Regression discontinuity design

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Outline

1. Introduction
2. The econometrics of RDD
3. The statistics of RDD
4. Sensitivity tests
5. Conclusion

The idea

Introduction

The econometrics of RDD

The statistics of RDD

Sensitivity tests

Conclusion

The goal: Estimate a treatment effect in a case where there might be self-selection/endogeneity.

The framework: Selection is based on a simple known (administrative) rule, based on a cutoff

The idea: treated and untreated are in general different, but those just around the cutoff should be similar.

The solution: make a local comparison, comparing just above with just below

Two cases

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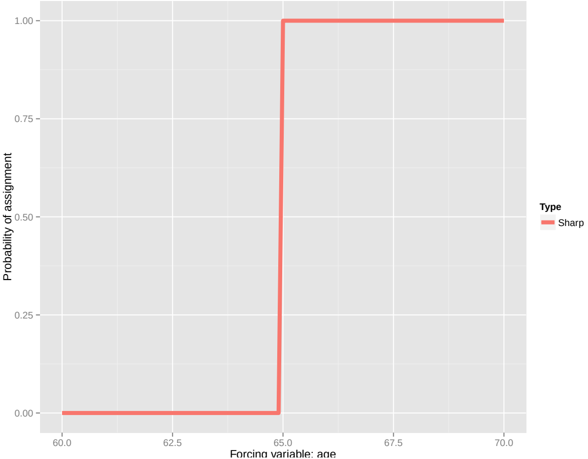
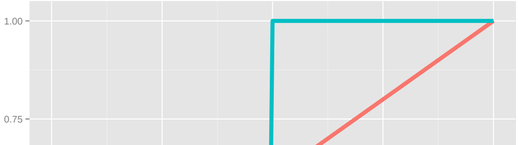
Two (three) cases:

- **Sharp RDD:** Assignment is strict (i.e. all above are treated, all below untreated)
- **Fuzzy RDD:** Assignment does not lead to full compliance
- **Mixed:** One sided-compliance

Figure 1: Assignment probabilities in sharp versus fuzzy





Examples

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Age: Retirement and Medicare (Card 2004). Alcohol in US and mortality (Carpenter et al 2007)

School score: Angrist et al. 2011, 2012...

Class size: If above 40 students, split 1 class into 2. (Angrist Lavy 1999)

Geo. location: Geo-wise treatment (for state, country): compare individuals at the border

Majority rule: For elections, Lee (2008)

Height: Military service ?

Poverty rate: Indian BPL? Super fuzzy...

The history

Majour contributions in eco:

- Thistlewaite and Campbell (1960)
- Angrist (1999) Fuzzy RDD, early application in economics
- Hahn et al (2001) Identification of ATE by RDD
- Porter (2003): Estimation of RDD
- Imbens and Lemieux (2008) State of the art in RDD
- Lee and Lemieux (2010) State of the art in RDD

cf Cook (2008) *Waiting for Life to Arrive: A History of the RDD*

Causality in econometrics

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Granger: X causes Y if X helps predict Y

Counterfactual: X causes Y if there is a difference between the Y observed when X occurred and the un-observed counterfactual Y *if X had not occurred*.

Refs: Granger (1969), Holland (1980)

Causality in econometrics

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Definitions:

Def: treatment variable

Individual i has treatment status:

$$D_i = \begin{cases} 1 & \text{if treated} \\ 0 & \text{if not treated} \end{cases}$$

Def: potential outcome

Individual i has outcome:

$$\begin{aligned} Y_i(D_i): &\quad \text{oberved outcome (treated or not)} \\ Y_i(1-D_i): &\quad \text{countefactual/hypothetical outcome} \end{aligned}$$

Term counter-factual

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Beware that counterfactual is used in two ways:

- **Hypothetical:** The unobserved outcome: $Y_i(0)|D_i = 1$ or $Y_i(1)|D_i = 0$
- **Approximation:** The closest factual: $Y_i(0)|D_i = 0$ for $Y_i(0)|D_i = 1$

Term has changed from: *we use **as** counter-factual* to *we use **the** counter-factual*

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Term counter-factual II

An example of the similarity of counter-factual notions:

Everyday life

Do you feel warmer with the jacket?

- Than you were before? $\Rightarrow$ closest factual
- Than you would be without? $\Rightarrow$ unobserved outcome

Counterfactual thinking is not new:

- Used every day, studied extensively in psychology
- Lewis (1973) "Causation", *Journal of Philosophy*
- Used in law, forensic medicine...

Term counter-factual III

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Another confusion in economics about Counterfactual analysis

CGE: What if policy A (country A reduces tarif by 3%)

Impact evaluation: Answers factual questions by asking counter-factual questions.

But IE answers counter-factual questions: treatment effect on the untreated.

Causal inference

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Def: Effects of the causes

The cause D causes the effect $Y_i(D=1) - Y_i(D=0)$

Fundamental Problem of Causal Inference (Holland 1980)

It is impossible to observe simultaneously the value of $Y_i(D=1)$ and $Y_i(D=0)$ on the same unit and, therefore, it is impossible to observe the effect of D for i.

Causal inference: the statistical approach

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In statistics, given the fundamental problem, we do inference for population, not individuals:

Definition: average treatment effect

$$ATE \equiv E[Y_i(1) - Y_i(0)]$$

Back to causalities

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We see now more fundamental differences between Granger and Counterfactual causality:

- Granger: Defines causes of effects: Among $X_1, X_2, \ldots, X_k$ which cause(s) Y?
- Counterfactual: Measures effects of causes: what is effect of X_1 on Y?

But is is really effect**s** of cause**s**? Most usually, is single effect of single cause.

Causal inference: the statistical approach

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Two more definitions

Definition

$$\text{ATE on treated: } ATT \equiv E[Y_i(1) - Y_i(0)|D = 1]$$
$$\text{ATE on untreated: } ATU \equiv E[Y_i(1) - Y_i(0)|D = 0]$$

Estimating the ATE

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One could think that ATE is estimated simply:

$$\begin{aligned} ATE &\equiv E[Y_i(1) - Y_i(0)] \\ &= E[Y_i(1)] - E[Y_i(0)] \end{aligned}$$

Unfortunately, we do oberve $E[Y_i(1)]$ only for the $\{j|D_j = 1\}$ i.e. we observe $E[Y_i(1)|D_i = 1]$!

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The frameworks in impact evaluation:

- **Sample selection:** Old-fashioned
- **RCT:** allocate D randomly, so $E[Y_i(1)|D_i = 1] = E[Y_i(1)]$ and hence just compare means!
- **Matching:** Assume D is based on observables: ATT and ATU, ATE
- **Diff-diff** Assume D is based only on time-invariant unobservables: ATT
- **IV:** Find exogeneous moves in D (estimating LATE/CATE)
- **RDD:** Assume D is random close to threshold: Local ATE

Identification in the sharp case

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In the sharp RDD, treatment is assigned based on whether the forcing variable x is below/above a threshold c.

$$D_i = 1(x_i \geq c) = \begin{cases} 1 & \text{if } x_i \geq c \\ 0 & \text{if } x_i < c \end{cases}$$

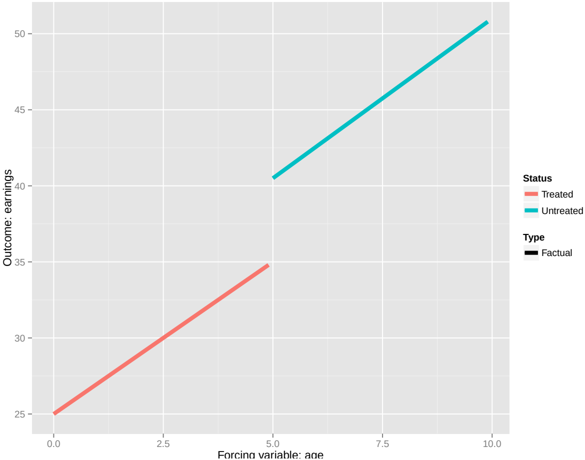
We will identify the ATE at a point:

Definition: point ATE

$$ATE(c) \equiv E[Y_i(1) - Y_i(0)|x_i = c]$$

Note here we have a local ATE in a well-defined sense (local in space of x)!

Figure 2: Assignment probabilities in sharp versus fuzzy



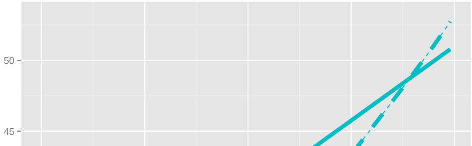
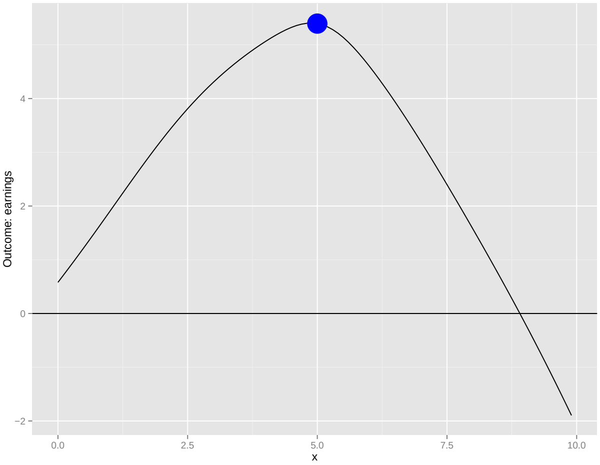


Figure 3: Assignment probabilities in sharp versus fuzzy



The RDD estimand

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To estimate:

$$ATE(c) \equiv E[Y_i(1) - Y_i(0)|x_i = c] = \overbrace{E[Y_i(1)|c]}^{\text{Factual}} - \overbrace{E[Y_i(0)|c]}^{\text{Counter-factual}}$$

we approximate the *counter-factual* untreated at c by the *factual* untreated just below c

$$\lim_{x \uparrow c} E[Y_i(0)|x_i = c] \approx E[Y_i(0)|x_i = c]$$

So end up using:

$$\overbrace{\lim_{x \downarrow c} E[Y_i(1)|x_i = c]}^{\text{Just treated}} - \overbrace{\lim_{x \uparrow c} E[Y_i(0)|x_i = c]}^{\text{Just untreated}}$$

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Assumptions

Main assumptions

- Identifying assumption: $E[Y(1)|c]$ and $E[Y(0)|c]$ are continuous in c.
- No manipulation of x around c

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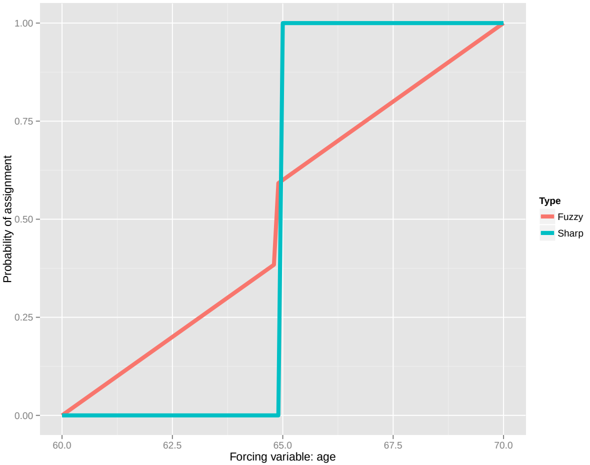
Assumption 2

Test 2: Manipulation of forcing variable

Check if individuals can manipulate their value around x?

- Age? (if *declared* age...)
- Declared income?
- Number of employees?
- Test score: can retake exam?

Figure 4: Assignment probabilities in sharp versus fuzzy



Cases

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Sharp case: $D_i = 1(x_i \geq c)$

Fuzzy case: $\lim_{x \downarrow c} Pr(D_i = 1|x) \neq \lim_{x \uparrow c} Pr(D_i = 1|x)$

Sharp new: $\overbrace{\lim_{x \downarrow c} Pr(D_i = 1|x)}^{=1} - \overbrace{\lim_{x \uparrow c} Pr(D_i = 1|x)}^{=0} = 1$

Mixed case: One-sided compliance only

Cases

In the fuzzy RDD, we use:

$$ATE \equiv \frac{\lim_{x\downarrow c} E[Y|X=x] - \lim_{x\uparrow c} E[Y|X=x]}{\lim_{x\downarrow c} E[D|X=x] - \lim_{x\uparrow c} E[D|X=x] \underbrace{\lim_{x\downarrow c} E[D|X=x] - \lim_{x\uparrow c}}_{=1 \text{ for sharp!}}}$$

This is just the form the IV takes for binary D and binary Z: Wald estimator! (see MHE 4.1.2)

$$IV_{\text{Wald}} \equiv \frac{E[Y|z_i=1] - E[Y|z_i=0]}{E[D|z_i=1] - E[D|z_i=0]}$$

Identification in the fuzzy case

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But in the fuzzy case, like with standard IV, we estimate only the complier-average effect (CATE/LATE)

And here, we only get a local version of that! $\Rightarrow$ local-local-average!

On the IV interpretation of RDD

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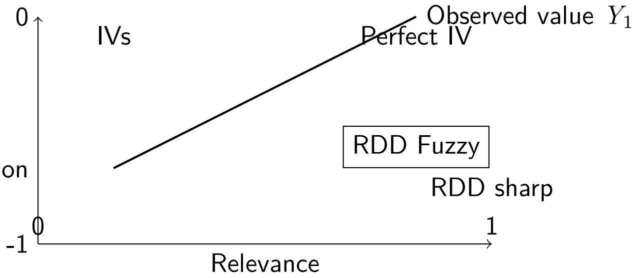
Remember the two conditions for IV with Y, X, Z:

Relevant: Should be relevant in first stage $(\beta \neq 0)$:
$X_i = \alpha + \beta Z_i + \varepsilon_i^X$

Exclusion: Should be excluded of structural form: $(\theta = 0)$:
$Y_i = \gamma + \delta X_i + \theta Z_i + \varepsilon_i^Y$

RDD recognises $\theta = 0$ only locally!



Parametric model

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We want to estimate: $\lim_{x \downarrow c} E[Y_i(1)|c] - \lim_{x \uparrow c} E[Y_i(0)|c]$

So can estimate separately on each side of each side of c:

Left: $(1-D)Y = \alpha_l + \beta_l(1-D)(X-c) + \epsilon_l$

Right: $DY = \alpha_r + \beta_r D(X-c) + \epsilon_r$

with $D = I(X \geq c)$, and test $H_0 : \quad \alpha_r = \alpha_l$

Assuming homoskedasticity ($\sigma_l = \sigma_r$, can estimate instead:

$$Y = \alpha + \tau D + \beta_1(X-c) + \beta_2 D(X-c) + \epsilon$$

Now we have $\tau = \alpha_r - \alpha_l$

Parametric model: extension II

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Eventually, we could assume the same slope on each side:

$$Y = \alpha + \tau D + \beta(X - c) + \epsilon$$

Parametric model: extension II

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Eventually, add higher-order polynomials:

$$Y = \alpha + \tau D + \beta_1^1 (X-c) + \beta_2^1 D(X-c) + \ldots + \beta_1^p (X-c)^p + \beta_2^p D(X-c)^p + \epsilon$$

What if use polynomial of order zero? **Group means test (=RCT!)**

What is parametric?

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What is parametric?

- Functional form
- Distributional assumptions (cf MLE)

What is non-parametric here?

- Local/pointwise estimation **This is exactly what we want!**

Local/pointwise estimation

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With kernel regression/smoother, want to estimate:

$$\mathrm{E}(Y|X) = m(X)$$

The idea is to estimate $\hat{y}(x_i)$ averaging over local neighbours.

$$\widehat{m}_h(x) = \sum_{i=1}^{n} w_i Y_i$$

Two approaches for w_i:

- use k nearest neighbours.
- use all neighbours within a window h.

Local/pointwise estimation

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The Nadaraya-Watson (1964) estimator use a window with weights:

$$\widehat{m}_h(x) = \frac{\sum_{i=1}^{n} \mathcal{K}_h(x - X_i)\, Y_i}{\sum_{i=1}^{n} \mathcal{K}_h(x - X_i)}$$

where $\mathcal{K}$ is a kernel with a bandwidth h.

Kernel and bandwidth choice

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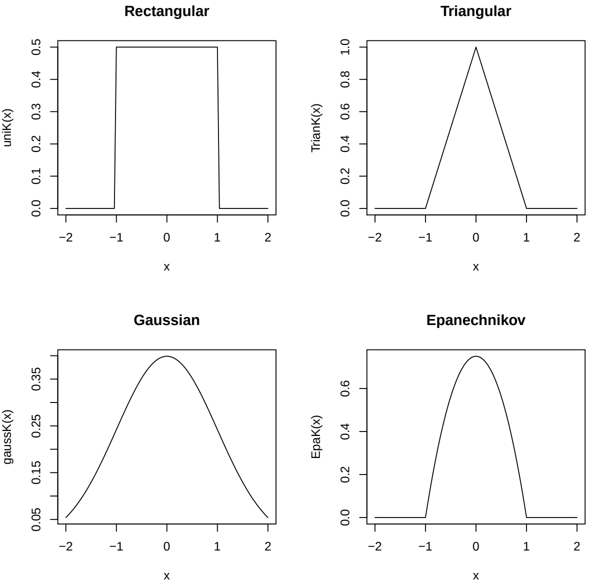
Parameters in non-parametric estimation:

Kernel $\mathcal{K}$: Rectangular, Gaussian...

Bandwidth h:
- $h \searrow$: bias $\searrow$, variance $\nearrow$
- $h \nearrow$: bias $\nearrow$, variance $\searrow$

Figure 5: various kernels



Problems

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Problems with Nadaraya-Watson:

- Biased
- Higher bias at boundary point **This is however what we want :-(**

Solution: Use local-linear regression

Local-linear regression

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Local-linear regression (LLR): fit a local weighted OLS at every point!
So we get $\hat{m}(x_i)$, and also for every point get a new $\hat{\alpha}(x), \hat{\beta}(x)$

It is a simple weighted regression, with weights obtained from kernel around x.

$$\hat{\alpha}(c), \hat{\beta}(c), \hat{\tau}(c) = \arg\min_{\alpha,\beta,\tau} \sum_{i=1}^{n} \left(Y_i - \alpha - \tau D - \beta(X_i - c)\right)^2 \mathcal{K}\left(\frac{X_i - c}{h}\right)$$

Local-linear regression

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Links with LLR:

OLS: If use LLR with $h \to \infty$ and rectangular $\mathcal{K}$: LLR $\to$ OLS!

Nadaraya-Watson: NW is a LLR with only a constant!

Choice of bandwidth

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Two procedures to choose bandwidth:

Plug-in: minimise (asymptotic) $\mathrm{MSE}(h) = \mathrm{bias}(h)^2 + \mathrm{var}(h)$
pluging-in estimates of bias and variance

Cross-validation: Minimise empirical MSE: $\frac{1}{N}\sum_{i=1}^{N}\left(Y_i - \hat{\mu}(X_i)\right)^2$

Inference

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Distribution: Normal!

Two types of variance estimators for non-parametric:

- Parametric: $\sigma \left(X' X \right)^{-1}$
- Non-parametric: Messy!!

Rate of convergence: slower than parametric!

Sensitivity tests

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- Bandwidth sensitivity
- Manipulation of forcing variable
- Placebo tests
- Discontinuity also for covariates?

Sensitivity tests: sensitivity to bandwidth

Test 1: sensitivity to bandwidth

Compute estimate at various bandwidths, compare

Sensitivity tests: Manipulation of forcing variable 1

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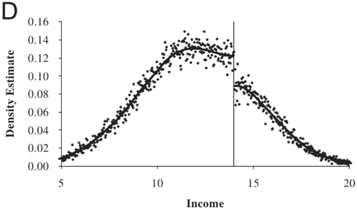
Test: look for discontinuity of density around threshold

McCrary (2008): look for discontinuity of the density of X at point c:



Sensitivity tests: Manipulation of forcing variable 2

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Actually, this approach helps also to check how a constraint is binding, in general.
Cf paper of Card and DellaVigna (2012) *How Do Authors Respond to Page Limits*

Figure 6: AER limits (Card DellaVigna 2012)

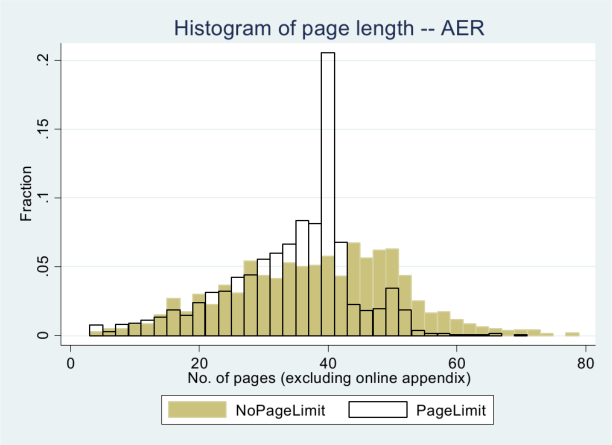
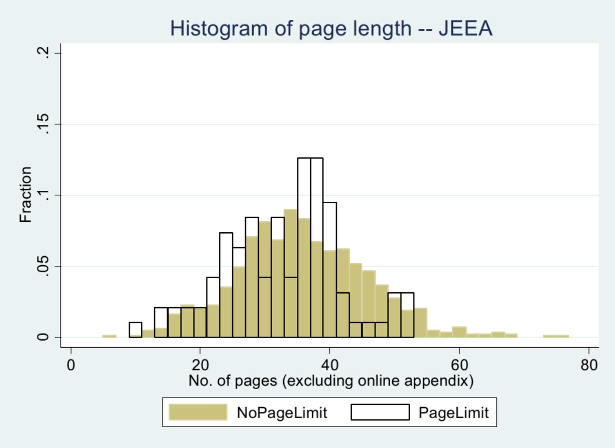


Figure 7: JEEA limits (Card DellaVigna 2012)



Sensitivity tests: Placebo tests

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Test 3: Placebo tests

Test for effect at a point known for known effect: estimate RDD at non-cutoff point.

Test 4: Discontinuity also for covariates

Like placebo, run RDD using as new y a covariate.

But passing test is not necessary! Just reduces credibility.

RDD

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RDD in a nutshell

- Estimating ATE just around cutoff
- So identify effect only around the cutoff, low external validity
- Step 1: Visual representation important
- Step 2: Estimation by parametric or non-parametric
- Step 3: Few sensitivity tests