Scapegoat Trees
AAPP 2017 Project

Analysis, C implementation and testing, C++ benchmarks

Team members:
- Crippa Mattia - 10397252
- Vetere Alessandro - 10425802

Introduction

- Scapegoat trees are **self balancing binary search trees**

- No extra data in the tree nodes

- Scapegoat scheme:
  - Tree:
    - Root node pointer
    - Total size
    - Maximum total size since last completely rebuilt
  - Node:
    - Key value
    - Left and right child nodes pointers

- Rebalancing operation:
  - On INSERT
  - On DELETE

Fundamental properties

- α-weight-balanced:

  $$\text{size(node.left)} \leq \alpha * \text{size(node)} \quad \wedge \quad \text{size(node.right)} \leq \alpha * \text{size(node)}$$

- α-height-balanced, n = size(root):

  $$h(T) \leq h_\alpha(n) = \lfloor \log_{1/\alpha} n \rfloor$$

- If T is an α-weight-balanced binary search tree, then T is α-height-balanced

- Scapegoat trees are **loosely** α-height-balanced, n = size(root):

  $$h(T) \leq h_\alpha(n) + 1$$

- Fixed α, a node of depth greater than $h_\alpha(n)$ is a **deep** node

Meaning of α

- Scapegoat node is an α-weight-unbalanced node used to perform a **rebalance** operation

- Subtree rooted at the scapegoat node becomes an ½-weight-balanced tree

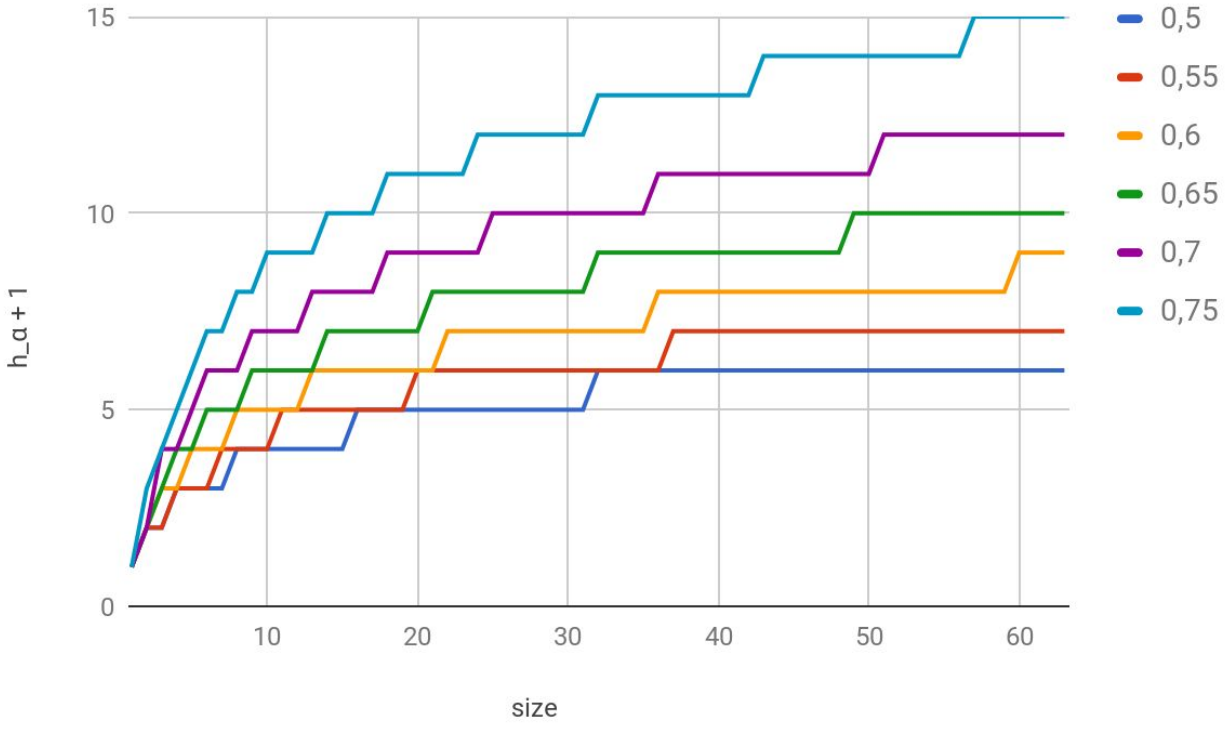
- Value of α should be in [0.5, 1):

    - High α → Few rebalance operations, quick INSERT, but slow SEARCH and DELETE

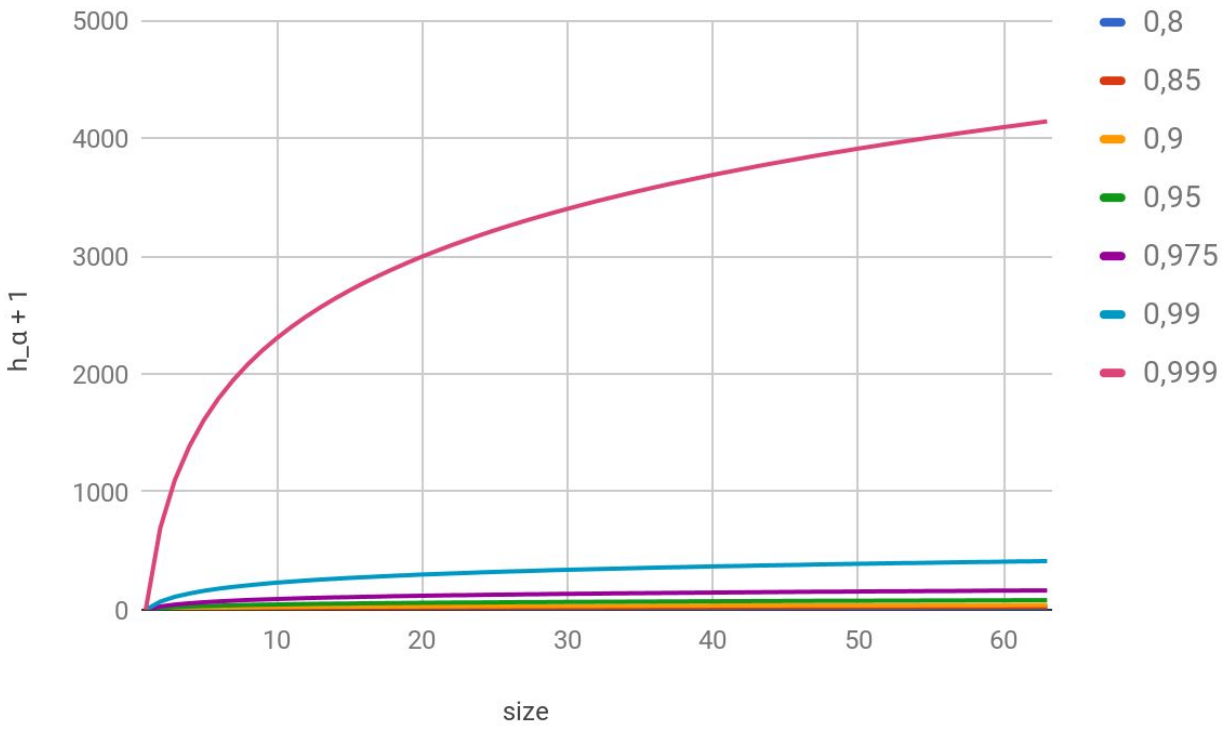
    - Low α → Many rebalance operations, quick SEARCH and DELETE, but slow INSERT

- Choose α depending on expected frequency of actions

Maximum tree depth given α



What happens as α approaches 1



SEARCH

- SEARCH proceeds as in a BST

- No restructuring is performed

- Due to loosely α-height-balanced property, worst case time:

$$O(h_\alpha(n)) = \mathbf{O(\log n)}$$

INSERT

- INSERT a node as into a BST, update size and max_size and record depth of new node
- If new node is deep, rebalance the tree:
    - Find scapegoat node
    - Rebalance the subtree rooted at the scapegoat
- Rebalancing operation will restore the α-height-balanced property
- Worst case time O(size(scapegoat)), but amortized time:

O(log n)

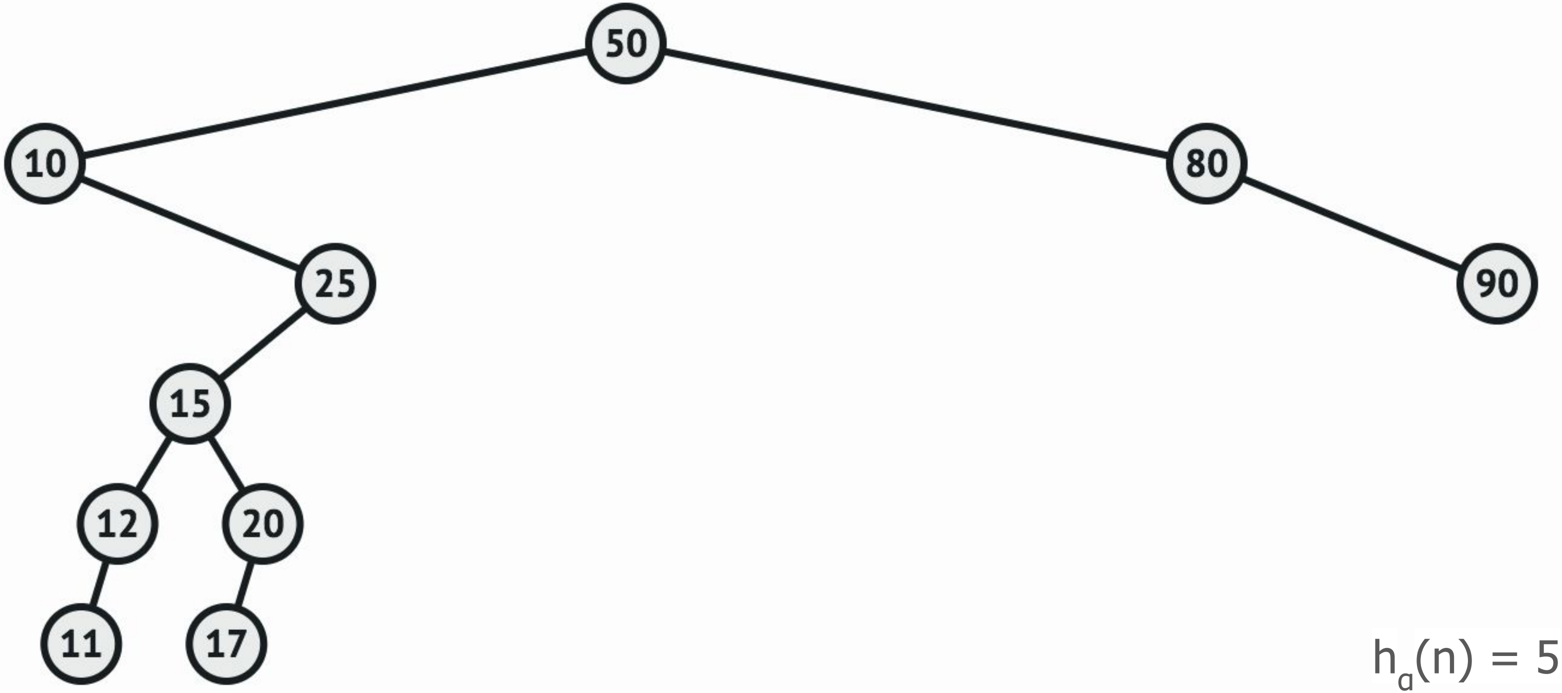
How to choose scapegoat node

- Climb the tree from the new node back up to the root and select the first node that isn't α-weight-balanced

- A better heuristic is choose the first node x_i satisfying the following condition, being i the minimum distance of x_i from the new node:

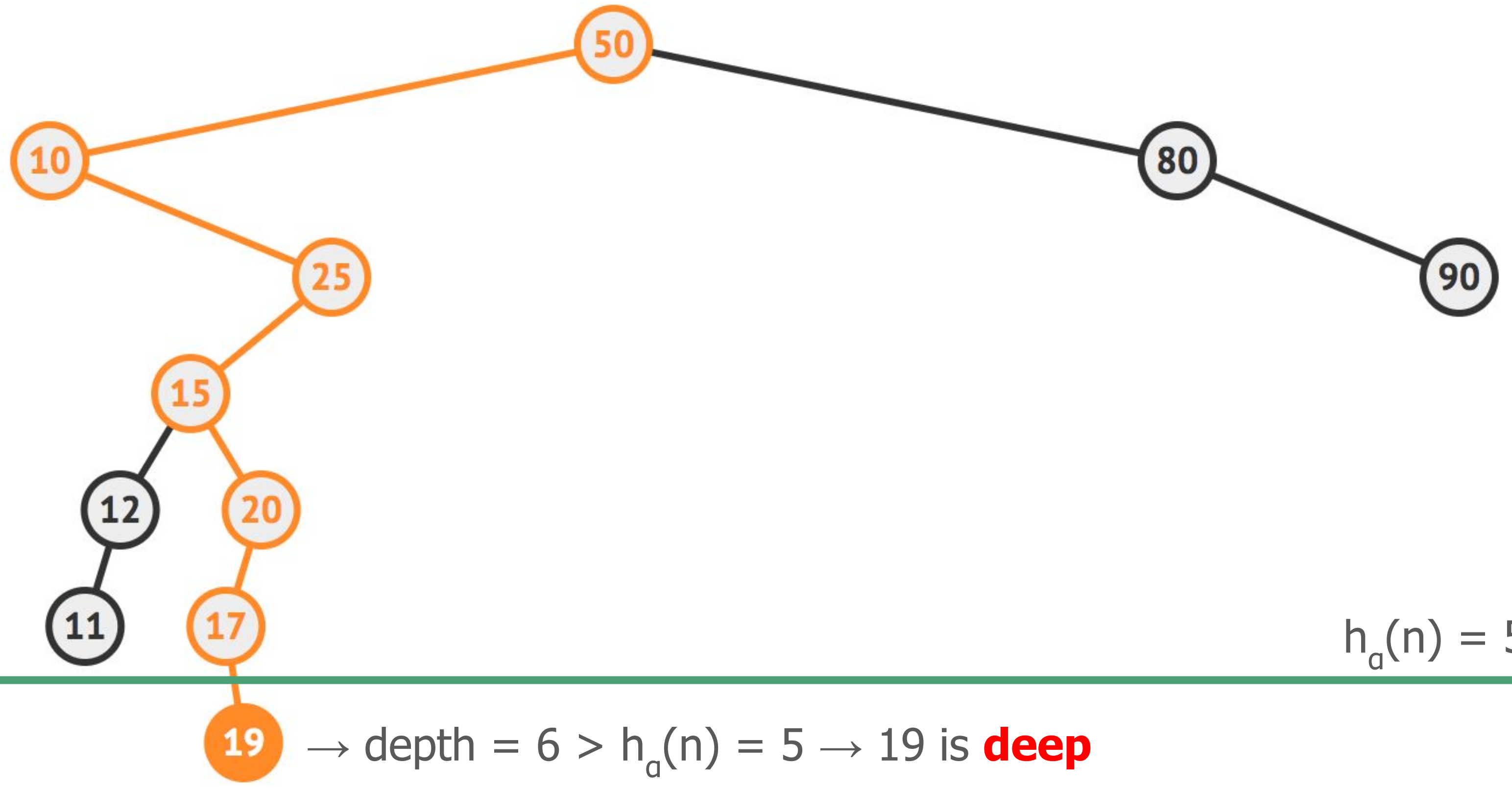
$$i > h_\alpha(size(x_i))$$

INSERT Example (1/5)



$h_\alpha(n) = 5$

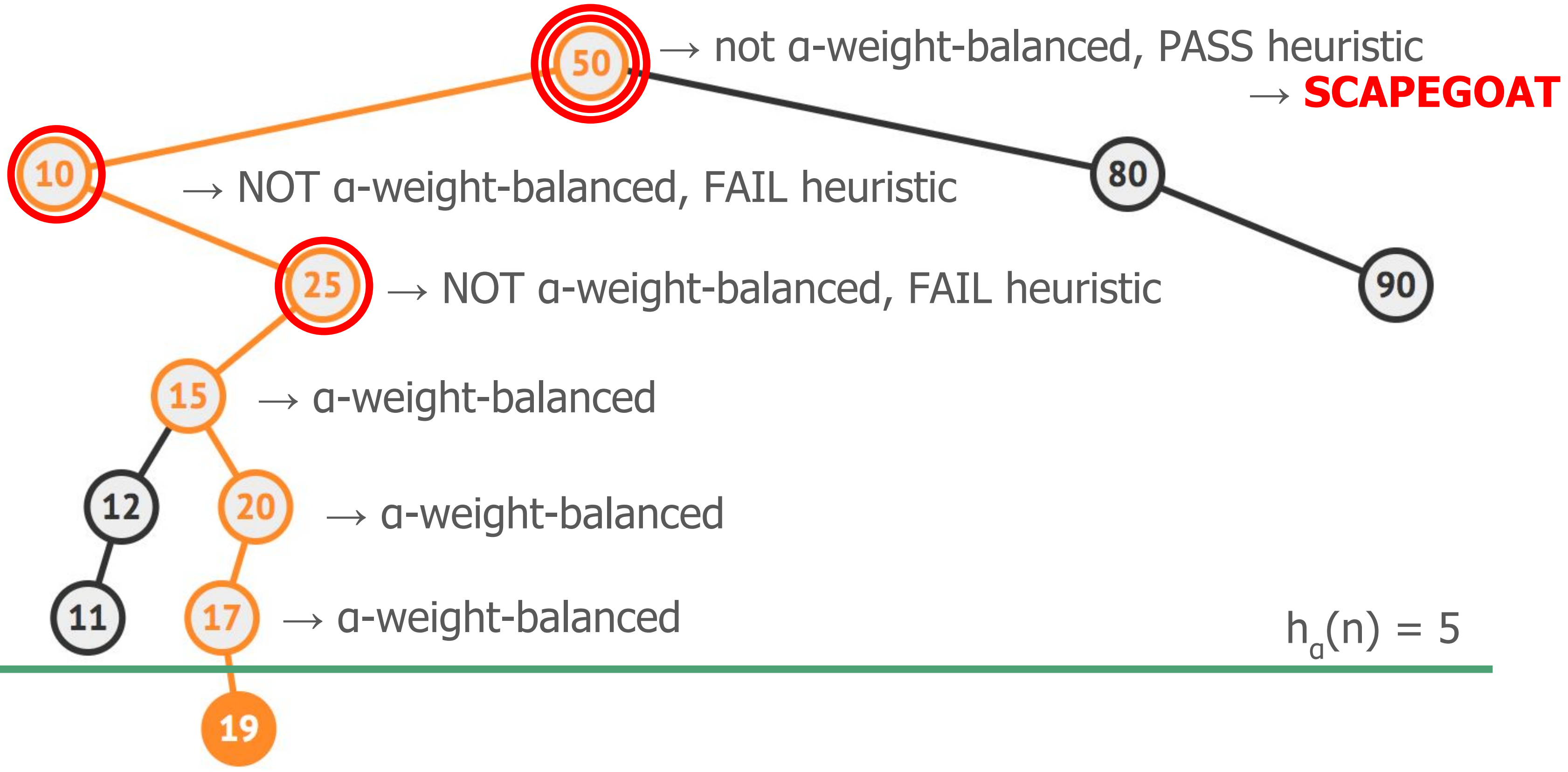
INSERT Example (2/5)



$h_\alpha(n) = 5$

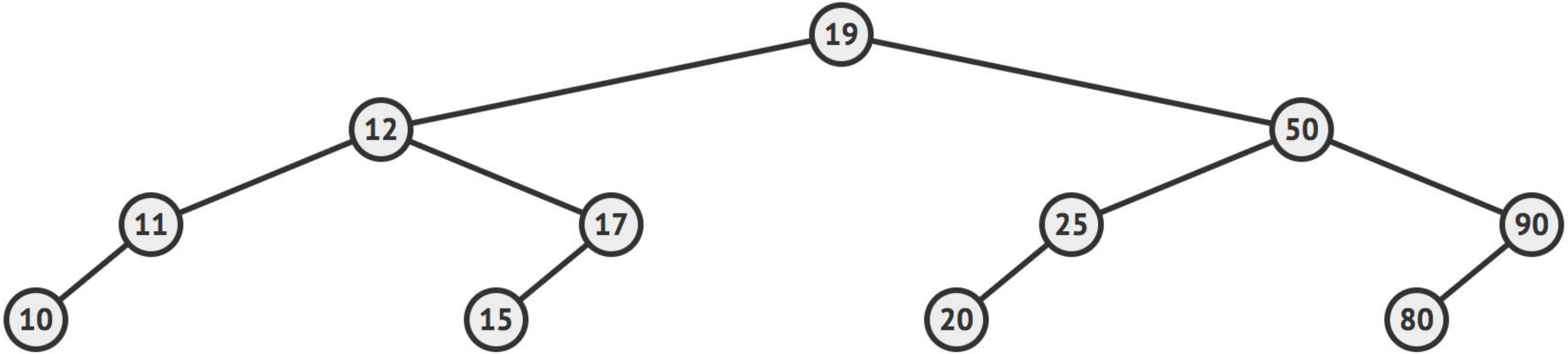
$\rightarrow$ depth = 6 > $h_\alpha(n)$ = 5 $\rightarrow$ 19 is **deep**

INSERT Example (3/5)



INSERT Example (4/5)

INSERT Example (5/5)



$h_\alpha(n) = 5$

DELETE

- DELETE proceeds as in a BST

- Uses max_size in the rebuilding condition:

$$\text{size} < \alpha * \text{max\_size}$$

- Worst case time O(n), but amortized time:

O(log n)

Pros

- SEARCH worst case time **O(log n)**
- INSERT and DELETE amortized time **O(log n)**
- No additional per node information

Cons

- Two extra values per tree
- α should be tuned for expected frequency of operations

Implementation in C

- Fast, robust and concise code
- Rich debug output at runtime if compiled using flag -DDEBUG
- Multiplatform, tested on Windows 10 64 bit and macOS Sierra
- Uses few external libraries:
    - stdio.h → printf
    - math.h → log, floor, ceil
    - stdlib.h → malloc, free
- Almost pedantic w.r.t. implementation described in the paper

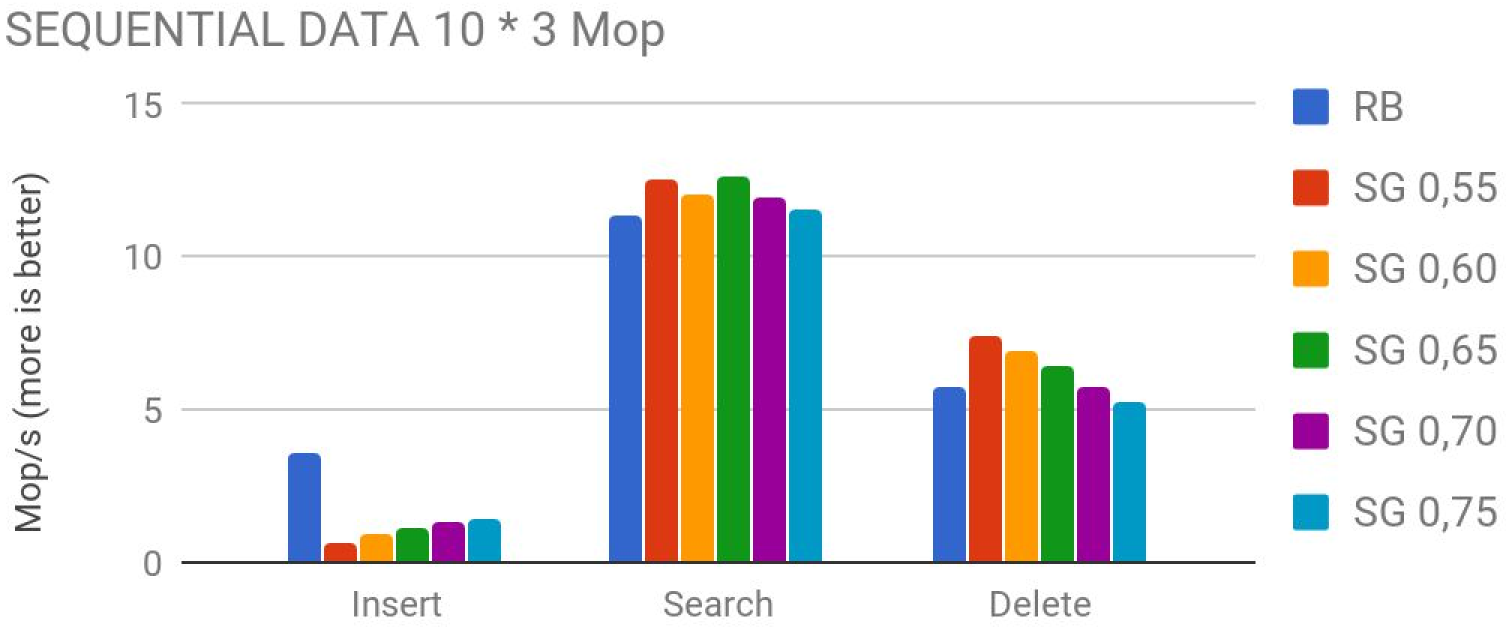
C API

```c
typedef struct sg_node {
	int key;
	struct sg_node* left;
	struct sg_node* right;
} t_sg_node;

typedef struct sg_tree {
	t_sg_node* root;
	unsigned int size;
	unsigned int max_size;
	double alpha;
	double log_one_over_alpha;
	unsigned int h_alpha;
} t_sg_tree;
```

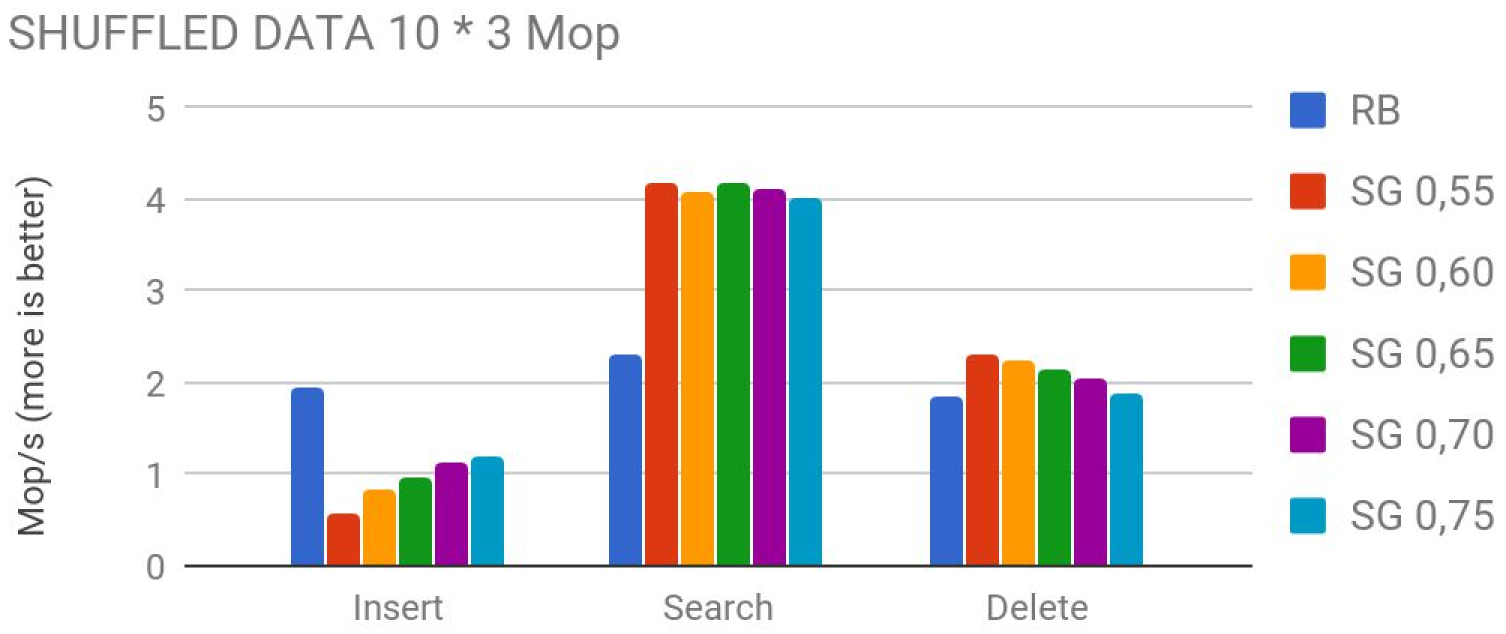
```c
t_sg_tree* sg_create_tree(double alpha);

void sg_delete_tree(t_sg_tree* tree);

t_sg_node* sg_search(t_sg_tree* tree, int key);

unsigned char sg_delete(t_sg_tree* tree, int key);

unsigned char sg_insert(t_sg_tree* tree, int key);

void sg_clear_tree(t_sg_tree* tree);
```

Benchmark in C++ (1/4)

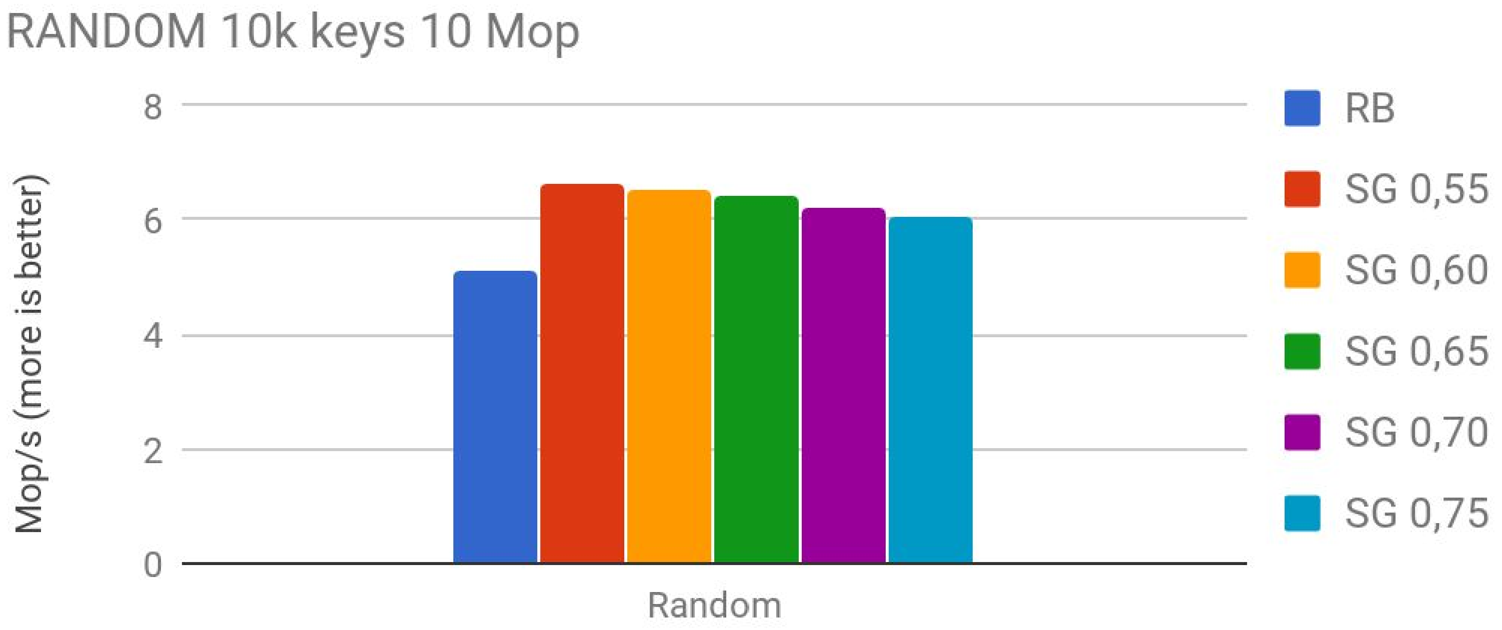
- Versus `std::set<int>` ➔ red-black tree



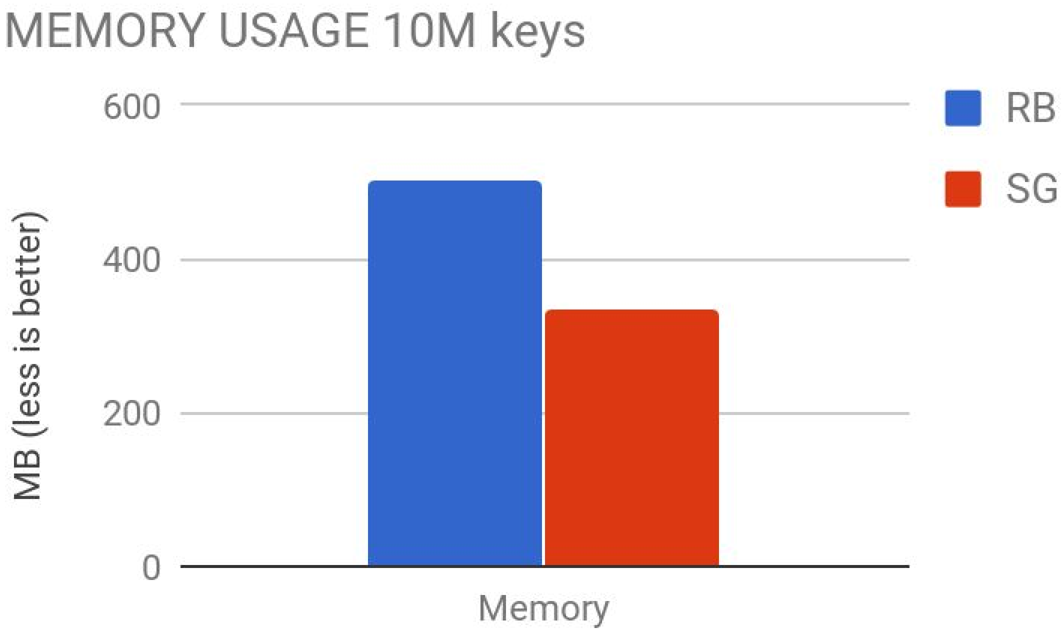
Benchmark in C++ (2/4)



Benchmark in C++ (3/4)



Benchmark in C++ (4/4)



Conclusions

- Sequential and shuffled INSERT faster on red-black tree, due to faster rebalancing operation
- Sequential SEARCH comparable, the two schemes yield balanced trees after sequential INSERTs
- Shuffled SEARCH much faster on scapegoat tree
- Sequential and shuffled DELETE comparable, a bit faster on scapegoat tree due to sporadic but complete rebalancing of the tree
- Random operations on random keys are faster on scapegoat tree, which demonstrates its strength as a general purpose ordered data structure
- Scapegoat tree is much more memory efficient than red-black tree