Proving mathematical statements with Lean

Lesson 9: Natural numbers - Multiplication and Power world

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Overview

1. Goals of today's meeting
2. Motivation
3. Exercises from sheet 6
4. Multiplication world
5. Power world
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1. Goals of today's meeting

- You get to know the natural number game.
- You learn how to build up the natural numbers from scratch.
- You finish the first two worlds of the natural number game.

2. Motivation

- After you did some hard work proving statements about relations and functions, it is time to lean back and enjoy implementing the natural numbers.
- You will start doing the natural number game, which is really addicting and fun.

3. Exercises from sheet 6

Today, we will solve the following exercise from sheet 6 in the natural number game [1]:

Exercise 1 (3pt) Define m^n for $m, n \in \mathbb{N}_0$ by:

$$m^0 = 1, \quad m^{n+1} := m^n m.$$

Prove by suitable induction arguments that:

$$m^{n+r} = m^n m^r, \quad (m^n)^r = m^{nr}, \quad (mn)^r = m^r n^r.$$

4. Multiplication world

Try to solve each level of multiplication world. Help each other out or ask me, if something is confusing you.

5. Power world

The levels 6, 7 and 8 are exactly the exercise 1 from your sheet 6. Try to finish the whole power world.

6. Voluntarily exercises for next week

- Finish both worlds.
- Solve exercise sheet 6 part one on paper.

Thank you for your cooperation!!

References

Argentieri Fernando (2023)
HS 2023 - MAT 115 Foundation of Mathematics Problem sheet 6
UZH

Kevin Buzzard, Jon Eugster (2023)
Natural Number Game
https://adam.math.hhu.de/ [29.11.2023]