```
> restart;
> with(LinearAlgebra) :
> Dmat := proc(se, pars)
  local DD1, i, j;
  description "Form the derivative matrix";
  with(LinearAlgebra) :
  DD1 := Matrix(1 ..Dimension(pars), 1 ..Dimension(se)) :
  for i from 1 to Dimension(pars)  do
      for j from 1 to Dimension(se) do
          DD1[i, j] := diff(se[j], pars[i])
      end do
   end do;
  DD1;
   end proc:
> Estpar := proc(DD1, pars, ret)
  local r, d, alphapre, alpha, PDE, FF, i, j, ans;
  description "Finds the estimable set of parameters for derivative matrix DD1. If ret = 1 returns
       alpha, PDEs, estimable parameter combinations. Otherwise returns estimable parameter
       combinations";
  with(LinearAlgebra) :
  r := Rank(DD1); d := Dimension(pars) - r :
  alphapre := NullSpace(Transpose(DD1)) : α := Matrix(d, Dimension(pars)) : PDE :=
       Vector(d) :
  FF := f(seq(pars[i], i = 1 ..Dimension(pars))) :
  for i from 1 to d do
        α[i, 1 ..Dimension(pars)] := alphapre[i] :
        PDE[i] := add(diff(FF, pars[j])·α[i, j], j = 1 ..Dimension(pars)) :
  end do:
   if ret = 1 then
           ans := ⟨pdsolve({seq(PDE[i] = 0, i = 1 ..d)}), {alpha}, {PDE}⟩ :
   else
          ans := pdsolve({seq(PDE[i] = 0, i = 1 ..d)}) :
   end if:
   ans :
  end proc:
> C := ⟨⟨ h[t]·r[t]|0|0|0|0|0⟩, ⟨ 0| h[t]·r[t]|0|0|0|0⟩, ⟨ 0|0| h[t]·r[t]|0|0|0⟩, ⟨ 0|0| 0| h[t]·r[t]|0
      |0⟩, ⟨ 0|0| 0| 0| h[t]·r[t]|0⟩, ⟨0| 0|0| 0| 0| h[t]·r[t]⟩⟩;

  A := ⟨⟨0|f[t − 1]|f[t − 1]|f[t − 1]|f[t − 1]|f[t − 1]⟩, ⟨(1 − h[t − 1])·s[t − 1]|0|0|0|0|0⟩,
      ⟨0|(1 − h[t − 1])·s[t − 1]|0|0|0|0⟩, ⟨0 |0|(1 − h[t − 1])·s[t − 1]|0|0|0⟩, ⟨0 |0|0|(1 − h[t
       − 1])·s[t − 1]|0|0⟩, ⟨0|0 |0|0|(1 − h[t − 1])·s[t − 1]|0⟩⟩; x1 := ⟨N[1, 1], N[1, 2], N[1,
      3], N[1, 4], N[1, 5], N[1, 6]⟩;
```

$$C := \begin{bmatrix} h_t r_t & 0 & 0 & 0 & 0 & 0 \\ 0 & h_t r_t & 0 & 0 & 0 & 0 \\ 0 & 0 & h_t r_t & 0 & 0 & 0 \\ 0 & 0 & 0 & h_t r_t & 0 & 0 \\ 0 & 0 & 0 & 0 & h_t r_t & 0 \\ 0 & 0 & 0 & 0 & 0 & h_t r_t \end{bmatrix}$$

$A := [[0, f_{t-1}, f_{t-1}, f_{t-1}, f_{t-1}, f_{t-1}],$
$[(1 - h_{t-1})\, s_{t-1}, 0, 0, 0, 0, 0],$
$[0, (1 - h_{t-1})\, s_{t-1}, 0, 0, 0, 0],$
$[0, 0, (1 - h_{t-1})\, s_{t-1}, 0, 0, 0],$
$[0, 0, 0, (1 - h_{t-1})\, s_{t-1}, 0, 0],$
$[0, 0, 0, 0, (1 - h_{t-1})\, s_{t-1}, 0]]$

$$x1 := \begin{bmatrix} N_{1,1} \\ N_{1,2} \\ N_{1,3} \\ N_{1,4} \\ N_{1,5} \\ N_{1,6} \end{bmatrix} \quad \textbf{(1)}$$

> $xt := \langle \mathrm{x}[t, 1], x[t, 2], x[t, 3], x[t, 4], x[t, 5], x[t, 6] \rangle;$

$$xt := \begin{bmatrix} x_{t,1} \\ x_{t,2} \\ x_{t,3} \\ x_{t,4} \\ x_{t,5} \\ x_{t,6} \end{bmatrix} \quad \textbf{(2)}$$

> *# Starting with 4 age categories:*

> $C := \langle \langle h[t] \cdot r[t] | 0 | 0 | 0 \rangle, \langle 0 | h[t] \cdot r[t] | 0 | 0 \rangle, \langle 0 | 0 | h[t] \cdot r[t] | 0 \rangle, \langle 0 | 0 | 0 | h[t] \cdot r[t] \rangle \rangle;$

$A := \langle \langle 0 | f[t-1] | f[t-1] | f[t-1] \rangle, \langle (1 - h[t-1]) \cdot s[t-1] | 0 | 0 | 0 \rangle, \langle 0 | (1 - h[t-1]) \cdot s[t-1] | 0 | 0 \rangle, \langle 0 | 0 | (1 - h[t-1]) \cdot s[t-1] | 0 \rangle \rangle; x1 := \langle \mathrm{N}[1, 1], N[1, 2], N[1, 3], N[1, 4] \rangle;$

$$C := \begin{bmatrix} h_t r_t & 0 & 0 & 0 \\ 0 & h_t r_t & 0 & 0 \\ 0 & 0 & h_t r_t & 0 \\ 0 & 0 & 0 & h_t r_t \end{bmatrix}$$

$$A := \begin{bmatrix} 0 & f_{t-1} & f_{t-1} & f_{t-1} \\ (1-h_{t-1})\, s_{t-1} & 0 & 0 & 0 \\ 0 & (1-h_{t-1})\, s_{t-1} & 0 & 0 \\ 0 & 0 & (1-h_{t-1})\, s_{t-1} & 0 \end{bmatrix}$$

$$x1 := \begin{bmatrix} N_{1,1} \\ N_{1,2} \\ N_{1,3} \\ N_{1,4} \end{bmatrix} \tag{3}$$

> # Components of the exhaustive summary

> # $C_1 x_1$

> *kappa11* := *MatrixMatrixMultiply*(*eval*(C, $t = 1$), *x1*);

$$\kappa 11 := \begin{bmatrix} h_1 r_1 N_{1,1} \\ h_1 r_1 N_{1,2} \\ h_1 r_1 N_{1,3} \\ h_1 r_1 N_{1,4} \end{bmatrix} \tag{4}$$

> # $C_2 A_2 x_1$

> *kappa12* := *MatrixMatrixMultiply*(*eval*(C, $t = 2$), *MatrixMatrixMultiply*(*eval*(A, $t = 2$), *x1*));

$$\kappa 12 := \begin{bmatrix} r_2 h_2 \left(f_1 N_{1,2} + f_1 N_{1,3} + f_1 N_{1,4}\right) \\ r_2 h_2 (1-h_1)\, s_1 N_{1,1} \\ r_2 h_2 (1-h_1)\, s_1 N_{1,2} \\ r_2 h_2 (1-h_1)\, s_1 N_{1,3} \end{bmatrix} \tag{5}$$

> # $C_3 A_3 A_2 x_1$

> *kappa13* := *MatrixMatrixMultiply*(*eval*(C, $t = 3$), *MatrixMatrixMultiply*(*eval*(A, $t = 3$),
MatrixMatrixMultiply(*eval*(A, $t = 2$), *x1*)));

(6)

$$\kappa 13 := \begin{bmatrix} r_3\, h_3\, (f_2\, (1-h_1)\, s_1\, N_{1,1} + f_2\, (1-h_1)\, s_1\, N_{1,2} + f_2\, (1-h_1)\, s_1\, N_{1,3}) \\ r_3\, h_3\, (1-h_2)\, s_2\, (f_1\, N_{1,2} + f_1\, N_{1,3} + f_1\, N_{1,4}) \\ r_3\, h_3\, (1-h_2)\, s_2\, (1-h_1)\, s_1\, N_{1,1} \\ r_3\, h_3\, (1-h_2)\, s_2\, (1-h_1)\, s_1\, N_{1,2} \end{bmatrix} \quad \textbf{(6)}$$

> $\#C_3 A_4 A_3 A_2 x_1$

> *kappa14* := *MatrixMatrixMultiply*(*eval*(*C*, *t* = 4), *MatrixMatrixMultiply*(*eval*(*A*, *t* = 4), *MatrixMatrixMultiply*(*eval*(*A*, *t* = 3), *MatrixMatrixMultiply*(*eval*(*A*, *t* = 2), *x1*))));

$$\kappa 14 := [[r_4\, h_4\, (f_3\, (1-h_2)\, s_2\, (f_1\, N_{1,2} + f_1\, N_{1,3} + f_1\, N_{1,4}) + f_3\, (1-h_2)\, s_2\, (1-h_1)\, s_1\, N_{1,1} + f_3\, (1-h_2)\, s_2\, (1-h_1)\, s_1\, N_{1,2})], [r_4\, h_4\, (1-h_3)\, s_3\, (f_2\, (1-h_1)\, s_1\, N_{1,1} + f_2\, (1-h_1)\, s_1\, N_{1,2} + f_2\, (1-h_1)\, s_1\, N_{1,3})], [r_4\, h_4\, (1-h_3)\, s_3\, (1-h_2)\, s_2\, (f_1\, N_{1,2} + f_1\, N_{1,3} + f_1\, N_{1,4})], [r_4\, h_4\, (1-h_3)\, s_3\, (1-h_2)\, s_2\, (1-h_1)\, s_1\, N_{1,1}]] \quad \textbf{(7)}$$

> # Building exhaustive summary up one component at a time

> *kappa1* := ⟨*kappa11*⟩;

$$\kappa 1 := \begin{bmatrix} h_1\, r_1\, N_{1,1} \\ h_1\, r_1\, N_{1,2} \\ h_1\, r_1\, N_{1,3} \\ h_1\, r_1\, N_{1,4} \end{bmatrix} \quad \textbf{(8)}$$

> *indets*(*kappa1*)

$$\{N_{1,1}, N_{1,2}, N_{1,3}, N_{1,4}, h_1, r_1\} \quad \textbf{(9)}$$

> *pars1* := $\langle h_1, r_1 \rangle$;

$$pars1 := \begin{bmatrix} h_1 \\ r_1 \end{bmatrix} \quad \textbf{(10)}$$

> *D1* := *Dmat*(*convert*(*kappa1*, *Vector*), *pars1*);

$$D1 := \begin{bmatrix} r_1\, N_{1,1} & r_1\, N_{1,2} & r_1\, N_{1,3} & r_1\, N_{1,4} \\ h_1\, N_{1,1} & h_1\, N_{1,2} & h_1\, N_{1,3} & h_1\, N_{1,4} \end{bmatrix} \quad \textbf{(11)}$$

> *nopars* := *Dimension*(*pars1*); *rr* := *Rank*(*D1*); *d* := *Dimension*(*pars1*) − *rr*;

$$nopars := 2$$
$$rr := 1$$
$$d := 1 \quad \textbf{(12)}$$

> *Estpar*(*D1*, *pars1*, 0);

$$\{f(h_1, r_1) = \_F1(r_1\, h_1)\} \quad \textbf{(13)}$$

> *kappa1* := ⟨*kappa11*, *kappa12*⟩;

$$\kappa 1 := \begin{bmatrix} h_1\, r_1\, N_{1,\,1} \\ h_1\, r_1\, N_{1,\,2} \\ h_1\, r_1\, N_{1,\,3} \\ h_1\, r_1\, N_{1,\,4} \\ r_2\, h_2\, (f_1\, N_{1,\,2} + f_1\, N_{1,\,3} + f_1\, N_{1,\,4}) \\ r_2\, h_2\, (1 - h_1)\, s_1\, N_{1,\,1} \\ r_2\, h_2\, (1 - h_1)\, s_1\, N_{1,\,2} \\ r_2\, h_2\, (1 - h_1)\, s_1\, N_{1,\,3} \end{bmatrix} \quad \textbf{(14)}$$

> *indets(kappa1)*

$$\{N_{1,\,1}, N_{1,\,2}, N_{1,\,3}, N_{1,\,4}, f_1, h_1, h_2, r_1, r_2, s_1\} \quad \textbf{(15)}$$

> $pars1 := \langle f_1, h_1, h_2, r_1, r_2, s_1 \rangle;$

$$pars1 := \begin{bmatrix} f_1 \\ h_1 \\ h_2 \\ r_1 \\ r_2 \\ s_1 \end{bmatrix} \quad \textbf{(16)}$$

> $D1 := Dmat(convert(kappa1, Vector), pars1);$

$$D1 := [[0, 0, 0, 0, r_2\, h_2\, (N_{1,\,2} + N_{1,\,3} + N_{1,\,4}), 0, 0, 0], \quad \textbf{(17)}$$
$$[r_1\, N_{1,\,1}, r_1\, N_{1,\,2}, r_1\, N_{1,\,3}, r_1\, N_{1,\,4}, 0, -r_2\, h_2\, s_1\, N_{1,\,1}, -r_2\, h_2\, s_1\, N_{1,\,2}, -r_2\, h_2\, s_1\, N_{1,\,3}],$$
$$[0, 0, 0, 0, r_2\, (f_1\, N_{1,\,2} + f_1\, N_{1,\,3} + f_1\, N_{1,\,4}), r_2\, (1 - h_1)\, s_1\, N_{1,\,1}, r_2\, (1 - h_1)\, s_1\, N_{1,\,2}, r_2\, (1 - h_1)\, s_1\, N_{1,\,3}],$$
$$[h_1\, N_{1,\,1}, h_1\, N_{1,\,2}, h_1\, N_{1,\,3}, h_1\, N_{1,\,4}, 0, 0, 0, 0],$$
$$[0, 0, 0, 0, h_2\, (f_1\, N_{1,\,2} + f_1\, N_{1,\,3} + f_1\, N_{1,\,4}), h_2\, (1 - h_1)\, s_1\, N_{1,\,1}, h_2\, (1 - h_1)\, s_1\, N_{1,\,2}, h_2\, (1 - h_1)\, s_1\, N_{1,\,3}],$$
$$[0, 0, 0, 0, 0, r_2\, h_2\, (1 - h_1)\, N_{1,\,1}, r_2\, h_2\, (1 - h_1)\, N_{1,\,2}, r_2\, h_2\, (1 - h_1)\, N_{1,\,3}]]$$

> $nopars := Dimension(pars1); rr := Rank(D1); d := Dimension(pars1) - rr;$

$$nopars := 6$$
$$rr := 3$$
$$d := 3 \quad \textbf{(18)}$$

> $Estpar(D1, pars1, 0);$

(19)

$$\left\{f\left(f_1, h_1, h_2, r_1, r_2, s_1\right) = \_F1\left(r_1\, h_1, r_2 f_1\, h_2, \frac{s_1\,(h_1-1)}{f_1}\right)\right\} \tag{19}$$

> $kappa1 := \langle kappa11, kappa12, kappa13 \rangle;$

$$\kappa 1 := \begin{bmatrix} h_1\, r_1\, N_{1,1} \\ h_1\, r_1\, N_{1,2} \\ h_1\, r_1\, N_{1,3} \\ h_1\, r_1\, N_{1,4} \\ r_2\, h_2\,(f_1\, N_{1,2} + f_1\, N_{1,3} + f_1\, N_{1,4}) \\ r_2\, h_2\,(1-h_1)\, s_1\, N_{1,1} \\ r_2\, h_2\,(1-h_1)\, s_1\, N_{1,2} \\ r_2\, h_2\,(1-h_1)\, s_1\, N_{1,3} \\ r_3\, h_3\,(f_2\,(1-h_1)\, s_1\, N_{1,1} + f_2\,(1-h_1)\, s_1\, N_{1,2} + f_2\,(1-h_1)\, s_1\, N_{1,3}) \\ r_3\, h_3\,(1-h_2)\, s_2\,(f_1\, N_{1,2} + f_1\, N_{1,3} + f_1\, N_{1,4}) \\ \vdots \end{bmatrix} \tag{20}$$

12 × 1 Matrix

> $indets(kappa1)$

$$\{N_{1,1}, N_{1,2}, N_{1,3}, N_{1,4}, f_1, f_2, h_1, h_2, h_3, r_1, r_2, r_3, s_1, s_2\} \tag{21}$$

> $pars1 := \langle f_1, f_2, h_1, h_2, h_3, r_1, r_2, r_3, s_1, s_2 \rangle :$

> $D1 := Dmat(convert(kappa1, Vector), pars1) :$

> $nopars := Dimension(pars1); rr := Rank(D1); d := Dimension(pars1) - rr;$

$$nopars := 10$$
$$rr := 5$$
$$d := 5 \tag{22}$$

> $Estpar(D1, pars1, 0);$

$$\left\{f\left(f_1, f_2, h_1, h_2, h_3, r_1, r_2, r_3, s_1, s_2\right) = \_F1\left(r_1\, h_1, r_2 f_1\, h_2, r_3\, h_3 f_2 f_1, \frac{s_1\,(h_1-1)}{f_1}, \frac{s_2\,(-1+h_2)}{f_2}\right)\right\} \tag{23}$$

> # Clear pattern

> #Estimable parameter combinations are $r_1\, h_1$, $r_2 h_2 f_1$, $r_3 h_3 f_1 f_2$, ..., $r_n h_n f_1 f_2 .. f_n$, $\frac{s_1\,(1-h_1)}{f_1}$, $\frac{s_2\,(1-h_2)}{f_2}$,..., $\frac{s_{n-1}\,(1-h_{n-1})}{f_{n-1}}$

> $pp := 4 \cdot n - 2$

$$pp := 4\, n - 2 \tag{24}$$

> $rr := 2 \cdot n - 1$

$$rr := 2\,n - 1 \tag{25}$$

> $dd := pp - rr;$

$$dd := 2\,n - 1 \tag{26}$$

> *# Apply extension Theomrem in two directions*

> *# First involves adding extra age classes - this would be a "trivual" application of the extension theorem as you add new exhaustive summary terms, but no new parameters. Note it is not completely straightforward, as model is parameter redundant. Would need to first reparameterise in terms of estimable parameter combinations. You can easily show that each additional terms can be written in terms of the estimable parameter combinations - and rest follows from this fact.*

> *# Second involves adding extra years. This would be more complicated, but still involves reparamerising in terms of the estimable parameter combinations.*

>

> # Hunter Survey data

> $kappa2 := \langle r[1] \cdot a[1], r[2] \cdot a[2], r[3] \cdot a[3] \rangle;$

$$\kappa 2 := \begin{bmatrix} r_1\, a_1 \\ r_2\, a_2 \\ r_3\, a_3 \end{bmatrix} \tag{27}$$

> $pars2 := \langle r[1], r[2], r[3] \rangle;$

$$pars2 := \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix} \tag{28}$$

> $D2 := Dmat(kappa2, pars2);$

$$D2 := \begin{bmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{bmatrix} \tag{29}$$

> $nopars := Dimension(pars2); rr := Rank(D2); d := Dimension(pars2) - rr;$

$$nopars := 3$$
$$rr := 3$$
$$d := 0 \tag{30}$$

>

>

> *# Radio Tracking data*

> $kappa3 := \langle h[1] \cdot v[1], (1 - h[1]) \cdot (1 - s[1]) \cdot v[1], h[2] \cdot v[2], (1 - h[2]) \cdot (1 - s[2]) \cdot v[2], h[3] \cdot v[3], (1 - h[3]) \cdot (1 - s[3]) \cdot v[3] \rangle;$

$$\kappa 3 := \begin{bmatrix} h_1\, v_1 \\ (1-h_1)\,(1-s_1)\, v_1 \\ h_2\, v_2 \\ (1-h_2)\,(1-s_2)\, v_2 \\ h_3\, v_3 \\ (1-h_3)\,(1-s_3)\, v_3 \end{bmatrix} \quad \textbf{(31)}$$

> $pars3 := \langle h_1, h_2, h_3, s_1, s_2, s_3 \rangle;$

$$pars3 := \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} \quad \textbf{(32)}$$

> $D3 := Dmat(kappa3, pars3);$

$$D3 := \begin{bmatrix} v_1 & -(1-s_1)\, v_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & v_2 & -(1-s_2)\, v_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & v_3 & -(1-s_3)\, v_3 \\ 0 & -(1-h_1)\, v_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -(1-h_2)\, v_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -(1-h_3)\, v_3 \end{bmatrix} \quad \textbf{(33)}$$

> $nopars := Dimension(pars3);\ rr := Rank(D3);\ d := Dimension(pars3) - rr;$

$$nopars := 6$$
$$rr := 6$$
$$d := 0 \quad \textbf{(34)}$$

>

> *# Considering different combinations of data sets:*

> $kappajoin := convert(\langle kappa1, kappa2 \rangle, Vector) :$

> $indets(kappajoin)$

$$\{N_{1,\,1}, N_{1,\,2}, N_{1,\,3}, N_{1,\,4}, a_1, a_2, a_3, f_1, f_2, h_1, h_2, h_3, r_1, r_2, r_3, s_1, s_2\} \quad \textbf{(35)}$$

> $pars := \langle f_1, f_2, h_1, h_2, h_3, r_1, r_2, r_3, s_1, s_2 \rangle :$

> $Djoin := Dmat(kappajoin, pars) :$

> $nopars := Dimension(pars);\ rr := Rank(Djoin);\ d := Dimension(pars) - rr;$

$$nopars := 10$$

$$rr := 8$$
$$d := 2 \qquad (36)$$

> $Estpar(Djoin, pars, 0);$

$$\left\{f\left(f_1, f_2, h_1, h_2, h_3, r_1, r_2, r_3, s_1, s_2\right) = \_F1\left(h_1, f_1\, h_2, f_2\, h_3 f_1, r_1, r_2, r_3, \frac{s_1}{f_1}, -\frac{s_2\left(-1+h_2\right)}{f_2}\right)\right\} \qquad (37)$$

> $pp := 4 \cdot n - 2$

$$pp := 4\, n - 2 \qquad (38)$$

> $rr := 3 \cdot n - 1$

$$rr := 3\, n - 1 \qquad (39)$$

> $dd := pp - rr;$

$$dd := n - 1 \qquad (40)$$

> $kappajoin := convert(\langle kappa1, kappa3\rangle, Vector):$

> $indets(kappajoin)$

$$\{N_{1,\,1}, N_{1,\,2}, N_{1,\,3}, N_{1,\,4}, f_1, f_2, h_1, h_2, h_3, r_1, r_2, r_3, s_1, s_2, s_3, v_1, v_2, v_3\} \qquad (41)$$

> $pars := \langle f_1, f_2, h_1, h_2, h_3, r_1, r_2, r_3, s_1, s_2, s_3\rangle :$

> $Djoin := Dmat(kappajoin, pars):$

> $nopars := Dimension(pars); rr := Rank(Djoin); d := Dimension(pars) - rr;$

$$nopars := 11$$
$$rr := 11$$
$$d := 0 \qquad (42)$$

> $kappajoin := convert(\langle kappa2, kappa3\rangle, Vector):$

> $indets(kappajoin)$

$$\{a_1, a_2, a_3, h_1, h_2, h_3, r_1, r_2, r_3, s_1, s_2, s_3, v_1, v_2, v_3\} \qquad (43)$$

> $pars := \langle h_1, h_2, h_3, r_1, r_2, r_3, s_1, s_2, s_3\rangle :$

> $Djoin := Dmat(kappajoin, pars):$

> $nopars := Dimension(pars); rr := Rank(Djoin); d := Dimension(pars) - rr;$

$$nopars := 9$$
$$rr := 9$$
$$d := 0 \qquad (44)$$

> $kappajoin := convert(\langle kappa1, kappa2, kappa3\rangle, Vector):$

> $indets(kappajoin)$

$$\{N_{1,\,1}, N_{1,\,2}, N_{1,\,3}, N_{1,\,4}, a_1, a_2, a_3, f_1, f_2, h_1, h_2, h_3, r_1, r_2, r_3, s_1, s_2, s_3, v_1, v_2, v_3\} \qquad (45)$$

> $pars := \langle f_1, f_2, h_1, h_2, h_3, r_1, r_2, r_3, s_1, s_2, s_3\rangle :$

> $Djoin := Dmat(kappajoin, pars):$

> $nopars := Dimension(pars); rr := Rank(Djoin); d := Dimension(pars) - rr;$

$$nopars := 11$$
$$rr := 11$$
$$d := 0 \qquad (46)$$

>