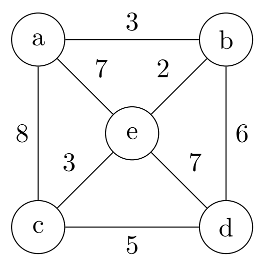
Weekly Assignment 7:
Greedy Algorithms

1. We consider the following graph.



   (a) Run Prim's algorithm on the graph. For each iteration, indicate: the vertex added to the spanning tree, the priority queue for the remaining vertices and, for each vertex, predecessor in the spanning tree and key.

   (b) Run Kruskal's algorithm on the same graph. For each iteration, indicate the vertex and edge added to the spanning tree.

   (c) Show that we can use Prim's algorithm to sort an array of size n.

2. Recall that the *closed (real) interval* $[x, y]$ is the set $\{z \in \mathbb{R} \mid x \leq z \leq y\}$, and its *length* is $y - x$. Given a set of real numbers $S = \{x_1, x_2, \ldots, x_n\}$, with $x_i \leq x_{i+1}$, you want to determine the smallest set of closed intervals of length 1 such that every $x \in S$ belongs to at least one interval.

   (a) Give a greedy algorithm to solve the problem.

   (b) Explain why your algorithm is correct.

3. We say that a sequence S' is a *subsequence* of S if there is a way to delete zero or more elements from S so that the remaining elements, in order, are equal to the sequence S'.

   Give a greedy algorithm that takes two sequences — S' of length m and S of length n, each possibly containing an element more than once — and decides in time $O(m + n)$ whether S' is a subsequence of S.

   Explain why your algorithm is correct.

4. Consider a container with capacity W together with a collection of n different materials. The value per unit of the i-th material is v_i, and the total available quantity of the i-th material is w_i. The problem is to choose an amount $x_i \leq w_i$ of each material such that it fits in the container

$$\sum_i x_i \leq W,$$

   while maximizing the total value

$$\sum_i x_i v_i.$$

   (We assume all quantities are nonnegative.) Give an $O(n \log n)$ algorithm that solves this problem.