Simple arithmetic in Lean

It can be quite tricky to manipulate formulas in Lean. This cheat sheet should give you and overview of the most important cases.

Note: We are using Lean 4 now, so all the tactics need to be written in brackets. E.g., rw [add_comm]

| Got | Want | Tactic | What is the rule? |
|--------------|----------------------------|--------------------------|--|
| a+b | b+a | rw add_comm | commutativity |
| a+b+c | a+(b+c) | rw add_assoc | associativity |
| a+b+c | a+c+b | rw add_assoc | higher commutativity |
| | | rw add_comm b c | |
| | | rw \l add_assoc | |
| a-b | a+(-b) | rw sub_eq_add_neg | subtraction = negative addition |
| a-a | 0 | rw sub_self | number - itself = 0 |
| a-b | (-b) + a | rw sub_eq_add_neg | negative commutativity |
| | | rw add_comm | |
| a-b-c | a-(b+c) | rw sub_sub | negative associativity |
| a-b+c | a-(b-c) | rw sub_add | negative associativity |
| a+b-c | a+(b-c) | rw add_sub | negative associativity |
| (-(-a)) | а | rw neg_neg | negative negative = positive |
| | | | |
| a*b | b*a | rw mul_comm | commutativity |
| a*b*c | a*(b*c) | rw mul_assoc | associativity |
| a*b*c | a*c*b | rw mul_assoc | higher commutativity |
| | | rw mul_comm b c | |
| | | rw \I mul_assoc | |
| a/b | a*b^-1 | rw div_eq_mul_inv | division = inverse multiplication |
| a*a^-1 | 1 | rw mul_inv_cancel | number * inverse = 1 |
| | | (you also need to | |
| | | prove that a =/= 0) | |
| (a*b)^-1 | a^-1 * b^-1 | rw mul_inv | |
| There is als | o stuff like div_div, div_ | mul and mul_div, but I r | ecommend you always work with inverses |
| (a^-1)^-1 | a | rw inv_inv | inv inv = number |
| | | | |
| a*(b+c) | a*b + a*c | rw mul_add | distribuity |
| (a+b)*c | a*c + b*c | rw add_mul | distribuity |
| a*(b-c) | a*b - a*c | rw mul_sub | distribuity |
| (a-b)*c | a*c - b*c | rw sub_mul | distribuity |
| | | | |
| a-b = 0 | a = b | rw sub_eq_zero | when is subtraction zero? |
| a*b = 0 | a = 0 \or b = 0 | rw mul_eq_zero | when is multiplication zero? |
| | | | |
| (a+b)^2 | a^2 + 2*a*b + b^2 | rw add_sq | binomial formulas |
| (a-b)^2 | a^2 - 2*a*b + b^2 | rw sub_sq | binomial formulas |
| a^2 - b^2 | (a+b)*(a-b) | rw sq_sub_sq | binomial formulas |