POLITECNICO
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“Prova finale: Introduzione all’analisi di missioni spaziali”

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1. Introduction

The aim of this paper is to find, select and analyse possible transfers between two given orbits. We developed possible tracks from a “low MEO” parking orbit to a “MEO” target orbit. We did not consider everything related to the launch and the atmospheric phase. In addition, we assumed all our manoeuvres as impulsive and our orbital transfers do not take into account any type of spatial environment conditions except for the parameters known and studied.

Our work strategy was to start from the characterisation of the orbits, as studied during the lessons, to better understand the data we had. Then, using the software program MATLAB we developed the basic functions to operate a so-called “Standard strategy”, so that we had a reference about cost and time.
From this point, we leveraged our knowledge related to orbit transfer to find and select the most cost-effective ways to reach our goal.
Firstly, we thought it could be a good idea to reuse the basic steps, but changing the order of them or the points of manoeuvre, due to theory reasons that could help us obtain a smaller Δv.
Secondly, we were looking for a transfer that would have reduced the time spent, so we decided to create a track closer to the attractor, having a bit higher cost, but reaching faster the target position.
Moreover, we strengthened our ideas more so that we could find a solution to reduce the high cost of plane change manoeuvre but keeping a low Δv of other ones throughout the transfer.

In the end, we compared the transfer strategies found with the standard one. All our results have been summarised in the ending tables. In order to have a better immediate understanding during an oral exposition of the analysis done, we have created a presentation, which resume all the meaningful information written here.

2. Initial orbit characterisation

2.1 Key orbital parameters

We wanted to characterise the initial orbit from the given data. We received the starting Cartesian coordinates of position and velocity expressed in a geocentric equatorial system as inputs:

$$\vec{r_i} = \begin{Bmatrix} -5829.6395 \\ -6946.1419 \\ 4067.7268 \end{Bmatrix} km \qquad \vec{v_i} = \begin{Bmatrix} 1.9470 \\ -4.5900 \\ -3.8490 \end{Bmatrix} km\ s^{-1}$$

It was possible to obtain Keplerian parameters using the MATLAB function ("rv2parorb"). Giving in input the two vectors and known the gravitational parameter we have received the following outputs:

$$a_i = 9832.9719\ km \qquad e_i = 0.0786 \qquad i_i = 49.8022°$$

$$\Omega_i = 72.2687° \qquad \omega_i = 45.2880° \qquad \theta_i = 102.3119°$$

2.2 Other relevant parameters

Then, we could completely characterise the orbit using the previous data to calculate:

$$p_i = 9772.2190\ km \qquad T_i = 9704\ s \qquad \mathcal{E}_i = -20.2685\ km^2\ s^{-2}$$

$$\begin{cases} r_{p_i} = 9059.8846\ km \\ r_{a_i} = 10606.0592\ km \end{cases} \qquad \begin{cases} v_{p_i} = 5.8845\ km\ s^{-1} \\ v_{a_i} = 6.8888\ km\ s^{-1} \end{cases}$$

In addition, known the average Earth radius, we could determine the altitude of the orbit and we observed that we were representing a "low MEO" parking orbit:

$$h_{p_i} = 2681.8846\ km \qquad h_{a_i} = 4228.0592\ km$$

2.3 Graphical representation

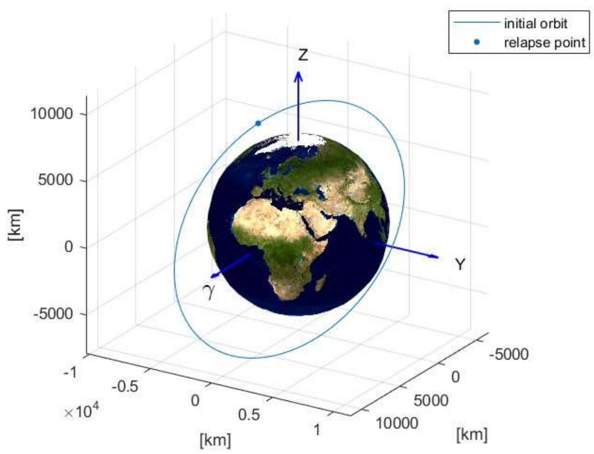

Figure 1. Initial orbit

3. Final orbit characterisation

3.1 Key orbital parameters

In relation to the final data given, we had the following Keplerian parameters:

$$a_f = 13790\ km \qquad e_f = 0.2719 \qquad i_f = 59.9887°$$

$$\Omega_f = 146.6199° \qquad \omega_f = 26.4363° \qquad \theta_f = 14.7709°$$

Thanks to the ("parorb2rv") MATLAB function we obtained, as results, the Cartesian coordinates of final position and velocity expressed in a geocentric equatorial system:

$$\vec{r_f} = \begin{Bmatrix} -8185.7893 \\ 1403.3120 \\ 5768.4319 \end{Bmatrix} km \qquad \vec{v_f} = \begin{Bmatrix} 2.1071 \\ -4.7207 \\ 4.8175 \end{Bmatrix} km\ s^{-1}$$

3.2 Other relevant parameters

Then, we have calculated the other important parameters:

$$p_f = 12770.5107\ km \qquad T_f = 16116\ s \qquad \mathcal{E}_f = -14.4525\ km^2 s^{-2}$$

$$\begin{cases} r_{p_f} = 10040.4990\ km \\ r_{a_f} = 17539.5010\ km \end{cases} \qquad \begin{cases} v_{p_f} = 7.1059\ km\ s^{-1} \\ v_{a_f} = 4.0677\ km\ s^{-1} \end{cases}$$

Similarly to what has been discussed for the first characterisation, we could also calculate the altitude of the orbital key points, and we observed we were in a "MEO" target orbit:

$$h_{p_f} = 3662.4990\ km \qquad h_{a_f} = 11161.5010\ km$$

3.3 Graphical representation

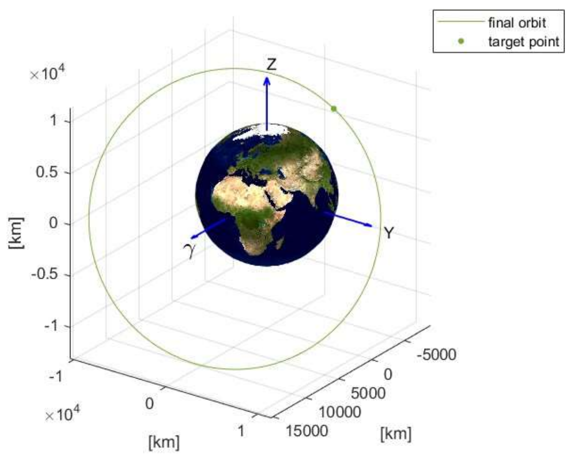

Figure 2. Final orbit

4. Transfer trajectory definition and analysis

4.1 Standard Strategy

The standard strategy implies a pre-fixed ordered number of steps to be followed:

- Plane change: $\{a_i, e_i, i_i, \Omega_i, \omega_i\} \Rightarrow \{a_i, e_i, i_f, \Omega_f, \omega_{cp}\}$
- Pericenter anomaly adjustment: $\{a_i, e_i, i_f, \Omega_f, \omega_{cp}\} \Rightarrow \{a_i, e_i, i_f, \Omega_f, \omega_f\}$
- Bitangent transfer to change orbital shape: $\{a_i, e_i, i_f, \Omega_f, \omega_f\} \Rightarrow \{a_f, e_f, i_f, \Omega_f, \omega_f\}$

We started from the initial position given and changed initial orbital plane reaching the final one at the first point of manoeuvre found. Then, at the first point possible we adjusted the pericenter anomaly and, in the end, we corrected the orbital shape.
There are multiple ways to complete the transfer based on the type of bitangent adopted. We solved all of them in MATLAB and discarded the pericenter-pericenter and apocenter-apocenter types due to, respectively, high Δv and time spent.
To choose the best one between the remaining ones we compared them and established as our standard strategy the one with an apocenter-pericenter bitangent transfer because, in our opinion, the time reduction was more significant, indeed:

- the apocenter-pericenter one has an increased Δv of 0.26% compared to the pericenter-apocenter
- this manoeuvre is characterized by a time reduction of 23.5%

We used the functions developed in MATLAB "changeOrbitalPlane", "changePericenterArg", "bitangentTransfer" and "TOF" to obtain the points, velocity difference of the manoeuvres and the time spent to reach each step.

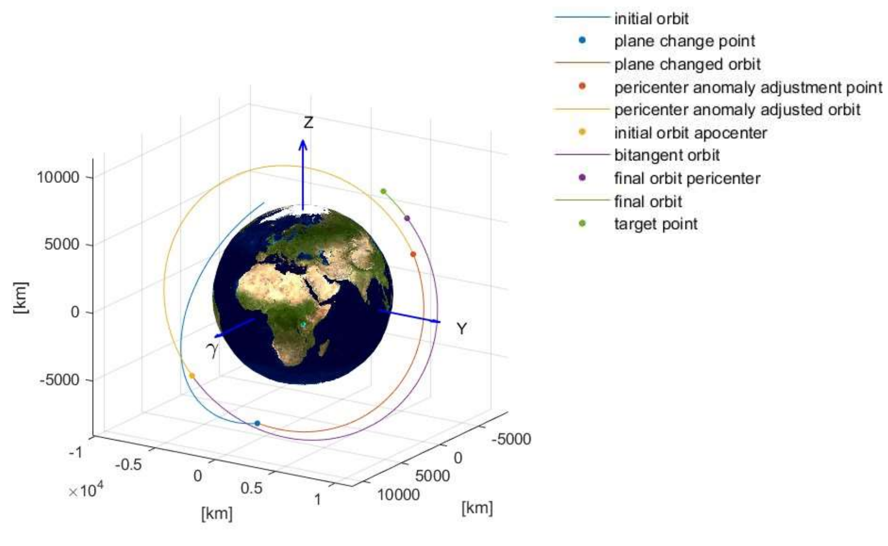

Figure 3. Standard strategy

4.2 Alternative Transfer1 - Step changed transfer

During the development of the standard strategy, we observed that we could have obtained a better result in terms of Δv inverting the order of the basic steps. Our idea was to operate a plane change a bit more distant from the attractor so to reduce the cost related to this manoeuvre.
So, we have changed the shape of initial orbit into the final one thanks to a bitangent manoeuvre, followed by a plane change and pericenter anomaly adjustment manoeuvres, both in the first possible point. Therefore, the transfer steps are:

- Bitangent shape change (pericenter to apocenter): $\{\boldsymbol{a_i}, \boldsymbol{e_i}, \boldsymbol{i_i}, \boldsymbol{\Omega_i}, \boldsymbol{\omega_i}\} \Rightarrow \{\boldsymbol{a_f}, \boldsymbol{e_f}, \boldsymbol{i_i}, \boldsymbol{\Omega_i}, \boldsymbol{\omega_i}\}$
- Plane change: $\{\boldsymbol{a_f}, \boldsymbol{e_f}, \boldsymbol{i_i}, \boldsymbol{\Omega_i}, \boldsymbol{\omega_i}\} \Rightarrow \{\boldsymbol{a_f}, \boldsymbol{e_f}, \boldsymbol{i_f}, \boldsymbol{\Omega_f}, \boldsymbol{\omega_{cp}}\}$
- Pericenter anomaly adjustment: $\{\boldsymbol{a_f}, \boldsymbol{e_f}, \boldsymbol{i_f}, \boldsymbol{\Omega_f}, \boldsymbol{\omega_{cp}}\} \Rightarrow \{\boldsymbol{a_f}, \boldsymbol{e_f}, \boldsymbol{i_f}, \boldsymbol{\Omega_f}, \boldsymbol{\omega_f}\}$

	Standard strategy	Alternative1	%
Δv [km/s]	**7.2571**	6.4341	-11.34
Time [s]	**18277**	23947	+31.02

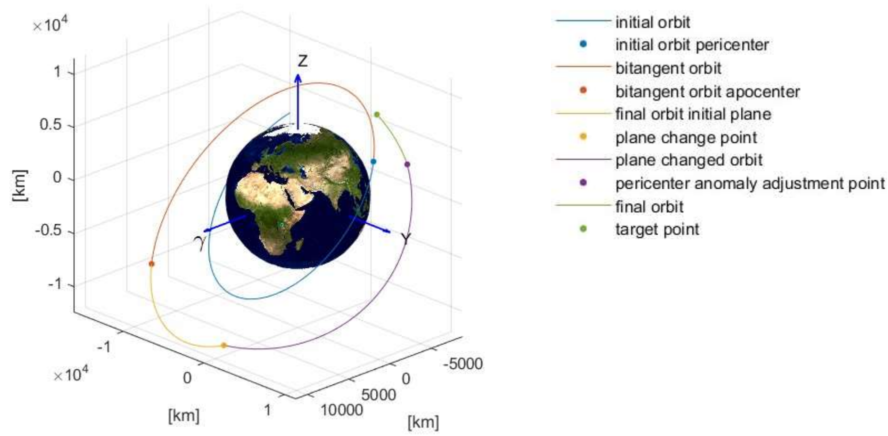

Figure 4. Alternative transfer1

4.3 Alternative Transfer2 – Time reduction

The purpose of this transfer was to reduce total time lapse spent to reach the final target point. We have thought at the advantages that moving in smaller orbits would have brought to time lapses for each manoeuvre.
Since final periapsis had a value included between initial periapsis and apoapsis, there was a point in the initial orbit that corresponds to a radius equal to final periapsis where was possible to make a tangent manoeuvre to have a new circular orbit with that radius.
After the plane change manoeuvre in the first possible point, it was necessary to change orbital shape again to the final one through a tangent manoeuvre, so that we maintained the final orbit periapsis.
Thanks to the nature of the circular orbit, we could choose that manoeuvre point in order to have the correct orientation of final orbit semi-major axis, avoiding an extra impulse to adjust pericenter anomaly.

The steps we obtained are:

- Circular Orbit (Final periapsis radius dimension): $\{\boldsymbol{a_i}, \boldsymbol{e_i}, \boldsymbol{i_i}, \boldsymbol{\Omega_i}, \boldsymbol{\omega_i}\} \Rightarrow \{\boldsymbol{r_{p_f}}, \boldsymbol{0}, \boldsymbol{i_i}, \boldsymbol{\Omega_i}, \boldsymbol{0}\}$
- Change plane: $\{\boldsymbol{r_{p_f}}, \boldsymbol{0}, \boldsymbol{i_i}, \boldsymbol{\Omega_i}, \boldsymbol{0}\} \Rightarrow \{\boldsymbol{r_{p_f}}, \boldsymbol{0}, \boldsymbol{i_f}, \boldsymbol{\Omega_f}, \boldsymbol{0}\}$
- Tangent manoeuvre: $\{\boldsymbol{r_{p_f}}, \boldsymbol{0}, \boldsymbol{i_f}, \boldsymbol{\Omega_f}, \boldsymbol{0}\} \Rightarrow \{\boldsymbol{a_f}, \boldsymbol{e_f}, \boldsymbol{i_f}, \boldsymbol{\Omega_f}, \boldsymbol{\omega_f}\}$

	Standard strategy	Alternative2	%
Δv [km/s]	**7.2571**	7.5779	+4.42
Time [s]	**18277**	7827	-57.18

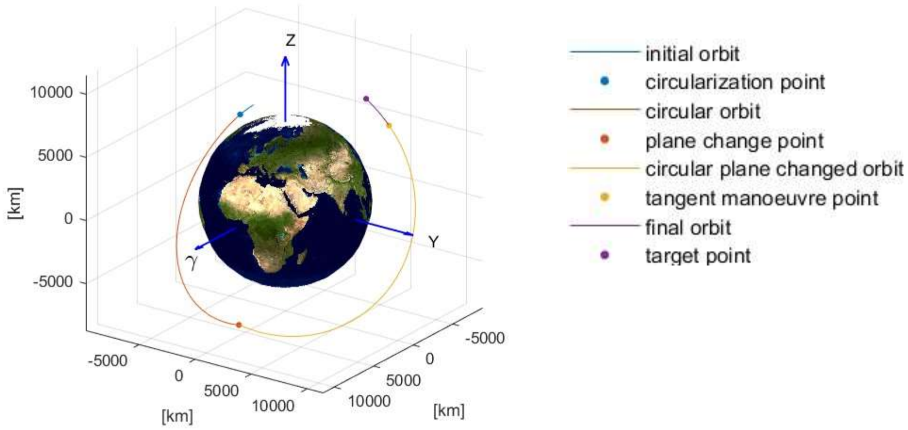

Figure 5. Alternative transfer2

4.4 Alternative Transfer3 - Reduced plane change Δv

The purpose of our final alternative transfer was to reduce as much as possible the Δv related to plane change, while keeping the convenience in total Δv. Knowing that this velocity difference decreases the further from the attractor the manoeuvre is done, we wanted to find an auxiliary orbit, that we called *star orbit*, that maintains his periapsis as the initial orbit's one and an apoapsis equal to what we have called *star radius*.

Even though moving too far from the attractor leads to a small Δv of plane change, the Δv of other manoeuvres, such as the change of orbital shape, grows with high values of orbital eccentricity. So, in order to obtain a convenient total Δv compared with the standard strategy's one, we used a MATLAB cycle to find the optimal apoapsis, id est *star radius*, of auxiliary orbit that corresponded to a minimum of total Δv.

Thanks to a first approximation we were able to find that, starting from a star radius equal to initial orbit's apoapsis to star radius equal to 100000 km with a step of 1 km, the optimal star radius was 17539.1 km *(view Figure 6a)*. Then, we refined step firstly to 0.1 km and then to 0.01 km reducing the radius range around the initial optimal value found, and certified that optimal star radius is equal to final orbit's apoapsis *(view Figure 6b)*.

We decided to circularize in *star orbit* apocenter because, thanks to the nature of circular orbit, we were able to keep the apocenter velocity, that is the lowest of the orbit, whatever is the manoeuvre point.

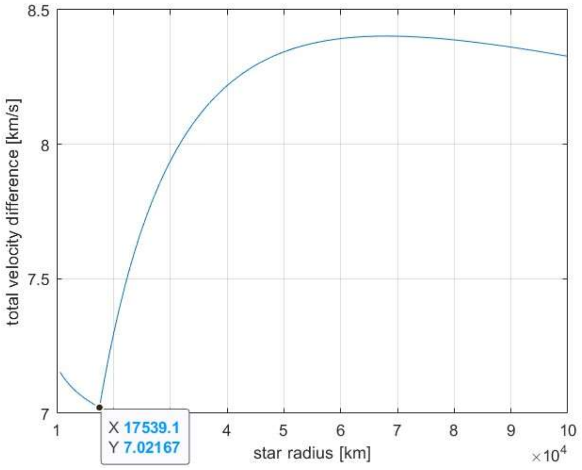

Figure 6a

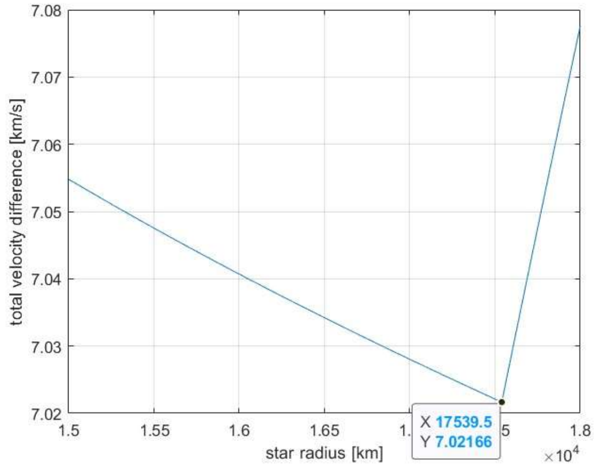

Figure 6b

Thus, the strategy followed is:

- Tangent-manoeuvre: $\{a_i, e_i, i_i, \Omega_i, \omega_i\} \Rightarrow \{a_{star}, e_{star}, i_i, \Omega_i, \omega_i\}$
- Circular Orbit (star radius dimension): $\{a_{star}, e_{star}, i_i, \Omega_i, \omega_i\} \Rightarrow \left\{r_{a_f}, 0, i_i, \Omega_i, 0\right\}$
- Plane change: $\left\{r_{a_f}, 0, i_i, \Omega_i, 0\right\} \Rightarrow \{r_{a_f}, 0, i_f, \Omega_f, 0\}$
- Final tangent-manoeuvre: $\{r_{a_f}, 0, i_f, \Omega_f, 0\} \Rightarrow \{a_f, e_f, i_f, \Omega_f, \omega_f\}$

	Standard strategy	Alternative3	%
Δv [km/s]	**7.2571**	7.0217	-3.24
Time [s]	**18277**	48186	+163.64

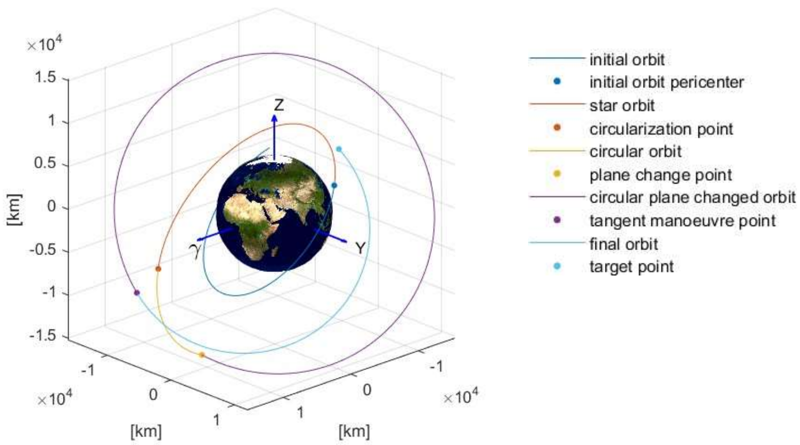

Figure 7. Alternative transfer3

5. Conclusions

After all these considerations we can assert that there are several ways to transfer from initial orbit to the final one, and every way could be suitable for different situations. Thanks to the lessons, we were capable to build different standard strategies and to establish which one had the best trade-off in both terms of cost and time spent.

As shown in the graph (*view Figure 8a*), in both *alternative transfer1* and *alternative transfer3*, we managed to obtain a lower Δv than standard strategy, even if the time lapses have grown. The most effective way to fully understand this advantage is to use the Tsiolkovsky equation to calculate the percentage amount of fuel saved with both transfers compared to the standard one.
Assuming the specific impulse value as 300 [s], we obtain:

- For *alternative transfer1* a fuel saving of about 11.33%
- For *alternative transfer3* a fuel saving of about 3.24%

The main question could be why choose *alternative transfer3* instead of *alternative transfer1*, since the latter has a higher global fuel save. The answer is that for some missions is more important to have the lowest manoeuvres' Δv possible on payloads due to different requirements, such as comfort reasons, structural points of view or type of power engines. Therefore, *alternative transfer3* perfectly fits this idea, since it has the lowest Δv related to plane change that keeps even the other manoeuvres' Δv as low as possible.
For what concern time lapses reduction, as shown in the graph (*view Figure 8b*), through *alternative transfer2* we managed to save about 57% of time compared to the standard strategy.
In terms of fuel used to complete this transfer, thanks to Tsiolkovsky equation, we are able to calculate that is required an extra 4.41% of fuel.
However, the advantages coming from the time lapse reduction makes the trade-off fully acceptable especially in missions whose main objective is to reach target position as soon as possible.

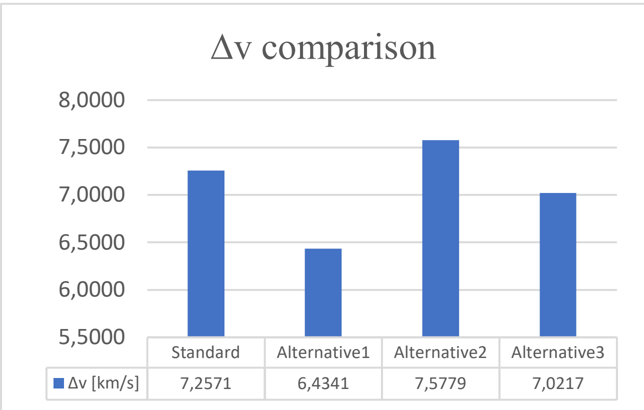

Figure 8a

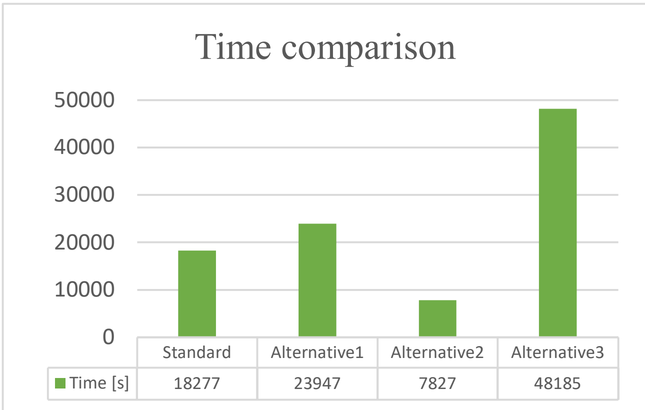

Figure 8b

As can be seen from the large number of existing maneuvers, it is impossible to determine a priori which one is the best. Everything is a function of the mission requirements, mission objectives, as well as the resources available.

6. Appendix

Transfer 1 (standard strategy)

t (s)	a (km)	e (-)	i (rad)	Ω (rad)	ω (rad)	θ (rad)	Δv (km/s)
0	9832.9719	0.0786	0.8692	1.2613	0.7904	1.7857	-
4174	9832.9719	0.0786	0.8692	1.2613	0.7904	4.1925	6.1296
	9832.9719	0.0786	1.0470	2.5590	6.2482	4.1925	
7513	9832.9719	0.0786	1.0470	2.5590	6.2482	0.2482	0.2467
	9832.9719	0.0786	1.0470	2.5590	0.4614	6.0350	
12692	9832.9719	0.0786	1.0470	2.5590	0.4614	3.1416	0.1614
	10323.2791	0.0274	1.0470	2.5590	0.4614	3.1416	
17911	10323.2791	0.0274	1.0470	2.5590	0.4614	0	0.7194
	13790	0.2719	1.0470	2.5590	0.4614	0	
18277	13790	0.2719	1.0470	2.5590	0.4614	0.2578	-

Transfer 2-Alternative1

t (s)	a (km)	e (-)	i (rad)	Ω (rad)	ω (rad)	θ (rad)	Δv (km/s)
0	9832.9719	0.0786	0.8692	1.2613	0.7904	1.7857	-
7186	9832.9719	0.0786	0.8692	1.2613	0.7904	0	0.7284
	13299.6928	0.3188	0.8692	1.2613	0.7904	0	
14818	13299.6928	0.3188	0.8692	1.2613	0.7904	3.1416	0.1332
	13790	0.2719	0.8692	1.2613	0.7904	3.1416	
18846	13790	0.2719	0.8692	1.2613	0.7904	4.1925	4.8263
	13790	0.2719	1.0470	2.5590	6.2482	4.1925	
23228	13790	0.2719	1.0470	2.5590	6.2482	0.2482	0.7463
	13790	0.2719	1.0470	2.5590	0.4614	6.0350	
23947	13790	0.2719	1.0470	2.5590	0.4614	0.2578	-

Transfer 3-Alternative2

t (s)	a (km)	e (-)	i (rad)	Ω (rad)	ω (rad)	θ (rad)	Δv (km/s)
0	9832.9719	0.0786	0.8692	1.2613	0.7904	1.7857	-
211	9832.9719	0.0786	0.8692	1.2613	0.7904	1.9176	0.4798
	10040.4990	0	0.8692	1.2613	0	2.7080	
3836	10040.4990	0	0.8692	1.2613	0	4.9830	6.2930
	10040.4990	0	1.0470	2.5590	0	4.9830	
7461	10040.4990	0	1.0470	2.5590	0	0.4614	0.8051
	13790	0.2719	1.0470	2.5590	0.4614	0	
7827	13790	0.2719	1.0470	2.5590	0.4614	0.2578	-

Transfer 4-Alternative3

t (s)	a (km)	e (-)	i (rad)	Ω (rad)	ω (rad)	θ (rad)	Δv (km/s)
0	9832.9719	0.0786	0.8692	1.2613	0.7904	1.7857	-
7186	9832.9719	0.0786	0.8692	1.2613	0.7904	0	0.7284
	13299.6928	0.3188	0.8692	1.2613	0.7904	0	
14818	13299.6928	0.3188	0.8692	1.2613	0.7904	3.1416	0.8326
	17539.5010	0	0.8692	1.2613	0	3.9320	
18685	17539.5010	0	0.8692	1.2613	0	4.9830	4.7613
	17539.5010	0	1.0470	2.5590	0	4.9830	
39762	17539.5010	0	1.0470	2.5590	0	3.6030	-0.6994
	13790	0.2719	1.0470	2.5590	0.4614	3.1416	
48186	13790	0.2719	1.0470	2.5590	0.4614	0.2578	-