NUMERICAL MODELING AND SIMULATION FOR ACOUSTICS
A.A. 2025/2026

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HOMEWORK 1

Consider the following wave propagation problem:

$$\begin{cases} \rho(x)\omega^2 u(x) + \dfrac{\partial}{\partial x}(\mu(x)\dfrac{\partial u}{\partial x})(x) = f(x) \qquad x \in (0, L), \\ \mu(0)\dfrac{\partial u}{\partial x}(0) = g_N, \\ u(L) - \sqrt{\dfrac{\mu(x)}{\rho(x)}}\dfrac{i}{\omega}\dfrac{\partial u}{\partial x}(L) = g_A, \end{cases} \tag{1}$$

where g_N and g_A are given constants, μ and ρ represent the (variable) stiffness and mass density of the medium, and i is the imaginary unit.

1. Write the weak formulation of problem (1) and the corresponding finite element discretization. Consider linear basis functions. Show that the Galerkin discretization leads to the following system
$$\omega^2 M\boldsymbol{u} - A\boldsymbol{u} = \boldsymbol{F}.$$
Define precisely the entries of the matrices M and A and of the right hand side $\boldsymbol{F}$.

2. Implement the solver in MATLAB and verify your implementation by considering:
$$\Omega = (0,1), \quad \rho = \mu = 1, \quad \text{and} \quad \mathrm{u_{ex}} = \sin(\omega \mathrm{x}),$$
with $\omega = 5\pi$ and compute the corresponding forcing terms f, g_N and g_A. Compute and report the L^2 norm $\|u - u_h\|_{L^2(0,L)}$ for different choices of the discretization parameter h (mesh size).

3. Consider now:
$$\omega = 5\pi, \quad \mu = \begin{cases} 4 & x \leq 0.5, \\ 1 & x > 0.5, \end{cases} \quad \text{and} \quad \rho = 1,$$
and use $f = g_A = 0$ and $g_N = 1$. Solve for and report: (i) the amplitude $|u(x)|$, (ii) the phase of $u(x)$, (iii) comments on the wave reflection at the interface $x = 0.5$.

4. Investigate the numerical dispersion of your FE discretization by computing the discrete wavenumber k_h (consider $f = g_N = 0$ and $u(L) = 0$ as a boundary condition for $x = L$). Plot the dispersion curves k_h/k as a function of the mesh parameter h/λ, being λ the wavelength. Discuss how the number of elements per wavelength affects accuracy.

5. (Optional) Implement a Perfectly Matched Layer (PML) on the right-end side of the domain and compare results obtained at point 3 with the absorbing boundary condition.