# Journey Through L-Functions

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## 1 Introduction

The aim of this piece of writing is to outline the journey through which one may encounter L functions in their various guises, to hopefully give some motivation on why they are of such interest, and to (again, hopefully) convey the beauty with which they meld together so many areas of maths. Indeed, L-functions dip their toes in Number Theory and Algebraic Geometry for sure, but also draw upon tools and techniques of analysis, representation theory, measure theory, combinatorics and functional analysis!

As of current, I'm not 100% sure where this journey will end up! I of course cannot cover everything about L-functions in depth there is simply too much. I mean, currently my list below doesn't really talk about automorphic L functions (eg. looking at those form eigenforms to start with) (we do mention the ideles and Hecke L functions so there is something Aut here)

Brief Topic list I want to look at

- Classical L functions encountered in ANT - Riemann and Dedekind Zeta functions, and Dirichlet L functions
- We can then generalise these to looking at Ray class characters when considering Abelian extensions and CFT
- Then we can further zoom out and look at the Ideles and Hecke L-functions (from an idelic point of view) - perhaps with this we should mention Tate's thesis - the change in perspective that it provokes - although I haven't yet read through his thesis.
- Next is natural to look at Artin L functions - this kinda introduces Galois representations.
- We can more generally look at Galois reps - then this proceeds to $\ell$ adic Galois reps - leads to geometry and Hasse weil zeta functions and the L functions coming from action on etale cohom.
- I've always had in my head this idea of the marriage of Artin L functions and Hecke (grossencharacter) L functions is obtained by passing to the Weil group - but shall have to look over this! - perhaps look at Number theoretic background again by Tate
- Then we should also mention modular forms and more automorphic ideas.

Clearly quite a loose plan - anyhoo good to write something down. I'm also reading about p adic L functions at the moment which itself is very interesting. What breadth! Perhaps I split into multiple different bits and bobs. Current issue is that I've scraped the surface of many of these topics - but to have a wide *and* deep view of things takes years.