Proving mathematical statements with Lean

Lesson 5: induction and some relations

Mattia L. Bottoni

Institute of Mathematics
University of Zurich

Universität Zürich[UZH]

01.11.2023

Overview

1. Goals of today's meeting
2. Motivation
3. Exercises from sheet 3
4. Relations
5. Voluntarily exercises for next week

1. Goals of today's meeting

- Solve exercise 3.2 with strong induction on paper like Lean would do it.
- Understanding the three characteristics of a relation and being able to repeat them.
- See how the three different characteristics of a relation are implemented in Lean 4 and understand it.

2. Motivation

- You start to apply your Lean knowledge on paper.
- You get to see how nicely and elegant one can prove statements about relations using Lean 4.

3. Exercises from sheet 3

Today, we will solve the following exercises from sheet 3 [1]:

Exercise 4 (2pt)

1. Define a relation on $\mathbb{Z}$ as xRy if $|x - y| < 1$. Is R reflexive? Symmetric? Transitive? If a property does not hold, say why. What familiar relation is this?
2. Define a relation R on $\mathbb{Z}$ as xRy if and only if $|x - y| \leq 1$. Say whether R is reflexive. Is it symmetric? Transitive?

Exercise 5 (2pt) Suppose $A \neq \emptyset$. Since $\emptyset \subseteq A \times A$, the set $R = \emptyset$ is a relation on A. Is R reflexive? Symmetric? Transitive? If a property does not hold, say why.

Exercise 6 (3pt)

1. Suppose R is a symmetric and transitive relation on a set A, and there is an element $a \in A$ for which aRx for every $x \in A$. Prove that R is reflexive.
2. Prove or disprove: If a relation is symmetric and transitive, then it is also reflexive.

4. Relations

Definition (relation)

Let A be a set. A relation R is a subset of $A \times A$. For an element $(a, b) \in A \times A$, we write aRb if $(a, b) \in R$

Relations

Definition (reflexive relation)

A relation R is called *reflexive*, if

$$\forall a \in A, aRa$$

Definition (symmetric relation)

A relation R is called *symmetric*, if

$$\forall a, b \in A, aRb \Rightarrow bRa$$

Definition (transitive relation)

A relation R is called *transitive*, if

$$\forall a, b, c \in A, aRb \wedge bRc \Rightarrow aRc$$

5. Voluntarily exercises for next week

- Finish the exercises from sheet 3 we could not finish today in Lean.

Thank you for your cooperation!!

References


Argentieri Fernando (2023)
HS 2023 - MAT 115 Foundation of Mathematics Problem sheet 3
UZH