

# Efficient Renaming in Sequence CRDTs

Matthieu Nicolas, G  rald Oster and Olivier Perrin

**Abstract**—The abstract goes here.

**Index Terms**—CRDTs, real-time collaborative editing, eventual consistency, memory-wise optimisation, performance.

## 1 INTRODUCTION

THIS demo file is intended to serve as a “starter file” for IEEE Computer Society journal papers produced under L  T  X using IEEEtran.cls version 1.8b and later. I wish you the best of success.

## 2 BACKGROUND

To deterministically solve conflicts and ensure convergence of all nodes, Conflict-free Replicated Data Types (CRDTs) rely on metadata. In the context of Sequence CRDTs, two different approaches were proposed, both trying to minimise the overhead introduced. The first one [1], [2], [3] attaches fixed size identifiers to each element in the sequence and uses them to represent the sequence as a linked list. The downside of this approach is an ever growing overhead, as it needs to keep removed elements to deal with potential concurrent updates, effectively turning them into tombstones. The second one [4], [5], [6], [7], [8], [9] avoids the need of tombstones by attaching identifiers from a dense totally ordered set to elements. Elements are ordered into the sequence by comparing their respective identifiers. However this approach suffers from an ever-increasing overhead, as the size of such dense totally ordered identifiers is unbounded and grows over time. In the context of this paper, we focus on the later approach.

### 2.1 LogootSplit

LogootSplit (LS) [7] is the state of the art of the variable-size identifiers approach of Sequence CRDT. As explained previously, it uses identifiers from a dense totally ordered set to position elements into the replicated sequence.

To this end, LogootSplit generates identifiers made of a list of tuples to elements. These tuples have four components: 1) a *position*, which embodies the intended position of the element 2) a *node identifier*, 3) a *node sequence number* and 4) an *offset*, which are combined to make identifiers unique. By comparing identifiers using the lexicographical order, LogootSplit is able to determine the position of the element relatively to others. In this paper, we represent identifiers using the following notation:  $position_{offset}^{node\ id\ node\ seq}$  where *position* is a lowercase letter, *node id* an uppercase one and both *node seq* and *offset* integers.

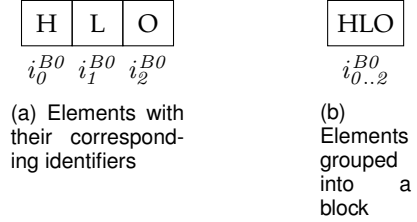


Fig. 1. Simulation results for the network.

Instead of storing an identifier for each element of the sequence, the main insight of LogootSplit is to aggregate dynamically elements into blocks. Grouping elements into blocks enables LogootSplit to assign logically an identifier to each element while effectively storing only the block length and the identifier of its first element. LogootSplit gathers elements with *contiguous* identifiers into a block. We call *contiguous* two identifiers that are identical except for their last offset, and with both offsets being consecutive. Figure 1 illustrates such a case: in 1a, the element identifiers form a chain of contiguous identifiers. LogootSplit is then able to group them into one block to minimise the metadata stored, as shown in 1b.

This feature allows to shift the root of metadata growth from the number of elements to the number of blocks. As blocks can contain an arbitrary number of elements, it enables to reduce significantly the memory overhead of the data structure.

### 2.2 Limits

As stated previously, the size of identifiers from a dense totally ordered set is variable. When nodes insert new elements between two others with the same *position* value, LogootSplit has no other option than to increase the size of the resulting identifiers. Figure 2 illustrates such cases. In this example, since node A inserts a new element between contiguous identifiers, LogootSplit is not able to generate a fitting identifier of the same size. To comply with the intended order, LogootSplit generates a new identifier by appending a new tuple to the identifier of the predecessor.

As a result, the size of identifiers tends to grow as the collaboration progresses. This growth impacts negatively performance of the data structure on several aspects. Since identifiers attached to values become longer, the memory overhead of the data structure increases accordingly. This

• The authors are with the Universit   de Lorraine, CNRS, Inria, LORIA, F-54500, Nancy, France.  
E-mail: {matthieu.nicolas, gerald.oster, olivier.perrin}@loria.fr.

Manuscript received ???; revised ???.

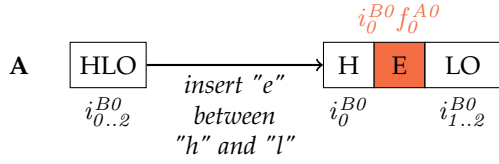


Fig. 2. Insertion leading to longer identifiers

also increases the bandwidth consumption as nodes have to broadcast identifiers to others.

Additionally, as the lifetime of the replicated sequence increases, the number of blocks composing it grows as well. Indeed, several constraints on identifier generation prevent nodes from adding new elements to existing blocks. For example, only the node that generated the block can append or prepend elements to it. These limitations cause the generation of new blocks. Since no mechanism to merge blocks a posteriori is provided, the sequence ends up fragmented into many blocks. The efficiency of the data structure decreases as each block introduces its own overhead.

In our benchmarks, we measure that the content eventually represents less than 1% of the data structure size, the remaining 99% being metadata. It is thus necessary to address the previously highlighted issues.

### 3 OVERVIEW

#### 3.1 Proposed approach

We propose a new Sequence CRDT belonging to the variable-size identifiers approach: *RenamableLogootSplit* (RLS) [10], [11].

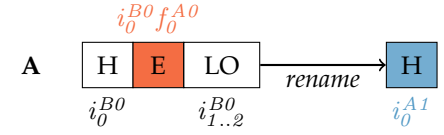
To address the limitations of LogootSplit, we embed in the data structure a renaming mechanism. The purpose of this mechanism is to reassign shorter identifiers to elements and to group them into blocks to minimise the memory overhead of the whole sequence.

To avoid costly and blocking consensus algorithms, we instead adopt the *optimistic replication* [12] approach for our mechanism. Nodes perform renaming without any coordination. However, this operation is not intrinsically commutative with others. If conflicts arise, we use *Operational Transformations* (OT) [13], [14] to enable nodes to resolve them deterministically.

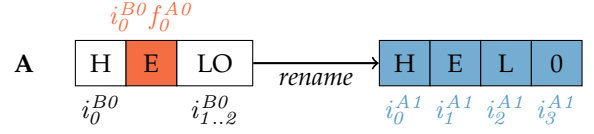
#### 3.2 System Model

The system is composed of a dynamic set of nodes, as nodes join and leave dynamically the collaboration during its lifetime. Nodes collaborate to build and maintain a sequence using *RenamableLogootSplit*. Each node owns a copy of the sequence and edit it without any coordination.

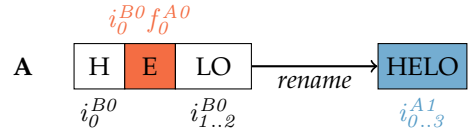
Nodes communicate through a Peer-to-Peer (P2P) network, which is unreliable. Messages can be lost, re-ordered or delivered multiple times. The network is also vulnerable to partitions, which split nodes into disjoint subgroups. To overcome the failures of the network, nodes rely on a message-passing layer. As *RenamableLogootSplit* is built on top of LogootSplit, it shares the same requirements for the operation delivery. This layer is thus used to deliver messages to the application exactly-once. The layer also ensures



(a) Selecting the new identifier of the first element



(b) Selecting the new identifiers of the remaining ones



(c) Final state obtained

Fig. 3. Renaming the sequence on node A

that *remove* operations are delivered after corresponding *insert* operations. Nodes use an anti-entropy mechanism [15] to synchronise in a pairwise manner, by detecting and re-exchanging lost operations.

*TODO? : Ajouter une sous-section présentant les propriétés que doit respecter l'opération rename (déterminisme, commutativité, respect de l'intention de l'utilisateur) – Matthieu*

## 4 RENAMABLELOGOOTSPILT IN CENTRALISED SETTINGS

### 4.1 rename operation

Our *rename* operation helps *RenamableLogootSplit* to reduce the overhead of nodes replica. This operation reassigns arbitrary identifiers to elements.

Its behaviour is illustrated in Figure 3. In this example, node A initiates a *rename* operation on its local state. First, node A reuses the id of the first element of the sequence ( $i_0^{B0}$ ) but modifies it with its own node id (A) and current sequence number (1). Also the offset is set to 0. Node A reassigns the resulting id ( $i_0^{A1}$ ) to the first element of the sequence as described in 3a. Then, node A derives contiguous identifiers for all remaining elements by successively incrementing the offset ( $i_1^{A1}$ ,  $i_2^{A1}$  and  $i_3^{A1}$ ), as shown in 3b. As we assign contiguous identifiers to all elements of the sequence, we eventually group them into one block as illustrated in 3c. It allows nodes to benefit the most from the block feature and to minimise the overhead of the resulting state.

To converge, other nodes have to rename their state identically. However, they can not simply replace their current state with the new renamed one. Indeed, they may have performed concurrent updates on their states. To not discard these updates, nodes have to process the *rename* operation themselves. To this end, the node issuing the *rename* operation broadcasts its former state to others. Using the former state, other nodes compute the new identifier of each renamed identifier. As for concurrently inserted

identifiers, we will explain in subsection 4.2 how nodes rename them in a deterministic way.

To limit bandwidth consumption of *rename* operations, we propose a compression technique to broadcast only necessary components to uniquely identify blocks instead of whole identifiers. It reduces the data to send to a fixed amount per block. Additionally, we can set an upper-bound to the size of *rename* operations by issuing them as soon as the state reaches a given number of blocks. *TODO? : Développer ce paragraphe en une sous-section de section 7 ? Je pourrais préciser les différents aspects du mécanisme de compression et de décompression (éléments permettant d'identifier un bloc de manière unique, algorithme de décompression, modèle de cohérence requis...) – Matthieu*

## 4.2 Dealing with concurrent updates

As *rename* operations can be issued without any kind of coordination, it is possible for other nodes to perform updates concurrently. Figure 4 illustrates such cases. In this example node B inserts the new element "L", assigns the id  $i_0^{B0} m_0^{B1}$  to it and broadcasts its update, concurrently to the *rename* operation described in Figure 3. Upon reception of the *insert* operation, node A adds the inserted element into its sequence, using the element id to determine its position. However, since identifiers were modified by the concurrent *rename* operation, node A inserts the new element at the end of its sequence (since  $i_3^{A1} < i_0^{B0} m_0^{B1}$ ) instead of at the intended position. As described by this example, applying naively concurrent updates would result in inconsistencies. It is thus necessary to handle concurrent operations to *rename* operations in a particular manner.

First, nodes have to detect concurrent operations to *rename* ones. To this end, we use an *epoch-based* system. Initially, the replicated sequence starts at the *origin* epoch noted  $\varepsilon_0$ . Each *rename* operation introduces a new epoch and enables nodes to advance their states to it from the previous epoch. The generated epoch is characterised using the node id and its current sequence number upon the generation of the *rename* operation. For example, the *rename* operation described in Figure 4 enables nodes to advance their states from  $\varepsilon_0$  to  $\varepsilon_{A1}$ .

As they receive *rename* operations, nodes build and maintain the *epoch chain*, a data structure ordering epochs according to their *parent-child* relation. Additionally, nodes tag every operation with their current epoch at the time the operation is generated. Upon the reception of an operation, nodes compare the operation epoch to their current one. If they differ, nodes have to transform the operation before applying it. Nodes determine against which *rename* operations to transform the received operation by computing the path between the operation epoch and their current one using the *epoch chain*. *TODO : Ajouter que les opérations doivent être livrées après l'opération rename qui introduit leur epoch – Matthieu*

Nodes use Figure 6 to transform *insert* or *remove* operations against *rename* ones. This algorithm maps identifiers from a given epoch to corresponding ones in the next epoch. The main idea of this algorithm is to use the predecessor of the given identifier to do so. An example of its usage is illustrated in Figure 5. This figure depicts the same scenario

as in Figure 4, except that this time node A uses Figure 6 to rename the concurrently generated id before inserting it in its state. The algorithm proceeds as follows. First, node A retrieves the predecessor of the given id  $i_0^{B0} m_0^{B1}$  in the former state:  $i_0^{B0} f_0^{A0}$ . Then it computes the counterpart of  $i_0^{B0} f_0^{A0}$  in the renamed state:  $i_1^{A1}$ . Finally, node A prepends it to the given id to generate the renamed id:  $i_1^{A1} i_0^{B0} m_0^{B1}$ . By reassigning this id to the concurrently added element, node A is able to insert it in its state while preserving the intended order.

As explained in subsection 4.1, some nodes may have applied concurrent *insert* operations to their state before receiving a given remote *rename* operation. Figure 6 also solves this case. It allows them to converge with nodes which processed the *rename* operations before the concurrent *insert* operations.

Since nodes rely on the former state to transform concurrent operations to a *rename* one, they have to store it. Nodes need it until each of them can no longer issue concurrent operations to the corresponding *rename* operation. In other words, nodes can safely garbage collect the former state once the *rename* operation became causally stable [16]. Meanwhile, nodes can offload it onto the disk as it is only required to handle concurrent operations.

*TODO? : Ajouter une sous-section sur la stratégie de renommage. À ce stade, me permettrait d'introduire le requirement de performance (temps d'intégration, mais aussi consommation en bande-passante ou même taille de la structure en mémoire). Permettrait aussi de faire le lien avec section 6. – Matthieu*

## 5 RENAMABLELOGOOTPLIT IN DISTRIBUTED SETTINGS

### 5.1 Concurrent *rename* operations

We now consider scenarios with concurrent *rename* operations. Figure 7 depicts such one. This figure expands the scenario previously described in Figure 5.

After broadcasting its *insert* operation, node B performs a *rename* operation on its state. This operation reassigns new identifiers to every element based on the id of the first element of the sequence ( $i_0^{B0}$ ), its node id (**B**) and current sequence number (2). This operation also introduces a new epoch :  $\varepsilon_{B2}$ . Since node A's *rename* operation was not yet delivered to node B at that moment, both *rename* operations are concurrent.

Node A and B then synchronise : each node applies the other one's *rename* operation. However, although they applied the same set of operations, their resulting states diverge. The identifiers attached to every element as well as the current epoch are now different. While the content of both sequences is still identical at that moment, the divergence will only widen. Subsequent operations will rely on these divergent identifiers to express at which position to insert new elements or which ones to remove. The same operations will thus produce different outputs on each copy.

The divergence depicted in Figure 7 occurs since the proposed *rename* operation is not commutative with itself. As concurrent operations may be delivered and applied in different orders to each nodes, conflicts may arise as a result. Nodes have to solve these conflicts to ensure the convergence of their state.

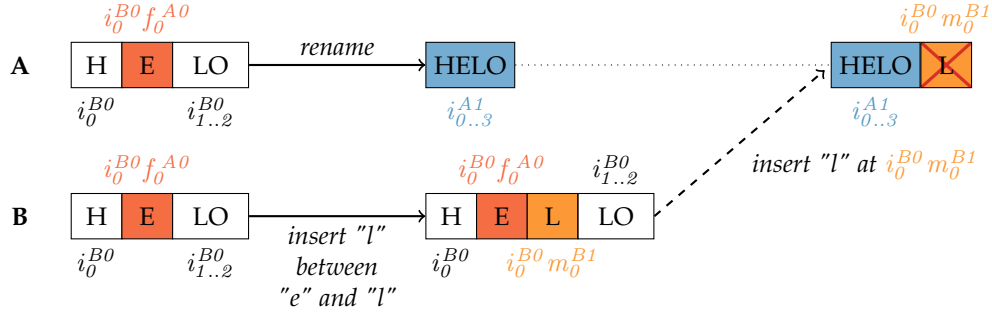


Fig. 4. Concurrent update leading to inconsistency

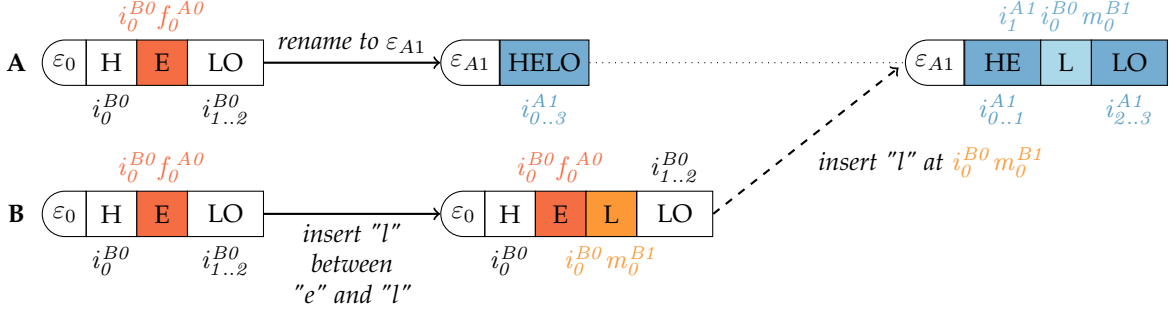


Fig. 5. Renaming concurrent update using Figure 6 before applying it to maintain intended order

```

function RENAMEID(id, renamedBlocks, nId, nSeq)
  length ← renamedBlocks.length
  firstId ← idBegin(renamedBlocks[0])
  lastId ← idEnd(renamedBlocks[length - 1])
  pos ← position(firstId)
  newFirstId ← new Id(pos, nId, nSeq, 0)
  newLastId ← new Id(pos, nId, nSeq, length - 1)
  if firstId < id and id < lastId then
    pred ← findPredecessor(id, renamedBlocks)
    indexOfPred ← findIndex(pred, renamedBlocks)
    newPred ← new Id(pos, nId, nSeq, indexOfPred)
    return concat(newPredecessor, id)
  else if lastId < id and id < newLastId then
    return concat(newLastId, id)
  else if newFirstId < id and id < firstId then
    predOfNewFirstId ← new Id(pos, nId, nSeq, -1)
    return concat(predOfNewFirstId, id)
  else
    return id ▷ Return the identifier unchanged as it
    does not conflict with the rename op
  end if
end function

```

Fig. 6. Rename concurrently generated identifier

To this end, we propose the following approach : nodes effectively apply only one *rename* operation from a set of concurrent *rename* operations. The insight behind this approach is that *rename* operations are system operations : they have no effect on the document content and they do not carry any user intention. Therefore nodes can ignore several *rename* operations from a set of concurrent *rename*

ones without disturbing the user experience. Thus, as long as every nodes apply one same *rename* operation from the set of concurrent *rename* ones, they will be able to benefit from its effects while avoiding conflicts.

We define the one *rename* operation that nodes apply from a set of concurrent *rename* operations the *primary* one. Other operations from the set are called *secondary* ones.

To make this approach feasible, we have to address two issues. First, nodes have to be able to select the *primary* *rename* operation to apply from a set of concurrent ones. In order to avoid performance issues due to coordination, nodes should select this operation in a coordination-free manner, i.e. solely using data from the set of concurrent *rename* operations. We propose such a mechanism in subsection 5.2. Second, nodes have to be able to revert the effect of applied *rename* operations. As nodes have only a partial knowledge of the system and of the issued operations, they may apply one *rename* operation at a moment only to receive afterward a concurrent *rename* operation which is deemed as the new *primary* one. We present a corresponding algorithm in subsection 5.3.

## 5.2 Breaking tie between concurrent *rename* operations

As stated in subsection 4.2, nodes build and maintain the *epoch chain*, a data structure ordering epochs according to their *parent-child* relation, as they receive *rename* operations. Using the *epoch chain*, nodes determine their current epoch as the latest one. However, in case of concurrent *rename* operations, this data structure is no longer suitable as several epochs may share the same parent epoch. Epochs now form the *epoch tree*. Several epochs may be considered as the latest

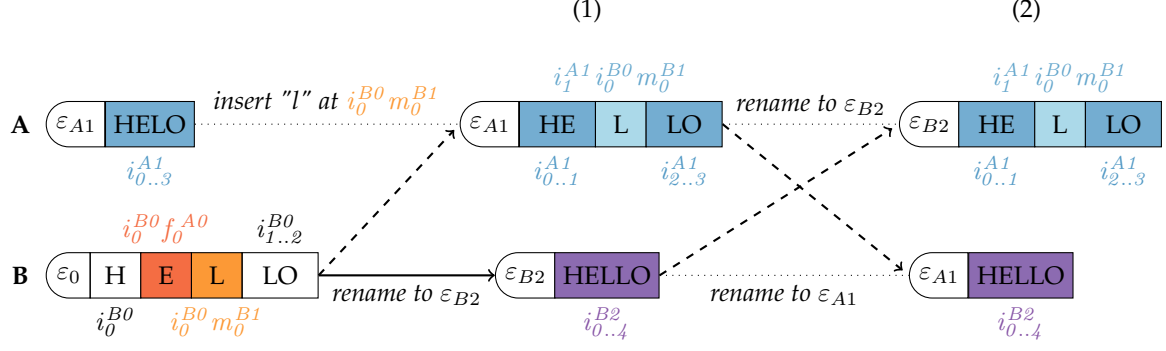


Fig. 7. Concurrent *rename* operations leading to divergent states

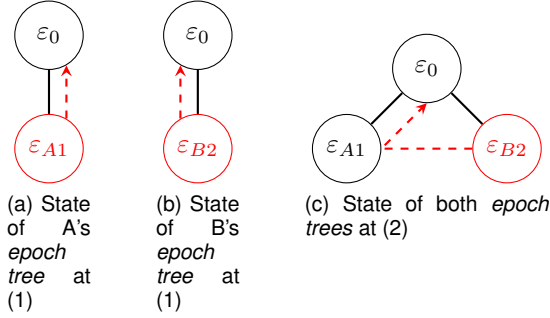


Fig. 8. Evolution of *epoch trees* during the scenario of Figure 7

ones. To converge, every node should eventually select the same epoch as the *primary* one.

To this end, we define *priority*, a total order relation between epochs. The goal of this relation is to enable nodes to designate the *primary rename* operation from a set of concurrent ones, but also the current epoch from the set of all issued *rename* operations, in a coordination-free manner.

To define the *priority* relation, we may actually select different strategies. In this work, we use the lexicographical order on the path of epochs in the *epoch tree*. Figure 8 provides an example of its use.

This figure displays the evolution of node A and B's *epoch trees* during the execution of Figure 7's scenario. Red dashed arrows represents the order between epochs according to the *priority* relation while current epochs are displayed as red nodes.

Initially, both nodes are only aware of their own *rename* operation. Node A's (resp. node B's) *epoch tree* is thus only composed of the epochs  $\varepsilon_0$  and  $\varepsilon_{A1}$  (resp.  $\varepsilon_0$  and  $\varepsilon_{B2}$ ). Using the *priority* relation, both nodes order epochs to determine the current one. According to the lexicographical order on the path of epochs in the *epoch tree*, node A (resp. node B) establishes that  $\varepsilon_0 < \varepsilon_0\varepsilon_{A1}$  (resp.  $\varepsilon_0 < \varepsilon_0\varepsilon_{B2}$ ). Therefore node A selects  $\varepsilon_{A1}$  as its current epoch while node B picks  $\varepsilon_{B2}$ .

Then, both nodes synchronise and their *epoch trees* converge. Using the *priority* relation, they are able to decide which *rename* operation to deem as the *primary* one in a coordination-free manner. Nodes establish that  $\varepsilon_0 < \varepsilon_0\varepsilon_{A1} < \varepsilon_0\varepsilon_{B2}$ . Therefore, they choose  $\varepsilon_{B2}$  as the new *primary* and current epoch. As  $\varepsilon_{B2}$  was already node B's current epoch, node B does not rename its state upon the reception of the

*rename* operation to  $\varepsilon_{A1}$ . On the other hand, node A has to *rename* its state from  $\varepsilon_{A1}$  to  $\varepsilon_{B2}$ . We explain how node A proceeds in subsection 5.3.

Other strategies could be proposed to define the *priority* relation. For example, *priority* could rely on metrics embedded in *rename* operations representing the accumulated work on the document. This topic will be further discussed in subsection 7.2.

### 5.3 Applying *primary rename* operations

As illustrated in subsection 5.2, nodes may have to *rename* their state from an epoch to a concurrent one. However, the proposed algorithm Figure 6 is only designed to *rename* to an epoch from its parent one. To address this issue, we propose that nodes having applied concurrent *secondary rename* operations first revert the effect of these concurrently applied *rename* operations.

We thus introduce Figure 9. The goals of Figure 9 are the following : (i) To revert identifiers generated causally before or concurrently to the reverted *rename* operation to their former value (ii) To assign new ids complying with the intended order to elements inserted causally after the reverted *rename* operation.

Upon the reception of a *rename* operation introducing a concurrent but *primary* epoch to their current one, nodes have first to determine which *rename* operations to revert. To this end, nodes use the *epoch tree*. By identifying the Lowest Common Ancestor (LCA) of their current epoch and of the new *primary* one, nodes can determine which operations to revert. Nodes have then to revert *rename* operations applied since the LCA epoch between their current epoch and the new *primary* one in the reverse order, using Figure 9. The Figure 10 illustrates such a scenario.

This figure describes the same scenario as Figure 7, except that nodes now use the *priority* to determine how to process the concurrent *rename* operation received.

Upon the reception of both *rename* operations, nodes compares introduced epochs to determine the *primary* one. According to the proposed *priority* relation, nodes deems  $\varepsilon_{B2}$  as *primary* one since  $\varepsilon_0 < \varepsilon_0\varepsilon_{A1} < \varepsilon_0\varepsilon_{B2}$ . Since node B's current epoch is already  $\varepsilon_{B2}$ , node B can simply ignore the received *rename* operation. Meanwhile, node A has to *rename* its state from  $\varepsilon_{A1}$  to  $\varepsilon_{B2}$ .

Node A first determines the LCA between  $\varepsilon_{A1}$  and  $\varepsilon_{B2}$  :  $\varepsilon_0$ . It then undoes each *rename* operations that separates its



```

function REVRENID(id, renamedIds, nId, nSeq)
  length  $\leftarrow$  renamedIds.length
  firstId  $\leftarrow$  renamedIds[0]
  lastId  $\leftarrow$  renamedIds[length - 1]
  pos  $\leftarrow$  getPosition(firstId)

  predOfNewFirstId  $\leftarrow$  new Id(pos, nId, nSeq, -1)
  newLastId  $\leftarrow$  new Id(pos, nId, nSeq, length - 1)

  if id < newFirstId then
    return revRenIdLessThanNewFirstId(id, firstId,
    newFirstId)
  else if isRenamedId(id, pos, nId, nSeq, length) then
    index  $\leftarrow$  getFirstOffset(id)
    return renamedIds[index]
  else if newLastId < id then
    return revRenIdGreaterThanNewLastId(id, lastId)
  else
    index  $\leftarrow$  getFirstOffset(id)
    return revRenIdFromPredId(id, renamedIds, index)
  end if
end function

function REVRENIDFROMPREDID(id, renamedIds, index)
  predId  $\leftarrow$  renamedIds[index]
  succId  $\leftarrow$  renamedIds[index + 1]
  tail  $\leftarrow$  getTail(id, 1)

  if tail < predId then
    return concat(predId, MIN_TUPLE, tail)
  else if succId < tail then
    offset  $\leftarrow$  getLastOffset(succId) - 1
    predOfSuccId  $\leftarrow$  createIdFromBase(succId, offset)
    return concat(predOfSuccId, MAX_TUPLE, tail)
  else
    return tail
  end if
end function

```

Fig. 9. Revert rename identifier

current epoch from  $\varepsilon_0$ , i.e. the *rename* operation from  $\varepsilon_0$  to  $\varepsilon_{A1}$ . To this end, node A uses Figure 9.

Figure 9 enables node A to retrieve fitting counterparts for every identifiers of its current state. For identifiers of the form  $i_{offset}^{A1}$ , it simply uses their offset to retrieve the original identifiers, as offsets correspond to the identifier indexes in *renamedIds*. For other identifiers such as  $i_1^{A1} i_0^{B0} m_0^{B1}$ , Figure 9 removes its prefix ( $i_1^{A1}$ ) to isolate its tail ( $i_0^{B0} m_0^{B1}$ ). The algorithm returns the tail if it fits between the identifier of its predecessor ( $i_0^{B0} f_0^{A0} < i_0^{B0} m_0^{B1}$ ) and the identifier of its successor ( $i_0^{B0} m_0^{B1} < i_1^{B0}$ ). If it would not, Figure 9 would use exclusive tuples of the renaming mechanism, *MIN\_TUPLE* and *MAX\_TUPLE*, to generate an identifier complying with the intended order.

Once node A reverted its state to the LCA epoch  $\varepsilon_0$  using Figure 9, it can successively apply *rename* operations leading to the new *primary* epoch  $\varepsilon_{B2}$  using Figure 6.

According to the *priority* relation used, nodes may have

to redo previously undo *rename* operations. In order to ensure the convergence in such cases, Figure 6 and Figure 9 must be inverse functions. We thus have to propose a new version of Figure 6, in particular to rename correctly ids assigned to elements inserted causally after the *rename* operation. This new version is presented in Figure 14.

#### 5.4 Garbage collection of former states

As explained in subsection 4.2 and subsection 5.3, nodes have to store epochs and corresponding *former states* to transform operations from previous or concurrent epochs to the current one, or to rename their state from their current epoch to the new *primary* one. However, once nodes become aware that some epochs can not possibly be required anymore to apply future operations, they can garbage collect these epochs and their corresponding *rename* operations. But, in systems which allow the generation of concurrent *rename* operations, nodes can not solely rely on causal stability of *rename* operations to determine which operations can be garbage collected. Therefore, we propose the two following rules to enable nodes to identify unnecessary epochs :

**Rule 1.** An epoch  $\varepsilon$  can be garbage collected if  $\varepsilon$  is a leaf of the epoch tree and a concurrent primary epoch  $\varepsilon'$  is causally stable.

**Rule 2.** An epoch  $\varepsilon$  can be garbage collected if  $\varepsilon$  is the root of the epoch tree, has only one child  $\varepsilon'$  and that  $\varepsilon'$  is causally stable.

Figure 11 illustrates a use case of Rule 1 and Rule 2. In 11a, we represent an execution in which two nodes A and B respectively issue two *rename* operation before eventually synchronising. The operations shown in 11a are solely *rename* operations, as only these operations are relevant to the problem at hand. In 11b, we represent the states of their respective *epoch trees* once they each generate their *rename* operations. In 11c, we represent the new states of their *epoch trees* once they each receive the first *rename* operation issued by each other. Epochs introduced by causally stable *rename* operations are represented in *epoch trees* using double circles. The *origin* epoch  $\varepsilon_0$  is considered as causally stable from the beginning by design. Epochs that are deemed no longer necessary by Rule 1 and Rule 2 are respectively displayed as blue dashed nodes and green dotted nodes.

Upon the delivery of the *rename* operation introducing epoch  $\varepsilon_{B2}$  to node A,  $\varepsilon_{B2}$  becomes causally stable. From this point, node A knows that every node switched to this epoch at least. Therefore nodes can no longer issue operations from  $\varepsilon_0$ ,  $\varepsilon_{A1}$  or  $\varepsilon_{A8}$ . Thus these epochs and the *rename* operations enabling nodes to switch between them can now be garbage collected. Rule 1 enables node A to garbage collect epochs  $\varepsilon_{A8}$  then  $\varepsilon_{A1}$ . Then Rule 2 enables node A to garbage collect  $\varepsilon_0$  and the *renaming* operation to switch to  $\varepsilon_{B2}$ .

On the other hand, upon the reception of the *rename* operation introducing  $\varepsilon_{A1}$ , node B can not garbage collect any epochs despite  $\varepsilon_{A1}$  being causally stable. Indeed, from its point of view, other nodes may still issue operations from epoch  $\varepsilon_{A1}$ . Since in that case node B would have to transform operations to apply them to  $\varepsilon_{B8}$ , node B has to retain all epochs forming the path between  $\varepsilon_{A1}$  and  $\varepsilon_{B8}$  and their corresponding *rename* operations.

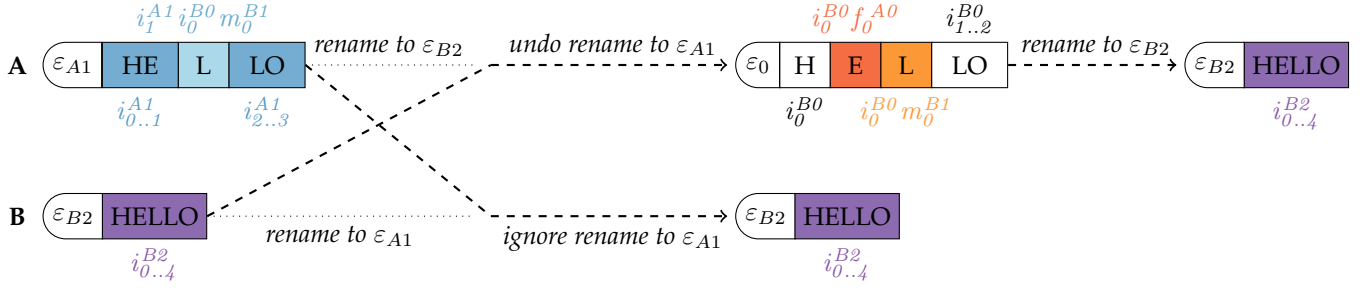


Fig. 10. Handling concurrent *rename* operations

Eventually, once the system becomes idle, the current *primary* epoch will become causally stable. Nodes will then be able to garbage collect all other epochs using Rule 1 and Rule 2, effectively suppressing the overhead of the renaming mechanism.

## 6 EVALUATION

### 6.1 Simulations and benchmarks

In order to validate the proposed approach, we proceed to an experimental evaluation. The aims of this evaluation are to measure (i) the memory overhead of the replicated sequence (ii) the computational overhead added to *insert* and *remove* operations by the renaming mechanism (iii) the cost of integrating *rename* operations.

Unfortunately, we were not able to retrieve an existing dataset of real-time collaborative editing sessions. We thus setup simulations to generate the dataset used to run our benchmarks. These simulations mimic the following scenario.

Several authors collaboratively write an article in real-time. First of all, the authors mainly specify the content of the article. Few *remove* operations are issued in order to simulate spelling mistakes. Once the document reaches an arbitrary given critical length, collaborators move on to the second phase of the simulation. During this phase, authors stop adding new content but instead focus on revamping existing parts. This is simulated by balancing the ratio between *insert* and *remove* operations. Every author has to issue a given number of *insert* and *remove* operations. The simulation ends once every collaborators received all operations. During the simulation, we take snapshots of the replicas' state at given steps to follow their evolution.

We ran simulations with the following experimental settings: we deployed 10 bots as separate Docker containers on a single workstation. Each container corresponds to a single mono-threaded Node.js process simulating an author. Bots share and edit collaboratively the document using either LogootSplit or RenamableLogootSplit according to the session. In both cases, each bot performs an *insert* or a *remove* operation locally every  $200 \pm 50$ ms and broadcasts it immediately to other nodes using a P2P full mesh network. During the first phase, the probability of issuing *insert* (resp. *remove*) operations is of 80% (resp. 20%). Once the document reaches 60k characters (around 15 pages), bots switch to the second phase and set both probabilities to 50%. After each local operation, the bot may move its cursor to another

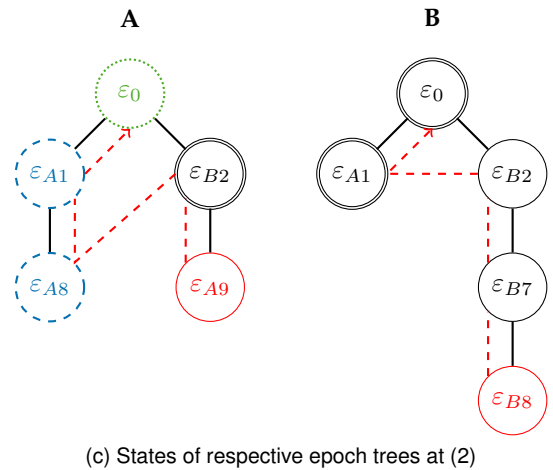
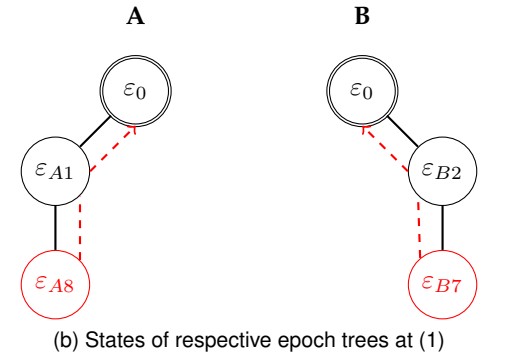
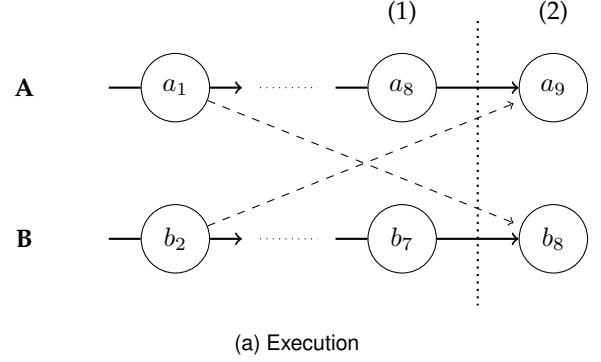


Fig. 11. Garbage collecting epochs and corresponding *former states*

random position in the document with a probability of 5%. Every bot generates 15k *insert* or *remove* operations and stops once it observed 150k operations. Snapshots of the state of bot are taken periodically every 10k observed

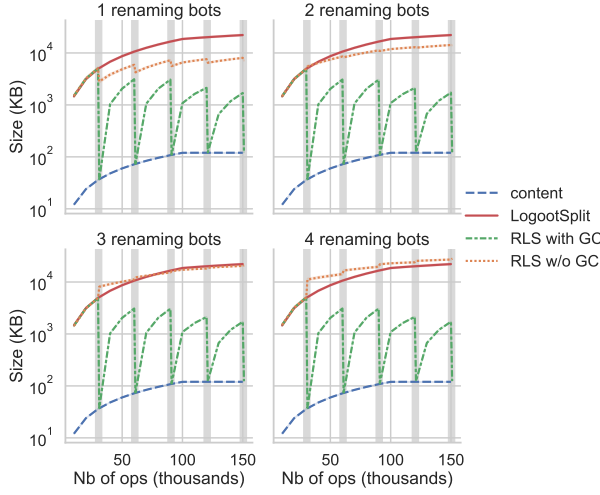


Fig. 12. Evolution of the size of the document

operations.

Additionally, in the case of RenamableLogootSplit, 1 to 4 bots are arbitrarily designated as *renaming bots* according to the session. *Renaming bots* issue *rename* operations every time they observe 30k operations overall. These *rename* operations are generated in a way ensuring that they are concurrent.

Code, benchmarks and results are available at: <https://github.com/coast-team/mute-bot-random/>.

## 6.2 Results

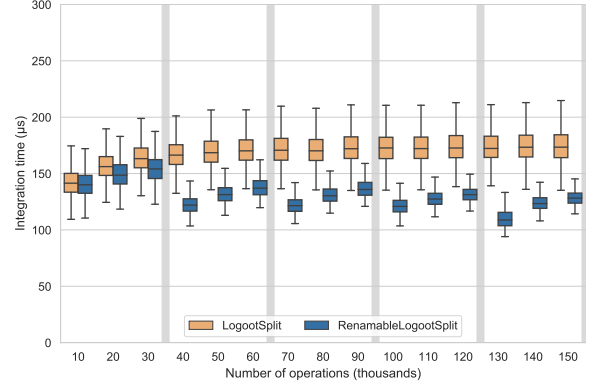
**6.2.0.1 Convergence:** We first proceeded to verify the convergence of nodes states at the end of simulations. For each simulation, we compared the final state of every nodes using their respective snapshots. We were able to confirm that nodes converged without any communication other than operations, thus satisfying the Strong Eventual Consistency (SEC) consistency model.

This result sets a first milestone in the validation of the correctness of RenamableLogootSplit. It is however only empirical. Further work to formally prove its correctness should be undertaken.

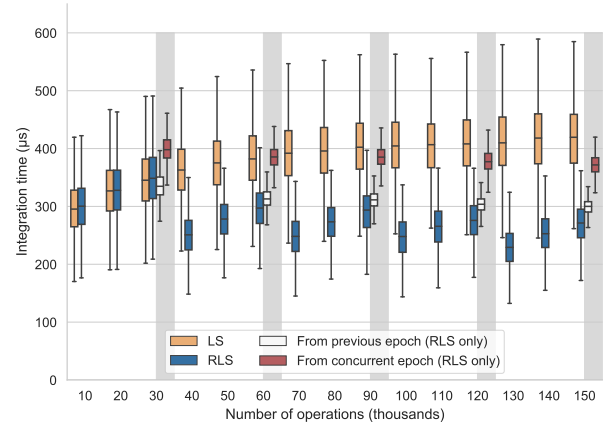
**6.2.0.2 Memory overhead:** We then proceeded to measure the evolution of the document’s memory consumption throughout the simulations, according to the CRDT used and the number of *renaming bots*. We present the obtained results in Figure 12.

For each plot displayed in Figure 12, we represent 4 different data. The blue dashed line illustrates the size of the actual content of the document, i.e. the text, while the red solid line corresponds to the size of the whole LogootSplit document.

The green dashed-dotted line represents the size of the RenamableLogootSplit document in the best case scenario. In this scenario, nodes assume that *rename* operations are garbage-collectable as soon as they receive them. Nodes are thus able to benefit the effects of the renaming mechanism while removing its own metadata, such as *former states* and *epochs*. In doing so, nodes are able to minimise periodically the metadata overhead of the data structure, independently



(a) Local operations



(b) Remote operations

Fig. 13. Integration time of standard operations

of the number of *renaming bots* and concurrent *rename* operations issued.

On the other hand, the orange dotted line represents the size of the RenamableLogootSplit document in the worst case scenario. In this scenario, nodes assume that *rename* operations never become causally stable and can thus never be garbage-collected. Nodes have to permanently store the metadata introduced by the renaming mechanism. The performances of RenamableLogootSplit thus decrease as the number of *renaming bots* and *rename* operations issued increases. Nonetheless, we observe that RenamableLogootSplit can outperform LogootSplit even in this worst case scenario while the number of *renaming bots* remains low (1 or 2). This result is explained by the fact that the renaming mechanism enables nodes to scrap the overhead of the internal data structure used to represent the document.

To summarise the results presented, the renaming mechanism introduces a temporary metadata overhead which increases with each *rename* operations. But the overhead will eventually subside once the system becomes quiescent and *rename* operations become causally stable. In subsection 7.1, we discuss that *former states* may be offloaded until causal stability is achieved to address the temporary memory overhead.

**6.2.0.3 Integration times of standard operations:** Next, we compared the evolution of integration times of respectively local and remote operations on LogootSplit and



RenamableLogootSplit documents. Figure 13 displays the results.

In these figures, orange boxplots correspond to integration times on LogootSplit documents while blue ones correspond to times on RenamableLogootSplit documents. While both are initially equivalent, integration times on RenamableLogootSplit documents are then reduced when compared to LogootSplit ones once *rename* operations have been applied. This improvement is explained by the fact that *rename* operations optimise the internal representation of the sequence.

Additionally, in the case of remote operations, we measured specific integration times for RenamableLogootSplit : integration times of remote operations from previous epochs and from concurrent epochs, respectively displayed as white and red boxplots in 13b. Operations from previous epochs are operations generated concurrently to the *rename* operation but applied after it. Since the operation has to be transformed beforehand using Figure 6, we observe a computational overhead compared to other operations. But this overhead is actually compensated by the optimisation of the internal representation of the sequence performed by *rename* operations.

Regarding operations from concurrent epochs, we observe an additional overhead as nodes have first to reverse the effect of the concurrent *rename* operation using Figure 9. Because of this overhead, RenamableLogootSplit’s performances for these operations are comparable to LogootSplit ones.

To summarise, transformation functions introduce an overhead with regard to integration times of concurrent operations to *rename* ones. Despite this overhead, RenamableLogootSplit achieves better performances than LogootSplit as long as the distance between the epoch of generation of the operation and the current epoch of the node remains limited. As the distance between both epochs increases, it leads to cases presenting worse performances than LogootSplit ones since the overhead is multiplied. Nonetheless, the renaming mechanism reduces the integration times of the majority of operations, i.e. the operations issued between two rounds of *rename* operations.

**6.2.0.4 Integration time of *rename* operation:** Finally, we measured the evolution of integration times of *rename* operation according to the number of operations since the last *rename* one. The results are displayed in Table 1.

The main outcome of these measures shows that *rename* operations are expensive when compared to others. Local *rename* operations take hundreds of milliseconds while remote ones and concurrent *primary* ones may last seconds if delayed for too long. It is thus necessary to take this result into account when designing strategies to trigger *rename* operations to prevent them from impacting negatively user experiences.

Another interesting result from this benchmark is that concurrent *secondary rename* operations are cheap to apply, as they only consist in storage of corresponding *former states*. Thus nodes can significantly reduce the overall computations of a set of concurrent *rename* operations by applying them in a specific order. We will discuss further this topic in subsection 7.3.

TABLE 1  
Integration time of rename operations

Parameters		Integration Time (ms)			
Type	Nb Ops (k)	Mean	Median	99 <sup>th</sup> Quant.	Std
Local	30	41.75	38.74	71.68	6.84
	60	78.32	78.16	81.42	1.24
	90	119.19	118.87	124.22	2.49
	120	143.75	143.57	148.59	2.16
	150	158.04	157.95	164.38	2.49
Remote	30	481.32	477.13	537.30	17.11
	60	981.62	978.24	1072.83	31.54
	90	1491.28	1481.83	1657.58	51.10
	120	1670.00	1663.85	1814.38	50.29
	150	1694.17	1675.95	1852.55	59.94
Prim. Remote	30	643.53	643.57	682.80	13.42
	60	1317.66	1316.39	1399.55	28.67
	90	1998.23	1994.08	2111.98	45.37
	120	2239.71	2233.22	2368.45	50.06
	150	2241.92	2233.61	2351.02	52.20
Sec. Remote	30	1.36	1.30	3.53	0.37
	60	2.82	2.69	4.85	0.45
	90	4.45	4.23	5.81	0.71
	120	5.33	5.10	8.78	0.90
	150	5.53	5.26	8.70	0.79

## 7 DISCUSSION

### 7.1 Offloading on disk unused *former states*

As explained in subsection 5.3, nodes have to store *former states* corresponding to *rename* operations to transform operations from previous or concurrent epochs. Nodes may receive such operations given 2 different cases : (i) nodes have recently issued *rename* operations (ii) nodes logged back in the collaboration. Between these specific events, *former states* are actually not needed to handle operations.

We can thus propose the following trade-off : to offload *former states* on the disk until their next use or until they can be garbage collected. It would enable nodes to mitigate the temporary memory overhead introduced by the renaming mechanism but increases integration times of operations requiring one of these *former states*. Nodes could adopt various strategies to deem *former states* offloadable and to retrieve them preemptively according to their constraints. The design of these strategies could be based on several heuristics : epochs of currently online nodes, number of nodes still able to issue concurrent operations, time elapsed since last use of the *former state*...

### 7.2 Designing a more effective *priority* relation

Although the *priority* relation proposed in subsection 5.2 is simple and ensures that nodes designate the same epoch as the *primary* one, it introduces a significant computational overhead in some cases. Notably it allows a single node, disjointed from the collaboration since a long time, to force every other nodes to revert *rename* operations they performed meanwhile because its own *rename* operation is deemed as the *primary* one.

The *priority* relation should thus be designed to ensure convergence, but also to minimise the overall amount of computations performed by nodes of the system. In order to design the *priority* relation, we could embed into *rename* operations metrics that represent the state of the system and the accumulated work on the document (number of

nodes currently at the *parent* epoch, number of operations generated at the *parent epoch*, length of the document...). This way, we can favour the branch from the *epoch tree* with the more and most active collaborators and prevent isolated nodes from overthrowing the existing order.

### 7.3 Postponing transition to *primary* epoch in case of high concurrency

As shown in Table 1, integrating *primary rename* operations is expensive as nodes have to browse and rename their whole current state. This process can introduce a signification computational overhead in some cases. Especially, a node may receive concurrent *rename* operations in the reverse order to the one defined by the *priority* relation. In this scenario, the node would successively consider every *rename* operation as the *primary* one and would rename its state each time. On the other hand *secondary rename* operations are cheap to integrate, as nodes simply add to their state a reference to the corresponding *former state*.

To mitigate the negative impact of this scenario, we can decompose the integration of *rename* operations into two parts in case of concurrency detection. First, nodes process *rename* operations as secondary ones. It enables nodes to integrate remote *insert* and *remove* operations, even from concurrent epochs, by transforming them. Then each node keeps track of a level of confidence in the current *primary* operation, computed for example from the time elapsed since the node received a concurrent *rename* operation and the number of online nodes still known as using the *parent* epoch. Once the level of confidence reaches a given threshold, the node renames its state according to the operation.

This strategy introduces a slight computational overhead for each *insert* or *remove* operations received during the period of uncertainty, as nodes may issue operations from different epochs during that time. In return, the strategy reduces the probability of erroneously integrating *rename* operations as *primary* ones.

## 8 RELATED WORK

Several works were proposed to address our problem of growth of identifiers in variable-size identifiers Sequence CRDTs. We present in this section the most relevant ones.

### 8.1 The core-nebula approach

The *core-nebula* approach [17], [18] was proposed to reduce the size of identifiers in Treedoc [4], another variable-size identifiers Sequence CRDT.

In this work, authors introduce a *rebalance* operation enabling nodes to reassign shorter identifiers to elements of the document. However, this *rebalance* operation is not commutative with *insert* and *remove* operations nor with itself. To achieve Eventual Consistency (EC) [19], the *core-nebula* approach prevents concurrent *rebalance* operations by regulating them using a consensus protocol. Operations such as *insert* and *remove* can still be issued without coordination and can thus be concurrent to *rebalance* ones. To deal with this issue, authors propose a *catch-up* protocol to transform these concurrent operations against the effects of *rebalance* ones.

Since consensus protocols do not scale well, the *core-nebula* approach proposes to split nodes among two groups : the *core* and the *nebula*. The *core* is a small set of stable and highly connected nodes while the *nebula* is an unbounded set of dynamic nodes. Only nodes from the *core* participate in the execution of the consensus protocol. Nodes from the *nebula* can still contribute to the document by issuing *insert* and *remove* operations.

Our work can be seen as an extension of this work. It adapts the *rebalance* mechanism and the *catch-up* protocol to LogootSplit and takes advantage of its block feature. Furthermore, it integrates a mechanism to deal with concurrent *rename* operations, hence removing the requirement of a consensus protocol. It makes this approach usable in systems without existing authorities providing nodes to the *core*.

However, systems can actually adopt the *core-nebula* approach to simplify the implementation of Renamable-LogootSplit. The use of a consensus protocol to regulate *rename* operations enables systems to discard all parts dedicated to the handling of concurrent *rename* operations, i.e. the design of a *priority* relation and the implementation of Figure 9 and Rule 1.

### 8.2 The LSEQ approach

The LSEQ approach [8], [9] is another approach proposed to address the growth of identifiers in variable-size identifiers Sequence CRDT. Instead of reducing periodically the identifier metadata using an expensive renaming mechanism, the authors define new identifier allocation strategies to reduce their growth rate.

In this work, authors observe that the identifier allocation strategy proposed in Logoot [5] is suited to a single editing pattern : from left to right, top to bottom. If insertions are made according to other patterns, generated identifiers quickly saturate the space of possible identifiers for a given size. Following insertions therefore trigger an increase of the identifier size. As a result, Logoot identifiers grow linearly with the number of insertions instead of the expected logarithmic progression.

LSEQ thus defines several identifier allocation strategy fitted to different editing pattern. Nodes pick randomly one of these strategies for each identifier size. Additionally LSEQ adopts an exponential tree model for identifiers : the range of possible identifiers doubles as the identifier size increases. *TODO: Voir comment mieux formuler ce passage sur le nombre de bits alloués à position qui double à chaque "profondeur" d'identifiant. – Matthieu* It enables LSEQ to fine-tune the size of identifiers according to needs. By combining the different allocation strategies to the exponential tree model, LSEQ achieves a polylogarithmic growth of identifiers according to the number of insertions.

While the LSEQ approach reduces the growth rate of identifiers in variable-size identifier Sequence CRDT, the sequence's overhead is still proportional to its number of elements. On the other hand, RenamableLogootSplit's renaming mechanism enables to reduce metadata to a fixed amount, independently of the number of elements.

These two approaches are actually orthogonal and can, as in the previous approach, be combined. The resulting system would reset the sequence's metadata periodically using

rename operations while LSEQ's identifier allocation strategies would reduce their growth in-between. This would also enable to reduce the frequency of rename operations, decreasing the system computations overall.

## 9 CONCLUSIONS AND FUTURE WORK

The conclusion goes here.

## APPENDIX A

### ALGORITHMS FOR CENTRALISED SETTINGS

*TODO: Décomposer et mettre en annexe l'algo de Figure 6 – Matthieu*

## APPENDIX B

### ALGORITHMS FOR DISTRIBUTED SETTINGS

### ACKNOWLEDGMENTS

The authors would like to thank...

## REFERENCES

- [1] G. Oster, P. Urso, P. Molli, and A. Imine, "Data Consistency for P2P Collaborative Editing," in *ACM Conference on Computer-Supported Cooperative Work - CSCW 2006*, ser. Proceedings of the 2006 20th anniversary conference on Computer supported cooperative work. Banff, Alberta, Canada: ACM Press, Nov. 2006, pp. 259 – 268. [Online]. Available: <https://hal.inria.fr/inria-00108523>
- [2] H.-G. Roh, M. Jeon, J.-S. Kim, and J. Lee, "Replicated abstract data types: Building blocks for collaborative applications," *Journal of Parallel and Distributed Computing*, vol. 71, no. 3, pp. 354 – 368, 2011. [Online]. Available: <http://www.sciencedirect.com/science/article/pii/S0743731510002716>
- [3] L. Briot, P. Urso, and M. Shapiro, "High Responsiveness for Group Editing CRDTs," in *ACM International Conference on Supporting Group Work*, Sanibel Island, FL, United States, Nov. 2016. [Online]. Available: <https://hal.inria.fr/hal-01343941>
- [4] N. Preguica, J. M. Marques, M. Shapiro, and M. Letia, "A commutative replicated data type for cooperative editing," in *2009 29th IEEE International Conference on Distributed Computing Systems*, June 2009, pp. 395–403.
- [5] S. Weiss, P. Urso, and P. Molli, "Logoot : A scalable optimistic replication algorithm for collaborative editing on P2P networks," in *Proceedings of the 29th International Conference on Distributed Computing Systems - ICDCS 2009*. Montreal, QC, Canada: IEEE Computer Society, Jun. 2009, pp. 404–412. [Online]. Available: <http://doi.ieeecomputersociety.org/10.1109/ICDCS.2009.75>
- [6] —, "Logoot-Undo: Distributed Collaborative Editing System on P2P Networks," *IEEE Transactions on Parallel and Distributed Systems*, vol. 21, no. 8, pp. 1162–1174, Aug. 2010. [Online]. Available: <https://hal.archives-ouvertes.fr/hal-00450416>
- [7] L. André, S. Martin, G. Oster, and C.-L. Ignat, "Supporting adaptable granularity of changes for massive-scale collaborative editing," in *International Conference on Collaborative Computing: Networking, Applications and Worksharing - CollaborateCom 2013*. Austin, TX, USA: IEEE Computer Society, Oct. 2013, pp. 50–59.
- [8] B. Nédelec, P. Molli, A. Mostéfaoui, and E. Desmontils, "LSEQ: an adaptive structure for sequences in distributed collaborative editing," in *Proceedings of the 2013 ACM Symposium on Document Engineering*, ser. DocEng 2013, Sep. 2013, pp. 37–46.
- [9] B. Nédelec, P. Molli, and A. Mostéfaoui, "A scalable sequence encoding for collaborative editing," *Concurrency and Computation: Practice and Experience*, p. e4108. [Online]. Available: <https://onlinelibrary.wiley.com/doi/abs/10.1002/cpe.4108>
- [10] M. Nicolas, "Efficient renaming in CRDTs," in *Middleware 2018 - 19th ACM/IFIP International Middleware Conference (Doctoral Symposium)*, Rennes, France, Dec. 2018. [Online]. Available: <https://hal.inria.fr/hal-01932552>

```

function RENIDFROMPREDID(id, renamedIds, nId, nSeq)
    index ← findIndexOfPred(id, renamedIds)
    predId ← renamedIds[index]
    newPredId ← new Id(pos, nId, nSeq, index)

    if predId.length + 1 < id.length then
        prefix ← concat(predId, MIN_TUPLE)
        tail ← getTail(id, prefix.length)
        if isPrefix(prefix, id) and tail < predId then
            return concat(newPredId, tail)
        end if
    end if

    succId ← renamedIds[index + 1]
    if succId.length + 1 < id.length then
        offset ← getLastOffset(succId) - 1
        predOfSuccId ← createIdFromBase(succId, offset)
        prefix ← concat(predOfSuccId, MAX_TUPLE)
        tail ← getTail(id, prefix.length)
        if isPrefix(prefix, id) and succId < tail then
            return concat(newPredId, tail)
        end if
    end if

    return concat(newPredId, id)
end function

function RENIDFROMINDEX(pos, nId, nSeq, index)
    return new Id(pos, nId, nSeq, index)
end function

function RENIDLESSTHANFIRSTID(id, firstId, newFirstId)
    offset ← getLastOffset(firstId) - 1
    predOfFirstId ← createIdFromBase(firstId, offset)
    prefix ← concat(predOfFirstId, MAX_TUPLE)
    predNewFirstId ← createIdFromBase(newFirstId, -1)
    if isPrefix(prefix, id) then
        tail ← getTail(id, prefix.length)
        return concat(predNewFirstId, tail)
    else if id < newFirstId then
        return id
    else
        return concat(predNewFirstId, id)
    end if
end function

function RENIDGREATERTHANLASTID(id, lastId, newLastId)
    prefix ← concat(lastId, MIN_TUPLE)
    if isPrefix(prefix, id) then
        tail ← getTail(id, prefix.length)
        return tail
    else if newLastId < id then
        return id
    else
        return concat(newLastId, id)
    end if
end function

```

Fig. 14. Updated algorithms to rename identifier

```

function REVRENIDLESSTHANNEWFIRSTID(id, firstId,
newFirstId)
    predNewFirstId ← createIdFromBase(newFirstId, -1)
    if predNewFirstId < id then
        tail ← getTail(id, 1)
        if tail < firstId then
            return tail
        else
            offset ← getLastOffset(firstId)
            predFirstId ← createIdFromBase(firstId, offset)
            return concat(predFirstId, MAX_TUPLE, tail)
        end if
    else
        return id
    end if
end function

function REVRENIDGREATERTHANNEWLASTID(id,
lastId)
    if id < lastId then
        return concat(lastId, MIN_TUPLE, id)
    else
        return id
    end if
end function

```



**Michael Shell** Biography text here.

Fig. 15. Remaining functions to revert identifier renaming

**John Doe** Biography text here.

- [11] M. Nicolas, G. Oster, and O. Perrin, "Efficient Renaming in Sequence CRDTs," in *7th Workshop on Principles and Practice of Consistency for Distributed Data (PaPoC'20)*, Heraklion, Greece, Apr. 2020. [Online]. Available: <https://hal.inria.fr/hal-02526724>
- [12] Y. Saito and M. Shapiro, "Optimistic replication," *ACM Comput. Surv.*, vol. 37, no. 1, p. 42–81, Mar. 2005. [Online]. Available: <https://doi.org/10.1145/1057977.1057980>
- [13] C. Sun and C. Ellis, "Operational transformation in real-time group editors: Issues, algorithms, and achievements," in *Proceedings of the 1998 ACM Conference on Computer Supported Cooperative Work*, ser. CSCW '98. New York, NY, USA: Association for Computing Machinery, 1998, p. 59–68. [Online]. Available: <https://doi.org/10.1145/289444.289469>
- [14] D. Sun and C. Sun, "Context-based operational transformation in distributed collaborative editing systems," *Parallel and Distributed Systems, IEEE Transactions on*, vol. 20, pp. 1454 – 1470, 11 2009.
- [15] D. S. Parker, G. J. Popek, G. Rudisin, A. Stoughton, B. J. Walker, E. Walton, J. M. Chow, D. Edwards, S. Kiser, and C. Kline, "Detection of mutual inconsistency in distributed systems," *IEEE Trans. Softw. Eng.*, vol. 9, no. 3, p. 240–247, May 1983. [Online]. Available: <https://doi.org/10.1109/TSE.1983.236733>
- [16] C. Baquero, P. S. Almeida, and A. Shoker, "Making operation-based crdts operation-based," in *Distributed Applications and Interoperable Systems*, K. Magoutis and P. Pietzuch, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 2014, pp. 126–140.
- [17] M. Letia, N. Preguiça, and M. Shapiro, "Consistency without concurrency control in large, dynamic systems," in *LADIS 2009 - 3rd ACM SIGOPS International Workshop on Large Scale Distributed Systems and Middleware*, ser. Operating Systems Review, vol. 44, no. 2. Big Sky, MT, United States: Assoc. for Computing Machinery, Oct. 2009, pp. 29–34. [Online]. Available: <https://hal.inria.fr/hal-01248270>
- [18] M. Zawirski, M. Shapiro, and N. Preguiça, "Asynchronous rebalancing of a replicated tree," in *Conférence Française en Systèmes d'Exploitation (CFSE)*, Saint-Malo, France, May 2011, p. 12. [Online]. Available: <https://hal.inria.fr/hal-01248197>
- [19] D. B. Terry, M. M. Theimer, K. Petersen, A. J. Demers, M. J. Spreitzer, and C. H. Hauser, "Managing update conflicts in bayou, a weakly connected replicated storage system," *SIGOPS Oper. Syst. Rev.*, vol. 29, no. 5, p. 172–182, Dec. 1995. [Online]. Available: <https://doi.org/10.1145/224057.224070>

**Jane Doe** Biography text here.