The blog starts with an axiomatic definition of sets. It will then continue to explore set theory, diving into relations, including orders, equivalences and functions. Once functions have entered the play, we introduce the axiom of choice and various equivalent axioms, and study their equivalence using the theory of well-orders.

When defining sets, the aim is not to explain "what" a set is, but rather to specify the axioms according to which sets behave. The notion of a set is simply too abstract to describe intuitively, moreover if we want to do rigorous mathematics with them then the last thing we want is only an intuitive notion: we need the laws according to which the object behaves to make rigorous derivations of other properties.

This separation between intuition and formalism will be taken a step further when we will discuss logic. In first order logic, we study theories, which are sets of logical sentences, another mathematical object, in two ways: one way is to interpret the sentences using a model, that is, we define what it means for a sentence to hold in a structure and if a structure satisfies all sentences of the theory, it "models" the sentence. This is a good way to create an intuition of the objects that are described by the theory. The other way of studying theories looks not at interpretations of sentences but at rules for recombining logical sentences into new logical sentences. The recombination leads us to definitions such as proof trees and ultimately to the notion of a formal "proof" of a "theorem".

If you have not seen mathematical logic yet, be aware that these proofs are different from a proof using natural language that is usual in mathematics texts. Formal proofs in logic are mathematical objects that we can again prove properties/theorems about in natural language. In a sense, we study the way we prove and reason about mathematical things from a meta-level.

Even though logic studies mathematical reasoning at the lowest possible level of abstraction, a rigorous course in model theory and proof trees is usually only given in the second year of a math degree. The reason is that students should be familiar with theories such as those of the natural, integer, rational and real numbers before being exposed to formal theories and models for them. I highly recommend reading through these posts before continuing. The theory of rings, groups, fields and ordered fields there is only a small subset of all that can really be explored in, say, a course such as Group Theory followed by one on Rings and Fields. It is unfortunate. If you do like to know, you should become a math major (advertising intended)!