MAE 157 Project Report

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Introduction

Lightweight structures are meant to maximize their strengths as materials based on criterion such as stress, deflection, and stiffness. The motivation behind this project is that most heavy duty car jacks that are able to lift trucks are large and unwieldy, requiring to be wheeled around on rollers and not normally able to be casually stored in smaller vehicles. Scissor jacks are common in most cars, but rare to find in larger trucks. These conflicting factors prompted us to create a lightweight structure that can be analyzed and designed to minimize the total mass. For the structural designing process of the scissor jack we will utilize material learned throughout the course to ensure that our scissor jack is lightweight, portable, easy to use, and most importantly will ensure convenience for unconventional vehicles, such as trucks. It is crucial to make sure that the scissor jack has the security and dependency needed. It will be accomplished in our design process, which will adhere to solving common flaws such as problems with reliability and control with heavier vehicles.

Project Objectives

The goal for this project is to design a scissor car jack with the lowest possible mass in order for functionality and ease of portability while also being able to lift 1/4th of any retail vehicle (as you'd need 4 contact points to lift the entire vehicle off the ground), with a mid-size truck being the largest.

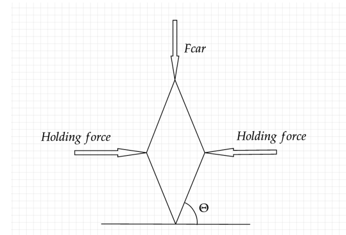
Structural Analysis Approach

We find the forces and displacements acting on each bar and use these to find the stress on each beam as a function of cross-sectional area and material properties (Young's Modulus, and yield stress). We can use this function to choose a material and area dimensions that minimize our mass.

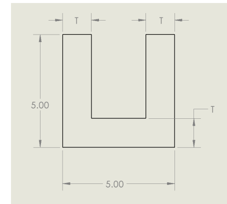
Initial assumptions	
Force of the car	4723.5 N
Beam height and width	5 cm
Beam length	18 cm

The initial assumptions are that the car weighs 18,894.4 Newtons, and the load applied to the jack is only ¼ that weight (4723.5 N) because the jack only has to lift one wheel of the ground. The average ground clearance of a sedan is about 15cm, so we first analyzed at this height because this is the minimum height where weight would be applied to the jack. We decided that the jack must lift the car up to 30cm for it to be usable, which is the second state we analyzed the structure. The length of the beam is set to 18cm, this is so that at full extension the beams do not collide with each other.

Our structure is composed of 4 beams arranged in a diamond pattern. The weight of the car is applied downwards to the node at the top of the diamond (denoted node 2). With the normal force of the floor being applied upwards to the bottom node (node 3). We tested



out three different materials, 6061 aluminum, 7075 aluminum, and 304 stainless steel. We chose these materials because they're common materials that are lightweight and strong. Each beam is a C-channel shape.



To simplify the model, we set the width and height of the c-channel at 5cm and solved for the minimum thickness of the beam. We also eliminated the center horizontal beam, and replaced it with horizontal forces at nodes 1 and 4, pointing inwards. We did this so we could focus on the C-channel beams.

The first formula used was Newton's Second Law, ΣF = ma = 0. We used this to find the axial forces in each beam by solving for the reaction forces, and then applying these forces at the nodes. Next I calculated the stiffness matrix of the beam K and then plugged it into the equation F = K * U to solve for the displacements of each node.

Failure Criteria

In our project, we used the buckling and the material failure criterion because the truss was under compression. Originally we were going to add the deflection criterion, but we decided that the values were not significant enough to add substance to our project, resulting in unnecessary work. Also, when calculating the buckling and material failure criterion, we came to the conclusion that all of the thicknesses that were calculated at full extension were going to be larger than the thicknesses at half. So we stopped calculating the values at full extension.

Using the yield stress and the axial force on the beam in half extension found it structural, we calculated the material failure criterion. We inserted these values into the matlab code to get the minimum thickness of each material. At the same time we also calculated the minimum thicknesses for the three materials using the same axial force and Young's modulus. Instead of using a critical length, we decided to use a constant one. This made the calculations so much easier, and it resulted in cleaner values. Below are the two main equations that we used to calculate the minimum mass values. All of the constants found to solve the equations, originated from the structural analysis of our project.

Material Failure Criterion: $\Phi_\sigma \left(\frac{F}{3Bt - 2t^2}\right) \leq \sigma^*$

Buckling Failure Criterion: $\Phi_b \frac{F}{3Bt - 2t^2} \leq \frac{\pi^2 E \frac{Bt^3 + 2t(B-t)^3}{3}}{(KL)^2}$

One problem that we encountered while solving for the minimum thickness is we noticed that our thicknesses were unrealistically small. At the beginning stages of the project we used a safety factor of 1.5, but that was resulting in measurements that were a thousandth of a centimeter. So as a group we decided to change our safety factor to 25, and that resulted in more acceptable results (which is a tenth of a centimeter).

Minimum Mass

The most important design variable was the minimum thickness because the minimum thickness allows us to minimize the mass.

Using previously derived equations from the structure analysis and the yielding failure criteria, the equation of minimum thickness is provided down below:

$$t = \frac{-3B \pm \sqrt{9B^2 - \left(\frac{8\Phi_{\sigma} F}{\sigma*}\right)}}{-4}$$

Along with that, three different densities were utilized, the densities were found based on the materials that were used to design the scissor car jack. The height and width of the beams were taken from the scissors car jack design dimensions which were required in order to solve for the minimum mass.

The beam height and width of the design were the same size (5cm), and in order to simplify the calculations, the same variable for both height and width was used in the area equation that was derived from the structural analysis. The beam length is evaluated as a constant when we calculate for the minimum mass. In order to avoid errors in calculations and produce the most precise values possible, Matlab was used. The Matlab script incorporated all elements present in the main minimum mass equation such as density, beam length, minimum thickness, and beam height and width. The only two design variables that changed were the densities and the minimum thicknesses for each material used.

The minimum mass calculations were done only for the half-extension aspect of the scissors jack because we realized that the thicknesses would be larger for the full-extension aspect, meaning that the overall minimized mass would not result from doing calculations for the full-extension scenario of the scissor car jack. The calculations for three different materials (6061 Aluminum, 7075 Aluminum, 304 stainless steel) were done to observe how the minimum mass was affected for each material.

The main equation used to solve for the minimum mass was:

$$m = (\rho \times L \times (3Bt - 2t^2)) \times 4$$

ρ: $density$

L: $Beam\ Length$

B: $Beam\ Height\ and\ Width$

t: $minimum\ thickness$

m: $minimum\ mass$

The equation inside the parenthesis is multiplied by four, to account for all four beams in the scissors jack design. If this is not done the mass of only one beam would be considered.

Results

For our results, we ended up validating our mass and stress requirements using the built-in FEA in Solidworks and Mass Analysis tool. Using the formulas in the analysis of displacement criterion and yield criterion, we validated our models using default values of 6061 Alloy, base/height of 5cm, and thickness of 1cm.

For deflection criterion, the simulation was run on the assembly model of the jack, and deflection was measured from the top plate (see Appendix Sec. 3, 2)

For yielding criterion, the simulation was run on an individual C-channel member in order to accurately determine the effects across the entire member and not be thrown off by the cutouts for the pins (see Appendix Sec. 3, 1).

For more detailed views of the models and FEA conditions used, please refer to Appendix Sec. 3. Using these default values and criteria, the simulation was validated to within a few hundredths of a millimeter for deflections, and to within a few kPa for stress.

Once that was validated, we proceeded to finish our hand calculations for the different materials and solving for minimum mass and thickness. Here are the results from our yielding criterion (we disregarded buckling because buckling had a far lower minimum thickness):

Yielding Criterion:

Half-Extension:

Material	Modulus of Elasticity(Pa)	Density (kg/m^3)	Force(N)	Minimum t (m)	Stress (N/m)	Minimum mass (kg)
6061	69 x 10^9	2700	5668.5	0.0035953623154333187256474461337743	1.1040*10^7	0.998148913043479
304	190 x 10^9	8000	5668.5	0.0048868396168957394631162588422097	8.2720*10^6	3.9471
7075	71.7 x 10^9	2810	5668.5	0.0019277818546926148521130836307237	2.0120*10^7	0.570005427435388

Then, after the models and FEA results were validated, the values gained from the hand calculations were input into the models for minimum thickness, and here were the results from Solidworks FEA:

Material	Yield Strength (Pa)	Stress (Max) (Pa)	Stress (Avg) (Pa)	Mass (kg)	Deflection (mm)
6061	2.75E+08	1.77E+07	1.10E+07	0.998	9.81E-02
304	2.07E+08	1.06E+07	7.81E+06	3.944	2.68E-02
7075	5.05E+08	2.58E+07	1.91E+07	0.57	9.40E-02

Here are the percent errors of our hand calculations compared to our FEA results:

Safety Factor (Max)	Safety Factor (Avg)	%Error (Max)	%Error (Avg)	%Error (Mass)
15.54	25.09	60.33	0.72	0
19.51	26.48	25.34	5.60	0.07853867396
19.56	26.51	28.46	5.32	0

What we can glean from these results is that from the average stresses, the simulation validates the hand-calculations with a percent error under 6% for all materials. Our safety factors are also consistent with what they should be according to our initial conditions setting up our design constraints. The error due to the max stresses analyzed is most likely due to the inherent properties of adding cutouts for pin connections that are not analyzed in truss analyses and failure criterion. However, we can be confident in our average stress results thanks to these percent errors from the FEA, and the mass percent error was extremely accurate as well.

Conclusions and Future Work

As a team, we concluded that the minimum mass would result from the material 7075 Aluminum when it is calculated using the material failure criterion. A way that we can add more variables to the project is to expand on the types of materials that are theoretically tested. This allows a larger range of values to choose the minimum mass from.

Also, another way that we can make the project be more complex is by using varying length instead of having it remain constant when strategically finding the minimum mass. Another way the length can be varied, is by using a critical length that can be calculated between the buckling and the material failure criterion. Instead of using a constant length for each trial, the critical length will be unique to each of the materials that we use. In addition, we would adjust the width and the height of the beams to two different distinct values rather than using the same measurement for both. This would also bring more complexity to the project design.

The last way that we could alter our project in the future is to use a larger car car weight resulting in a larger compressive force on the truss. This will ultimately allow more use for any consumer because the car jack can be used on a larger variety of cars. In addition, we can also manipulate the angle at which the car jack extension is tested at.

Criteria Met by Project

The criterion that we met by the project is that our results were verified by the solidworks as shown in the results section of the project. Another met criterion was the numerical values for the project (ie. material properties, geometry, and the boundary conditions) are meant to serve purpose for a specific application. The application is to lift a car of specific weight and ground clearance off the ground a certain amount. We also used different material models than those used in class, by using specific materials, such as 6061 and 7075 aluminum and 304 stainless steel, because they are strong and they can handle the forces exerted by the car. Our beams are also more geometrically complex than those used in class. We chose a c-channel cross sectional area for our beam because most jacks of this type use this beam shape. This made our minimum thickness and mass calculations more complex.

<u>Appendix</u>

Section 1) Structural Analysis Math (Curtis Abe):

- Solving for axial force in each beam

$$\Sigma F_y = ma = 0 = -F_c - F_{12}cos(\theta) - F_{24}cos(\theta)$$
$$\Sigma F_x = ma = 0 = F_{24}sin(\theta) - F_{12}sin(\theta)$$
$$F_{24} = F12$$
$$F_{12} = F_{24} = -\frac{F_c}{2cos(\theta)}$$

-
- Solving for the axial stress in each beam

$$\sigma = \frac{F}{A}$$
$$\sigma = \frac{F_c}{2Asin(\theta)}$$

- Solving for the stiffness matrix (do this for each beam)

$$K = \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$
$$c = cos(\theta)$$
$$s = sin(\theta)$$
$$\theta_{12} = 56.4427$$
$$\theta_{13} = -56.4427$$
$$\theta_{24} = -56.4427$$
$$\theta_{34} = 56.4427$$

- Stiffness matrix of the entire truss

$$K = \frac{EA}{L}\begin{pmatrix} 0.611111 & 0 & -0.305555 & -0.460642 & -0.305555 & 0.460642 & 0 & 0 \\ 0 & 1.38889 & -0.460642 & -0.694445 & 0.460642 & -0.694445 & 0 & 0 \\ -0.305555 & -0.460642 & 0.611111 & 0 & 0 & 0 & -0.305555 & 0.460642 \\ -0.460642 & -0.694445 & 0 & 1.38889 & 0 & 0 & 0.460642 & -0.694445 \\ -0.305555 & 0.460642 & 0 & 0 & 0.611111 & 0 & -0.305555 & -0.460642 \\ 0.460642 & -0.694445 & 0 & 0 & 0 & 1.38889 & -0.460642 & -0.694445 \\ 0 & 0 & -0.305555 & 0.460642 & -0.305555 & -0.460642 & 0.611111 & 0 \\ 0 & 0 & 0.460642 & -0.694445 & -0.460642 & -0.694445 & 0 & 1.38889 \end{pmatrix}$$

- Solving for the node displacements

$$\begin{bmatrix} f_{x1} \\ 0 \\ 0 \\ -Fcar \\ f_{x3} \\ f_{y3} \\ f_{x4} \\ 0 \end{bmatrix} = K \begin{bmatrix} 0 \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ 0 \\ 0 \\ 0 \\ u_{y4} \end{bmatrix}$$

$$u_{y1} = -\frac{48974.273}{3.0\,E\,t - 40.0\,E\,t^2}$$

$$u_{x2} = 0$$

$$u_{y2} = -\frac{97948.547}{3.0\,E\,t - 40.0\,E\,t^2}$$

$$u_{y4} = -\frac{48974.273}{3.0\,E\,t - 40.0\,E\,t^2}$$

$$t = thickness$$

Section 2.1) Failure Criterion

1) Yield Criterion (Matthieu Lu)

```
clear
clc
%Yield Criterion
syms sigma sigmastar t B F A SF
sigmastar = 5.515e7
SF = 25
B = 0.05
F = 5668.53
A = 3*B*t - 2*t^2
sigma = F/A

eqn = SF*sigma == sigmastar
S = vpa(solve(eqn,t))
```

2) Individual Stress (Allison Eiler)

```
clear
clc
%Yield Criterion
syms sigma t B F A
B = 0.05;
F = 5668.5;
t = 0.004886839616895739463116258842209 7;
A = 3*B*t - 2*t^2;
sigma = F/A
```

Derivation combining equations from the structural analysis and buckling failure criterion to find minimum thickness: (Allison Eiler)

$$\sigma = \frac{F}{A} \qquad (1)$$

$$A = 3Bt - 2t^2 \qquad (2)$$

Combine (1) and (2) →

$$\sigma = \frac{F}{3Bt - 2t^2} \ (3)$$

$$\Phi_b \sigma \leq \sigma_{cr} \ (4)$$

$$\sigma_{cr} = \frac{\pi^2 EI}{(KL)^2} \quad (5) \qquad I = \frac{Bt^3 + 2t(B-t)^3}{3} \ (6)$$

Combine (3), (4), (5), and (6) →

$$\Phi_b \frac{F}{3Bt - 2t^2} \leq \frac{\pi^2 E \frac{Bt^3 + 2t(B-t)^3}{3}}{(KL)^2}$$

**Solve for t using matlab code because work by hand was complicated

Matlab Code Used to get precise values for minimum mass:

```
clear
clc
%Buckling Criterion
syms L t SF B F E I K A Sigma Sigmacr
SF = 25;
B = 0.05;
F = 5668.53;
E = 69e9;
K = 1;
L = .18;
I = (B*(t^3)+2*t*((B-t)^3))/3;
A = 3*B*t - 2*t^2;
Pcrit = ((pi^2)*E*I/((K*L)^2));
Sigma = F/A;
Sigmacr = Pcrit/A

eqn = SF*Sigma == Sigmacr
solutions = vpa(solve(eqn,t))
```

Section 2.2) Minimum Mass Calculations (Denesse Gomez)

Derivation combining equations from the structural analysis and failure criteria(yielding) to find minimum thickness:

$$\sigma = \frac{F}{A} \qquad (1)$$

$$A = 3Bt - 2t^2 \qquad (2)$$

Combine (1) and (2) →

$$\sigma = \frac{F}{3Bt - 2t^2} \quad (3)$$

$$\Phi_\sigma * \sigma \leq \sigma^* \quad (4)$$

Combine (3) and (4)→

$$\Phi_\sigma\left(\frac{F}{3Bt - 2t^2}\right) \leq \sigma^*$$

Multiply both sides by (3Bt - 2t ^{2}) and divide both sides by . Then use quadratic equation to solve for t:

$$t = \frac{-3B \pm \sqrt{9B^2 - \left(\frac{8\Phi_\sigma F}{\sigma^*}\right)}}{-4}$$

B: $Beam\ Height\ and\ Width$

Φ_σ: $Safety\ Factor\ for\ Yielding$

σ^*: $Yielding\ Strength$

F: $Force$

$t = minimum\ mass$

Deriving the equation used to find minimum mass:

$$m = \rho \times L \times A \quad (1)$$

$$A = 3Bt - 2t^2 \quad (2)$$

Combining (1) and (2) →

$$m = \rho \times L \times (3Bt - 2t^2) \ (3)$$

Equation (3) only accounts for the mass of one beam, so multiply equation (3) by 4 to get the total mass of the four beams found in the scissors car jack design:

$$m = (\rho \times L \times (3Bt - 2t^2)) \times 4$$

ρ: $density$

L: $Beam\ Length$

B: $Beam\ Height\ and\ Width$

t: $minimum\ thickness$

m: $minimum\ mass$

Matlab Code Used to get precise values for minimum mass:

```
clear
clc
% Minimum Mass
syms Density L A B t m_i;
Density = 8000;
L = 0.18;
t = 0.004886839616895739463116258842209 7;
B = 0.05;
A = 3*B*t - 2*t^2;
m_i = Density*L*A;

TotalMinimumMass= 4*m_i
```

Section 2.3) Table Results for the Minimum Mass and the Minimum Thicknesses (Allison Eiler and Denesse Gomez)

Buckling Criterion:

Half-Extension:

Material	Modulus of Elasticity(Pa)	Density (kg/m^3)	Force(N)	Minimum t (m)	Minimum mass (kg)
6061	69 x 10^9	2700	5668.5	0.000081302759739195838916600688960829	0.023682184520524
304	190 x 10^9	8000	5668.5	0.000029433926396039654531780700136904	0.025420931984792
7075	71.7 x 10^9	2810	5668.5	0.00007888997874023550350201161763 1838	0.023916347456730

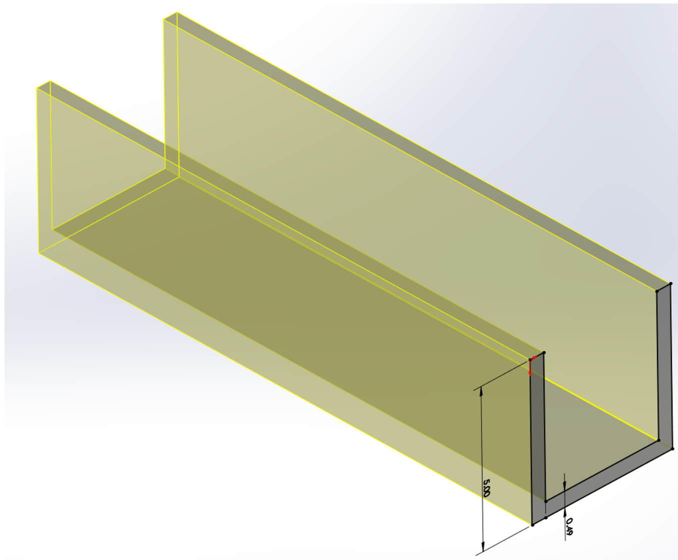
Yielding Criterion:

Half-Extension:

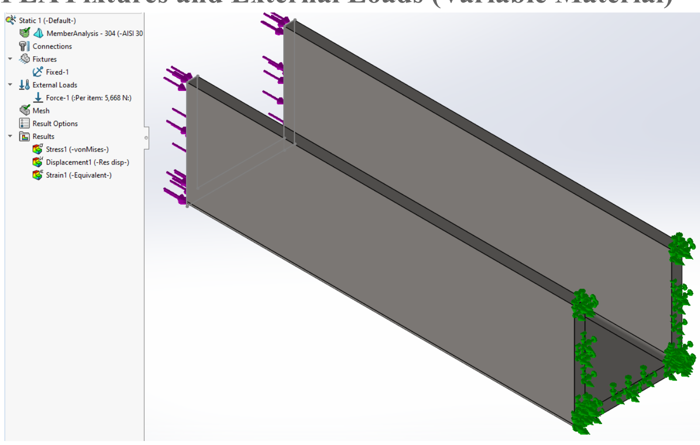
Material	Modulus of Elasticity(Pa)	Density (kg/m^3)	Force(N)	Minimum t (m)	Stress (N/m)	Minimum mass (kg)
6061	69 x 10^9	2700	5668.5	0.0035953623154333187256474461337743	1.1040*10^7	0.998148913043479
304	190 x 10^9	8000	5668.5	0.0048868396168957394631162588422097	8.2720*10^6	3.9471
7075	71.7 x 10^9	2810	5668.5	0.0019277818546926148521130836307237	2.0120*10^7	0.570005427435388

Section 3) CAD Models and FEA Results (Matthieu Lu)

1. Beam Model used for Stress Analysis
    a. **CAD Model (Variable thickness)**

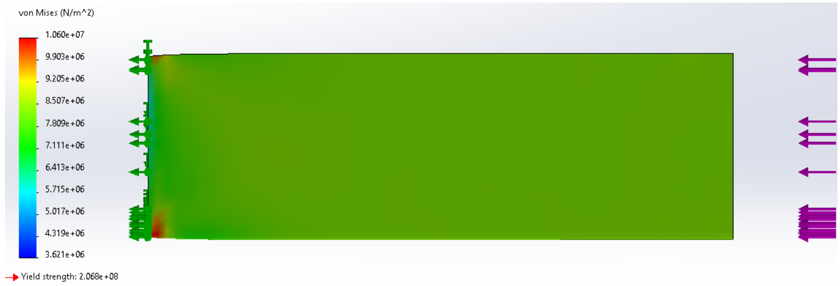


    b. **FEA Fixtures and External Loads (Variable Material)**



    c. FEA Results
        i. *6061-T6*

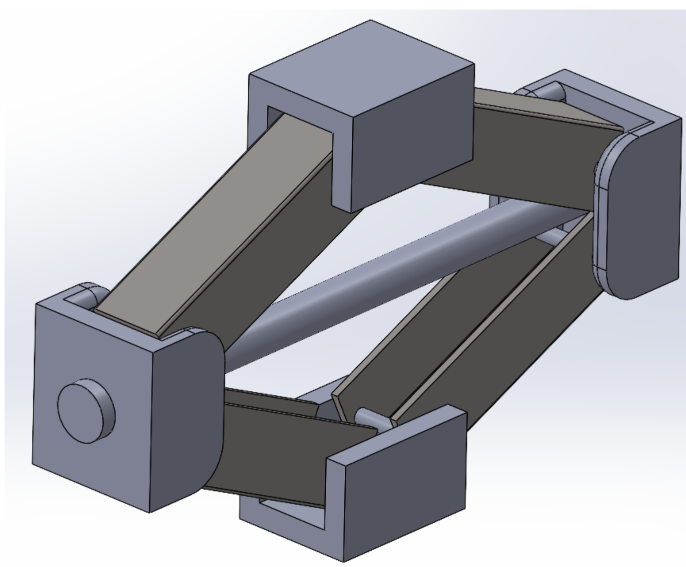
ii. *AISI 304*



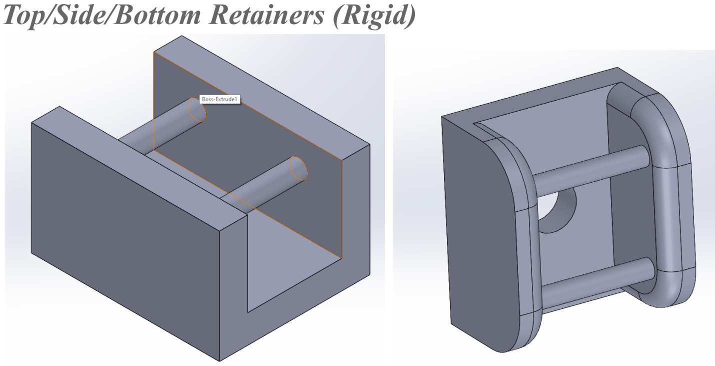
iii. *7075-T6 (SN)*

2. Jack Assembly

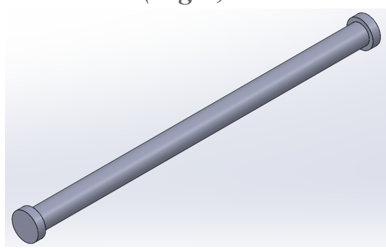
a. **CAD Model**



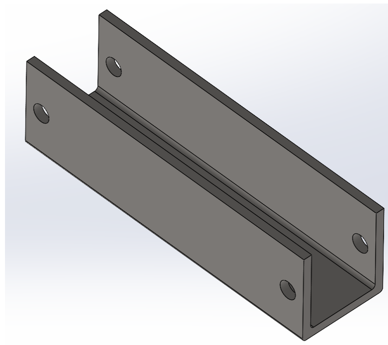
i. ***Top/Side/Bottom Retainers (Rigid)***



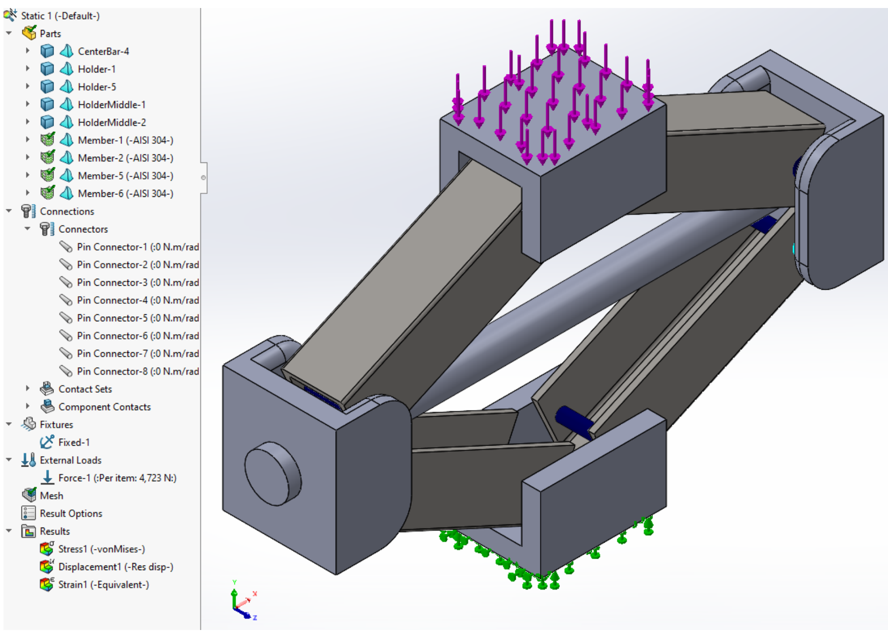
ii. ***Center Bar (Rigid, Variable Length affects height of Jack extension)***



iii. ***Beams (Variable Thickness/Material, cutouts for pin connections)***



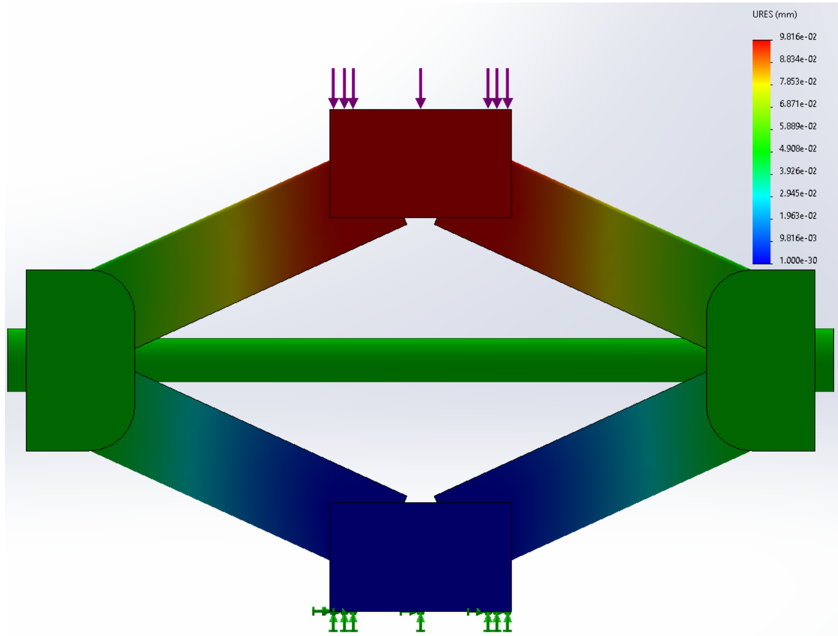
b. **FEA Fixtures and External Loads**



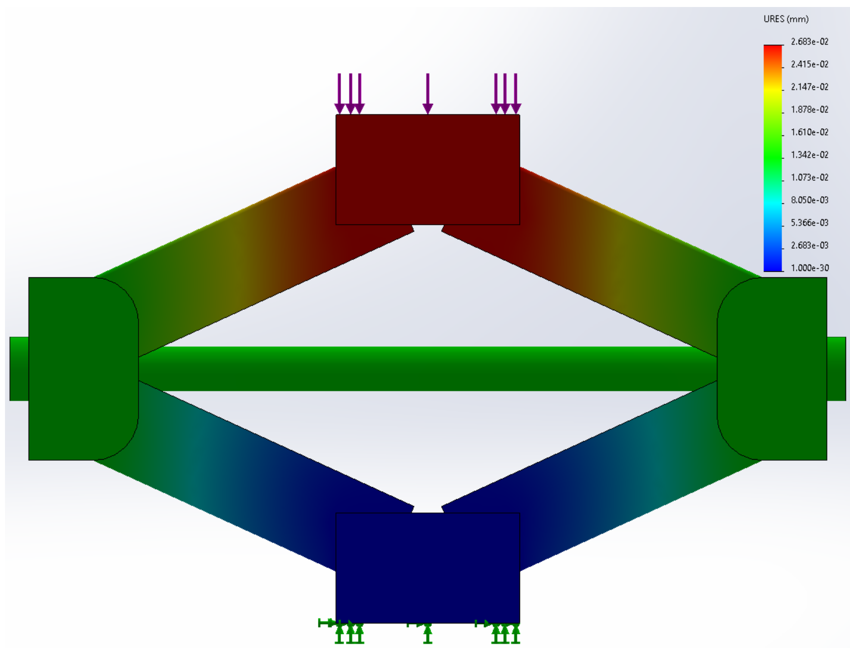
i. Dark blue highlighted members are pin connections.

c. FEA Results

i. 6061-T6



ii. AISI 304



iii. 7075-T6 (SN)