Proving mathematical statements with Lean

Lesson 4: induction

Mattia L. Bottoni

Institute of Mathematics
University of Zurich

25.10.2023

Overview

1. Goals of today's meeting
2. Motivation
3. Exercises from sheet 3
4. Induction and strong induction
5. Voluntarily exercises for next week

1. Goals of today's meeting

- Use induction in Lean 4,
- Understand how to use induction to solve the first three exercises of sheet 3.
- Knowing the difference between induction and strong induction.

2. Motivation

- Not only is induction a rather "funny" proving method, it is also fundamental to introducing the natural numbers (as you will see at a later time).

3. Exercises from sheet 3

Today, we will solve the following exercises from sheet 3 [1]:

Exercise 1 (4pt) Prove that

$$(1 + 2 + \cdots + n)^2 = 1^3 + 2^3 + \cdots + n^3,$$

for every $n \in \mathbb{N}^*$.

Exercise 2 (4pt) Prove that $6 | (n^3 - n)$ for every integer $n \geq 0$. (Hint: Strong induction)

Exercise 3 (5pt)(Binomial theorem) Prove that for $n \in \mathbb{N}$:

$$(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k.$$

(Hint: use the identity $\binom{n}{k} + \binom{n}{k-1} = \binom{n+1}{k}$).

4. Induction and strong induction

Definition (induction)

Assume you want to prove a statement $P(n)\ \forall n \in \mathbb{N}$. With induction, you can prove this in two steps:

- the base case

$$P(k = 0) \Rightarrow \mathit{true}$$

- and the induction step: Assume $P(n)$ is true, then

$$P(k = n) \Rightarrow P(k = n + 1)$$

Definition (strong induction)

The strong induction assumes more in the hypothesis. This does not mean, that it is a stronger form of the induction: Assume $P(n)$ is true $\forall n \in \mathbb{N}$, then

$$P(k = n) \Rightarrow P(k = n + 1)$$

5. Voluntarily exercises for next week

- Solve the exercises from sheet 3 on paper. Write down questions if there are any.
- Hardcore: If you want to, try to prove the exercise 1 from sheet 2 in Lean 4.

Thank you for your cooperation!!

References

Argentieri Fernando (2023)
HS 2023 - MAT 115 Foundation of Mathematics Problem sheet 3
UZH