Proving mathematical statements with Lean

Lesson 1: introduction and propositional logic

Mattia L. Bottoni

Institute of Mathematics
University of Zurich

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1. Goals of today's meeting

- Getting used to Lean.
- Knowing the basic tactics used for proving statements about propositional logic.
- Being able to prove some first statements with Lean.

2. Motivation

- Proofs are a fundamental concept in mathematics. Yet a lot of undergraduate students struggle with it (including me).
- Lean gives a more "programming-based" and "trial and error" approach to proving statements, which for me and hopefully also you, is something very refreshing.
- In the future, theorem provers will certainly have a large impact on mathematics. At least in some fields. Learning this at an "early age" is certainly a good investment.
- Propositional logic gives the first insight into mathematical proofs. We will start to develop patterns that help us to prove statements.

3. Exercises from sheet 0

In exercise sheet 0, there is only one exercise that makes sense to solve with Lean [1]:

Exercise 5. Show the following statements are always true, no matter what truth values of P, Q, R are. Such statements are called "tautologies".

1. (Distributive Law). $P \wedge (Q \vee R) \iff (P \wedge Q) \vee (P \wedge R)$.
2. (Contrapositive). $P \implies Q \iff \neg Q \implies \neg P$.

4. Exercises from sheet 1

These are the exercises from sheet 1 we will have a look at [2]:

Exercise 6. (4pts) Show the following statements are logically equivalent.

1. $P \Leftrightarrow \neg Q = (\neg P \Rightarrow Q) \wedge (Q \Rightarrow \neg P)$
2. $\neg P \Rightarrow Q = (\neg P \wedge \neg Q) \Rightarrow Q \wedge \neg Q$

Exercise 7.(Extra) (2pts) Use the laws of logical equivalences (Commutative laws, associative laws, etc.) to verify the following logical equivalences. Supply a reason for each step.

1. $(P \wedge (\neg(\neg P \vee Q))) \vee (P \wedge Q) = P$
2. $(\neg(P \vee \neg Q)) \vee (\neg P \wedge \neg Q) = \neg P$

5. Tactics for propositional logic

We will learn a few tactics today. But don't worry, we will learn them one at a time.

- *intro*, *apply* and *exact*
- *split*, *left and right* and *cases*
- maybe also *by contra*

To do so, we will now switch to Lean's web editor.

6. Voluntarily exercises for next week

- Think about differences between Lean-based and written proofs.
- Try to install Lean on your computer with the help of the instructions.
- Optional exercise: Try to implement and prove the exercises from sheet 0 and 1.

Thank you for your cooperation!

References

Argentieri Fernando (2023)
HS 2023 - MAT 115 Foundation of Mathematics Problem sheet 0
UZH

Argentieri Fernando (2023)
HS 2023 - MAT 115 Foundation of Mathematics Problem sheet 1
UZH