CDS PRICING



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- A credit default swap (CDS) is a financial derivative which is used to transfer the credit risk of a reference entity from one party to another
- In a standard CDS contract one party purchases credit protection from another party, to cover the loss of the face value of an asset following a credit event
- CDS are expressed in basis points (one b-point is 0.01% and therefore 100 b-points are equal to 1%).



When Are Credit Default Swaps Used?

Speculation

Because swaps are traded, they naturally have fluctuating market values that a CDS trader can profit from. Investors buy and sell CDSs from each other, attempting to profit from the difference in prices.

Hedging

A credit default swap by itself is a form of hedging. A bank might purchase a CDS to hedge against the risk of the borrower defaulting. Insurance companies, pension funds, and other securities holders can purchase CDSs to hedge credit risk.

Arbitrage

Arbitrage generally involves purchasing a security in one market and selling it in another. CDSs can be used in arbitrage—an investor can purchase a bond in one market, then buy a CDS on the same reference entity on the CDS market.



Key Words: CDS Spreads and Probability of Default

CDS Spreads

It is the amount that the buyer must pay to the seller during the contract period according to a specific schedule (annual, quarterly, ...) for protection, and this amount is a percentage of the actual amount. Payments will be made until the CDS contract expires or until the credit event occurs.

Default Probability

The Probability of Default is a forward-looking Expectation Measure, which assigns a value between zero and one to the likelihood of a defined Credit Event (such as default, bankruptcy ecc..), within a specified time horizon.

If the market expects a specific asset to default, its price in the market will fall (everyone would be trying to sell the asset). Therefore, the market's expectation of an asset's probability of default can be obtained by analyzing the market for credit default swaps of the asset.

From default probability to CDS spread

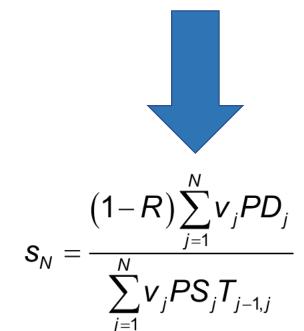
Where:

- s = spread
- R = recovery rate
- v = discount factor
- PS = probability of survival
- PD = probability of default

$$s\sum_{j=1}^{N} v_{j}PS_{j}T_{j-1,j} + s\sum_{j=1}^{N} v_{j}PD_{j}\frac{T_{j-1,j}}{2} = (1-R)\sum_{j=1}^{N} v_{j}PD_{j}$$

Expected PV of stream of premium payments if no default occurs Expected PV of accrued premium payment in period when default occurs

Expected PV of default payment in period when default occurs



From CDS spread to default probability

$$s = \frac{(1-R)\sum_{j=1}^{N} v_{j}PD_{j}}{\sum_{j=1}^{N} v_{j}PS_{j}T_{j-1,j} + \sum_{j=1}^{N} v_{j}PD_{j}\frac{T_{j-1,j}}{2}}$$



$$PD_{N} = \frac{s_{N} \left[\sum_{j=1}^{N-1} v_{j} T_{j-1,j} PS_{j} + v_{N} T_{N-1,N} PS_{N-1} \right] - (1-R) \sum_{j=1}^{N-1} v_{j} PD_{j}}{v_{N} \left[s_{N} T_{N-1,N} + (1-R) \right]}$$



Implementation in python

	В1	D1	В2	D2	в3	D3	В4	D4	В5	D5
Company										
Fiat	750	850	740	840	720	820	670	740	630	680
Ericsson	275	375	335	415	450	500	435	485	470	480
British Airways	450	550	450	550	450	550	450	550	400	500
Merrill Lynch	30	34	30	35	33	37	35	39	39	43

DATASET

SPREAD FROM THE MARKET

```
#spread is (bid+ask)/2
mid_spread=[(df.iloc[credit,maturity]+df.iloc[credit,maturity+1])/2 for maturity in
df_CDS=pd.DataFrame()
j=0
for credit in df.Company:
    df_CDS[credit]=[mid_spread[i] for i in range(j,20,4)]
    j+=1

df_CDS
```

	Fiat	Ericsson	British Airways	Merrill Lynch
0	800.0	325.0	500.0	32.0
1	790.0	375.0	500.0	32.5
2	770.0	475.0	500.0	35.0
3	705.0	460.0	500.0	37.0
4	655.0	475.0	450.0	41.0

FROM SPREAD TO DEFAULT PROBABILITIES

```
#2. from spread to default probabilities
def prob_def(credit_name, r=.04, Recovery=.4, PD=np.zeros(len(df_CDS["Fiat"])+1)):
  PS=1-PD
  Discount_Factors=[(1+r)**-t for t in range(1,len(df_CDS["Fiat"])+1)]
  maturity=np.arange(0,len(df CDS["Fiat"]))
  for N in maturity:
    sum_PS=sum([(Discount_Factors[j]*PS[j]) for j in range(0,N)])
    sum_PD=sum([(Discount_Factors[j]*PD[j]) for j in range(0,N)])
    CDS_spread=df_CDS.loc[N, credit_name]/10000
    numerator=CDS_spread*(sum_PS+(Discount_Factors[N]*PS[N-1]))-(1-Recovery)*sum_PD
    denominator=Discount_Factors[N]*(CDS_spread+(1-Recovery))
    PD[N]=numerator/denominator
    PS[N]=1-sum([PD[t] for t in range(0,N+1)])
  PD= PD[:-1]
  return 100*PD
PD_df= pd.DataFrame({"PD %": ["PD 1", "PD 2", "PD 3", "PD 4", "PD 5"],
                          'Fiat': prob_def("Fiat"),
                          'Ericsson':prob def("Ericsson"),
                          'British Airways':prob_def("British Airways"),
                          'Merrill Lynch':prob_def("Merrill Lynch")})
PD_df.set_index("PD %")
```

	Fiat	Ericsson	British Airways	Merrill Lynch
PD %				
PD 1	11.764706	5.138340	7.692308	0.530504
PD 2	10.130815	6.353871	7.100592	0.544465
PD 3	8.361473	9.499034	6.554392	0.660903
PD 4	4.848875	4.965161	6.050208	0.708620
PD 5	3.877607	6.295851	2.240401	0.950390

FROM DEFAULT PROBABILITIES TO SPREAD

```
#from default probability to spread
def calc_spread(PD, r=.04, Recovery=.4):
  Spread=np.zeros(len(df_CDS["Fiat"]))
  PS=1-PD/100
  Discount_Factors=[(1+r)**-t for t in range(1,len(df_CDS["Fiat"])+1)]
  maturity=np.arange(1,len(df_CDS["Fiat"])+1)
  for N in maturity:
   sum_PS=sum([(Discount_Factors[j]*PS[j]) for j in range(0,N)])
   sum_PD=sum([(Discount_Factors[j]*PD[j])/100 for j in range(0,N)])
   Spread[N-1]=((1-Recovery)*sum_PD)/sum_PS
   PS[N]=1-sum([PD[t]/100 for t in range(0,N)])
  return Spread*10000
Spread_bps_df= pd.DataFrame({"Spread in bps": ["Spread 1", "Spread 2", "Spread 3", "Spread 4", "Spread 5"],
                          'Fiat': calc_spread(PD_df["Fiat"]),
                          'Ericsson':calc_spread(PD_df["Ericsson"]),
                          'British Airways':calc_spread(PD_df["British Airways"]),
                          'Merrill Lynch':calc_spread(PD_df["Merrill Lynch"])})
Spread_bps_df
```

	Spread in bps	Fiat	Ericsson	British Airways	Merrill Lynch
0	Spread 1	800.000000	325.000000	500.000000	32.000000
1	Spread 2	745.536978	362.687428	481.146305	32.412797
2	Spread 3	715.153241	448.596467	475.201652	34.861433
3	Spread 4	655.025644	433.736874	472.304574	36.825403
4	Spread 5	609.684979	445.788365	426.822661	40.768973

