The Design of a Supersonic Turbofan Engine for Commercial Air Travel

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Abstract

This project aims to provide design parameters for a supersonic turbofan engine for the UCI-Supersonic 2021. In order to achieve performance specifications that would make it an improvement over previous supersonic commercial aircraft, namely the Concorde.

In order to design this engine, the bypass ratio and fan pressure ratio must be chosen based on TSFC and ST requirements. Once these two properties are determined, they can be used to plot a carpet plot of TSFCs and STs based on either a fixed compressor ratio or maximum burner temperature, and a maximum burner temperature and compressor can be chosen based on performance characteristics.

In the end, the engine design has an inlet diameter of 1.8m, a bypass ratio of 0.5, a fan pressure ratio of 1.2, a maximum temperature of 1650K, and a compressor pressure ratio of 20. With these values, it meets the cruising design requirements for both thrust and efficiency, making it a suitable candidate for the UCI-Supersonic 2021.

With these designed values, the ST is 0.6918, the TSFC is 0.0291, the thrust produced is 83000kN, and the overall efficiency is 37%.

Introduction

The task at hand is to design a supersonic turbofan engine alongside Rolls-Royce for the Boom Technologies Overture jet.

This engine must be able to cruise at Mach 1.7 at an altitude of 60,000 feet, have a range of 8000km, have a cruise thrust of 80kN, and have a minimum take-off thrust of 100kN.

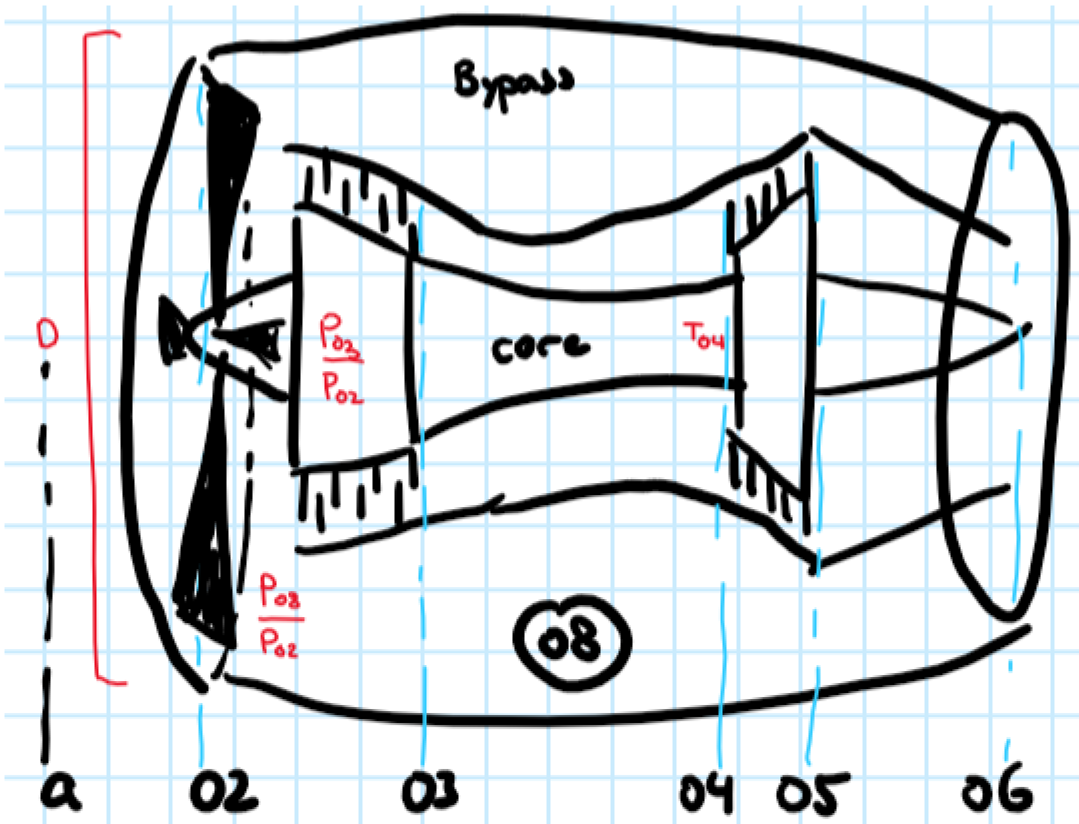
Some design limitations include a maximum inlet diameter of 2 meters and a maximum turbine inlet stagnation temperature of 1800K due to material tolerances.

To design such an engine, there are a few key parameters that must be determined: the bypass ratio, the fan pressure ratio, the turbine inlet stagnation temperature, the compressor pressure ratio, and the inlet diameter. These design parameters will allow designers to design the rotors and stators properly to meet the performance requirements. However, there are a few difficulties regarding designing a turbofan engine for supersonic flight, which is normally achieved with just a turbojet engine.

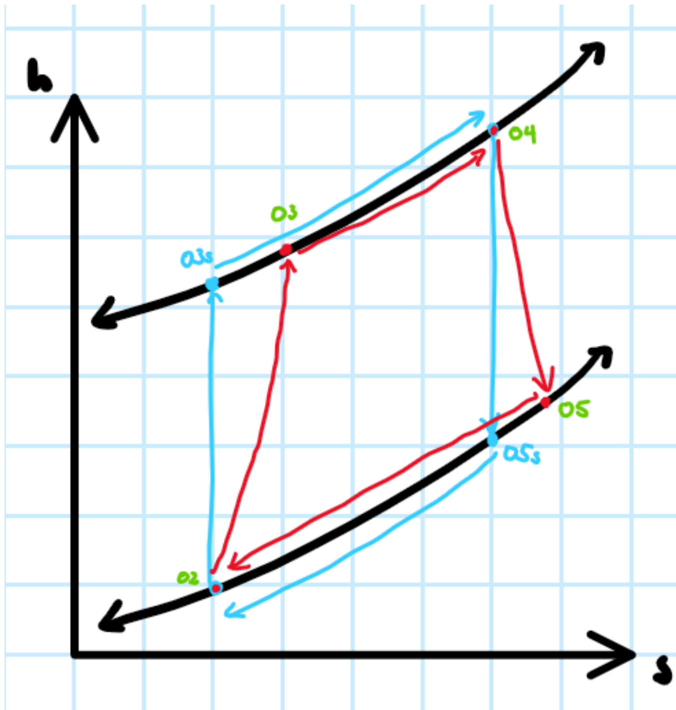
These properties are highly dependent on each other, and more detail will be provided in the next section.

Design Method

Here is a rough diagram of the engine we'll be designing.



The primary parameters we'll be optimizing are in red.



This is an h-s diagram demonstrating the Brayton cycle that this machine will be performing, as well as the adjusted values given the different polytropic and adiabatic efficiencies of each stage.

To begin calculating the design parameters, we'll begin by starting with cruise conditions. Given a Mach number and altitude for ambient conditions, stagnation temperature and pressure can be determined for the inlet. Assuming near-maximum turbine inlet temperature and a reasonable compressor pressure ratio, we'll be able to determine specific thrust and thrust specific fuel consumption contour distribution for various bypass ratios and fan pressure ratios. These contours will be based on these equations:

$$T_{02} = T_a\left(1 + \frac{\gamma - 1}{2}M^2\right)$$

$$p_{02} = p_a\left[1 + \eta_d\left(\frac{T_{02}}{T_a} - 1\right)\right]^{\gamma_d/(\gamma_d - 1)}$$

$p_{08} = p_{02} p_{rf}$, bypass stagnation pressure

$$T_{08} = T_{02}\left[1 + \frac{1}{\eta_f}\left(p_{rf}^{(\gamma_f - 1)/\gamma_f} - 1\right)\right]$$, bypass stagnation temperature

$$u_{ef} = \sqrt{2\eta_{fn}\frac{\gamma_f}{\gamma_\gamma - 1}RT_{08}\left[1 - (p_a/p_{08})^{(\gamma_f - 1)/\gamma_f}\right]},$$, fan exhaust velocity

$$u_e = \sqrt{2\eta_n\frac{\gamma_n}{\gamma_n - 1}RT_{06}\left[1 - \left(\frac{p_7}{p_{06}}\right)^{\gamma_n - 1/\gamma_n}\right]}$$, core exhaust velocity

$$\frac{\mathcal{T}}{\dot{m}_a} = (1 + f)u_e + \mathcal{B}u_{ef} - (1 + \mathcal{B})u$$, isolate T, which is T_bare

$$\mathcal{T} = \frac{\mathcal{T}_{bare}}{1.04 + 0.01\beta^{1.2}}$$, effective thrust

$$ST = \frac{\mathcal{T}}{\dot{m}_a(1 + \beta)}$$

$$\text{TSFC} = \frac{\dot{m}_f}{\mathcal{T}} = \frac{f}{(1 + f)u_e + \mathcal{B}u_{ef} - (1 + \mathcal{B})u}$$

P_{rf} (Fan Pressure Ratio) and beta (Bypass Ratio) are generated in a meshgrid, and using matrix manipulation, we can then plot contours of ST and TSFC. From these contours, we'll select a bypass ratio and fan pressure ratio that best balances between specific thrust and thrust specific fuel consumption.

Once the bypass ratio and fan pressure ratio have been reasonably chosen, we'll return to choosing the maximum turbine inlet stagnation temperature and compressor pressure ratio.

Using the equations from above as well as the ones listed below and two “for” loops, we’ll be able to generate a carpet plot of fixed compressor pressure ratios (P_{rc}) and maximum turbine inlet stagnation temperature (T04).

$$T_{03} = T_{02}\left[1 + \frac{1}{\eta_c}\{P_{rc}^{(\gamma_c-1)/\gamma_c} - 1\}\right]$$

$$p_{03} = p_{02}p_{rc}$$

$$f = \frac{T_{04}/T_{03} - 1}{Q_R/c_p T_{03} - T_{04}/T_{03}}$$

$$p_{04} = p_{03}(p_{04}/p_{03})$$

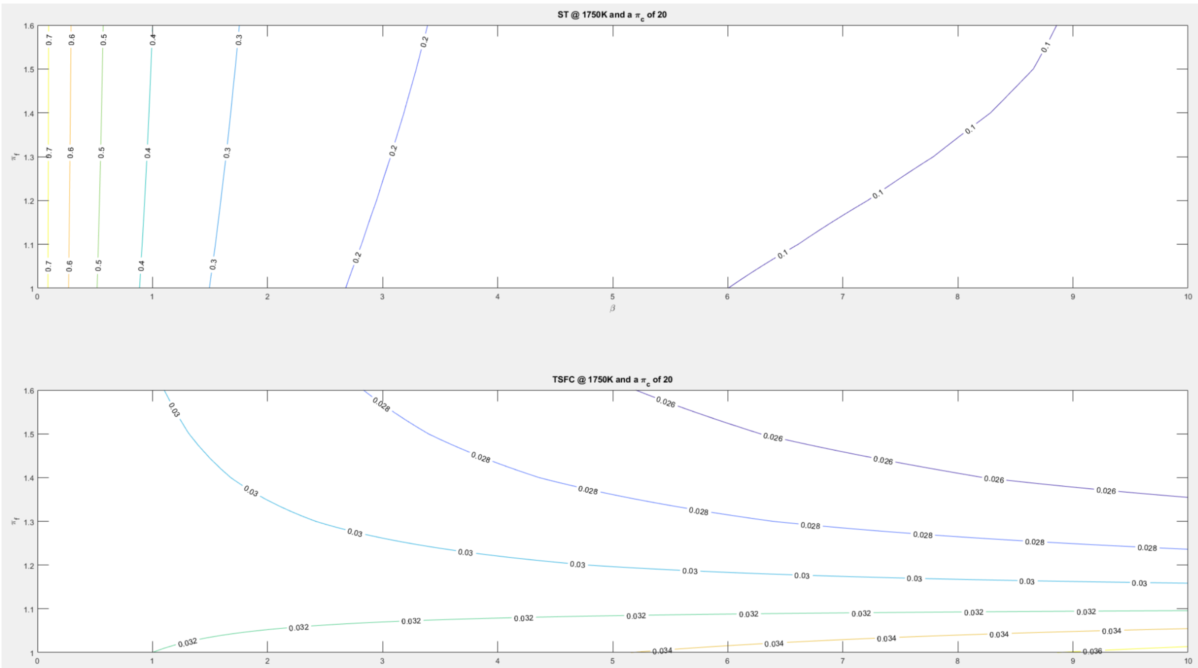
$$T_{05} = T_{04} - (T_{03} - T_{02}) - \mathcal{B}(T_{08} - T_{0a})$$

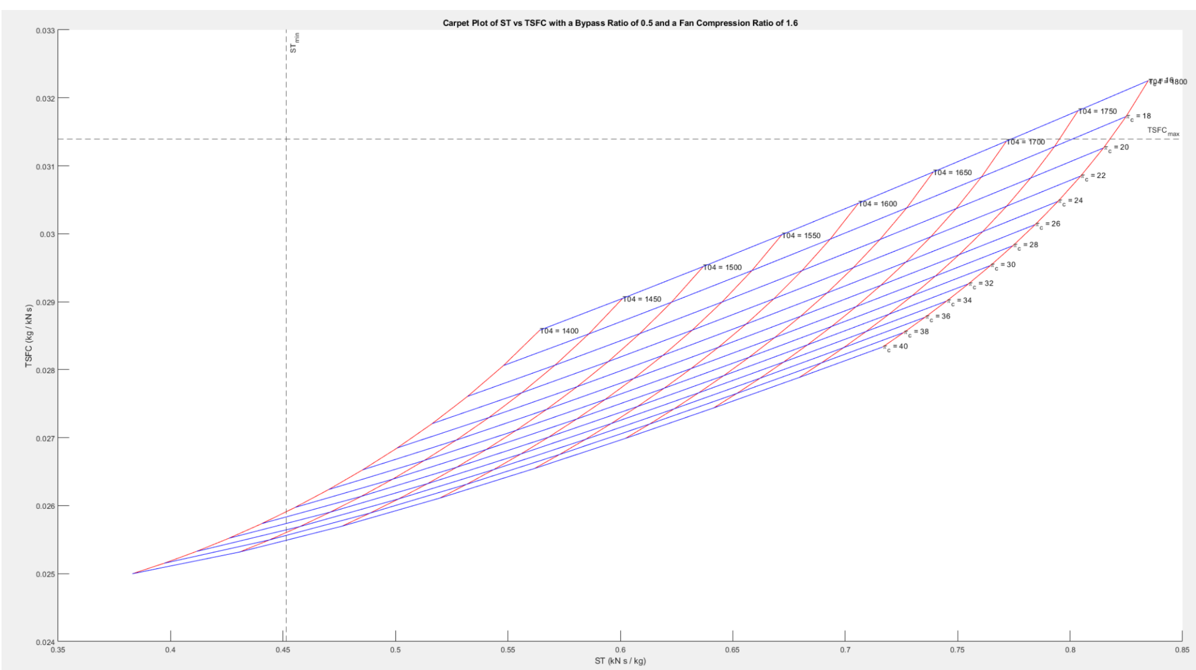
$$p_{05} = p_{04}\left[1 - \frac{1}{\eta_t}\left(1 - \frac{T_{05}}{T_{04}}\right)\right]^{\gamma_t/(\gamma_t-1)}$$

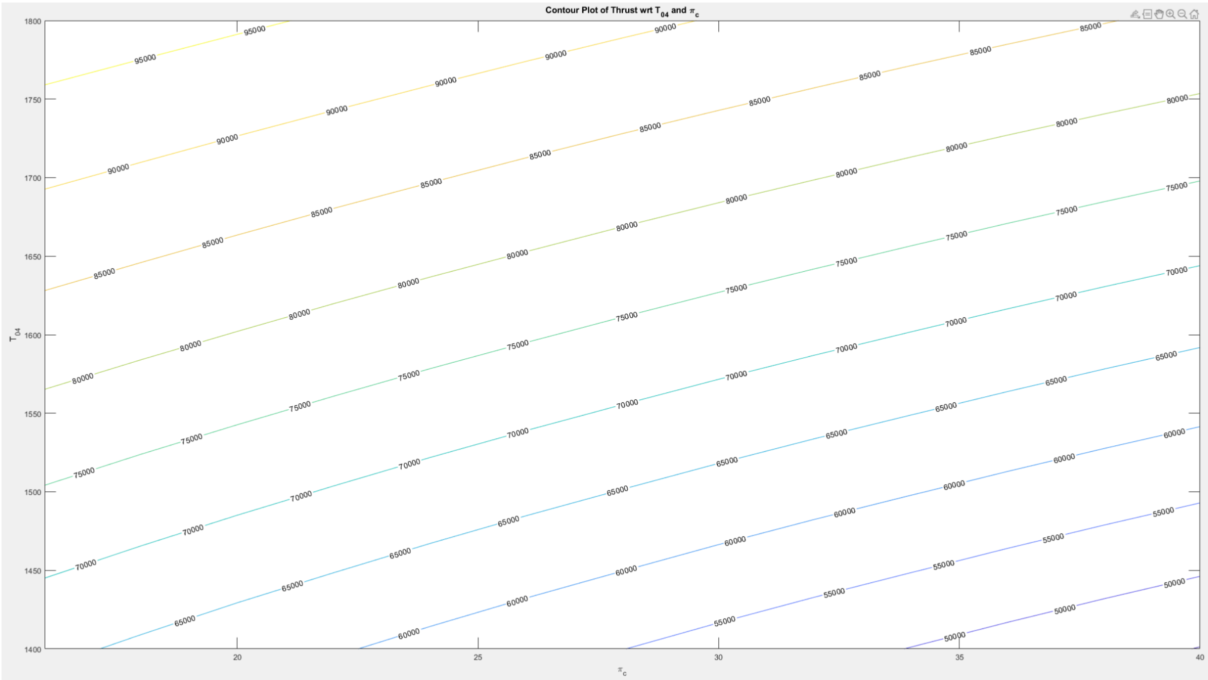
When this carpet plot is generated, we can then determine a minimum ST requirement and a maximum TSFC requirement based on the maximum T04, maximum T03, inlet diameter, and required cruise thrust.

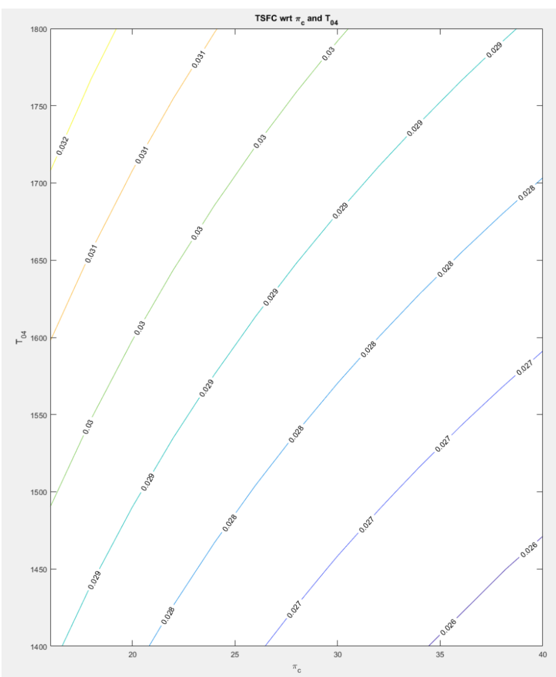
Once those requirements have been plotted on the carpet plot, we can choose the best T04 and P_{rc} to optimize either the lowest TSFC, the highest ST, or a balance of both, for our engine.

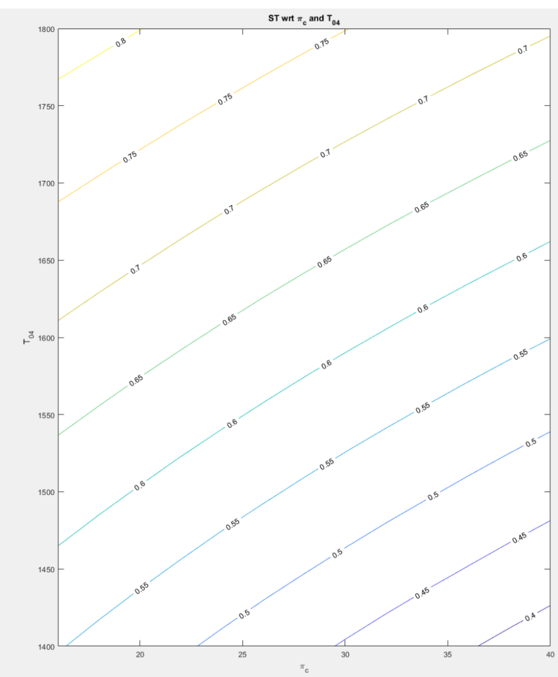
Results

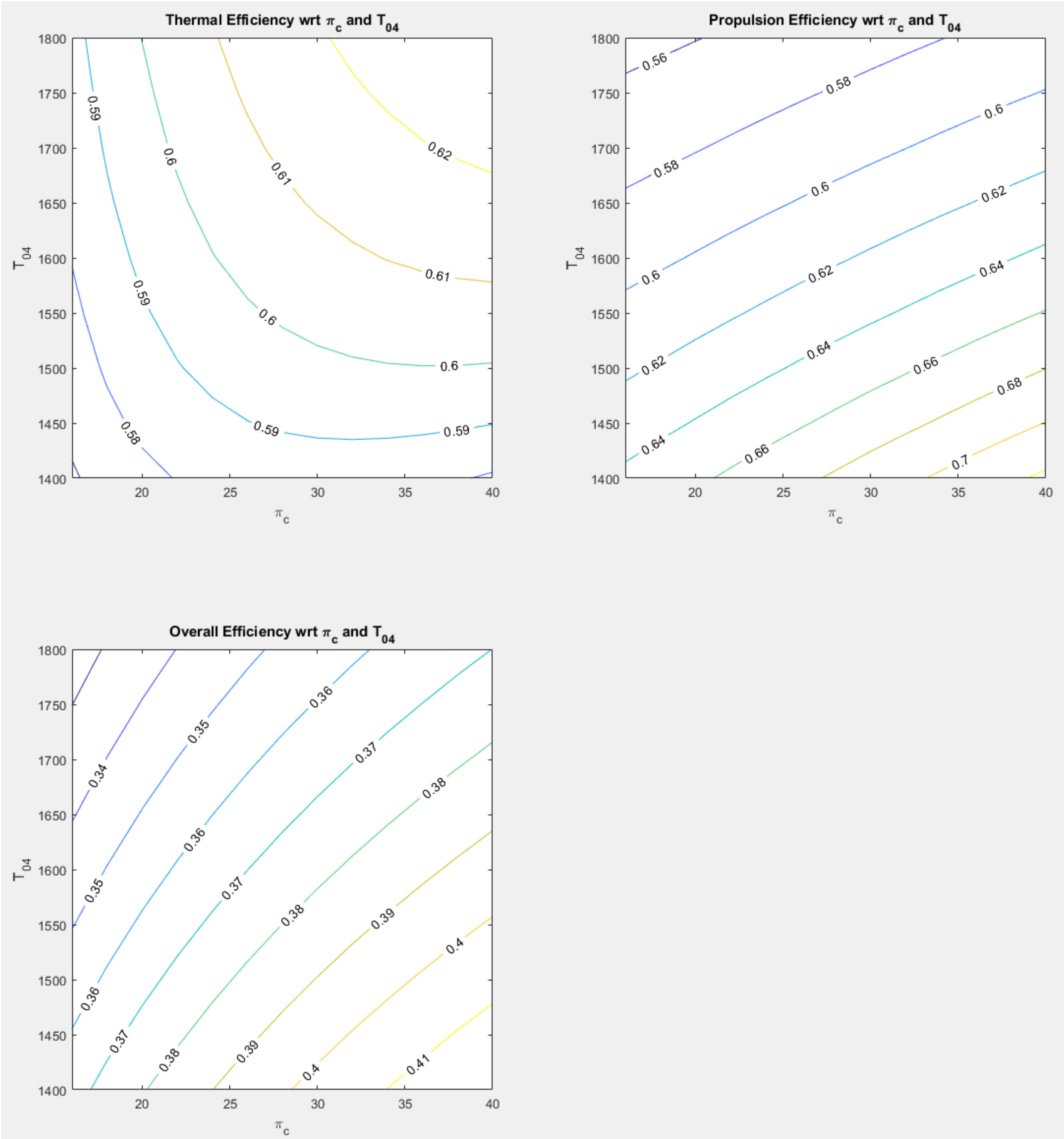

ST @ 1750K and a π_c of 20
π_f
β
TSFC @ 1750K and a π_c of 20
π_f
β


Carpet Plot of ST vs TSFC with a Bypass Ratio of 0.5 and a Fan Compression Ratio of 1.6
TSFC (kg / kN s)
ST (kN s / kg)
ST_{min}
$TSFC_{max}$
T04 = 1400
T04 = 1450
T04 = 1500
T04 = 1550
T04 = 1600
T04 = 1650
T04 = 1700
T04 = 1750
T04 = 1800
π_c = 18
π_c = 20
π_c = 22
π_c = 24
π_c = 26
π_c = 28
π_c = 30
π_c = 32
π_c = 34
π_c = 36
π_c = 38
π_c = 40


Contour Plot of Thrust wrt T_{04} and π_c
T_{04}
π_c
1400
1450
1500
1550
1600
1650
1700
1750
1800
20
25
30
35
40
50000
55000
60000
65000
70000
75000
80000
85000
90000
95000


TSFC wrt π_c and T_{04}
T_{04}
π_c
1400
1450
1500
1550
1600
1650
1700
1750
1800
20
25
30
35
40
0.026
0.027
0.028
0.029
0.03
0.031
0.032


ST wrt π_c and T_{04}
T_{04}
π_c
1400
1450
1500
1550
1600
1650
1700
1750
1800
20
25
30
35
40
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8



Thermal Efficiency wrt π_c and T_{04}
Propulsion Efficiency wrt π_c and T_{04}
Overall Efficiency wrt π_c and T_{04}
T_{04}
π_c

Analysis

When looking at the beta-pi_f (Bypass Ratio/Fan Pressure Ratio) graphs plotting the contours of ST and TSFC, we can see that we end up with the best ST with a smaller bypass ratio but diminishing returns on TSFC. However, ST is abysmal when selecting a high bypass ratio, and because it's a supersonic aircraft, increasing the bypass ratio would only increase drag, so the selected bypass ratio will remain low (approximately 0.5). In regards to fan pressure ratio, it can be seen that with higher fan pressure ratios, TSFC improves while not really affecting ST, so a relatively high fan pressure ratio will be used (approximately 1.6).

When using those values when using variable T04 and pi_c's in order to calculate the thrust and efficiencies, we can see that higher T04 and pi_c's are typically associated with higher engine efficiency. However, higher pi_c's generate lower thrust. Analyzing the carpet plot, higher pi_c increases TSFC and a higher T04 increases ST. Balancing these two, we want to choose a relatively high pi_c (approximately 24) and a relatively high T04 (approximately 1650K) in order to have the best of both worlds.

Inlet diameter of 1.85 will be selected because otherwise, the effective thrust of the engine is not high enough at the previously mentioned design specs to satisfy the requirement of 80kN cruise thrust.

It should be noted that an afterburner was not designed due to the fact that afterburners typically only increase the specific thrust of the engine while decreasing the fuel efficiency. This makes it an unnecessary design component for a commercial aircraft that does not require additional maneuverability and is primarily concerned with fuel efficiency.

Impact

Compared to the Concorde, this engine is more efficient with a TSFC of 0.0291 kg/(kN·s) instead of 0.039 kg/(kN·s) which allows for a more viable commercial vehicle thanks to decreased fuel costs. However, beyond that, the impact of this engine is dubious at best.

The main issues with supersonic commercial flight haven't been the efficiency of the engines, but rather the restrictions on flight routes due to sonic booms. With the EU and United States having restricted supersonic flight over land, the main routes where this type of craft would be most useful aren't actually flyable without some way to mitigate the sonic boom.

Summary

By generating contour and carpet plots and knowing the performance requirements of the aircraft, we were able to deduce design parameters (bypass ratio, fan pressure ratio, maximum turbine inlet stagnation temperature, compressor pressure ratio, and inlet diameter) that would allow for a good balance between thrust, fuel efficiency, and overall efficiency. Values can be fine-tuned and adjusted, depending on whether performance or efficiency is more highly valued, but for this design, it's been approximated that the design parameters ought to be an inlet diameter of 1.85 meters, a bypass ratio of 0.5, a fan pressure ratio of 1.6, a T04 of 1650K, and a compressor pressure ratio of 24.

With this design methodology and chosen design values, the ST is 0.6918, the TSFC is 0.0291, the thrust produced is 83000kN, and the overall efficiency is 37%. This satisfies the overall design specifications of the engine and is more efficient than its predecessors in the supersonic commercial flight category.

Appendix

```
clear all
close all
clc

%% Physical Variables
Qr = 45e06;
cp = 1004;
Ta = 217;
Pa = 7172;
Pe = Pa;
gamma = 1.4;
R = 287;
M = 1.7;
Pa_sl = 101.3e3;
Ta_sl = 288;

%% Fixed Engine Parameters
%Inlet/Diffuser
nd  = .95;
gd  = 1.4;
cpd = gd*R/(gd-1);
%Compressor (Polytropic)
npc = .9;
gpc = 1.37;
cppc = gpc*R/(gpc-1);
%Fan (Adiabatic)
nf  = .92;
gf  = 1.4;
cpf = gf*R/(gf-1);
%Burner
nb  = .97;
bpr = .95; %Burner Pressure Recovery
gb  = 1.35;
cpb = gb*R/(gb-1);
%Turbine (Polytropic)
npt = .92;
gpt = 1.33;
cppt = gpt*R/(gpt-1);
%Jet Nozzle
nn1 = .98;
gn1 = 1.36;
cpn1 = gn1*R/(gn1-1);
%Fan Nozzle
nn2 = .99;
gn2 = 1.4;
cpn2 = gn2*R/(gn2-1);


%% Chosen Variables
D = 1.85;
beta = 0.5;
pi_f = 1.6;

%% Turbojet
u = M*sqrt(1.4*287*Ta);

%Inlet/Diffuser
T02 = Ta * (1+((gd-1)/2)*(M^2));
P02 = Pa * ((1 + nd*((T02/Ta)-1))^(gd/(gd-1)));
T0a = Ta * (1+((gamma-1)/2)*M^2);

%Mass Flow Rate
sigma_0 = P02/Pa_sl;
theta_0 = T02/Ta_sl;
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```
A = pi*(D^2)/4;
mdot_inlet = 231.8*(sigma_0/sqrt(theta_0))*A;
mdot_a = mdot_inlet ./(beta+1);

%Min ST and Max TSFC
thrust_min = 80
D_max = 1.8;
mdot_max = 231.8*(sigma_0/sqrt(theta_0))*(pi*(D_max/2)^2);
T04_max = 1800;
T03_max = 1000; %max from design specs
f_max = (T04_max - T03_max)/((nb*Qr/cpb)-T04_max);
mdot_a_max = mdot_max/(1+beta);
mdot_f_max = mdot_a_max*f_max;
ST_min = thrust_min/(mdot_a_max*(1+beta))
TSFC_max = mdot_f_max/thrust_min

figure
hold on
for i = 1:9
    T04(i) = 1400 + 50*(i-1);

    for j = 1:13
        pi_c(i,j) = 16 + 2*(j-1);

        %Post-Compressor (03)
        T03(i,j) = T02 * pi_c(i,j)^((gpc-1)/(npc*gpc));
        P03 = pi_c(i,j) * P02; %polytropic

        %Pre-Turbine (Burner, 04)
        T04(i) = T04(i);
        P04 = bpr * P03;

        %Bypass (08)
        T08 = T02 * (1 + (1/nf)*(pi_f^((gf-1)/gf) - 1));
        P08 = P02 * pi_f;

        %Post-Turbine (05)
        T05 = T04(i) - (T03(i,j) - T02) - beta*(T08 - T0a);
        pi_t = (T05/T04(i)).^(gpt*npt/(gpt-1)); %polytropic
        P05 = P04 * pi_t;

        %Nozzle Inlet (06)
        T06 = T05;
        P06 = P05;

        %Nozzle Exit (e)
        ue(i,j) = sqrt(2*nn1*(gn1/(gn1-1))*R*T06*(1-(Pa/P06)^((gn1-1)/gn1)));

        %Fan Exit (ef)
        uef(i,j) = sqrt(2*nn2*(gn2/(gn2-1))*R*T08*(1-(Pa/P08)^((gn2-1)/gn2)));

        %Fuel
        f = (T04(i) - T03(i,j))/((nb*Qr/cpb)-T04(i));
        mdot_f(i,j) = f * mdot_a;

        %ST
        ST(i,j) = ((1+f)*ue(i,j) + beta*uef(i,j) - (1+beta)*u)/1000;

        %TSFC
        TSFC(i,j) = f/ST(i,j);

        %Total Exhaust Mass Flow Rate
        mdot_e(i,j) = mdot_f(i,j) + mdot_inlet;

        %Thrust
        T_bare(i,j) = (((1+f)*ue(i,j) + beta*uef(i,j) - (1+beta)*u))*mdot_a;
        T_eff(i,j) = T_bare(i,j)/(1.04+(0.01*beta^1.2));

        plot(ST(:,j), TSFC(:,j), 'b')
```

```
    end
    text(ST(i,1),TSFC(i,1),['T04 = ' num2str(T04(i))])
    plot(ST(i, :), TSFC(i, :), 'r')

end

%Thermal Efficiency
nth = ((0.5*ue.^2.*(mdot_f+mdot_a) + 0.5*uef.^2.*(mdot_inlet-mdot_a-mdot_f))-(0.5*u^2*mdot_inlet))./(mdot_f*Qr);

%Propulsion Efficiency
np = (T_eff*u)./((0.5*ue.^2.*(mdot_f+mdot_a) + 0.5*uef.^2.*(mdot_inlet-mdot_a-mdot_f))-(0.5*u^2*mdot_inlet));

%Overall Efficiency
no = nth.*np;

for x = 1:j
    text(ST(i,x),TSFC(i,x),['\pi_c = ' num2str(pi_c(i,x))])
end
title(['Carpet Plot of ST vs TSFC with a Bypass Ratio of ' num2str(beta) ' and a Fan Compression Ratio of ' num2str(pi_f)])
xlabel('ST (kN s / kg)')
ylabel('TSFC (kg / kN s)')
xline(ST_min,'--','ST_m_i_n')
yline(TSFC_max,'--','TSFC_m_a_x')

hold off

%Thrust Graph
figure
contour(pi_c(1,:),T04,T_eff,'ShowText','on')
title('Contour Plot of Thrust wrt T_0_4 and \pi_c')
xlabel('\pi_c')
ylabel('T_0_4')

%TSFC and ST Contours
figure
subplot(1,2,1)
contour(pi_c(1,:),T04,TSFC,'ShowText','on')
title('TSFC wrt \pi_c and T_0_4')
xlabel('\pi_c')
ylabel('T_0_4')

subplot(1,2,2)
contour(pi_c(1,:),T04,ST,'ShowText','on')
title('ST wrt \pi_c and T_0_4')
xlabel('\pi_c')
ylabel('T_0_4')

%nth, np, and no Contours
figure
subplot(2,2,1)
contour(pi_c(1,:),T04,nth,'ShowText','on')
title('Thermal Efficiency wrt \pi_c and T_0_4')
xlabel('\pi_c')
ylabel('T_0_4')

subplot(2,2,2)
contour(pi_c(1,:),T04,np,'ShowText','on')
title('Propulsion Efficiency wrt \pi_c and T_0_4')
xlabel('\pi_c')
ylabel('T_0_4')

subplot(2,2,3)
contour(pi_c(1,:),T04,no,'ShowText','on')
title('Overall Efficiency wrt \pi_c and T_0_4')
xlabel('\pi_c')
ylabel('T_0_4')


%% Beta and Pi_f Contours
clear T03
```

```
[beta,pi_f] = meshgrid(0:.1:10,1:0.1:1.6);
T04 = 1750;
pi_c = 20;
D = 1.8;

%Post-Compressor (03)
T03 = T02 * pi_c^((gpc-1)/(npc*gpc));
P03 = pi_c * P02; %polytropic

%Pre-Turbine (Burner, 04)
T04 = T04;
P04 = bpr * P03;

%Bypass (08)
T08 = T02 * (1 + (1/nf).*(pi_f.^((gf-1)/gf) - 1));
P08 = P02 .* pi_f;

%Post-Turbine (05)
T05 = T04 - (T03 - T02) - beta.*(T08 - T0a);
pi_t = (T05./T04).^(gpt*npt/(gpt-1)); %polytropic
P05 = P04 .* pi_t;

%Nozzle Inlet (06)
T06 = T05;
P06 = P05;

%Nozzle Exit (e)
ue = sqrt(2*nn1*(gn1/(gn1-1))*R.*T06.*(1-(Pa./P06).^((gn1-1)/gn1)));

%Fan Exit (ef)
uef = sqrt(2*nn2*(gn2/(gn2-1))*R.*T08.*(1-(Pa./P08).^((gn2-1)/gn2)));

%Fuel
f = (T04 - T03)/((nb*Qr/cpb)-T04);
mdot_f = f * mdot_a;

%ST
ST = ((1+f).*ue + beta.*uef - (1+beta).*u)./(1000*(1+beta));

%TSFC
TSFC = f./real(((1+f).*ue + beta.*uef - (1+beta).*u)/1000);

figure
subplot(2,1,1);
contour(beta,pi_f,ST,'ShowText','on');
title(['ST @ ' num2str(T04) 'K and a \pi_c of ' num2str(pi_c)])
xlabel('\beta');
ylabel('\pi_f');
subplot(2,1,2);
contour(beta,pi_f,TSFC,'ShowText','on');
xlabel('\beta');
ylabel('\pi_f');
title(['TSFC @ ' num2str(T04) 'K and a \pi_c of ' num2str(pi_c)])

clear all
clc

%% Physical Variables
Qr = 45e06;
cp = 1004;
Ta = 217;
Pa = 7172;
Pe = Pa;
gamma = 1.4;
R = 287;
M = 1.7;
Pa_sl = 101.3e3;
Ta_sl = 288;
```

```
%% Fixed Engine Parameters
%Inlet/Diffuser
nd  = .95;
gd  = 1.4;
cpd = gd*R/(gd-1);
%Compressor (Polytropic)
npc = .9;
gpc = 1.37;
cppc = gpc*R/(gpc-1);
%Fan (Adiabatic)
nf  = .92;
gf  = 1.4;
cpf = gf*R/(gf-1);
%Burner
nb  = .97;
bpr = .95; %Burner Pressure Recovery
gb  = 1.35;
cpb = gb*R/(gb-1);
%Turbine (Polytropic)
npt = .92;
gpt = 1.33;
cppt = gpt*R/(gpt-1);
%Jet Nozzle
nn1 = .98;
gn1 = 1.36;
cpn1 = gn1*R/(gn1-1);
%Fan Nozzle
nn2 = .99;
gn2 = 1.4;
cpn2 = gn2*R/(gn2-1);

%% Chosen Variables
D = 1.85;
beta = 0.5;
pi_f = 1.6;
T04 = 1650;
pi_c = 24;

%% Turbojet
u = M*sqrt(1.4*287*Ta);

%Inlet/Diffuser
T02 = Ta * (1+((gd-1)/2)*(M^2));
P02 = Pa * ((1 + nd*((T02/Ta)-1))^(gd/(gd-1)));
T0a = Ta * (1+((gamma-1)/2)*M^2);

%Mass Flow Rate
sigma_0 = P02/Pa_sl;
theta_0 = T02/Ta_sl;
A = pi*(D^2)/4;
mdot_inlet = 231.8*(sigma_0/sqrt(theta_0))*A;
mdot_a = mdot_inlet/(beta+1);

%Post-Compressor (03)
T03 = T02 * pi_c^((gpc-1)/(npc*gpc));
P03 = pi_c * P02; %polytropic

%Pre-Turbine (Burner, 04)
T04 = T04;
P04 = bpr * P03;

%Bypass (08)
T08 = T02 * (1 + (1/nf)*(pi_f^((gf-1)/gf) - 1));
P08 = P02 * pi_f;

%Post-Turbine (05)
T05 = T04 - (T03 - T02) - beta*(T08 - T0a);
pi_t = (T05/T04)^(gpt*npt/(gpt-1)); %polytropic
P05 = P04 * pi_t;
```

```
%Nozzle Inlet (06)
T06 = T05;
P06 = P05;

%Nozzle Exit (e)
ue = sqrt(2*nn1*(gn1/(gn1-1))*R*T06*(1-(Pa/P06)^((gn1-1)/gn1)));

%Fan Exit (ef)
uef = sqrt(2*nn2*(gn2/(gn2-1))*R*T08*(1-(Pa/P08)^((gn2-1)/gn2)));

%Fuel
f = (T04 - T03)/((nb*Qr/cpb)-T04);
mdot_f = f * mdot_a;

%ST
ST = ((1+f)*ue + beta*uef - (1+beta)*u)/1000

%TSFC
TSFC = f/ST

%Total Exhaust Mass Flow Rate
mdot_e = mdot_f + mdot_inlet;

%Thrust
T_bare = (((1+f)*ue + beta*uef - (1+beta)*u))*mdot_a;
T_eff = T_bare/(1.04+(0.01*beta^1.2))

%Thermal Efficiency
nth = ((0.5*ue.^2.*(mdot_f+mdot_a) + 0.5*uef.^2.*(mdot_inlet-mdot_a-mdot_f))-(0.5*u^2*mdot_inlet))./(mdot_f*Qr);

%Propulsion Efficiency
np = (T_eff*u)./((0.5*ue.^2.*(mdot_f+mdot_a) + 0.5*uef.^2.*(mdot_inlet-mdot_a-mdot_f))-(0.5*u^2*mdot_inlet));

%Overall Efficiency
no = nth.*np
```