# Supersonic Wind Tunnel Design

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# Abstract:

The goal of this project is to design a wind tunnel to generate a flow at a certain Mach number in a certain test section with specified specifications. Other specifications of the design are determined off of physical constraints and properties of the wind tunnel. While determining these parameters, the various properties of the fluid flow will also be analyzed/determined, and these things will also be accounted for in the design of the shape of the inlet and outlet.

By determining a few design aspects based on real-world buildings, such as maximum outlet size and inlet size, and assuming isentropic flow within the wind tunnel itself with no losses due to friction etc, the rest of the specs can be found. By finding the critical area of the wind tunnel based on the specs of the test section and assuming the local properties at the exit are equal to ambient properties, we can use the equations of isentropic flow to find the stagnation properties at the exit. This is a snowball effect, as it allows the determination of the Mach number and various other parameters along the rest of the wind tunnel.

In the end, the wind tunnel designed is approximately contained within a 2 x 0.5 x 10 $m^3$ enclosure, making it possible to fit within a typical portable used by many educational institutions. It must be ran using a motor with more than 110 horsepower, and has a max speed of approximately Mach 2 within the test section if all specs are followed.

# Introduction

Wind tunnels are devices designed to simulate airflow at high velocities in order to provide another layer of testing for products between development and deployment. It provides real world data to double check the results from hand calculations and CFD simulations, as at the end of the day, no calculation is perfect and the real world will inevitably reveal things that could not be accounted for. These wind tunnels are used for performance testing of vehicles such as race cars, aircrafts (commercial planes, jets, etc), and spacecrafts (rockets). However, these wind tunnels are normally used to test scaled down versions of these vehicles because the wind tunnels themselves cannot be that large. This is due to the fact that it takes an enormous amount of power to run these wind tunnels, especially when trying to test vehicles in very high Mach numbers. This power draw issue is why the design of a wind tunnel is critical, as it can save a lot of power while still generating extremely high flow rates if designed correctly. Most wind tunnels are designed as converging-diverging nozzles due to the flow characteristics that help accelerate the flow in the choke of such nozzles with minimal power use.

However, the size of the inlets and outlets of such designs are usually limited by the size of the building that they can be housed in. This is the basis on what is limiting certain design aspects of this wind tunnel. The fundamental criteria for this wind tunnel design is that it must reach Mach 2 within the testing section with a cross sectional area of .25 $m^2$. Such a small testing section allows for the power rating of the fan to be much lower, and for other properties of the tunnel to be scaled down even though the Mach number is so high. The arbitrary properties chosen were the outlet size and the Mach number right after the fan. Using these properties, and using the Isentropic Properties Table (Table A.1), the stagnation properties of the flow can be determined. With that, the local properties of the flow can be determined throughout the wind tunnel.

# Design Method

Firstly, it is assumed flow within the wind tunnel after the fan is isentropic flow. Using that assumption, knowing that the area within the test section is 1 $m^2$ ($A_3$) and flow is at 0.6 Mach number ($M_3$), assuming exit area to be 4 $m^2$, and the Mach number after the fan is 0.15, we can begin solving for the various properties of the wind tunnel and its flows at certain points. Assuming that we want it to fit in a portable classroom, an arbitrary size of 8 meters is selected (3m for the inlet, 2m for the test section, and 3m for the outlet)

One property that is carried throughout the flow is the critical area (A*), which can be obtained using the Table A.1. However, the critical area is different between the sections prior to the shock and after the shock, and must be calculated separately.

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

Figure 1.1: Critical Area Calculation

In order to calculate critical areas before and after the shock, normal shock properties are used, and it is assumed that the shock happens essentially right after the test section. Given that we know M3 (Mach number in the test section), we also know the Mach number after the shock, which is a function of Mach number before the shock.

$$M_2^2 = \frac{1+\frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}}$$

Figure 1.2: Mach Number across Normal Shock

Given that we now know the Mach number before and after the shock, as well as the areas where those Mach numbers are, we can calculate the critical areas of both sections, and from there solve for stagnation properties.

It is assumed that at the exit, the local properties of the flow (pressure, temperature, and density) are equal to ambient properties. Assuming the exit area is 4 $m^2$ ($A_4$), we can then determine the Mach number ($M_4$) by relating $A_4/A^*$ to Mach number on Table A.1 again and interpolating between points.

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2}M^2$$

$$\frac{p_0}{p} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{1}{\gamma-1}}$$

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1 + \frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

Figure 1.3: Isentropic Relations

Knowing these properties, we can then also find the stagnation properties of the flow ($P_0$, $T_0$, and $rho_0$) thanks to the assumption that the local properties ($P_4$, $T_4$, and $rho_4$) at the exit is equal to ambient/atmospheric. Now that the stagnation properties are known, the isentropic flow equations can be used at each point to find the local properties because Mach numbers are known.

Also, because the local Mach numbers have been calculated/assumed as well as the local temperatures, we can also determine the speed of sound (a) at each point as well as the velocity of the flow (v).

After all the local flow properties were determined after the shock, we can then use the normal shock properties to find the stagnation and local flow properties before the shock using the following equations.

$$M_2^2 = \frac{1 + \frac{\gamma-1}{2} M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}}$$

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1) M_1^2}{2 + (\gamma-1) M_1^2}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)\left(\frac{\rho_1}{\rho_2}\right) = \frac{h_2}{h_1} = \left[1 + \frac{2\gamma}{\gamma+1}\left(M_1^2 - 1\right)\right]\left[\frac{2 + (\gamma-1) M_1^2}{(\gamma+1) M_1^2}\right]$$

$$\frac{p_{0_2}}{p_{0_1}} = \left[\frac{(\gamma+1) M_1^2}{2 + (\gamma-1) M_1^2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{\gamma+1}{2\gamma M_1^2 - (\gamma-1)}\right]^{\frac{1}{\gamma-1}}$$

$$\frac{p_{0_2}}{p_1} = \left[\frac{(\gamma+1) M_1^2}{2 + (\gamma-1) M_1^2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{\gamma+1}{2\gamma M_1^2 - (\gamma-1)}\right]^{\frac{1}{\gamma-1}}\left[1 + \frac{\gamma-1}{2} M_1^2\right]^{\frac{\gamma}{\gamma-1}}$$

Figure 1.4: Normal Shock Relations

Once we know all the properties within the tunnel, we could then begin to calculate the properties of the fan, the first being the pressure ratio of the fan ($P_{02}/P_{01}$).

$$P_{02}/P_{01} = \pi$$

Figure 1.4: Pressure Ratio Calculation

To calculate the power needed by the fan, we use the following equation to determine the power rating needed to drive the wind tunnel to its maximum speed (converted to horsepower). This is the power rating for an ideal fan with no losses.

$$P_{fan} = \frac{1}{2} V_4^2 \, \dot{m}_4$$

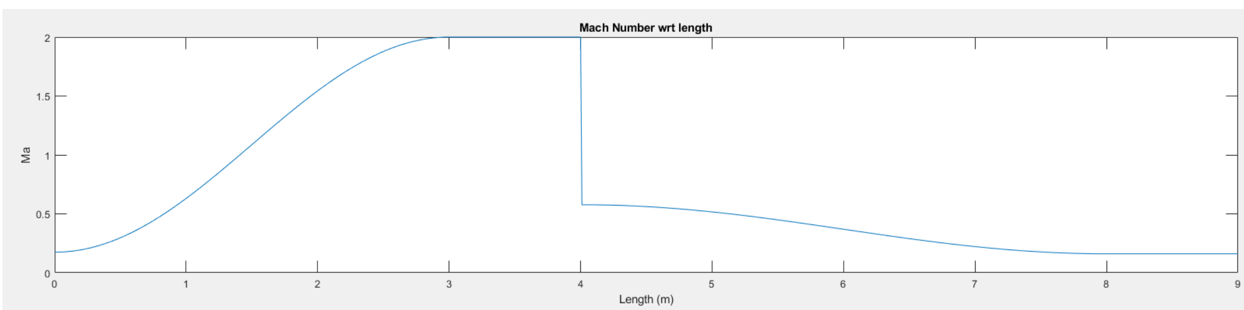
Figure 1.5: Power Calculations for Fan

The Reynolds number for the flow within the test section can be calculated using the following equation and density that we found earlier.

$$Re = \frac{\rho V D}{\mu}$$

Figure 1.6: Calculations for Reynolds Number within the Test Section

Once all of that has been done, we then move on to the physical design of the wind tunnel. To do this, the method selected was to have Mach numbers move linearly through the inlet and outlet portions of the wind tunnel, and using that, generate an area using the area relation of isentropic flow in Matlab.




$$\frac{A}{A^*} = \left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \frac{\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M}$$

Figures 1.7 and 1.8: Mach Number Function wrt Length, and Area Mach Relationship Equation Used to Calculate Shape

This was done in order to avoid the issue of there having 2 Mach numbers (supersonic and subsonic) for every area ratio, and to generate smooth walls to prevent boundary layer separation at the outlet due to drastic changes in geometry. The sharp drop in Mach number is directly correlated with the normal shock within the tunnel.

To validate the results, we then plug in all the local properties calculated into mass and energy conservation equations, and if all assumptions/calculations made are correct, we should conserve energy and mass throughout the entire flow.

$$h_0 = h + \frac{1}{2}V^2 \quad , h = C_p T$$

$$\dot{m} = \Gamma \frac{p_0}{\sqrt{RT_0}} q(M) A$$

Figures 1.9 and 1.10: Internal Energy and Mass Flow Rate Equations Used to Validate Flow within the Tunnel

The thrust of the system will be calculated using mass flow rate and velocity.

# Results and Discussions

Results =

5×10 table

| Point | StagPres | StagTemp | StagDens | MachNumber | Area | Velocity | SoundSpeed | Pressure | Temperature |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 1.0133e+05 | 296 | 1.225 | 0.2508 | 0.5 | 85.954 | 342.72 | 96987 | 292.32 |
| 2 | 1.4313e+05 | 297.53 | 1.7214 | 0.1747 | 0.5 | 60.22 | 344.71 | 1.4011e+05 | 295.73 |
| 3 | 1.4313e+05 | 297.53 | 1.7214 | 2 | 0.25 | 515.43 | 257.71 | 18293 | 165.3 |
| 0 | 1.0318e+05 | 297.54 | 1.241 | 0.5774 | 0.25 | 193.3 | 334.78 | 82313 | 278.94 |
| 4 | 1.0318e+05 | 297.54 | 1.241 | 0.1611 | 0.75 | 55.558 | 344.87 | 1.0133e+05 | 296 |

Figure 2.1: Properties of Flow at Points 1, 2, 3, s2, and 4

FanProperties =

1×5 table

| TotalPressureRatio | Power | ForceofThrust | mdotflow | Re |
|---|---|---|---|---|
| 1.4126 | 111.78 | 2760.5 | 49.686 | 5.7047e+06 |

Figure 2.2: Fan Properties

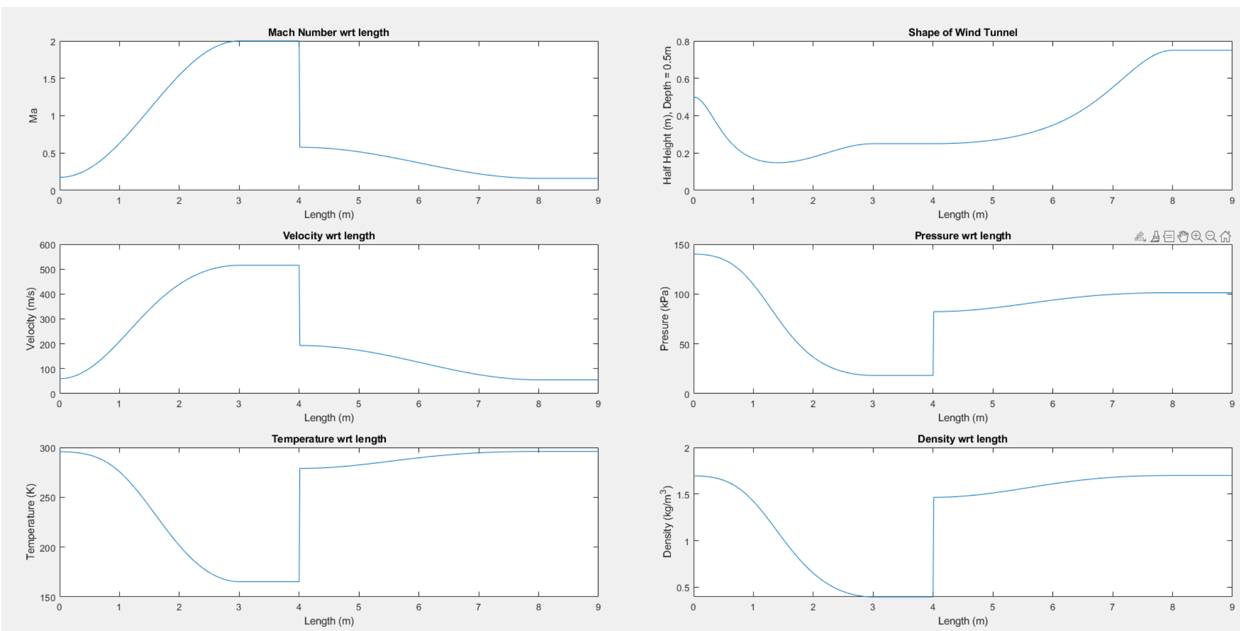



Figure 2.3: Local Properties of Flow Within the Wind Tunnel

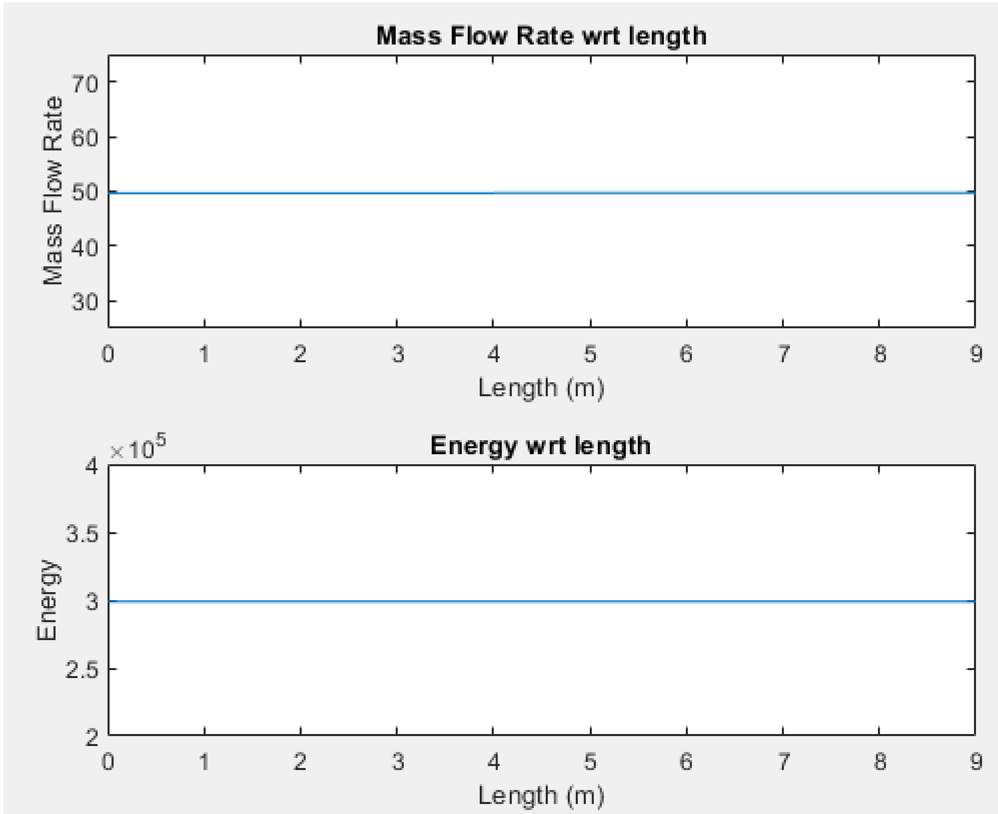



Figure 2.4: Mass Flow Rate and Internal Energy Throughout the Wind Tunnel

After following the procedure and equations laid out in "Design Method", we end up with Figures 2.1-2.4. In the design process, the goal is to design a wind tunnel with a maximum Mach number of 2 in a .5x.5 meter test section that is 1 meter long. Applying the arbitrary physical constraint of wanting to make the wind tunnel fit within a typical classroom portable, the constraints of a .75 $m^2$ (1.5x.5 meter) diffuser, a 1 x 0.5 meter inlet, and a total length of 8 meters was chosen. These properties allow the wind tunnel to fit within a typical portable (14 x 4 meters, or about 600-700 square feet) while still having some extra space to allow air entering not be turbulent due to disturbances and the exit be largely free of backpressure.

Once all the equations are solved, we end up with the values in Figure 2.1. In this table, we can see that the flow within the tunnel is isentropic due to the fact that the stagnation properties are constant throughout Points 2 and 3, and then constant again at s2 and 4. It can also be seen that the fan does work from Points 1 and 2 from the slight increases in stagnation pressure and temperature. It is interesting to note that technically/theoretically, the flow at Point 1 (right before the fan) has a higher Mach number and velocity than the flow after the fan.

The amount of work that the fan does and the power needed is also calculated. This allows us to determine that we need a fixture that can withstand approximately 2800 Newtons of force due to the thrust generated by a 110 hp fan (Figure 2.2). This power rating for a fan makes sense for a wind tunnel with such a large test section, and such a motor could probably be repurposed from a car engine. I find it interesting that the power rating of the fan and the thrust generated by a supersonic wind tunnel is lower than the previously designed subsonic wind tunnel, but things like test section cross sectional area being smaller, thus lower mass flow rate most likely plays a large part in that.

After the fan, we can refer to Figure 2.3 to analyze the local properties of the flow. By plugging in the linear piecewise function for Mach number into the various isentropic flow equations (knowing the stagnation properties), we can generate plots of all the local properties. All of the properties follow the logical progression of the flow within the wind tunnel with a supersonic choke. With regards to the shape of the wind tunnel, the walls are smooth and seem to be gradual enough in transitions to prevent boundary layer separation. We see that as velocity increases to match the Mach number in the wind tunnel, properties such as pressure and temperature decrease as the energy is presumably being converted to kinetic energy within the test section, and they increase again as the flow exits. However, some of the energy at the exit is conserved as kinematic energy (the energy originally inputted by the fan) and that's reflected by the non-zero Mach number despite local pressure and temperature returning to ambient.

# Assessment

After completing the design for this wind tunnel, the values of the flow can be easily altered by changing the area of the outlet. The outlet affects the values of both the stagnation properties within the wind tunnel and the fan properties. The larger the outlet area, the lower the stagnation properties end up being, and the lower the power needed for the fan.

Currently, the design is a relatively effective compact wind tunnel for the size of test section needed, and the motor requirement for the fan can be relatively easily acquired if one found a sedan’s engine and repurposed it. The shape is difficult to manufacture, given the specific constraints of the choke being the right size and in the right place to properly accelerate the flow to supersonic levels.

# Conclusion

In the end, a 2 x 0.5 x 10meter Mach 2 wind tunnel was designed with a power rating of 110 hp. This design allows the wind tunnel to be placed in most classrooms, and should be relatively easy to build and manufacture as long as a powerful enough fan can be acquired.. Analyzing this wind tunnel design, it could be more efficient if depth wasn't limited to 1 meter, but otherwise, is a very effective tool. The flow follows the laws of physics and fluid mechanics, although it hasn't been fully tested as to whether or not the physical flow follows the requirements for laminar flow within the test section or if the inlet/outlets will induce turbulence, but that analysis can come from the CFD analysis arriving shortly.

# Appendix

## Hand Calculations

1) $A_1 = A_2 = 2m \times 2m = 4m^2$
$A_4 = 3 \times 3 = 9m^2$
2) 6m, 6m, 13m total length

| | Section 1 | Section 2 | Throat | Section 3 | Section 5' | Section 4 |
|---|---|---|---|---|---|---|
| | $M_1$ : | $M_2$ : .025 | $M^*$ : 1 | $M_3$ : 2 | $M_5$ : .5774 | $M_4$ : .0132250 |
| $m^2$ | $A_1$ : .75 | $A_2$ : .75 | $A^*$ : .1737 | $A_3$ : .25 | $A_5$ : .25 | $A_4$ : 1.5 |
| Pa | $P_o$ : 101325 | $P_o$ : | $P_o$ : | $P_o$ : 140553 | $P_o$ : 101325 | $P_{oe}$ : 101325 |
| K | $T_o$ : 296 | $T_o$ : | $T_o$ : | $T_o$ : 296.033 | $T_o$ : 296 | $T_{oe}$ : 296 |
| $kg/m^3$ | $\rho_o$ : | $\rho_o$ : | $\rho_o$ : | $\rho_o$ : | $\rho_o$ : | $\rho_o$ : |
| | $P_1$ : | $P_2$ : | $P^*$ : | $P_3$ : 17962.7 | $P_5$ : 80816.8 | $P_e$ : 101325 |
| | $T_1$ : | $T_2$ : | $T^*$ : | $T_3$ : 164.476 | $T_5$ : 277.47 | $T_e$ : 296 |
| | $\rho_1$ : | $\rho_2$ : | $\rho^*$ : | $\rho_3$ : | $\rho_5$ : | $\rho_e$ : |
| m/s | $a_1$ : | $a_2$ : | $a^*$ : | $a_3$ : | $a_5$ : | $a_e$ : |
| | $\dot{m}$ : | $\dot{m}$ : | $\dot{m}$ : | $\dot{m}$ : | $\dot{m}$ : | $\dot{m}$ : |

Section 3:

$A_3/A^* = 1.439$

$P_{oe}/P_o = .7209$

$P_o = 140553$

$T_o/T_1 = 1.687$

$T_3 = 164.476$

$P_3/P_o = .1278$

$P_3 = 17962.7$

$T_3/T_o = .5556$

$T_o = 296.033$

Section 5':

$A_5/A_5^* = 1.216$

$A^*_5 = .2056$

$P_5/P_{oe} = .7976$

$T_5/T_{oe} = .9374$

Section 4:

$P_e/P_{oe} = .9999$

$P_{oe} = 101325$

$T_e/T_{oe} = 1$

$T_{oe} = 296$

$A = .5(2_y)$

# Code Used to Generate Plots

```
clear
clc
sympref('FloatingPointOutput',true);

%Constants
gamma = 1.4;
R = 287;
Cp = (gamma*R)/(gamma-1);

%Point 1
syms v1 P1 T1 P01 T01 M1 A1 a1 rho1
%v1  = 0;
P1  = 101325;
T1  = 296;
P01 = 101325;
T01 = 296;
%M1  = 0;
A1  = .5;
a1  = sqrt(gamma*R*T1);
rho1= 1.225;
rho01 = 1.225;

%Point 1
syms v2 P2 T2 P02 T02 M2 A2 a2 rho2
%v2  = 0;
%P2  = 101000;
%T2  = 296;
%P02 = 101000;
%T02 = 296;
% M2  = 0.025;
A2  = A1;
%a2  = sqrt(gamma*R*T1);
%rho2= 1.225;

%Point Crit
syms vc Pc Tc P0c T0c Mc Ac ac rhoc
%vc  = 0;
%Pc  = 101000;
%Tc  = 296;
%P0c = 101000;
%T0c = 296;
Mc  = 1;
%Ac  = ;
%ac  = sqrt(gamma*R*T1);
%rhoc= 1.225;

%Point 3
syms v3 P3 T3 P03 T03 M3 A3 a3 rho3
%v3  = 0;
% P3  = 17962.7;
% T3  = 164.476;
```

```
% P03 = 140553;
% T03 = 296.033;
M3  = 2;
A3  = .25;
%a3  = sqrt(gamma*R*T1);
%rho3= 1.225;
Ac = A3/1.687;

%Point Post Shock
syms vs2 Ps2 Ts2 P0s2 T0s2 Ms2 As2 as2 rhos2 s2
%vs2  = 0;
%Ps2  = 101000;
%Ts2  = 296;
%P0s2 = 101000;
%T0s2 = 296;
Ms2  = .5774;
As2  = .25;
%as2  = sqrt(gamma*R*T1);
%rhos2= 1.225;
Asc = .2056;

%Point 4
syms v4 P4 T4 P04 T04 M4 A4 a4 rho4
%v4  = 0;
P4  = 101325;
T4  = 296;
% P0e = 101325;
% T0e = 296;
% M4  = .013225;
A4  = .75;
%a4  = sqrt(gamma*R*T1);
rho4= 1.225;

%Section 4
AMR4 = A4/Asc == ((5 + M4^2)^3)/(216*M4);
M4 = min(double(vpasolve(AMR4,M4,[0 Inf])));
PMR4 = (1+(((gamma-1)/2)*(M4^2)))^(gamma/(gamma-1));
P04 = PMR4*P4;
TMR4 = 1+(((gamma-1)/2)*(M4^2));
T04 = TMR4*T4;
RMR4 = (1+(((gamma-1)/2)*(M4^2)))^(1/(gamma-1));
rho04 = rho4*RMR4;
a4 = sqrt(gamma*R*T4);
v4 = M4*a4;

%Section Post Shock
P0s2 = P04;
T0s2 = T04;
rho0s2 = rho04;
PMRs2 = (1+(((gamma-1)/2)*(Ms2^2)))^(gamma/(gamma-1));
Ps2 = P0s2/PMRs2;
```

```
TMRs2 = 1+(((gamma-1)/2)*(Ms2^2));
Ts2 = T0s2/TMRs2;
RMRs2 = (1+(((gamma-1)/2)*(Ms2^2)))^(1/(gamma-1));
rhos2 = rho0s2/RMRs2;
as2 = sqrt(gamma*R*Ts2);
vs2 = Ms2*as2;

%Section 3
P0shock = ((((gamma+1)*(M3^2))/(2+((gamma-1)*(M3^2))))^(gamma/(gamma-1)))*(((gamma+1)/
((2*gamma*(M3^2))-(gamma-1)))^(1/(gamma-1)));
P03 = P0s2/P0shock;
PMR3 = (1+(((gamma-1)/2)*(M3^2)))^(gamma/(gamma-1));
P3 = P03/PMR3;
Tshock = (1+((2*gamma*((M3^2)-1))/(gamma+1)))*((2+((gamma-1)*(M3^2)))/((gamma+1)*
(M3^2)));
T3 = Ts2/Tshock;
TMR3 = 1+(((gamma-1)/2)*(M3^2));
T03 = T3*TMR3;
Rshock = ((gamma+1)*(M3^2))/(2+((gamma-1)*(M3^2)));
rho3 = rhos2/Rshock;
RMR3 = (1+(((gamma-1)/2)*(M3^2)))^(1/(gamma-1));
rho03 = rho3*RMR3;
a3 = sqrt(gamma*R*T3);
v3 = M3*a3;

%Section 2
P02 = P03;
T02 = T03;
rho02 = rho03;
AMR2 = A2/Ac == ((5 + M2^2)^3)/(216*M2);
M2 = min(double(vpasolve(AMR2,M2,[0 Inf])));
PMR2 = (1+(((gamma-1)/2)*(M2^2)))^(gamma/(gamma-1));
P2 = P02/PMR2;
TMR2 = 1+(((gamma-1)/2)*(M2^2));
T2 = T02/TMR2;
RMR2 = (1+(((gamma-1)/2)*(M2^2)))^(1/(gamma-1));
rho2 = rho02/RMR2;
a2 = sqrt(gamma*R*T2);
v2 = M2*a2;

%Mass Flow Rate
GammaM = sqrt(gamma*(2/(gamma+1))^((gamma+1)/(gamma-1)));
qM3 = M3*((2/(gamma+1))*(1+(((gamma-1)/2)*(M3^2))))^(-(gamma+1)/(2*(gamma-1)));
mdotflow = GammaM*qM3*A3*(P03/sqrt(R*T03));

%Fan Properties
TotalPressureRatio = P02/P01;
Power = (0.5*mdotflow*((M4*a4)^2))/.92
Power = Power*.001341022; %horsepower
ForceofThrust = mdotflow*v4;
Re = (rho3*v3*.5)/(1.789*(10^(-5)))
```

```
FanProperties = table(TotalPressureRatio,Power,ForceofThrust,mdotflow,Re)

%Section 1
qM1 = (mdotflow*sqrt(R*T01))/(GammaM*P01*A1);
AMR1 = 1/qM1 == ((5 + M1^2)^3)/(216*M1);
M1 = min(double(vpasolve(AMR1,M1,[0 Inf])));
P1 = P01/((1+(((gamma-1)/2)*(M1^2)))^(gamma/(gamma-1)));
T1 = T01/(1+(((gamma-1)/2)*(M1^2)));
a1 = sqrt(gamma*R*T1);
v1 = M1*a1;



%Display
Point = [1,2,3,0,4]';
MachNumber = [M1,M2,M3,Ms2,M4]';
Area = [A1,A2,A3,As2,A4]';
StagPres = [P01,P02,P03,P0s2,P04]';
StagTemp = [T01,T02,T03,T0s2,T04]';
StagDens = [rho01,rho02,rho03,rho0s2,rho04]';
Velocity = [v1,v2,v3,vs2,v4]';
SoundSpeed = [a1,a2,a3,as2,a4]';
Pressure = [P1,P2,P3,Ps2,P4]';
Temperature = [T1,T2,T3,Ts2,T4]';
Results = table(Point,StagPres,StagTemp,StagDens,MachNumber,Area,Velocity,SoundSpeed,↙
Pressure,Temperature)

%--------------Tunnel Calculations--------------%


%Length of Tunnel
x = [0:0.01:9];
inlength = 3;
outlength = 5;

%Ma Number Piecewise
for n=1:length(x);
   if x(n) < inlength;
       M(n)=(M3-M2)/2*cos(1/3*pi*x(n)+pi)+(M3-M2)/2+M2;
   elseif x(n) >=inlength & x(n)<=(inlength+1);
       M(n)=M3;
%     elseif x(i) ==(inlength+1);
%         M(i)=M3s;
   elseif x(n) >(inlength+1) & x(n)<=(inlength+1+outlength-1);
       M(n)=(Ms2-M4)/2*cos(1/4*pi*x(n)-pi)-(Ms2-M4)/2+Ms2;
   else
       M(n)=M4;
   end
end

for n = 1:length(x);
```

```
    if x(n) <= (inlength+1)
        %Area
        Aratio(n) = (((2/(gamma+1))*(1+((gamma-1)/2)*(M(n)^2)))^((gamma+1)/(2*(gamma-↙
1))))/M(n);
        A(n) = Aratio(n)*Ac;
        halfheight(n) = A(n);
        %Pressure
        Pratio(n) = (1+(((gamma-1)/2)*(M(n)^2)))^(gamma/(gamma-1));
        P(n) = (P03/Pratio(n))/1000;
        %Temperature
        Tratio(n) = 1+(((gamma-1)/2)*(M(n)^2));
        T(n) = T03/Tratio(n);
    else
        %Area
        Aratio(n) = (((2/(gamma+1))*(1+((gamma-1)/2)*(M(n)^2)))^((gamma+1)/(2*(gamma-↙
1))))/M(n);
        A(n) = Aratio(n)*Asc;
        halfheight(n) = A(n);
        %Pressure
        Pratio(n) = (1+(((gamma-1)/2)*(M(n)^2)))^(gamma/(gamma-1));
        P(n) = (P04/Pratio(n))/1000;
        %Temperature
        Tratio(n) = 1+(((gamma-1)/2)*(M(n)^2));
        T(n) = T04/Tratio(n);
    end
    %Velocity
    v(n) = sqrt(R*gamma*T(n))*M(n);
    %Density
    rho(n) = rho03/(1+(((gamma-1)/2)*(M(n)^2)))^(1/(gamma-1));
end

%--------------Validation--------------%
for n = 1:length(x)
    qM(n) = 1/((((2/(gamma+1))*(1+((gamma-1)/2)*(M(n)^2)))^((gamma+1)/(2*(gamma-1))))/M↙
(n));
    if x(n) <= (inlength+1)
        mdot(n) = GammaM*qM(n)*A(n)*(P03/sqrt(R*T03));
    else
        mdot(n) = GammaM*qM(n)*A(n)*(P04/sqrt(R*T04));
    end
    h0(n) = (Cp*T(n))+(0.5*(v(n)^2));
end

figure(1)
tiledlayout(2,1);

nexttile;
plot(x,mdot);
axis([0 9 25 75])
title('Mass Flow Rate wrt length');
xlabel('Length (m)');
```

```
ylabel('Mass Flow Rate');

nexttile;
plot(x,h0);
axis([0 9 200000 400000])
title('Energy wrt length');
xlabel('Length (m)');
ylabel('Energy');

%--------------Results--------------%
figure(2)
tiledlayout(3,2);

nexttile;
plot(x,M);
% axis([0 8 0 inf])
title('Mach Number wrt length');
xlabel('Length (m)');
ylabel('Ma');

nexttile;
plot(x,halfheight);
% axis([0 8 0 inf])
title('Shape of Wind Tunnel');
xlabel('Length (m)');
ylabel('Half Height (m), Depth = 0.5m');

nexttile;
plot(x,v);
% axis([0 8 30 220])
title('Velocity wrt length');
xlabel('Length (m)');
ylabel('Velocity (m/s)');

nexttile;
plot(x,P);
% axis([0 8 75 105])
title('Pressure wrt length');
xlabel('Length (m)');
ylabel('Presure (kPa)');

nexttile;
plot(x,T);
% axis([0 8 275 300])
title('Temperature wrt length');
xlabel('Length (m)');
ylabel('Temperature (K)');

nexttile;
plot(x,rho);
% axis([0 8 1 1.25])
```

```
title('Density wrt length');
xlabel('Length (m)');
ylabel('Density (kg/m^3)');
```