To find the expected value $E[Y_i]$, we need to sum over all possible values of Y_i weighted by their probabilities.

Given the joint probability distribution $P(y)$, we have: $E[Y_i] = \sum_{y_i} y_i \cdot P(Y_i = y_i)$

Substitute the joint probability distribution $P(y)$ into the formula: $E[Y_i] = \sum_{y_i} y_i \cdot \frac{n!}{y_1!y_2!...y_k!} p_1^{y_1} p_2^{y_2} ... p_k^{y_k}$

Since Y_i equals the number of trials for which the outcome falls into cell i, we have $Y_i = y_i$.

Therefore, $E[Y_i] = \sum_{y_i} y_i \cdot \frac{n!}{y_1!y_2!...y_k!} p_1^{y_1} p_2^{y_2} ... p_k^{y_k}$

Since $Y_i = y_i$, we can simplify the expression to: $E[Y_i] = \sum_{y_i} y_i \cdot \frac{n!}{y_i!(n-y_i)!} p_i^{y_i} (1 - p_i)^{n-y_i}$

Now, we can simplify the expression further by recognizing that this is the probability mass function of a binomial distribution with parameters n and p_i: $E[Y_i] = n \cdot p_i$

Therefore, the expected value $E[Y_i] = n \cdot p_i$.