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Part IV: Motion Tracking

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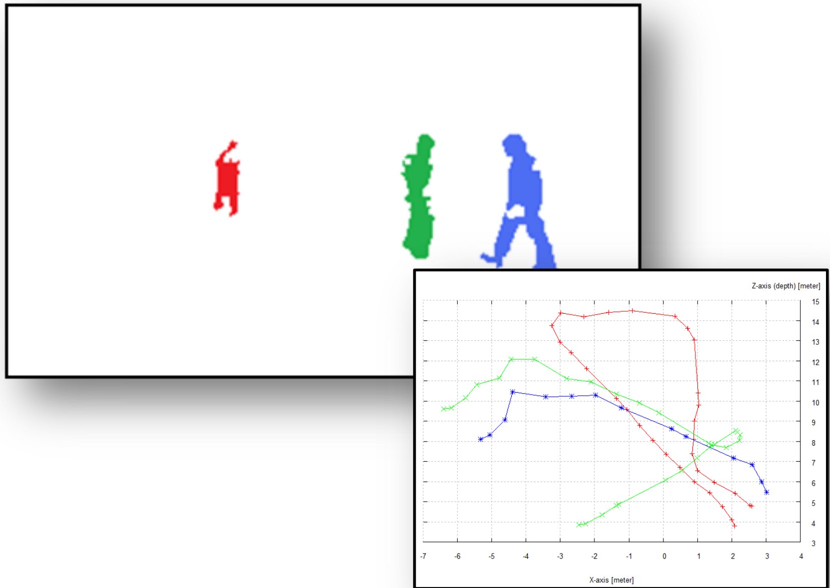
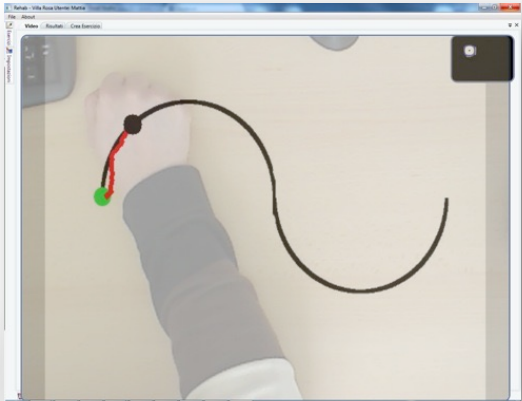
Introduction and motivations

- Follow one or more moving objects for:
  - People monitoring
  - Traffic monitoring and analysis
  - Biological applications (cells tracking)
- High Level analysis for
  - Discovery of activities
  - Behavior understanding
  - Detection of threats
- More in general
  - Understand WHAT is moving in a scene
  - Understand HOW it moves/interacts with the environment

Object Tracking

- On the basis of the applications requirements
  - Track the 2D coordinates (centroid)
  - Track in 3D (more cameras are required)
  - Determine the position of complex objects (human body articulations)
- Examples:





Object Tracking: applications

- Monitoring and surveillance
  - Motion classification
  - Identification of anomalous/suspicious behaviors
  - Follow a trajectory
- Human Machine Interfaces
  - Interact with a device removing physical barrier (mouse, keyboard)
  - Natural language understanding
- Virtual Reality
  - Immersive presence
  - Animation of virtual characters
- Mining and retrieval
  - Browse databases containing specific motion patterns

Benefits

- In HCI, control PC (or systems in general) → no need for additional tools
- In surveillance, Automated / Semi-automated systems → reduce the stress of human operators
- Virtual reality, computer animation → animate and drive the avatar
- But also
  - Training of athletes,
  - Gait disorders detection
  - Medical applications
  - …

2D Tracking

- Motion in the image plane
- Sometimes it is enough
- Different approaches
  - **Region-based** → set of pixels that share similar features (color)
  - **Contour-based** → determine position and shape of an object over time. Useful to track deformable objects.
  - **Feature-based** → select meaningful points (contours, corners)
  - **Template-based** → use specific models (hands, faces, eyes)

Tracking: Region-based

- Tracking regions with uniform appearance is a good method for real-time applications:
  - Fast (> 30 fps)
  - Good tradeoff quality/speed
- A region can be represented by the projection of an area with similar color on the image plane
- For example obtained from segmentation (e.g. after background suppression)

Tracking: Region-based

- **Colors** of regions **must be different** in order to be distinguished
- It is **unstable in presence of variable illumination**
- If applied to situations with variable illumination, appropriate compensation techniques have to be applied, i.e.:
  - Use HS of HSV
  - Use a normalized RGB space
- Ok indoor, troubles outdoor

Tracking: Region-based

- What do I want to track?
  - Any type of moving object
  - Skin vs non-skin (hand and face tracking)
  - Areas with certain colors
- How?
  - Color thresholding if uniform
  - Color histograms
- Problems
  - Color changes over time (illumination, posture)
  - Acquired models of objects need to be updated

Tracking: Region-based

- A possible approach:
  - Divide the object to be tracked into regions
  - Each region is associated to a color vector (average for all pixels in the region) or histogram
  - Compute the color at each frame
  - If the ratio between the reference and actual values is close to 1, the match is good

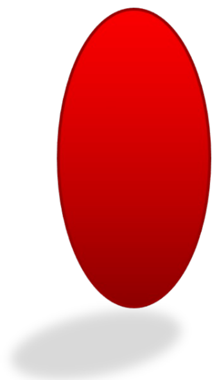
Tracking: Region-based

- How to use histograms
  - For each moving object compute the histogram
  - At each step evaluate the histogram of the tracked region O^t and compare it with the reference model O^r for each region *i*
  - Similarity can be evaluated using:
    - Bin-by-bin comparison (intersection) $$\bigcap(O_i^t, O_i^r) = \sum_{n=1}^{U} \min\{O_{i,n}^r, O_{i,n}^t\}$$
    - SSD $$SSD(O_i^t, O_i^r) = \sum_{n=1}^{U} (O_{i,n}^r - O_{i,n}^t)^2$$
  - Bins should be neither to few, nor too many

See A. Bovik, Chapter 7

Note: Shadows

- Shadows are source of noise → false positives
- A shadow does not correspond to the motion of a real object



- Variation of the luminance
- Chrominance remains (ideally) unaltered
- → for a proper tracking shadows should be removed before tracking using a suitable algorithm

Blobs extraction

- Aggregation of a set of pixels that share common features
- An object can be made up by several blobs (head, torso, legs, …)
- Features include also position

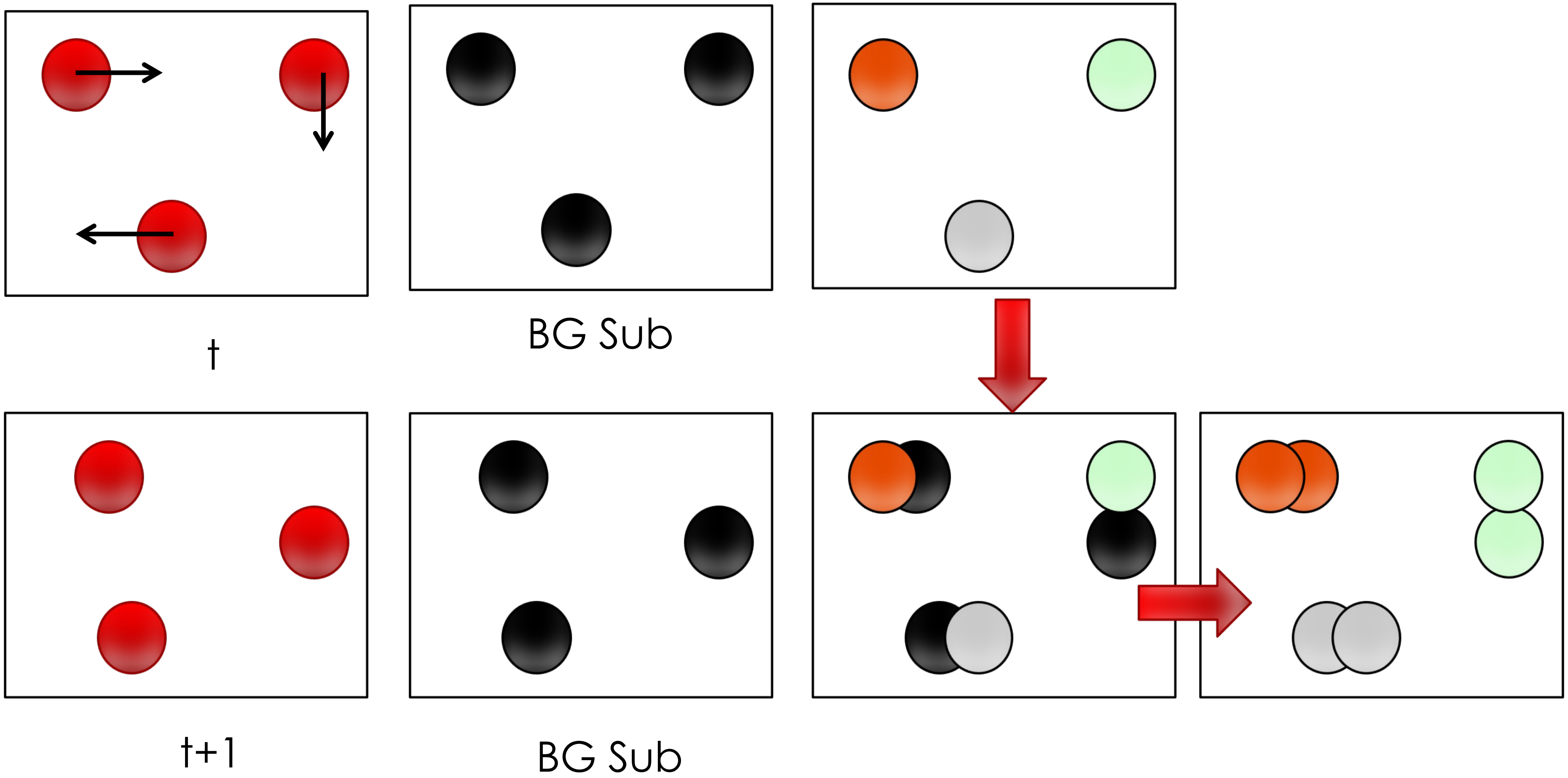
  → pixels with similar color but far (in x,y) from the object must be discarded

- Typical application in combination with background suppression

Target association

- Procedure **common to all trackers**, not only region-based
- In general it is worth noting that **detection is not carried out on a frame-basis**
- This could be too demanding in terms of computational resources
- ONCE detected, targets are followed on a **proximity** basis
- Example:
  1. Background subtraction informs about the presence of motion
  2. Histograms characterize each moving objects
  3. Unless occlusions occur, the target in the next frame *should be* the closest blob

Target association



Target association

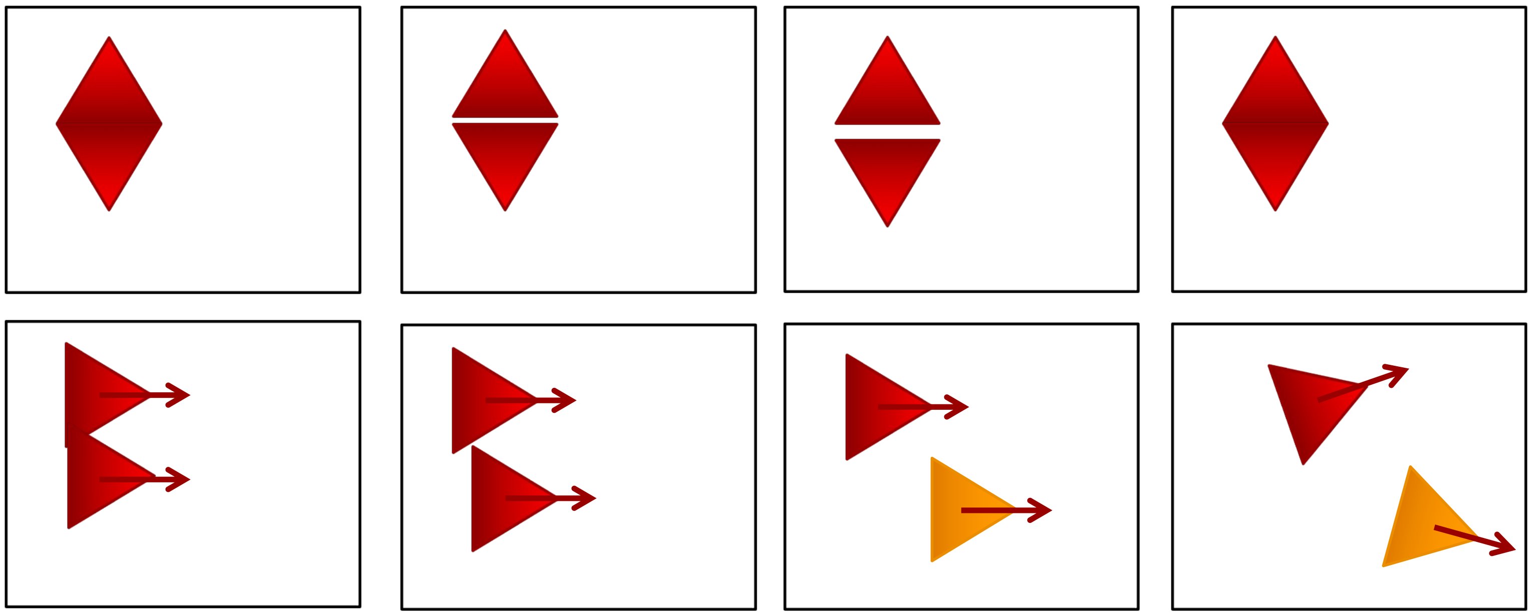
- Association can be performed as:
  - Overlapping blob (issues with scale → depends on objects size)
  - Centroid with minimum distance (as above)
  - Overlapping bounding boxes (may fail due to perspective)
  - Bounding box with minimum distance
  - Bounding box to centroid distance
- When association has been completed

  → For each object/blob update the appearance model to account for small variations

  → In presence of occlusions, the last saved model can be used to disambiguate

Splitting

- If objects are identified as single blobs, no problem!
- However, background suppression may return ambiguous results
  1. Objects are split into several small blobs (not enough separation btw BG and FG)
  2. Two objects enter the scene together, then separate

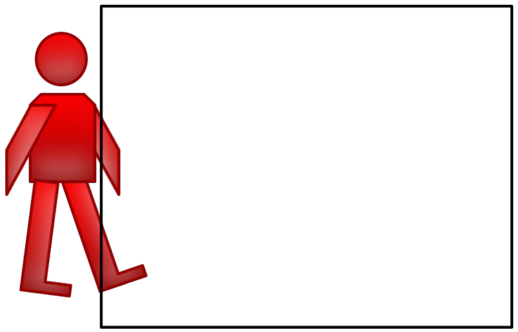


Splitting

- Before saying it is case 1. or 2. evidence needs to be accumulated
  → Impossible to understand what's going on immediately
  → A temporal interval is needed to tell if:
    → It is necessary to keep the object merged even though it is fragmented
    → A new object has to be created

Merging

- When two objects move together consistently, then perhaps they're the same object.
- Example:
  - A person's arm and foot enter the scene first and are detected as two well-separated FG blobs
  - Then also the rest of the body enters and a single blob is created
- Blobs are close and they might share similar properties

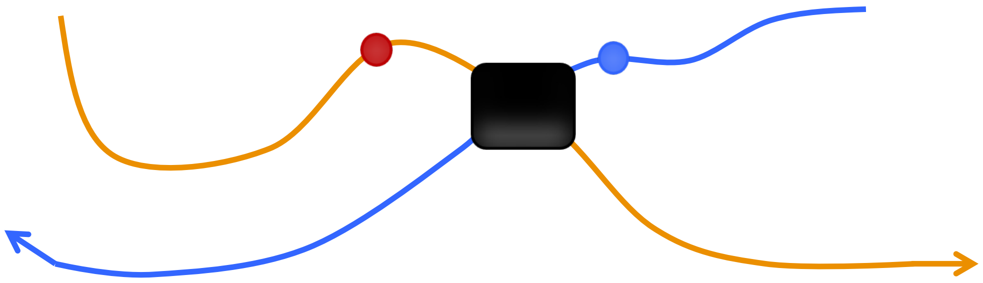


Criteria for splitting and merging

- Observation is the key
- Need to monitor the regions of interest and evaluate consistency in terms of:
  - Direction of motion
  - Distance between centroids/bounding boxes
  - Temporal range in which the phenomenon is observed
  - Velocity
  - Matching in features

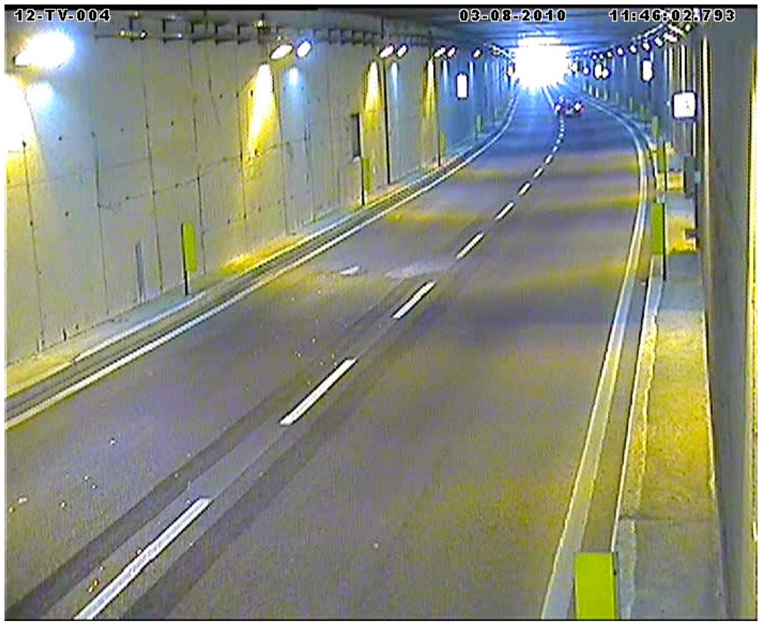
Occlusions

- Moving objects occlude each other while crossing
  - One or more objects disappear from the scene
  - Bigger blobs appear as a result of the occlusion, with properties that do not belong to any of the models acquired previously
- How to resolve the occlusion?
  - Need to re-associate “A to A” and “B to B”
  - Histograms are a *good* way out



Occlusions

- It's an "anomalous" (though very frequent) situation
- Objects overlap and the acquired models are not reliable anymore
- Model update should be **avoided** during occlusions



Tracking: Feature-based

- The objective is to retrieve the motion information of a set of features

- Considering:
  - $A=\{A(0), A(1), \ldots, A(j-1)\}$ is a set of images
  - $m_i(x_i, y_i)$ $i=[0, j-1]$ the position of the feature in the image plane in each frame

- Objective:
  - determine the displacement vector $d_i=(dx_i, dy_i)$ that best estimates the position of the feature in the next frame $m_{i+1}(x_{i+1}, y_{i+1})=m_i+d_i$

- If needed, points can be grouped and objects can be represented using the bounding box or the convex hull

What features?

- A good feature point has distinctive characteristics:
  - Brightness
  - Contrast
  - Texture
  - Edges
  - Corners
  - Points with high curvature

Good features to track

- For each candidate point, compute:

$$Z = \begin{bmatrix} \sum_W J_x^2 & \sum_W J_x J_y \\ \sum_W J_y J_x & \sum_W J_y^2 \end{bmatrix}$$

- J_x and J_y are the gradients evaluated on the point in x and y direction within W (nxn window)
- A good feature point is where the smallest eigenvalue of Z is larger than a specified threshold
- In practice, it highlights corner points and textures

Good features to track

- Eigenvalues should be above the image noise
- Small eigenvalues imply strong similarity within the window
- A large and a small eigenvalue means unidirectional patterns
- If both eigenvalues are large, the point is of high interest (salt&pepper texture, corners)

How to track them?

- Must ensure that the same points are tracked throughout the video!
- Ideally we would expect that

$$A_i(Dm - d) = A_{i+1}(m)$$

- where
  - A_i, A_{i+1} successive frames
  - m is the 2D position of the feature point
  - D is the deformation matrix (affine transformation model)
  - d is the displacement vector (translational model)

How to track them?

- However:
  - Due to noise the equality does usually not hold
  - Motion across successive frames is assumed to be small → a translational model is a good approximation

- Feature dissimilarity measure is used to quantify the change of appearance between the first and current frame

$$\varepsilon = \iint_W [A_i(m-d) - A_{i+1}(m)]^2 \omega(m)\, dm$$

- ω is a weighting function (e.g., Gaussian to emphasize the center of the window)
- When the feature dissimilarity grows too large, the feature point should be abandoned

How to track them?

- To minimize the residual we differentiate w.r.t. unknowns (*d*)

$$e = 2\iint_W [A_i(m) - A_{i+1}(m)]g(m)\omega(m)\,dm$$

- and

$$g(m) = \begin{bmatrix} \dfrac{\partial(A_i(m) - A_{i+1}(m))}{\partial x} \\ \dfrac{\partial(A_i(m) - A_{i+1}(m))}{\partial y} \end{bmatrix}$$

- In this case the solution for the displacement vector can be expressed by the 2x2 linear system of equations (see paper for details):

$$Zd = e$$

The Lucas-Kanade optical flow

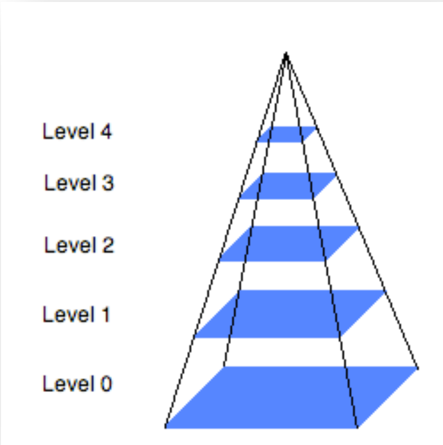
- Two-frame differential method for optical flow estimation developed by Bruce D. Lucas and Takeo Kanade (1981)
- Consider $u=[u_x, u_y]$ in frame I and $v=[v_x, v_y]$ in frame J
- The goal is to find **d** that satisfies $v=u+d$ such as I and J are similar (translational model)
- Because of the aperture problem, **similarity** must be defined in 2D
- **d** is the vector that minimizes

$$\epsilon(\mathbf{d}) = \epsilon(d_x, d_y) = \sum_{x=u_x-\omega_x}^{u_x+\omega_x} \sum_{y=u_y-\omega_y}^{u_y+\omega_y} \left(I(x,y) - J(x+d_x, y+d_y)\right)^2 .$$

- We use an integration window

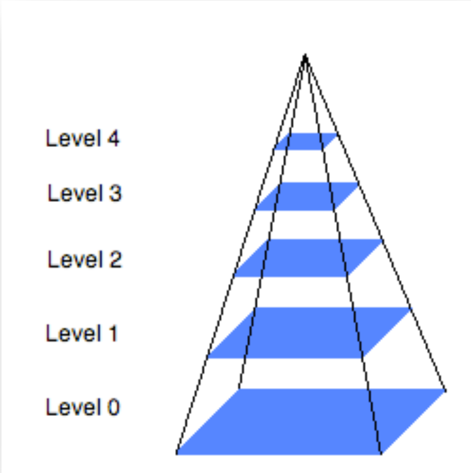
Pyramidal implementation

- The two key components to any feature tracker are **accuracy** and **robustness**
- **Accuracy** relates to the local sub-pixel accuracy attached to tracking → small integration window preferable to limit smoothness and preserve detail information (two image patches moving rapidly in different directions)
- **Robustness** relates to the sensitivity of tracking with respect to changes of light and big motions → a large window is preferable
- → Pyramidal implementation



Pyramidal implementation

- Level 0 is the image at original resolution
- Level 4 (in this example) is the image at lowest resolution
- The L-th level is defined as a linear combination of the elements in the previous level



$$
\begin{aligned}
I^L(x,y) \;=\; & \frac{1}{4} I^{L-1}(2x,2y) + \\
& \frac{1}{8}\left(I^{L-1}(2x-1,2y) + I^{L-1}(2x+1,2y) + I^{L-1}(2x,2y-1) + I^{L-1}(2x,2y+1)\right) + \\
& \frac{1}{16}\left(I^{L-1}(2x-1,2y-1) + I^{L-1}(2x+1,2y+1) + I^{L-1}(2x-1,2y+1) + I^{L-1}(2x+1,2y+1)\right).
\end{aligned}
$$

Pyramidal implementation

- At each level of the pyramid an initial guess **g** of the flow is computed at the lower level, which is then refined at the current level.

$$\epsilon^L(\mathbf{d}^L) = \epsilon^L(d_x^L, d_y^L) = \sum_{x=u_x^L-\omega_x}^{u_x^L+\omega_x} \sum_{y=u_y^L-\omega_y}^{u_y^L+\omega_y} \left(I^L(x,y) - J^L(x+g_x^L+d_x^L, y+g_y^L+d_y^L)\right)^2$$

- **g** is used to pre-translate the image patch in the second image **J**

- **d** should be small

- The information is then propagated at the upper level

$$\mathbf{g}^{L-1} = 2\,(\mathbf{g}^{\mathbf{L}} + \mathbf{d}^L).$$

- Overall the displacement becomes $\mathbf{d} = \sum_{L=0}^{L_m} 2^L\,\mathbf{d}^L.$

Bayesian tracking

- Idea:
  - Estimate the state of a system over discrete time steps
  - At each step, noisy measurements
  - The state is represented by the x-y coordinates, velocity and acceleration along each dimension
  - 6D vector
  - Noise is typically smaller than the information about the state

Bayesian tracking

- The state can be defined as

$$x_k = f_k(x_{k-1}, w_{k-1})$$

- Linking the measurement with the state vector:

$$z_k = h_k(x_k, v_k)$$

- w_k is the process noise
- v_k is the measurement noise
- f_k and z_k are in general nonlinear functions defined in

$$f_k : \Re^{n_x} \times \Re^{n_w} \rightarrow \Re^{n_x} \text{ and } h_k : \Re^{n_x} \times \Re^{n_v} \rightarrow \Re^{n_z}$$

Bayesian tracking

- System and measurement should be available in probabilistic form
- Every time the measurement is available, the estimation can be computed
- It is an online method

→ for every step *k* an estimate can be computed based on the previous observations z_k up to instant *k*

Bayesian tracking

- The initial *pdf* of the state vector is given, $p(x_0|z_0)$
- z_0 contains no measurement
- Goal is to compute $p(x_k|z_k)$ at time k
- The process consists of two steps
  - Prediction
  - Update

Bayesian tracking - Prediction

- From the previous representation:

$$x_k = f_k(x_{k-1}, w_{k-1}) \rightarrow p(x_k \mid x_{k-1})$$

$$z_k = h_k(x_k, v_k) \rightarrow p(z_k \mid x_k)$$

- The posterior *pdf* at *k-1* is propagated forward in time using the **system model**

$$p(x_k \mid z_{k-1}) = \int p(x_k \mid x_{k-1}) p(x_{k-1} \mid z_{k-1}) dx_{k-1}$$

Prior at time step k

System model

Posterior pdf at k-1

Bayesian tracking - Update

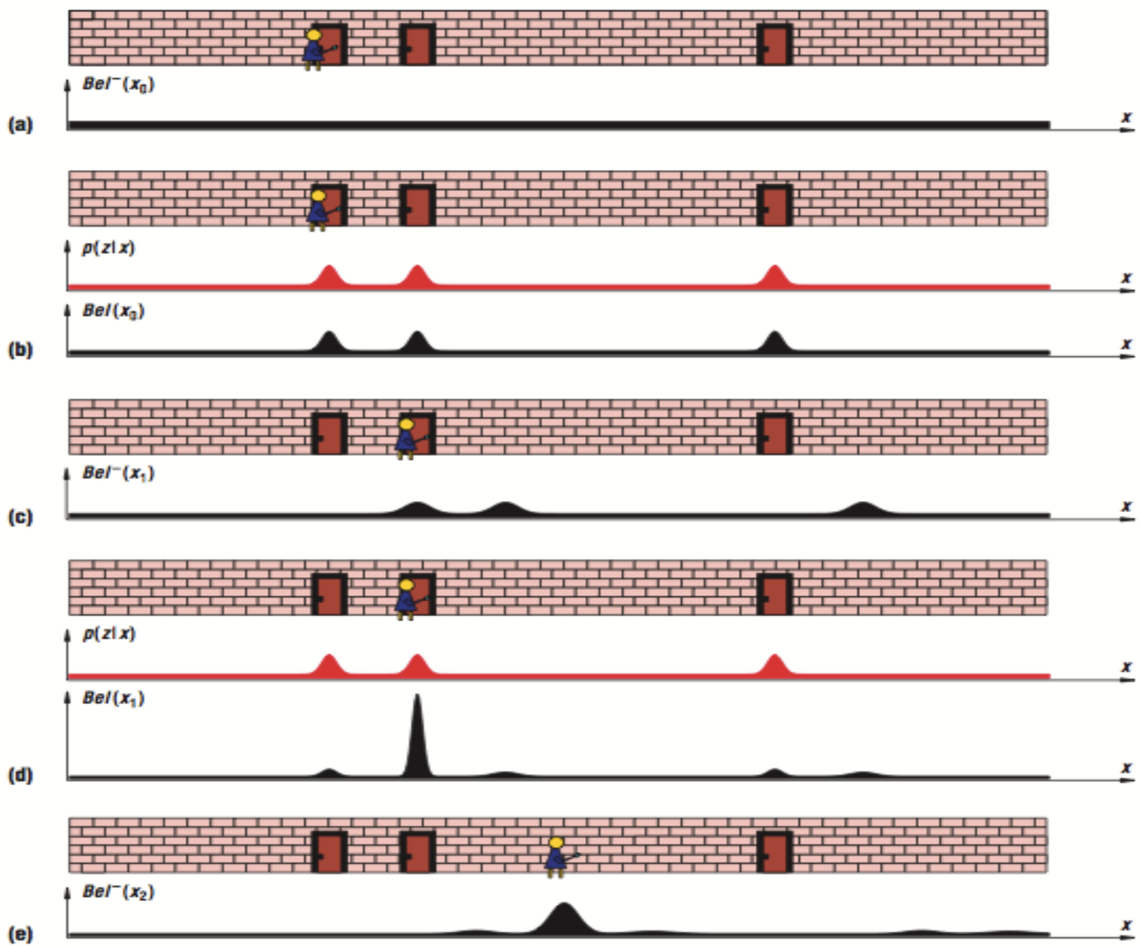
- Using the Bayes theorem it is possible to obtain the desired pdf $p(x_k|z_k)$

$$p(x_k \mid z_k) = \frac{p(z_k \mid x_k)\, p(x_k \mid z_{k-1})}{p(z_k \mid z_{k-1})}$$

- Where $p(z_k|z_{k-1})$ is used for normalization and computed as

$$p(z_k \mid z_{k-1}) = \int p(z_k \mid x_k)\, p(x_k \mid z_{k-1})\, dx_k$$

Bayesian tracking – Toy Example



- User has a door "sensing device"
- User walks at typical *walking speed*
- (a) position is unknown
- (b) the sensor senses a door
- User can be in any of the three positions
- (c) User moves
- (d) another door is sensed
- (e) user moves again

D. Fox, J. Hightower, L. Liao, D. Schulz, G. Borriello, "Bayesian Filters for Location Estimation", IEEE Pervasive Computing, 2003.

The Kalman filter in a nutshell

- In line with Bayesian tracking:
  - Take a measurement
  - Measurement is subject to error
  - Derive the state of the system from the measurement

- We start from:

$$z_1, \sigma_{z1}^2$$

$$\hat{x}_1 = z_1$$

$$\sigma_1^2 = \sigma_{z1}^2$$

The Kalman filter in a nutshell

- Add a second measurement
- Combine the two
- Iterate…

$$z_2, \sigma_{z2}^2$$
$$\hat{x}_2 = ?$$
$$\sigma_2^2 = ?$$

$$\hat{x}_2 = \hat{x}_1 + K_2(z_2 - \hat{x}_1)$$
$$K_2 = \frac{\sigma_{z1}^2}{\sigma_{z1}^2 + \sigma_{z2}^2}$$

$$\frac{1}{\sigma_2^2} = \frac{1}{\sigma_{z1}^2} + \frac{1}{\sigma_{z2}^2}$$

- The algorithm works online
- It's a weighted average

$$\sigma_2^2 = \frac{\sigma_{z1}^2 \sigma_{z2}^2}{\sigma_{z1}^2 + \sigma_{z2}^2}$$

The Kalman filter in a nutshell

- Not all the difference between two measurements is noise
- Motion occurs
- Need to include a motion model, taking into account for example position and velocity
- The process:

  Loop {
  1. Predict the new state and the uncertainty
  2. Correct using the new measurement

  }

KF in practice

- KF provides a computationally efficient solution to the least squares method
- Assuming that w_k and v_k are normal distributions (zero-mean, Q_k and R_k covariance), then
- The state and measurement model can be written as:

$$x_k = A_k x_{k-1} + B_k u_k + w_{k-1}$$
$$z_k = H_k x_k + v_k$$

KF in practice

$$x_k = A_k x_{k-1} + B_k u_k + w_{k-1}$$

$$z_k = H_k x_k + v_k$$

- x_k is the current state
- x_{k-1} is the previous state
- A_k is the state transition matrix
- w_k is the process noise
- z_k is the actual measurement
- H_k is the measurement matrix
- v_k is the measurement noise
- B_k and u_k refer to additional and optional control input

KF in practice

- We assume that the process noise and measurement noise are not changing over time, plus
  - Gaussian
  - Zero mean
  - $p(w) = N(0, Q)$
  - $p(v) = N(0,R)$

Process noise covariance

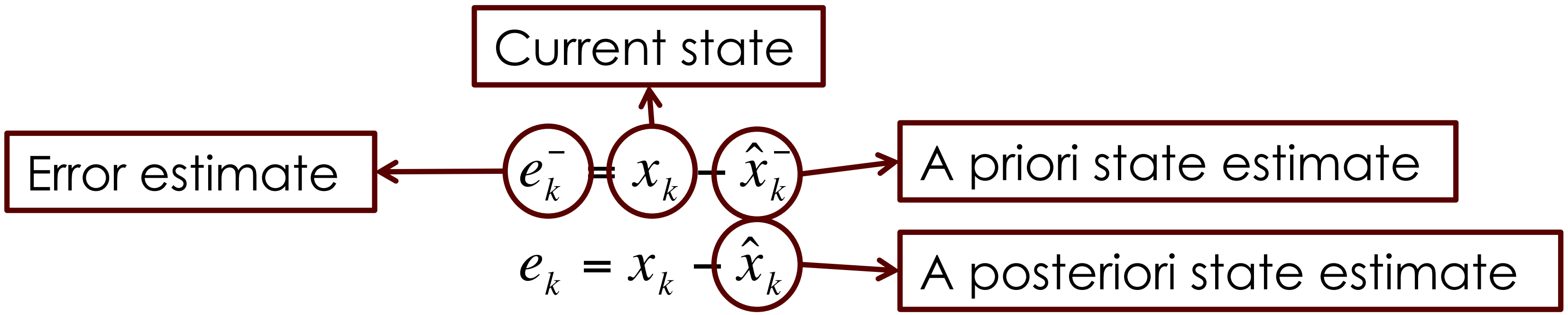
Measurement noise covariance

Predict-and-correct stages

- During the **first** phase the **current state estimate** together with the **error estimate** are propagated forward in time

- In the second stage a new measurement is taken to modify the two estimations

- Evaluation of
  - a priori estimate (based on the past measurements)
  - a posteriori estimate (as soon as measurement z_k is available)



Predict-and-correct stages

- Once the error estimates have been computed, determine the error covariance by:

$$P_k^- = E[e_k^- e_k^{-T}]$$
$$P_k = E[e_k e_k^T]$$

- For the prediction stage we have then (discarding the optional control parameter):

$$\hat{x}_k^- = A_k \hat{x}_{k-1}$$
$$P_k^- = A_k P_{k-1} A_k^T + Q_{k-1}$$

Predict-and-correct stages

- The update stage begins with the computation of the "gain" of the KF, which minimizes the a posteriori error covariance

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}$$

- The gain is used to modify the a priori estimate and to compute the a posteriori state estimate:

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H_k \hat{x}_k^-)$$

- And to compute the a posteriori error covariance:

$$P_k = (I - K_k H_k) P_k^-$$

How to:

- Predict the position of a point following a certain trajectory in (x,y) plane

$$x_k = A_k x_{k-1} + w_{k-1}$$

$$z_k = H_k x_k + v_k$$

- x, w, z, and v are 2x1 vectors
- A and H are 2x2
- Given w and v compute Q and R (in some cases determined empirically)
- During initialization
  - $x_0 = Hz_0$
  - P_0 diagonal matrix with reasonable values for covariance (maximum allowed displacement?)

How to:

- Now predict … and correct:

$$\hat{x}_k^- = A_k \hat{x}_{k-1}$$

$$P_k^- = A_k P_{k-1} A_k^T + Q_{k-1}$$

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R_k)^{-1}$$

$$\hat{x}_k = \hat{x}_k^- + K_k (z_k - H_k \hat{x}_k^-)$$

$$P_k = (I - K_k H_k) P_k^-$$

Implementation of the KF

- Estimate a constant function y=c (voltage)

- Measurements we take are corrupted by white noise (quantization error due to conversion)

- Problem statement is easy, assuming A and H are both =1

$$x_k = x_{k-1} + w_k$$

$$z_k = x_k + v_k$$

Example taken from “An introduction to the Kalman Filter” by Greg Welch and Gary Bishop

Filter equations

- Our a priori state estimate is given by the previous state

$$\hat{x}_k^- = \hat{x}_{k-1}$$

$$P_k^- = P_{k-1} + Q$$

- And the measurement update

$$K_k = P_k^-(P_k^- + R)^{-1}$$

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - \hat{x}_k^-)$$

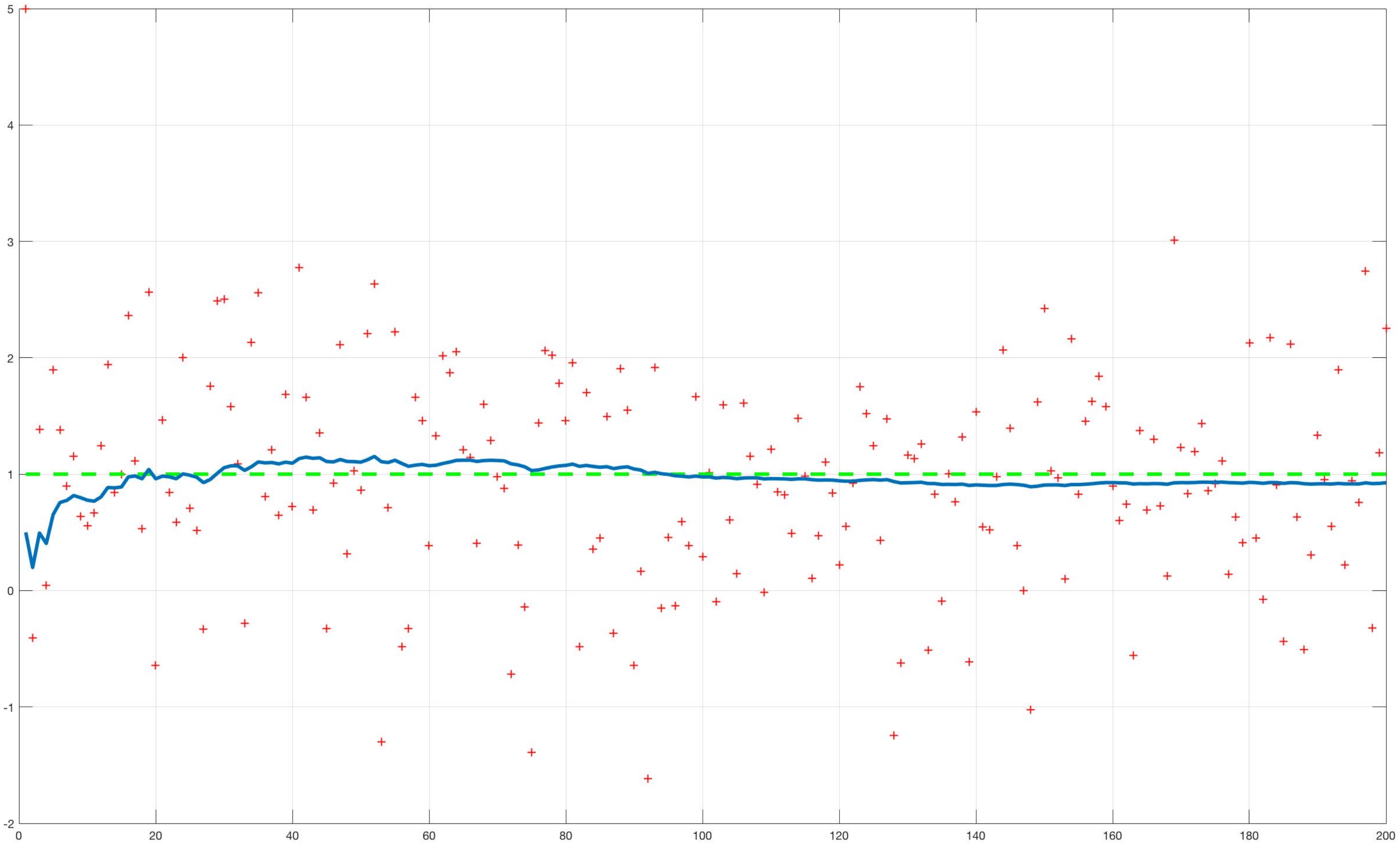
$$P_k = (1 - K_k)P_k^-$$

- We assume the process noise is small
- To make the algorithm converge we have to set a “wrong” initial value
- P≠0 must be chosen as well

Matlab code

```
% State transition matrix A and measurement matrix H are unitary
% Assume the measurement error is normal (white) with sigma=0.1
% Assume the process error variance is small 1e-5

x=1*ones(1,200); % the constant value to predict is 1

R=1; Q=1e-5;
N=size(x);  % Let's take a number of measurements N

% Set priors
x_est = zeros(1,size(x,2));
P_est = zeros(1,size(x,2));
x_est(1) = 0.5;
P_est(1) = 0.5;

z = zeros(1,size(x,2)); % Set vector for measurements
z(1)=5;

for n=2:size(x,2)
   x_prior = x_est(n-1);
   P_prior = P_est(n-1) + randn*Q;
   z(n) = x(n) + randn*sqrt(R);

   K = P_prior / (P_prior + R);
   x_est(n) = x_prior + K*(z(n) - x_prior);
   P_est(n) = (1-K)*P_prior;
end

plot(1:size(x,2),z, 'r+');
hold on;
grid on;
plot(1:size(x,2),[x], 'g--'); plot(1:size(x,2),x_est, '-');
```

Matlab code



The extended KF (EKF)

- Assumption in KF: state and measurement are linear
- In general wrong in object tracking
- "Adapt" the filter to linearize mean and covariance
- The process is still defined through f_k and h_k, which are in this case not linear

$$x_k = f_k(x_{k-1}, w_{k-1})$$
$$z_k = h_k(x_k, v_k)$$

The EKF

- Use partial derivatives to linearize the estimation
- Partial derivatives of:
  - Process (A)
  - Measurements (H)
- Since we don't know the values of noise we can assume here for simplicity that state and measurement are

$$\tilde{x}_k = f(\hat{x}_{k-1}, 0)$$

$$\tilde{z}_k = h(\tilde{x}_k, 0)$$

EKF prediction

- Using the same notation as for KF, the prediction stage is given by:

$$\tilde{x}_k = f(\hat{x}_{k-1}, 0)$$

$$P_k^- = A_{k-1} P_{k-1} A_{k-1}^T + W_{k-1} Q_{k-1} W_{k-1}^T$$

- A_k is the Jacobian matrix of partial derivatives of *f* with respect to x_k
- W_k is the Jacobian matrix of partial derivatives of *f* with respect to w_k
- Q_k is the process noise covariance matrix

EKF update

- The update step becomes:

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + V_k R_k V_k^T)^{-1}$$

$$\hat{x}_k = \tilde{x}_k + K_k (z_k - h_k(\tilde{x}_k, 0))$$

$$P_k = (I - K_k H_k) P_k^-$$

- H_k is the Jacobian matrix of partial derivatives of *h* with respect to x_k
- V_k is the Jacobian matrix of partial derivatives of *h* with respect to v_k
- R_k is the measurement noise covariance matrix

EKF predict-update

Time Update ("Predict")

(1) Project the state ahead

$$\hat{x}_k^- = f(\hat{x}_{k-1}, u_{k-1}, 0)$$

(2) Project the error covariance ahead

$$P_k^- = A_k P_{k-1} A_k^T + W_k Q_{k-1} W_k^T$$

Measurement Update ("Correct")

(1) Compute the Kalman gain

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + V_k R_k V_k^T)^{-1}$$

(2) Update estimate with measurement z_k

$$\hat{x}_k = \hat{x}_k^- + K_k(z_k - h(\hat{x}_k^-, 0))$$

(3) Update the error covariance

$$P_k = (I - K_k H_k) P_k^-$$

Initial estimates for $\hat{x}_{k-1}$ and P_{k-1}

Particle filters

- KF and EKF assume the posterior probability to be Gaussian
- In case it is not Gaussian performances are reduced
- PFs can do the job
- Main concept:
  - Represent alternative solutions as a set of samples
  - Each sample with a weight
  - The more the samples, the closer the optimal Bayesian estimate

Properties

- While KF is an optimal solution to the estimation problem, PFs provide an approximate solution
- Can be applied to either linear and non-linear problems
- Can handle multimodal distributions
- Multiple hypothesis on the process state

How PFs work

- Take a set of points associated with the corresponding weights:

$$\{x_k^i\}_{i=1}^N, \ \{w_k^i\}_{i=1}^N$$

- The a posteriori pdf is represented by:

$$p(x_k \mid z_k) \approx \sum_{i=1}^{N} w_{k-1}^i \delta(x_k - x_{k-1}^i)$$

PFs – SIR algorithm

- How do we select $\{x_k^i\}_{i=1}^N, \ \{w_k^i\}_{i=1}^N$ for the *pdf* approximation?

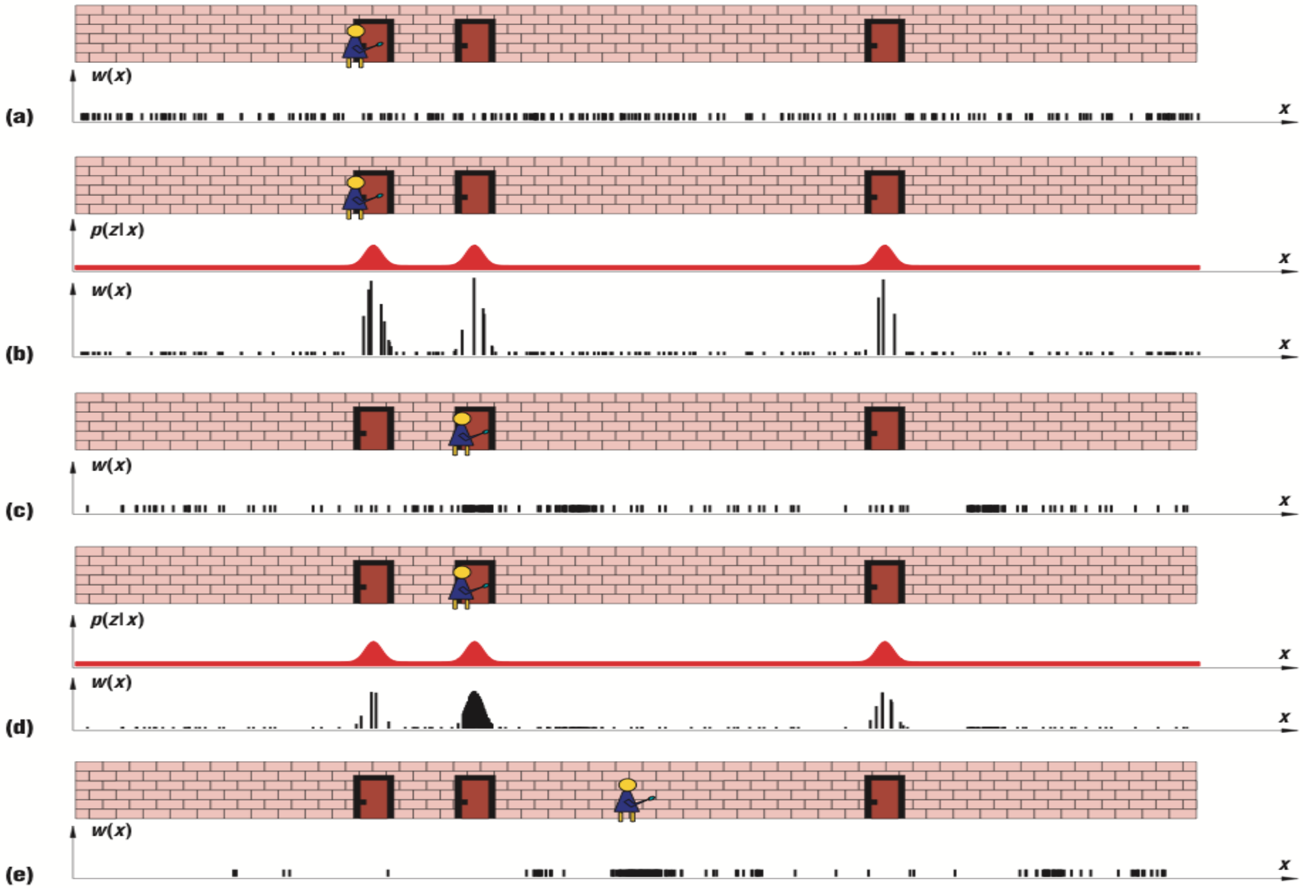
- Select ***PROPOSAL DISTRIBUTION*** $x_k^i \approx \pi(x_k \mid x_{k-1}^i)$ with $i = 1 \ldots N_s$
- Draw $\pi(x_k \mid x_{k-1})$ $\left(Usually \ = p(x_k \mid x_{k-1})\right)$
- Update the weights $\hat{w}_k^i = w_{k-1}^i p(\mathrm{z}_\mathrm{k} | \mathrm{x}_\mathrm{k}^\mathrm{i})$
- Normalize $w_k^i = \hat{w}_k^i / \sum_{i=1}^{Ns} \left(w_k^i\right)^2$
- Resample:

(i) draw *Ns* particles with probability proportional to their weight;

(ii) set uniform weight for the new set

$$w_k^i = 1 / N_s$$

PFs – Toy Example



D. Fox, J. Hightower, L. Liao, D. Schulz, G. Borriello, "Bayesian Filters for Location Estimation", IEEE Pervasive Computing, 2003.