
# Model identification for a multi-rotor UAV

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## Laboratory tasks

The project deals with the dynamics of a multirotor UAV (longitudinal dynamics), represented by the ANT-X 2DOF drone, and involves the following two tasks.

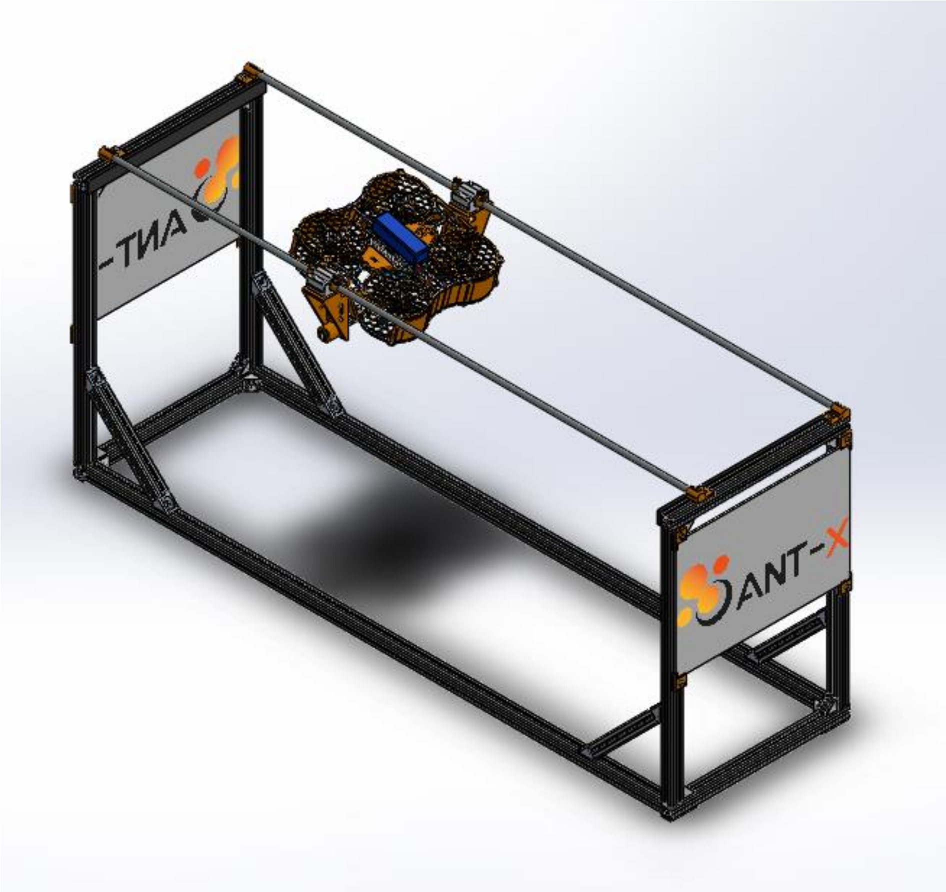
*Task I: 05/12/2024*

- System identification of **closed-loop** longitudinal dynamics

*Task II: 12/12/2024*

- Pitch estimation from pitch rate and acceleration measurements

# A small 2DOF laboratory quadrotor




The ANT-X 2DoF Drone is a laboratory test-bed designed to be employed for control research and educational purposes.

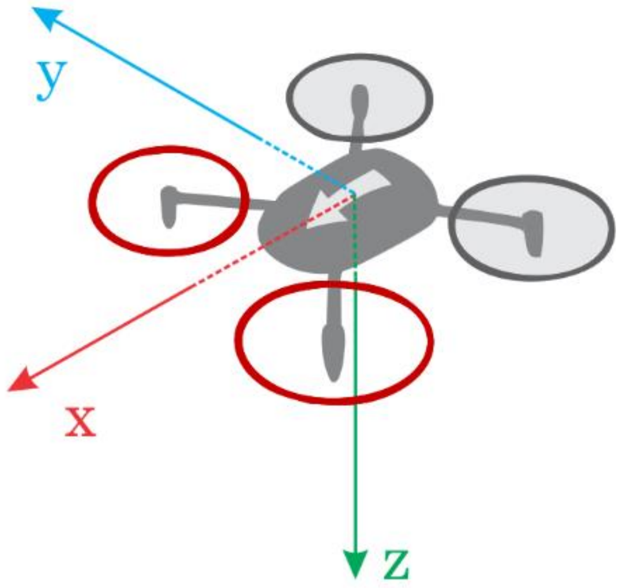
Two degrees of freedom:

- Longitudinal translation
- Pitch attitude

The 2DoF Drone is designed to run experiments in a safe and controlled environment, and in a repeatable way.

# Reference frames

- Body frame:




- Inertial frame: North-East-Down (NED) convention

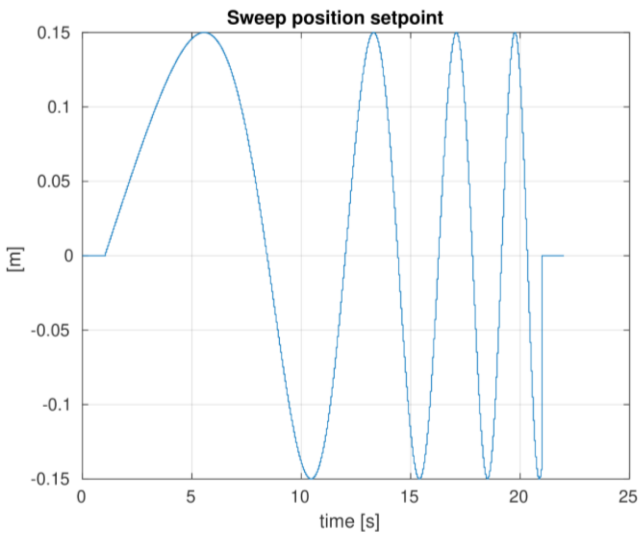
# Task 1: model identification

- The first lab activity consists in **identifying a model for the closed-loop longitudinal dynamics** of the multirotor. The model shall have as input the position setpoint and as output the position and the acceleration in the NED frame.

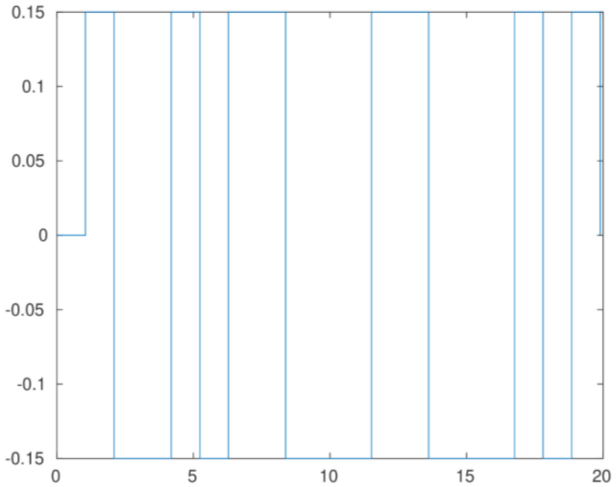
In greater detail:

- Choose an **input sequence** to be applied to the system to collect data both for identification and validation.
- Conduct the **experiment**.
- Define a **model structure** and a suitable **identification method** for the closed-loop longitudinal dynamics of the multirotor.
- **Identify a model** using the collected response data.
- **Assess the uncertainty** of the identified model.
- **Evaluate the performance** of the identified model using the **validation dataset**.

## Input Sequence



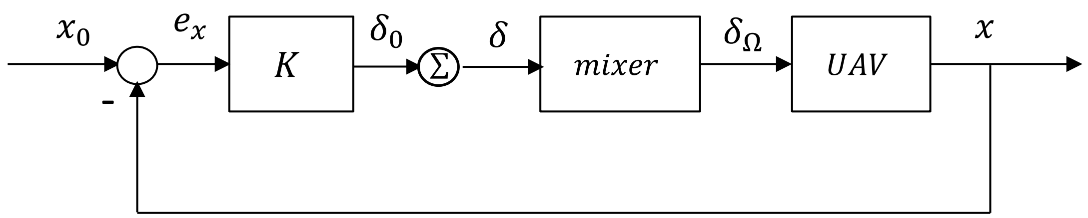

Sinesweep

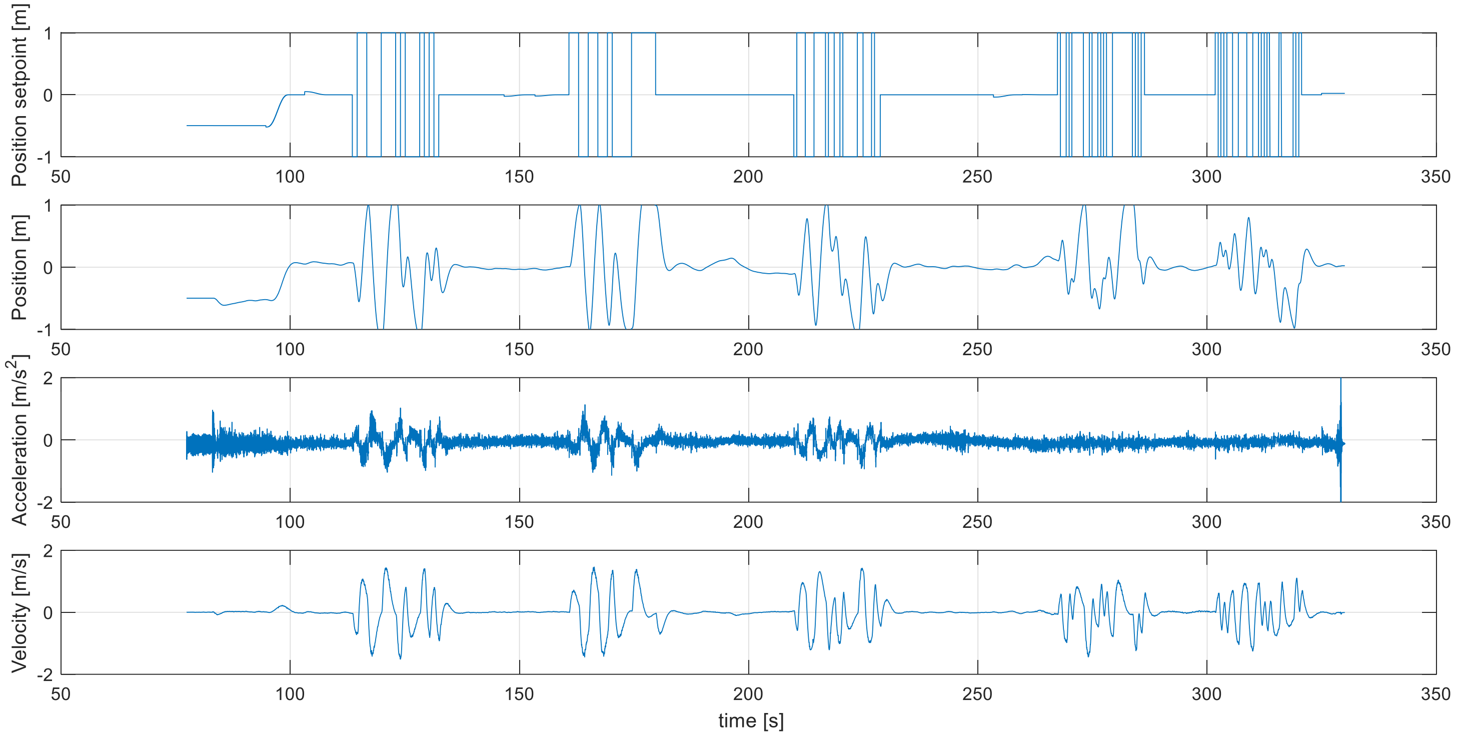



PRBS

- **Design parameters**: Sinesweep $\omega_{min},\ \omega_{max},\ T$
  PRBS $\omega_{min},\ \omega_{max},\ A,\ T$
- **Intervals of Interest**: $0 < \omega < 7\ \mathrm{rad/s},\ 0 < T < 60\ \mathrm{s},\ 0 < A < 0.25\ \mathrm{m}$

# Identification experiments and I/O estimation data



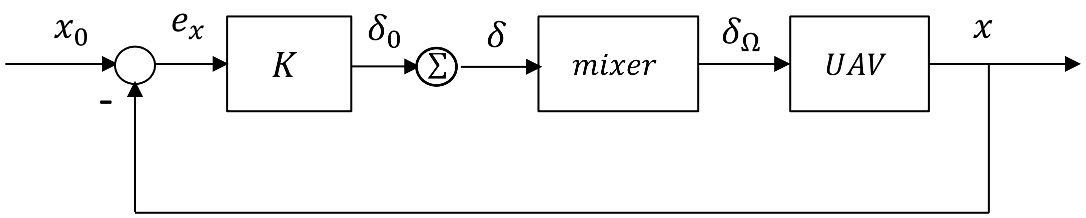




***Decoupled dynamics***

- *Longitudinal dynamics*
  - Input: $x_0$
  - Output: $y = [x \; \ddot{x}]^\top$
- *The definition of the model class relating $x_0$ to y is part of the lab activity*

# Model Structure

- The close-loop dynamics of the quadrotor is, at first approximation, comparable to the dynamics of a second order model.




$$\ddot{x} + 2\xi\omega_n\dot{x} + \omega_n^2 x = \omega_n^2 x_0$$

- Tunable parameters: natural frequency $\omega_n$

  damping factor $\xi$

- Construct the state-space system to retrieve $y = [x \ \ddot{x}]^\top$

# Identification Strategy

- **Black-Box Approach**: no internal knowledge of the system, rely only on input and output data.

  → `tfest`

- **Grey-Box Approach**: partial knowledge of the system, rely on a model containing partial information, and I/O data.

  → `greyest`

- **White-Box Approach**: perfect knowledge of the system, rely on the physical model.

# Validation Metrics

- Variance Accounted For

$$\mathrm{VAF} = \max\left(1 - \frac{\mathrm{Var}(y_m - \hat{y})}{\mathrm{Var}(y)}, 0\right) 100$$

- FITting percentage

$$\mathrm{FIT} = \max\left(1 - \frac{\|y_m - \hat{y}\|^2}{\|y_m - E(y_m)\|^2}, 0\right) 100$$

- Prediction Error Cost

$$\mathrm{PEC} = \frac{1}{\sqrt{N}}\|y_m - \hat{y}\|^2$$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# State estimation for a multi-rotor UAV

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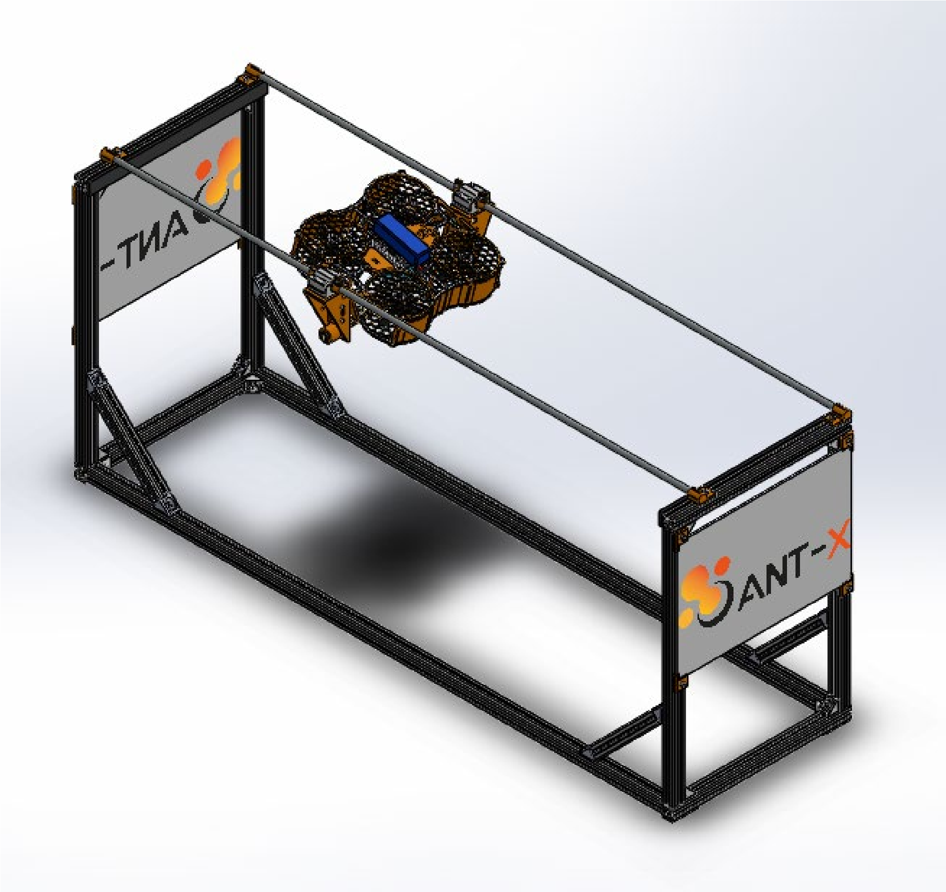


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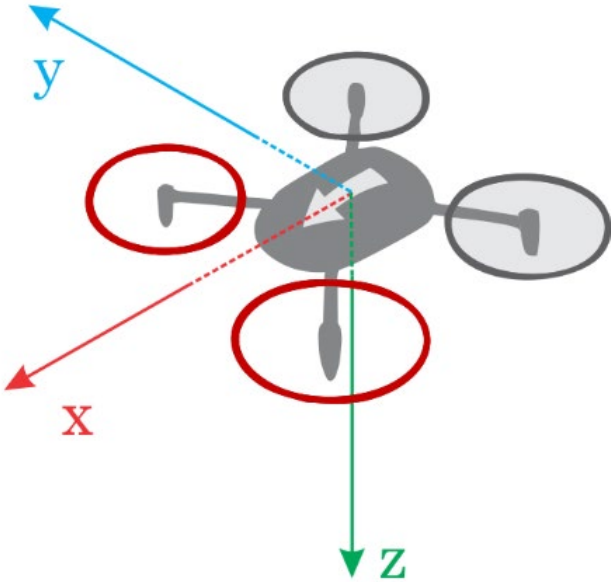


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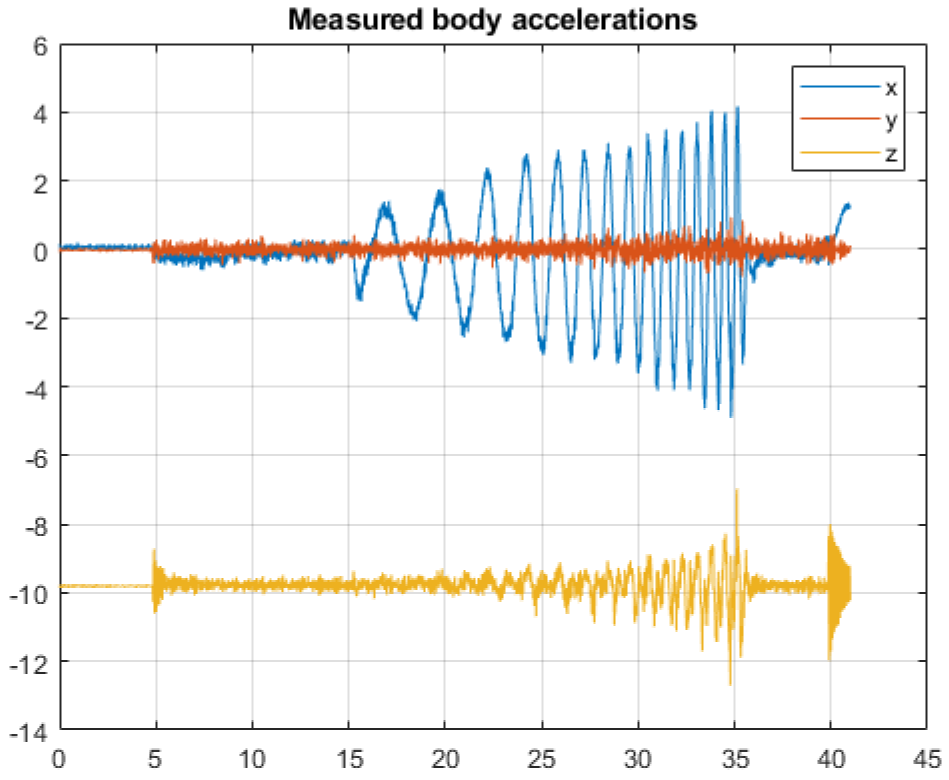
# Task 2: state estimation

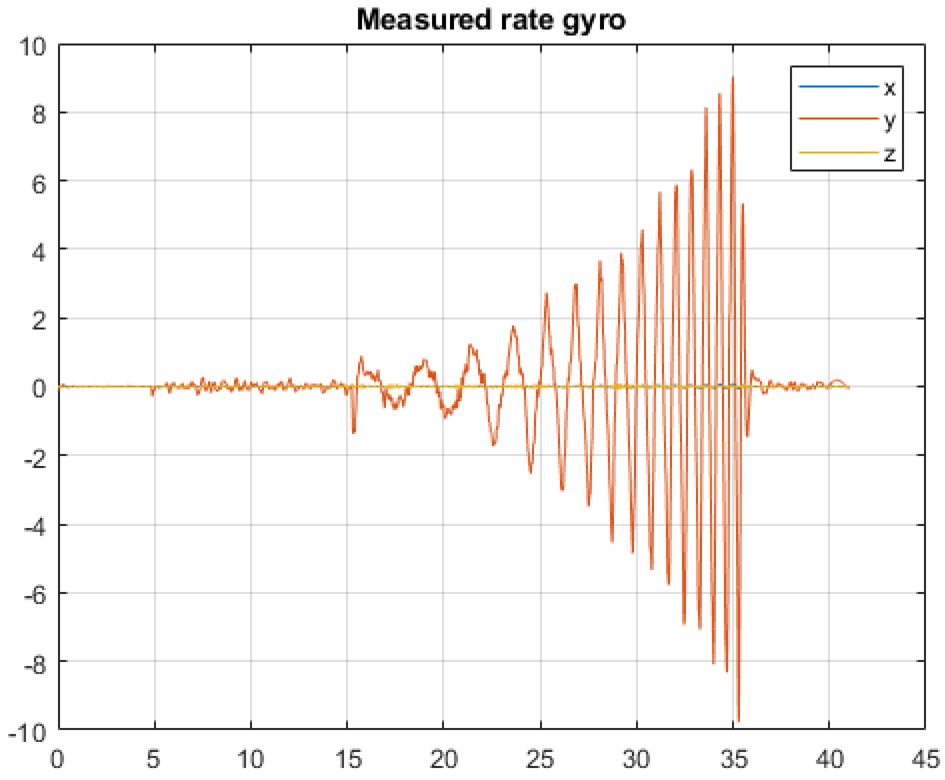
- The second lab activity aims at:
  - Designing a **Kalman filter for pitch estimation** of the 2DOF drone.
  - **Validating its performance** against measurements collected in small angles and large angles operations.

- Specifically:
  - A **DT/DT filter** is to be produced.
  - Care should be taken in the design of the filter, justifying all choices, including variance of process and measurement noise.
  - The filter shall be implemented in MATLAB in predictor/corrector form *without* using existing functions.
  - **Uncertainty** of the estimates shall be assessed.

# Measurements Collection

- Collect measurements of pitch rate and acceleration in experiments on the 2dof drone.
- Interpolate data to be consistent with filter time.



# Problem Formulation

- Formulate the problem of estimating the attitude from pitch rate and longitudinal acceleration measurements.

- Kinematic model: $x_{k+1} = Fx_k + Gv_k$

$$F = e^{A \cdot dt} \qquad A = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix} \qquad G = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$x_k = \begin{bmatrix} \delta\theta \\ \Delta\beta \end{bmatrix} \qquad v_k \sim N(0, Q_k) \qquad Q_k = \begin{bmatrix} \sigma_v^2 \cdot dt & 0 \\ 0 & \sigma_u^2 \cdot dt \end{bmatrix}$$

- Sensitivity matrix: $h(x_k) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \end{bmatrix} \underbrace{\begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}}_{r_a}$

# MEKF Design

- Design and implement in MATLAB a filter to estimate pitch (single-axis version of MEKF).

- Prediction:

$$\hat{x}_k^- = F x_{k-1}^+ + B\underbrace{(\tilde{\omega}_{k-1} - \Delta\beta_{k-1}^+)}_{\text{Dynamic Replacement}} \qquad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\hat{P}_k^- = F \hat{P}_{k-1}^+ F^\top + G Q_{k-1} G^\top$$

- Accelerometer correction:

$$\hat{x}_k^+ = \hat{x}_k^- + K_k(\varepsilon_k - H_a \hat{x}_k^-)$$

$$P_k^+ = P_k^- - K_k H_a P_k^-$$

Kalman Gain

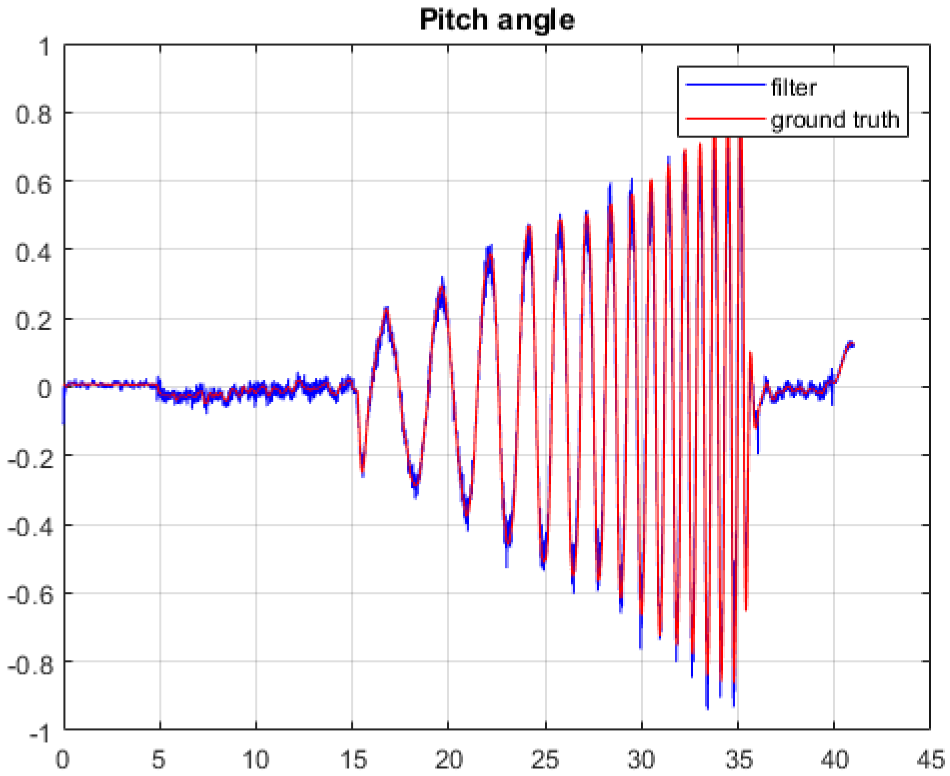
$$K_k = \hat{P}_k^- H_a^\top (H_a \hat{P}_k^- H_a^\top + R_a)^{-1} \qquad R_a = \sigma_a^2$$

Residual

$$\varepsilon_k = \underbrace{\tilde{a}_k}_{\text{Measured acc.}} - \underbrace{a_k}_{\text{Estimated acc.}}$$

# Filter Validation

- Validate the performance of the filter using the collected data.
  - Comparison with pitch angle obtained from ground truth

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