Simulating Quantum Drude Oscillators on a photonic quantum computer

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I. INTRODUCTION

II. DEFINITION OF THE MODEL

$$\begin{aligned} H &= \frac{1}{2}\sum_{i=1}^{N}\left(x_i^2 + p_i^2\right) + \sum_{i<j}\gamma_{ij}x_i x_j \\ &= \sum_{i=1}^{N}\left(a_i^\dagger a_i + \frac{1}{2}\right) + \sum_{i<j}\gamma_{ij}x_i x_j \\ &= \sum_{i=1}^{N}\left(a_i^\dagger a_i + \frac{1}{2}\right) + \frac{1}{2}\sum_{i<j}\gamma_{ij}\left(a_i + a_i^\dagger\right)\left(a_j + a_j^\dagger\right) \end{aligned} \tag{1}$$

with

$$a_i = \frac{x_i + ip_i}{\sqrt{2}}\,, \qquad a_i^\dagger = \frac{x_i - ip_i}{\sqrt{2}}\,. \tag{2}$$

III. PHOTONIC CIRCUIT

The circuit implements a unitary $U(\theta)$ acting on an input state that we simply take to be the vacuum state $|0\rangle$. The state prepared by the circuit is therefore given by

$$|\psi(\theta)\rangle = U(\theta)|0\rangle\,. \tag{3}$$

IV. VARIATIONAL ALGORITHM

We define the following loss function:

$$\mathcal{C}(\theta) := \langle\psi(\theta)|H|\psi(\theta)\rangle \tag{4}$$

with the Hamiltonian defined in eq. (1). In order to compute this loss, one therefore has to measure both the photon number operator on each channel, as well as the position quadrature operator on each channel.

Algorithm 1: Computation of the energy

Parameters: reference statevector $|0\rangle$, circuit U
Result: Value of the energy E

Prepare statevector $|\psi\rangle = U|0\rangle$;
Measure the position quadratures x_i;
Prepare statevector $|\psi\rangle = U|0\rangle$;
Measure the photon numbers n_i;
Compute the energy E with eq. (1);
return E.

Algorithm 2: Computation of the loss

Parameters: $M \in \mathbb{N}$
Result: Value of the loss $\mathcal{C}$

Initialize $\mathcal{C} \leftarrow 0$;
for $j = 1$ *to* M **do**
  Compute the energy E with alg. (1);
  Update the loss $\mathcal{C} \leftarrow \mathcal{C} + E$;
  end for
$\mathcal{C} \leftarrow \mathcal{C}/M$;
return $\mathcal{C}$.

Algorithm 3: Training of the parameterized photonic circuit

Parameters: $N_{\text{steps}} \in \mathbb{N}$, initial parameters $\theta_0 \in \mathbb{R}^K$, learning rate $\eta \in \mathbb{R}_+$
Result: Optimized hyperparameters $\theta \in \mathbb{R}^K$

Initialize hyperparameters $\theta \leftarrow \theta_0$;
for $i = 1$ *to* N_{steps} **do**
  Compute the loss $\mathcal{C}$ with alg. (2);
  Compute the gradient $\nabla_\theta \mathcal{C}$ with shift rule and alg. (2);
  Update the parameters $\theta \leftarrow \theta - \eta\nabla_\theta\mathcal{C}$;
  end for
return θ.

Questions that should be addressed:

- Is alg. (1) the best since one needs to prepare the state twice to measure both the position quadratures and the photon numbers. Maybe a single measure in the coherent states basis would be possible? This can be achieved by a heterodyne mea-

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surement. With strawbery fields it can only be performed in the Gaussian or Bosonic backends, but not the tensorflow backend...

- What is the role of the parameter M in alg. (2)? Could one safely set $M = 1$ and still hope for convergence, a bit like in reinforcement learning? Setting $M = 1$ is of course a very rough estimate of the expected value, but since it is embedded in the training loop, maybe this estimate is actually enough?

- In strawberryfields the training procedure is managed by the tensorflow backend. Is it actually implementing the shift rule? Double check that.

V. CONCLUSION

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DATA AVAILABILITY

CODE AVAILABILITY

The reader will find an open source python code accompanying this paper following this github repository.

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