Proving mathematical statements with Lean

Lesson 11: Repetition

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Overview

1. Goals of today's meeting
2. Motivation
3. Repetition of important tactics in Lean
   ring_nf, nlinarith and norm_num
   have
   apply? and rw?
4. What will happen next

1. Goals of today's meeting

- Sheet 5 had exercises that where so complex to solve in Lean, that I decided to make this repetition level for you. Try to solve it.
- You get a better overview of the new tactics we learned in the last weeks and you get better at applying them.

2. Motivation

- I know the topics around relations and functions have been a bit overwhelming, with this repetition level you will get a better overview.
- After you solve this level, you will already be very good at using Lean.

3. Repetition of important tactics in Lean

For all of the following definitions, you will find examples in the repetition-level on GitHub `https://github.com/MattiaBottoni/Lean-meetings/tree/main/Lean4ExerciseSheets`

3.1. ring_nf, nlinarith and norm_num

Definition (ring_nf)

If you think that your goal is a trivial equation that could be solved by simple arithmetic, you can use the *ring_nf* tactic.

Definition (linarith and nlinarith)

If you have a goal in the form of an inequality and you have a hypothesis that helps you prove this inequality, you can use the *linarith* tactic. If the inequality is not linear, use the *nlinarith* tactic instead.

Definition (norm_num)

If you have a goal that just contains numbers, with *norm_num* you can easily solve the goal.

3.2. have

Definition (have)

Whenever you need a hypothesis, which is trivial to prove, in order to solve your goal, you can use the *have* tactic to introduce this new hypothesis.

3.3. apply?, rw? and simp?

Whenever you are completely stuck and out of ideas, one of these tactics is the way to go.

Definition (apply?)

If you think that you have simplified your goal as far as possible, you can use *apply?* and check the output theorems that Lean will give you. Choose one of the theorems by clicking on them.

Definition (rw?)

If the goal needs to be simplified further, use *rw?*. Just like *apply?*, Lean will give you possible rewrite tactics to use. You can also use e.g *rw? at h* to see how to simplify a hypothesis.

Definition (simp?)

Sometimes using *simp* simplifies a lot for you. But you don't know what actually happened. To find out, type a question mark after the *simp* and read the Lean infoview.

4. What will happen next

- Between Christmas and New Year's eve I will upload all levels with comments so that you can solve them on your own.
- I hope that some of you will participate in the interviews with me.
- Whenever you will need some assistance during your studies, I will always be there for you.

Thank you for your cooperation!!