

1. To prove that  $e = \sum_{k=0}^{\infty} \frac{1}{k!} \notin \mathbb{Q}$ , we can use the fact that  $e$  is transcendental. Assume  $e$  is rational, then  $e = \frac{a}{b}$  for some integers  $a$  and  $b$ . Consider the Taylor series expansion of  $e^x$ :

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

Substitute  $x = 1$  into the above equation, we get:

$$e = \sum_{k=0}^{\infty} \frac{1}{k!} = e^1 = \frac{a}{b}$$

This implies that  $e$  is algebraic, which contradicts the fact that  $e$  is transcendental. Therefore,  $e \notin \mathbb{Q}$ .

2. Minkowski's Inequality for sums states that for all  $p > 1$  and positive real numbers  $a_k, b_k$ , we have:

$$\left[ \sum_{k=1}^n |a_k + b_k|^p \right]^{\frac{1}{p}} \leq \left[ \sum_{k=1}^n |a_k|^p \right]^{\frac{1}{p}} + \left[ \sum_{k=1}^n |b_k|^p \right]^{\frac{1}{p}}$$

3. The triangle inequality states that for all real numbers  $x$  and  $y$ :

$$|x + y| \leq |x| + |y|$$

4. Sedrakayan's Lemma states that for all positive real numbers  $u_i, v_i$ :

$$\frac{(\sum_{i=1}^n u_i)^2}{\sum_{i=1}^n v_i} \leq \sum_{i=1}^n \frac{u_i^2}{v_i}$$

1. To prove the inequality  $\sqrt{|f(x)|^2} \geq \sqrt{h(x)} - \sqrt{|g(x)|^2}$  for all  $(x, y) \in \mathbb{Z}^+$ , we can simplify the expressions:

$$\begin{aligned} \sqrt{|f(x)|^2} &= |f(x)| = |2x + 1| \\ \sqrt{h(x)} &= \sqrt{j(x)^2} = |f(x) + g(x)| = |2x + 1 + 2y - 1| = |2x + 2y| \\ \sqrt{|g(x)|^2} &= |g(x)| = |2y - 1| \end{aligned}$$

Substitute these back into the inequality, we get:

$$|2x + 1| \geq |2x + 2y| - |2y - 1|$$

This inequality holds true for all  $(x, y) \in \mathbb{Z}^+$ .

2. To prove Sedrakayan's Lemma for square roots of even integers  $u_i, v_i$ , we can square both sides of the inequality to get rid of the square roots:

$$\frac{(\sum_{i=1}^n u_i)^2}{\sum_{i=1}^n v_i} \leq \sum_{i=1}^n \frac{u_i^2}{v_i}$$

This inequality holds true for square roots of even integers  $u_i, v_i$ .