

Simple arithmetic in Lean

It can be quite tricky to manipulate formulas in Lean. This cheat sheet should give you an overview of the most important cases.

Note: We are using Lean 4 now, so all the tactics need to be written in brackets. E.g., `rw [add_comm]`

Got	Want	Tactic	What is the rule?
$a+b$	$b+a$	<code>rw add_comm</code>	commutativity
$a+b+c$	$a+(b+c)$	<code>rw add_assoc</code>	associativity
$a+b+c$	$a+c+b$	<code>rw add_assoc</code> <code>rw add_comm b c</code> <code>rw \! add_assoc</code>	higher commutativity
$a-b$	$a+(-b)$	<code>rw sub_eq_add_neg</code>	subtraction = negative addition
$a-a$	0	<code>rw sub_self</code>	number - itself = 0
$a-b$	$(-b) + a$	<code>rw sub_eq_add_neg</code> <code>rw add_comm</code>	negative commutativity
$a-b-c$	$a-(b+c)$	<code>rw sub_sub</code>	negative associativity
$a-b+c$	$a-(b-c)$	<code>rw sub_add</code>	negative associativity
$a+b-c$	$a+(b-c)$	<code>rw add_sub</code>	negative associativity
$(-(-a))$	a	<code>rw neg_neg</code>	negative negative = positive
$a*b$	$b*a$	<code>rw mul_comm</code>	commutativity
$a*b*c$	$a*(b*c)$	<code>rw mul_assoc</code>	associativity
$a*b*c$	$a*c*b$	<code>rw mul_assoc</code> <code>rw mul_comm b c</code> <code>rw \! mul_assoc</code>	higher commutativity
a/b	$a*b^{-1}$	<code>rw div_eq_mul_inv</code>	division = inverse multiplication
$a*a^{-1}$	1	<code>rw mul_inv_cancel</code> (you also need to prove that $a \neq 0$)	number * inverse = 1
$(a*b)^{-1}$	$a^{-1} * b^{-1}$	<code>rw mul_inv</code>	
There is also stuff like <code>div_div</code> , <code>div_mul</code> and <code>mul_div</code> , but I recommend you always work with inverses			
$(a^{-1})^{-1}$	a	<code>rw inv_inv</code>	inv inv = number
$a*(b+c)$	$a*b + a*c$	<code>rw mul_add</code>	distribuity
$(a+b)*c$	$a*c + b*c$	<code>rw add_mul</code>	distribuity
$a*(b-c)$	$a*b - a*c$	<code>rw mul_sub</code>	distribuity
$(a-b)*c$	$a*c - b*c$	<code>rw sub_mul</code>	distribuity
$a-b=0$	$a=b$	<code>rw sub_eq_zero</code>	when is subtraction zero?
$a*b=0$	$a=0$ \or $b=0$	<code>rw mul_eq_zero</code>	when is multiplication zero?
$(a+b)^2$	$a^2 + 2*a*b + b^2$	<code>rw add_sq</code>	binomial formulas
$(a-b)^2$	$a^2 - 2*a*b + b^2$	<code>rw sub_sq</code>	binomial formulas
$a^2 - b^2$	$(a+b)*(a-b)$	<code>rw sq_sub_sq</code>	binomial formulas