97

```
23    B C_3       8.76
24    B G_2       9.68
8       G_2       9.95
50      B^2      14.86
2         B      17.95
28    D C_1      34.92
14    A C_1      34.93
29    D G_3      41.93
0     const      49.42
9       A B     201.52
19      B D     204.53
3         D     296.76
49      A^2     792.09
51      D^2     852.58
37      H I    1082.65
30      E H    1086.49
42    I G_3    1309.96
34    E G_3    1317.58
41    I C_3    1403.09
33    E C_3    1409.09
40    I C_1    1555.06
32    E C_1    1561.34
10      A D    3157.24
53      I^2    3929.62
31      E I    3964.13
22      B I    4455.20
20      B E    4481.91
5         I    5466.76
4         E    5532.17
26      D E   33625.23
27      D I   33672.05
11      A E   33819.41
13      A I   33840.01
16    A G_1         inf
46  C_3 G_2         inf
47  C_3 G_3         inf
7       C_3         inf
17    A G_2         inf
18    A G_3         inf
1         A         inf
45  C_3 G_1         inf
```

```
C:\Users\matth\anaconda3\lib\site-
packages\statsmodels\stats\outliers_influence.py:193: RuntimeWarning: divide by
zero encountered in double_scalars
  vif = 1. / (1. - r_squared_i)
```

[23]:
```
X_train_p_r_2 = X_train_p_r.copy()
X_test_p_r_2 = X_test_p_r.copy()
for column in X_train_p_r.columns:
    if results_3.pvalues.loc[column] > 0.20:
        X_train_p_r_2.drop(column,axis=1, inplace=True)
        X_test_p_r_2.drop(column,axis=1, inplace=True)
```

[24]:
```
X_train_p_r_2_c = sm.add_constant(X_train_p_r_2)
model_4 = sm.OLS(y_train, X_train_p_r_2_c)
results_4 = model_4.fit()
results_4.summary()
```

```
C:\Users\matth\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142:
FutureWarning: In a future version of pandas all arguments of concat except for
the argument 'objs' will be keyword-only
  x = pd.concat(x[::order], 1)
```

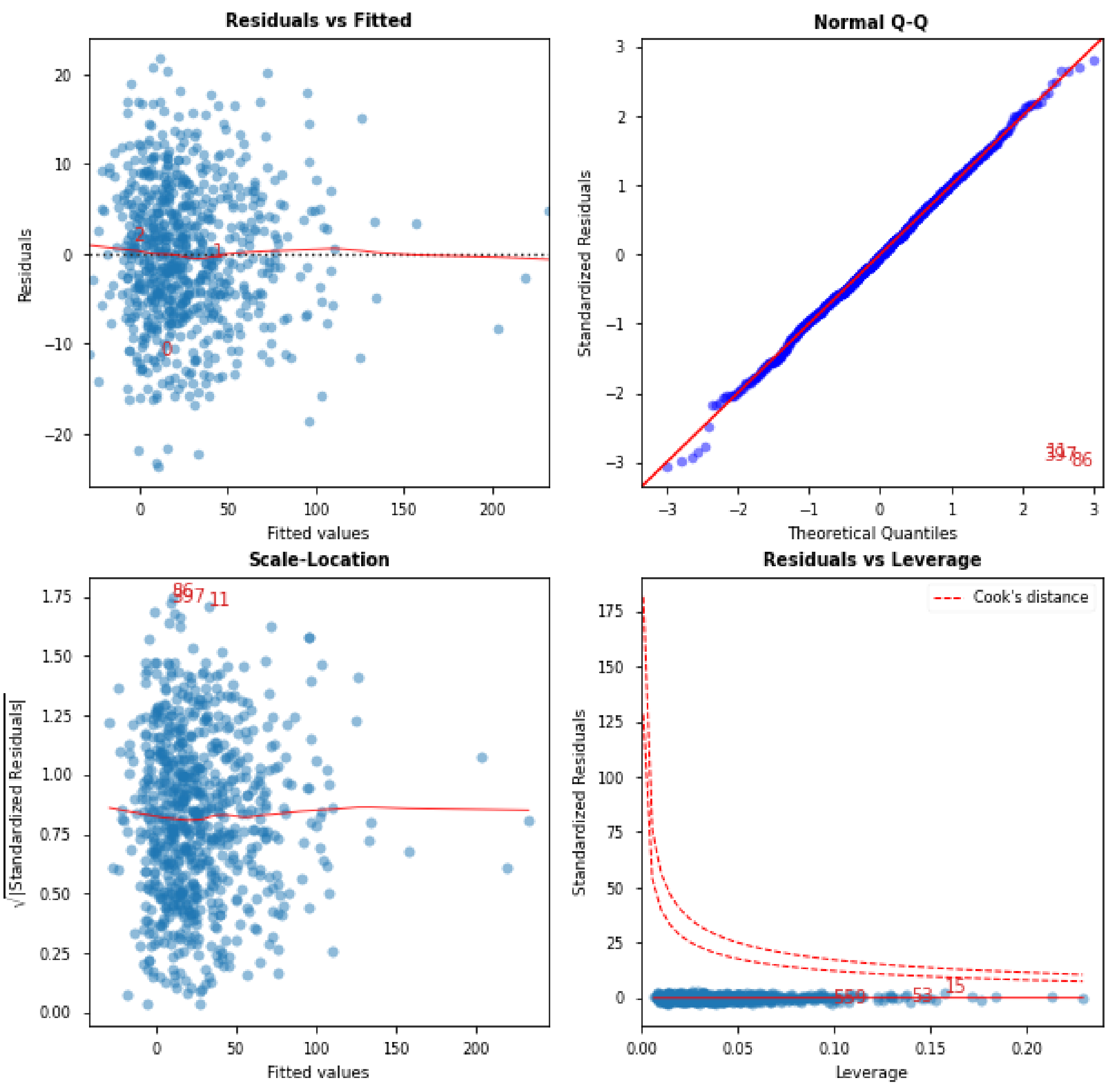
[24]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:               response   R-squared:                       0.939
Model:                            OLS   Adj. R-squared:                  0.936
Method:                 Least Squares   F-statistic:                     352.6
Date:                Mon, 24 Oct 2022   Prob (F-statistic):               0.00
Time:                        18:52:12   Log-Likelihood:                -2582.6
No. Observations:                 747   AIC:                             5229.
Df Residuals:                     715   BIC:                             5377.
Df Model:                          31
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -1.2644      0.850     -1.488      0.137      -2.933       0.404
A              3.9325      1.054      3.733      0.000       1.864       6.001
D             -1.3406      1.422     -0.943      0.346      -4.133       1.451
C_2           -2.2669      0.729     -3.111      0.002      -3.697      -0.836
C_3            7.6656      2.303      3.329      0.001       3.144      12.187
G_2            2.6079      1.835      1.421      0.156      -0.995       6.211
A B            0.1927      0.288      0.669      0.504      -0.373       0.758
A E           12.3640      4.801      2.575      0.010       2.938      21.790
A I          -12.0402      4.810     -2.503      0.013     -21.484      -2.596
A G_1          1.0235      0.500      2.046      0.041       0.041       2.006
A G_2          0.7052      0.495      1.424      0.155      -0.267       1.677
A G_3          2.2037      0.718      3.068      0.002       0.793       3.614
B D           -0.2300      0.281     -0.820      0.413      -0.781       0.321
B E           -0.3219      1.095     -0.294      0.769      -2.471       1.827
B H           -0.2553      0.054     -4.717      0.000      -0.362      -0.149
```

```
B I              0.1629      1.090      0.149      0.881      -1.978       2.304
B C_3            0.6906      0.436      1.584      0.114      -0.165       1.546
B G_2            0.4175      0.392      1.066      0.287      -0.351       1.186
D E            -11.3615      4.697     -2.419      0.016     -20.583      -2.140
D I             11.0402      4.703      2.347      0.019       1.807      20.274
D G_3           -1.6571      0.939     -1.764      0.078      -3.501       0.187
E H             -0.8567      0.066    -13.076      0.000      -0.985      -0.728
E I              5.5581      1.293      4.297      0.000       3.019       8.098
E C_3           15.1282      6.727      2.249      0.025       1.920      28.336
F H             -0.0264      0.018     -1.438      0.151      -0.062       0.010
H G_3            0.5509      0.294      1.876      0.061      -0.026       1.128
I C_3          -15.0915      6.729     -2.243      0.025     -28.303      -1.880
C_1 G_2          4.2964      1.365      3.148      0.002       1.617       6.976
C_3 G_2          1.1338      1.675      0.677      0.499      -2.154       4.422
C_3 G_3          0.8347      1.468      0.569      0.570      -2.048       3.717
B^2              0.0546      0.034      1.629      0.104      -0.011       0.120
F^2             -0.0030      0.002     -1.227      0.220      -0.008       0.002
I^2             -3.3008      1.286     -2.568      0.010      -5.825      -0.777
==============================================================================
Omnibus:                        0.060   Durbin-Watson:                   1.919
Prob(Omnibus):                  0.970   Jarque-Bera (JB):                0.129
Skew:                          -0.002   Prob(JB):                        0.937
Kurtosis:                       2.936   Cond. No.                     1.35e+16
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The smallest eigenvalue is 1.02e-25. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.
"""
```

[25]:
```python
cls = Linear_Reg_Diagnostic(results_4)
fig, ax = cls()
```

```
C:\Users\matth\anaconda3\lib\site-packages\statsmodels\graphics\gofplots.py:993:
UserWarning: marker is redundantly defined by the 'marker' keyword argument and
the fmt string "bo" (-> marker='o'). The keyword argument will take precedence.
  ax.plot(x, y, fmt, **plot_style)
```



	Features	VIF Factor
31	F^2	1.09
3	C_2	1.32
24	F H	1.53
27	C_1 G_2	1.54
25	H G_3	1.55
29	C_3 G_3	1.93
21	E H	2.17
30	B^2	2.43
28	C_3 G_2	2.70
14	B H	2.77
17	B G_2	8.32
16	B C_3	8.64
0	const	8.76

```
5       G_2       8.85
4       C_3      11.94
20    D G_3      40.31
6       A B     188.39
12      B D     190.06
2         D     253.21
26    I C_3    1210.30
23    E C_3    1212.60
15      B I    1296.32
13      B E    1301.26
32      I^2    2960.24
22      E I    2968.38
18      D E   31761.15
19      D I   31835.97
7       A E   31974.90
8       A I   32037.87
9     A G_1        inf
10    A G_2        inf
11    A G_3        inf
1         A        inf
```

```
C:\Users\matth\anaconda3\lib\site-
packages\statsmodels\stats\outliers_influence.py:193: RuntimeWarning: divide by
zero encountered in double_scalars
  vif = 1. / (1. - r_squared_i)
```

[26]:

```
X_train_p_r_3 = X_train_p_r_2.copy()
X_test_p_r_3 = X_test_p_r_2.copy()
for column in X_train_p_r_2.columns:
    if results_4.pvalues.loc[column] > 0.20:
        X_train_p_r_3.drop(column,axis=1, inplace=True)
        X_test_p_r_3.drop(column,axis=1, inplace=True)
```

[27]:

```
X_train_p_r_3_c = sm.add_constant(X_train_p_r_3)
model_5 = sm.OLS(y_train, X_train_p_r_3_c)
results_5 = model_5.fit()
results_5.summary()
```

```
C:\Users\matth\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142:
FutureWarning: In a future version of pandas all arguments of concat except for
the argument 'objs' will be keyword-only
  x = pd.concat(x[::order], 1)
```

[27]:

```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:               response   R-squared:                       0.938
```

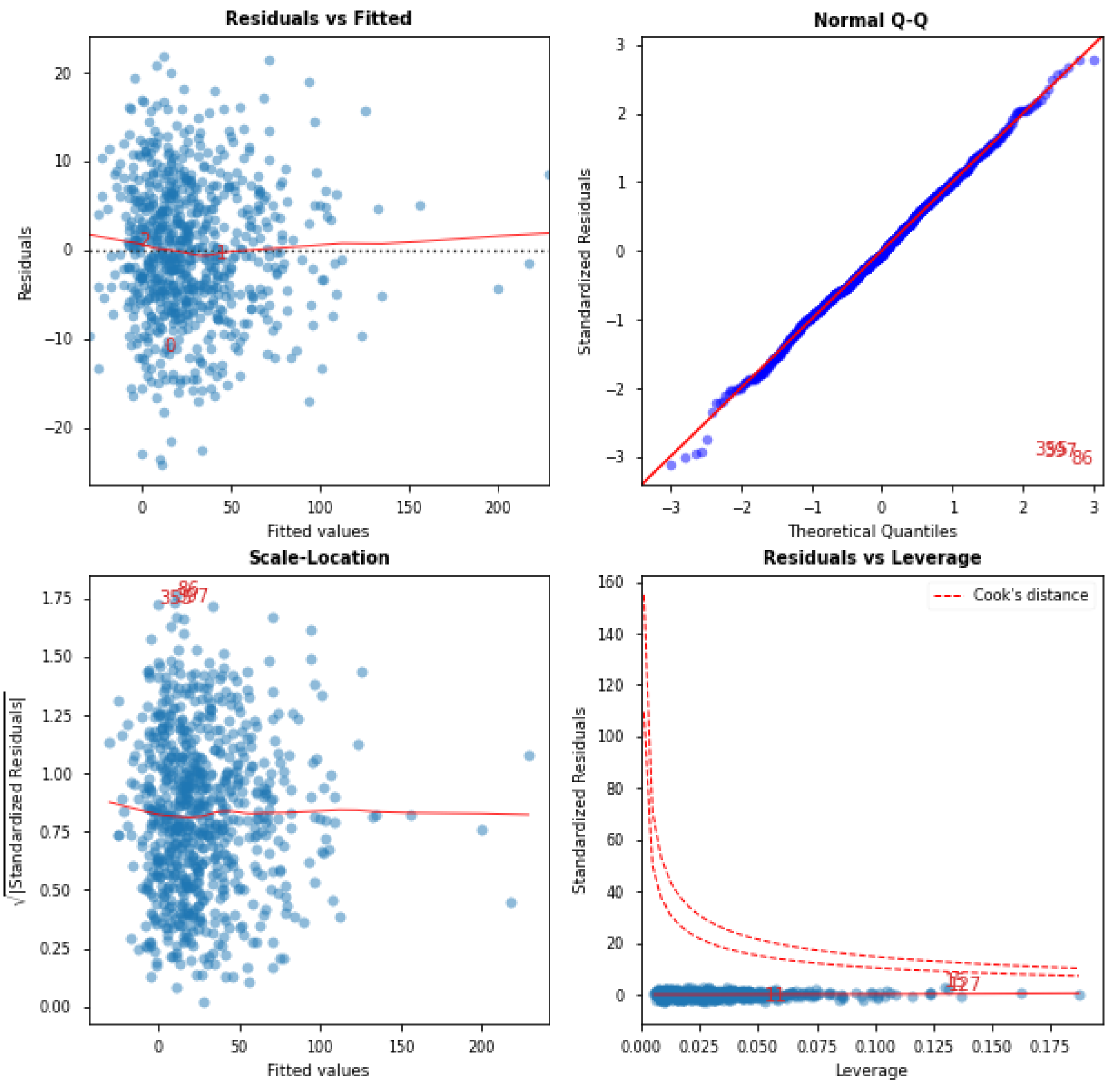
```
Model:                            OLS   Adj. R-squared:                  0.936
Method:                 Least Squares   F-statistic:                     494.1
Date:                Mon, 24 Oct 2022   Prob (F-statistic):               0.00
Time:                        18:52:13   Log-Likelihood:                -2588.8
No. Observations:                 747   AIC:                             5224.
Df Residuals:                     724   BIC:                             5330.
Df Model:                          22
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -1.4414      0.735     -1.960      0.050      -2.885       0.002
A              3.0724      0.249     12.325      0.000       2.583       3.562
C_2           -2.2765      0.722     -3.155      0.002      -3.693      -0.860
C_3            9.0147      2.124      4.245      0.000       4.846      13.184
G_2            1.1884      0.740      1.606      0.109      -0.265       2.641
A E           12.7765      4.786      2.670      0.008       3.381      22.172
A I          -12.5752      4.789     -2.626      0.009     -21.978      -3.173
A G_1          0.7192      0.255      2.825      0.005       0.219       1.219
A G_2          0.3836      0.254      1.508      0.132      -0.116       0.883
A G_3          1.9696      0.662      2.975      0.003       0.670       3.269
B H           -0.2651      0.053     -4.984      0.000      -0.370      -0.161
B C_3          0.7798      0.432      1.806      0.071      -0.068       1.627
D E          -11.5579      4.676     -2.472      0.014     -20.739      -2.377
D I           11.3598      4.677      2.429      0.015       2.177      20.543
D G_3         -1.7344      0.859     -2.019      0.044      -3.420      -0.048
E H           -0.8497      0.064    -13.181      0.000      -0.976      -0.723
E I            5.1486      0.928      5.548      0.000       3.327       6.971
E C_3         15.8171      6.359      2.487      0.013       3.332      28.302
F H           -0.0233      0.018     -1.278      0.202      -0.059       0.012
H G_3          0.5186      0.293      1.772      0.077      -0.056       1.093
I C_3        -15.6163      6.365     -2.453      0.014     -28.112      -3.120
C_1 G_2        4.0043      1.302      3.075      0.002       1.448       6.561
B^2            0.0042      0.025      0.168      0.867      -0.045       0.053
I^2           -2.9762      0.923     -3.226      0.001      -4.787      -1.165
==============================================================================
Omnibus:                        0.093   Durbin-Watson:                   1.930
Prob(Omnibus):                  0.955   Jarque-Bera (JB):                0.144
Skew:                          -0.023   Prob(JB):                        0.930
Kurtosis:                       2.950   Cond. No.                     4.80e+16
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
[2] The smallest eigenvalue is 2.92e-28. This might indicate that there are
strong multicollinearity problems or that the design matrix is singular.
```

"""

[28]:
```
cls = Linear_Reg_Diagnostic(results_5)
fig, ax = cls()
```

```
C:\Users\matth\anaconda3\lib\site-packages\statsmodels\graphics\gofplots.py:993:
UserWarning: marker is redundantly defined by the 'marker' keyword argument and
the fmt string "bo" (-> marker='o'). The keyword argument will take precedence.
  ax.plot(x, y, fmt, **plot_style)
```




```
   Features  VIF Factor
2       C_2        1.29
22      B^2        1.35
21  C_1 G_2        1.40
4       G_2        1.43
```

```
18       F H        1.51
19     H G_3        1.54
15       E H        2.09
10       B H        2.67
0      const        6.53
11     B C_3        8.44
3        C_3       10.11
14     D G_3       33.55
20     I C_3     1078.20
17     E C_3     1078.95
23       I^2     1517.79
16       E I     1521.57
12       D E    31349.21
13       D I    31354.09
6        A I    31624.02
5        A E    31631.82
8      A G_2         inf
7      A G_1         inf
1          A         inf
9      A G_3         inf
```

```
C:\Users\matth\anaconda3\lib\site-
packages\statsmodels\stats\outliers_influence.py:193: RuntimeWarning: divide by
zero encountered in double_scalars
  vif = 1. / (1. - r_squared_i)
```

[29]:

```python
X_train_p_r_4 = X_train_p_r_3.copy()
X_test_p_r_4 = X_test_p_r_3.copy()
for column in X_train_p_r_3.columns:
    if results_5.pvalues.loc[column] > 0.05:
        X_train_p_r_4.drop(column,axis=1, inplace=True)
        X_test_p_r_4.drop(column,axis=1, inplace=True)
```

[30]:

```python
X_train_p_r_4_c = sm.add_constant(X_train_p_r_4)
model_6 = sm.OLS(y_train, X_train_p_r_4_c)
results_6 = model_6.fit()
results_6.summary()
```

```
C:\Users\matth\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142:
FutureWarning: In a future version of pandas all arguments of concat except for
the argument 'objs' will be keyword-only
  x = pd.concat(x[::order], 1)
```

[30]:

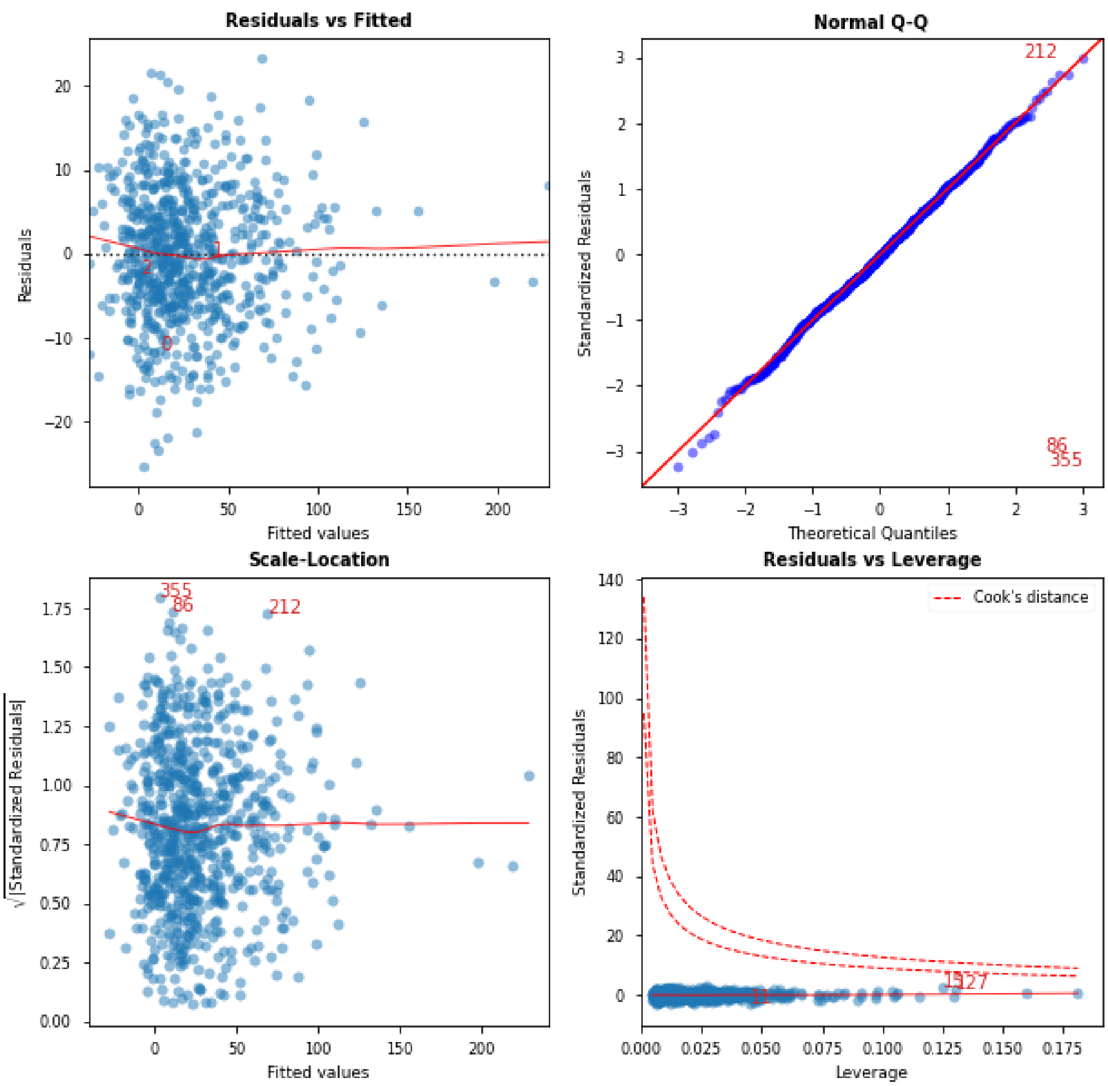
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:               response   R-squared:                       0.937
```

```
Model:                            OLS   Adj. R-squared:                  0.935
Method:                 Least Squares   F-statistic:                     632.6
Date:                Mon, 24 Oct 2022   Prob (F-statistic):               0.00
Time:                        18:52:13   Log-Likelihood:                -2595.0
No. Observations:                 747   AIC:                             5226.
Df Residuals:                     729   BIC:                             5309.
Df Model:                          17
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -1.1726      0.570     -2.057      0.040      -2.292      -0.054
A              3.5285      0.197     17.928      0.000       3.142       3.915
C_2           -2.1416      0.715     -2.993      0.003      -3.546      -0.737
C_3            5.9581      1.153      5.166      0.000       3.694       8.223
A E           12.0627      4.792      2.517      0.012       2.654      21.471
A I          -11.8842      4.797     -2.478      0.013     -21.301      -2.467
A G_1          0.2341      0.222      1.053      0.293      -0.202       0.671
A G_3          1.5530      0.891      1.743      0.082      -0.196       3.302
B H           -0.3260      0.047     -6.950      0.000      -0.418      -0.234
D E          -10.9865      4.679     -2.348      0.019     -20.172      -1.801
D I           10.8033      4.680      2.308      0.021       1.615      19.992
D G_3         -1.7922      0.861     -2.081      0.038      -3.483      -0.102
E H           -0.8613      0.064    -13.406      0.000      -0.987      -0.735
E I            5.1221      0.932      5.496      0.000       3.293       6.952
E C_3         14.6040      6.325      2.309      0.021       2.186      27.022
I C_3        -14.4318      6.333     -2.279      0.023     -26.865      -1.999
C_1 G_2        4.8182      1.182      4.076      0.000       2.497       7.139
I^2           -2.9534      0.926     -3.188      0.001      -4.772      -1.135
==============================================================================
Omnibus:                        0.001   Durbin-Watson:                   1.941
Prob(Omnibus):                  1.000   Jarque-Bera (JB):                0.019
Skew:                          -0.002   Prob(JB):                        0.991
Kurtosis:                       2.976   Cond. No.                         883.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
"""
```

[31]:
```python
cls = Linear_Reg_Diagnostic(results_6)
fig, ax = cls()
```

```
C:\Users\matth\anaconda3\lib\site-packages\statsmodels\graphics\gofplots.py:993:
UserWarning: marker is redundantly defined by the 'marker' keyword argument and
the fmt string "bo" (-> marker='o'). The keyword argument will take precedence.
  ax.plot(x, y, fmt, **plot_style)
```



	Features	VIF Factor
16	C_1 G_2	1.14
2	C_2	1.25
6	A G_1	2.02
8	B H	2.05
12	E H	2.06
3	C_3	2.95
0	const	3.89
1	A	4.55
11	D G_3	33.41
7	A G_3	34.49
15	I C_3	1057.05
14	E C_3	1057.13
17	I^2	1515.83

```
13        E I     1519.51
9         D E    31082.04
10        D I    31091.62
4         A E    31415.52
5         A I    31415.80
```

1.10 Reduced Model

[32]:
```
X_train_p_r_5 = X_train_p_r_4.copy()
X_test_p_r_5 = X_test_p_r_4.copy()
for column in X_train_p_r_4.columns:
    if results_6.pvalues.loc[column] > 0.05:
        X_train_p_r_5.drop(column,axis=1, inplace=True)
        X_test_p_r_5.drop(column,axis=1, inplace=True)
```

[33]:
```
X_train_p_r_5_c = sm.add_constant(X_train_p_r_5)
model_reduced = sm.OLS(y_train, X_train_p_r_5_c)
results_reduced = model_reduced.fit()
results_reduced.summary()
```

```
C:\Users\matth\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142:
FutureWarning: In a future version of pandas all arguments of concat except for
the argument 'objs' will be keyword-only
  x = pd.concat(x[::order], 1)
```

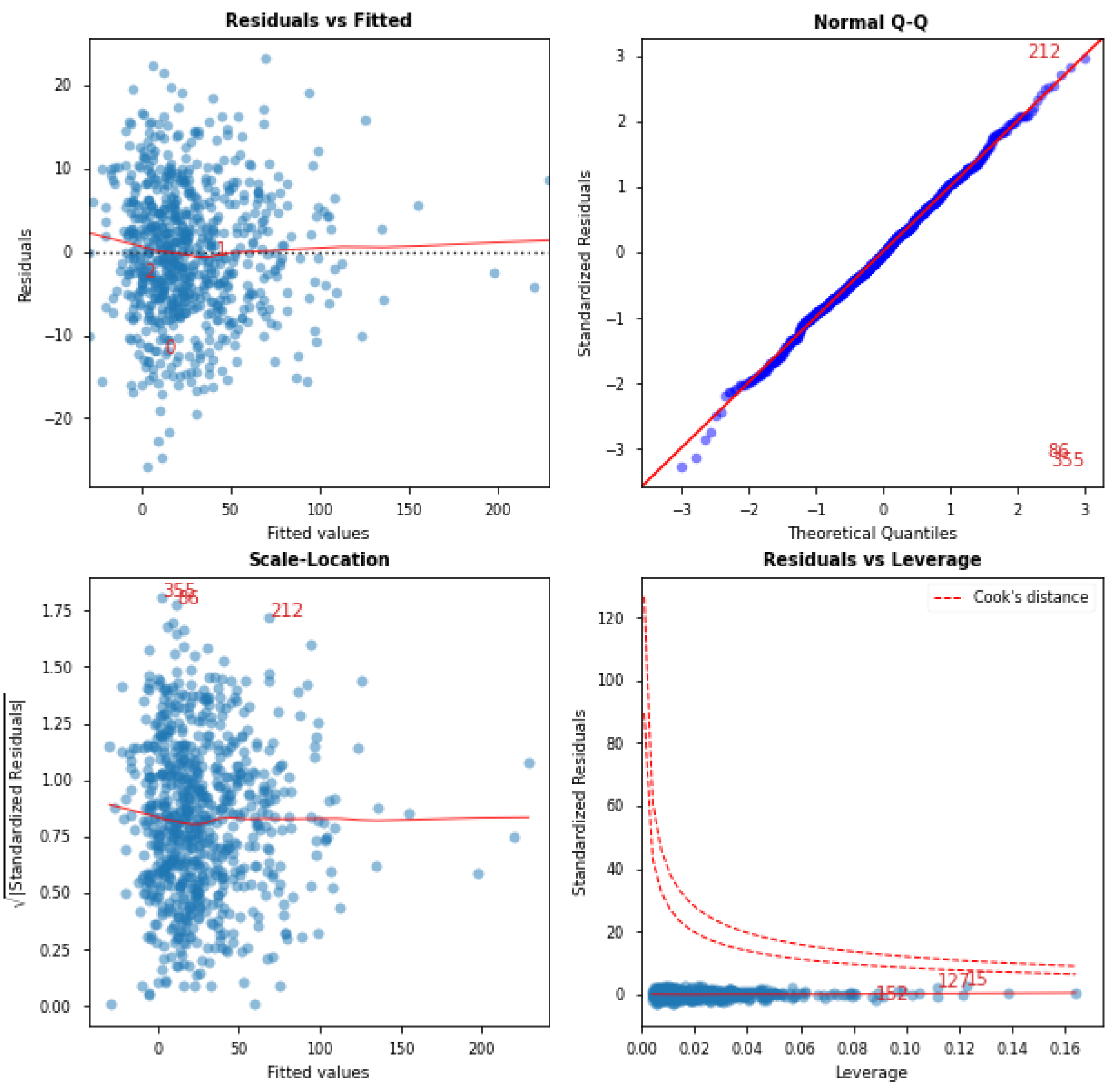
[33]:
```
<class 'statsmodels.iolib.summary.Summary'>
"""
                            OLS Regression Results
==============================================================================
Dep. Variable:               response   R-squared:                       0.936
Model:                            OLS   Adj. R-squared:                  0.935
Method:                 Least Squares   F-statistic:                     715.0
Date:                Mon, 24 Oct 2022   Prob (F-statistic):               0.00
Time:                        18:52:14   Log-Likelihood:                -2596.9
No. Observations:                 747   AIC:                             5226.
Df Residuals:                     731   BIC:                             5300.
Df Model:                          15
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -1.0936      0.568     -1.925      0.055      -2.209       0.022
A              3.6477      0.169     21.637      0.000       3.317       3.979
C_2           -2.2053      0.714     -3.089      0.002      -3.607      -0.804
C_3            5.7405      1.146      5.007      0.000       3.490       7.991
A E           11.8973      4.778      2.490      0.013       2.516      21.278
A I          -11.8318      4.786     -2.472      0.014     -21.228      -2.436
B H           -0.3279      0.047     -6.987      0.000      -0.420      -0.236
```

```
D E          -10.7803      4.668     -2.309      0.021    -19.946     -1.615
D I           10.7010      4.674      2.290      0.022      1.526     19.876
D G_3         -0.4240      0.180     -2.352      0.019     -0.778     -0.070
E H           -0.8636      0.064    -13.471      0.000     -0.989     -0.738
E I            5.1271      0.933      5.495      0.000      3.295      6.959
E C_3         14.5780      6.324      2.305      0.021      2.163     26.993
I C_3        -14.4629      6.330     -2.285      0.023    -26.890     -2.035
C_1 G_2        4.6393      1.175      3.949      0.000      2.333      6.946
I^2           -2.9652      0.927     -3.197      0.001     -4.786     -1.144
==============================================================================
Omnibus:                        0.004   Durbin-Watson:                   1.930
Prob(Omnibus):                  0.998   Jarque-Bera (JB):                0.006
Skew:                          -0.002   Prob(JB):                        0.997
Kurtosis:                       2.987   Cond. No.                         878.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
"""
```

[34]:
```
cls = Linear_Reg_Diagnostic(results_reduced)
fig, ax = cls()
```

```
C:\Users\matth\anaconda3\lib\site-packages\statsmodels\graphics\gofplots.py:993:
UserWarning: marker is redundantly defined by the 'marker' keyword argument and
the fmt string "bo" (-> marker='o'). The keyword argument will take precedence.
  ax.plot(x, y, fmt, **plot_style)
```



	Features	VIF Factor
14	C_1 G_2	1.12
2	C_2	1.24
9	D G_3	1.46
10	E H	2.05
6	B H	2.05
3	C_3	2.91
1	A	3.33
0	const	3.85
13	I C_3	1053.59
12	E C_3	1054.16
15	I^2	1515.54
11	E I	1519.34
7	D E	30867.66

```
8        D I     30926.69
4        A E     31154.42
5        A I     31203.94
```

The Resudial Plot no longer contains a quadratic trend and is now horizontal compared to part b.

1.11 Validation Set Testing

[35]:
```python
X_test_p_c = sm.add_constant(X_test_p)
X_test_p_r_5_c = sm.add_constant(X_test_p_r_5)
```

```
C:\Users\matth\anaconda3\lib\site-packages\statsmodels\tsa\tsatools.py:142:
FutureWarning: In a future version of pandas all arguments of concat except for
the argument 'objs' will be keyword-only
  x = pd.concat(x[::order], 1)
```

[36]:
```python
y_pred_f = results_full.predict(X_test_p_c)
y_pred_r = results_reduced.predict(X_test_p_r_5_c)
```

[37]:
```python
mean_squared_error(y_test, y_pred_f)
```

[37]: 79.79269272903343

[38]:
```python
mean_squared_error(y_test, y_pred_r)
```

[38]: 80.93076620252883

MSE for the reduced and full models are very close. This does not indicate overfitting of the full model, but it is still preferable to use the model with 15 features instead of the one with 103 features if the performance on the validation set is similar. This could prevent future overfitting.

1.12 Interval Testing

[39]:
```python
prediction_reduced = results_reduced.get_prediction(X_test_p_r_5_c)
```

[40]:
```python
Interval = prediction_reduced.conf_int()
```

[41]:
```python
obs = len(Interval)
```

[42]:
```python
i = 0
count = 0
for value in Interval:
    if y_test.iloc[i].values <= value[1] and y_test.iloc[i].values >= value[0]:
        count += 1
    i += 1
```

[43]:
```python
count
```

[43]: 48

[44]:
```
count/obs
```

[44]: 0.1927710843373494