Task 1:
Question 1: To find the expected value $E[Y_i]$, we need to sum over all possible values of Y_i weighted by their probabilities.

Given the joint probability distribution $P(y)$, we have: $E[Y_i] = \sum_{y_i} y_i \cdot P(Y_i = y_i)$

Substitute the joint probability distribution $P(y)$ into the formula: $E[Y_i] = \sum_{y_i} y_i \cdot \frac{n!}{y_1!y_2!...y_k!} p_1^{y_1} p_2^{y_2} ... p_k^{y_k}$

Since Y_i equals the number of trials for which the outcome falls into cell i, we have $Y_i = y_i$.

Therefore, $E[Y_i] = \sum_{y_i} y_i \cdot \frac{n!}{y_1!y_2!...y_k!} p_1^{y_1} p_2^{y_2} ... p_k^{y_k}$

Since $Y_i = y_i$, we can simplify the expression to: $E[Y_i] = \sum_{y_i} y_i \cdot \frac{n!}{y_i!(n-y_i)!} p_i^{y_i} (1-p_i)^{n-y_i}$

Now, we can simplify the expression further by recognizing that this is the probability mass function of a binomial distribution with parameters n and p_i: $E[Y_i] = n \cdot p_i$

Therefore, the expected value $E[Y_i] = n \cdot p_i$.

Question 2: To find the probability that the sample contains 100 persons between 18-24, 200 between 25-34, and 200 between 45-64, we need to multiply the proportions for each age group together.

The probability of selecting 100 persons between 18-24 is 0.18^{100}. The probability of selecting 200 persons between 25-34 is 0.23^{200}. The probability of selecting 200 persons between 45-64 is 0.27^{200}.

Therefore, the probability of this specific sample is $0.18^{100} \times 0.23^{200} \times 0.27^{200}$.

To find the expected value to obtain a person in the 65 and above age group, we need to multiply the proportion of that age group by the total number of samples: Expected value $= 0.16 \times 500 = 80$.

Task 2:
Question 1: The probability distribution of the experiment can be derived using the binomial distribution formula:

$$P(Y = y) = \binom{n}{y} p^y q^{n-y}$$

where y is the number of successes observed during the n trials.

Question 2: The expectation of the probability distribution is calculated as:

$$\begin{aligned}
E[Y] &= \sum_{y=0}^{n} y \cdot P(Y = y) \\
&= \sum_{y=0}^{n} y \cdot \binom{n}{y} p^y q^{n-y} \\
&= \sum_{y=0}^{n} y \cdot \frac{n!}{y!(n-y)!} p^y q^{n-y} \\
&= \sum_{y=0}^{n} n \cdot \frac{(n-1)!}{(y-1)!(n-y)!} p^y q^{n-y} \\
&= np \sum_{y=0}^{n} \binom{n-1}{y-1} p^{y-1} q^{n-y} \\
&= np \sum_{k=0}^{n-1} \binom{n-1}{k} p^k q^{n-1-k} \\
&= np(p+q)^{n-1} \\
&= np
\end{aligned}$$

Question 3: Given that the medication is worthless, the probability of success (p) is actually 0.3. The probability of at least nine out of ten recovering is:

$$P(Y \geq 9) = P(Y = 9) + P(Y = 10) = \binom{10}{9}(0.3)^9(0.7)^1 + \binom{10}{10}(0.3)^{10}(0.7)^0$$

Question 4: The probability of observing at least one defective fuse in a sample of 5 can be calculated as the complement of the probability of observing zero defective fuses:

$$\begin{aligned}
P(\text{at least one defective}) &= 1 - P(\text{no defective}) \\
&= 1 - \binom{5}{0}(0.05)^0(0.95)^5
\end{aligned}$$

Task 3:

Question 1: The expected value of the distribution is given by:

$$\begin{aligned} E(y) &= \sum_{y=0}^{\infty} y \cdot p(y) \\ &= \sum_{y=0}^{\infty} y \cdot \frac{\lambda^y}{y!} e^{-\lambda} \\ &= \sum_{y=1}^{\infty} \frac{\lambda^y}{(y-1)!} e^{-\lambda} \\ &= e^{-\lambda} \sum_{y=1}^{\infty} \frac{\lambda^y}{(y-1)!} \\ &= e^{-\lambda} \lambda \sum_{y=1}^{\infty} \frac{\lambda^{y-1}}{(y-1)!} \\ &= e^{-\lambda} \lambda \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} \\ &= e^{-\lambda} \lambda e^{\lambda} \\ &= \lambda \end{aligned}$$

Therefore, the general formula for the expected value of this distribution is $E(y) = \lambda$.

Question 2: The probability that a region does not contain any seedlings is $p(0) = \frac{\lambda^0}{0!} e^{-\lambda} = e^{-\lambda}$. The probability that none of the ten regions contain seedlings is $(e^{-\lambda})^{10} = e^{-10\lambda}$.

Given that the mean density of seedlings is five per square yard, $\lambda = 5$. Therefore, the probability that none of the regions will contain seedlings is $e^{-10 \times 5} = e^{-50}$.