MovieLens Recommendation System Project

Contents

1 Introduction

Recommendation Systems are popular algorithms that enable businesses to personalise their value propositions. Google and Facebook use recommendation algorithms to personalise the ads you'll be shown. Entertainment companies such as Netflix are now using recommendations to transform their digital movie platform into an entertainment space that appeals to your specific interests. Netflix's personalised strategy not only includes what movies will be recommended to a user, but also how such movies will be advertised to each. For example, two users may receive the same movie recommendation, yet for one user the movie is pitched as an action film and for the other the movie is classified as a romance rom-com. We will create a recommendation system similar to the Netflix system using the MovieLens 10M dataset provided by Grouplens.

1.1 Objective

The goal of this project will be to create a model that predicts user ratings of a movie using different features values. To calculate the accuracy of our algorithm, we will use a loss function called the residual mean squared error or RMSE on a test set. The RMSE is defined as follow:

$$RMSE = \sqrt{\frac{1}{N}\sum_{u,i}(\hat{y}_{u,i} - y_{u,i})^2}$$

Where $y_{u,i}$ represents the rating for movie i by user u, and $\hat{y}_{u,i}$ denotes the prediction for movie i by user u.

```r
#RMSE function
RMSE <- function(true_ratings, predicted_ratings){
  sqrt(mean((true_ratings - predicted_ratings)^2))
}
```

Our aim we will be to develop an algorithm with a RMSE < 0.86490.

In order to create this algorithm, we will need to first separate the data into two different sets one for training and one for testing. Then we will explore and analyse the data, develop methods to predict ratings, compare the different methods using the results and finally conclude which one is the best algorithm.

1.2 Dataset

To download MovieLens 10M, we use the code provided edx. This code also adds libraries that we will be using to conduct analysis and splits the data into two different sets, one for training, the edx dataset, and one for testing, the validation dataset.

```r
##########################################################
# Create edx set, validation set (final hold-out test set)
##########################################################

# Note: this process could take a couple of minutes

if(!require(tidyverse)) install.packages("tidyverse", repos =
                                            "http://cran.us.r-project.org")
if(!require(caret)) install.packages("caret",
                                      repos = "http://cran.r-project.org")
if(!require(data.table)) install.packages("data.table", repos =
                                             "http://cran.us.r-project.org")
if(!require(lubridate)) install.packages("lubridate", repos =
                                            "http://cran.us.r-project.org")
if(!require(gridExtra)) install.packages("gridExtra", repos =
                                            "http://cran.us.r-project.org")

library(tidyverse)
library(caret)
library(data.table)
library(lubridate)
library(gridExtra)

# MovieLens 10M dataset:
# https://grouplens.org/datasets/movielens/10m/
# http://files.grouplens.org/datasets/movielens/ml-10m.zip

dl <- tempfile()
download.file("http://files.grouplens.org/datasets/movielens/ml-10m.zip", dl)

ratings <- fread(text = gsub("::", "\t", readLines(unzip(dl, "ml-10M100K/ratings.dat"))),
                 col.names = c("userId", "movieId", "rating", "timestamp"))

movies <- str_split_fixed(readLines(unzip(dl, "ml-10M100K/movies.dat")), "\\::", 3)
colnames(movies) <- c("movieId", "title", "genres")

# if using R 3.6 or earlier:
movies <- as.data.frame(movies) %>% mutate(movieId = as.numeric(levels(movieId))[movieId],
                                            title = as.character(title),
                                            genres = as.character(genres))

# if using R 4.0 or later:

movielens <- left_join(ratings, movies, by = "movieId")

# Validation set will be 10% of MovieLens data
set.seed(1, sample.kind="Rounding") # if using R 3.5 or earlier, use `set.seed(1)`
test_index <- createDataPartition(y = movielens$rating, times = 1, p = 0.1, list = FALSE)
edx <- movielens[-test_index,]
```

```r
temp <- movielens[test_index,]

# Make sure userId and movieId in validation set are also in edx set
validation <- temp %>% semi_join(edx, by = "movieId") %>% semi_join(edx, by = "userId")

# Add rows removed from validation set back into edx set
removed <- anti_join(temp, validation)
edx <- rbind(edx, removed)

rm(dl, ratings, movies, test_index, temp, movielens, removed)
```

2 Methods and Analysis

2.1 Data Exploration and Cleaning

2.1.1 Overview

Let's first explore the data.

Here is what the first 6 data points look like:

```
##    userId movieId rating timestamp                         title
## 1:      1     122      5 838985046              Boomerang (1992)
## 2:      1     185      5 838983525               Net, The (1995)
## 3:      1     292      5 838983421               Outbreak (1995)
## 4:      1     316      5 838983392               Stargate (1994)
## 5:      1     329      5 838983392 Star Trek: Generations (1994)
## 6:      1     355      5 838984474       Flintstones, The (1994)
##                           genres
## 1:                Comedy|Romance
## 2:         Action|Crime|Thriller
## 3:  Action|Drama|Sci-Fi|Thriller
## 4:       Action|Adventure|Sci-Fi
## 5: Action|Adventure|Drama|Sci-Fi
## 6:       Children|Comedy|Fantasy
```

```
dim(edx)
```

```
## [1] 9000055       6
```

```
names(edx)
```

```
## [1] "userId"    "movieId"   "rating"    "timestamp" "title"     "genres"
```

We can see that there are 9000055 ratings and 6 columns: "userId", "movieId","rating", "timestamp","title" and "genres". MovieId and userId represent distinctive Id numbers representing in the following order a particular movie and particular user. The 'timestamp' is the time in seconds since midnight Coordinated Universal Time (UTC) of January 1, 1970, when the movie was rated. The 'title' field is simply the name of the movie and the 'genres' define the genres of the movie. The 'rating' is a whole or half number between 0 and 5 where 0 means that the user hated the movie and 5 the user loved that movie.

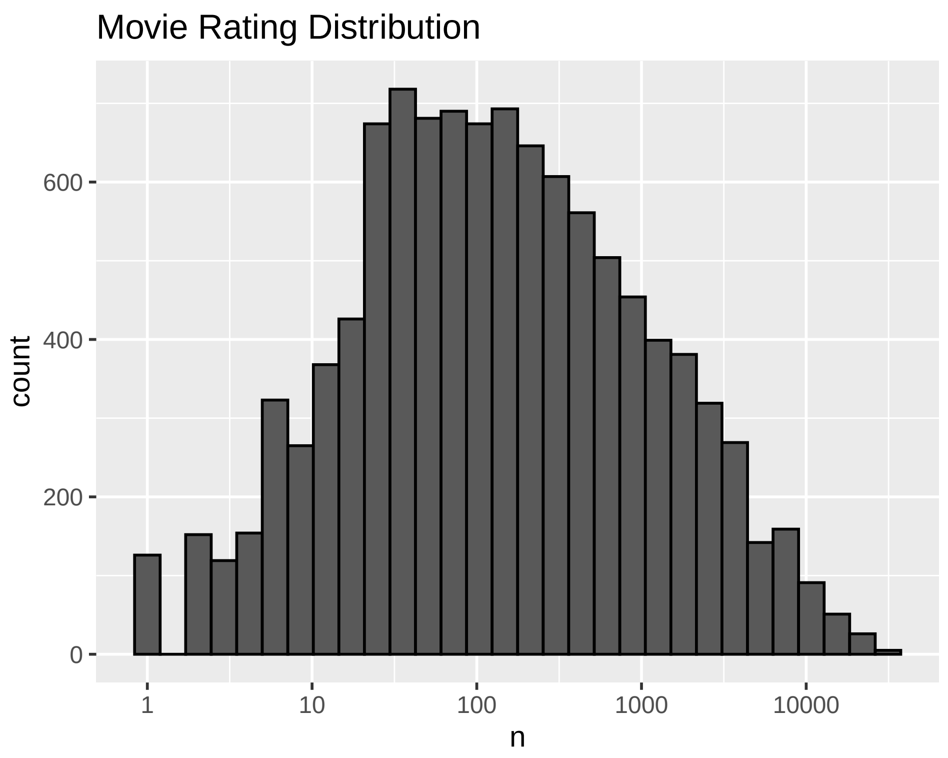
There are 69878 unique users and 10677 different movies:

```
edx %>% summarize(n_users = n_distinct(userId), n_movies = n_distinct(movieId))
```

```
##   n_users n_movies
## 1   69878    10677
```

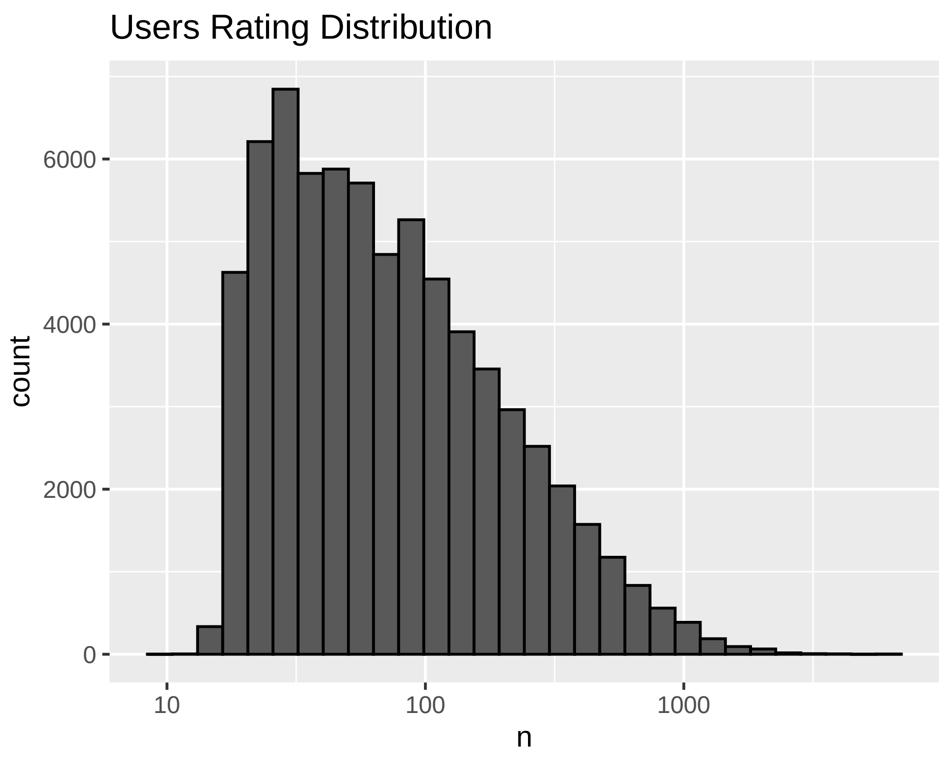
We can observe that not all movie are rated as often. Some are rated more and some less.

```
edx %>%
  dplyr::count(movieId) %>%
  ggplot(aes(n)) +
  geom_histogram(bins = 30, color = "black") +
  scale_x_log10() +
  ggtitle("Movie Rating Distribution")
```



Similarly we can observe that not all users rate at the same reccurence. Some rate often, some rate less.

```
edx %>%
  dplyr::count(userId) %>%
  ggplot(aes(n)) +
  geom_histogram(bins = 30, color = "black") +
  scale_x_log10() +
  ggtitle("Users Rating Distribution")
```



The 5 most given ratings are:

```
## Selecting by count
```

```
## # A tibble: 5 x 2
##   rating   count
##    <dbl>   <int>
## 1    4   2588430
## 2    3   2121240
## 3    5   1390114
## 4    3.5  791624
## 5    2    711422
```

The top 10 most ranked movies are:

```
## # A tibble: 10 x 2
##    title                                                        count
##    <chr>                                                        <int>
##  1 Pulp Fiction (1994)                                          31362
##  2 Forrest Gump (1994)                                          31079
##  3 Silence of the Lambs, The (1991)                             30382
##  4 Jurassic Park (1993)                                         29360
##  5 Shawshank Redemption, The (1994)                             28015
##  6 Braveheart (1995)                                            26212
##  7 Fugitive, The (1993)                                         25998
##  8 Terminator 2: Judgment Day (1991)                            25984
##  9 Star Wars: Episode IV - A New Hope (a.k.a. Star Wars) (1977) 25672
## 10 Apollo 13 (1995)                                             24284
```

2.1.2 Genres Observations

We can observe that every movie can be of different genres:

```
##                            title                         genres
## 1:               Boomerang (1992)                 Comedy|Romance
## 2:                Net, The (1995)          Action|Crime|Thriller
## 3:                Outbreak (1995)   Action|Drama|Sci-Fi|Thriller
## 4:                Stargate (1994)        Action|Adventure|Sci-Fi
## 5: Star Trek: Generations (1994) Action|Adventure|Drama|Sci-Fi
## 6:        Flintstones, The (1994)        Children|Comedy|Fantasy
```

There are exactly 20 different genres and 797 different combinations of genres.

```
popular_genres <- edx %>% group_by(genres) %>% summarize(count = n()) %>% arrange(desc(count))
different_genres <- popular_genres %>% select(genres) %>%
  separate_rows(genres, sep = "\\|")%>% unique()
number_different_genres <- nrow(different_genres)

different_combination <- edx %>% summarize(n_genre = n_distinct(genres)) %>% .$n_genre
genres <- data_frame(n_genres = number_different_genres, n_combination_genres = different_combination)
```

```
## Warning: `data_frame()` was deprecated in tibble 1.1.0.
## Please use `tibble()` instead.
```

```
genres %>% knitr::kable()
```

n_genres	n_combination_genres
20	797

Every movie has a genre, and if the genre is not defined it is marked as even if it is not listed "(no genres listed)"

```
#There is no NA genres
edx %>% filter(genres == NA)
```

```
## Empty data.table (0 rows and 6 cols): userId,movieId,rating,timestamp,title,genres
```

```
edx %>% select(title, genres) %>% filter(genres == "(no genres listed)") %>% slice(1:1)
```

```
##                     title             genres
## 1: Pull My Daisy (1958) (no genres listed)
```

The top 10 genres are:

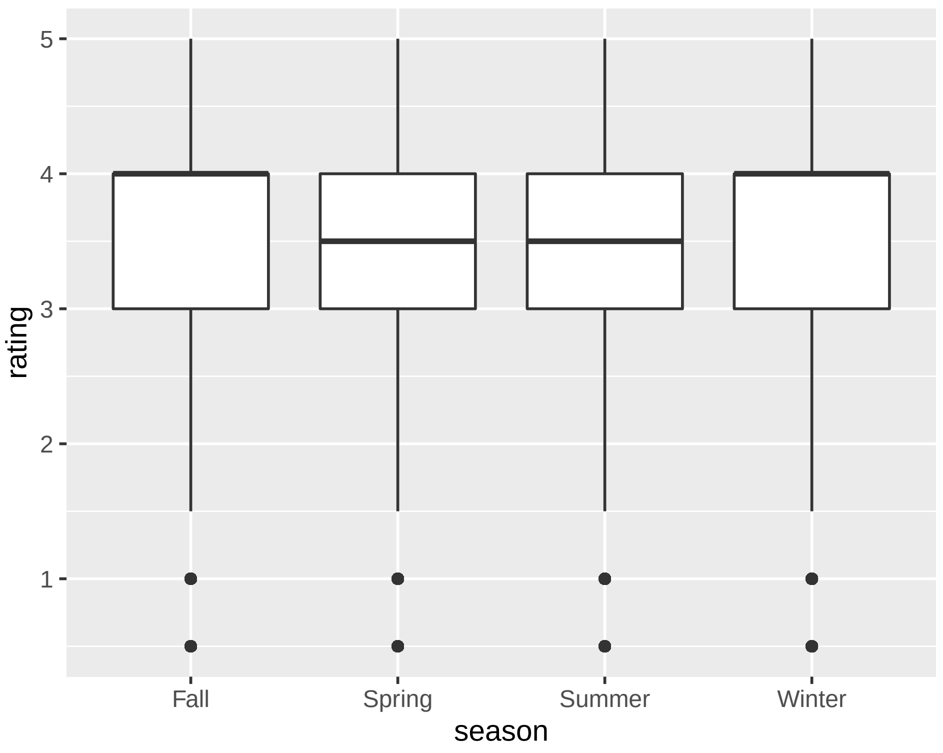
```
## # A tibble: 10 x 2
##    genres                      count
##    <chr>                       <int>
##  1 Drama                      733296
##  2 Comedy                     700889
##  3 Comedy|Romance             365468
##  4 Comedy|Drama               323637
##  5 Comedy|Drama|Romance       261425
##  6 Drama|Romance              259355
##  7 Action|Adventure|Sci-Fi   219938
##  8 Action|Adventure|Thriller 149091
##  9 Drama|Thriller             145373
## 10 Crime|Drama                137387
```

2.1.3 Timestamp Observations

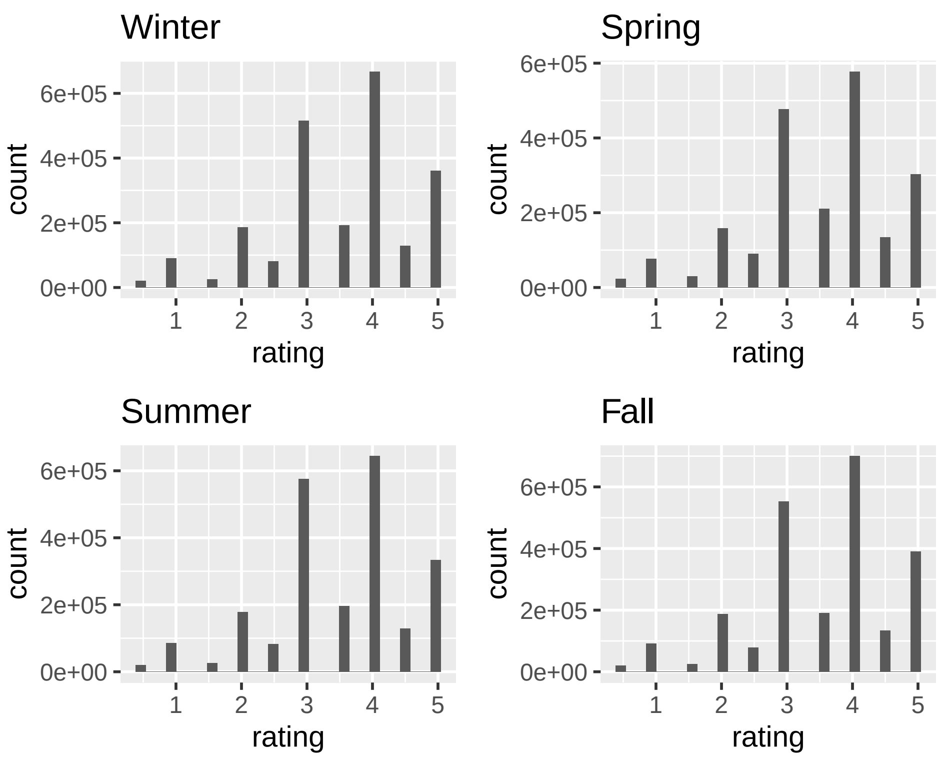
Firstly, let's convert the timestamp into a time that is more readible. We will create a new dataset from edx to add new fields. The field time represents the new time as a date time.From the 'time' field, we will extract the year, month and season, assuming that all the ratings were done in the northern hemisphere.

```r
new_edx <- edx %>% mutate(time=as_datetime(timestamp))
new_edx <- new_edx %>% mutate(year=year(time), month=month(time))
new_edx <- new_edx %>% mutate(season=case_when(.$month%in%c(12,1,2)~"Winter",
                                               .$month%in%c(3,4,5)~"Spring",
                                               .$month%in%c(6,7,8)~"Summer",
                                               .$month%in%c(9,10,11)~"Fall"))
```

Let's now analyze the distribution of ratings per season and see if the season might influence the way a user rates a movie.



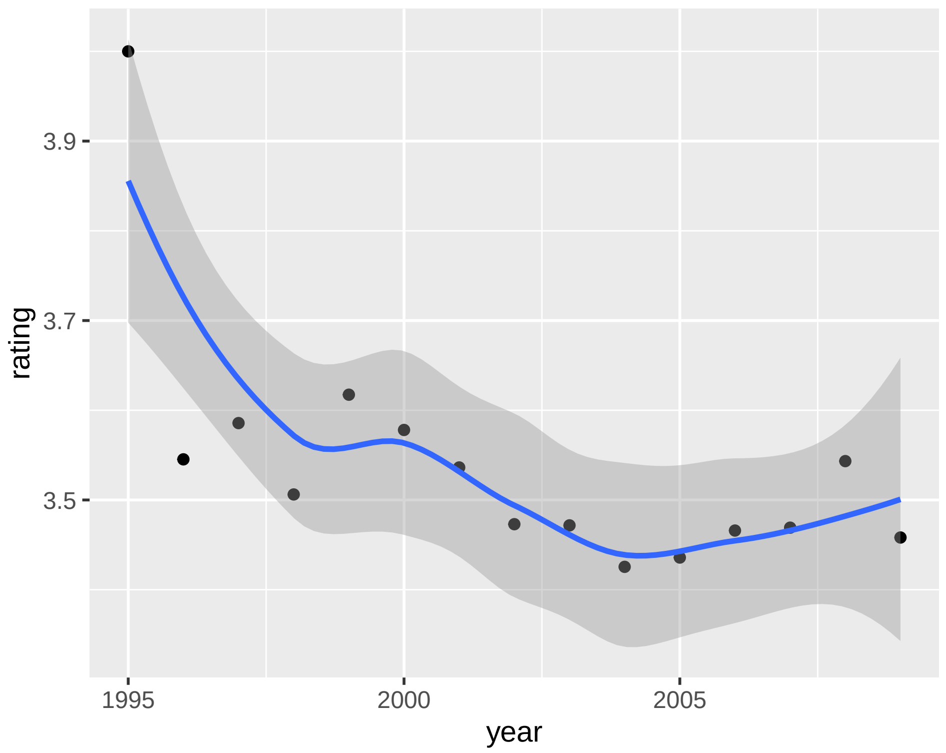
There does not seem to be not of variability between seasons but let's make a plot counting the number of different ratings per season:



We can observe that the season is not a good indicator in predicting the rating of a movie as it seems to be pretty consistant every season. We will then ignore the season field.

Let's investigate if the year a movie was rated influences the ratings metric.

```
new_edx %>% group_by(year) %>%
  summarize(rating = mean(rating)) %>%
  ggplot(aes(year, rating)) + geom_point() + geom_smooth()
```



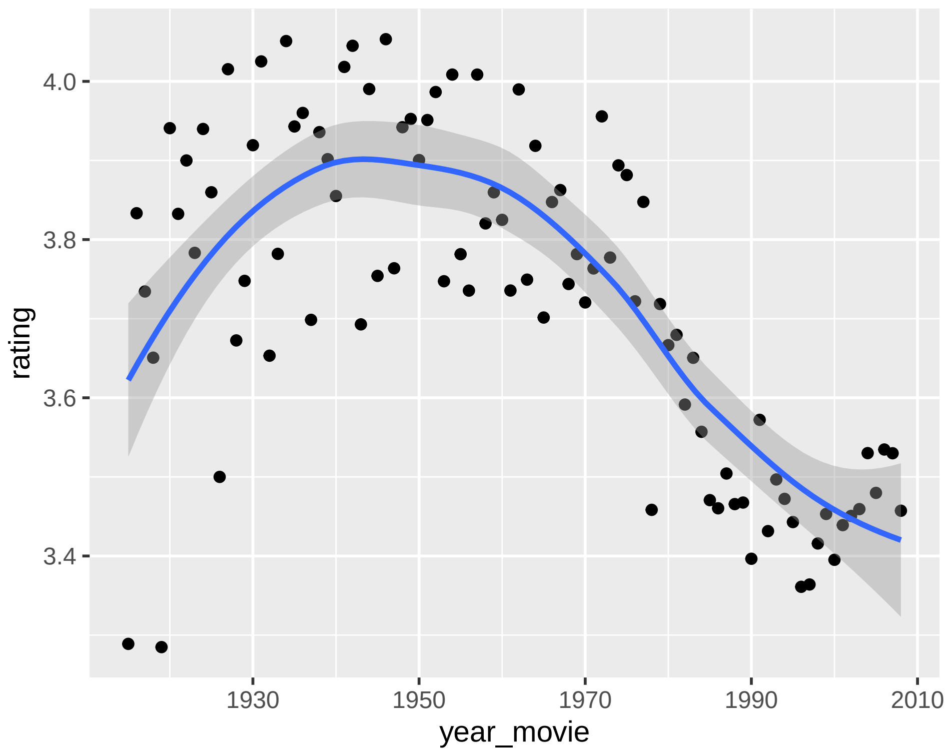
We can definitely see more variability that ranges between 4 and 3.4, we will use this metric to predict the rating of a movie.

2.1.4 Title and Year

Each movie title has in its title the year the movie came out. Let's extract that year and see if it has any influence on the rating of the movie.

```
new_edx<-new_edx%>%mutate(year_movie = substr(title,nchar(as.character(title))-4,nchar(as.character(titl
#we first need to convert this as a date, so we will add jan 1st to the year. This will allow us to ext
new_edx<- new_edx %>% mutate(year_movie=paste(year_movie,"-01-01", sep = ""))
new_edx<- new_edx %>% mutate(year_movie=year(year_movie))
new_edx %>%  group_by(year_movie) %>%
  summarize(rating = mean(rating)) %>%
  ggplot(aes(year_movie, rating)) +  geom_point()  + geom_smooth()
```

```
## `geom_smooth()` using method = 'loess' and formula 'y ~ x'
```



We can observe that the year the movie was released influences the rating so we will use it in our models.

2.1.5 Data Cleaning

Now let's only select the data we will use for our models from the new edx dataset and apply the same logic to the validation dataset.

```
#new edx dataframe
new_edx<- new_edx%>% select(-time,-timestamp,-season,-month)

#new validation dataframe
new_validation <- validation %>% mutate(time=as_datetime(timestamp))
new_validation <- new_validation %>% mutate(year=year(time))
new_validation<-new_validation %>%
  mutate(year_movie = substr(title,nchar(as.character(title))-4,nchar(as.character(title))-1))
new_validation<-new_validation%>%mutate(year_movie=paste(year_movie,"-01-01", sep = ""))
new_validation<-new_validation%>%mutate(year_movie=year(year_movie))
new_validation <- new_validation%>%select(-time,-timestamp)
```

2.2 Models

For the following models we will use the RMSE function that we defined earlier to calculate the accuracy of our model. We can interpret the RMSE similarly to a standard deviation: it is the typical error we make when predicting a movie rating. If this number is larger than 1, it means our typical error is larger than one star, which is not good.

2.2.1 The Naive Average Rating Movie Method

Let's predict that the rating of a movie is similar for all movies regardless of the user. This model will assume the same rating for all movies and users with all the differences explained by random variation. The function is defined as follow:

$$Y_{u,i} = \mu + \epsilon_{u,i}$$

where $\epsilon_{u,i}$ is independent errors sampled from the same distribution centered at 0 and μ the "true" rating for all movies.

```
#calculating the average of all ratings
mu_hat <- mean(new_edx$rating)
mu_hat
```

```
## [1] 3.512465
```

```
#calculate the RMSE
naive_rmse <- RMSE(new_validation$rating, mu_hat)
naive_rmse
```

```
## [1] 1.061202
```

```
#Store the value of the RMSE in a table
rmse_results <- data_frame(method = "Just the average rating", RMSE = naive_rmse)
rmse_results %>% knitr::kable()
```

method	RMSE
Just the average rating	1.061202

We get an RMSE of 1.06 which means that we predict with an error of one star which is pretty significant. Let's see if we can do better.

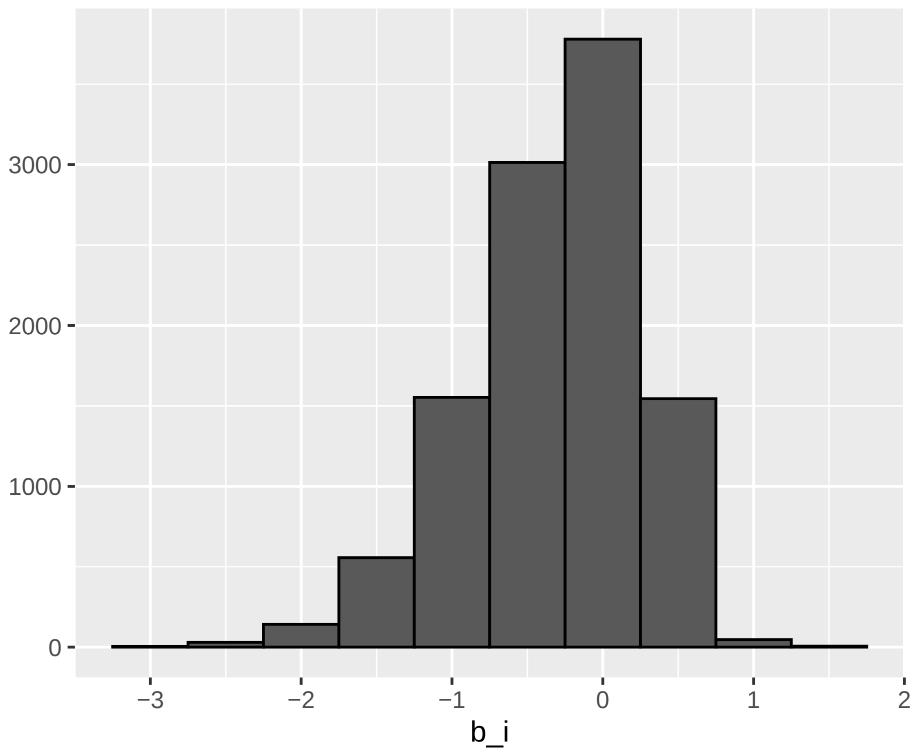
2.2.2 Movie Effect Model

As we have observed earlier, we know that some movies are rated higher than others. Let's include this in our model by adding a bias b_i to represent average ranking for movie i :

$$Y_{u,i} = \mu + b_i + \epsilon_{u,i}$$

```
mu <- mean(new_edx$rating)
#caculate b_i
movie_avgs <- new_edx %>%
  group_by(movieId) %>%
  summarize(b_i = mean(rating - mu))
```

We can see that b_i varies between approximately -3 and 1.5.



Please note that μ was equal to roughly 3.5, which implies that a perfect star would be given when b_i is equal to 1.5.

Let's now evaluate our model:

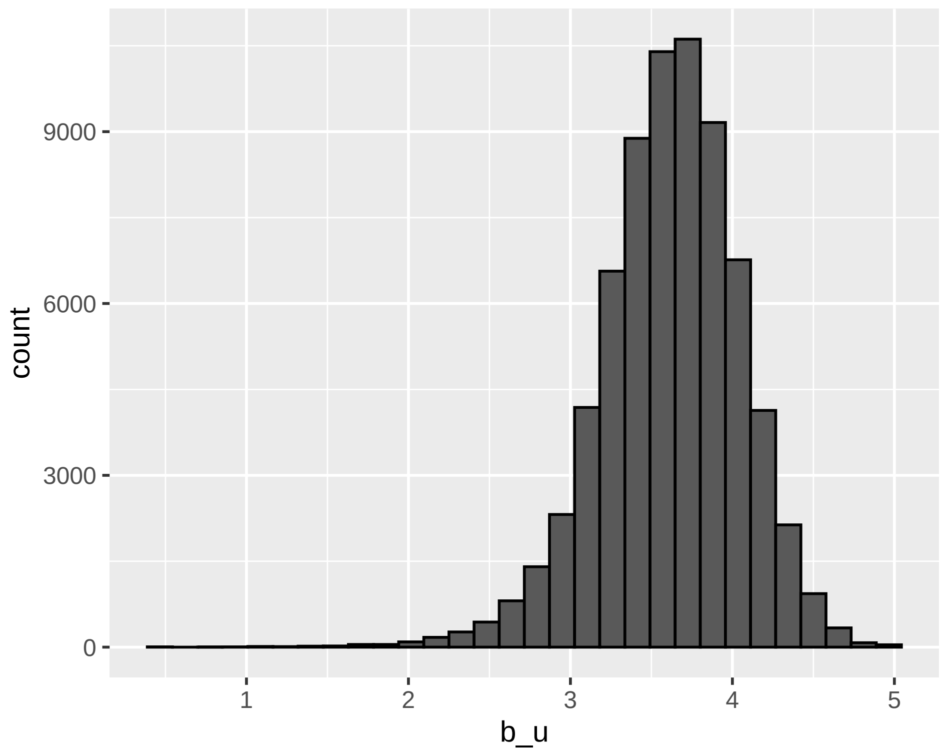
```r
#calculate b_i for new_validation
predicted_ratings <- mu + new_validation %>%
  left_join(movie_avgs, by='movieId') %>%
  .$b_i

#calculate the RMSE
model_1_rmse <- RMSE(predicted_ratings, new_validation$rating)
#store result
rmse_results <- bind_rows(rmse_results,
                          data_frame(method="Movie Effect Model",
                                     RMSE = model_1_rmse ))
#show result
rmse_results %>% knitr::kable()
```

method	RMSE
Just the average rating	1.0612018
Movie Effect Model	0.9439087

Our RMSE is still pretty high, let's see if we can do better.

2.2.3 Movie + User Effect Model

If we compute the average rating for user u for those that have rated 100 or more movies, we can observe some variability too here:



Let's incorporate this finding and see if we can improve our model by adding a user specific effect $b_u$to represent average rating given by user u:

$$Y_{u,i} = \mu + b_i + b_u + \epsilon_{u,i}$$

```r
#calculating b_u for new_edx
user_avgs <- new_edx %>%
  left_join(movie_avgs, by='movieId') %>%
  group_by(userId) %>%
  summarize(b_u = mean(rating - mu - b_i))

#calculating b_u for new_validation
predicted_ratings <- new_validation %>%
  left_join(movie_avgs, by='movieId') %>%
  left_join(user_avgs, by='userId') %>%
  mutate(pred = mu + b_i + b_u) %>%
  .$pred

#calculating the RMSE for this model
model_2_rmse <- RMSE(predicted_ratings, new_validation$rating)

#Storing the RMSE for this model
rmse_results <- bind_rows(rmse_results,
                          data_frame(method="Movie + User Effects Model",
                                     RMSE = model_2_rmse ))
```

method	RMSE
Just the average rating	1.0612018
Movie Effect Model	0.9439087
Movie + User Effects Model	0.8653488

We can observe that we have signigicantly improve our model and reduced our RMSE to 0.8653488.

2.2.4 Movie + User + Year_Movie Effect Model

Earlier we saw that the year a movie was released could influence the rating of a movie. Let's add it to the function where b_{iym} represents the average rating per year when movie i was released:

$$Y_{u,i} = \mu + b_i + b_u + b_{iym} + \epsilon_{u,i}$$

```r
#calculating b_iym for new_edx
year_movie_avgs <- new_edx %>%
  left_join(movie_avgs, by='movieId') %>%
  left_join(user_avgs, by='userId') %>%
  group_by(year_movie) %>%
  summarize(b_iym = mean(rating - mu - b_i- b_u))

#calculating b_iym for new_validation
predicted_ratings <- new_validation %>%
  left_join(movie_avgs, by='movieId') %>%
  left_join(user_avgs, by='userId') %>%
  left_join(year_movie_avgs, by='year_movie') %>%
  mutate(pred = mu + b_i + b_u +b_iym) %>%
  .$pred

#calculating the RMSE for this model
model_3_rmse <- RMSE(predicted_ratings, new_validation$rating)

#Storing the RMSE for this model
rmse_results <- bind_rows(rmse_results,
                          data_frame(method="Movie + User + Year_Movie Effects Model",
                                     RMSE = model_3_rmse ))
```

Our model has improved but not significantly.

method	RMSE
Just the average rating	1.0612018
Movie Effect Model	0.9439087
Movie + User Effects Model	0.8653488
Movie + User + Year_Movie Effects Model	0.8650043

2.2.5 Movie + User + Year_Movie + Year rated Effects Model

Earlier we saw that the year a movie was rated could influence the rating of a movie. Let's add it to the function where b_{iuy} represents the average rating per year when movie i was rated by user u:

$$Y_{u,i} = \mu + b_i + b_u + b_{iym} + b_{iuy} + \epsilon_{u,i}$$

```r
#calculating b_iuy for new_edx
year_avgs <- new_edx %>%
  left_join(movie_avgs, by='movieId') %>%
  left_join(user_avgs, by='userId') %>%
  left_join(year_movie_avgs, by='year_movie') %>%
  group_by(year) %>%
  summarize(b_iuy = mean(rating - mu - b_i- b_u-b_iym))
```

```
#calculating b_iuy for new_validation
predicted_ratings <- new_validation %>%
  left_join(movie_avgs, by='movieId') %>%
  left_join(user_avgs, by='userId') %>%
  left_join(year_movie_avgs, by='year_movie') %>%
  left_join(year_avgs, by='year') %>%
  mutate(pred = mu + b_i + b_u +b_iym +b_iuy) %>%
  .$pred

#calculating the RMSE for this model
model_4_rmse <- RMSE(predicted_ratings, new_validation$rating)

#Storing the RMSE for this model
rmse_results <- bind_rows(rmse_results,
                          data_frame(method = "Movie + User + Year_Movie
                                     + Year rated Effects Model",
                                     RMSE = model_4_rmse ))
```

Our model has improved but not significantly.

method	RMSE
Just the average rating	1.0612018
Movie Effect Model	0.9439087
Movie + User Effects Model	0.8653488
Movie + User + Year_Movie Effects Model	0.8650043
Movie + User + Year_Movie + Year rated Effects Model	0.8649285

2.2.6 Movie + User + Year_Movie + Year rated + Genres Effects Model

Earlier we saw that the genres of a movie could influence the rating of a movie. Let's add it to the function where b_g represents the average rating per genres:

$$Y_{u,i} = \mu + b_i + b_u + b_{iym} + b_{iuy} + b_g + \epsilon_{u,i}$$

```
#calculating b_g for new_edx
genre_avgs <- new_edx %>%
  left_join(movie_avgs, by='movieId') %>%
  left_join(user_avgs, by='userId') %>%
  left_join(year_movie_avgs, by='year_movie') %>%
  left_join(year_avgs, by='year') %>%
  group_by(genres) %>%
  summarize(b_g = mean(rating - mu - b_i- b_u - b_iym - b_iuy))

#calculating b_g for new_validation
predicted_ratings <- new_validation %>%
  left_join(movie_avgs, by='movieId') %>%
  left_join(user_avgs, by='userId') %>%
  left_join(year_movie_avgs, by='year_movie') %>%
  left_join(year_avgs, by='year') %>%
  left_join(genre_avgs, by='genres') %>%
  mutate(pred = mu + b_i + b_u +b_iym +b_iuy +b_g) %>%
  .$pred

#calculating the RMSE for this model
model_5_rmse <- RMSE(predicted_ratings, new_validation$rating)
#Storing the RMSE for this model
rmse_results <- bind_rows(rmse_results,
                          data_frame(method="Movie + User + Year_Movie +
                                     Year rated + Genres Effects Model",
                                     RMSE = model_5_rmse ))
```

Our model has improved but not significantly.

method	RMSE
Just the average rating	1.0612018
Movie Effect Model	0.9439087
Movie + User Effects Model	0.8653488
Movie + User + Year_Movie Effects Model	0.8650043
Movie + User + Year_Movie + Year rated Effects Model	0.8649285
Movie + User + Year_Movie + Year rated + Genres Effects Model	0.8645943

2.2.7 Regularized Movie + User Effect Model

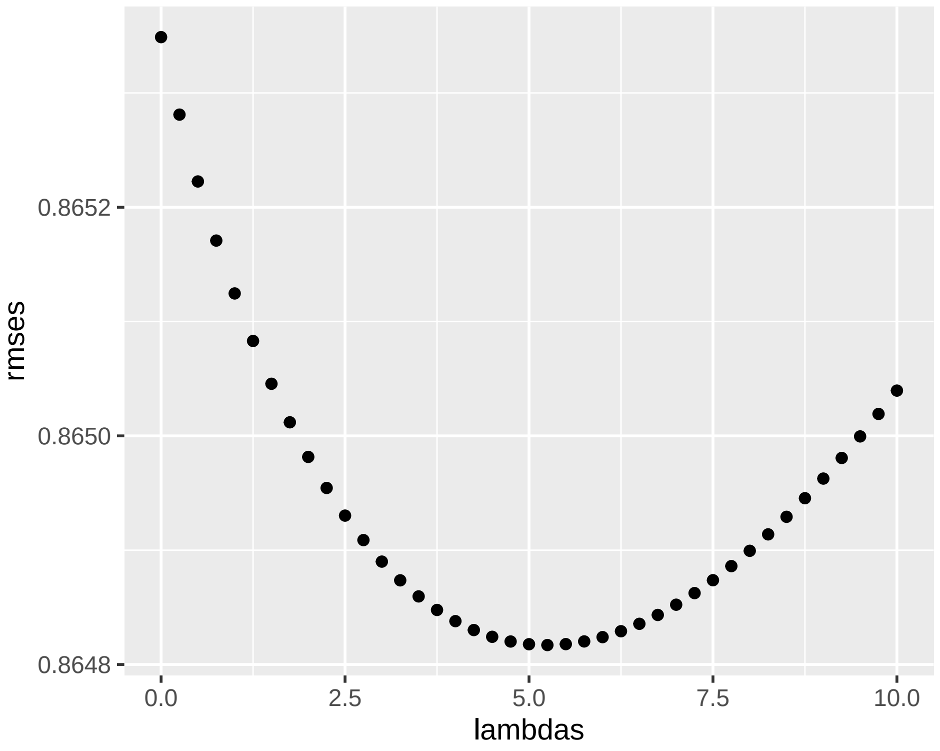
We know that some movies are rated more or less than others and some users rate more or less often than others. In our last models, this was not incorporated and could strongly overestimate or underestimate a rating of a movie by user u. We will regularization technique to penalise that aspect. For this model we will try to minmize the following equation :

$$\sum_{u,i}(y_{u,i} - \mu - b_i - b_u)^2 + \lambda((\sum_{i}(b_i^2) + \sum_{u}(b_u^2)))$$

We will use cross validation to find λ, the tuning parameter using the following code:

```
#define the different lambdas
lambdas <- seq(0, 10, 0.25)

#cross validation algorithm
rmses <- sapply(lambdas, function(l){
  #average rating
  mu <- mean(new_edx$rating)

  #finding b_i for new_edx using l
  b_i <- new_edx  %>%
    group_by(movieId) %>%
    summarize(b_i = sum(rating - mu)/(n()+l))

  #finding b_u for new_edx  using l
  b_u <- new_edx %>%
    left_join(b_i, by="movieId") %>%
    group_by(userId) %>%
    summarize(b_u = sum(rating - b_i - mu)/(n()+l))

  #Calculating the RMSE with l
  predicted_ratings <-
    new_validation %>%
    left_join(b_i, by = "movieId") %>%
    left_join(b_u, by = "userId") %>%
    mutate(pred = mu + b_i + b_u) %>%
    .$pred
  return(RMSE(predicted_ratings, new_validation$rating))
})
```

Here is a graph of all the different lambdas for our regularized model:



```
#find minimum lambda
lambda <- lambdas[which.min(rmses)]
lambda
```

```
## [1] 5.25
```

```
#pick the lambda that minimises the RMSE
rmse_results <- bind_rows(rmse_results,
                          data_frame(method="Regularized Movie + User Effect Model",
                                     RMSE = min(rmses)))
```

We can observer that our Regularized Movie + User Effect Model is more effective than our simple Movie + User Effects Model.

method	RMSE
Just the average rating	1.0612018
Movie Effect Model	0.9439087
Movie + User Effects Model	0.8653488
Movie + User + Year_Movie Effects Model	0.8650043
Movie + User + Year_Movie + Year rated Effects Model	0.8649285

method	RMSE
Movie + User + Year_Movie + Year rated + Genres Effects Model	0.8645943
Regularized Movie + User Effect Model	0.8648170

3 Results

From our RMSE table:

method	RMSE
Just the average rating	1.0612018
Movie Effect Model	0.9439087
Movie + User Effects Model	0.8653488
Movie + User + Year_Movie Effects Model	0.8650043
Movie + User + Year_Movie + Year rated Effects Model	0.8649285
Movie + User + Year_Movie + Year rated + Genres Effects Model	0.8645943
Regularized Movie + User Effect Model	0.8648170

We can observe that the most effective model is the Movie + User + Year_Movie + Year rated + Genres Effects Model with an RMSE equal to 0.8645943 which is below our objective of 0.86490.

4 Conclusion

In conclusion, our most optimal model to predict the rating of a movie is the Movie + User + Year_Movie + Year rated + Genres Effects Model. We could use regularization on this model and add more effects to this model but this would make the calculation too long. We can observe that the more effects we add the smaller the RMSE becomes. However, the RMSE does not decrease more significantly and stays around approximately 0.86, which means that we need either more data or new methods if we want our RMSE to drop significantly. Hopefully, soon we will be able to use quantum computers which will allow us data scientist to create better models with more data.