Proving mathematical statements with Lean

Lesson 8: Repetition

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Overview

- 1. Goals of today's meeting
- 2. Motivation
- 3. Repetition of important tactics in Lean ring_nf, nlinarith and norm_num have apply? and rw?
- 4. Voluntarily exercises for next week

1. Goals of today's meeting

- By repeating what we learned so far about relations and functions, you deepen your Lean proving skills.
- You get a better overview of the new tactics we learned in the last weeks.

2. Motivation

- I know the last few weeks have been a bit overwhelming, with this repetition level you will get a better overview.
- After you solve this level, you will already be very good at using Lean.

3. Repetition of important tactics in Lean

For all of the following definitions, you will find examples in the repetition-level on GitHub (add Link!)

3.1. ring_nf, nlinarith and norm_num

Definition (ring_nf)

If you think that your goal is a trivial equation that could be solved by simple arithmetic, you can use the *ring_nf* tactic.

Definition (linarith and nlinarith)

If you have a goal in the form of an inequality and you have a hypothesis that helps you prove this inequality, you can use the *linarith* tactic. If the inequality is not linear, use the *nlinarith* tactic instead.

Definition (norm_num)

If you have a goal that just contains numbers, with norm_num you can easily solve the goal.

3.2. have

Definition (have)

Whenever you need a hypothesis, which is trivial to prove, in order to solve your goal, you can use the *have* tactic to introduce this new hypothesis.

3.3. apply?, rw? and simp?

Whenever you are completely stuck and out of ideas, one of these tactics is the way to go.

Definition (apply?)

If you think that you have simplified your goal as far as possible, you can use *apply?* and check the output theorems that Lean will give you. Choose one of the theorems by clicking on them.

Definition (rw?)

If the goal needs to be simplified further, use rw?. Just like apply?, Lean will give you possible rewrite tactics to use. You can also use e.g rw? at h to see how to simplify a hypothesis.

Definition (simp?)

Sometimes using *simp* simplifies a lot for you. But you don't know what actually happened. To find out, type a question mark after the *simp* and read the Lean infoview.

4. Voluntarily exercises for next week

- Solve all the exercises of the repetition level.
- Finish exercise sheet 5.

Thank you for your cooperation!!