

Improving Replicated Sequences Performances

Matthieu Nicolas, Gérald Oster, and Olivier Perrin

Université de Lorraine, CNRS, Inria, LORIA, F-54500, France

1 Introduction

2 Replicated Sequences

2.1 Sequence

- Abstract Data Type (ADT) which allows to represent a list of values
- Provide two operations to update the sequence, *insert* and *remove*
- $insert(S, index, elt) = S'$ inserts the value *elt* at the index *index* into the sequence *S* and returns the updated sequence *S'*
- $remove(S, index) = S'$ removes from the sequence *S* the value at the index *index* and returns the updated sequence *S'*

2.2 Conflict-free Replicated Data Types (CRDTs) [1, 2]

- Optimistically replicated data structure
- Updates performed without coordination between nodes
- Ensure Strong Eventual Consistency (SEC) [2]

2.3 Logoot [3]

- Element-wise Sequence CRDT
- Key insight is to replace changing and unreliable *indexes* with universal and immutable *positions*
 - *indexes* are adapted for sequential executions
 - They are closely tied to the current state of the sequence, as updating the later shift them
 - However, they can not be used in a distributed setting, in which the order of the execution of the updates can be different from one node to another, to embody the user's intention
 - Conversely, Logoot's *positions* are designed to be independent of the state
 - This allows the definition of commutative operations, providing a *Sequence* suited for distributed settings

TODO: Insérer un exemple de manipulation concurrente d'une séquence (ins/remove en parallèle), sans mécanisme de résolution de conflits, pour illustrer la divergence du résultat si on ne se base que sur des index – Matthieu

- When inserting an element into the sequence, a fitting *position* is computed by the node and associated to the element

- These *positions* perform several roles
 - They identify uniquely an element
 - They order the elements relatively to each other
- To be able to perform these roles, *positions* need to comply with several constraints
 - Be globally unique: several nodes should not be able to compute the same *position* concurrently
 - Be temporarily unique: a node should not be able to associate the same *position* to different elements during the lifetime of the sequence
 - Be totally ordered: an order relation must exist over *positions* so a node can order two elements given their respective *position*
 - Belong to a dense set: a node should always be able to generate a new *position* between two others
- To comply with these constraints, Logoot *positions* are composed of one or several of the following tuples:

$$\langle priority, id_{site}, seq_{site} \rangle$$

where:

NOTE: on utilise actuellement des entiers pour représenter les éléments des tuples à l'heure actuelle mais pourrait utiliser n'importe quel type disposant d'une relation d'ordre, donc je ne sais pas quel type indiquer ici – Matthieu

- *priority* allows to determine the location of this position relatively to others
- *id_{site}* refers to the node's identifier, assumed to be unique
- *seq_{site}* refers to the node's logical clock, which increases monotonically with local updates

TODO: Reprendre l'exemple précédent, mais en remplaçant les index par des positions Logoot, pour illustrer la convergence – Matthieu

- It is worth to notice that: *J'illustre dans les prochains points comment la composition d'une position permet d'assurer l'ensemble des contraintes stipulées précédemment, mais l'explication concernant l'espace dense se révèle être particulièrement complexe et nécessiter de rentrer dans des détails. À garder ? – Matthieu*
 - The couple $\langle id_{site}, seq_{site} \rangle$ of the last tuple of the position ensures its uniqueness as
 - * No other node can generate a position using the same *id_{site}* as it is unique
 - * No other position can be generated by the same node using the same *seq_{site}* as it is increasing monotonically with local updates
 - Every part of positions can be used to define a total order based on the lexicographical order
 - To ensure that positions form a dense set, we have to reserve exclusively the usage of the minimal value of *priority* to a specific tuple, *minTuple*. By doing this we have, for every other tuple *t*, *minTuple* < *t*. *minTuple* is then used only as an intermediary tuple, the default value when a node has to generate a tuple less than another one, but is not able to. Thus, given two positions *p1* and *p2* such as *p1* < *p2*, any node is always able to generate a new position *p3* such as *p1* < *p3* < *p2* by reusing the tuples of *p1*, appending *minTuple* as many times as required and finally adding its own new tuple *t*.
- Updates the definition of the operations *insert* and *remove* to take into account these changes
 - *insert*(*S*, *pos*, *elt*) = *S'* inserts the value *elt* at the position *pos* into the sequence *S* and returns the updated sequence *S'*

- $remove(S, pos) = S'$ removes from the sequence S the value at the position pos and returns the updated sequence S'
- Provides some functions to nodes so they can obtain *positions* from *indexes*
 - $generatePos(S, index) = pos$ generates a new position pos corresponding to the index $index$ of the current sequence S
 - $getPos(S, index) = pos$ retrieves the position pos of the element at the index $index$ in the current sequence S

2.4 LogootSplit [4]

- Block-wise Sequence CRDTs
- Goal is to improve further the performances of Logoot
- It is expensive to generate and associate a position to each element of the sequence
- André et al. [4] proposes to aggregate dynamically elements into blocks
- It allows to reduce the metadata per element as we only need to store a *position interval* for a block, independently of how many elements it contains
- To do so, add a new component to positions tuples: the *offset* *Préciser que offset doit appartenir à un ensemble énumérable, de façon à ce que l'on puisse déterminer son prédécesseur, successeur ou si une valeur est manquante ? – Matthieu*
- LogootSplit *positions* are thus composed of one or several of the following tuples:

$$< priority, id_{site}, seq_{site}, offset >$$

- Define positions as aggregable if all components but the *offsets* of their last tuple are identical and if their respective *offsets* are consecutive
- Define *position interval* as the following couple:

$$< posBegin, end >$$

where:

- $posBegin$ refers to the first position of the interval
- end refers to the value of the offset of the last position of the interval
- Given these two values, we are able to recreate all positions from the interval if needed
- It is worth to notice that
 - The uniqueness of a LogootSplit position is ensured by the triple $< id_{site}, seq_{site}, offset >$
 - As such, an *offset* can not be used twice for the same block
 - It is then necessary to prevent such a case from happening when a node prepend or append a new element to an existing block
 - It can be achieved by keeping track of used *offset* per block
 - * To be more precise, only need to keep track of the minimum and maximum *offsets* ever used for this block
- Updates the definition of the operations *insert* and *remove* to make them block-wise

- $insert(S, pos, elts) = S'$ inserts into the sequence S the values $elts$ from the position pos . Returns the updated sequence S'
- $remove(S, posIntervals) = S'$ removes from the sequence S the values with their position contained in one of the interval of $posIntervals$ and returns the updated sequence S'
- Also introduce the following function:
 - $getPosIntervals(S, index, length) = posIntervals$ retrieves the list of *position intervals* at the index $index$ in the current sequence S

2.5 Limits

- As shown previously, LogootSplit positions' size is not bounded in order to comply with the dense set constraint
- Positions will become longer as the collaboration progresses, downgrading the performances of the application *NOTE: trouver un autre terme que "collaboration"? – Matthieu*
 - Since nodes have to broadcast positions, store them but also compare them to determine their order
- *Ici j'aimerais aussi ajouter quelques mots pour expliquer que le nombre de blocs influe négativement sur les performances de l'application (augmente le temps de parcours de la liste, et donc le temps d'exécution d'une insertion ou d'une suppression; doit conserver un position interval pour chaque bloc). Une séquence est donc optimale si elle composée d'un unique bloc. Mais en raison des contraintes définissant les positions agrégeables (insertions à des index consécutifs par un même noeud, sans suppression intermédiaire) et de l'absence de mécanisme pour fusionner à posteriori les blocs existants, les séquences issues d'une collaboration sont généralement morcelées en de nombreux blocs. – Matthieu*

3 Overview

- We build up on top of LogootSplit
- To address its limitations, we introduce a renaming mechanism
- The purpose of this mechanism is to reassign shorter positions to each element such as all of them can be aggregated into one unique block
- This allows to reduce the metadata per element, the computation time required to apply next updates and also the bandwidth used to broadcast future updates
- *TODO: Expliquer que toutefois le mécanisme de renommage ajoute un coût et qu'on cherche à le minimiser. Mais j'ai du mal à voir comment introduire la notion de cet overhead sans rentrer dans les détails qui en sont la source (règles de réécriture, arborescence des epochs, transformation des opérations concurrentes), que je préférerais introduire un par un dans la suite du papier – Matthieu*
- For simplicity purposes, will first present the renaming mechanism in the context of a centralised system, with only one node able to trigger the renaming mechanism
 - Will describe the functioning of this initial version and discuss its limitations
- Will then present a more elaborated version of the renaming mechanism, overcoming the limitations of the centralised version and allowing its use in a distributed setting

4 Renaming in a centralised setting

4.1 System Model

NOTE: Ça me paraît correct mais confusant de parler d'un système centralisé pour un système P2P où seulement une fonctionnalité (le renommage) n'est disponible que pour un noeud particulier. Voir pour mieux présenter cet aspect – Matthieu

- Peer-to-peer network
 - Nodes join and leave dynamically
- Nodes build and maintain a sequence collaboratively using LogootSplit
 - Each node owns a copy of the sequence and can edit it without any kind of coordination with others
- The network is unreliable
 - Messages can be lost, re-ordered and delivered multiple times
- To overcome the faults of the network, a message-passing layer is used to deliver messages to the application exactly-once, in the correct order
 - At each node, the insertion of a position happens before its removal
- An anti-entropy mechanism is used by nodes to synchronise in a pairwise manner, by detecting and re-exchanging lost messages
- One node is arbitrarily designed as the leader
- This node only is able to trigger renamings

4.2 Specification

- Introduce a new operation *rename*
- This operation allows nodes to map each *position* of their current sequence to a new one
- Must be defined for all *positions* of the current sequence of the node triggering the renaming...
- ... but also for all *positions* which can be added concurrently by other nodes
- To ensure the correctness of the resulting sequences, this operation must comply with several properties
 - *position uniqueness preservation* : as *positions* must be unique, a *rename* operation should not map several *positions* to the same resulting one
 - *order preservation* : as *positions* ensures the order of their respective element, a *rename* operation should not shuffle this order
- To ensure the eventual consistency of the system, the *rename* operation must also be commutative with the other operations *insert* and *delete* as nodes may receive and apply them in different orders
 - As no concurrent *rename* operation can be issued, since only node is able to trigger the renaming mechanism, there is no need to design the *rename* operation as commutative with itself

4.3 Proposition

- Decompose the *rename* operation into the following components and steps

4.3.1 renamingMap

- The first step is to generate the *renamingMap*.
- This map is computed by the node which triggers the *rename* operation
- It associates each existing position of its current sequence to a new one
- It will be used by every nodes to apply the renaming on their state
- As it needs to be broadcasted to other nodes, the size of the *renamingMap* impacts the efficiency of the proposed solution
- A mechanism to compress the *renamingMap* is introduced in section 7.4
- To compute this map, the node uses the algorithm 1

Algorithm 1 Generate renamingMap

```

function GENERATERENAMINGMAP( $S, id', seq'$ )
   $renamingMap \leftarrow Map()$ 

   $firstPos \leftarrow S.getFirstPos()$ 
   $firstTuple \leftarrow firstPos.getFirstTuple()$ 
   $priority \leftarrow firstTuple.priority$   $\triangleright$  Retrieve the priority of the first tuple of the first position

  for all  $(pos, index) \in S$  do
     $pos' \leftarrow new\ Pos(< priority, id', seq', index >)$   $\triangleright$  Generate the new corresponding position
     $renamingMap.set(pos, pos')$ 
  end for

  return  $renamingMap$ 
end function

```

- This algorithm has the following behavior:
 - Consist in generating a *Map* associating each position of the sequence S to a new position
 - To optimise the resulting sequence, the new positions should be aggregable into one block
 - i.e. the new positions must form a *position interval*
 - First, have to pick a position as the beginning of the interval
 - This choice is arbitrary
 - In our case, pick the following position pos' :

$$pos' = < priority, id', seq', 0 >$$

where:

- * $priority$ is the same value as the $priority$ of the first tuple of the first element of the sequence.
We explain this choice in section 4.3.3.
- * id' is the id_{site} of the node performing the *rename* operation
- * seq' is the current seq_{site} of the node performing the *rename* operation
- The first position pos of the current sequence will be map to pos'
- Then, all former positions are mapped to the consecutive ones of the new position according to the existing order
- All positions of the current sequence will be mapped to a set of positions which effectively form the following position interval $posInterval'$:

$$posInterval' = \langle pos', length - 1 \rangle$$

where:

* $length$ is the length of the sequence

- *TODO: Ajouter une phrase expliquant qu'on perd de l'information avec le renommage. Par exemple, on remplace le id_{site} contenu dans la position qui identifie l'auteur de cet élément par id' qui est l'identifiant de l'auteur du renommage – Matthieu*

4.3.2 Epoch-based mechanism

- A renaming can be seen as a change of frame of reference, applied to positions
- Positions come from different frames according to if a renaming occurred between their generation
- We should not compare positions from different frames as it is meaningless
- It is thus necessary
 - to define the frame of reference in which a given position is valid
 - add information to the system to model this frame
- To achieve this, we introduce the notion of *epochs*
- The sequence has a default epoch : the *origin* one
- When a renaming is performed, a new epoch is generated and replaces the current epoch of the sequence
- The set of these epochs forms a chain in the case of the centralised setting
- The renamingMaps presented previously allow nodes to move their sequence from one epoch to the next one
- By tagging operations with the current epoch at the time of generation, we can encapsulate its scope of validity
- Upon reception of an operation, a node is able to determine by comparing its current epoch to the epoch embedded in the operation if it can be applied or if additional steps are required beforehand
- To represent epochs, we use the following data structure:

$$epoch = \langle \langle epochNumber, id_{site} \rangle, parentId \rangle$$

where:

- $epochNumber$ is the successor of the $epochNumber$ of the previous epoch
- id_{site} is the identifier of the node which generated the epoch
- $\langle epochNumber, id_{site} \rangle$ form the identifier of the epoch
- $parentId$ refers to the previous epoch by its identifier

Algorithm 2 Rename position

```
function RENAMEFORWARDPOS(renamingMap, pos)
  if renamingMap.has(pos) then
    return renamingMap.get(pos)
  end if

  renamedPositions  $\leftarrow$  keysOf(renamingMap)
  firstPos  $\leftarrow$  renamedPositions[0]
  lastPos  $\leftarrow$  renamedPositions[renamedPositions.length - 1]
  newFirstPos  $\leftarrow$  renamingMap.get(firstPos)
  newLastPos  $\leftarrow$  renamingMap.get(lastPos)
  minFirstPos  $\leftarrow$  min(firstPos, newFirstPos)
  maxLastPos  $\leftarrow$  max(lastPos, newLastPos)

  if pos < minFirstPos or maxLastPos < pos then
    return pos  $\triangleright$  Return the position unchanged as it does not conflict with the renaming
  end if

  if newFirstPos < pos < firstPos then
     $\langle$  priority, id, seq, offset  $\rangle \leftarrow$  newFirstPos  $\triangleright$  Retrieve all components of newFirstPos
    predecessorOfNewFirstPos  $\leftarrow$   $\langle$  priority, id, seq, offset - 1  $\rangle$ 
    return concat(predecessorOfNewFirstPos, pos)
  end if

  predecessorOfPos  $\leftarrow$  findPredecessor(renamedPositions, pos)
  newPredecessorOfPos  $\leftarrow$  renamingMap.get(predecessorOfPos)
  return concat(newPredecessorOfPos, pos)
end function
```

4.3.3 Rename positions

- To compute the new position corresponding to a given position in the new epoch, define the following function $renameForwardPos(renamingMap, pos) = pos'$
- Its behavior, which is detailed in algorithm 2, can be described as follow:
 - If pos was one of the positions belonging to the state when the $renamingMap$ was computed, then its renaming has already been decided and its new value is stored in the $renamingMap$. We just return it.
 - If that is not the case, we need to compute the new position corresponding to it in the new frame of reference
 - To be able to preserve the existing order between pos and other positions through the renaming, we use different strategies according to the position's value:
 - * We define respectively as $firstPos$ and $lastPos$ the first and last positions contained in the entries of the $renamingMap$, and $newFirstPos$ and $newLastPos$ their images in the new frame of reference.
 - * If pos is actually outside of the range impacted by the renaming, i.e $pos < \min(firstPos, newFirstPos)$ or $\max(lastPos, newLastPos) < pos$, we can return it unchanged as it will not conflict with other renamed positions
 - * If we have $newFirstPos < pos < firstPos$, we rename pos to shift it just before $newFirstPos$ by concatenating pos to the predecessor of $newFirstPos$. The predecessor of $newFirstPos$ is obtained by subtracting 1 to its *offset* part.
 - * In the remaining case, it means that we have $firstPos < pos < newLastPos$. In this case, we look for $predecessorOfPos$, the predecessor of pos among the keys of $renamingMap$. Then, we retrieve the image of this position in the new state, $newPredecessorOfPos$. By concatenating pos to $newPredecessorOfPos$, we are able to generate a new position while preserving the existing order.
- The main idea of this approach is that we preserve the existing order between positions by concatenating the former positions to given prefixes to form the new positions.
- The greater the original position, the greater the prefix used to compose the new position.
- So, with $p1$ and $p2$ two positions such as $p1 < p2$ and $p1'$ and $p2'$ their respective renamed versions, we have
 - either $p1'$ and $p2'$ sharing the same prefix. In that case, comparing $p1'$ and $p2'$ effectively comes down to comparing $p1$ and $p2$.
 - either the prefix of $p2'$ is greater than the prefix of $p1'$.
- In both cases, we have $p1' < p2'$.
- *NOTE: Ce que je dis au-dessus concerne le cas où on a $firstPos < p1 < p2 < newLastPos$, mais est vrai aussi pour le cas où on a $newFirstPos < p1 < firstPos < p2$ de façon moins évidente. Reste à montrer que l'ordre est conservé dans les cas limites ($p1 < newFirstPos < p2 < firstPos$; $lastPos < p1 < newLastPos < p2$). Renvoyer à la section validation. – Matthieu*

4.3.4 Putting it all together

TODO: Expliquer qu'on propose un nouveau CRDT, <insérer un nom ici> (RenamableLogootSplit?). Cette structure de données encapsule une liste répliquée en utilisant LogootSplit, mais maintient aussi l'époque courante, une map des epochs et les renamingMaps permettant d'avancer d'une epoch à l'autre. Dispose des fonctions présentées précédemment, generateRenamingMap() et renameForwardPos(). Propose la fonction rename() qui retourne la séquence renommée en appliquant renameForwardPos à

chaque position de l'état courant.

Surcharge le traitement des opérations insert et delete pour:

- Déléguer le traitement de l'opération à l'instance de `LogootSplit` si l'epoch de génération correspond à l'epoch courante

- Ou transformer l'opération au préalable à l'aide de `renameForwardPos()` (potentiellement à travers plusieurs epochs) si les epochs ne correspondent pas.

Indiquer qu'on a une contrainte supplémentaire pour la livraison des opérations : elles doivent être livrées dans l'ordre causal par rapport à la dernière opération de renommage observée (doit être à la même epoch que celle de génération de l'opération ou à une epoch suivante pour pouvoir appliquer une opération). –

Matthieu

4.4 Garbage collection

- As stated previously, the renaming mechanism generates and stores additional metadata: the epochs and the renamingMaps used to transform concurrent operations against the renaming
- However, we do not need to keep this additional metadata forever
- Since they are used to handle concurrent operations to the renaming, they are not required anymore once no additional concurrent operation can be issued by a node
- i.e. we can safely garbage collect rewriting rules once the corresponding renaming operation is causally stable [5]
- Nodes need thus to keep track of the progress of others to detect when this condition is met
- This can be done in a coordination-free manner by exploiting the epochs attached to operations:
 - Each node stores a vector of epochs, with one entry for each node
 - Upon the reception of an operation, the node updates the entry of the sender with the epoch of the operation
 - As nodes collaborates, epochs in the vector will progress
 - By retrieving the minimum epoch from the vector, we can identify which epoch has been reached by all nodes
 - We can then safely garbage collect all previous epochs and corresponding renamingMaps

4.5 Limits

4.5.1 Size of concurrently generated positions

TODO: Ajouter quelques lignes sur le fait que le renommage a pour effet d'augmenter la taille des positions insérées en concurrence. Tempérer ce problème en argumentant que ce nombre de positions concurrentes devrait s'avérer faible par rapport au nombre total de positions contenues dans la séquence et qu'elles seront de toute façon réduites au cours du renommage suivant. – Matthieu

4.5.2 Fault-tolerance

- The system is vulnerable to failures, as only one particular node is able to trigger renamings
 - A failure of this node would prevent the renaming mechanism from being triggered ever again
 - But other nodes would still be able to continue their collaboration in such scenario
 - The failure of the renaming mechanism does not impede the liveness of the system
- To address this fault-tolerance issue, can set up a consensus-based system
 - Require nodes to perform a consensus to trigger a renaming

- But consensus algorithms are expensive and not suited for dynamic systems
- Can adapt the idea introduced in [6]
- In this paper, authors propose to divide a distributed system into two tiers: the *Core*, a small set of controlled and stable nodes, and the *Nebula*, an uncontrolled set of nodes
 - * Only nodes from the *Core* would participate in the consensus leading to a renaming
- Provide a trade-off between the cost of performing a renaming and the resilience of the system
- But this approach is not suited for all kind of applications
- In fully distributed systems, there is no central authority to provide a set of stable nodes acting as the *Core*

5 Renaming in a fully distributed setting

5.1 System Model

5.2 Intuition

5.3 Strategy to determine leading epoch in case of concurrency

5.4 Transitioning from a losing epoch to the leading one

5.5 Garbage collection

6 Evaluation

7 Discussion

7.1 Offloading on disk unused renaming rules

- As stated previously, nodes have to keep renamingMaps as long as another nodes may issue operations which would require to be transformed to be applied
- Thus nodes need to keep track of the progress of others to determine if such operations can still be issued or if it is safe to garbage collect the renaming rules
- In a fully distributed setting, this requirement is difficult to reach as nodes may join the collaboration, perform some operations and then disconnect
- Other nodes, from their point of view, are not able to determine if they disconnected temporarily or if it left definitely the collaboration
- However, as the disconnected nodes stopped progressing, they hold back the whole system and keep the current active nodes from garbage collecting old renaming rules
- To limit the impact of stale nodes on active ones, we propose that nodes offload unused renamingMaps by storing them on disk

Présenter une méthode pour déterminer les règles de renommage non-utilisées (conserver uniquement les règles utilisées pour traiter les x dernières opérations ?) – Matthieu

7.2 Alternative strategy to determine leading epoch

7.3 Postponing transition between epochs in case of instability

- May reach a situation in which several nodes keep generating concurrent renaming operations on different epoch branches
- In such case, switching repeatedly between these concurrent branches may prove wasteful *"costly" plutôt? – Matthieu*
- However, as long as nodes possess the required renamingMaps, they are able to rewrite operations from the other side and to integrate them into their copy, even if they are not on the latest epoch of their branch
 - At the cost of an overhead per operation
- Thus not moving to the new current epoch does not impede the liveness of the system
- Nodes can wait until one branch arise as the leading one then move to this epoch
- To speed up the emergence of such a branch, communications can be increased between nodes in such case to ease synchronisation

7.4 Compressing the renaming operation

TODO: Retravailler pour y ajouter la notion d'offset. Par contre, faire remarquer qu'on a pas besoin de l'offset pour identifier de manière unique "la base" d'une position (toute la position sauf l'offset) – Matthieu

- Propagating the renaming operation consists in broadcasting the list of blocks on which the renaming was performed, so that other nodes are able to compute the same rewriting rules
- This could prove costly, as the state before renaming can be composed of many blocks, each using long positions
- We propose an approach to compress this operation to reduce its bandwidth consumption at the cost of additional computations to process it
- Despite the variable length of positions, the parts required to identify a position uniquely are fixed
 - We only need the *siteId* and the *seq* of the last tuple of the position to do so
- Instead of broadcasting the list of whole positions, the node which performs the renaming can just broadcast the list of tuples $\langle siteId, seq \rangle$
- On reception of a compressed renaming operation, a node needs first to regenerate the list of renamed blocks to be able to apply it
- To achieve so, it can browse its current state looking for positions with corresponding tuples $\langle siteId, seq \rangle$
- If some positions are missing from the state, it means that they were deleted concurrently
- The node can thus browse the concurrent remove operations to the renaming one to find the missing blocks
- Once all positions has been retrieved and the list of blocks computed, the renaming operation can be processed normally

7.5 Operational Transformation

Ajouter une section sur OT pour expliquer que gérer les opérations concurrentes aux renommages consiste en finalité à transformer ces opérations, mais qu'on a décidé de ne pas présenter et formaliser l'approche comme étant de l'OT dans ce papier pour des raisons de simplicité ? – Matthieu

8 Conclusion

References

- [1] Marc Shapiro et al. *A comprehensive study of Convergent and Commutative Replicated Data Types*. Research Report RR-7506. Inria – Centre Paris-Rocquencourt ; INRIA, Jan. 2011, p. 50. URL: <https://hal.inria.fr/inria-00555588>.
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- [3] Stéphane Weiss, Pascal Urso, and Pascal Molli. “Logoot : A Scalable Optimistic Replication Algorithm for Collaborative Editing on P2P Networks”. In: *Proceedings of the 29th International Conference on Distributed Computing Systems - ICDCS 2009*. Montreal, QC, Canada: IEEE Computer Society, June 2009, pp. 404–412. DOI: 10.1109/ICDCS.2009.75. URL: <http://doi.ieeecomputersociety.org/10.1109/ICDCS.2009.75>.
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