Property Graph Model

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Fundamentals of Information Systems
Master Degree in Data Science

Data Model

Property Graph Model

- The Property Graph Model (PGM) represents data as a directed, attributed multi-graph

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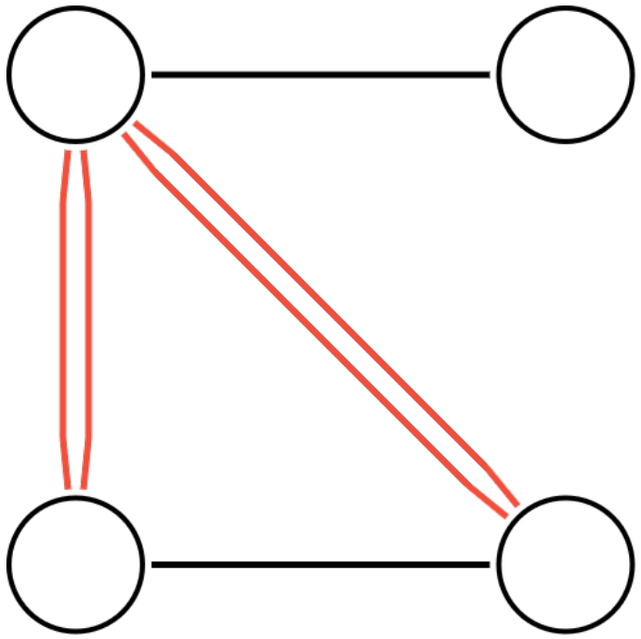

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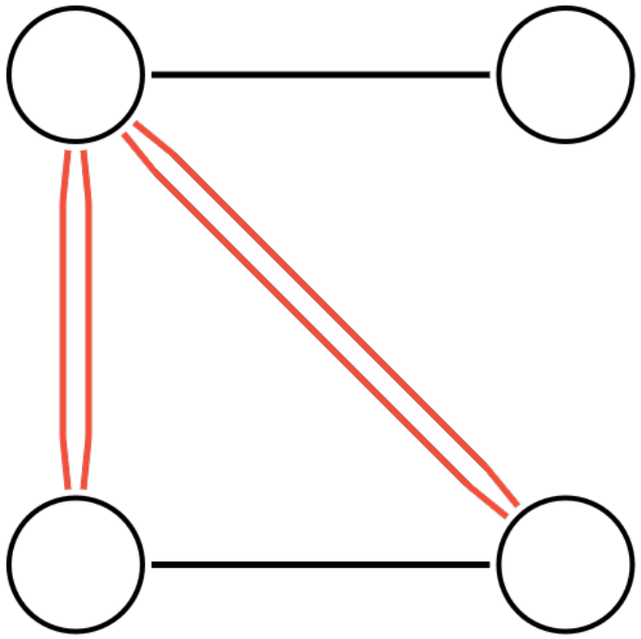

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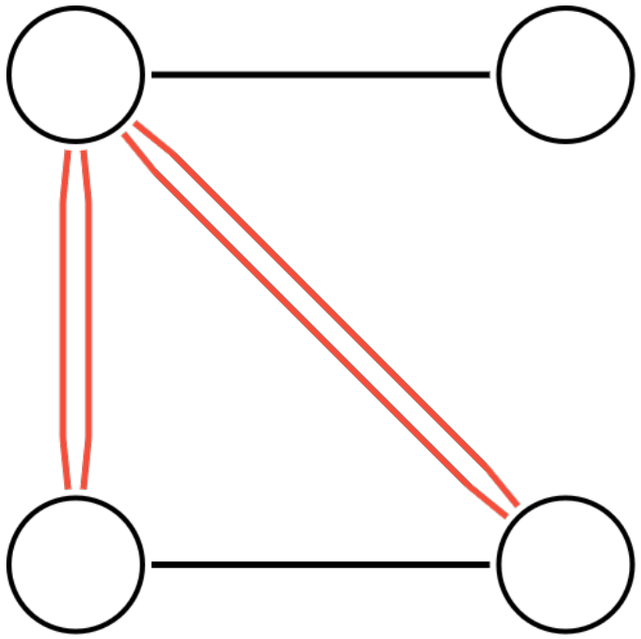

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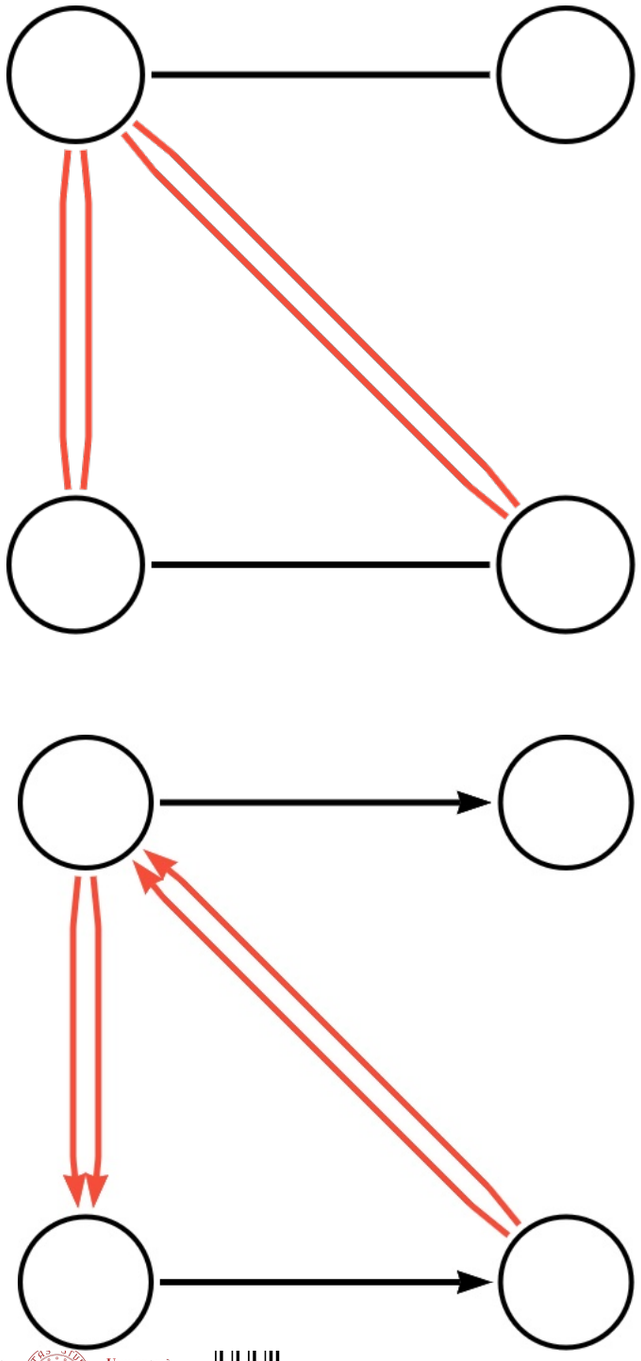

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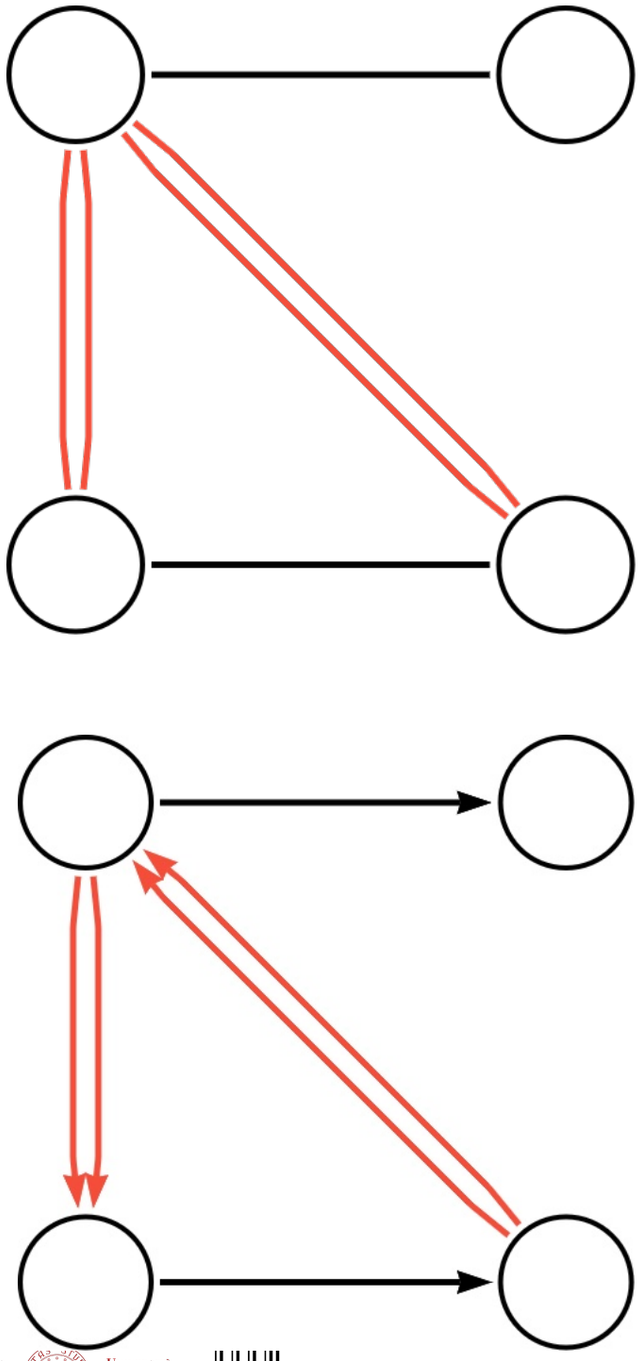

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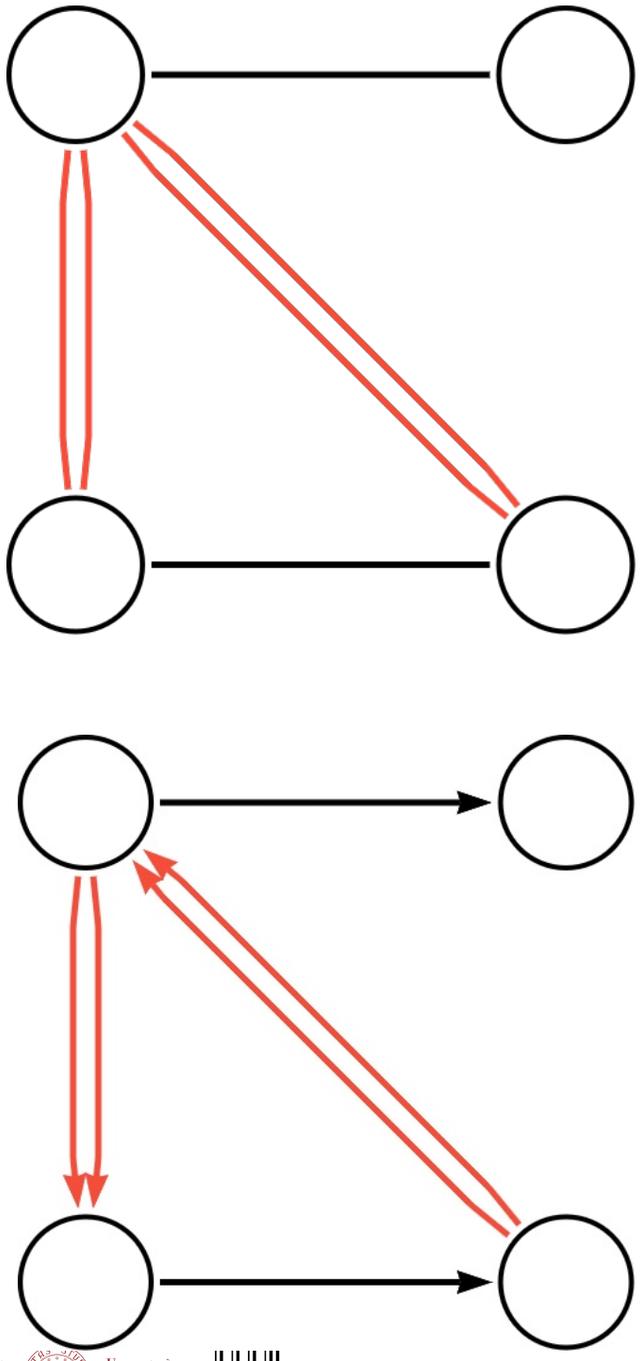

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A directed graph may have multiple directed edges from a vertex to a second (possibly the same) vertex are called as directed multigraphs

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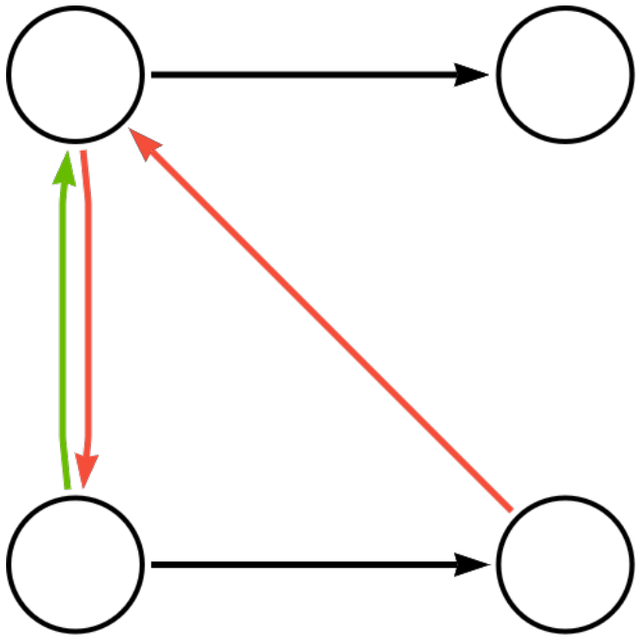

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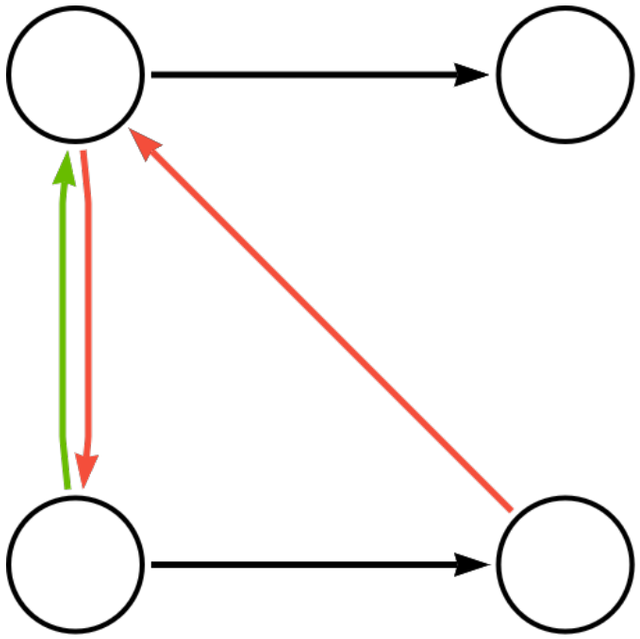

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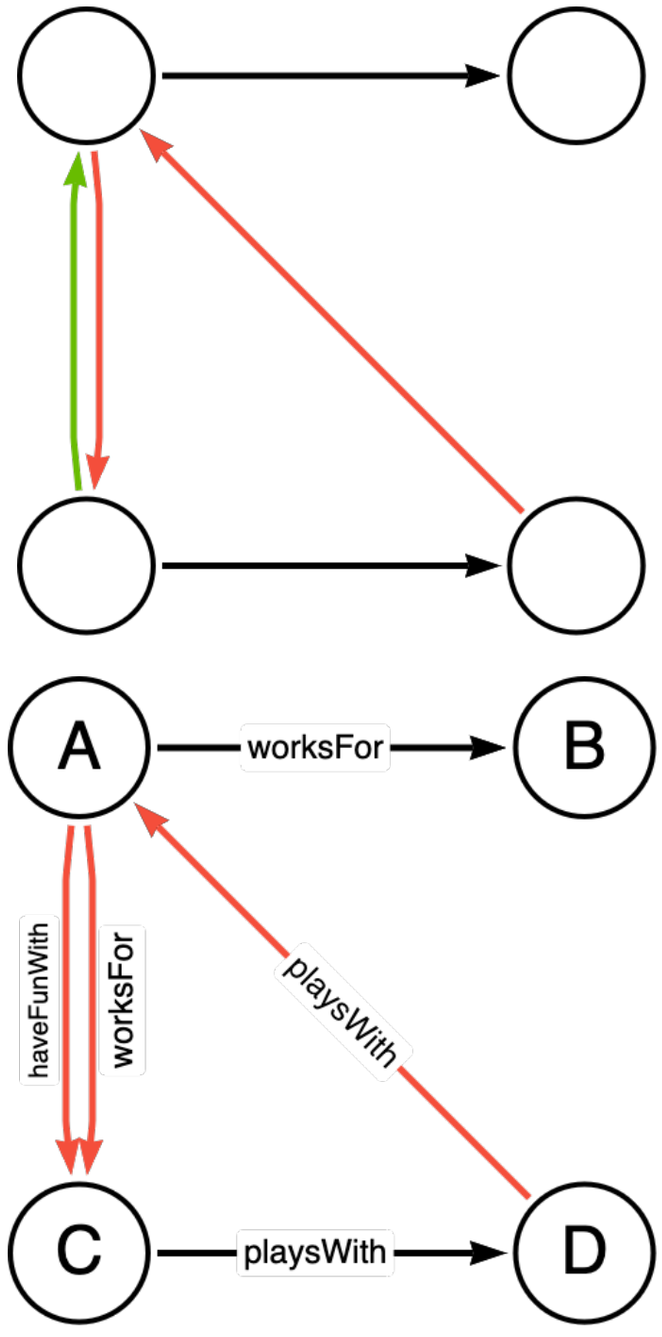

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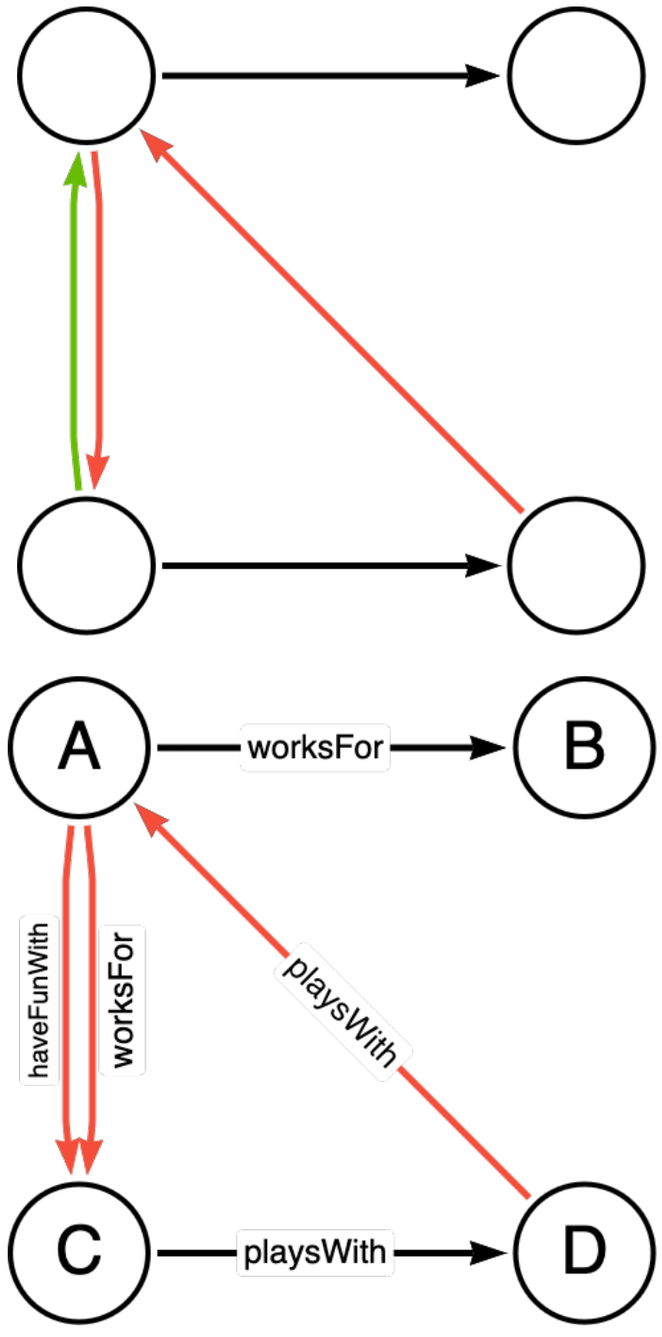

This is not a multi-graph

A directed labeled multi-graph is a graph allowing multiple edges with the same starting and end nodes and where nodes and vertices are labelled

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- In a PGM, vertices and edges are rich objects with a set of keys, a set of labels, and a set of values
- The pair (key, value) are called properties. E.g., name:'Jason'

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property keys = {name, born, middleInitial, twitter}

property key = 10

property values = {'Jason', 1995, 'J', '@jj05'}

property label = {:Novice}

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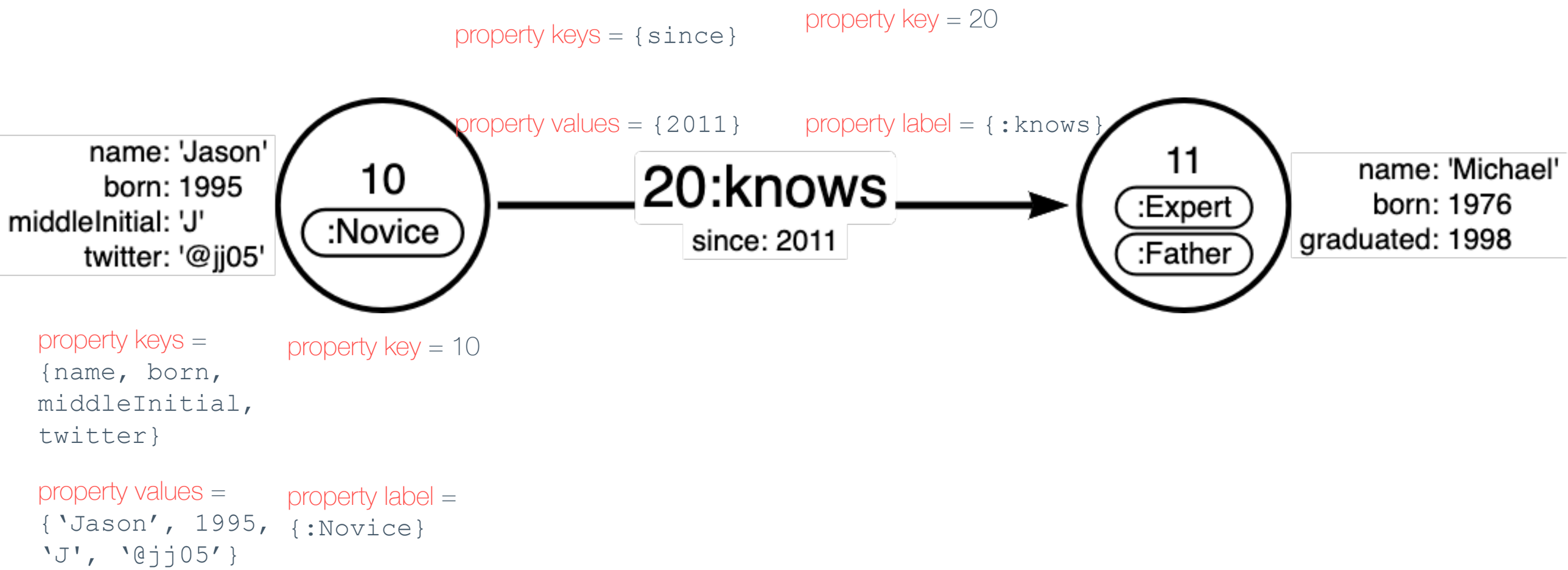

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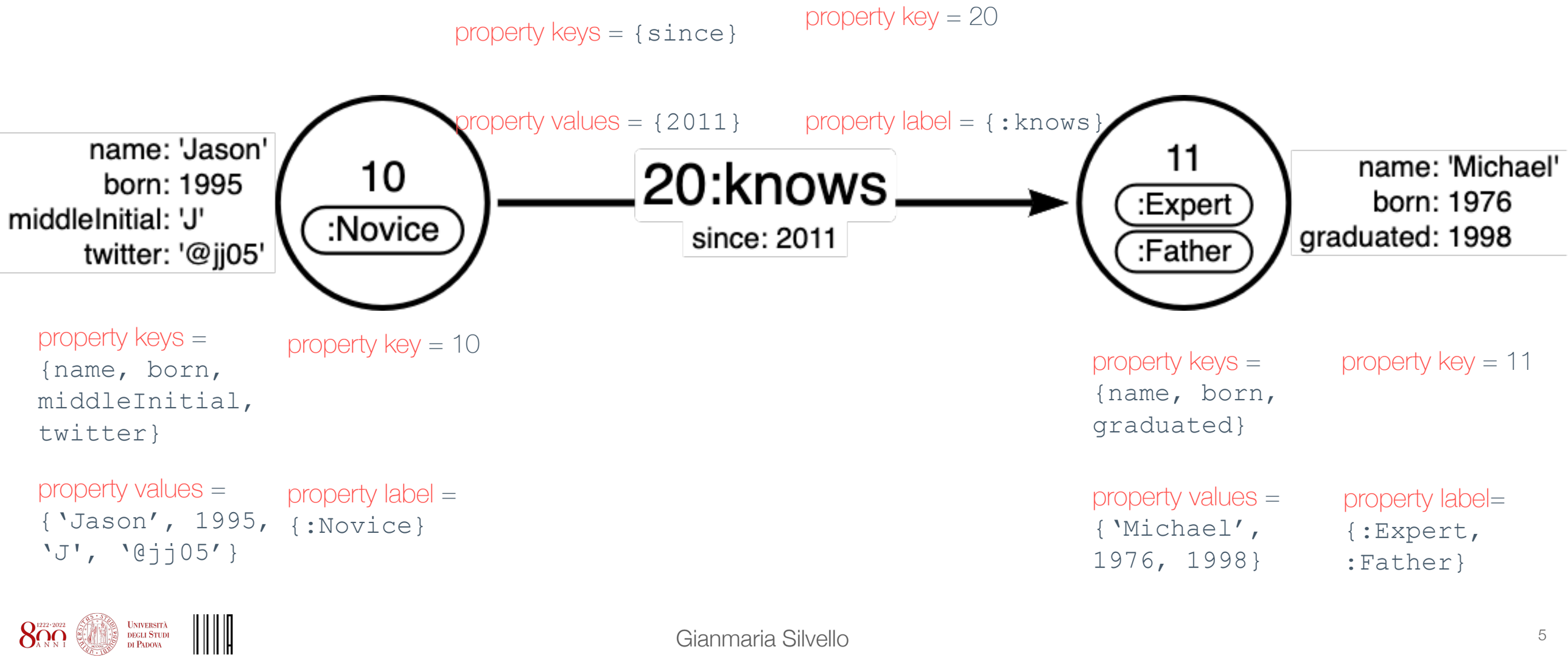

PGM

- We say that vertex 11 (or the vertex with key 11) has two labels: Expert and Father
- Labels are preceded by a colon and they are purely descriptive
- Properties contain actual data and are composed of a key and a value (may have a complex data type)
  - For instance, edge 20 has one label (knows) and one property (since) with a value (2011)

PGM definition

$\mathcal{O}$ is a set of objects

$\mathcal{L}$ is a finite set of labels

$\mathcal{K}$ is a set of property keys

$\mathcal{N}$ is a set of values

A property graph is a structure $(V, E, \eta, \lambda, \nu)$ where

(eta: η; ni: ν; lambda: λ)

- $V \subseteq \mathcal{O}$ is a finite set of objects, called vertices;
- $E \subseteq \mathcal{O}$ is a finite set of objects, called edges;
- $\eta : E \to V \times V$ is a function assigning to each edge an ordered pair of vertices;
- $\lambda : V \cup E \to \mathcal{P}(\mathcal{L})$ is a function assigning to each object a finite set of labels; and,

  $V \cap E = \emptyset$
- $\nu : V \cup E \times \mathcal{K} \to \mathcal{N}$ is a partial function assigning values for properties to objects.

PGM example


name: 'Jason'
born: 1995
middleInitial: 'J'
twitter: '@jj05'
10
:Novice
20:knows
since: 2011
11
:Expert
:Father
name: 'Michael'
born: 1976
graduated: 1998

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$$V = \{10, 11\}$$

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$$\eta(20) = (10, 11) \neq \{10, 11\}$$

PGM example

V (v1,v2) is an ordered set defining an order relation amongst the set members.

E

Let P be a set and ⊑ a partial order over P such that a,b ∈ P ⇒ a ⊑ b or b ⊑ a or a || b (incomparable).

λ

ν If all the elements of P are comparable then P is a totally ordered set.

$$\eta(20) = (10, 11) \neq \{10, 11\}$$

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Notation

π_i projects an n-tuple to its i-th element

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is the set of incoming edges of v

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$$\mathtt{src}(e) = \pi_1(\eta(e))$$

is the source vertex of e

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is the source vertex of e

$$\mathtt{trg}(e) = \pi_2(\eta(e))$$

is the target vertex of e

Query Languages

Regular path queries (RPQ)

Regular path queries express reachability queries

RPQ ask for all pair of vertices connected by at least one path where the sequence of edge labels along the path forms a word in the language of a given regular expression

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set of labels

if $a \in \mathcal{L} \Rightarrow a \in RPQ$

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e^- is the inverse of e

↳ it goes from the object to the subject

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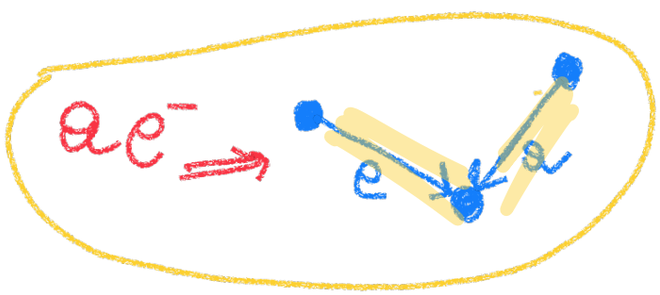

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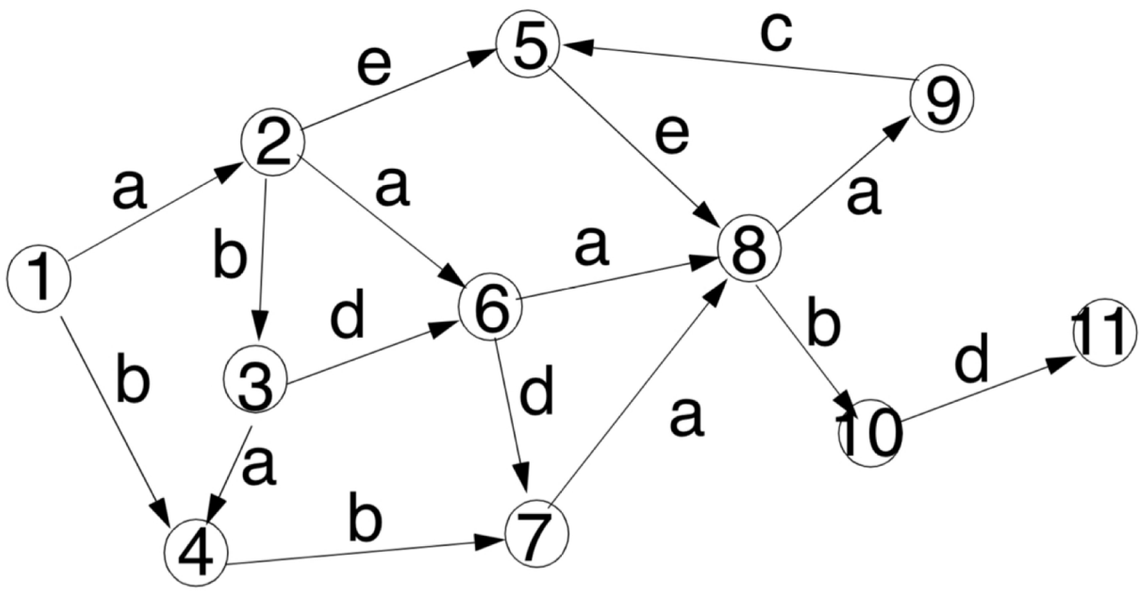
↳ one or more e

Example

$R = a^{+}(b|d)\,ab$

- a^{+}: one or more repetitions of a
- $(b|d)$: one occurrence of b OR d
- ab: one occurrence of a AND b



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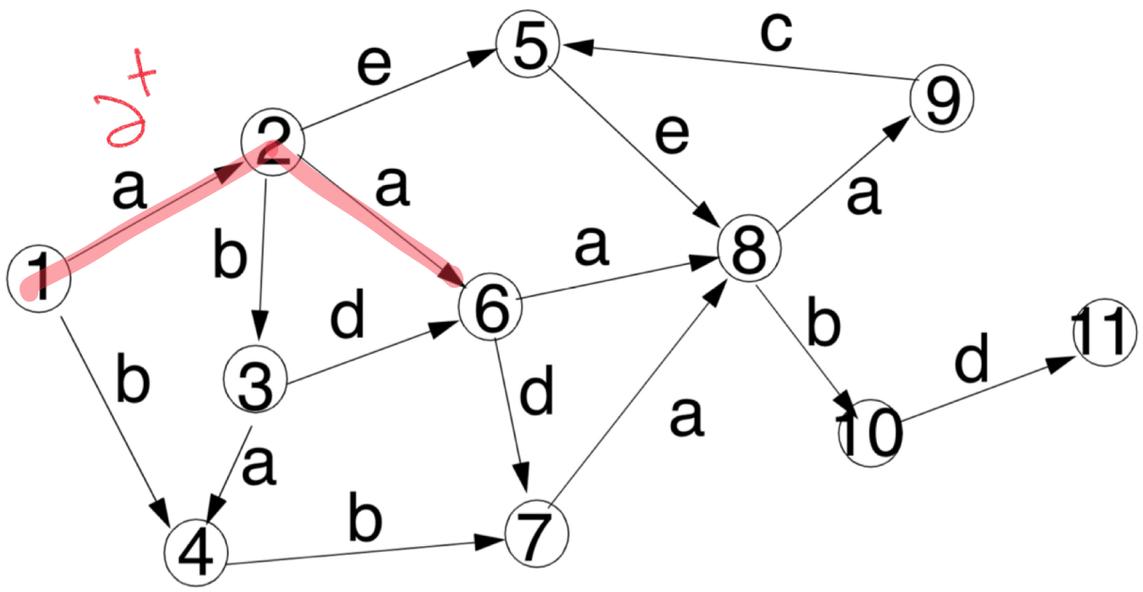

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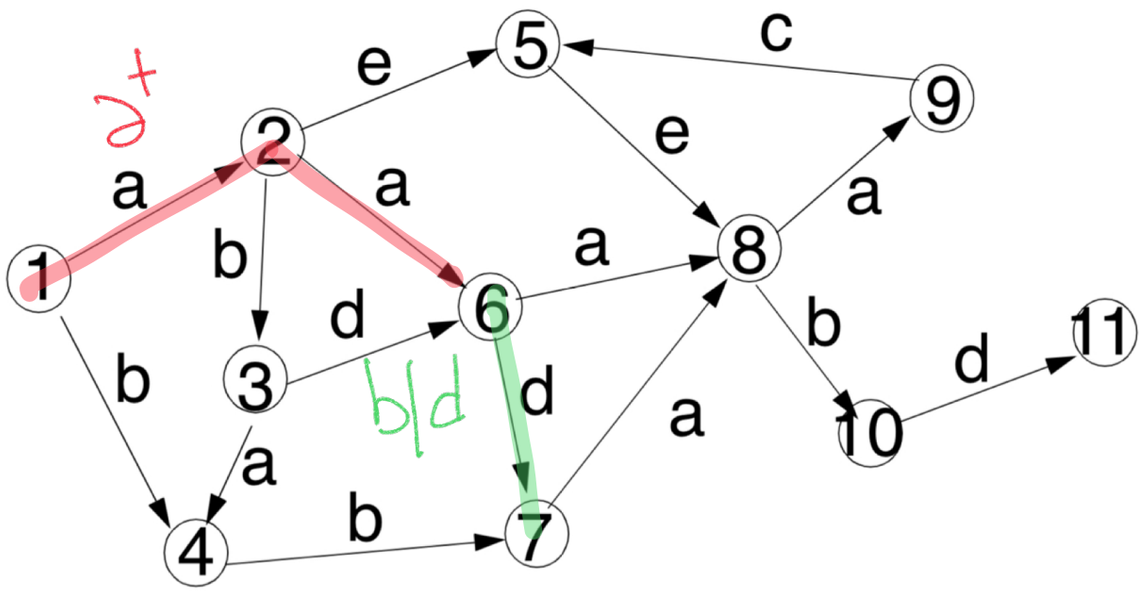

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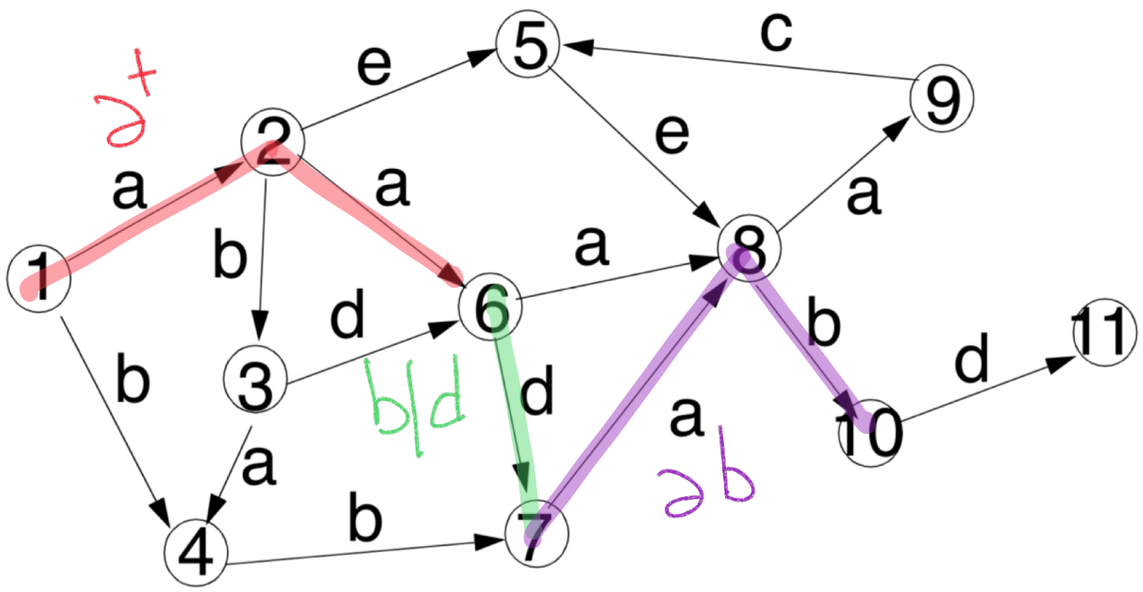

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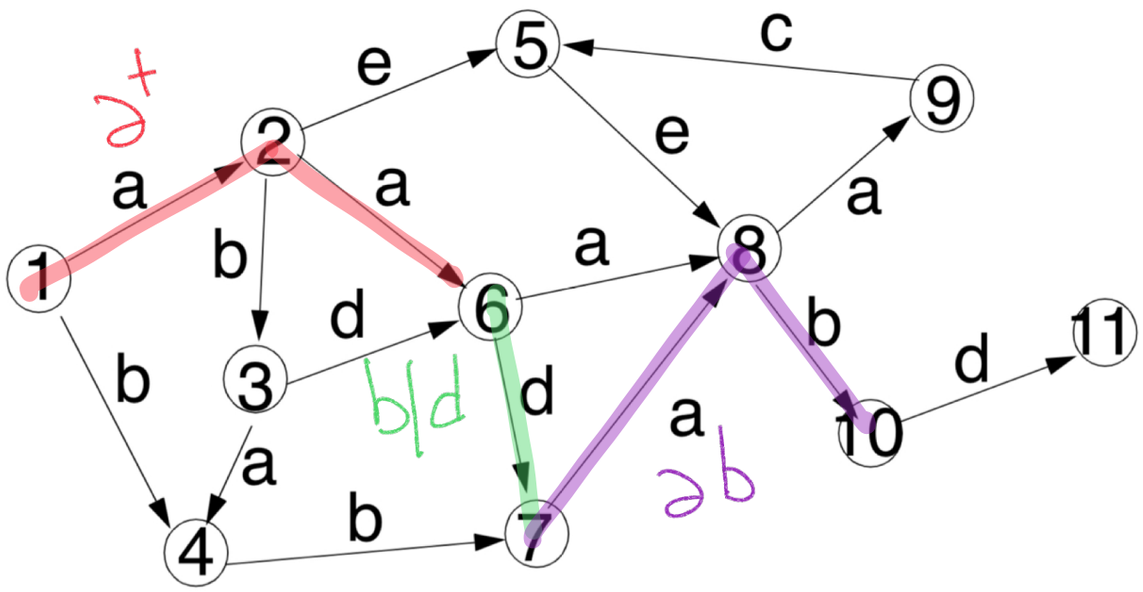

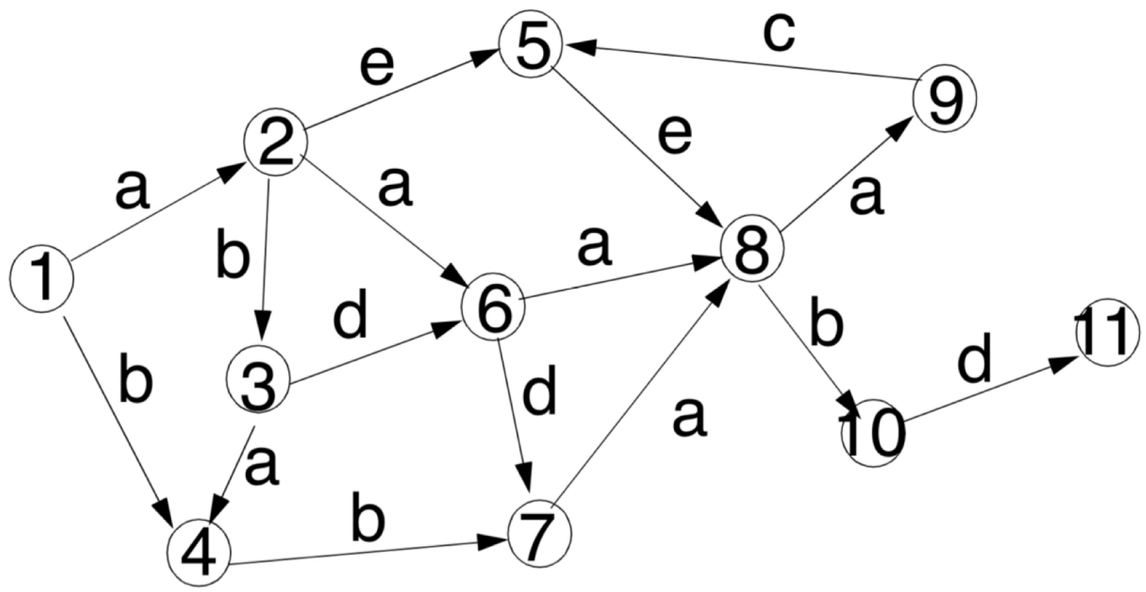

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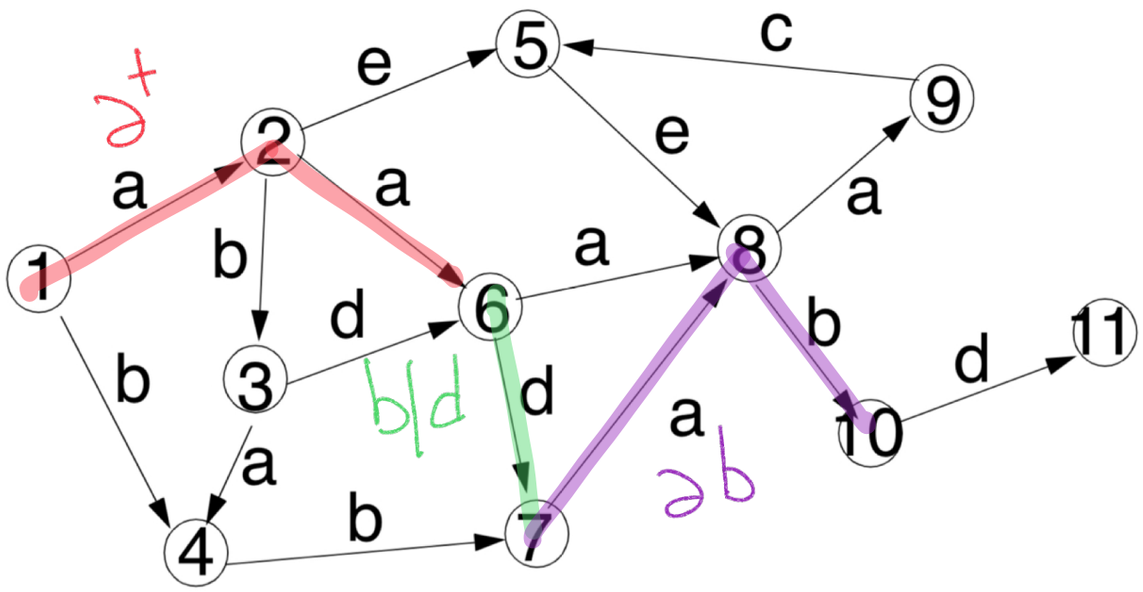

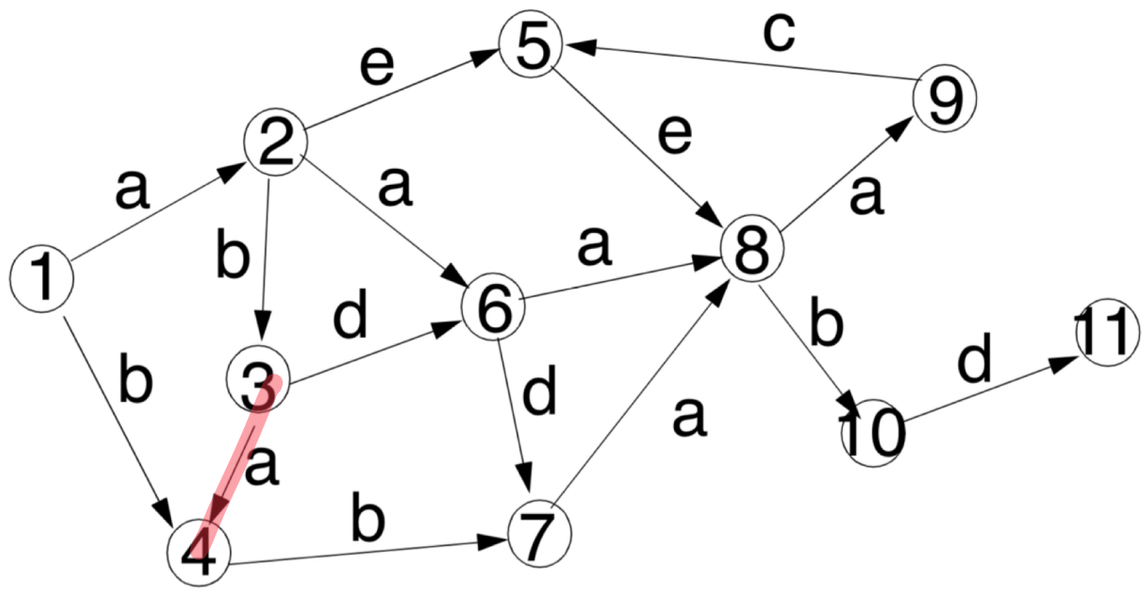

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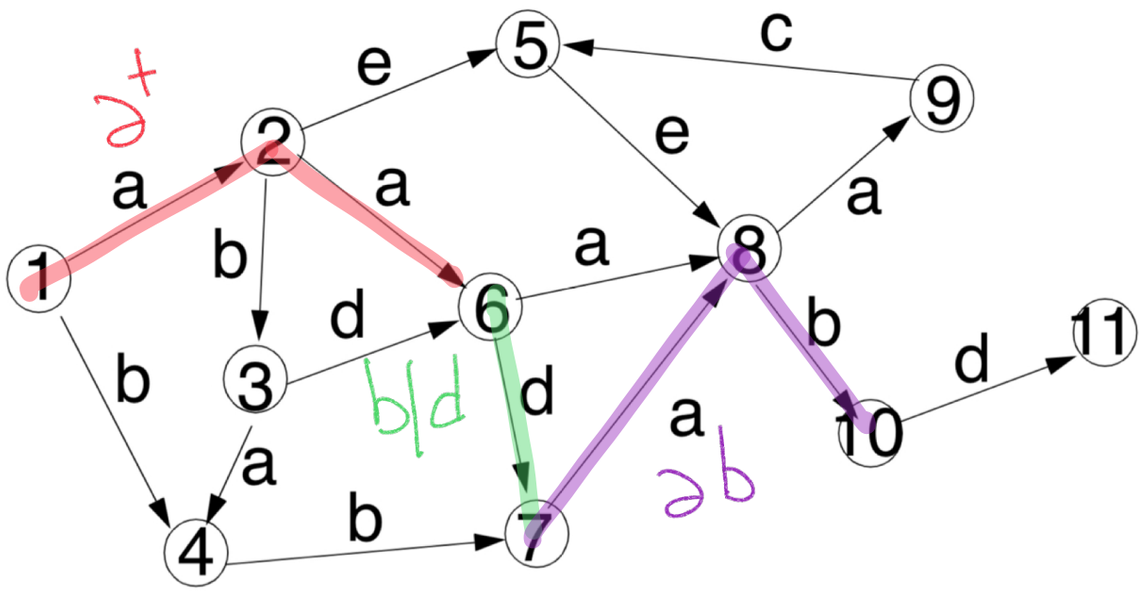

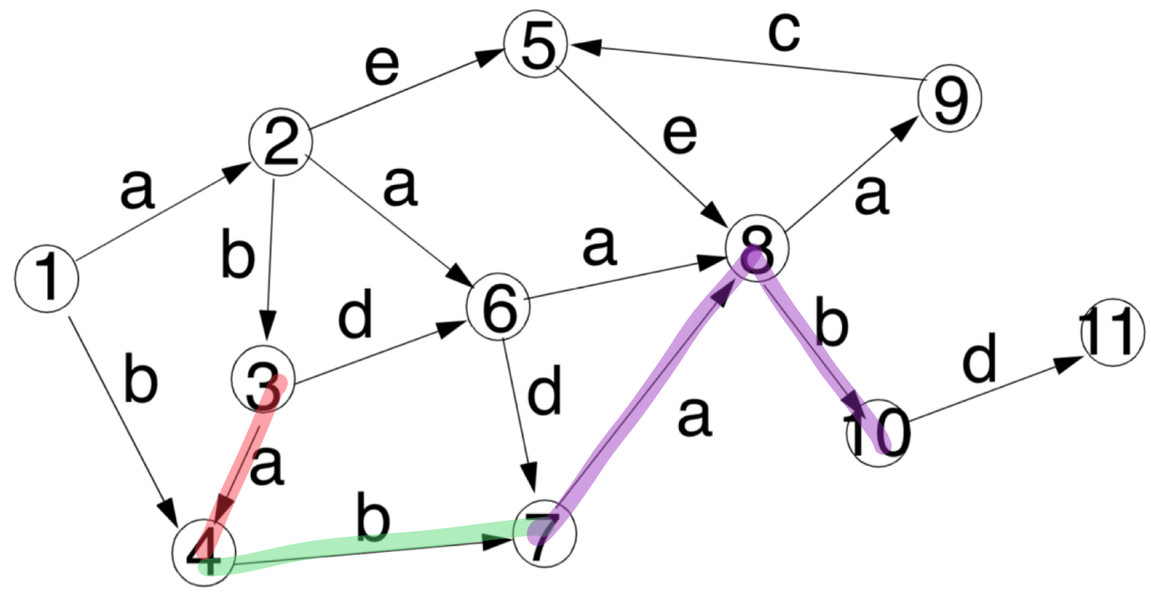

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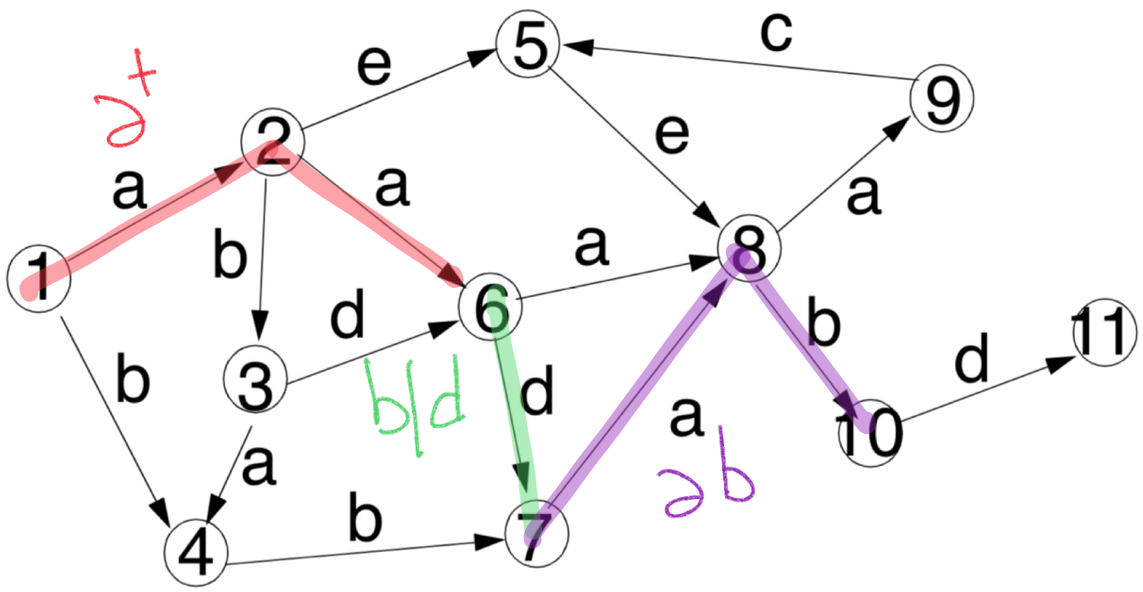

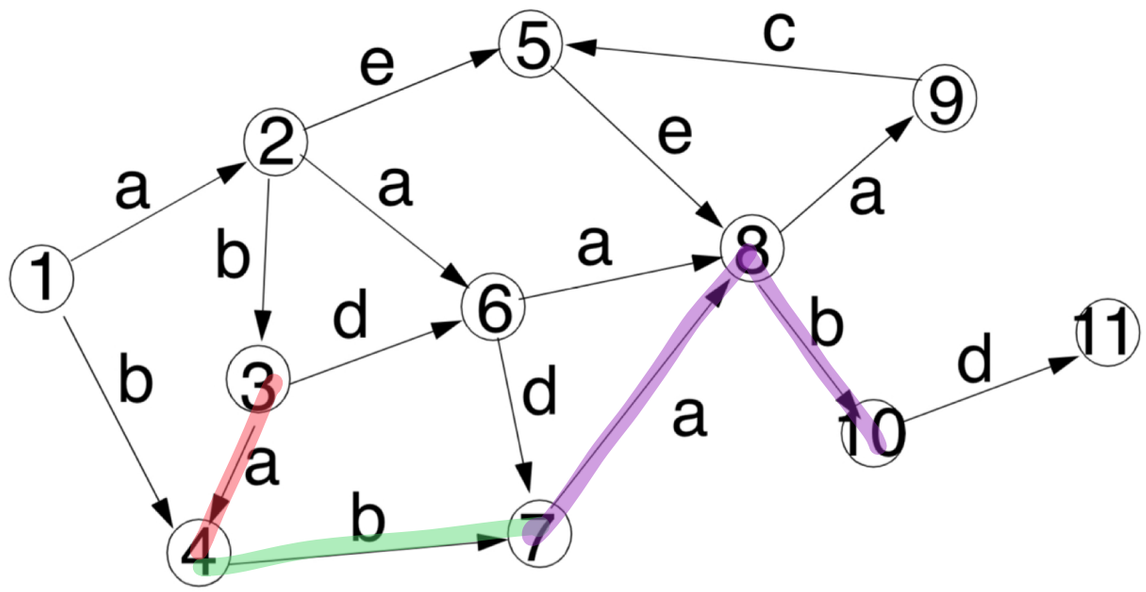

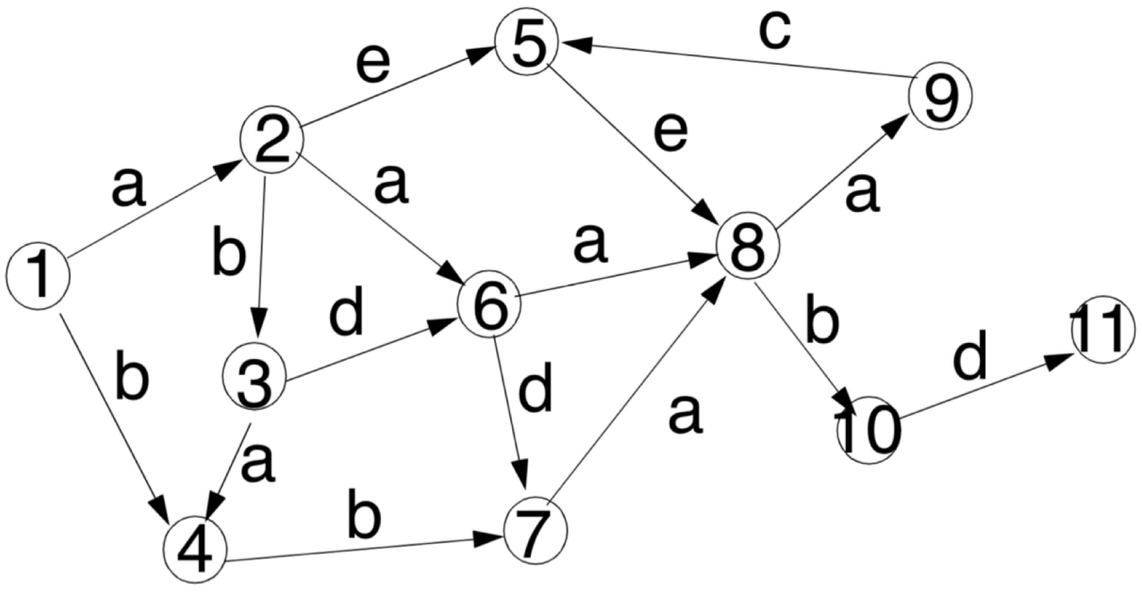

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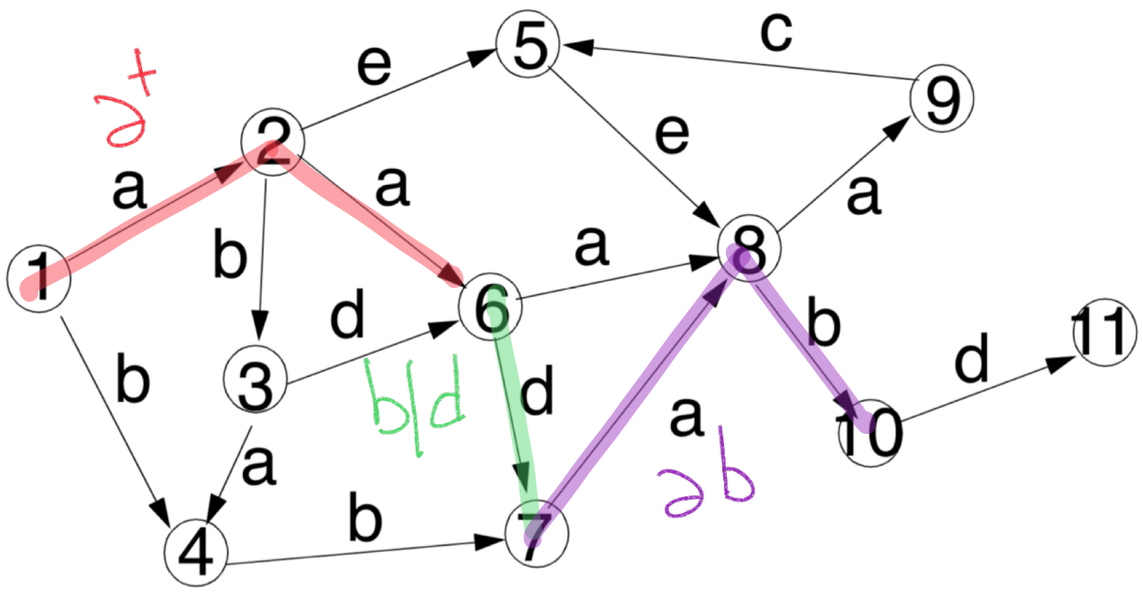

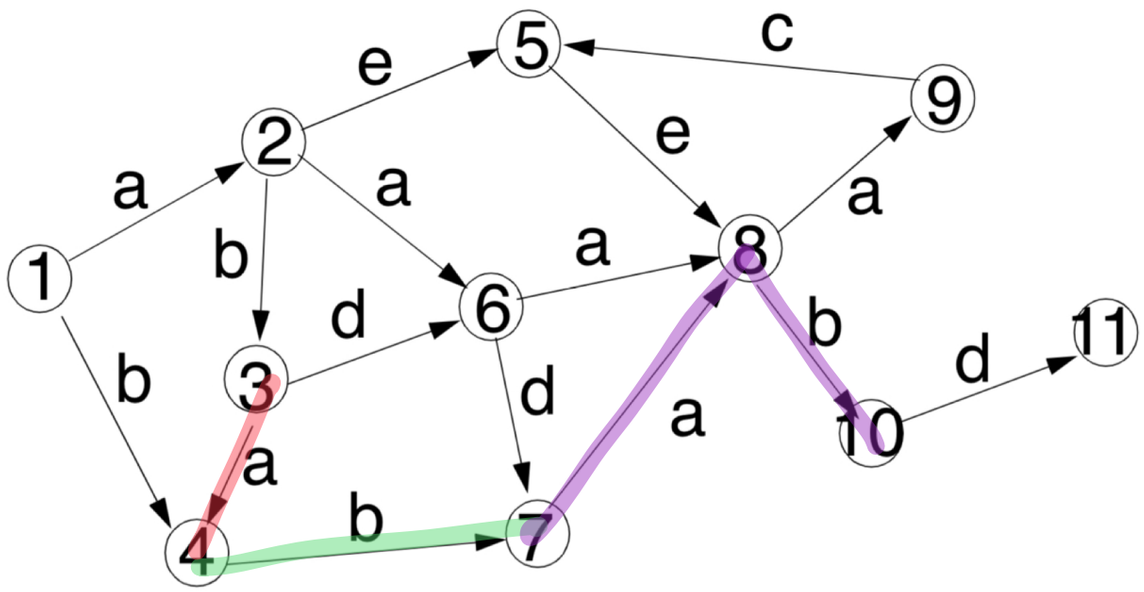

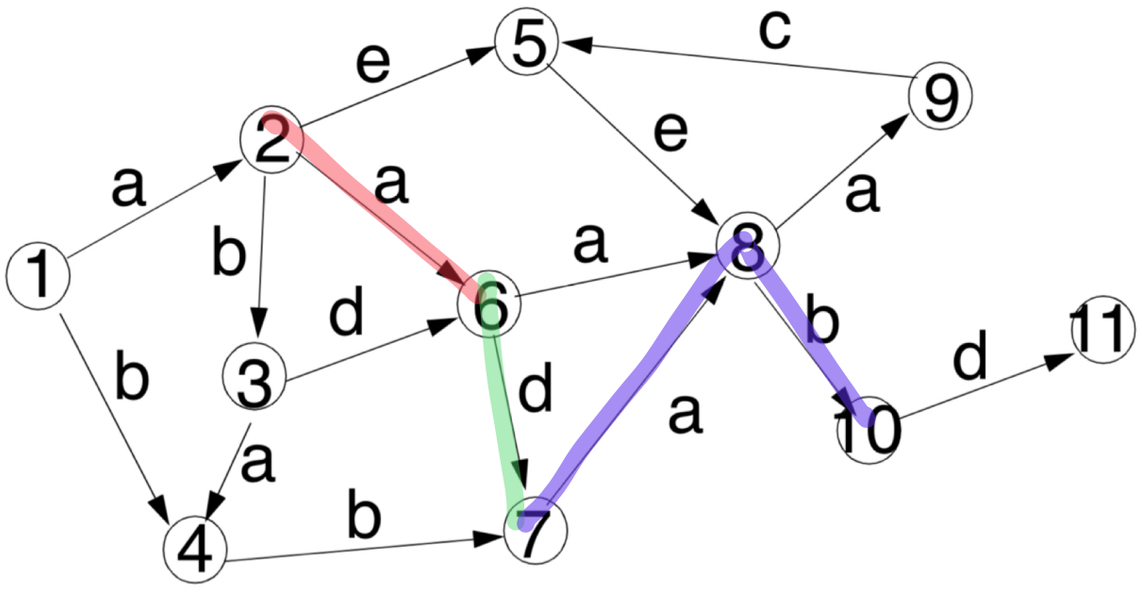

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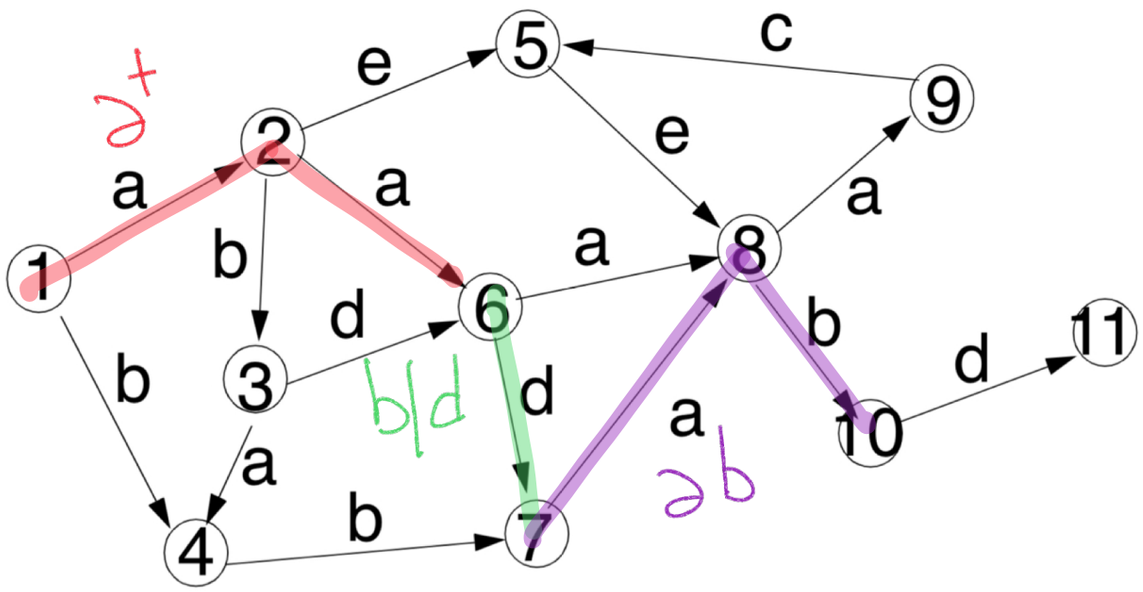

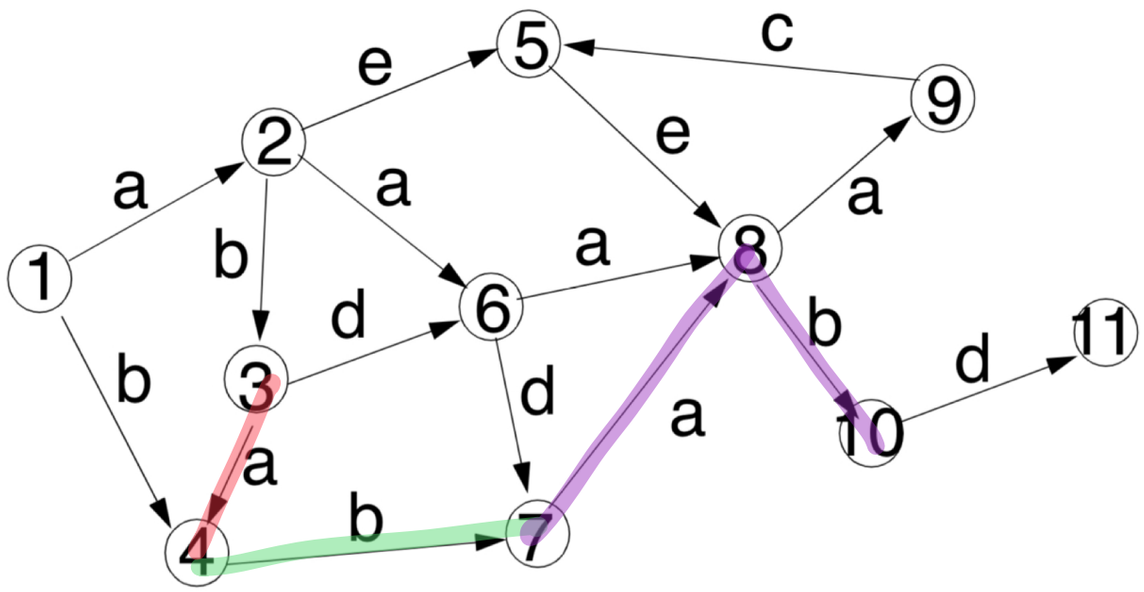

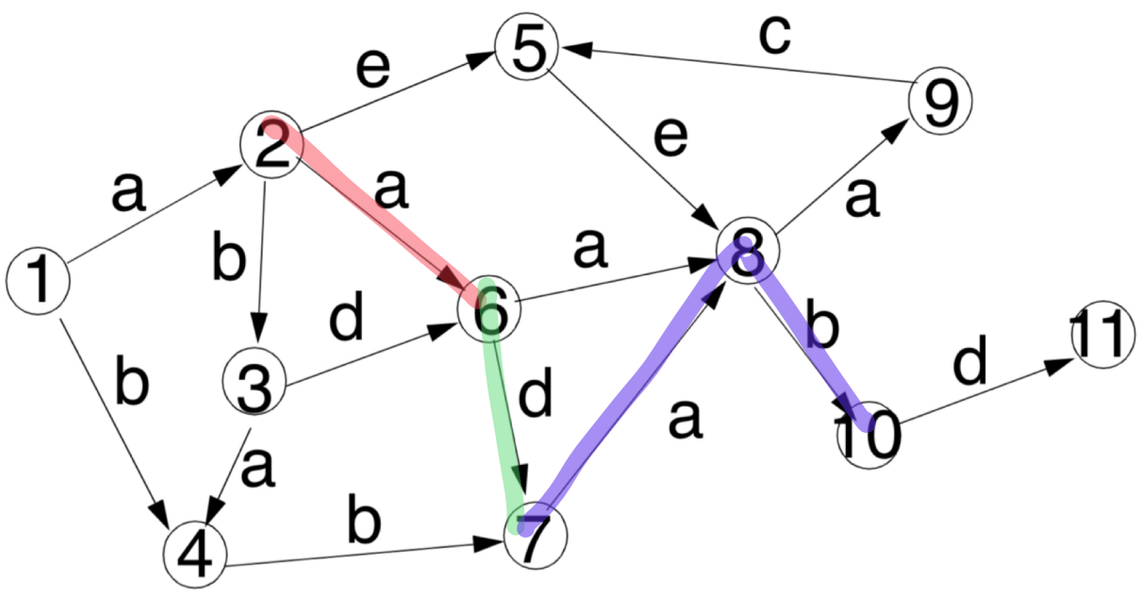

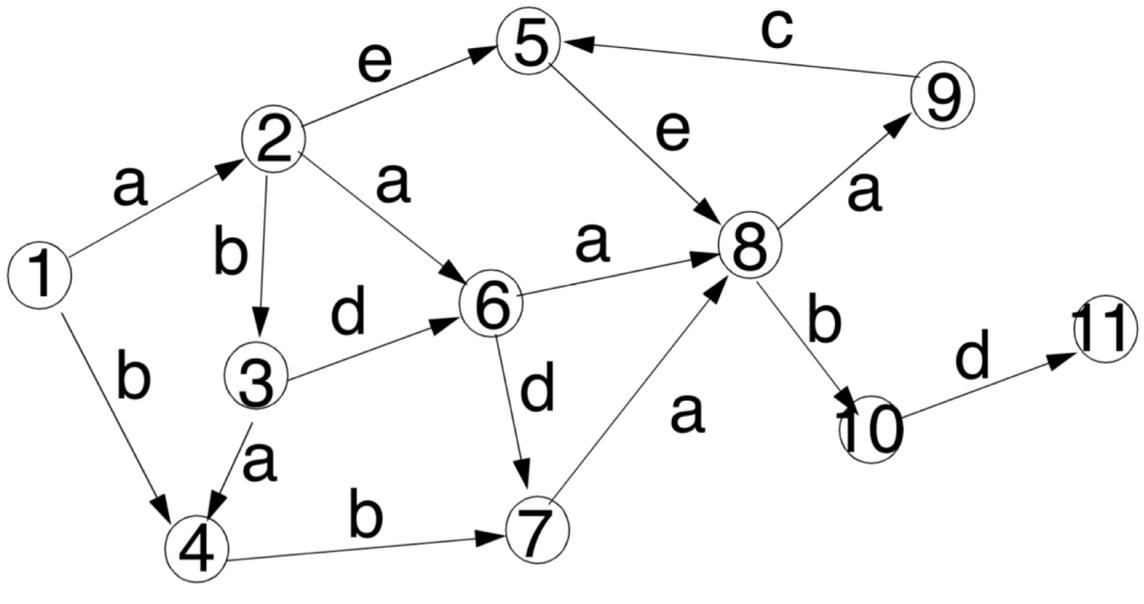

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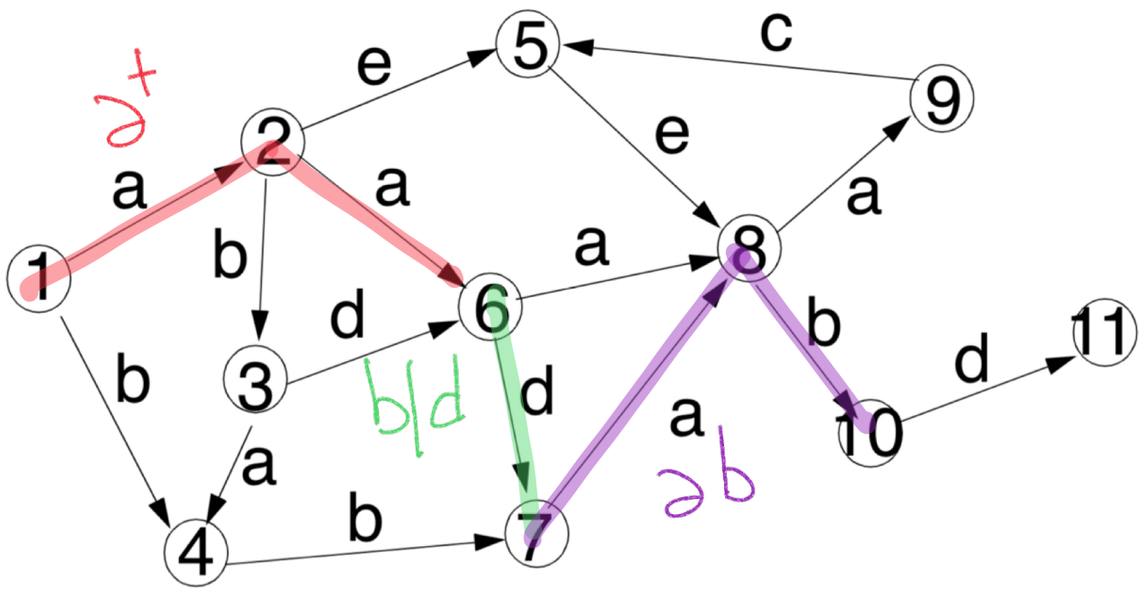

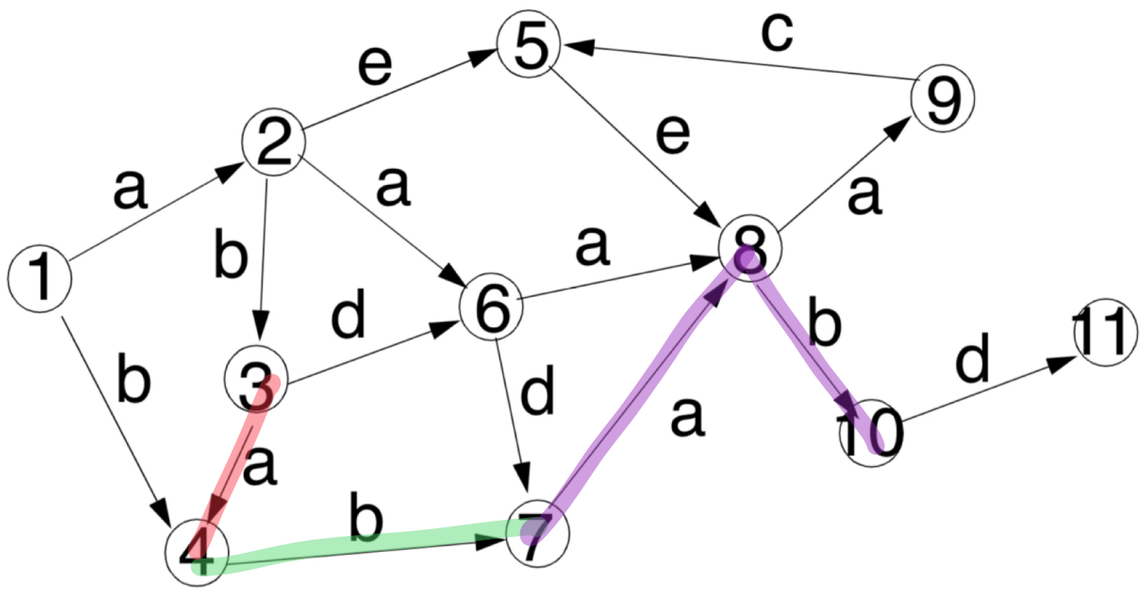

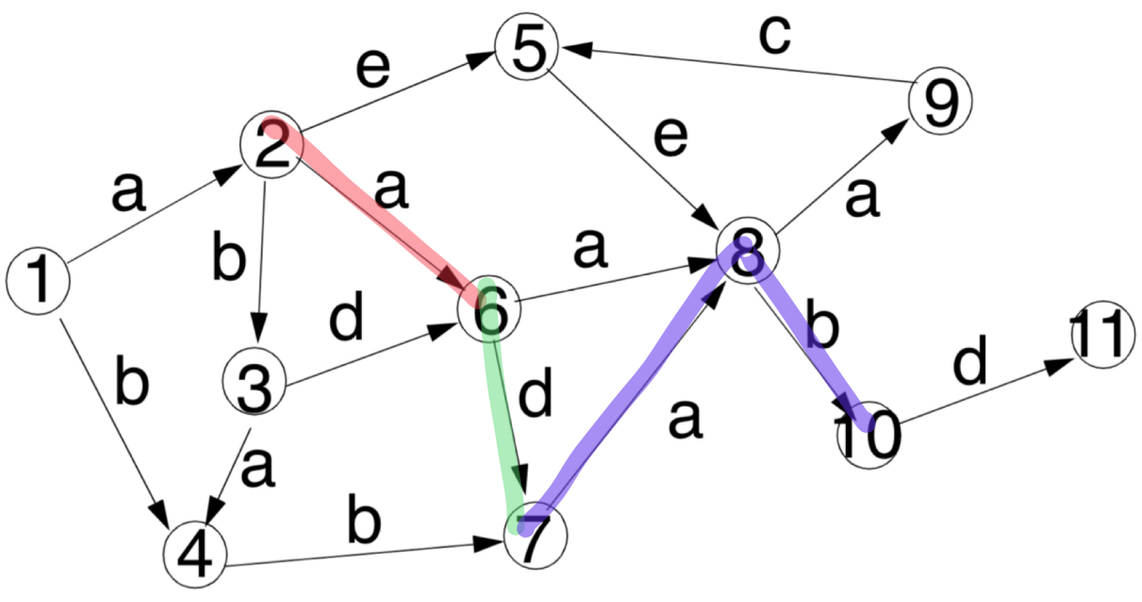

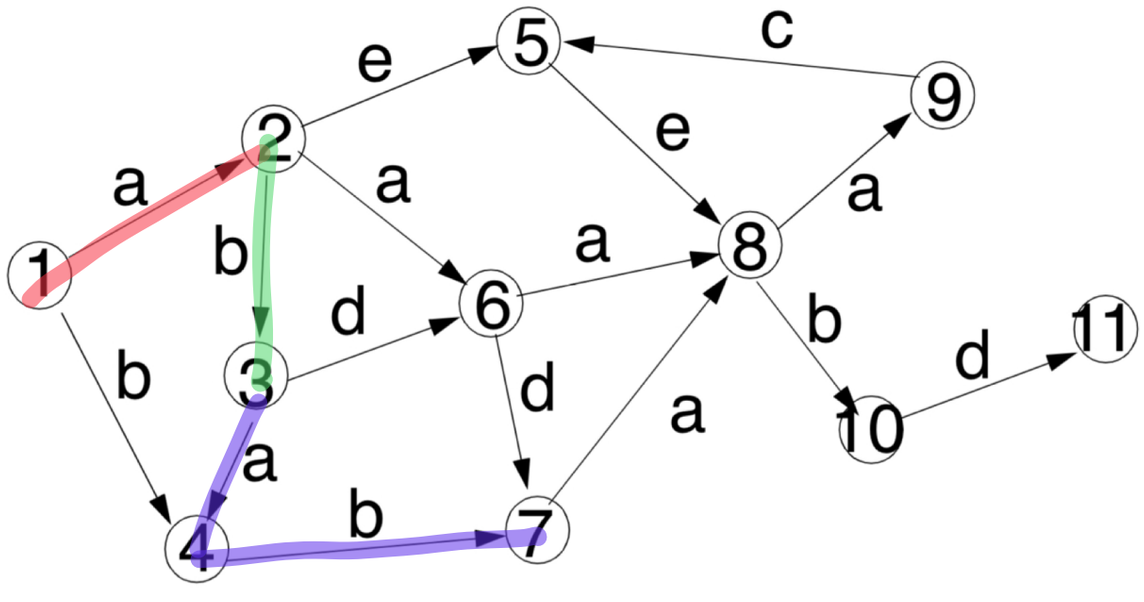

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d^{-} → reverse d

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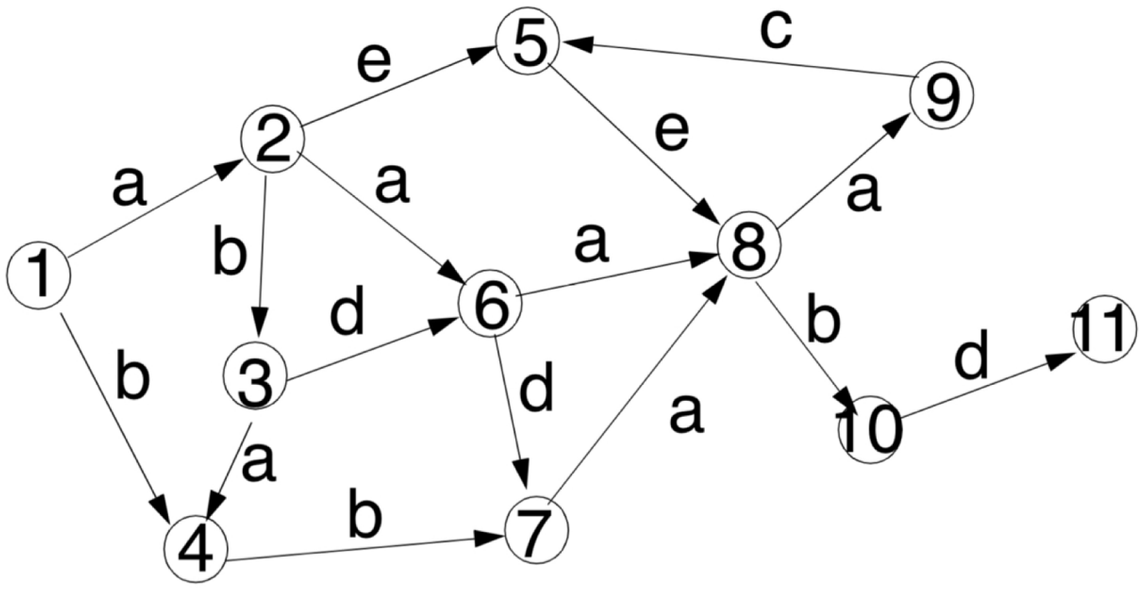

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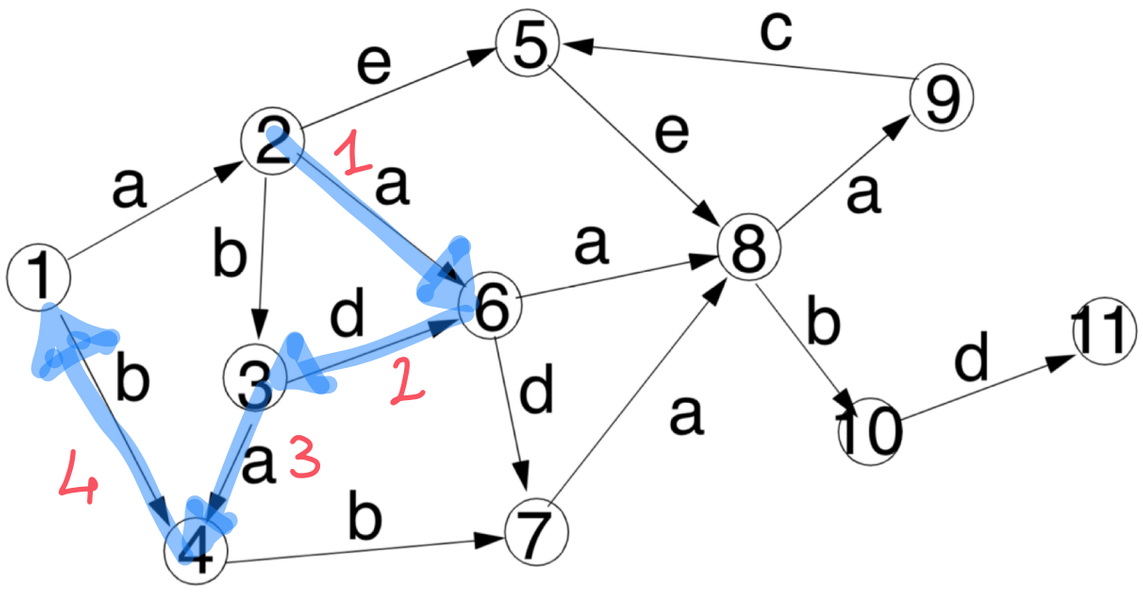

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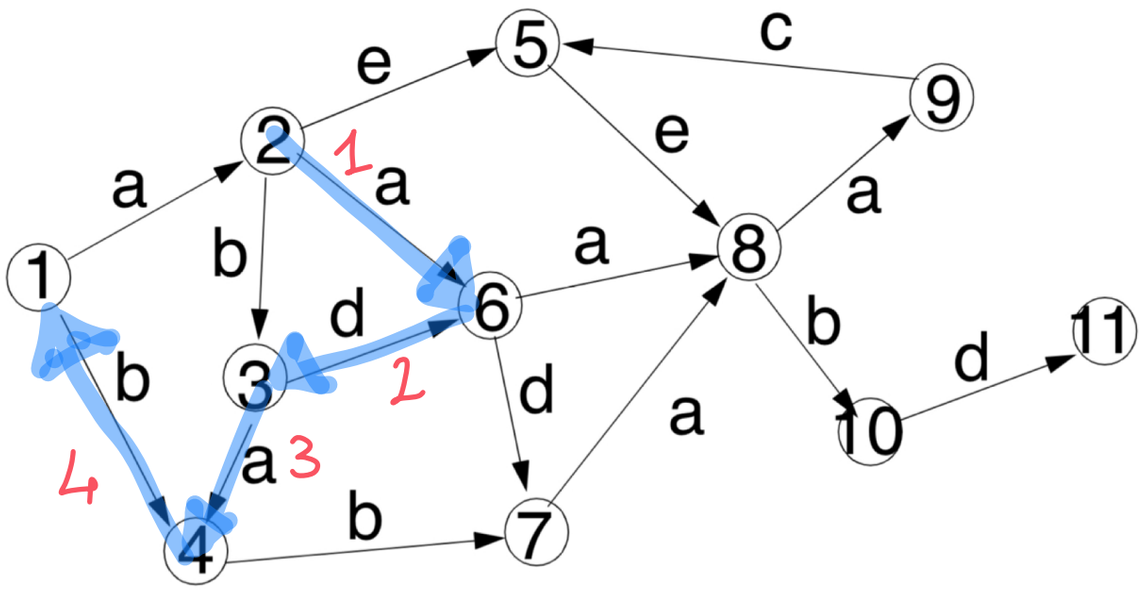

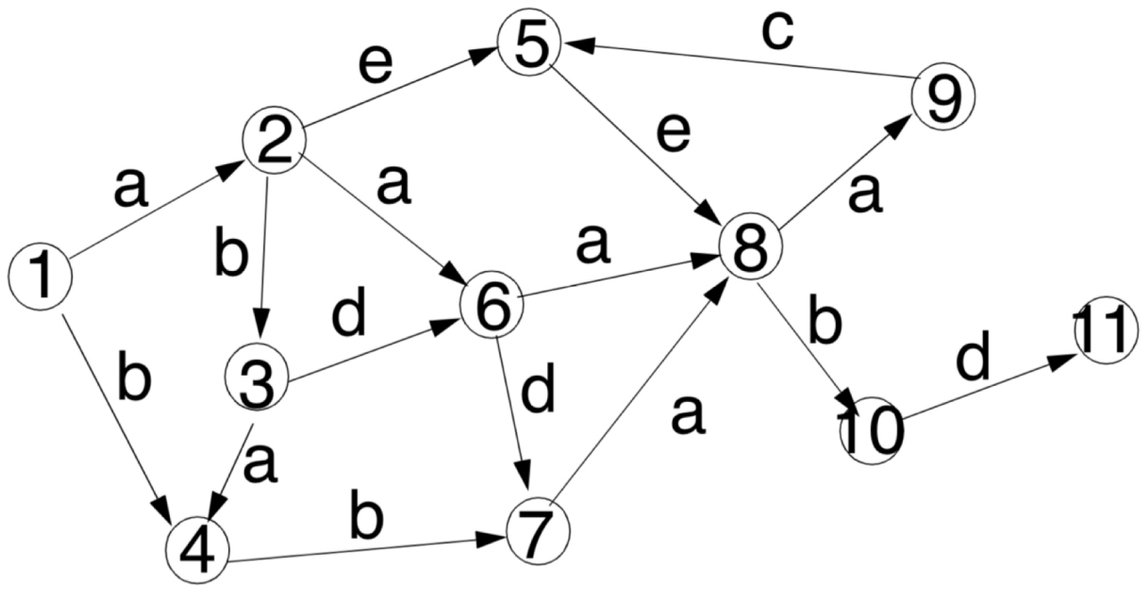

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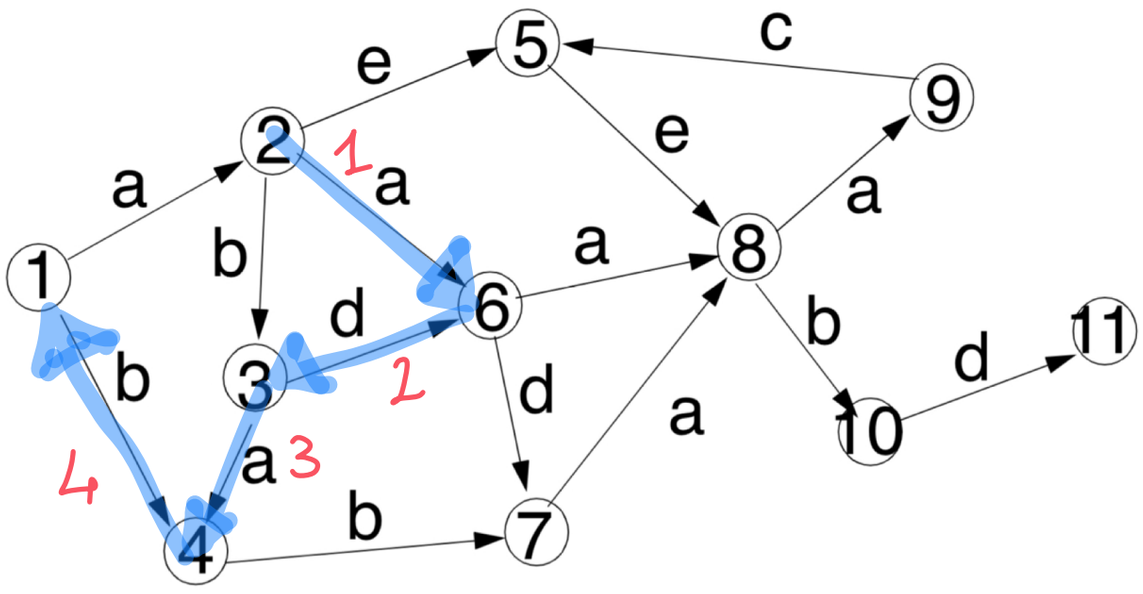

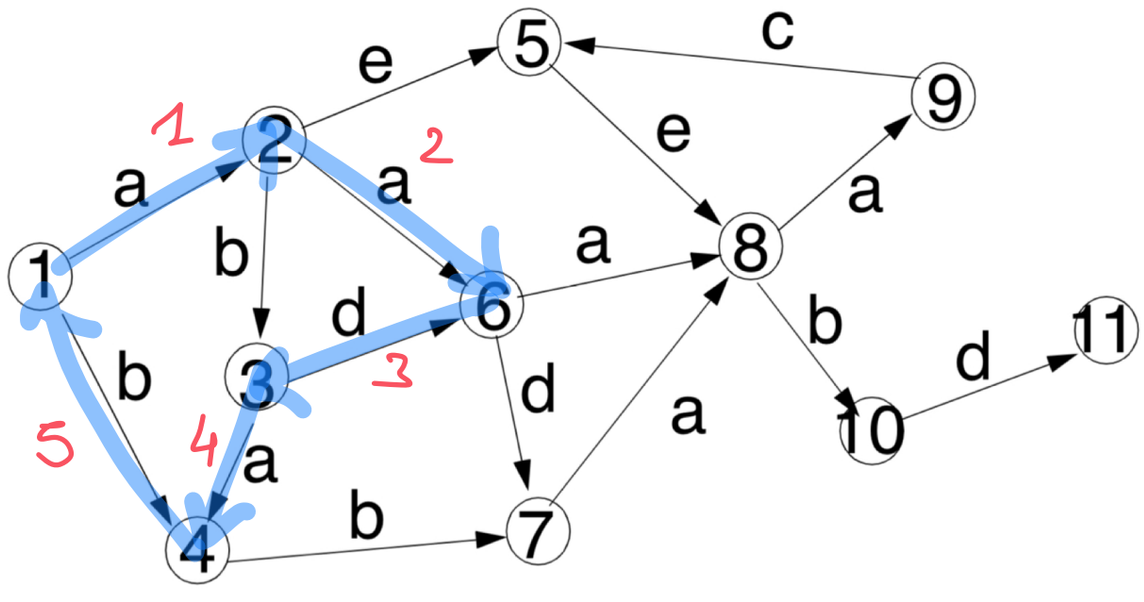

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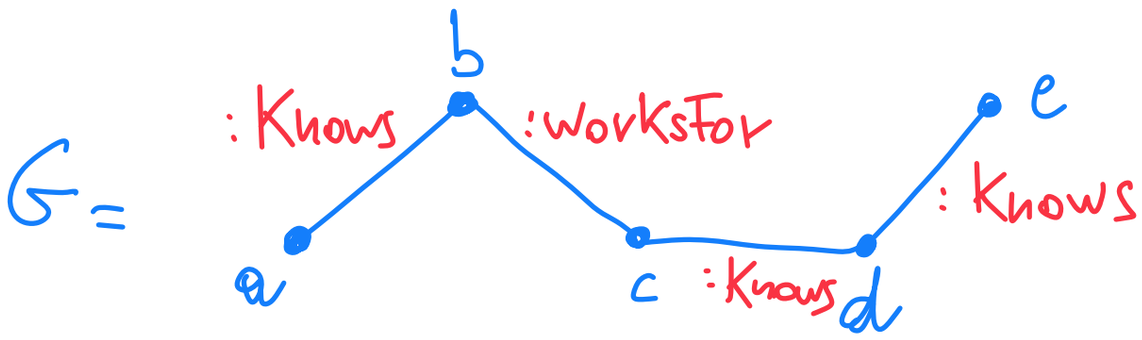
4) if $g = e + f \in RPQ$, then $[\![e]\!]_G \cup [\![f]\!]_G$

5) if $g = (e)^+ \in RPQ$, then $[\![g]\!]_G = \{(s,t) \mid (s,t) \in TC([\![e]\!]_G)$ where TC is the transitive closure of binary relation R

Example 1

$q = (:Knows)/(:worksFor)/(:Knows)^+$



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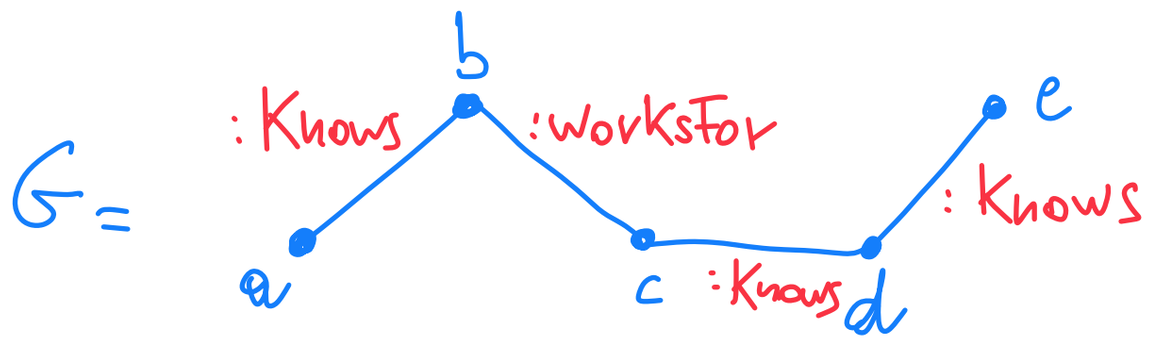

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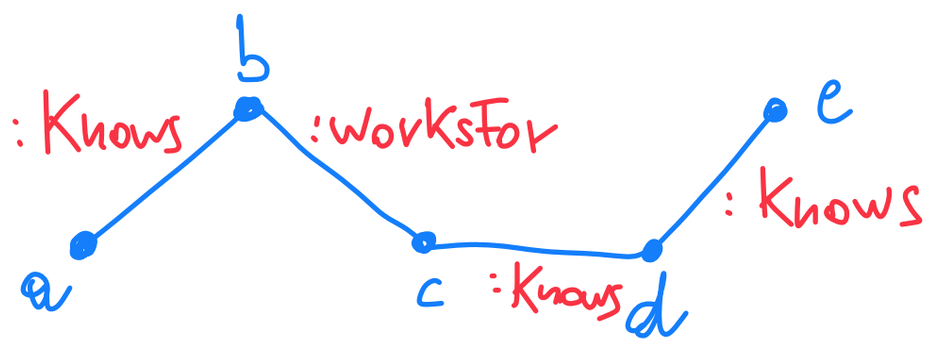

$$[\![q]\!]_G = \{(a,d), (a,e)\}$$

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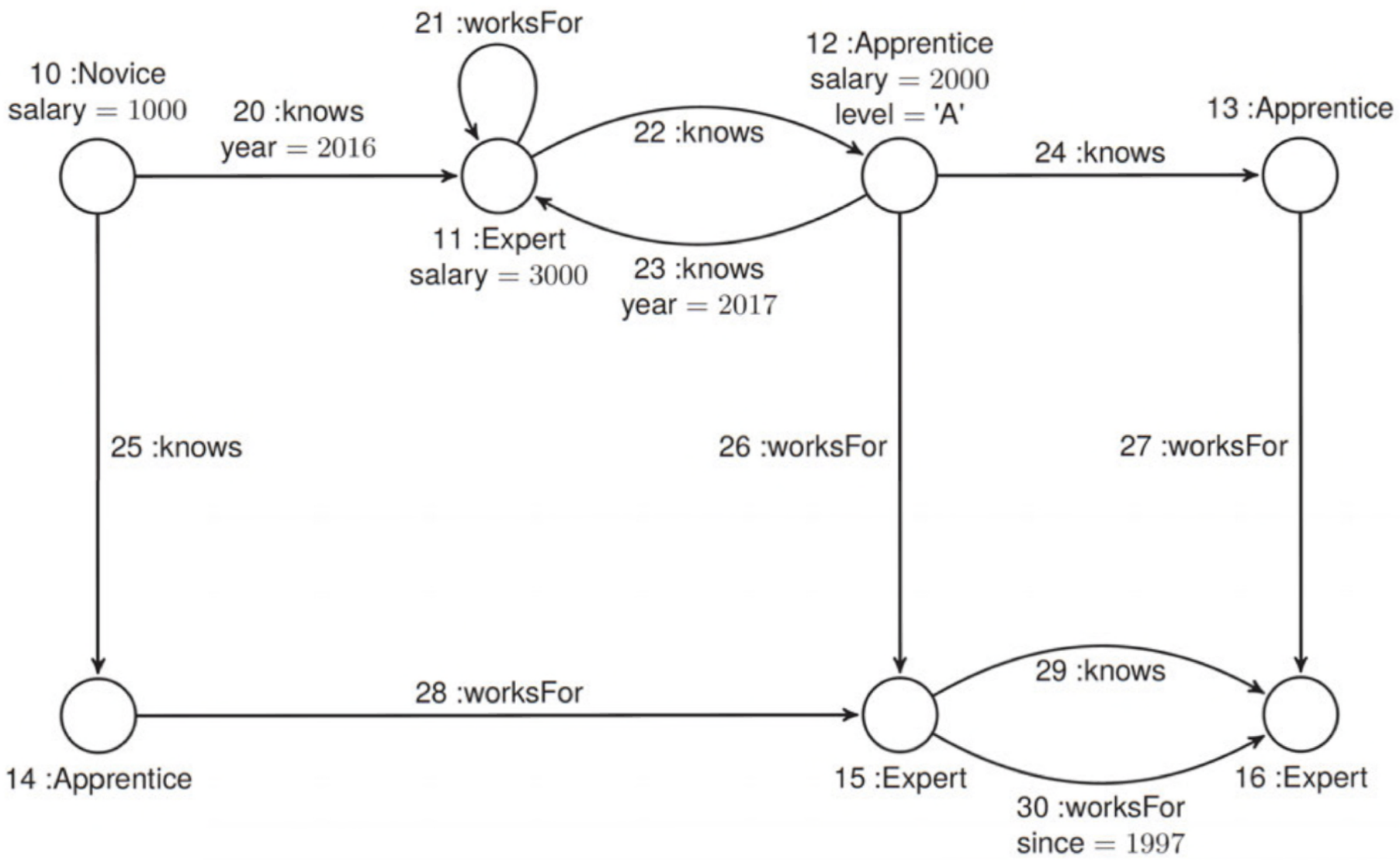
Given the query $q = :knows / :worksFor / :knows^+$ evaluated on G_{ex} get $[\![q]\!]_{G_{ex}}$.

G_{ex}



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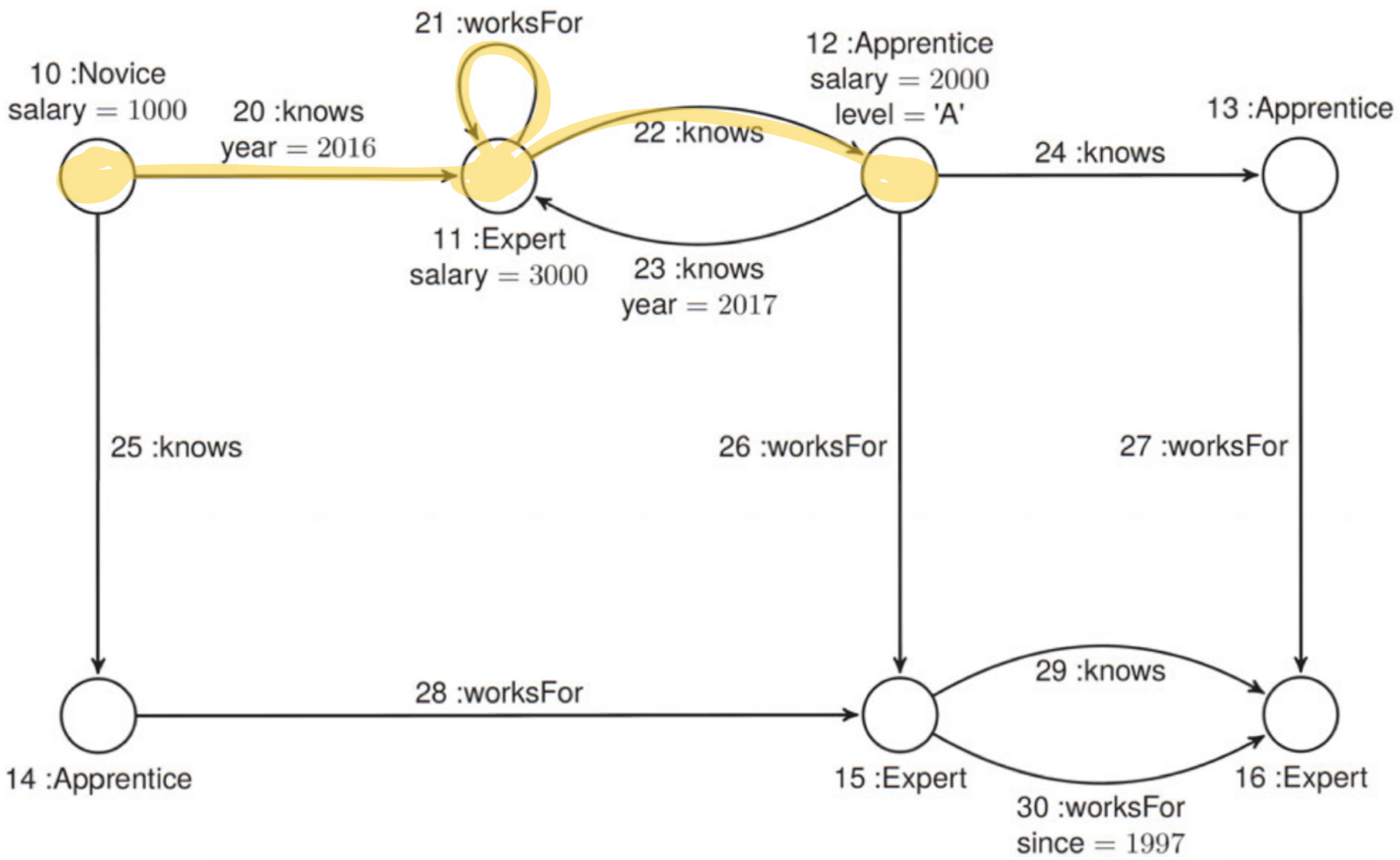

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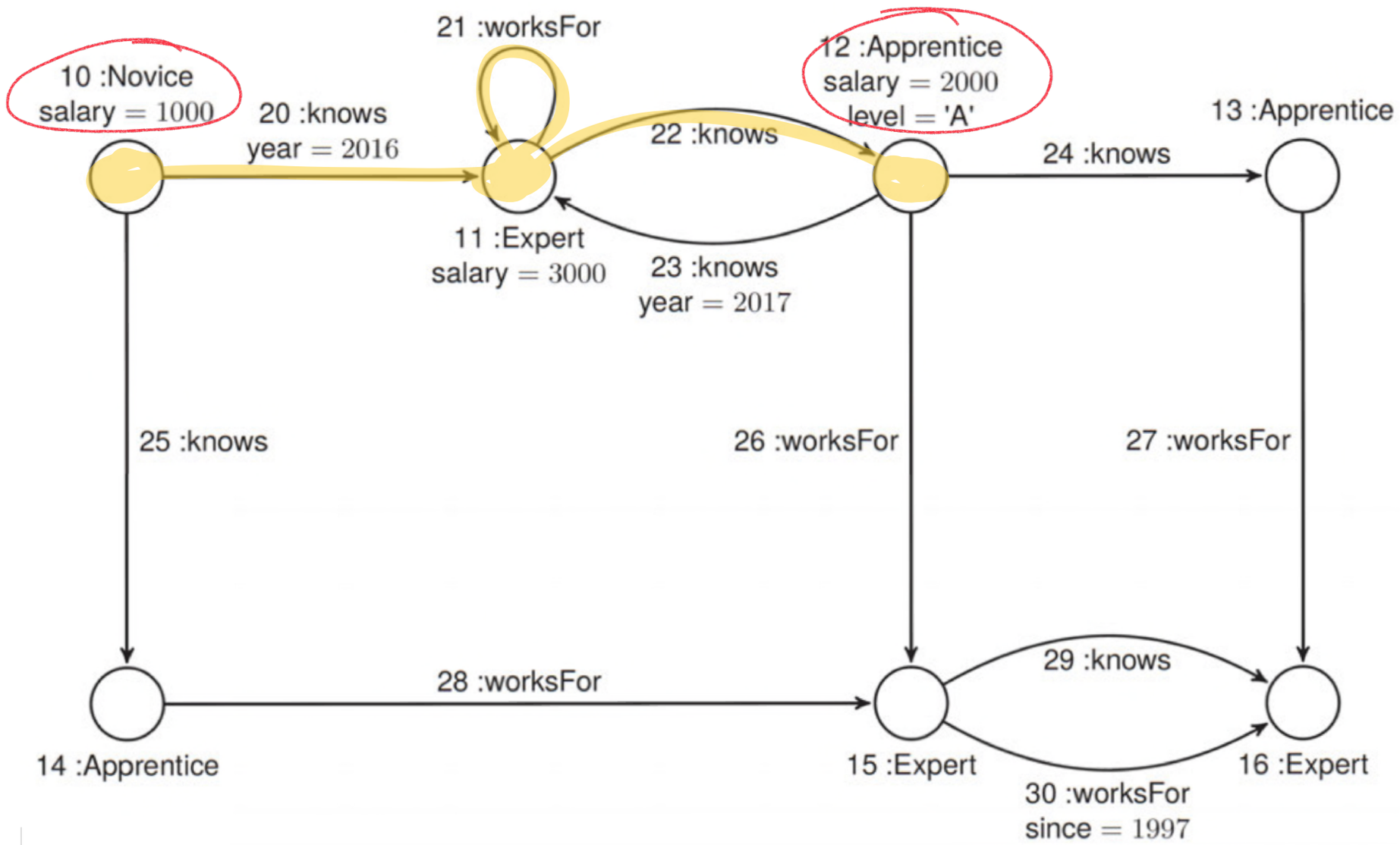

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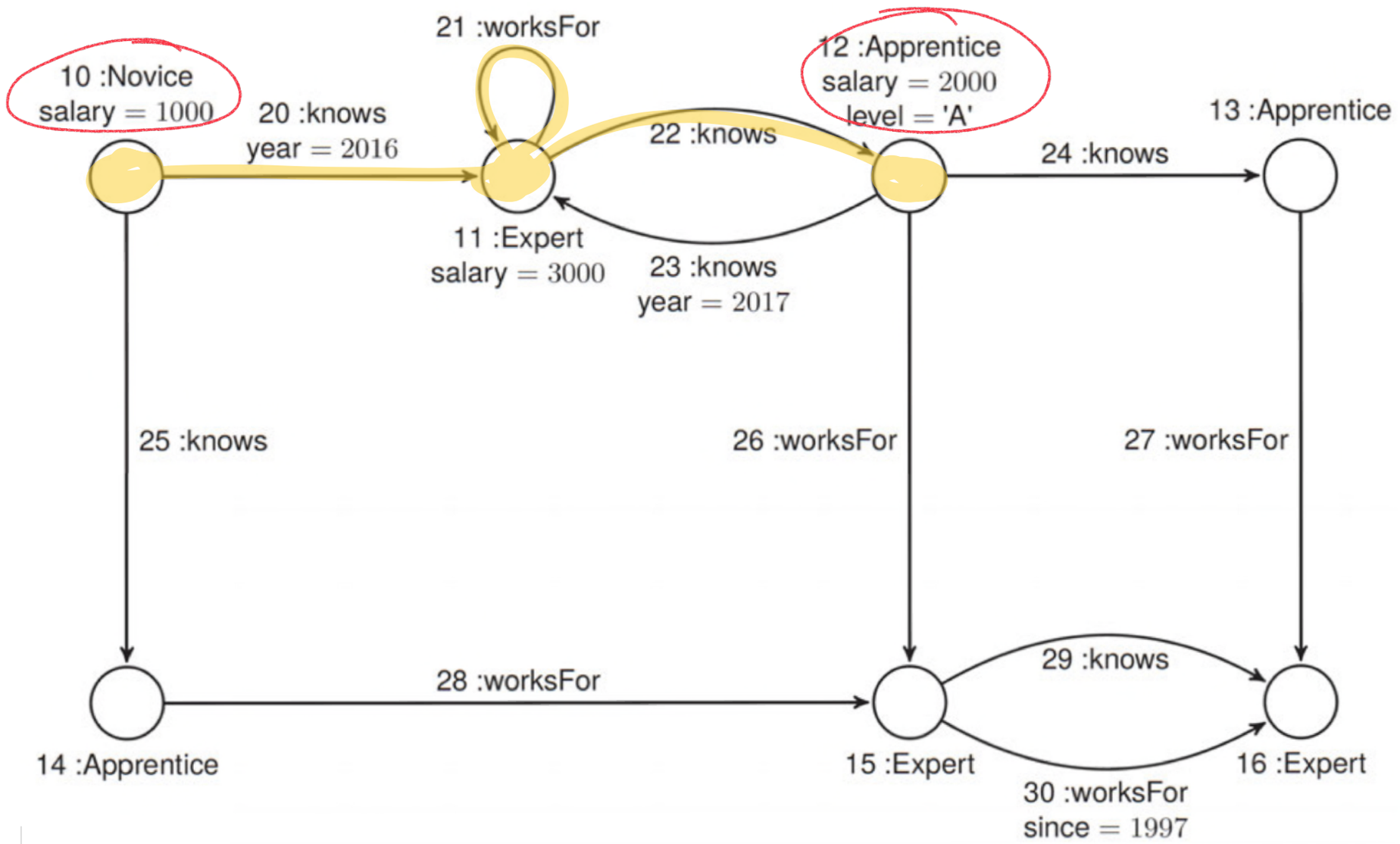

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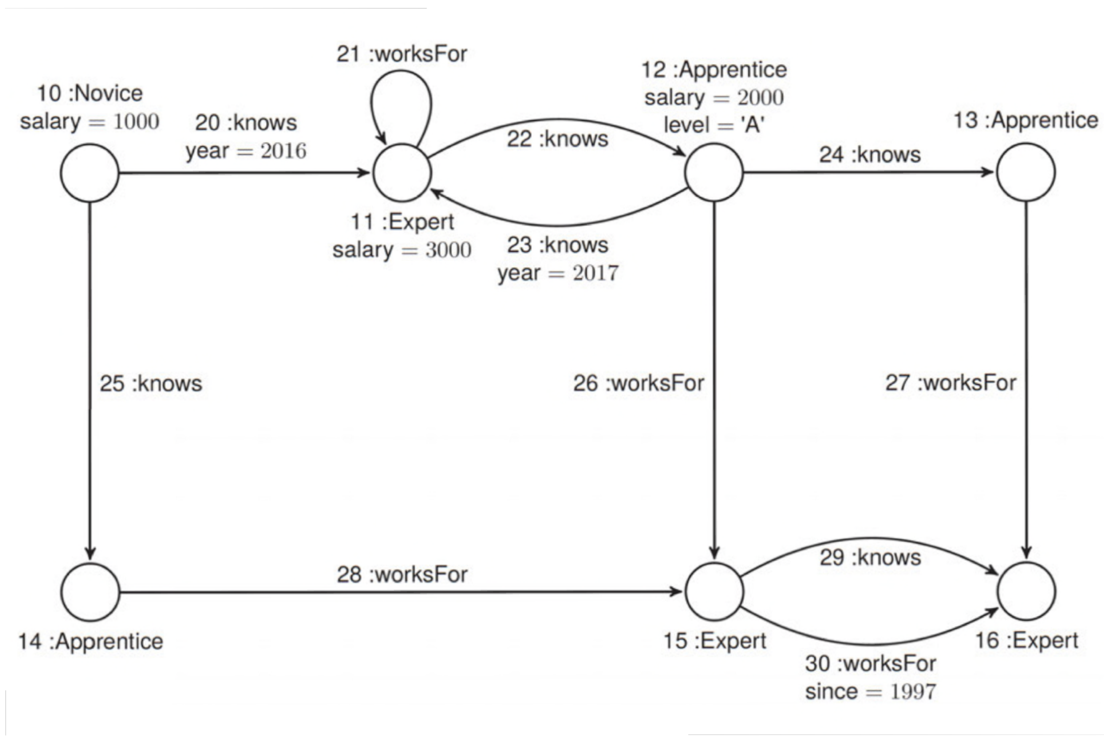

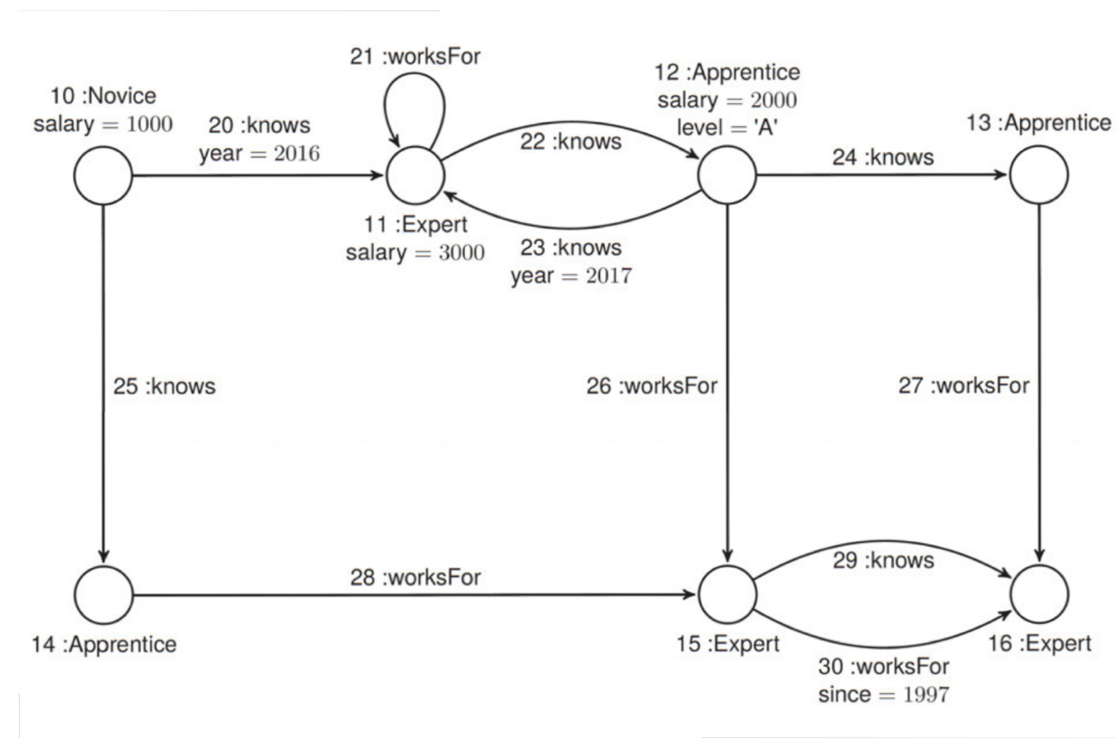

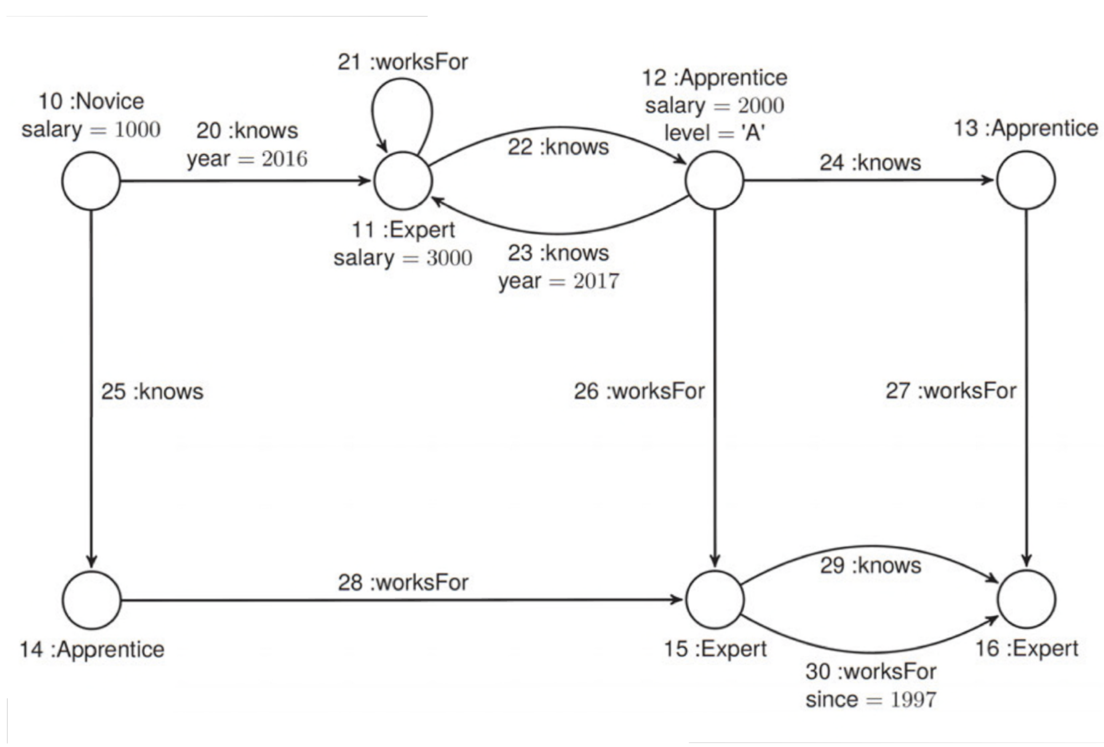

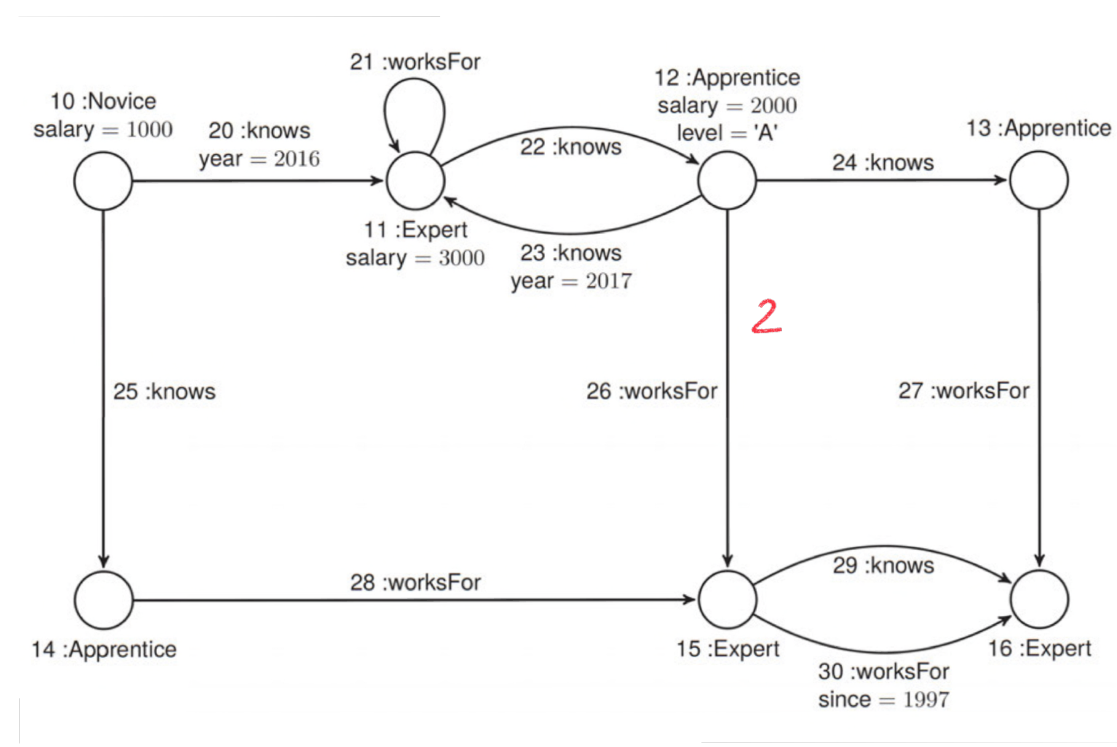

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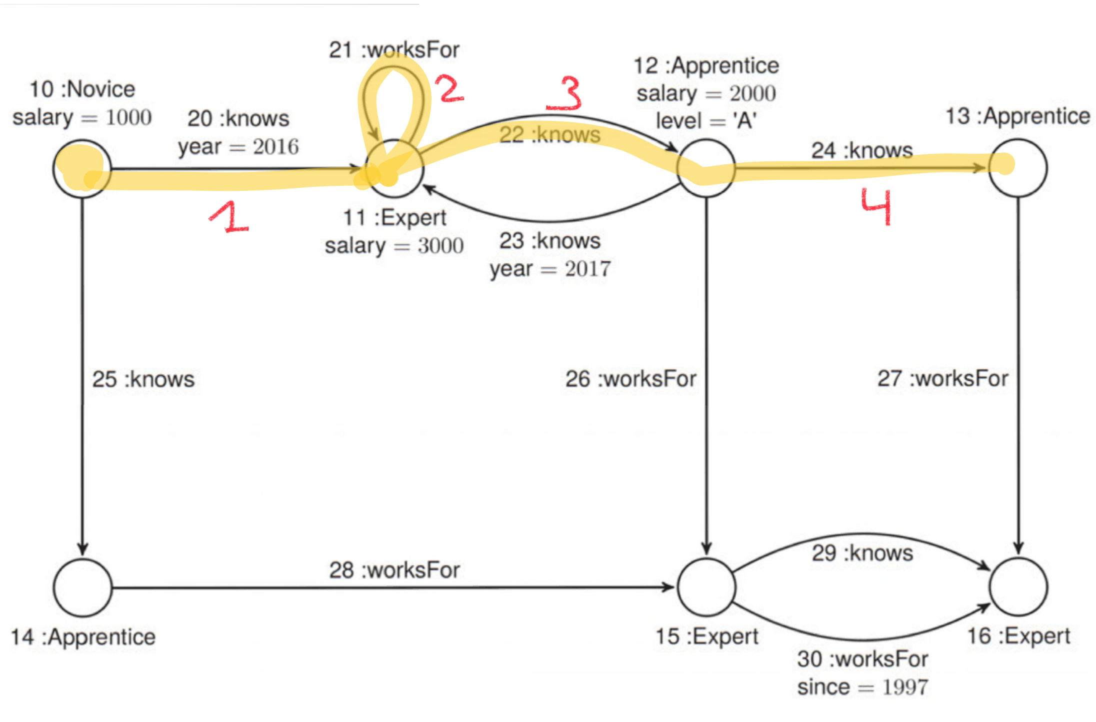

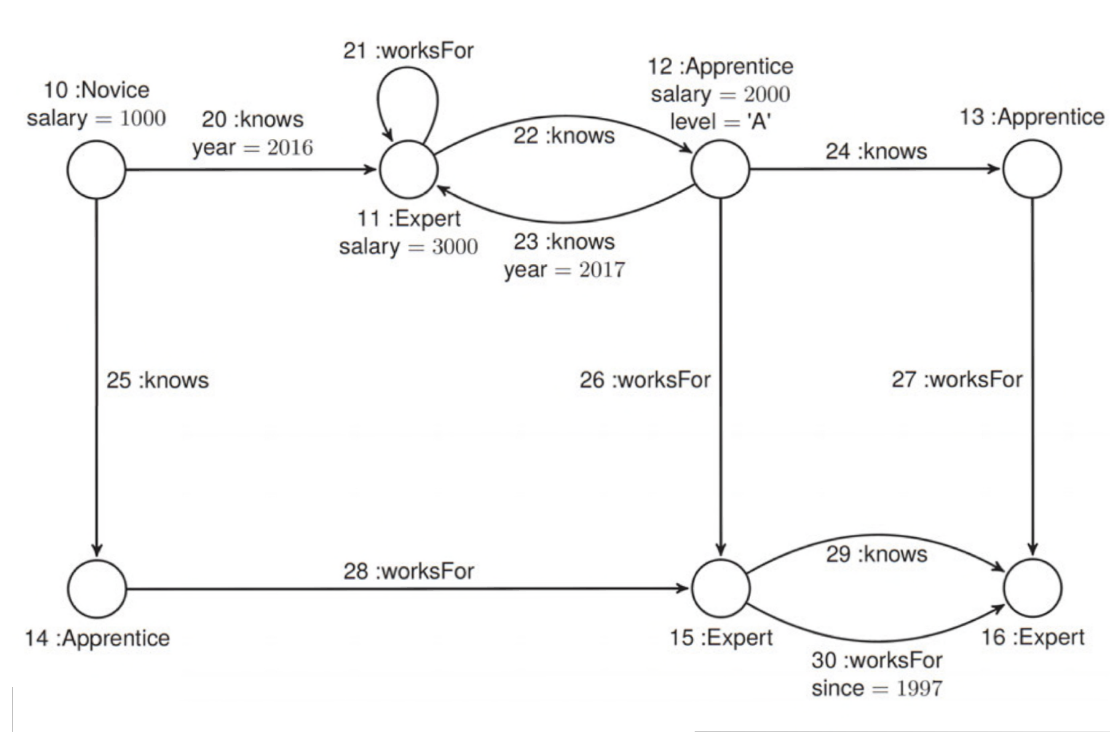

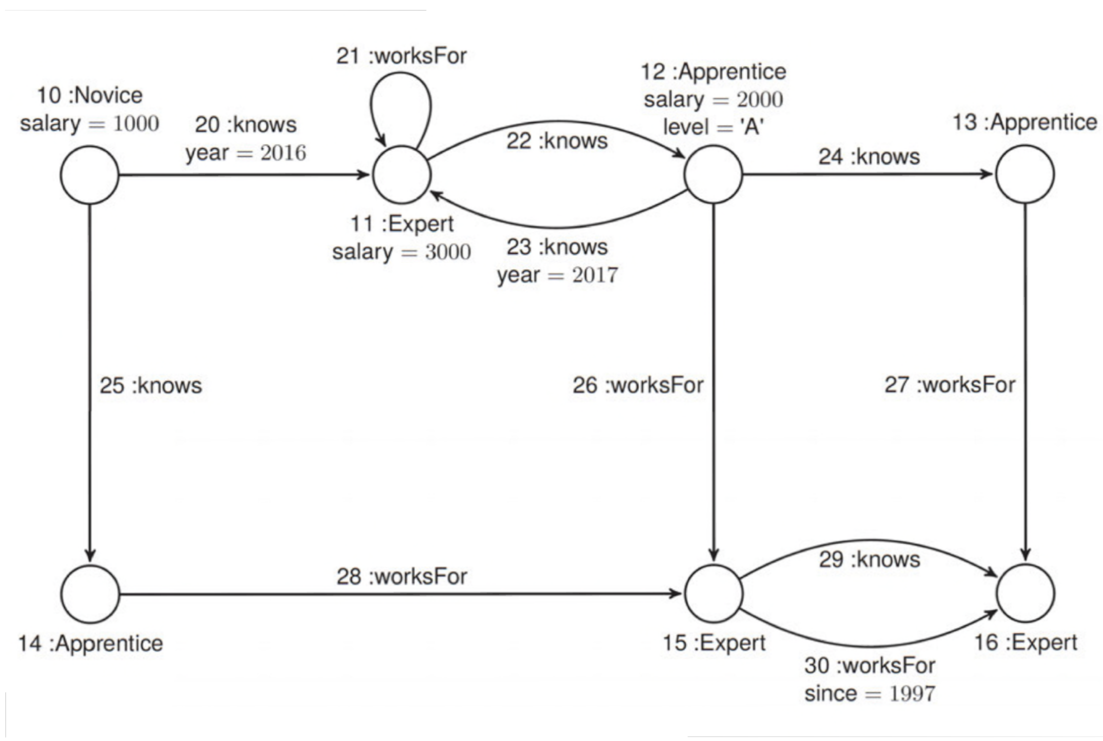

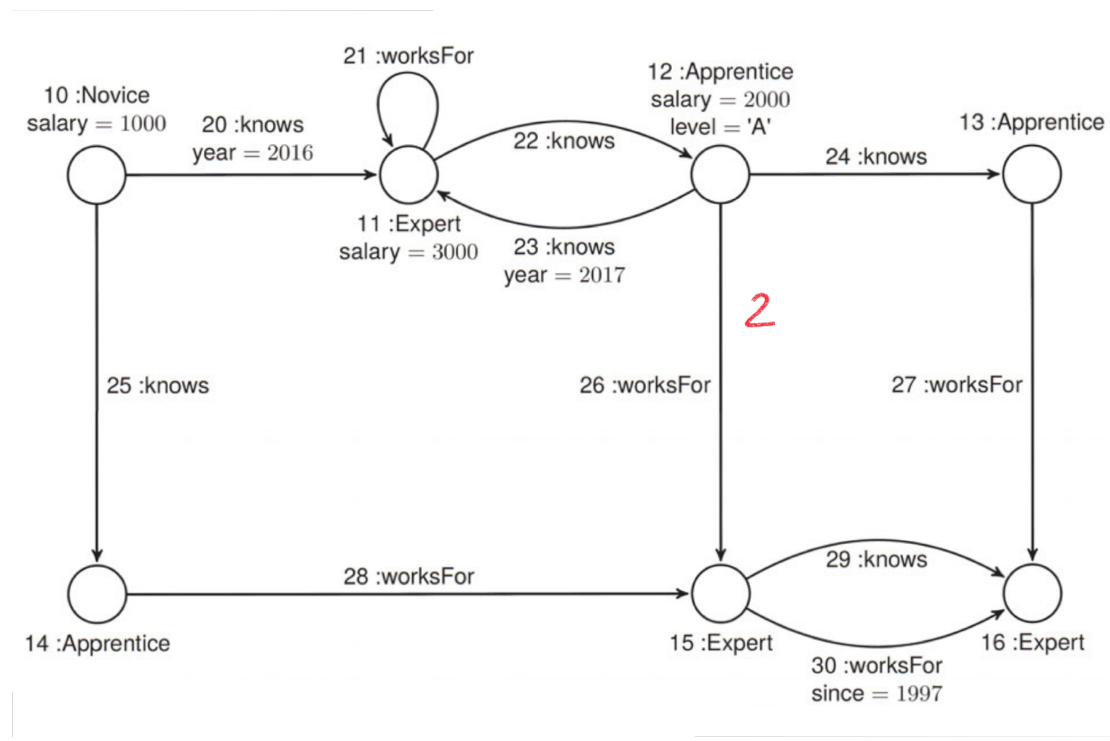

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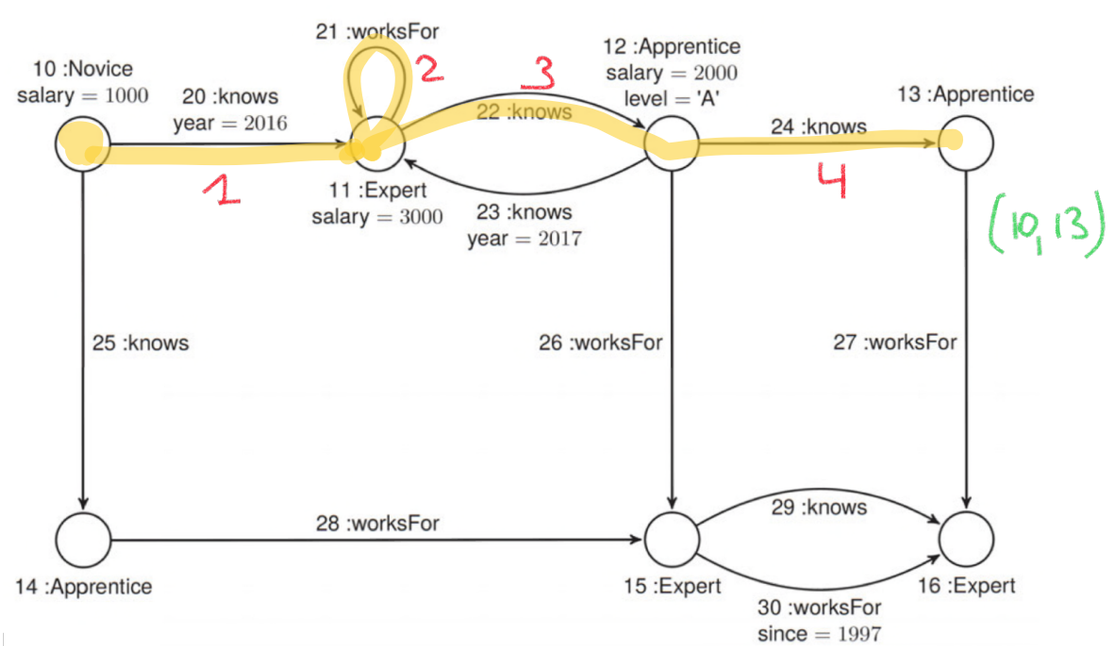

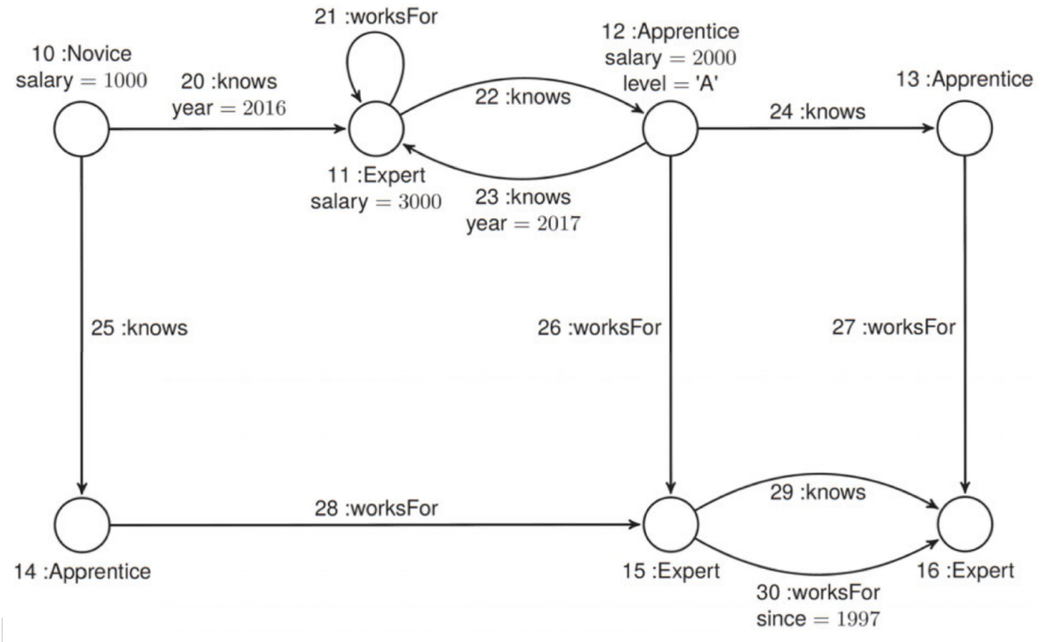

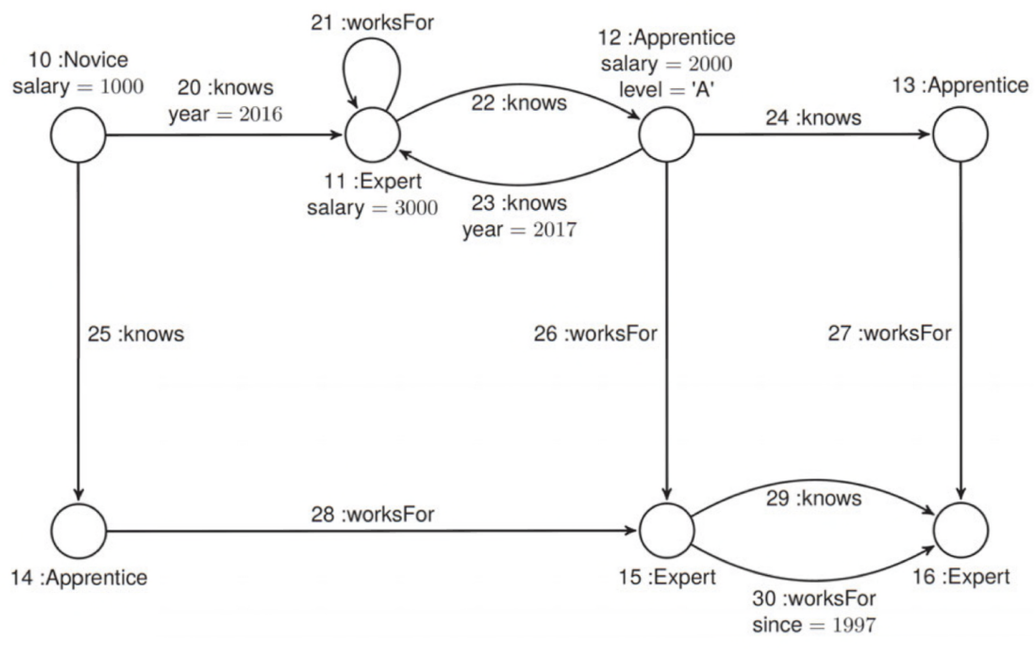

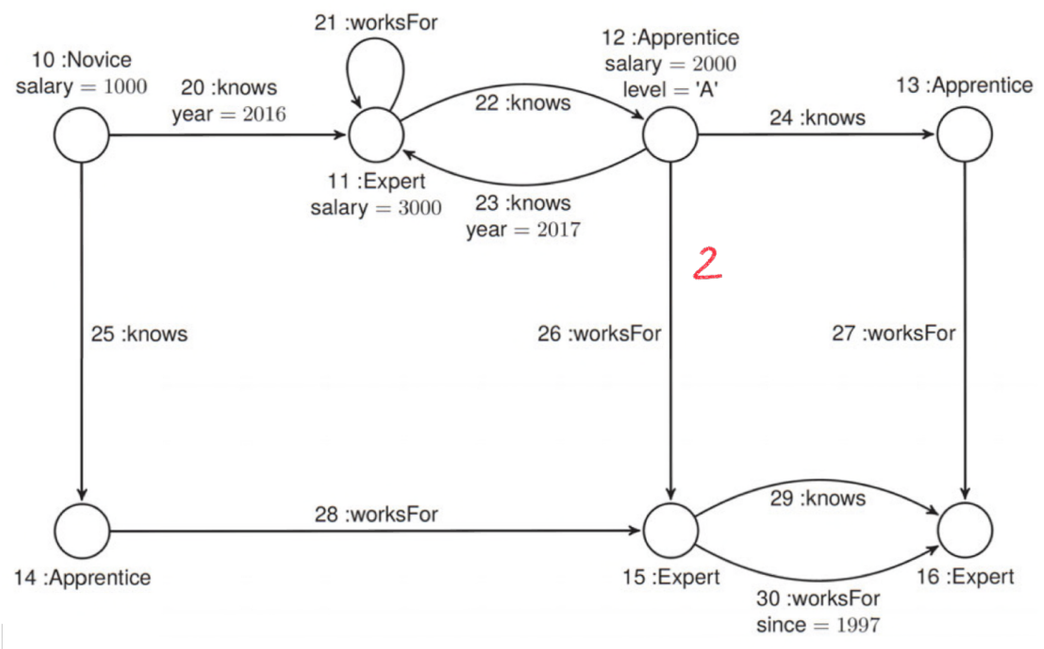

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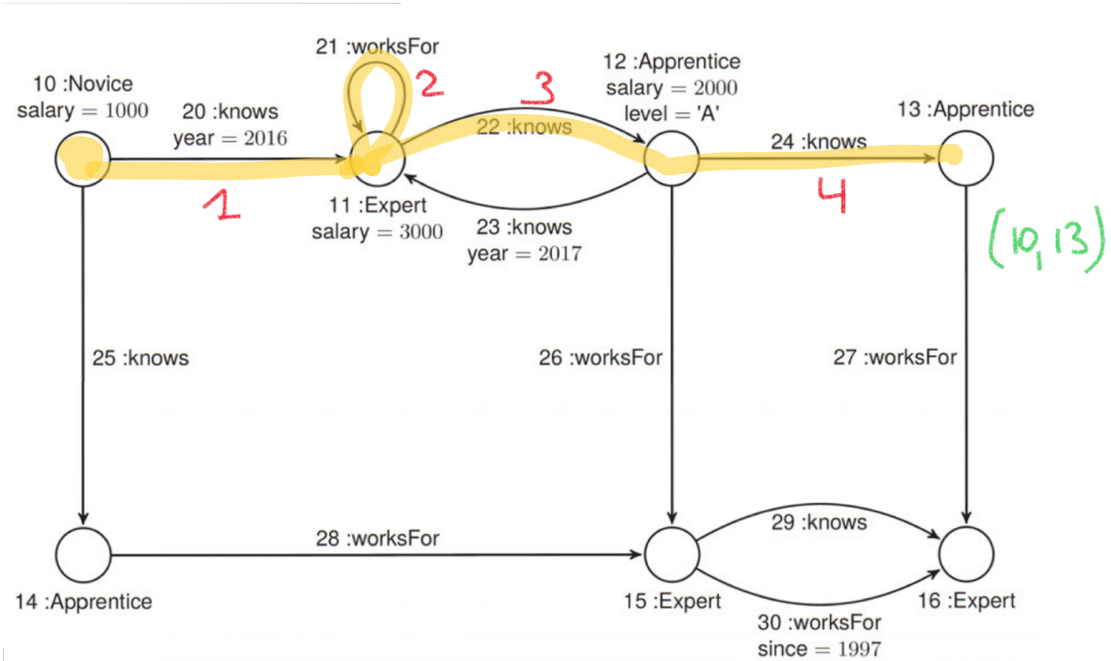

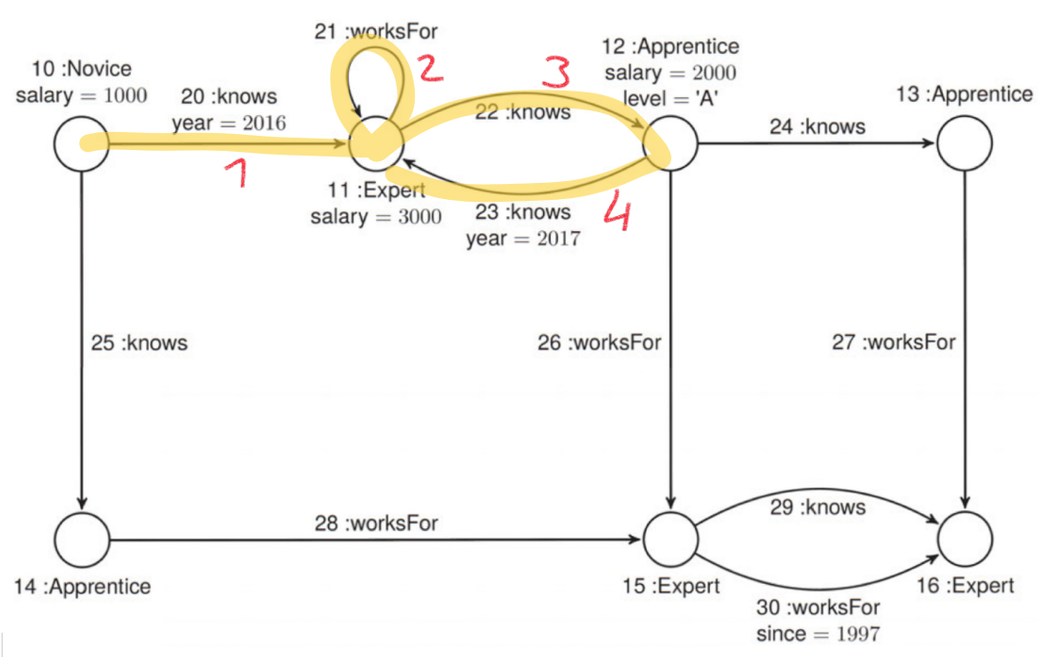

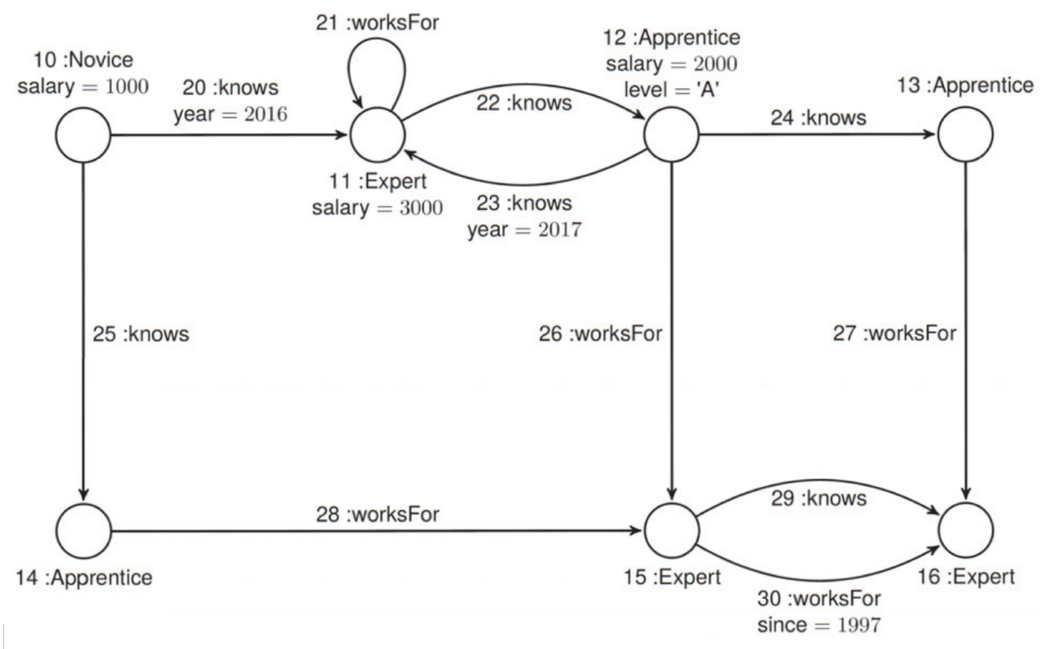

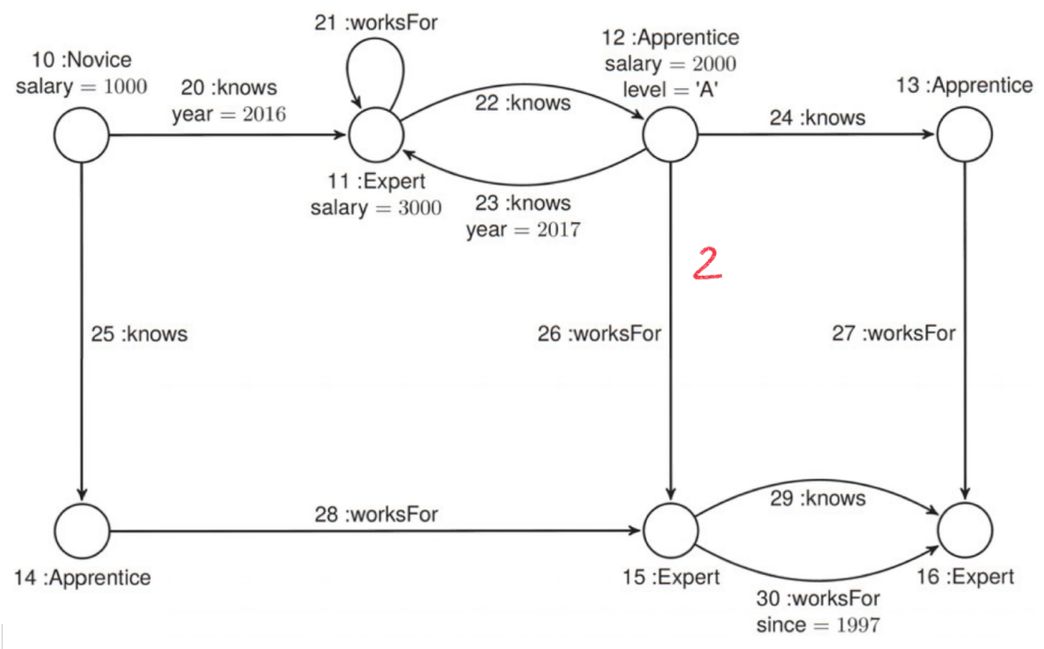

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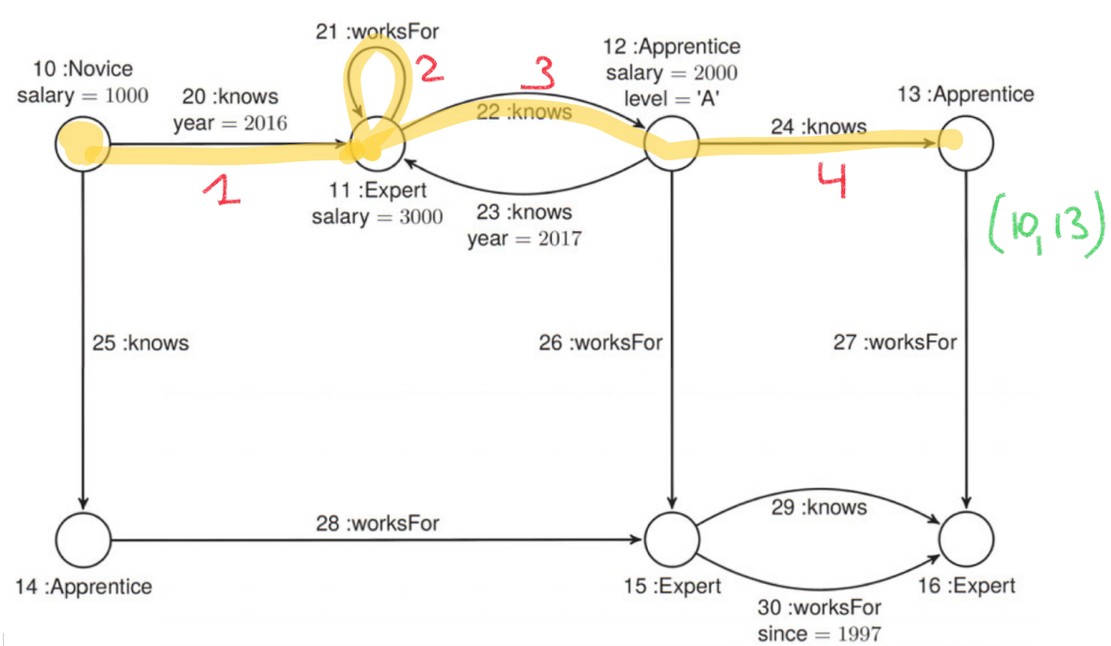

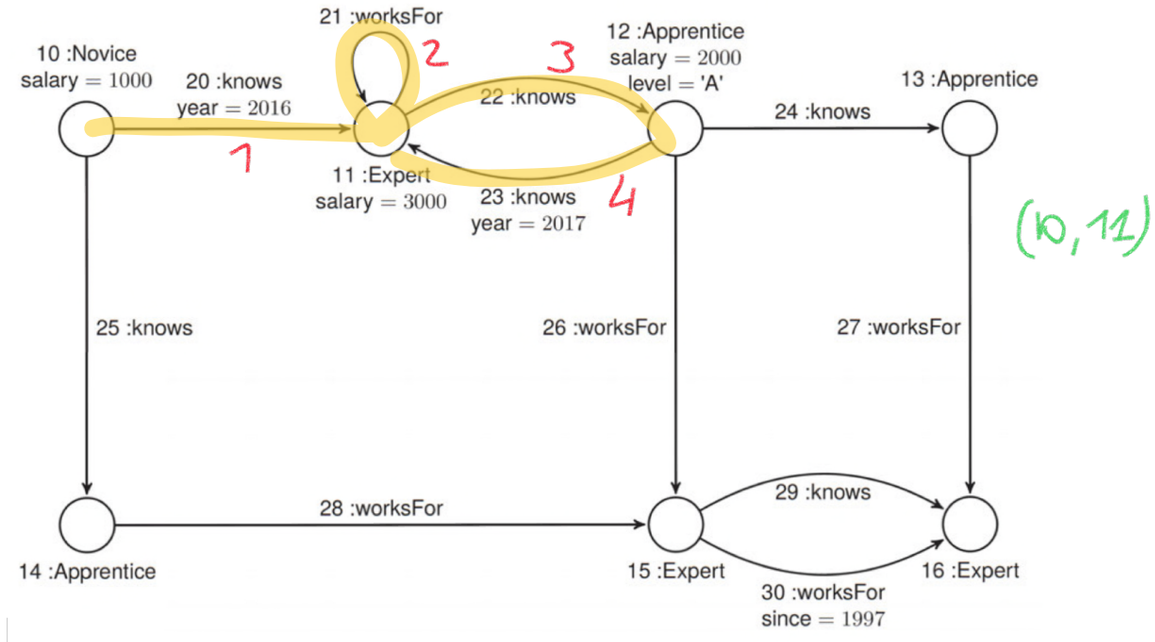

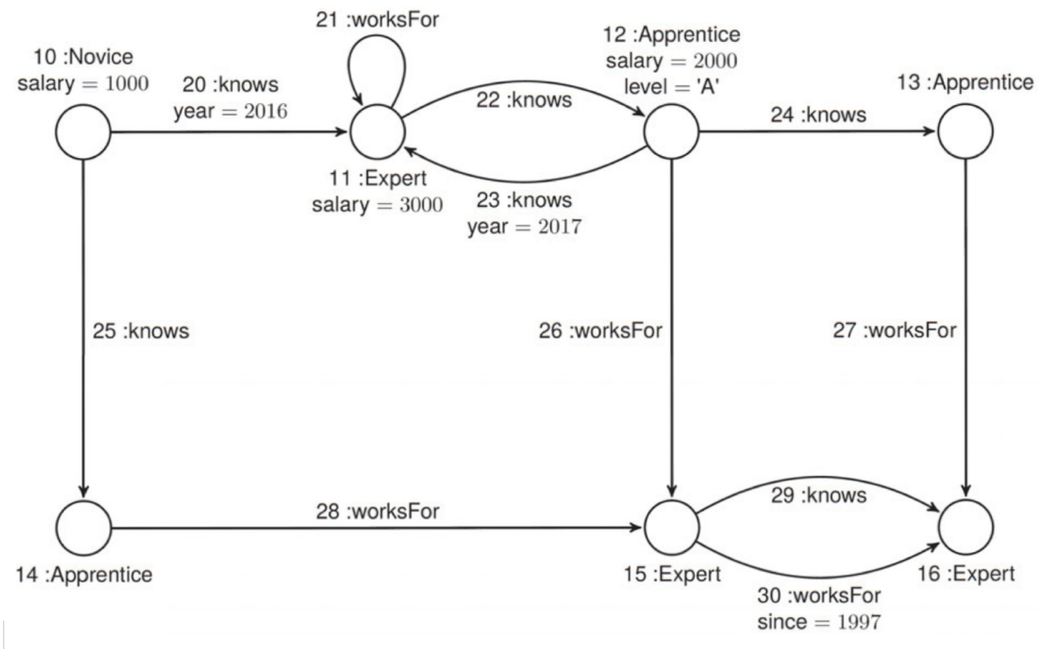

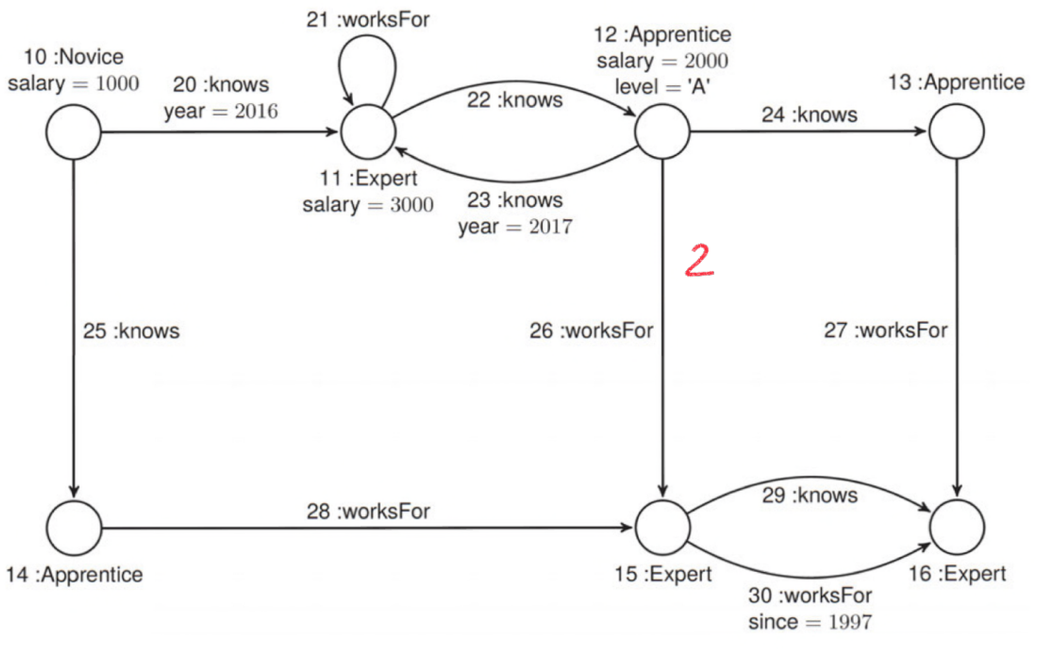

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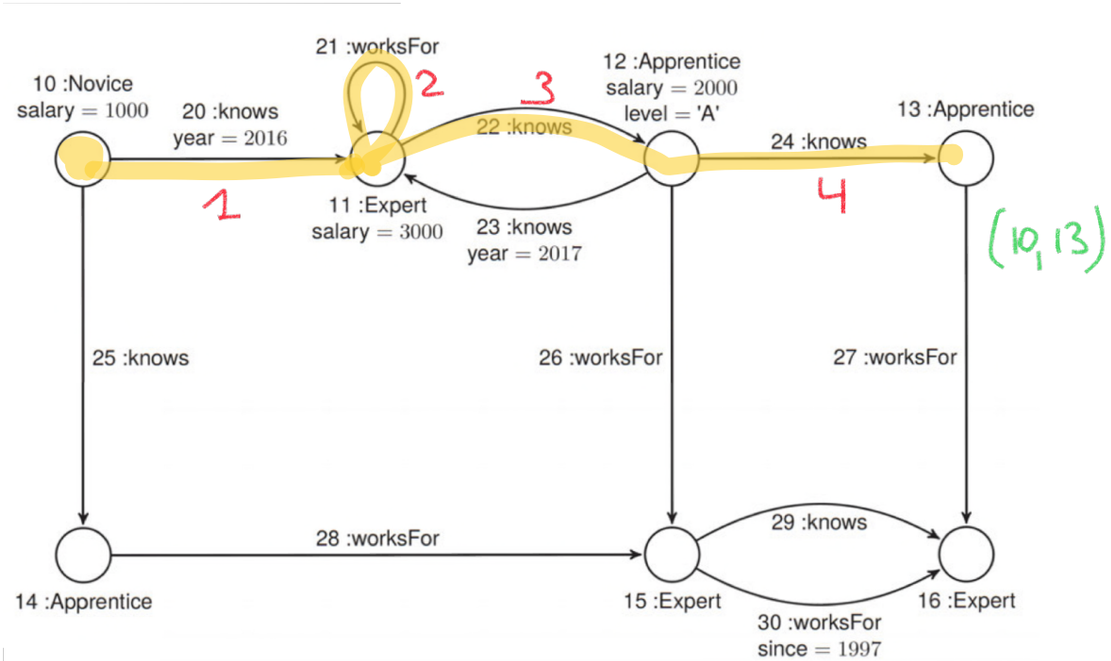

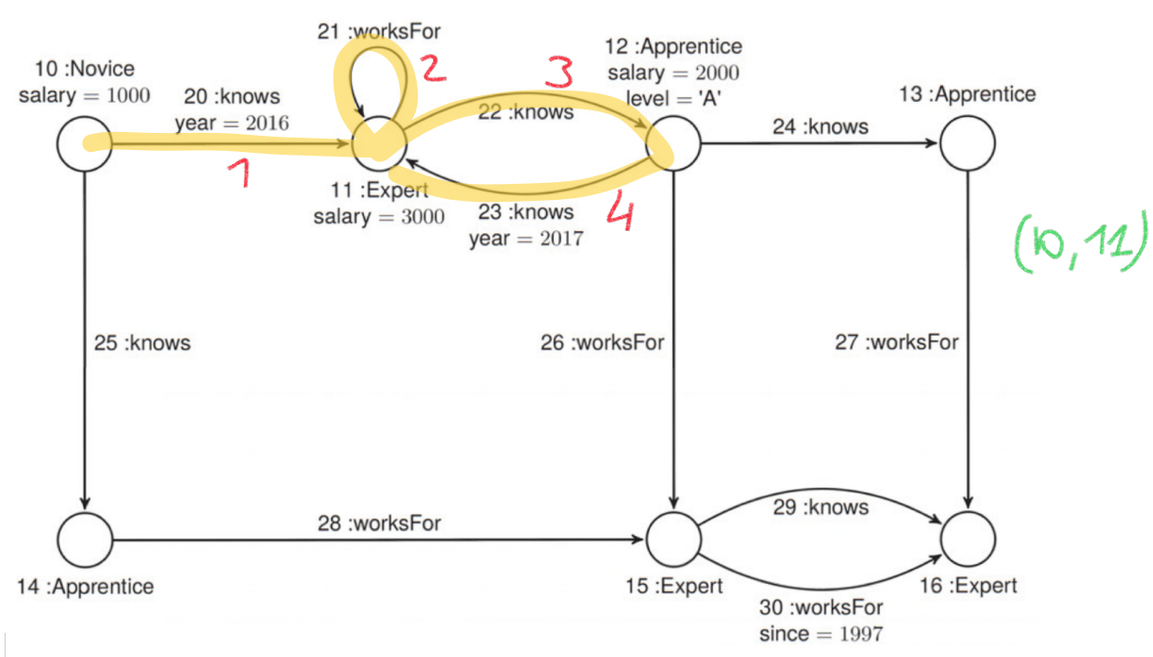

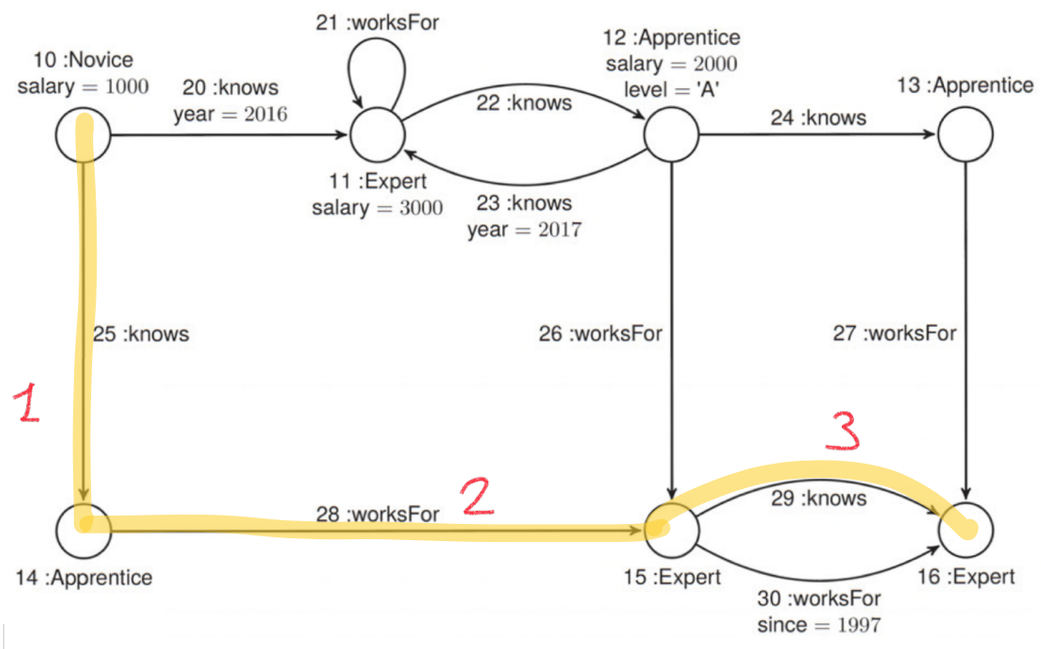

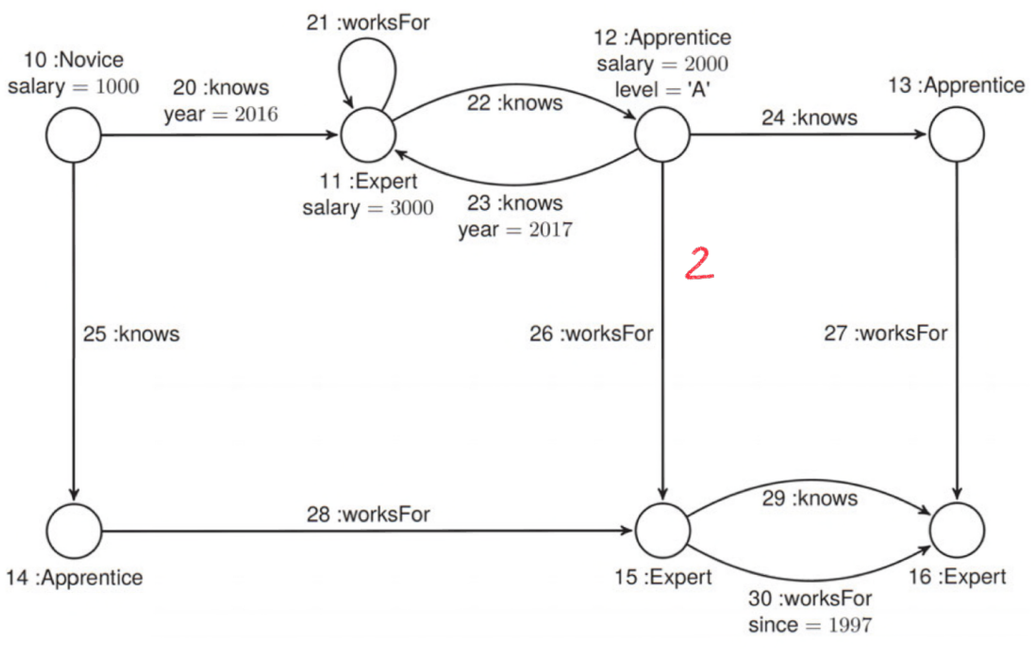

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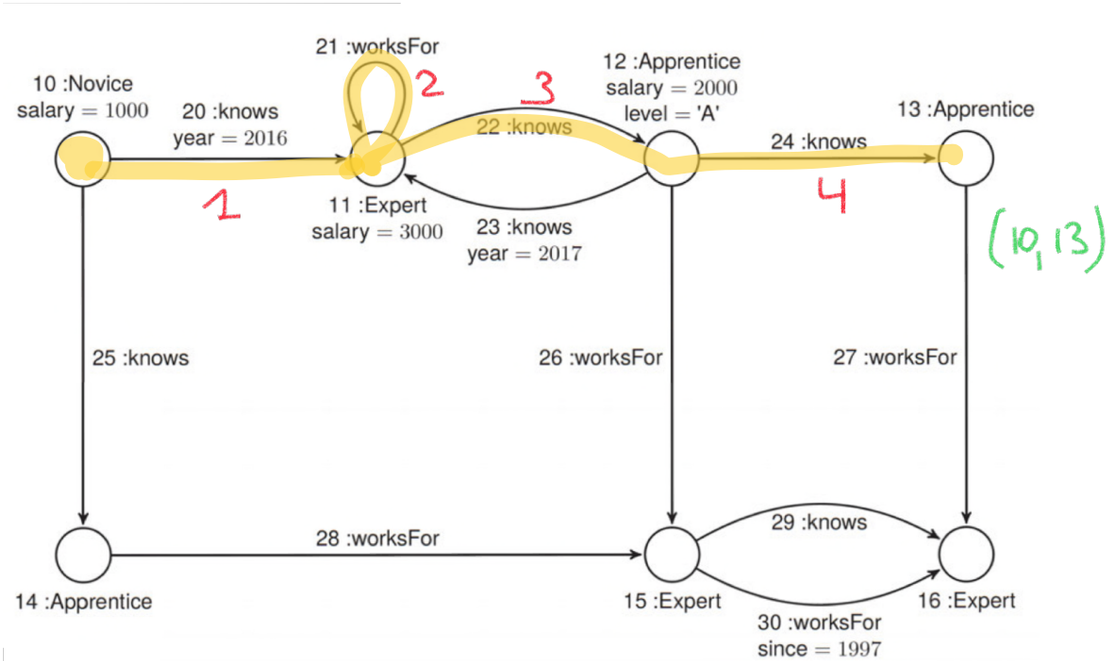

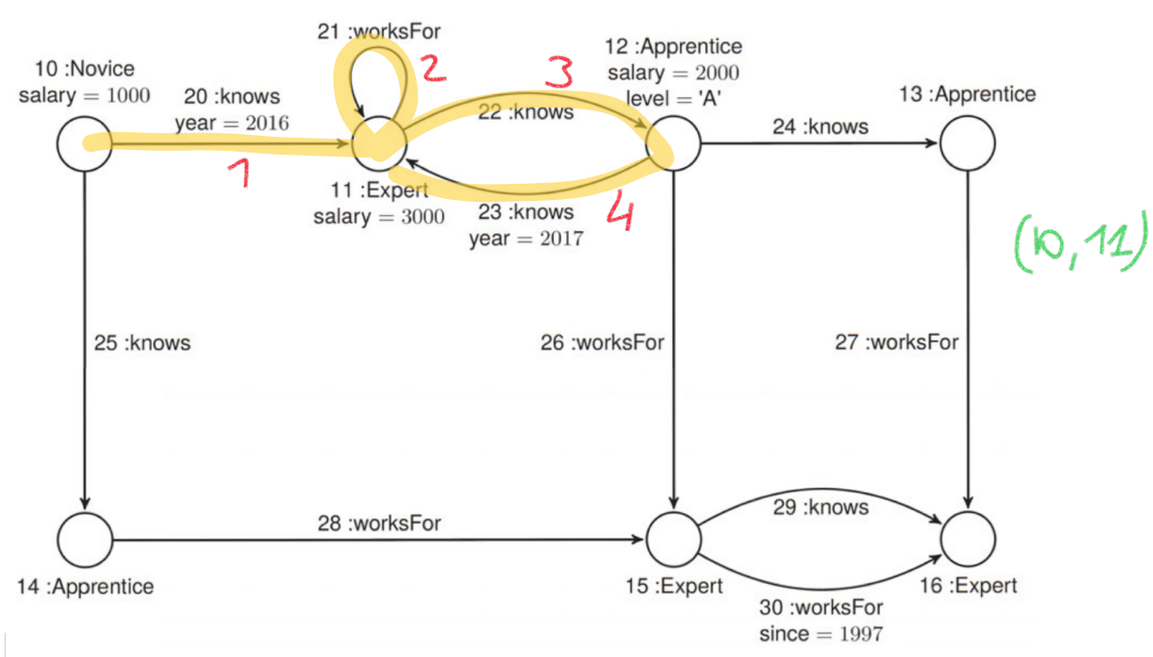

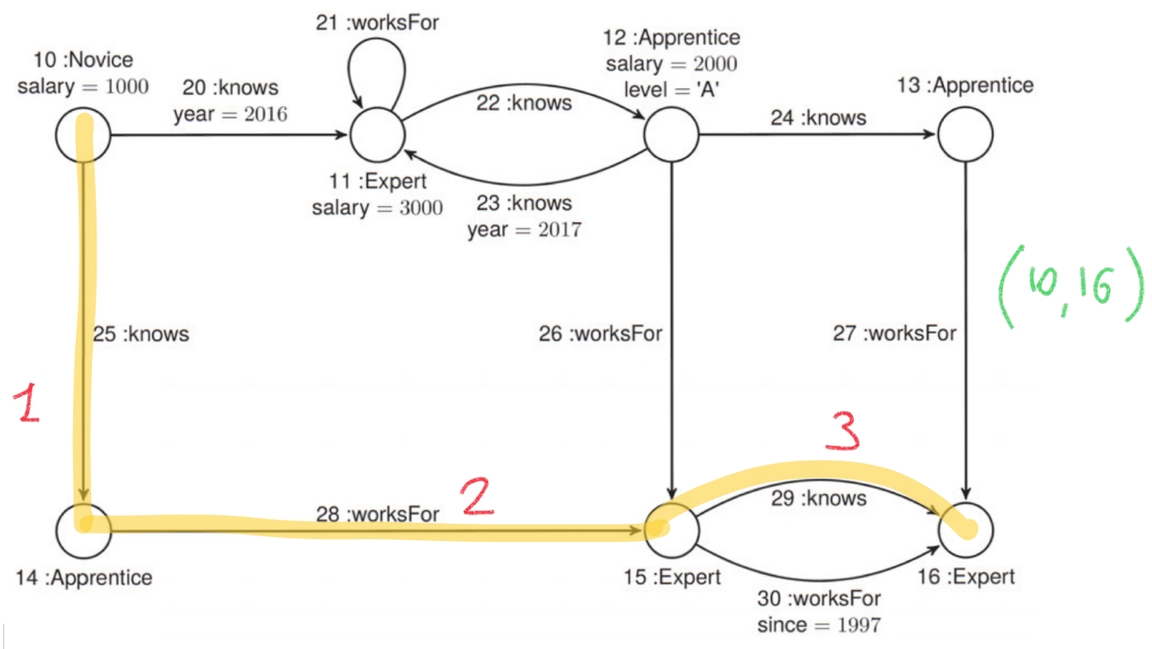

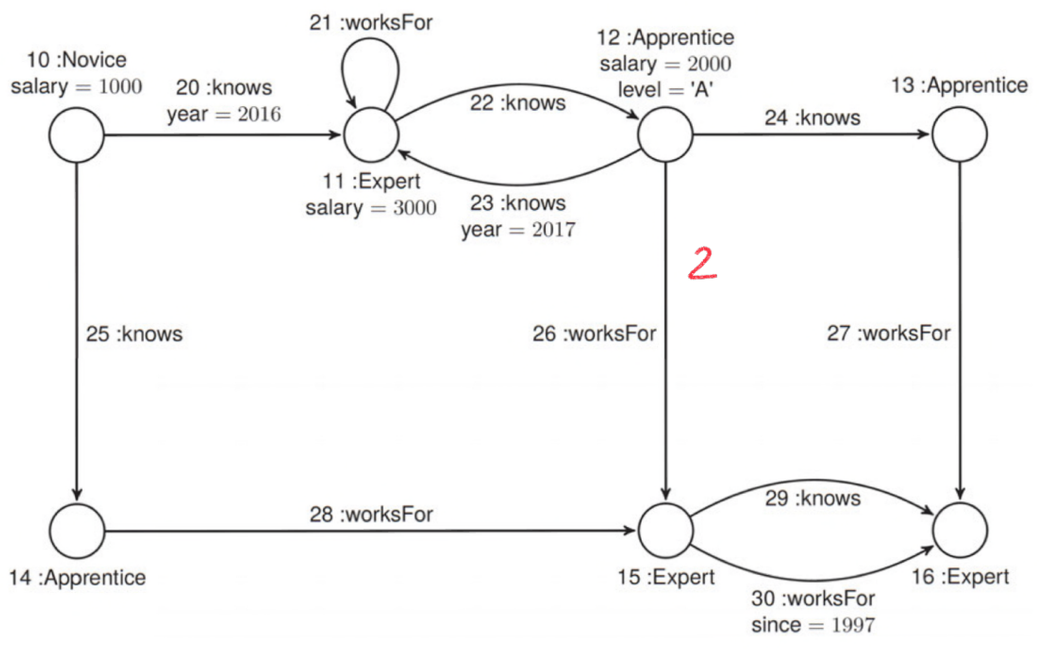

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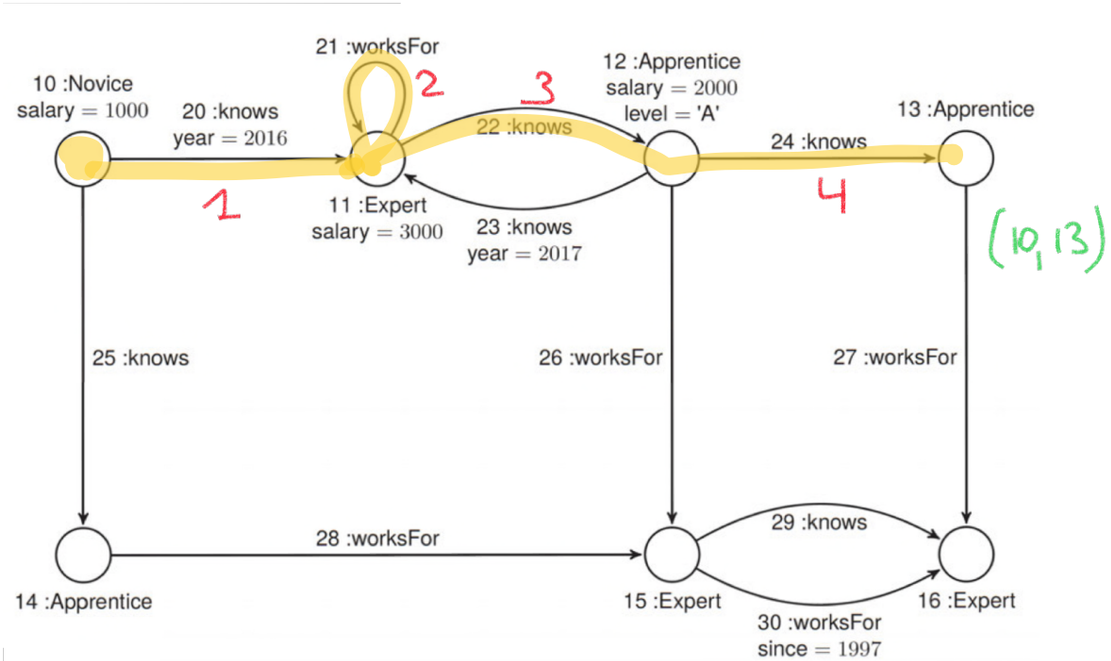

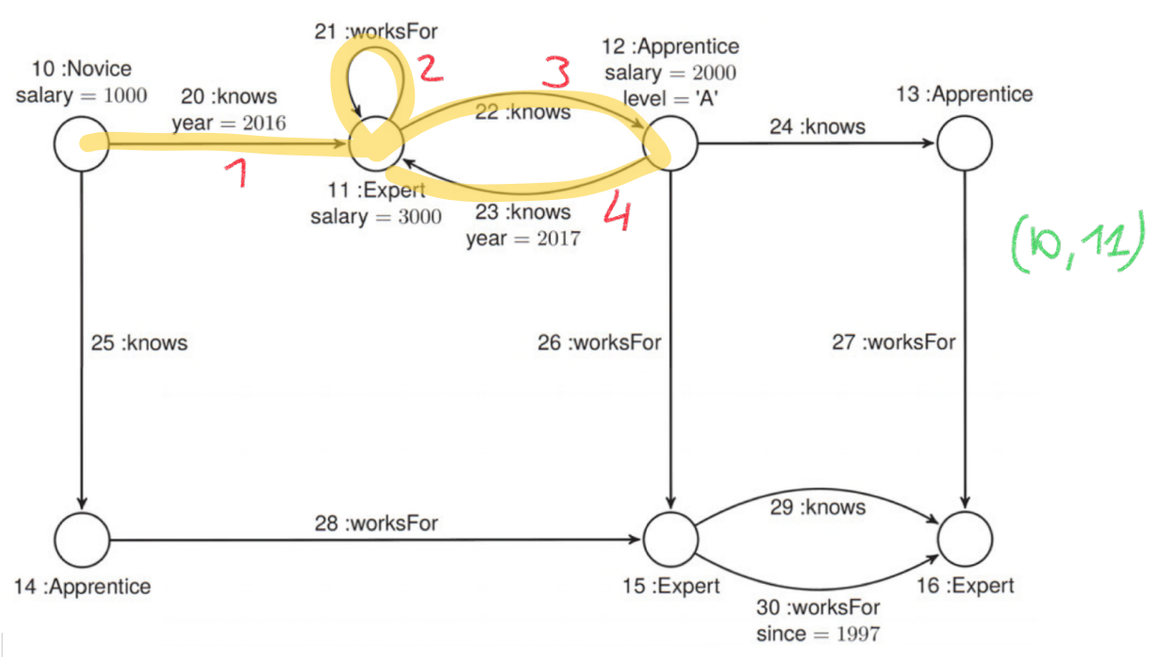

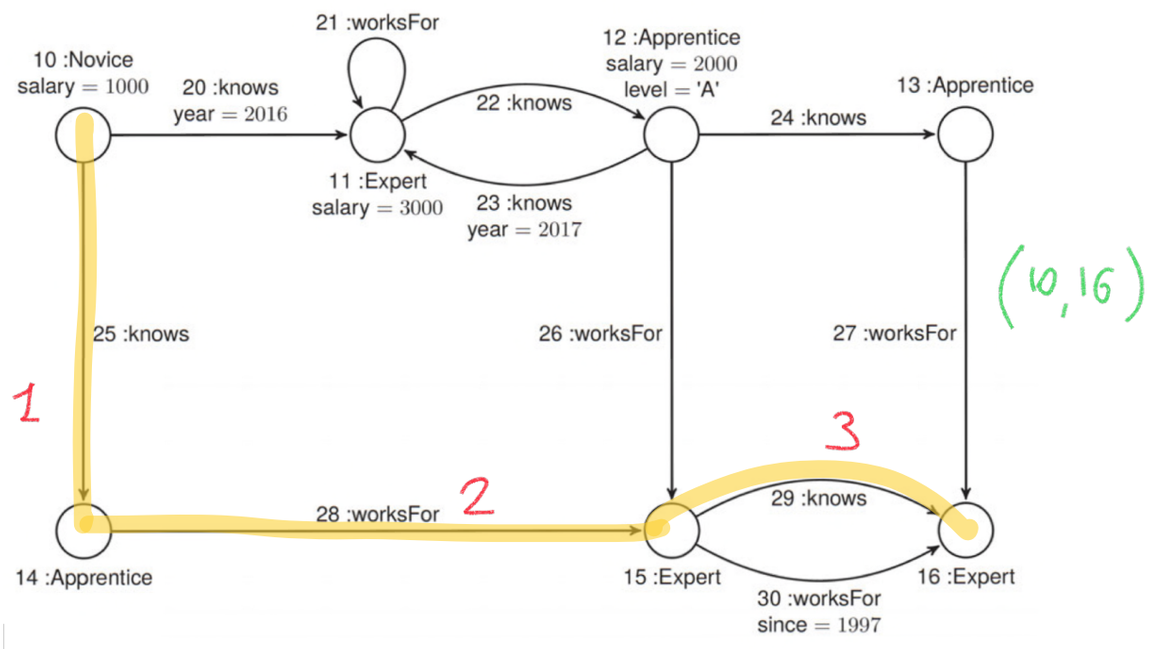

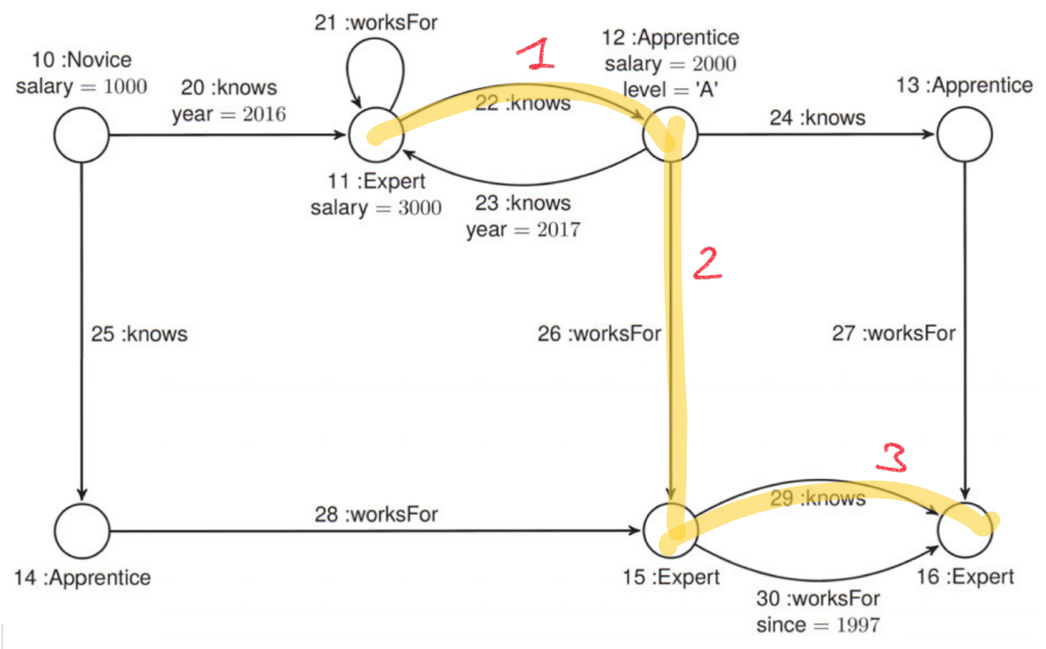

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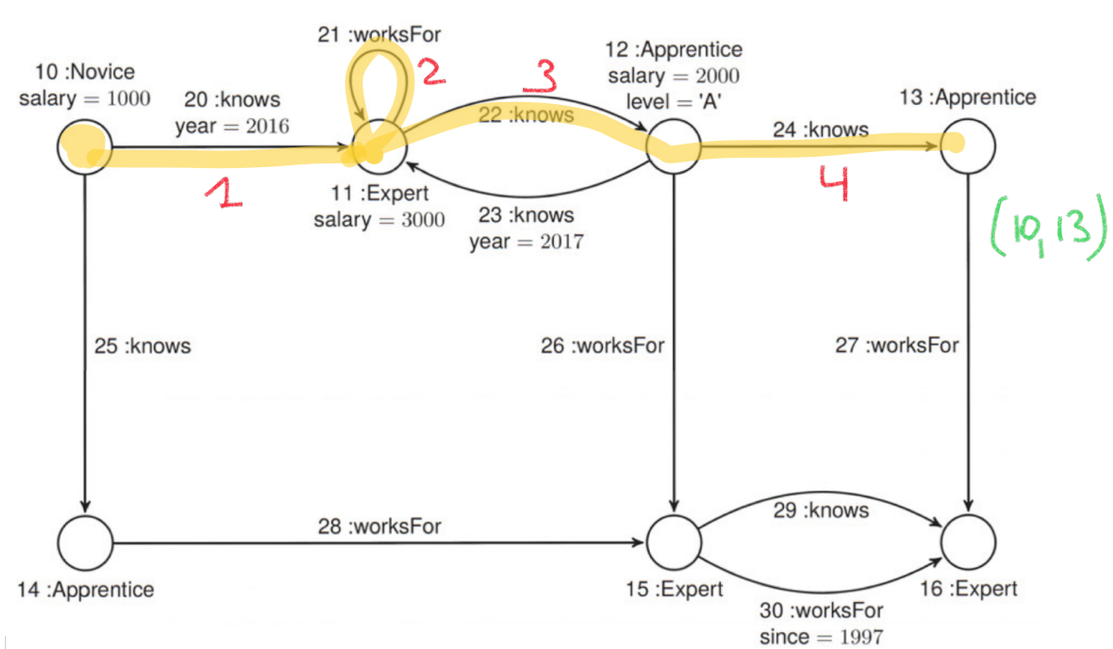

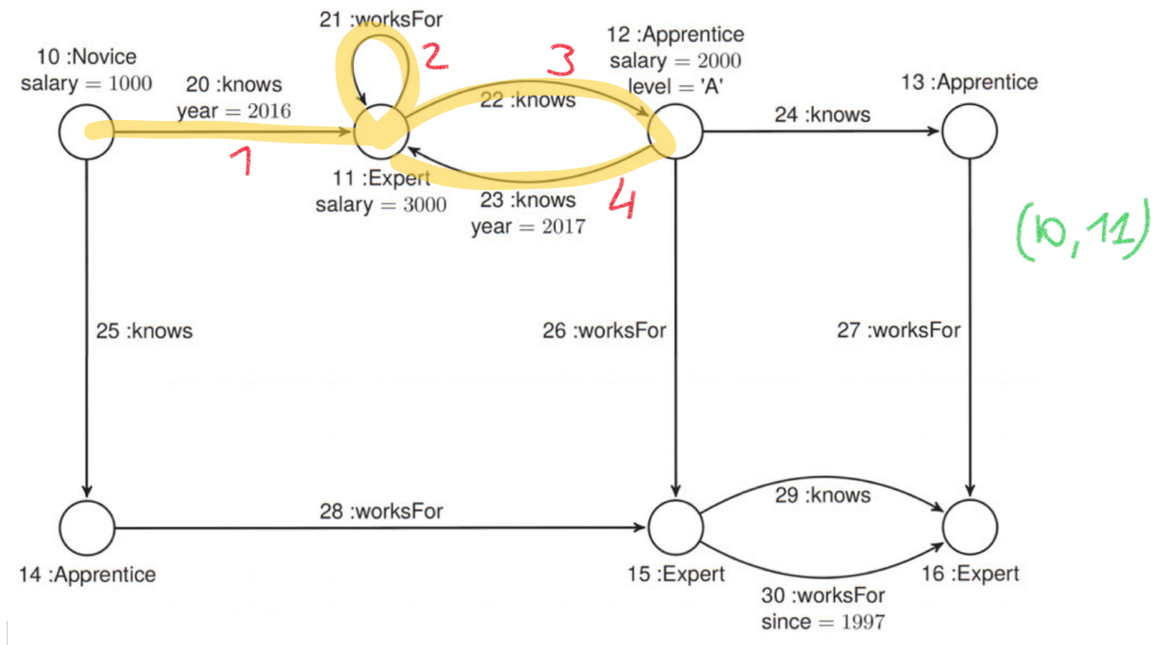

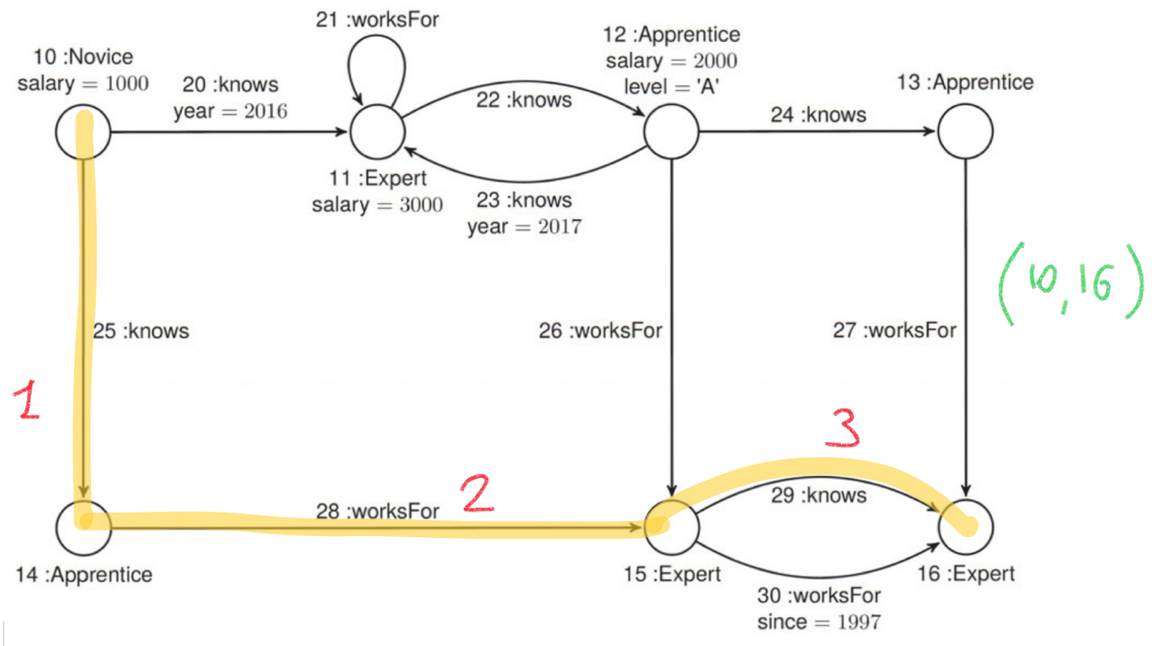

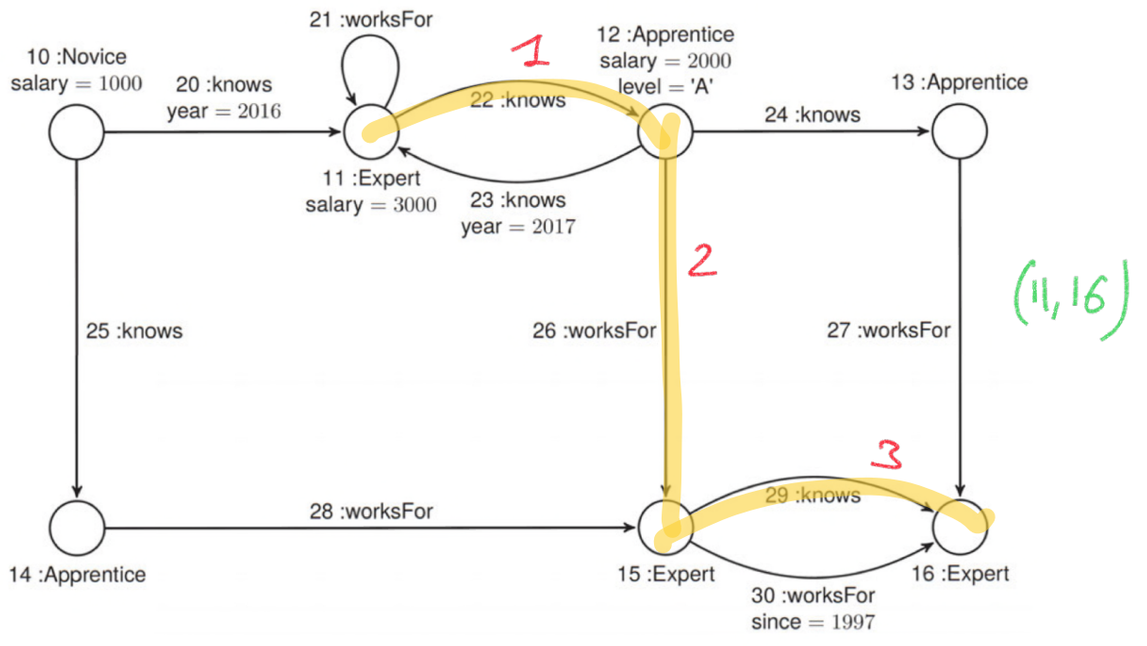

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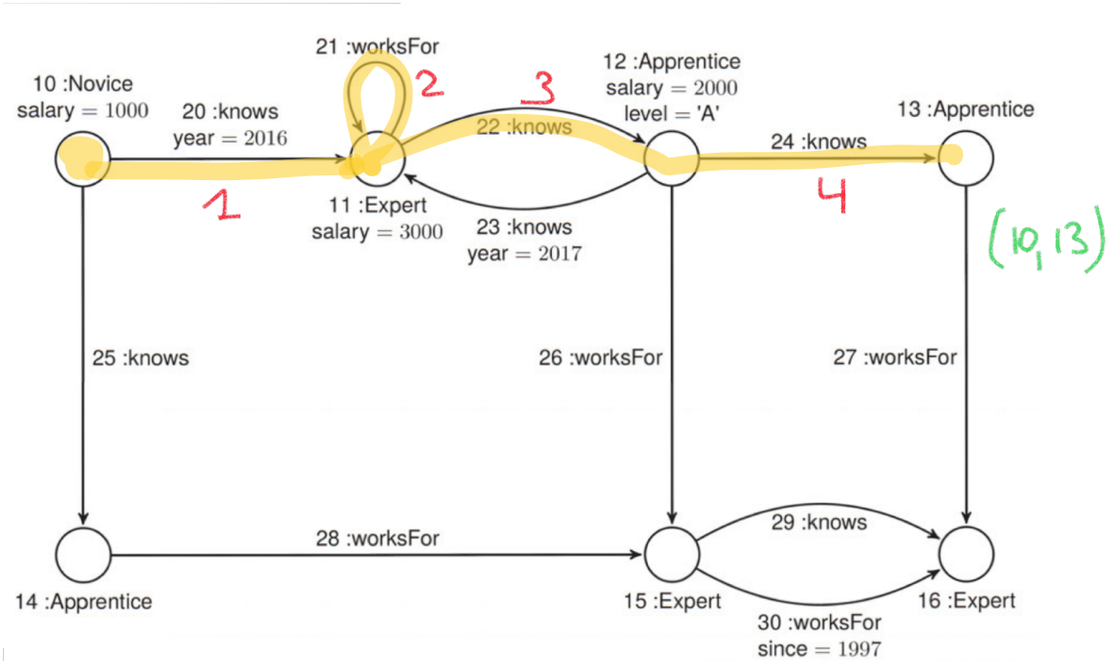

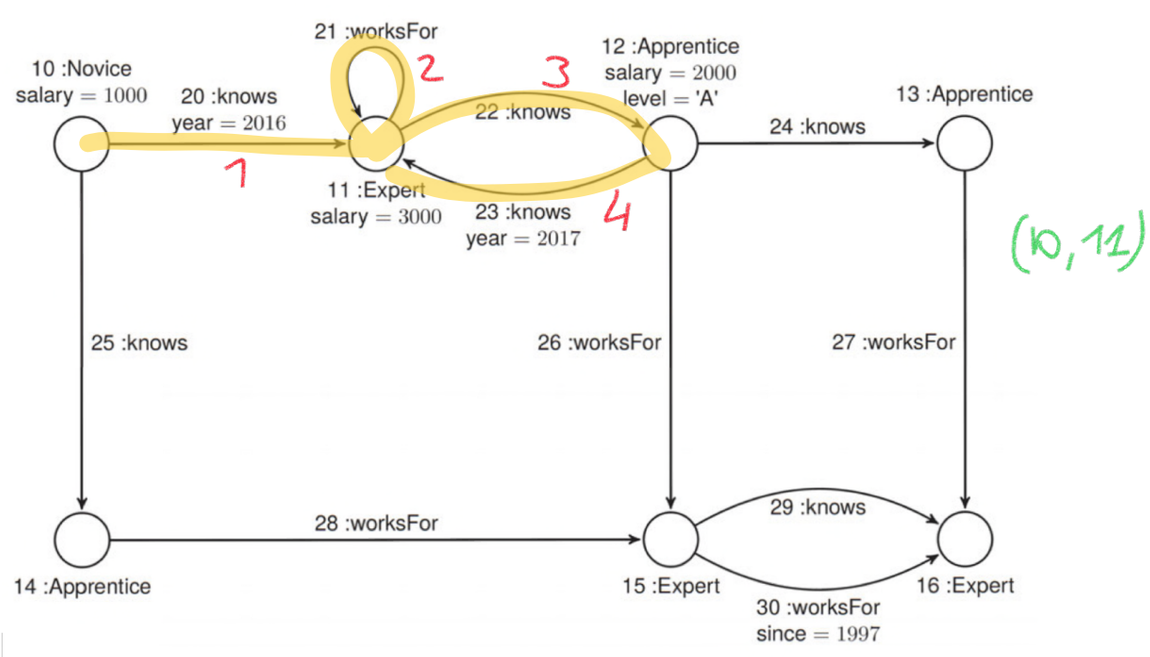

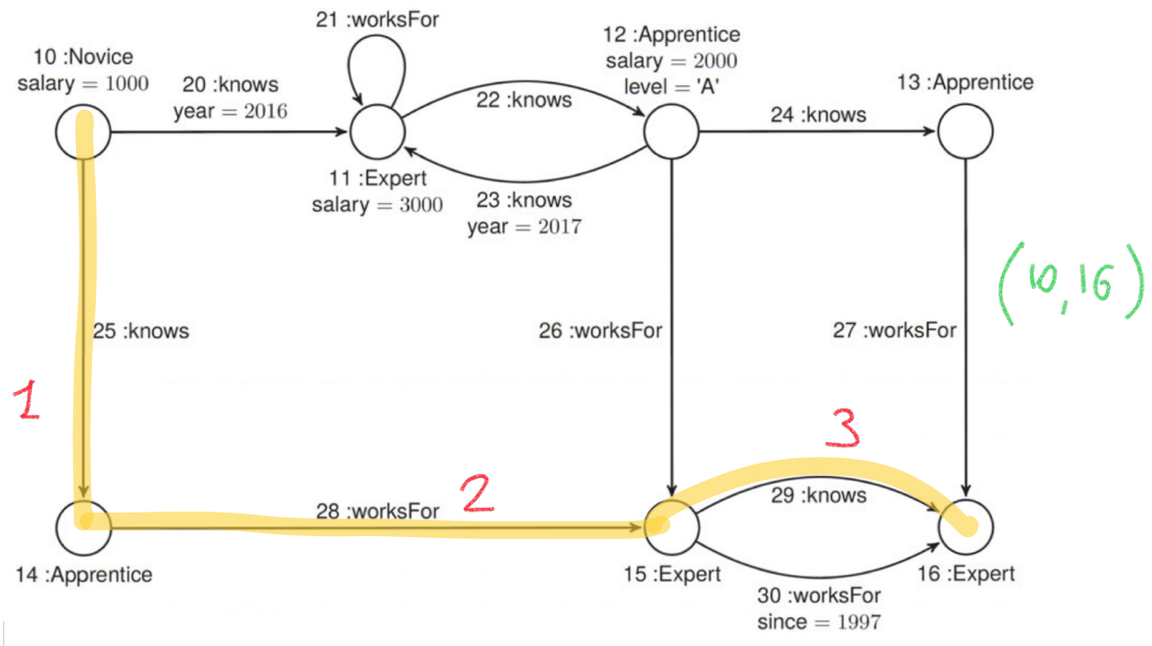

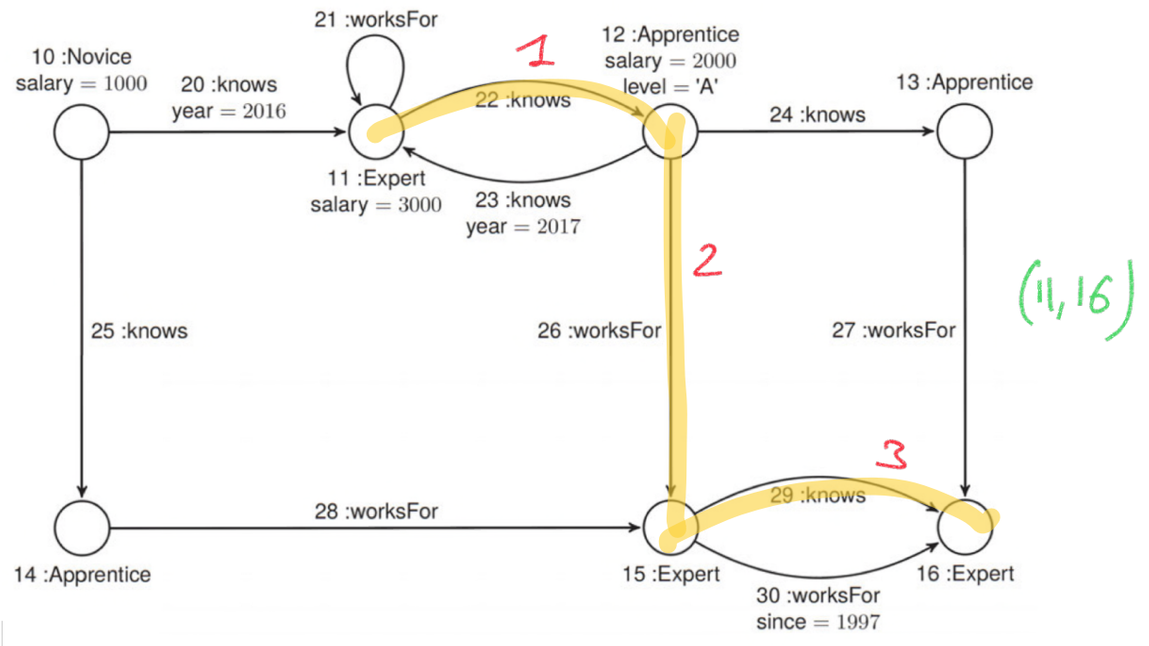

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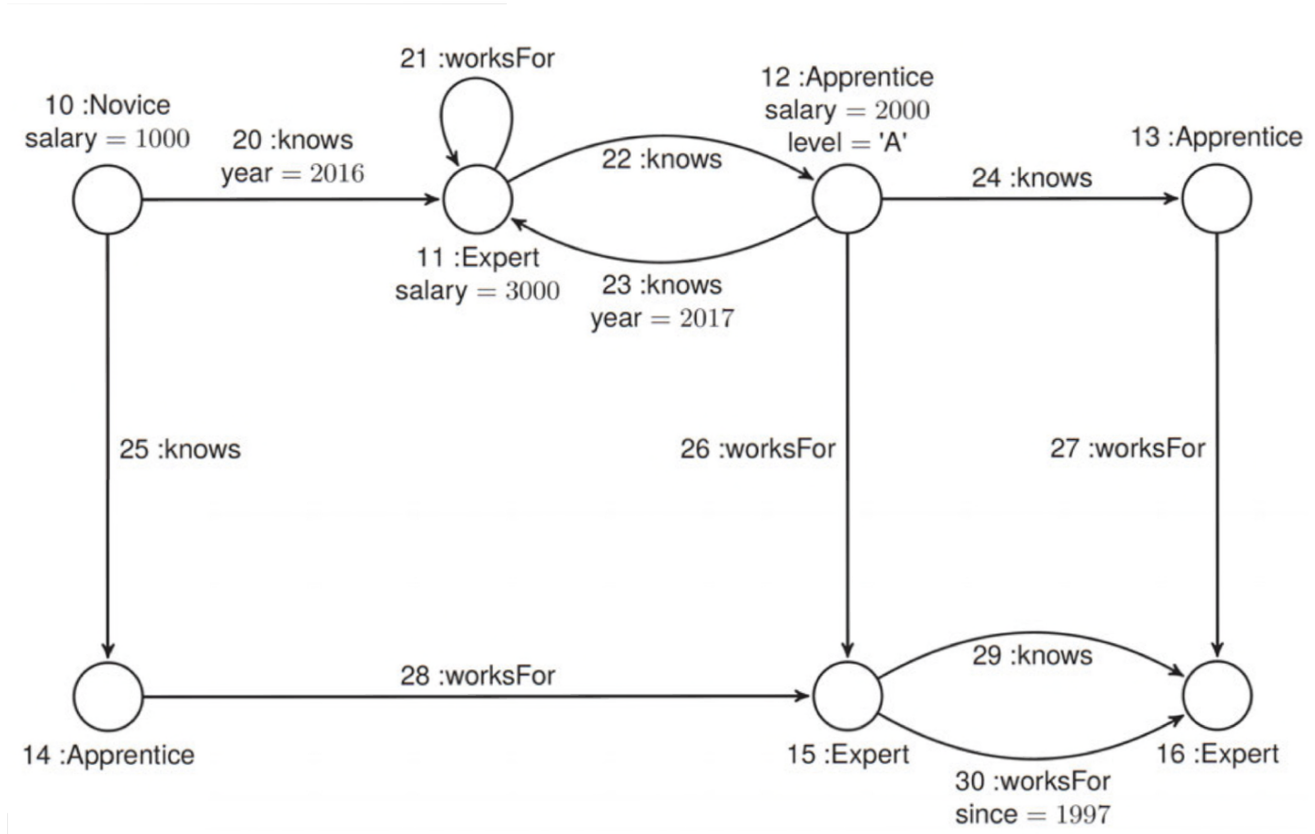

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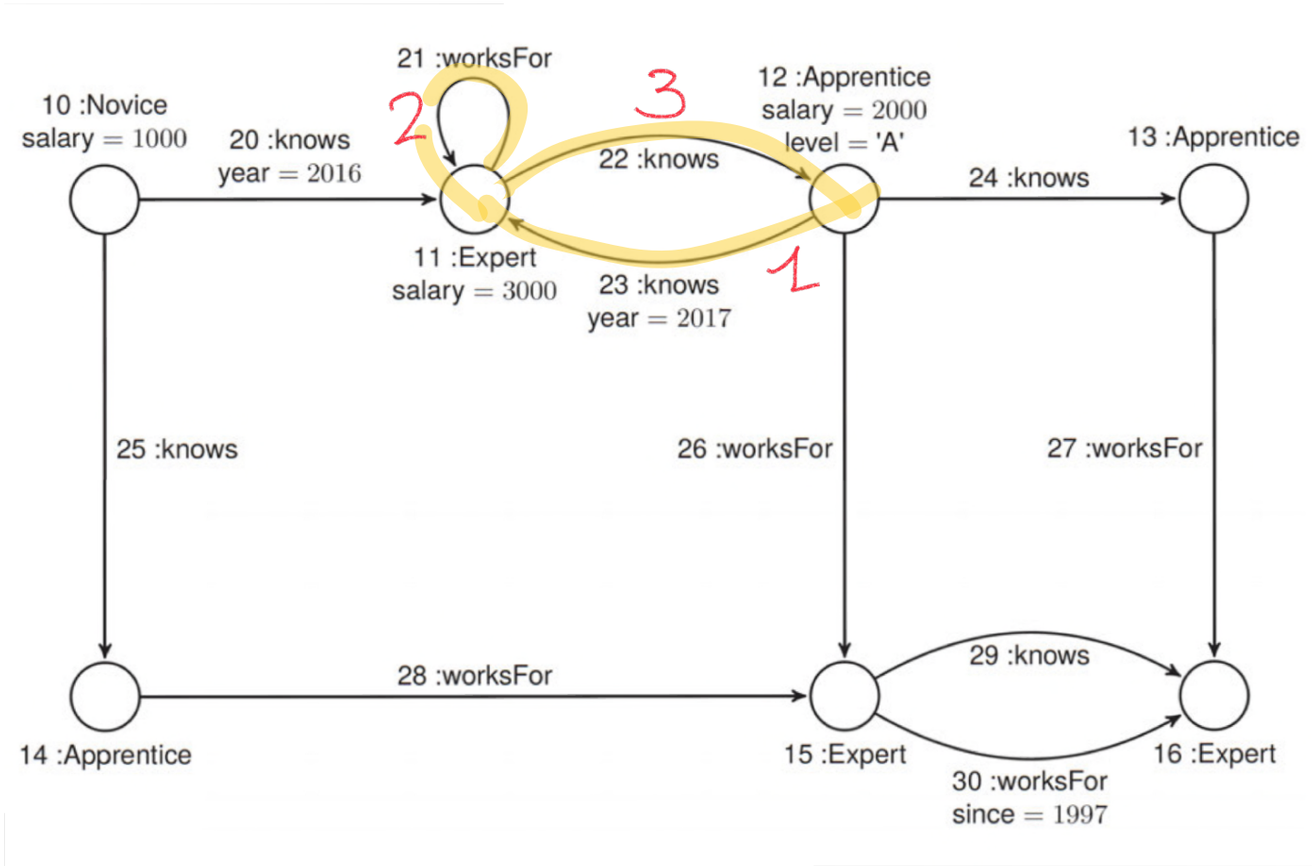

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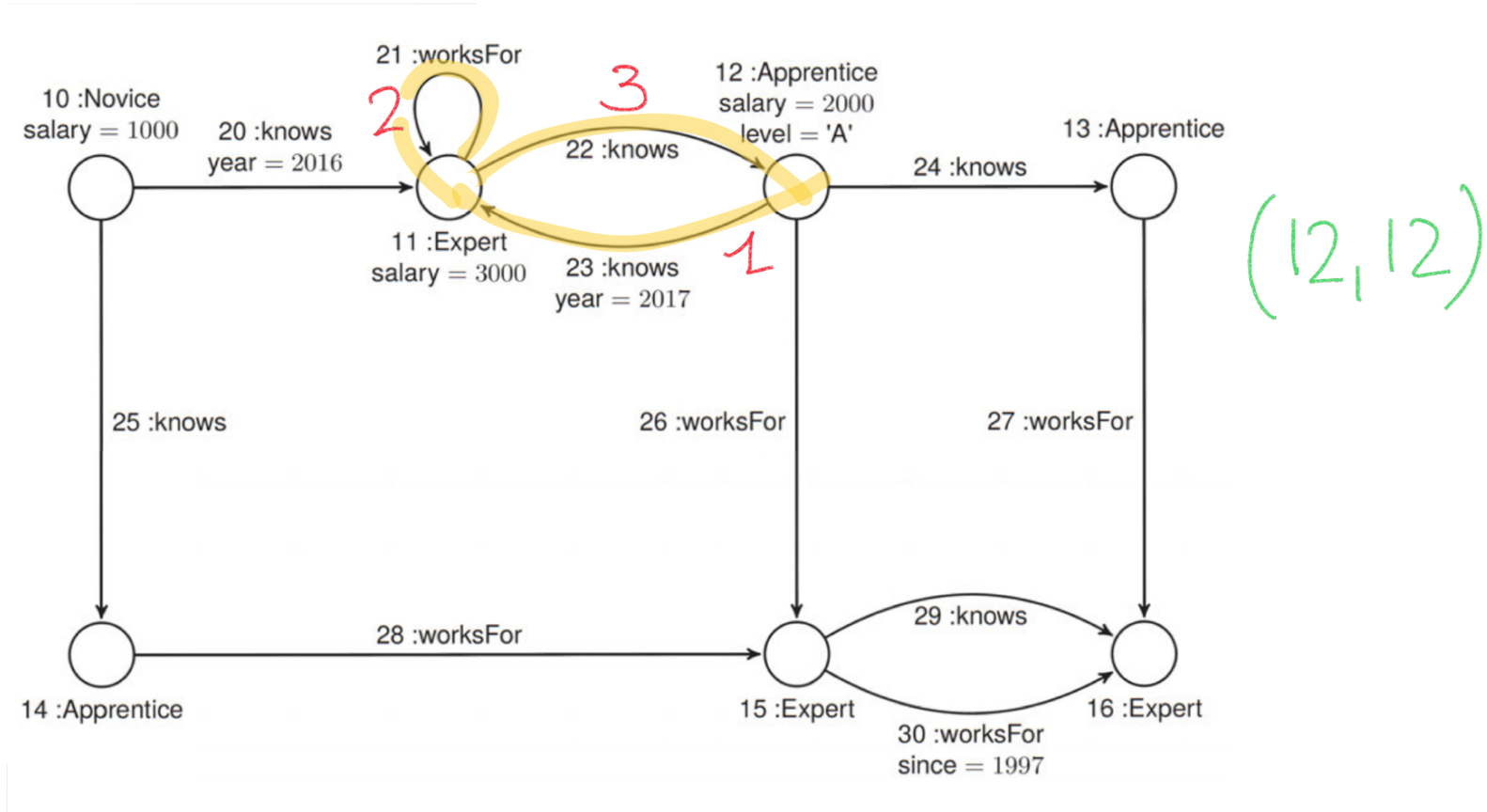

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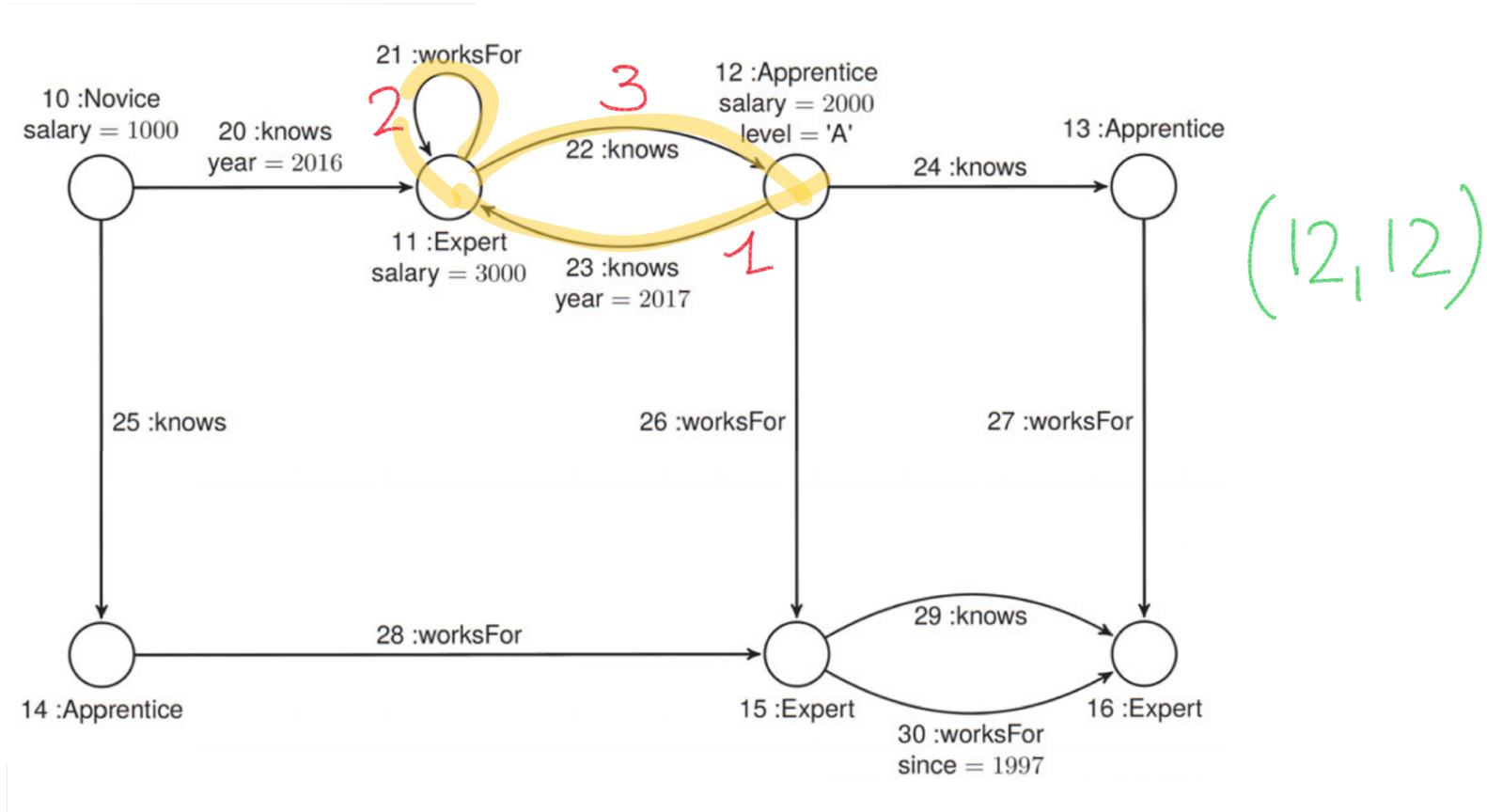

(12,12)

Conjunctive graph queries (CGQ)

CGQs identify sub structures in a graph

Let $\mathcal{V}$ be a set of vertex variables, a conjunctive query is defined as

$$\underbrace{(z_1, \dots, z_m)}_{\text{set of nodes}} \leftarrow \underbrace{a_1}_{\text{edge label}} \underbrace{(x_1, y_1)}_{\text{vertex variables}}, \dots, a_n\ (x_n, y_n)$$

where

- $m \geq 0, n \geq 0$
- $x_1, y_1, \dots, x_n, y_n \in \mathcal{V}$
- $a_1, \dots, a_n \in \mathcal{L}$
- for each $0 \leq i \leq m$, it holds that $z_i \in \{x_1, y_1, \dots, x_n, y_n\}$
  where m is the arity of the expression

Conjunctive graph queries

Given $r = (z_1, \dots, z_m) \leftarrow a_1(x_1, y_1), \dots, a_n(x_n, y_n)$

A mapping for r on G is a function $\mu : V \to V$ s.t.
for each $1 \leq i \leq n$, $\exists\ e_i \in E$ where

$$\eta(e_i) = (\mu(x_i), \mu(y_i)) \wedge a_i \in \lambda(e_i)$$

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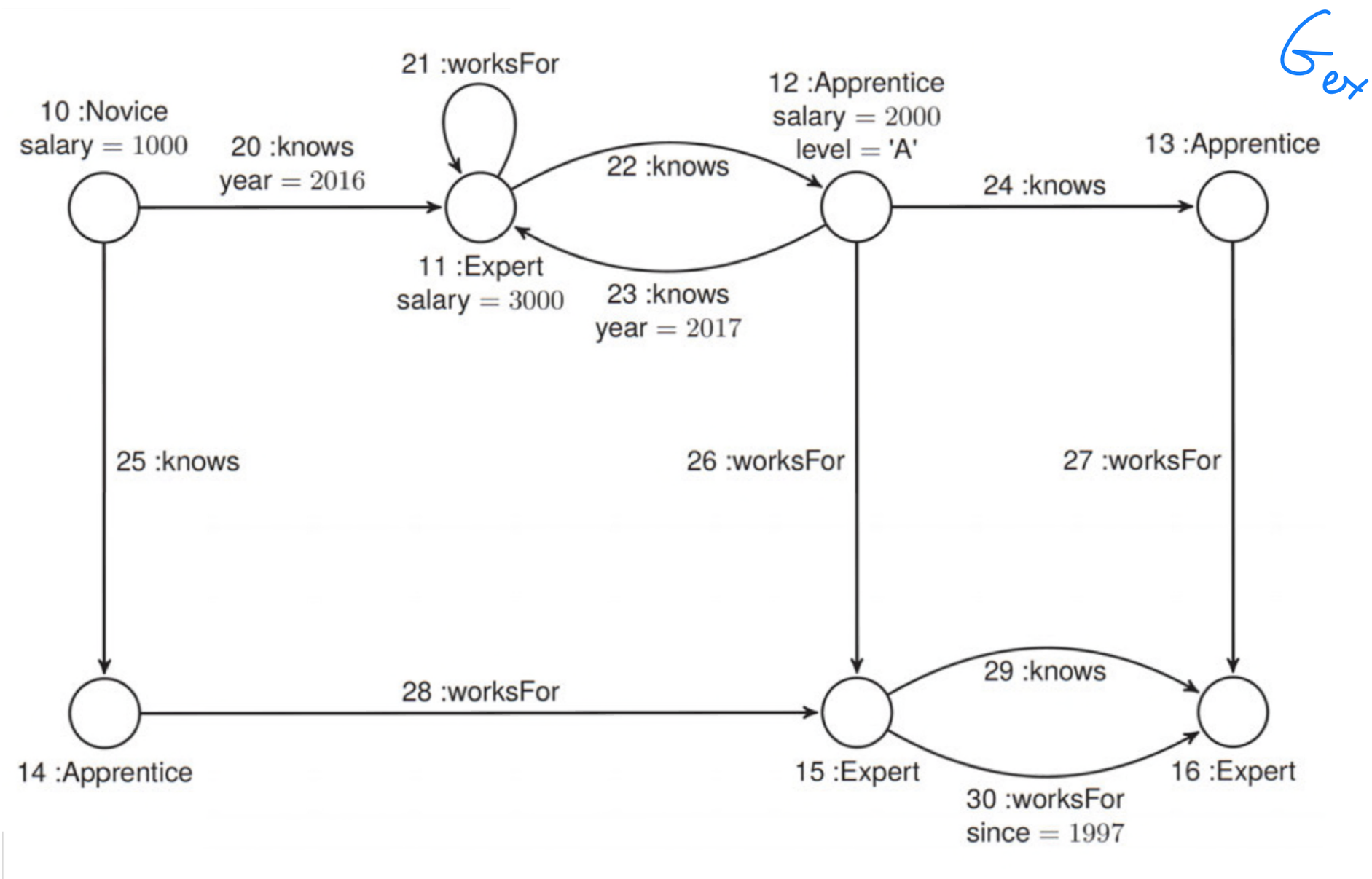
The evaluation of r on G is

$$[\![r]\!]_G = \{ \mu(z_1), \dots, \mu(z_m) \mid \mu \text{ is a mapping for } r \text{ on } G \}$$

Example 1

$q_1 = (a, b) \leftarrow :knows(a, b)$



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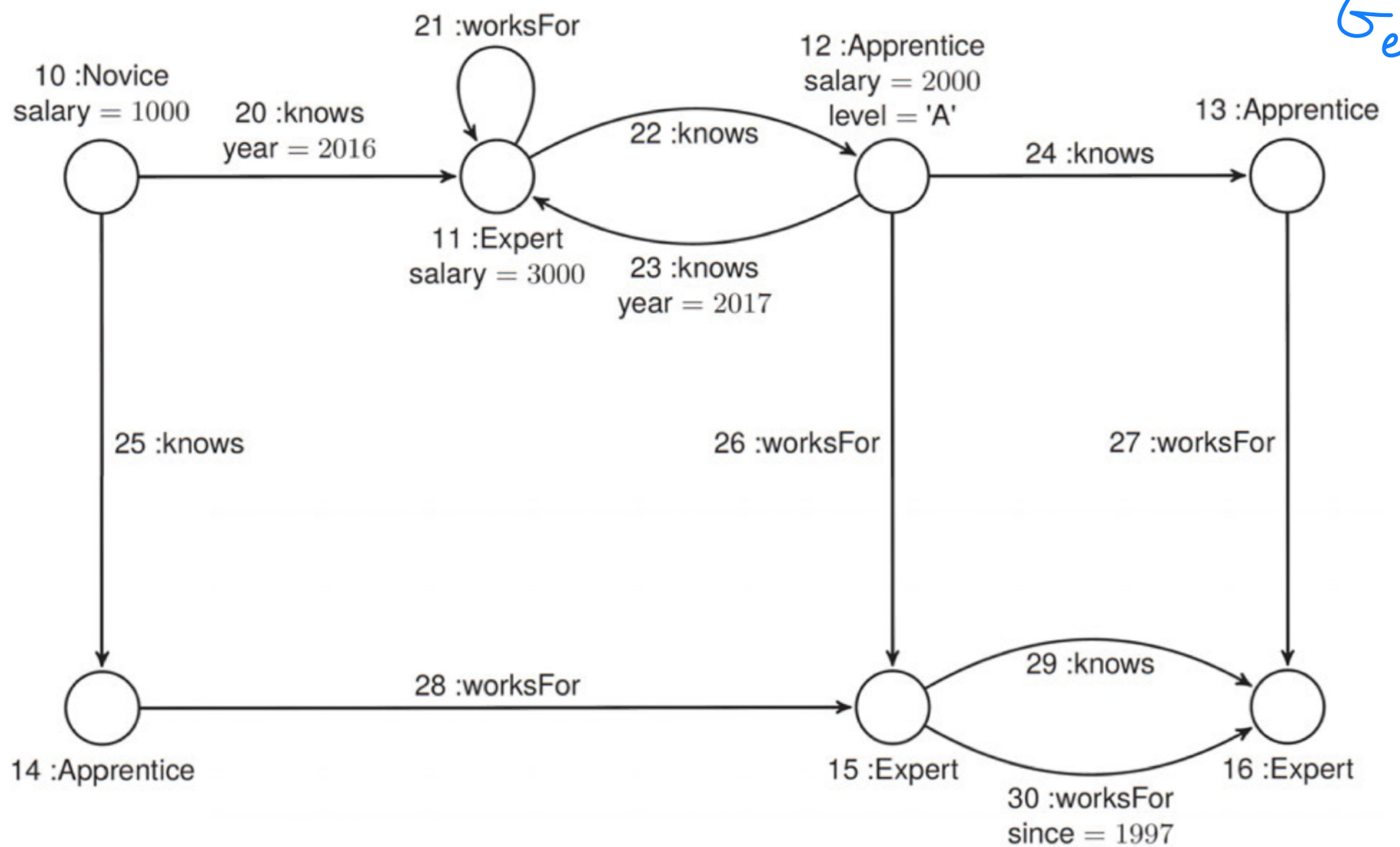
$q_1 = (a, b) \longleftarrow :knows(a, b)$

We need to find a function μ mapping a and b to $V_{ex} = \{10, 11, 12, \ldots, 16\}$

G_{ex}



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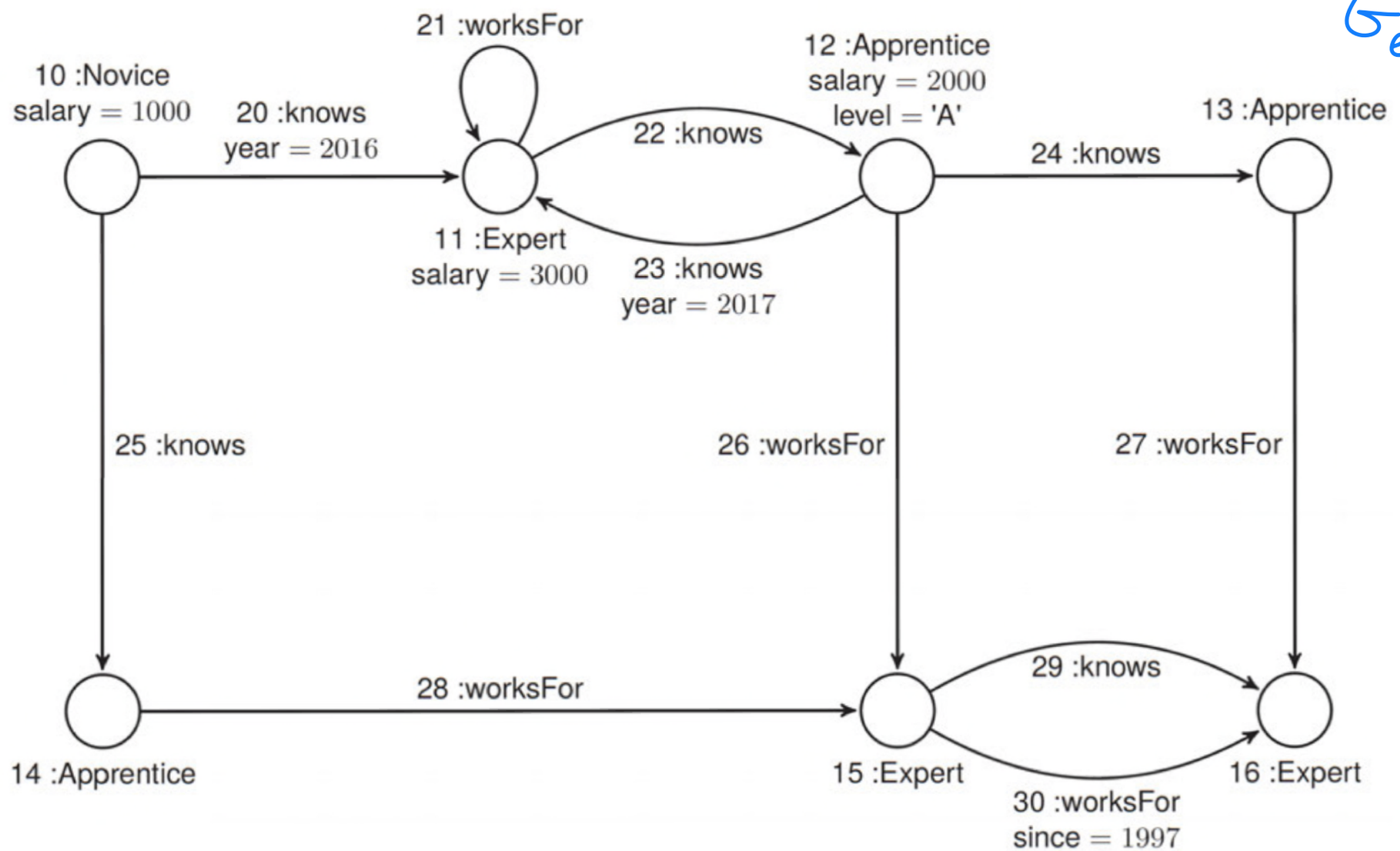

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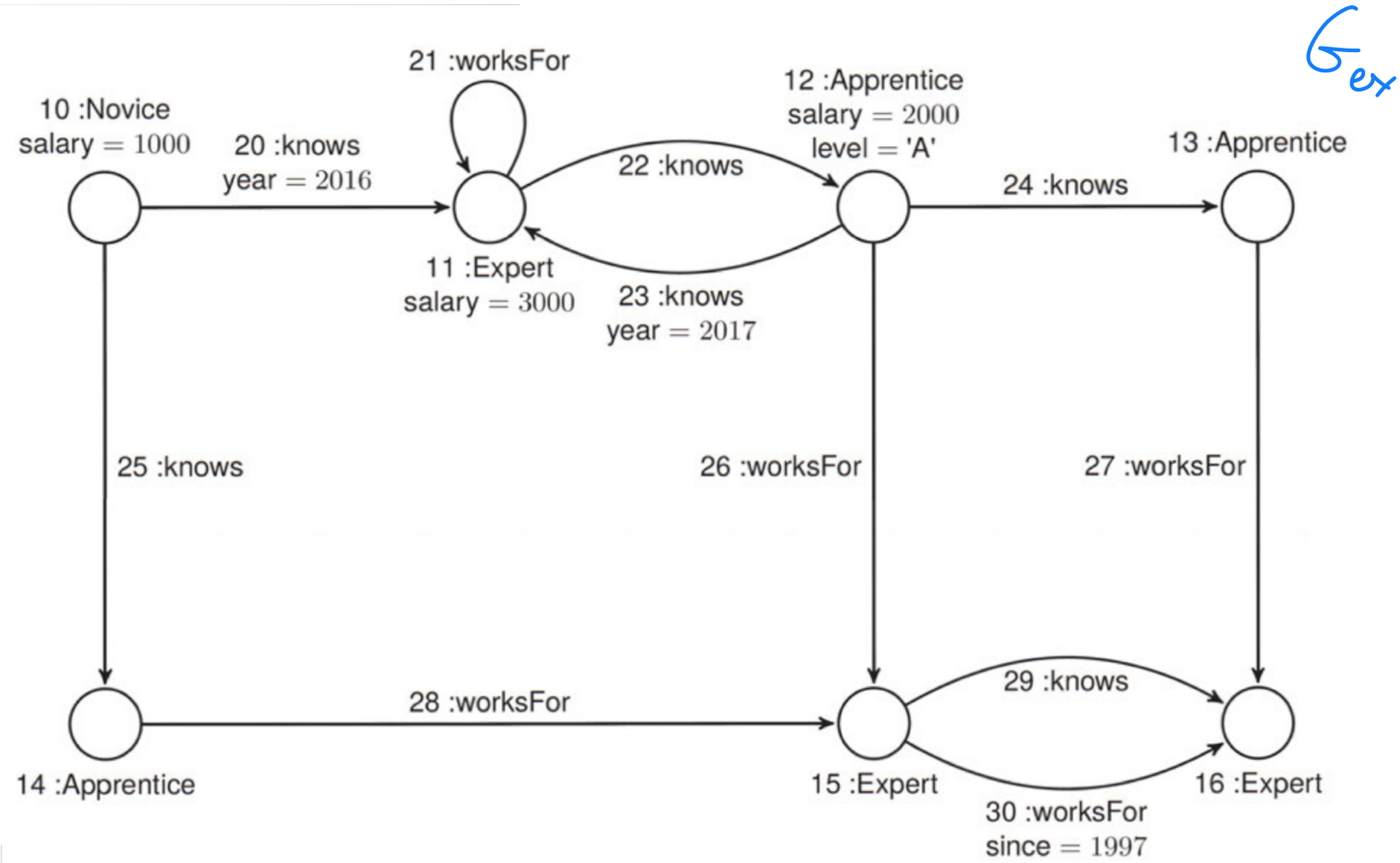

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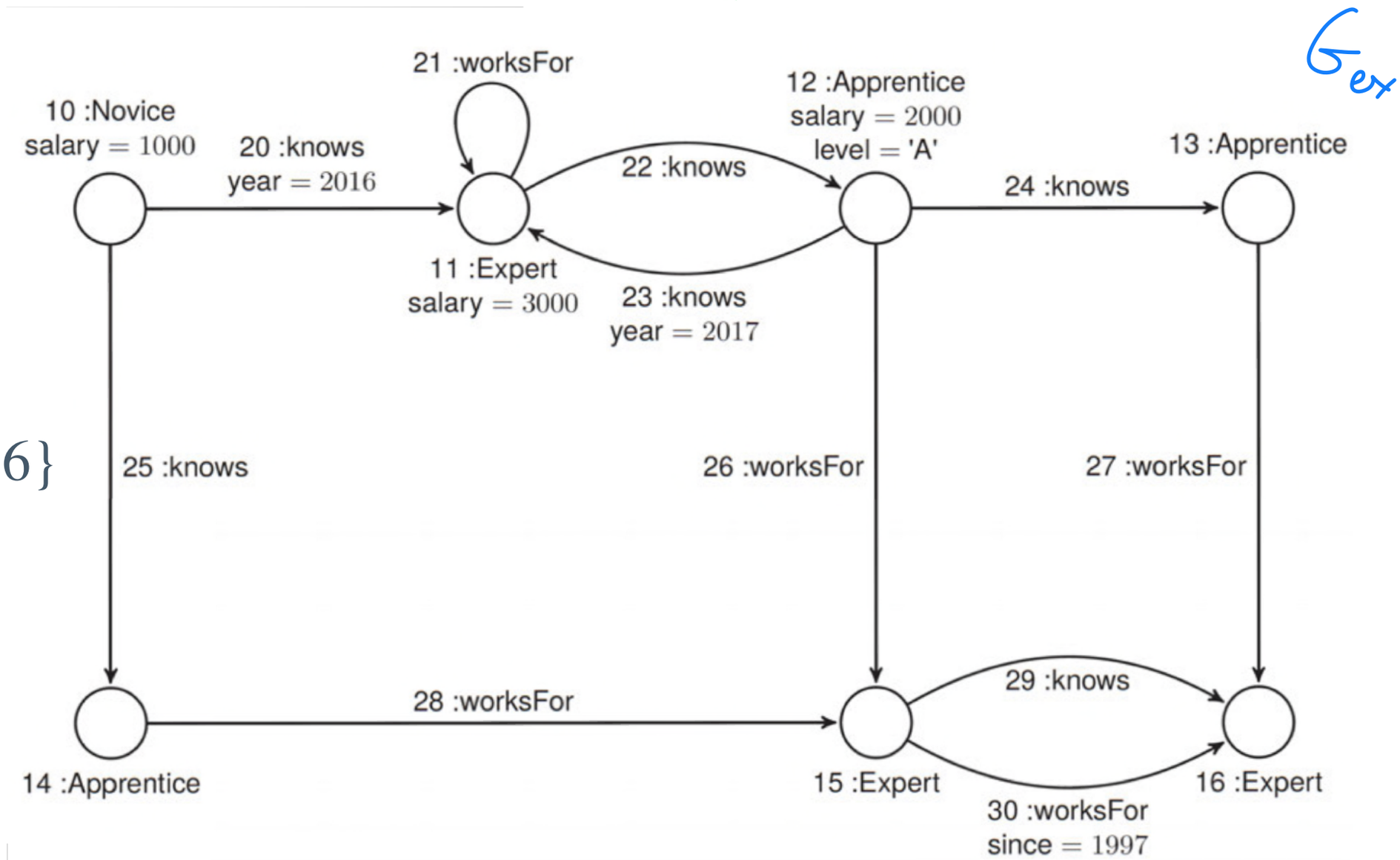

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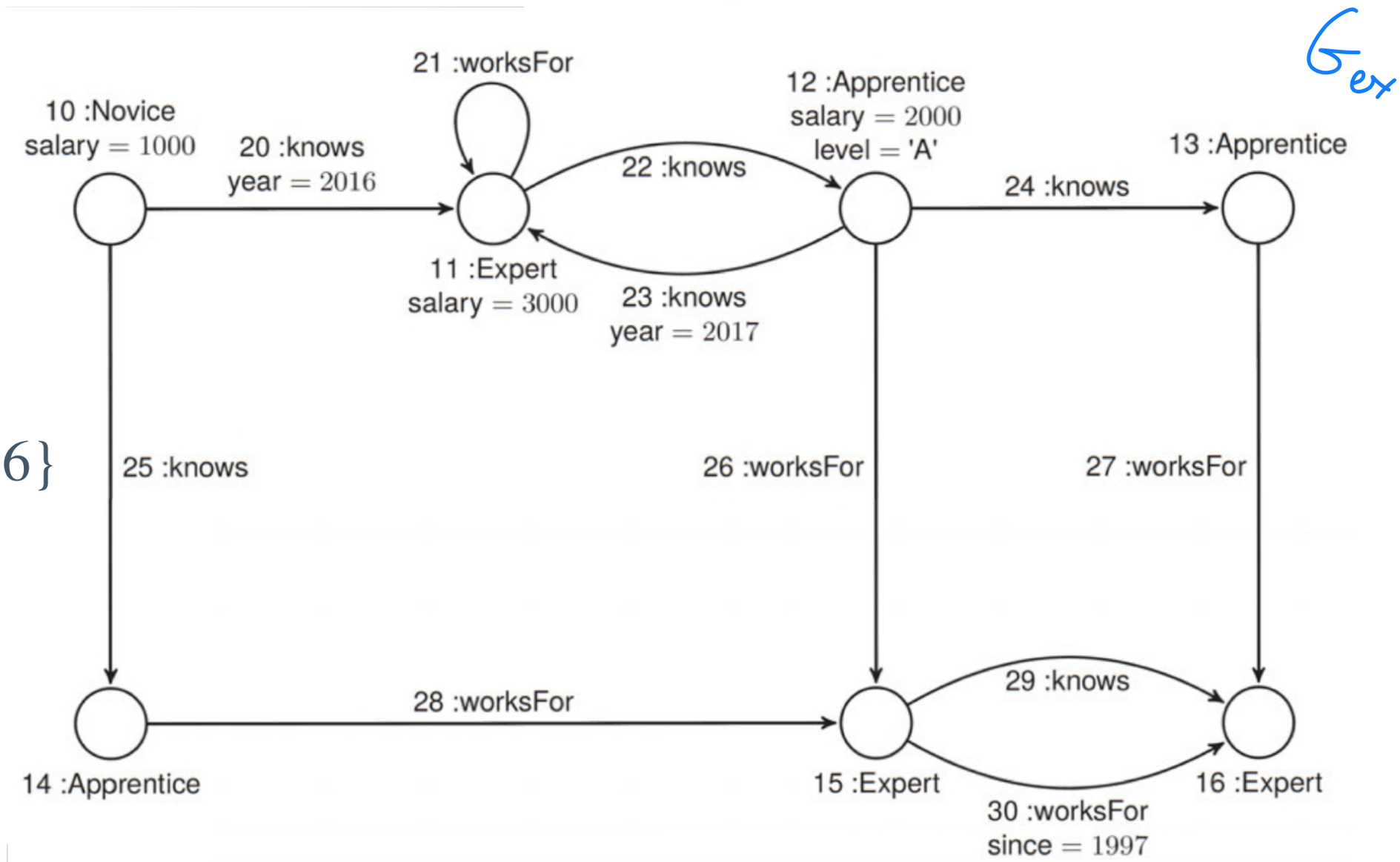
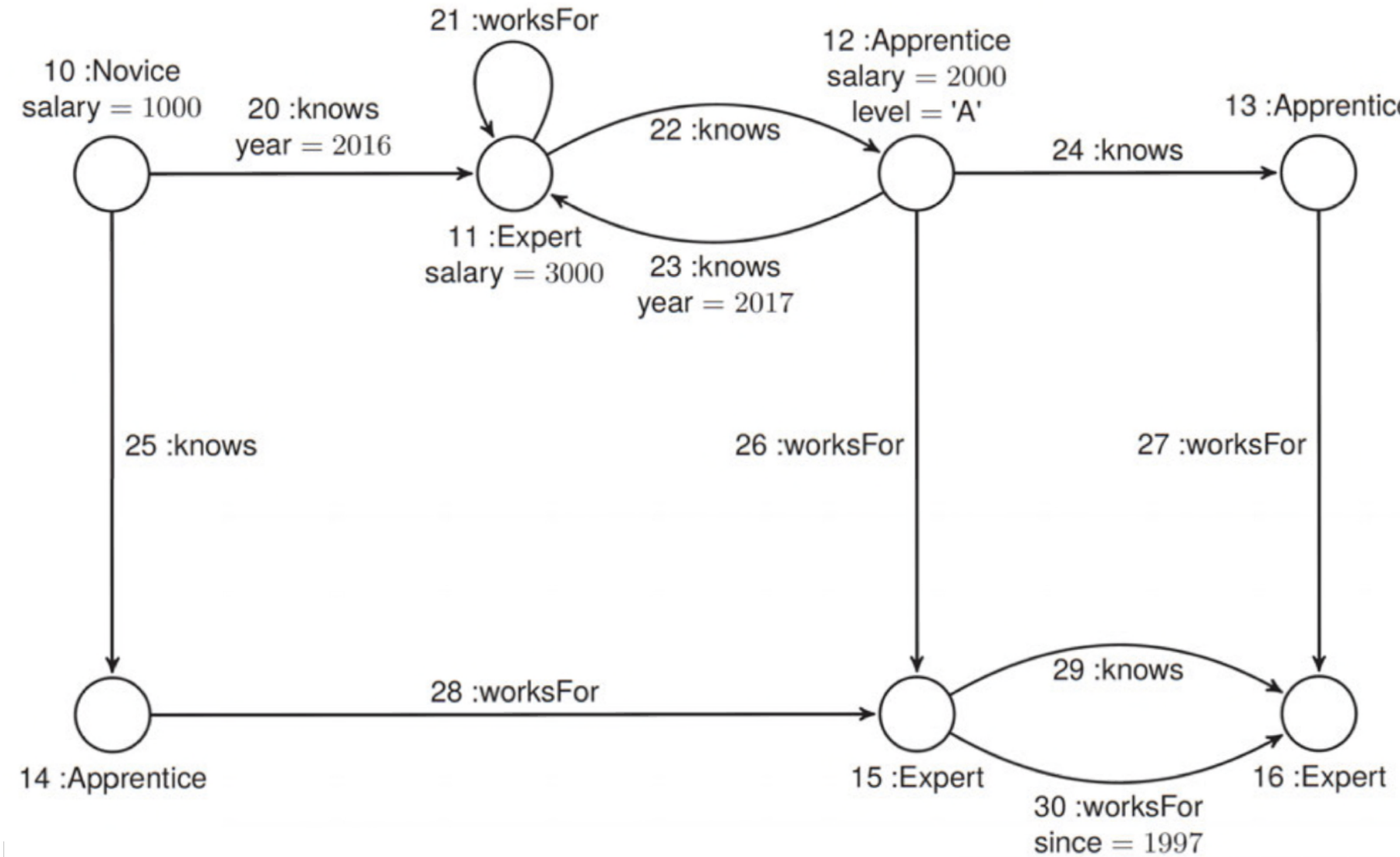


$[\![q_1]\!]_{G_{ex}} = (10,11), (11,12), (12,11), (12,13), (10,14), (15,16)$

Example 2

$$q_2 = (a_1, e_2) \leftarrow \text{:knows}(a_1, a_2), \text{:worksFor}(a_1, e_1), \text{:knows}(e_1, e_2), \text{:worksFor}(a_2, e_2)$$



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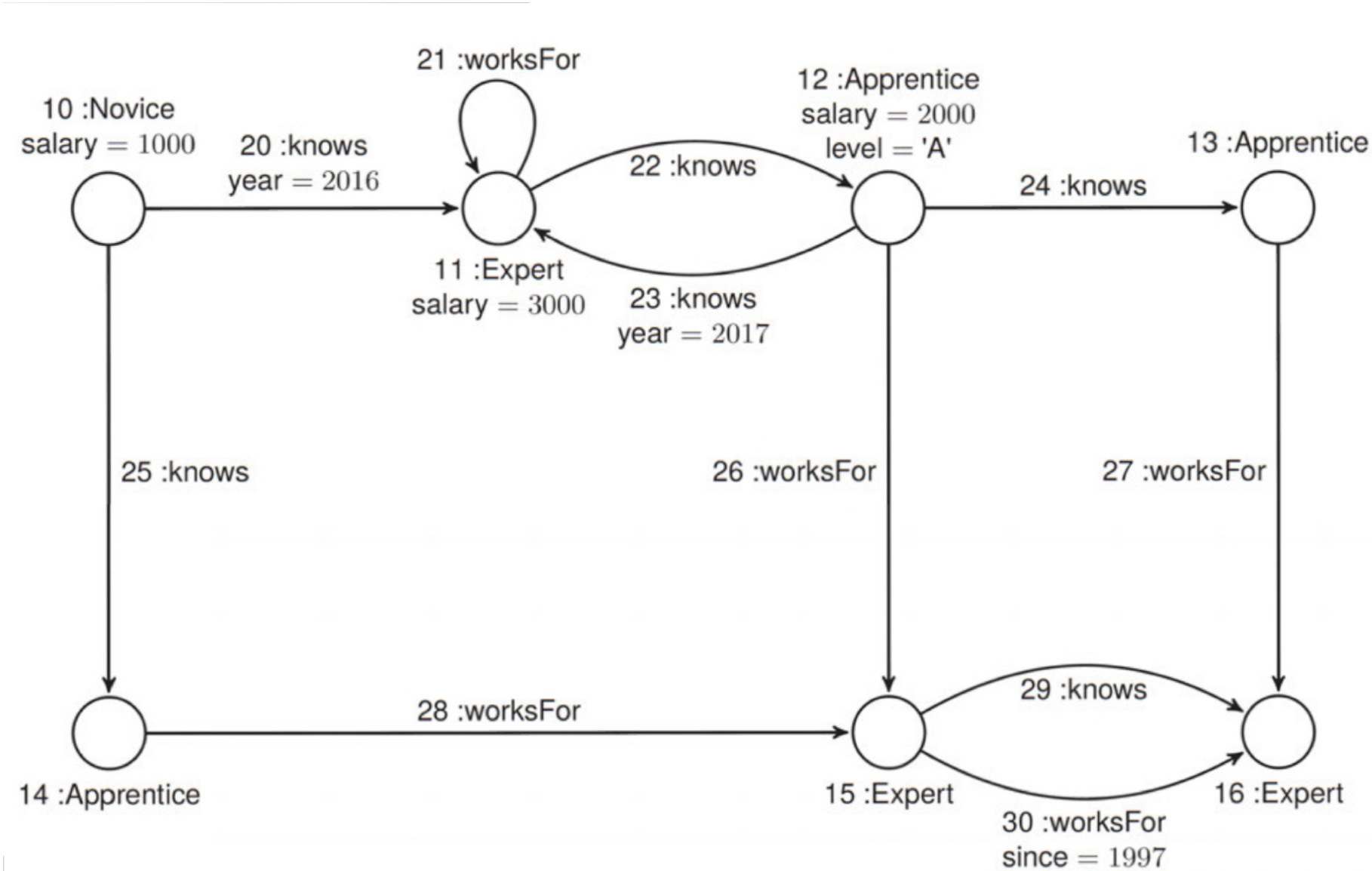
Check all source vertices for one edge labeled “knows” and one edge labeled “worksFor”



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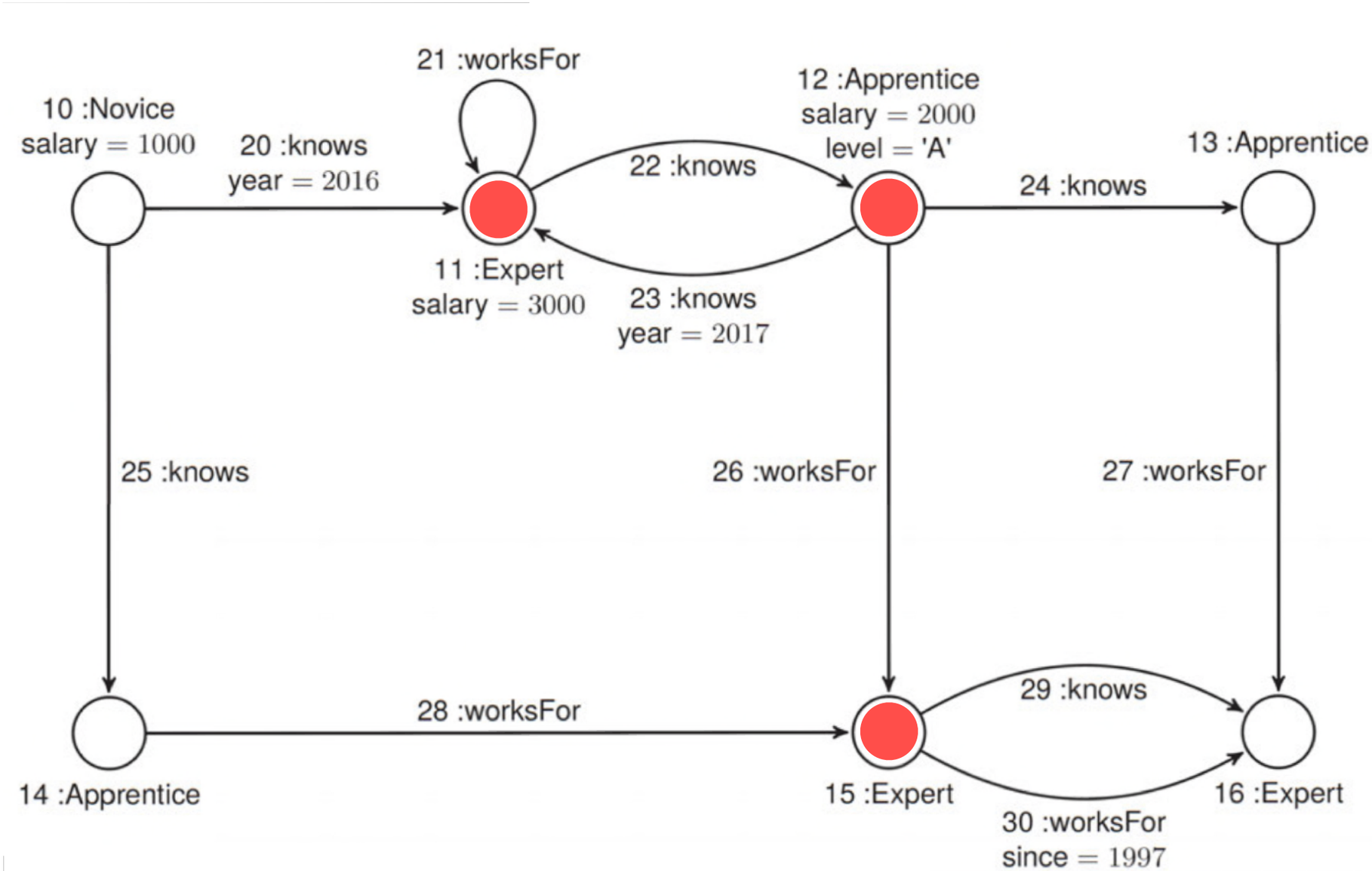

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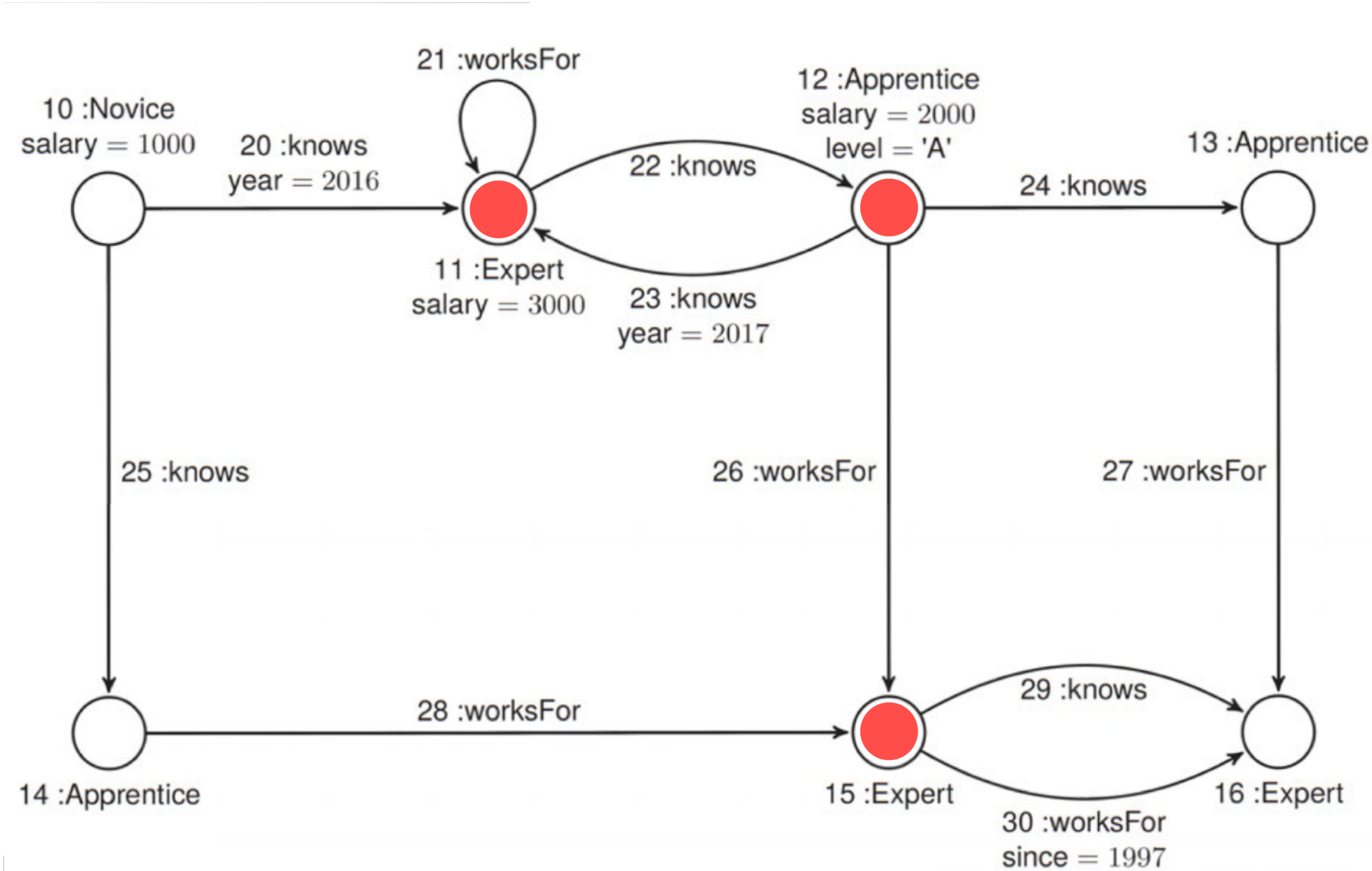
Check all source vertices for one edge labeled “knows” and one edge labeled “worksFor”

Check all target vertices for an edge labeled “worksFor” that are also source vertices for an edge labeled “knows”



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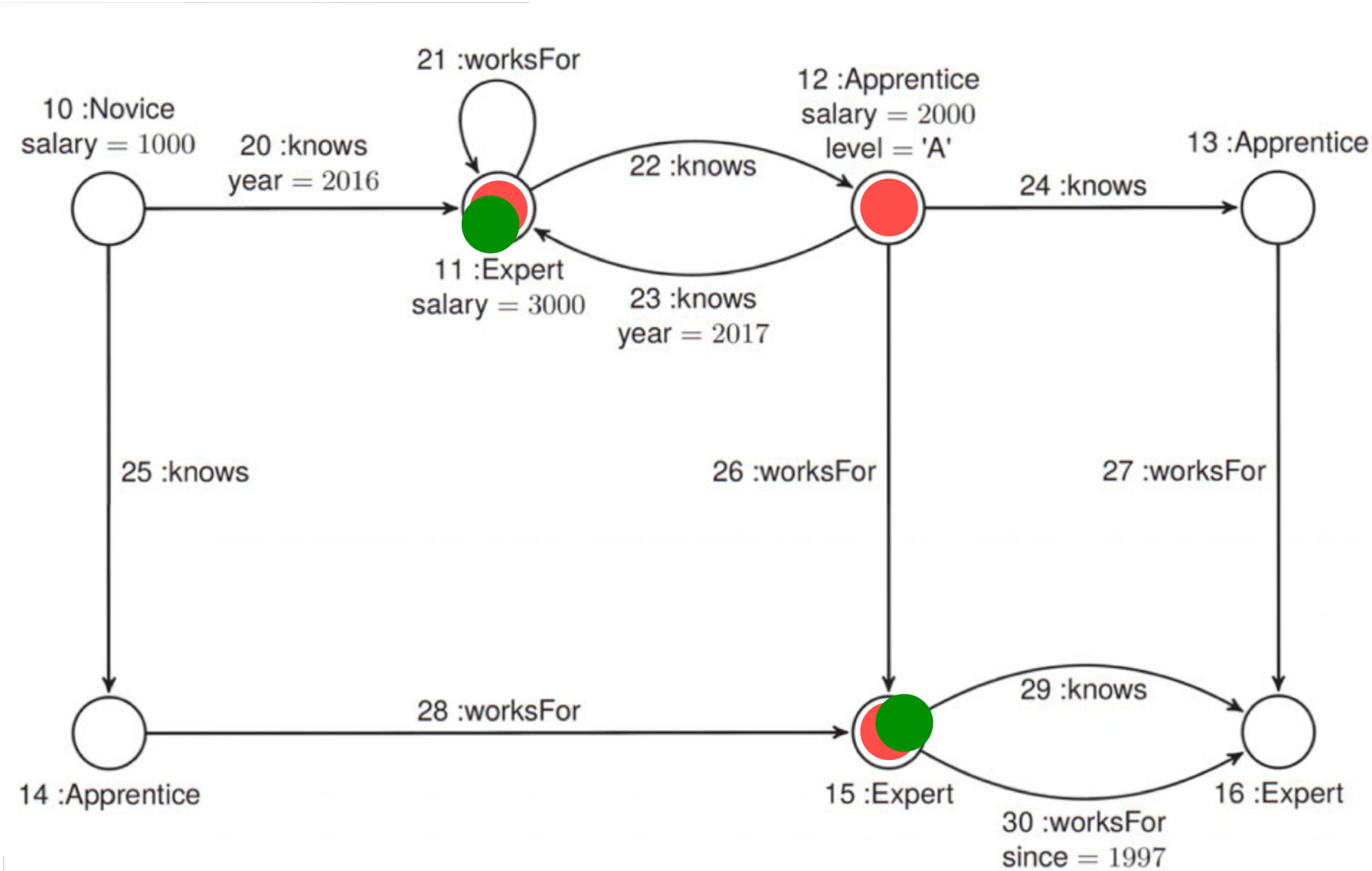

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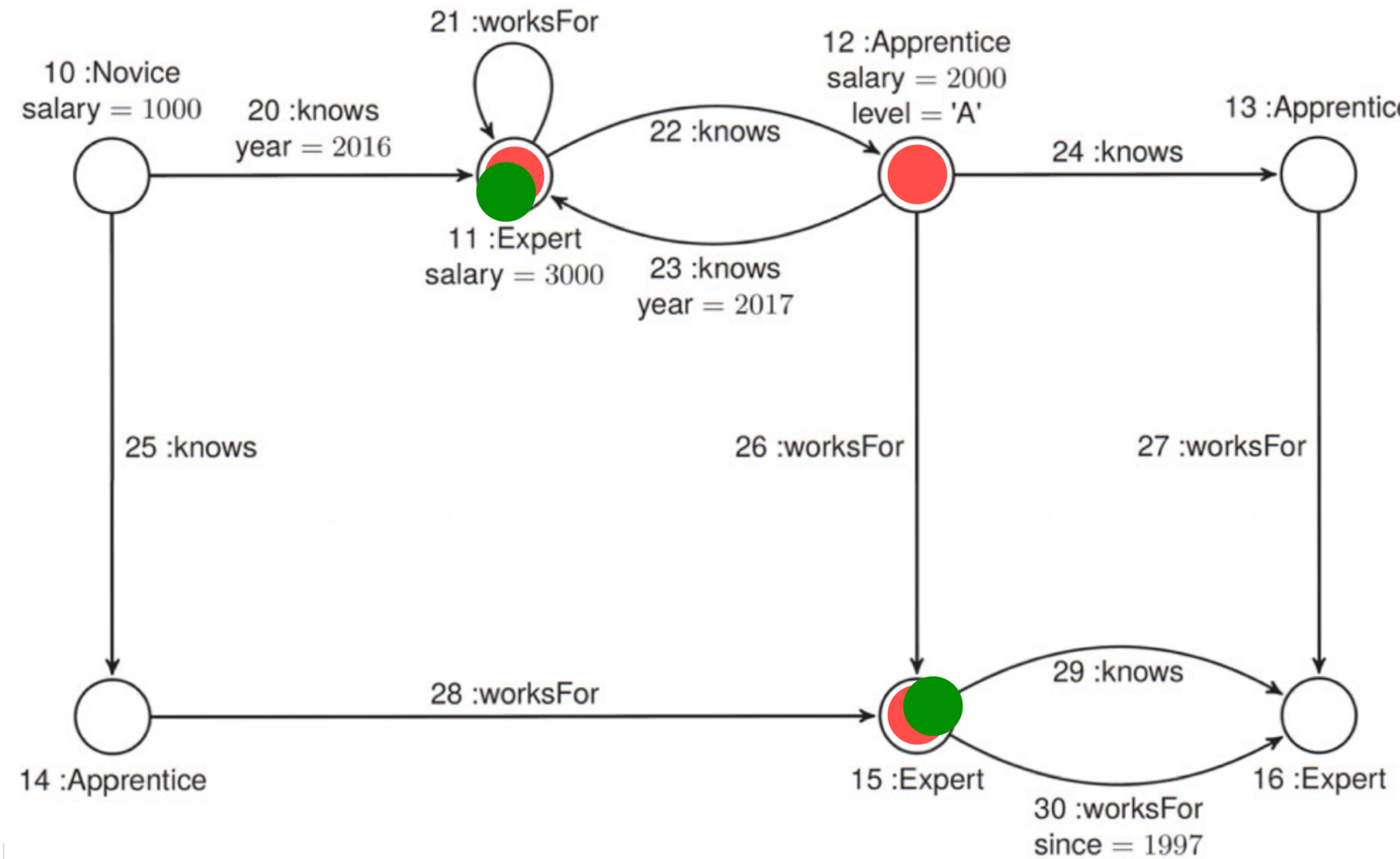

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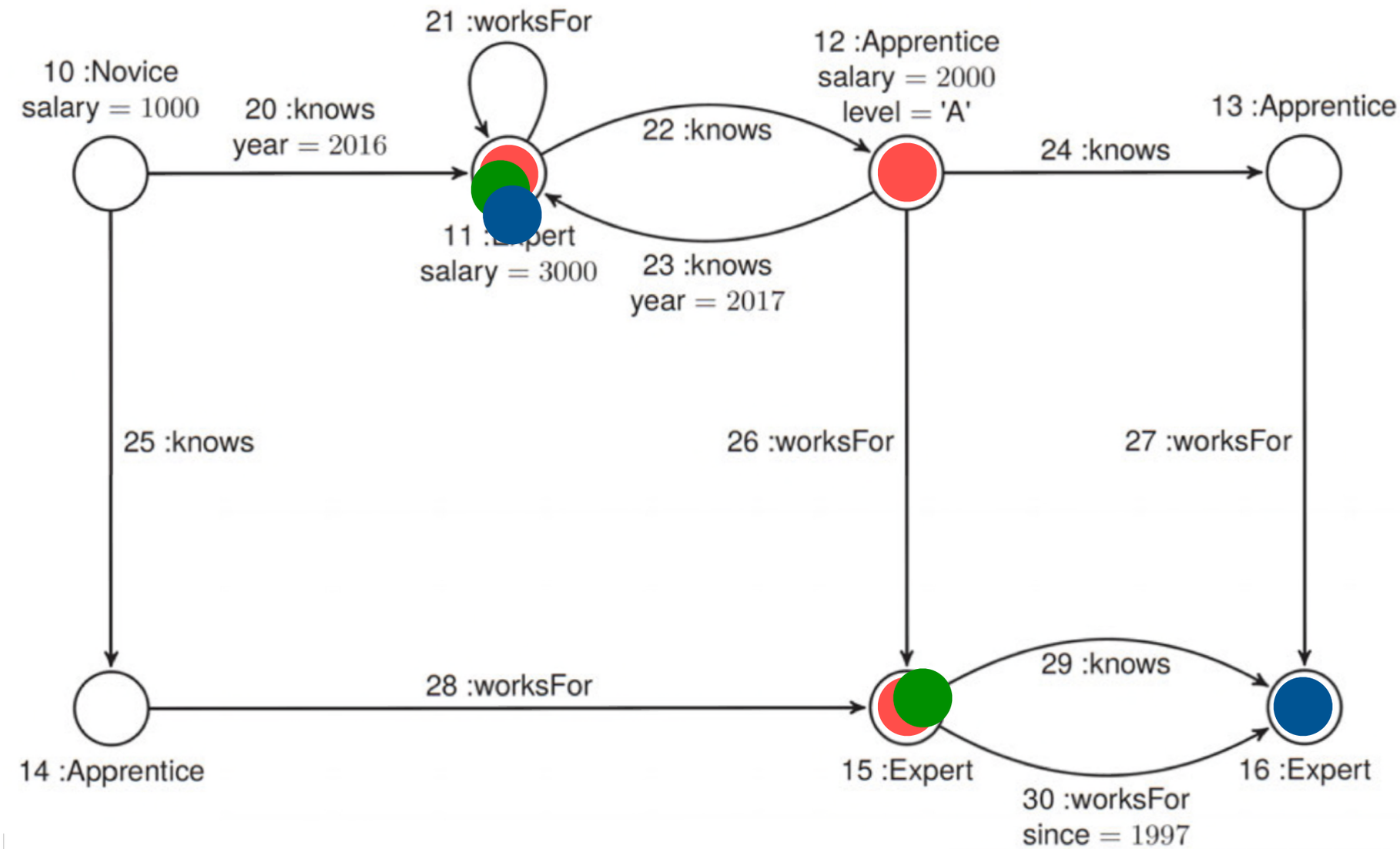

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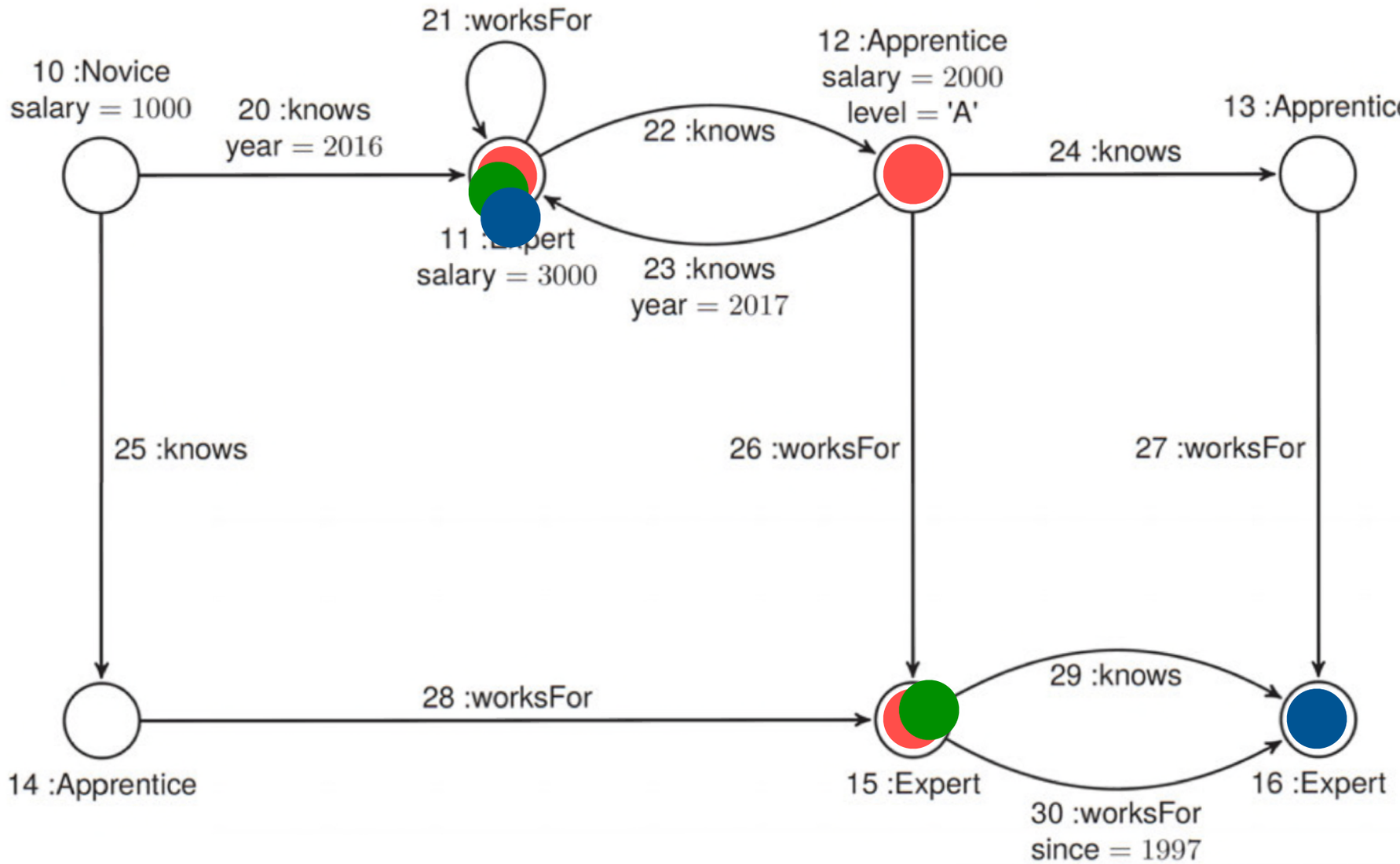

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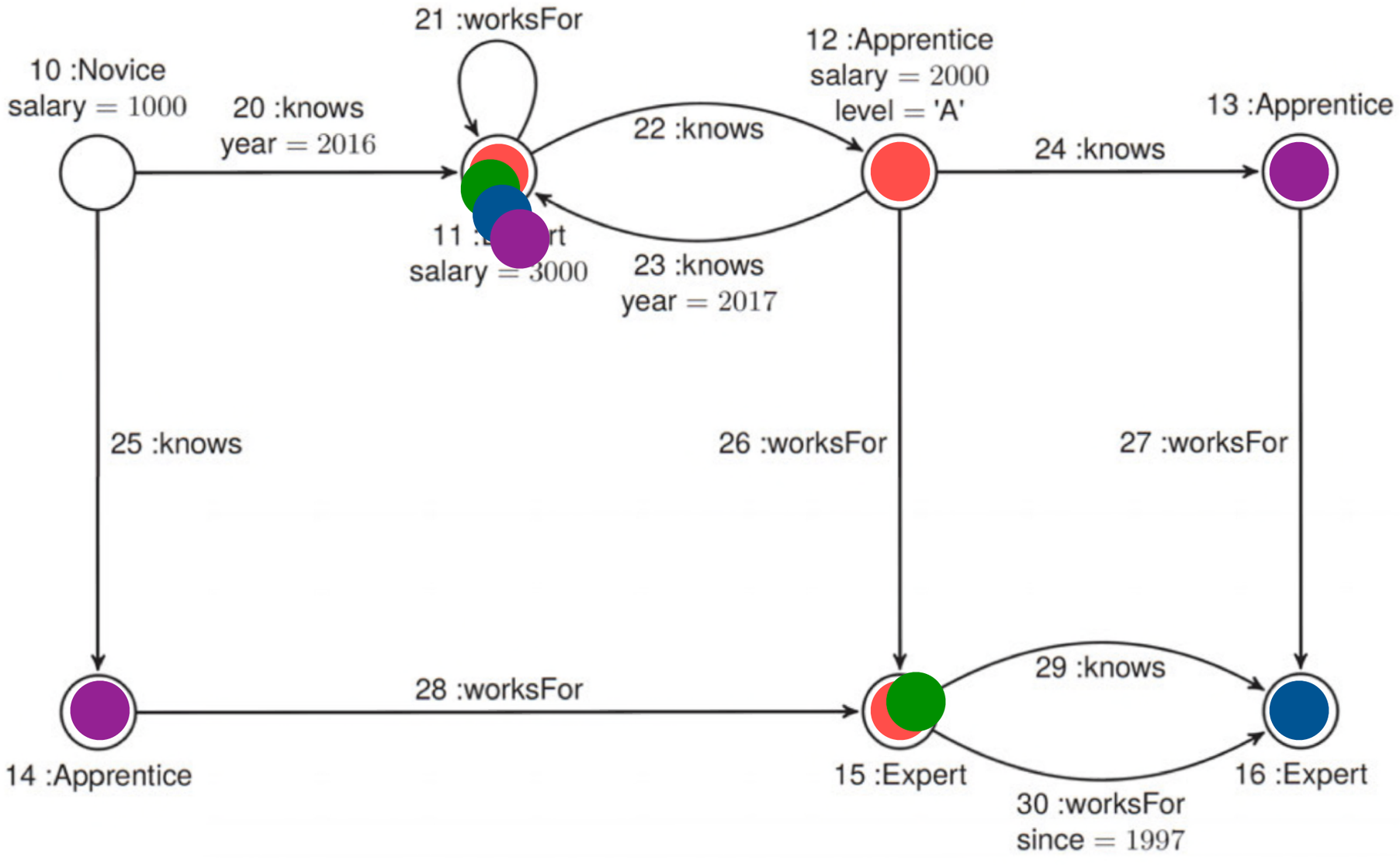

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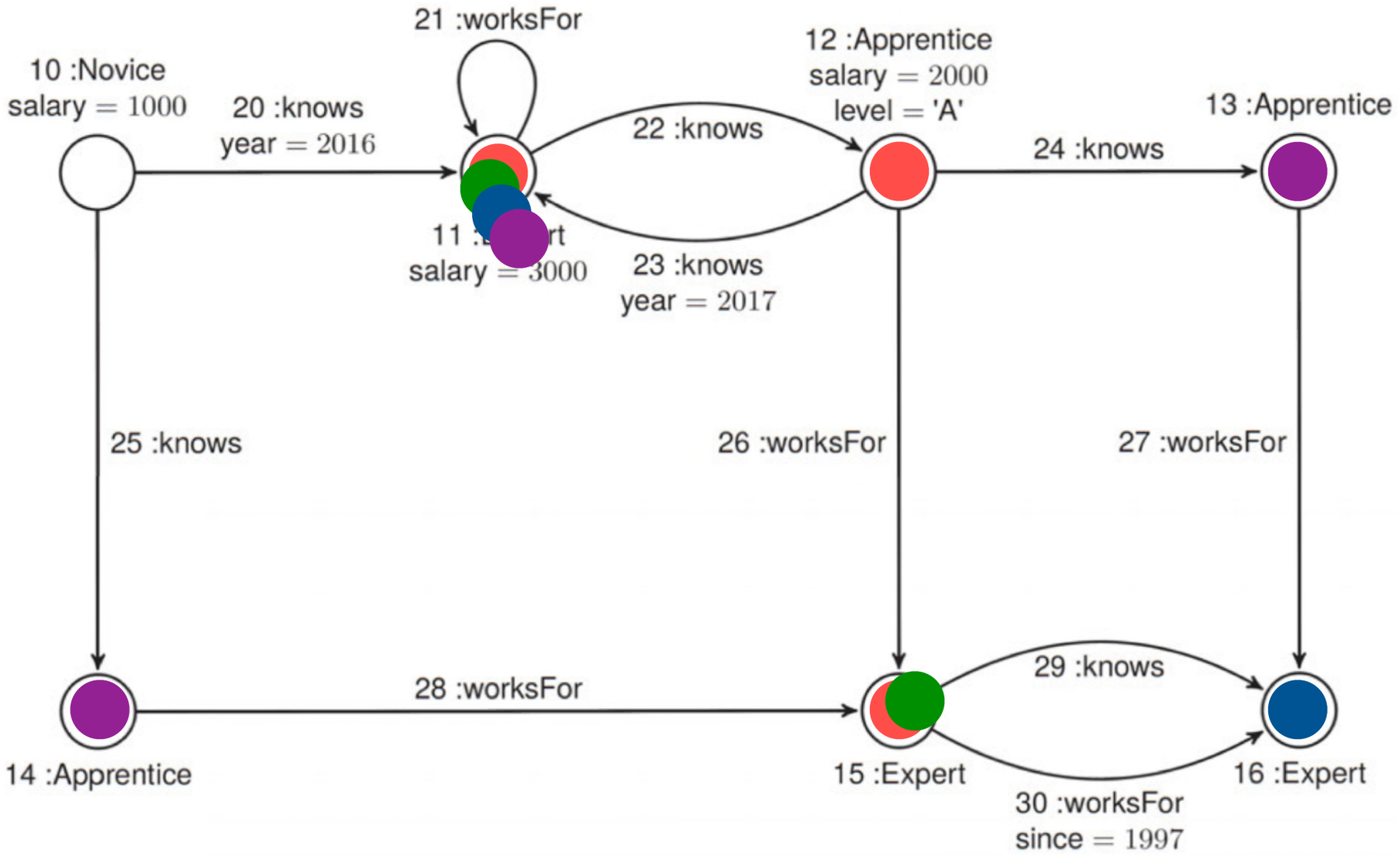

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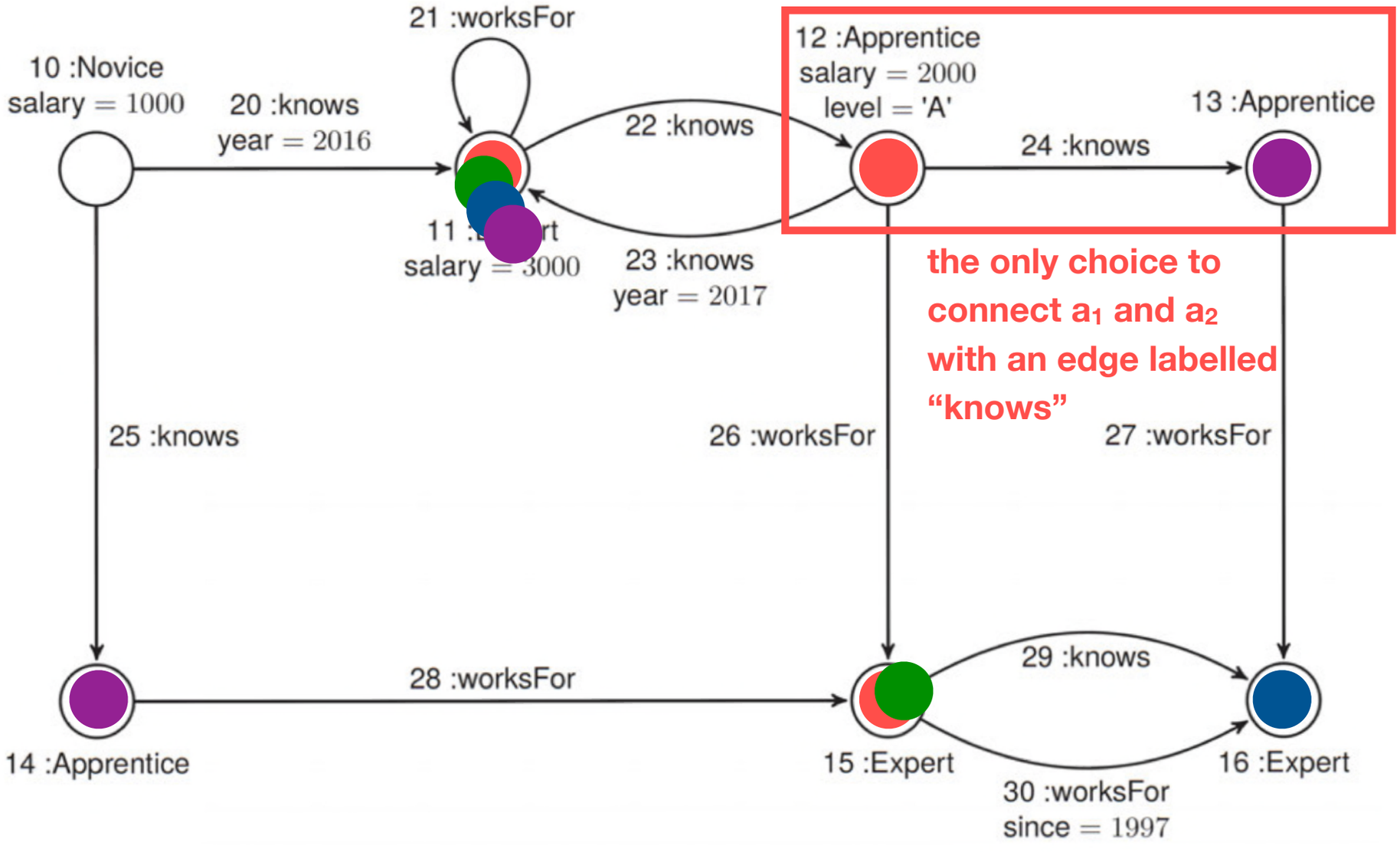

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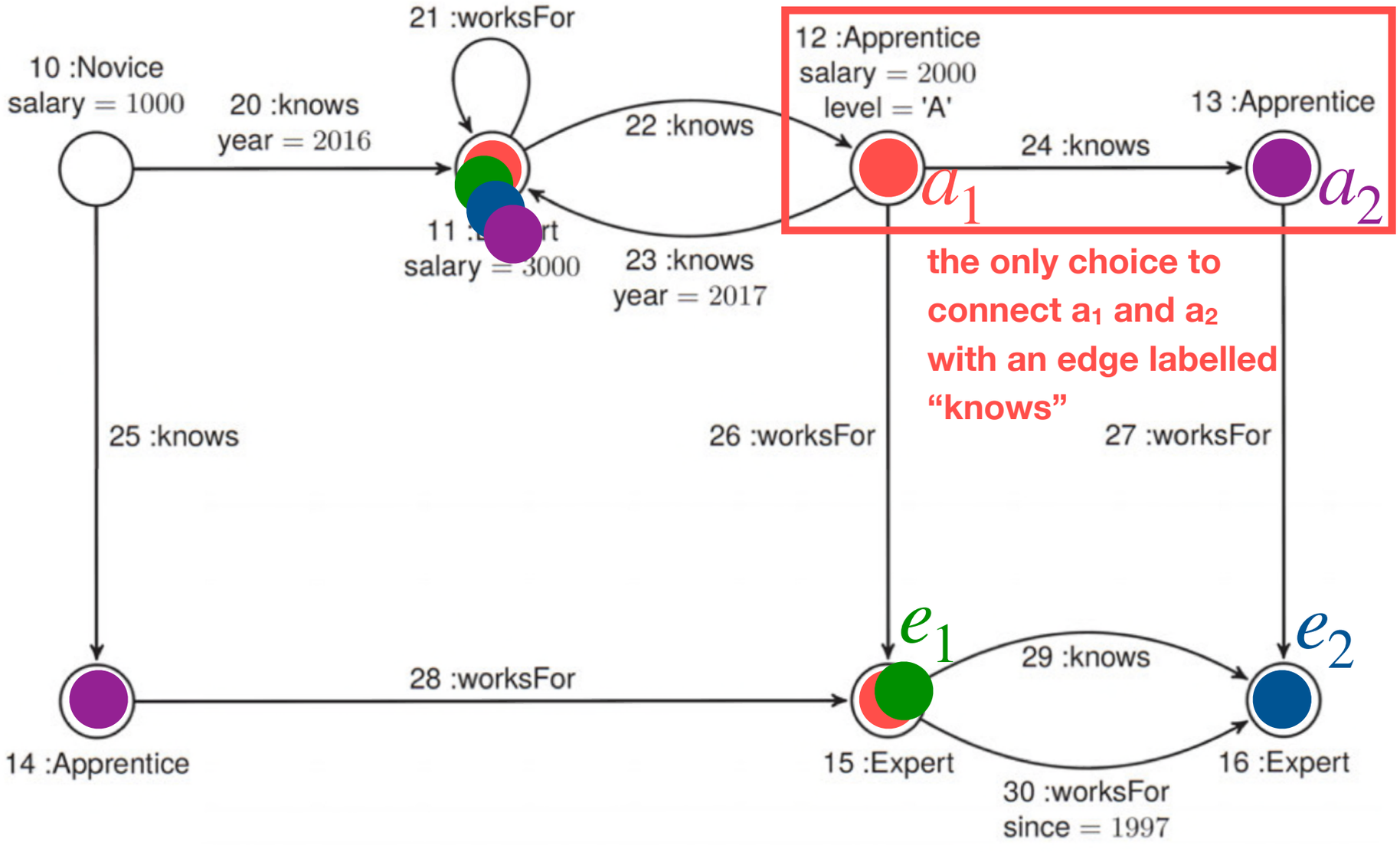

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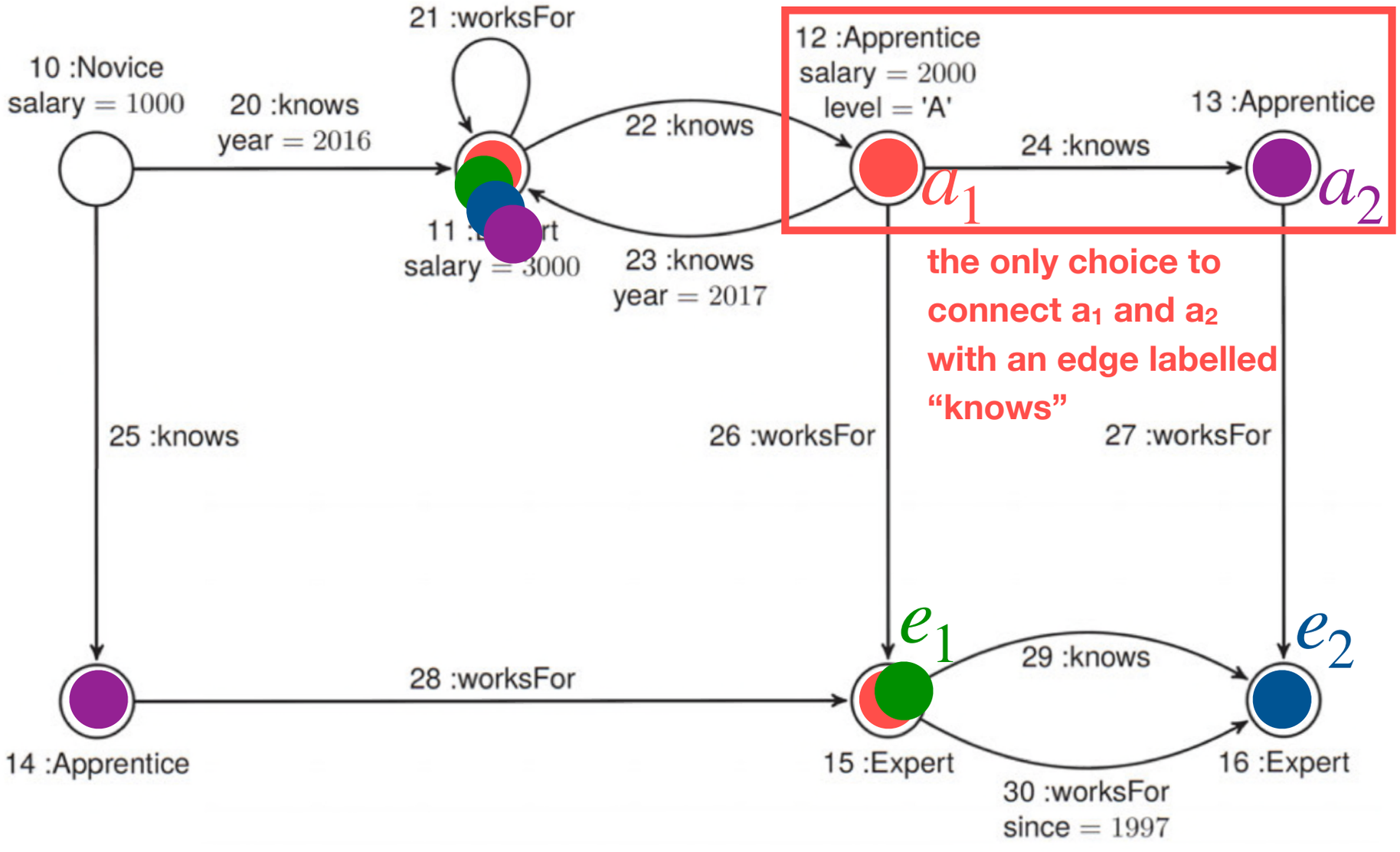

Solution

$$[\![q_2]\!]_{G_{ex}} = (12,16)$$

Conjunctive Regular Path Queries (CRPQ)

$(z_1, \ldots, z_m) \leftarrow \alpha_1(x_1, y_1), \ldots, \alpha_n(x_n, y_n)$, where $\alpha_1, \ldots, \alpha_n$ are RQ

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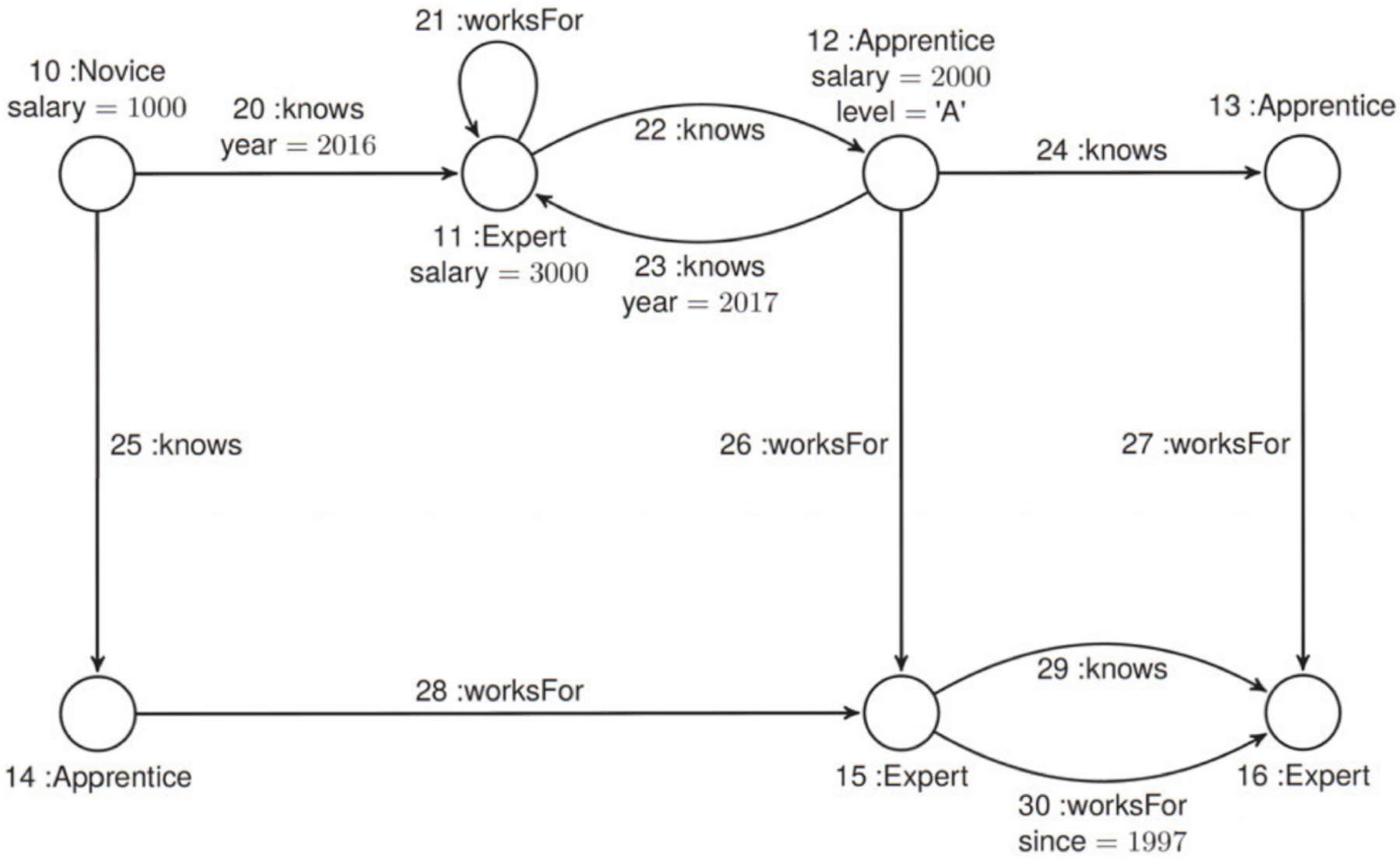
Example $q = (a, b) \leftarrow$:knows/:worksFor/:knows$^+(a, b)$



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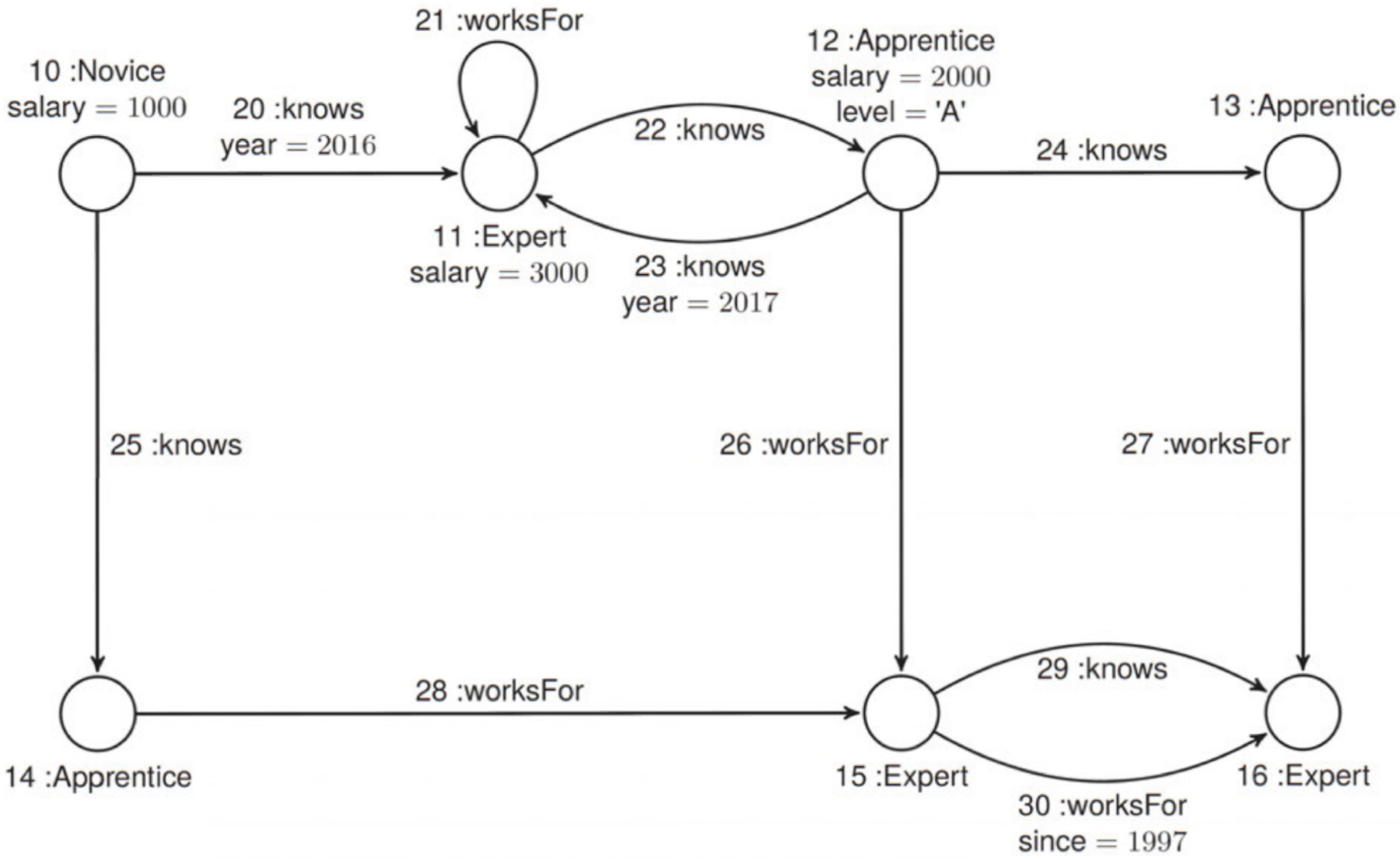
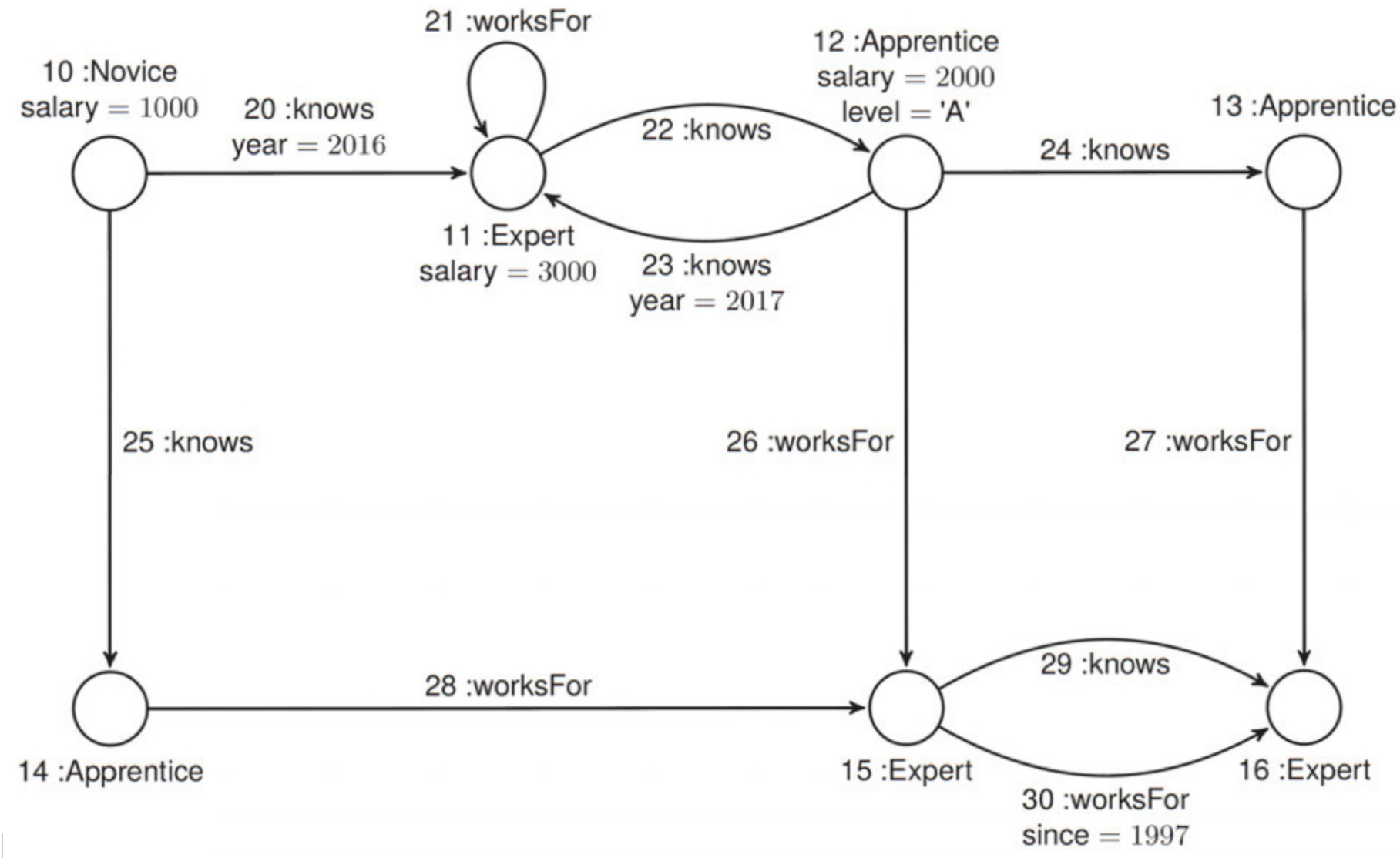


Solution: it is the same as Example 2 on Slide 9

Example 3

$$q = (n, e) \leftarrow :\text{knows}^{+}(n, a_1), :\text{worksFor}(a_1, e), :\text{knows}(n, a_2), :\text{worksFor}(a_2, e)$$



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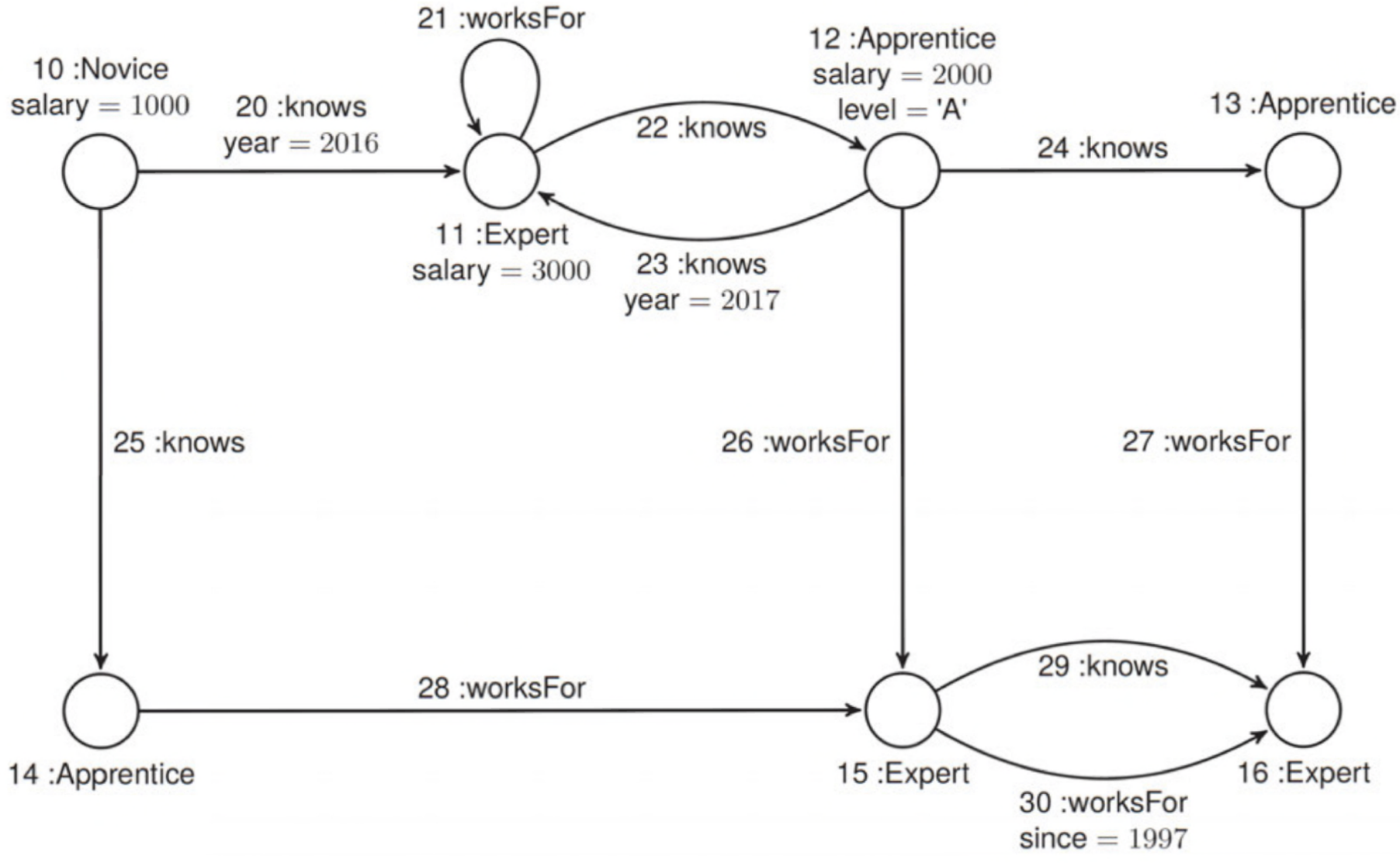
$n = \{10,11,12\}$

$a_1 = \{11,13,12,14\}$

$a_2 = \{14,11,12\}$

$e = \{11,16,15\}$



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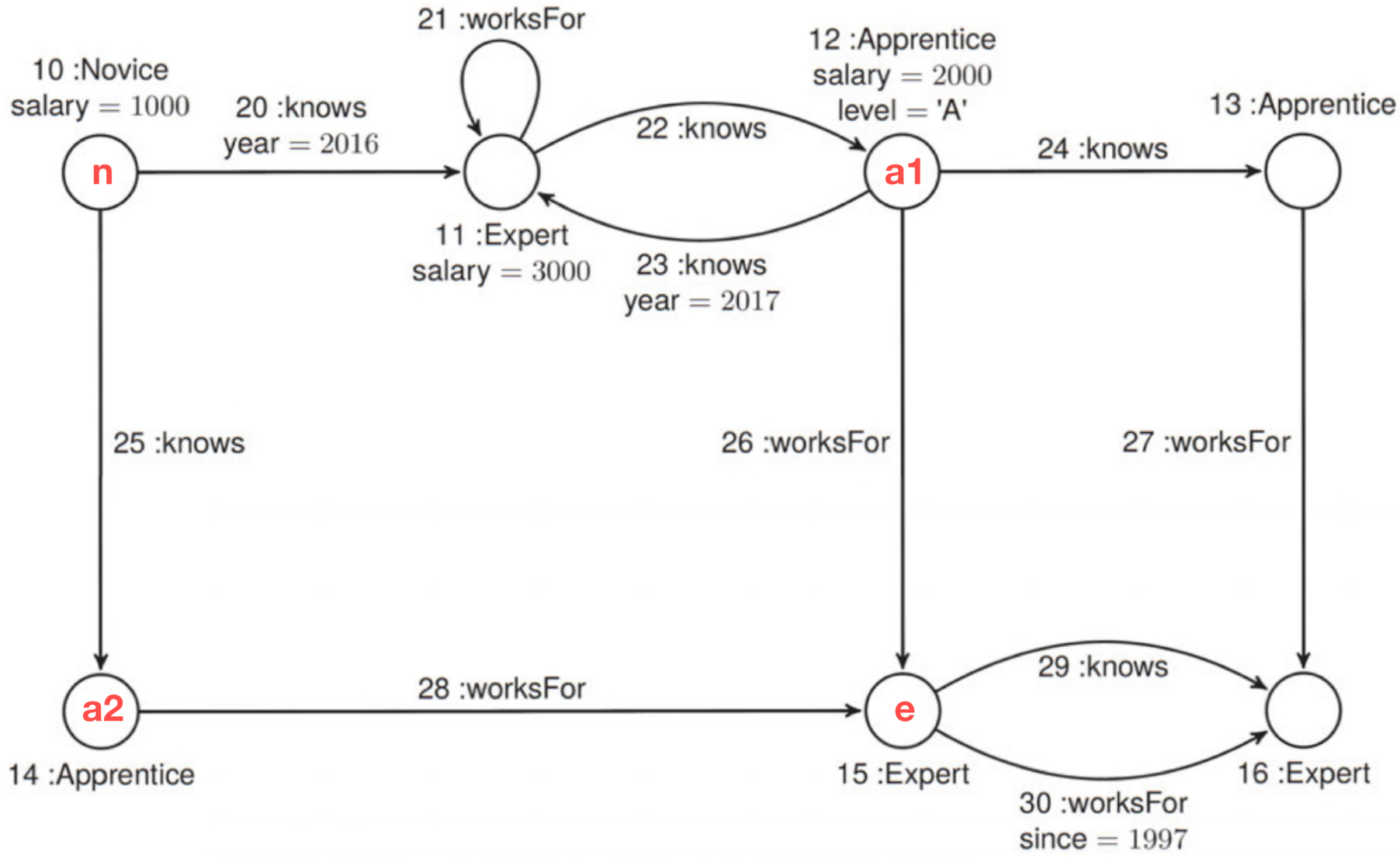

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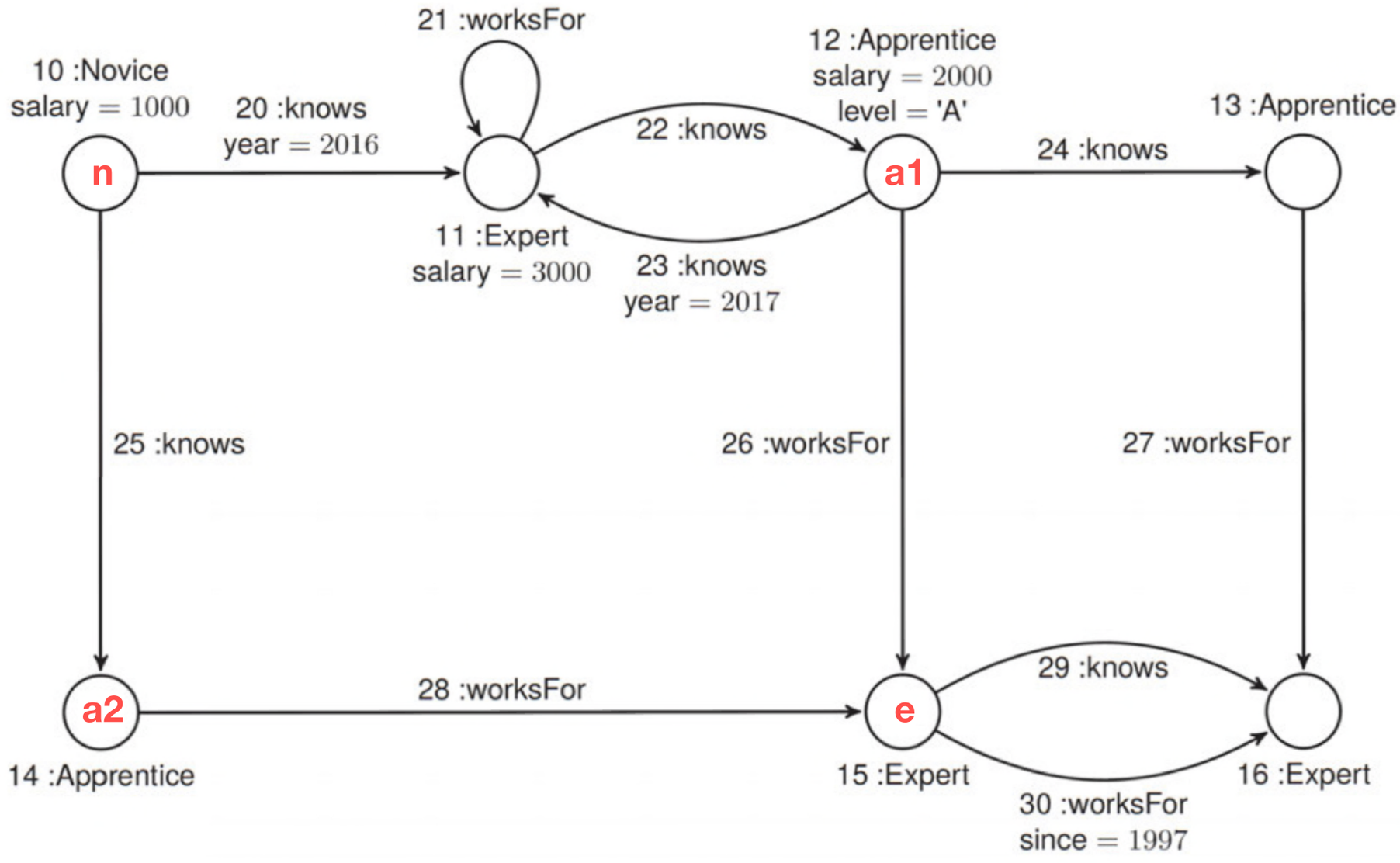

$[\![q]\!]_{G_{ex}} = \{(10,15)\}$

Union of CRP queries

Union of conjunctive regular queries (UCRPQ) is a finite non-empty set R ⊆ CRPQ. each element of which is of the same arity m

Given $R \in$ UCRPQ of arity m, the semantics of evaluating R over G is the m-ary relation $[\![R]\!]_G \subseteq V \times \ldots \times V$

defined as $[\![R]\!]_G = \bigcup_{r \in R} [\![r]\!]_G$

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Example

$(x) \leftarrow$:knows/:worksFor(x, y)

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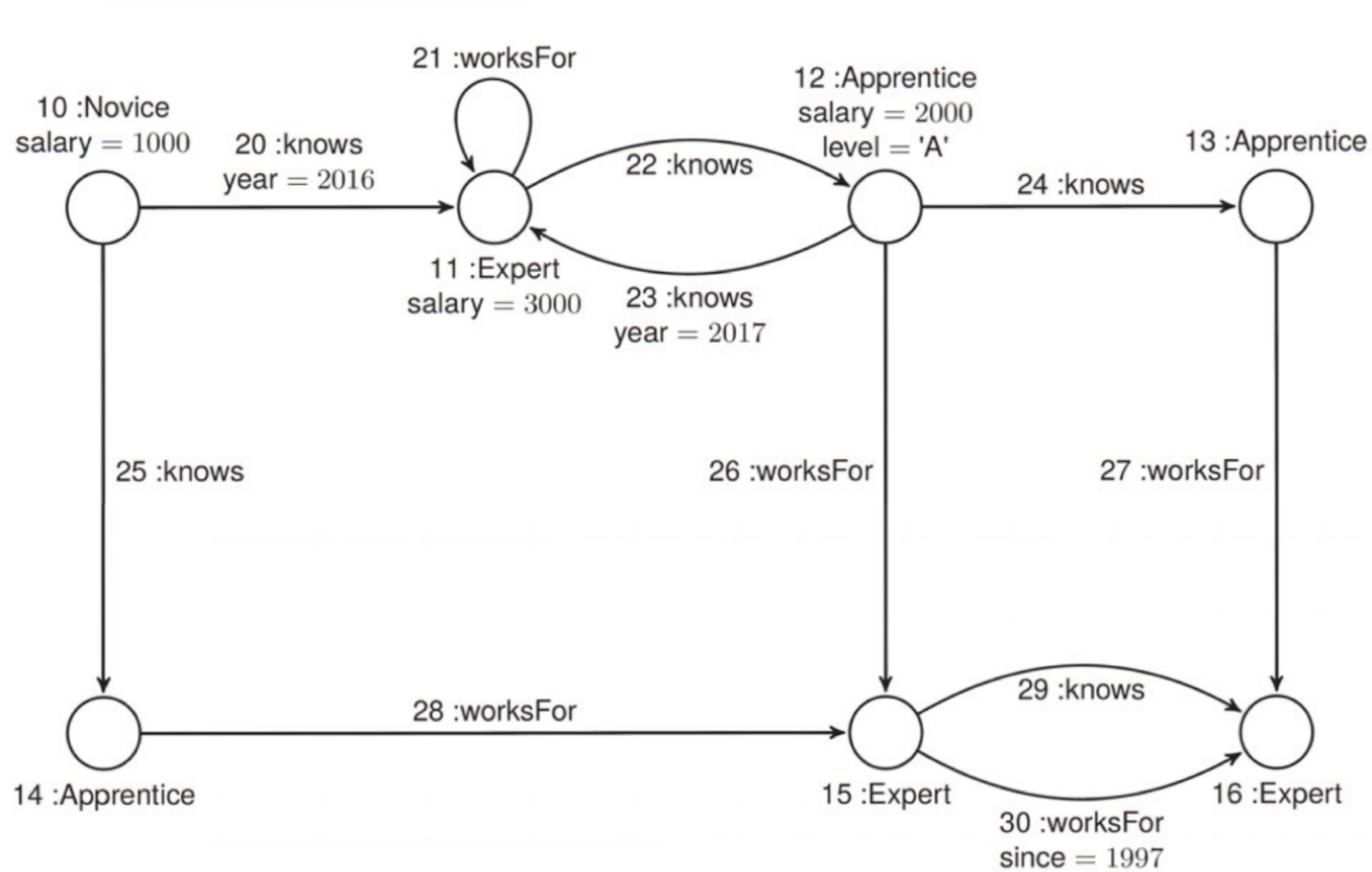

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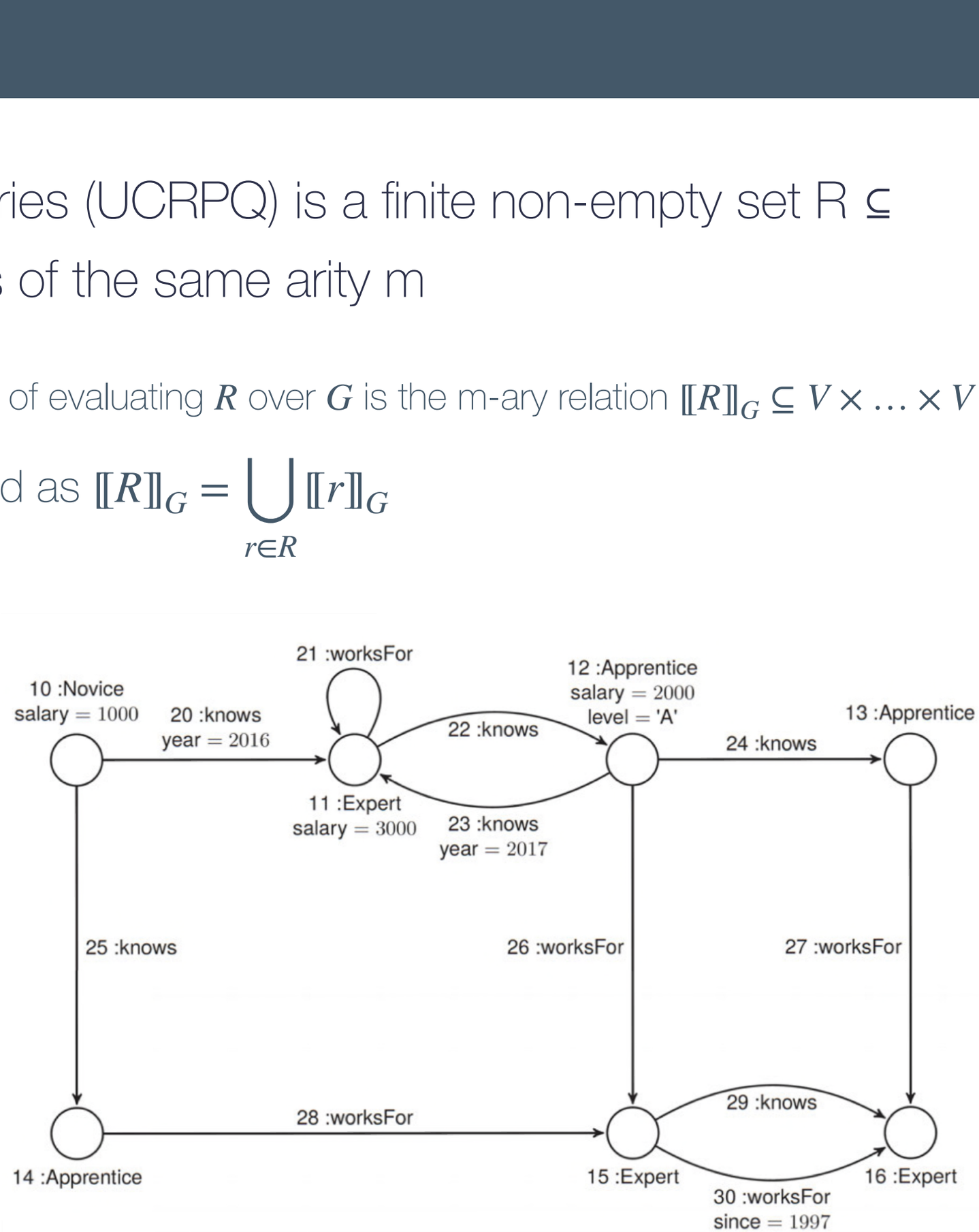

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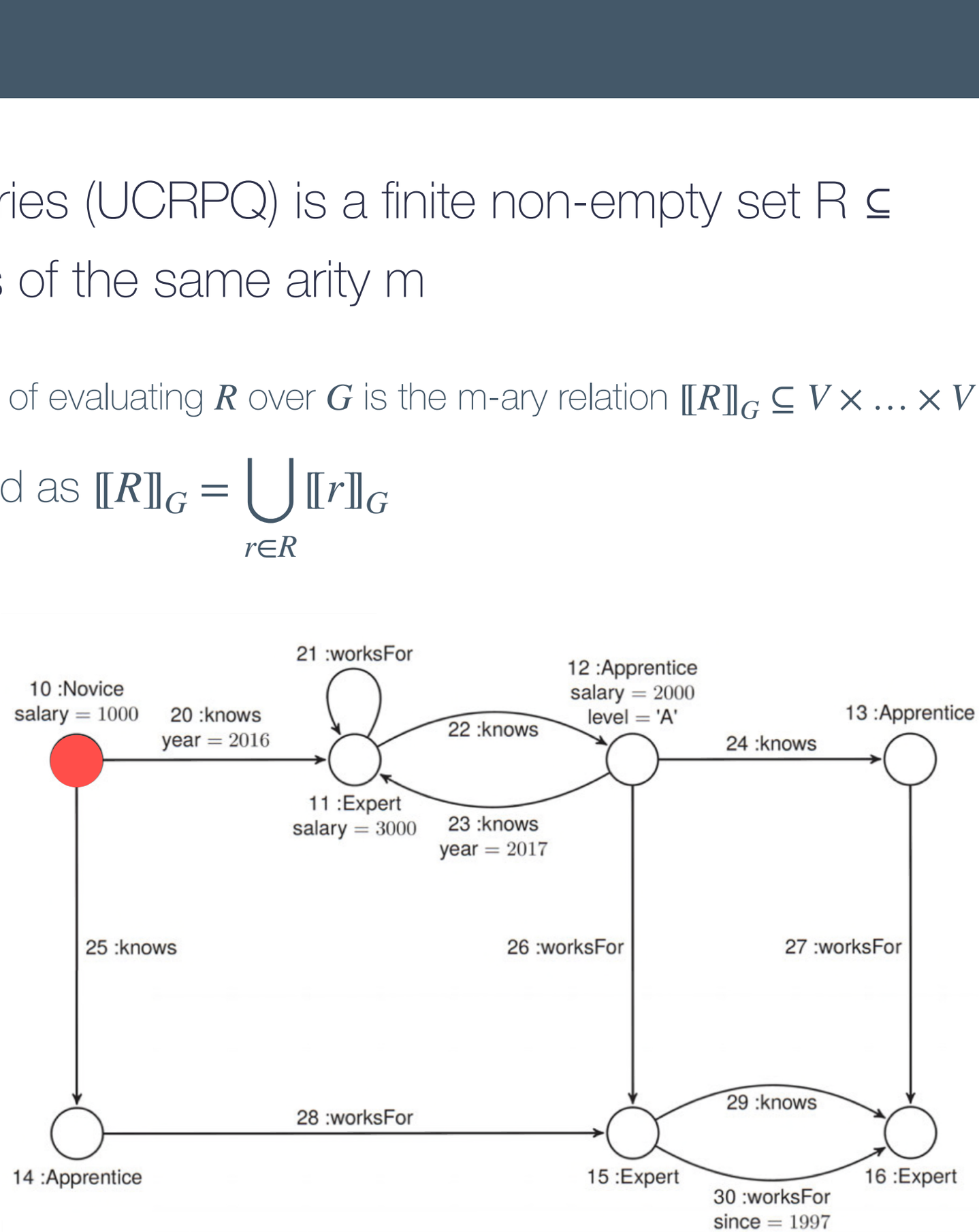

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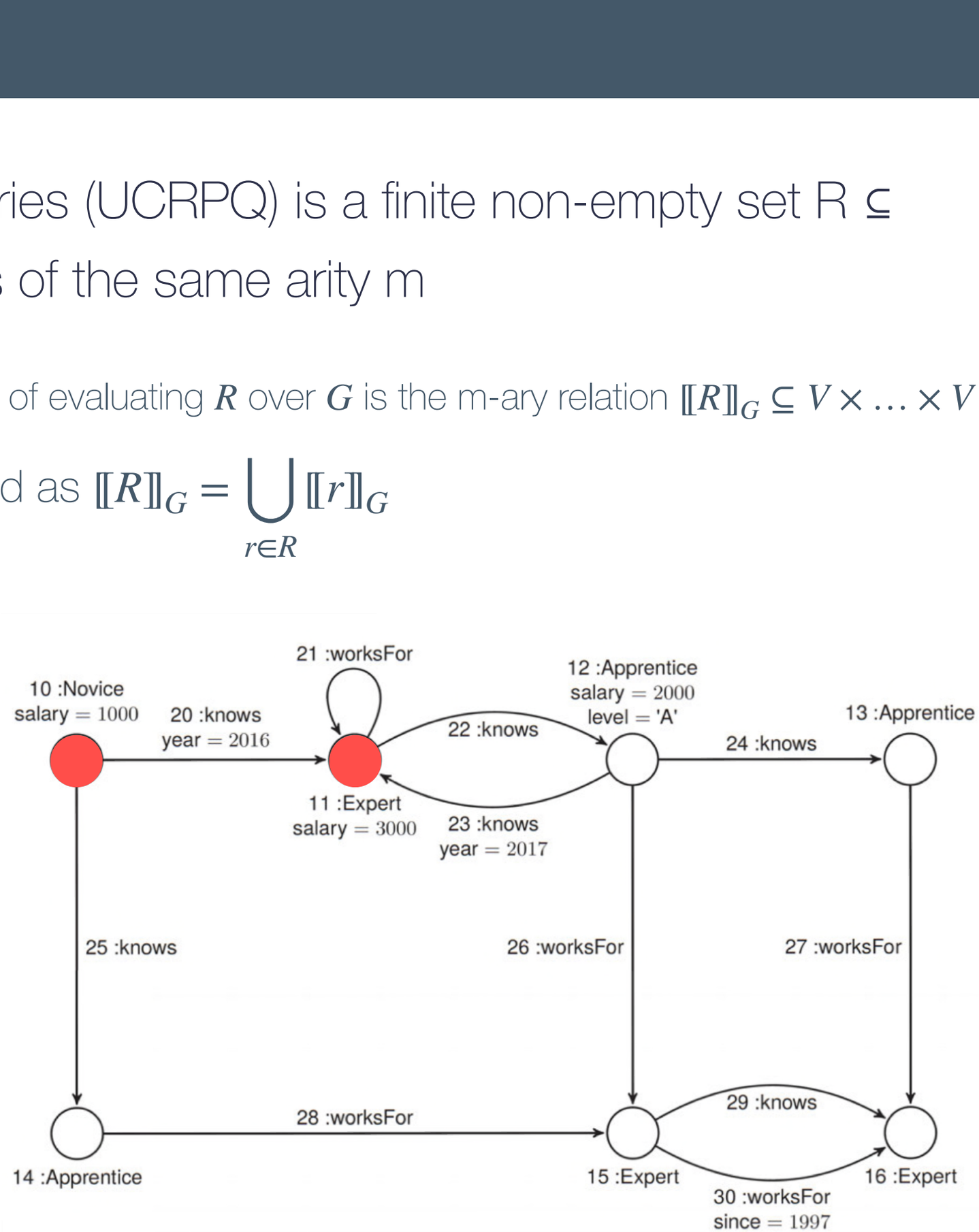

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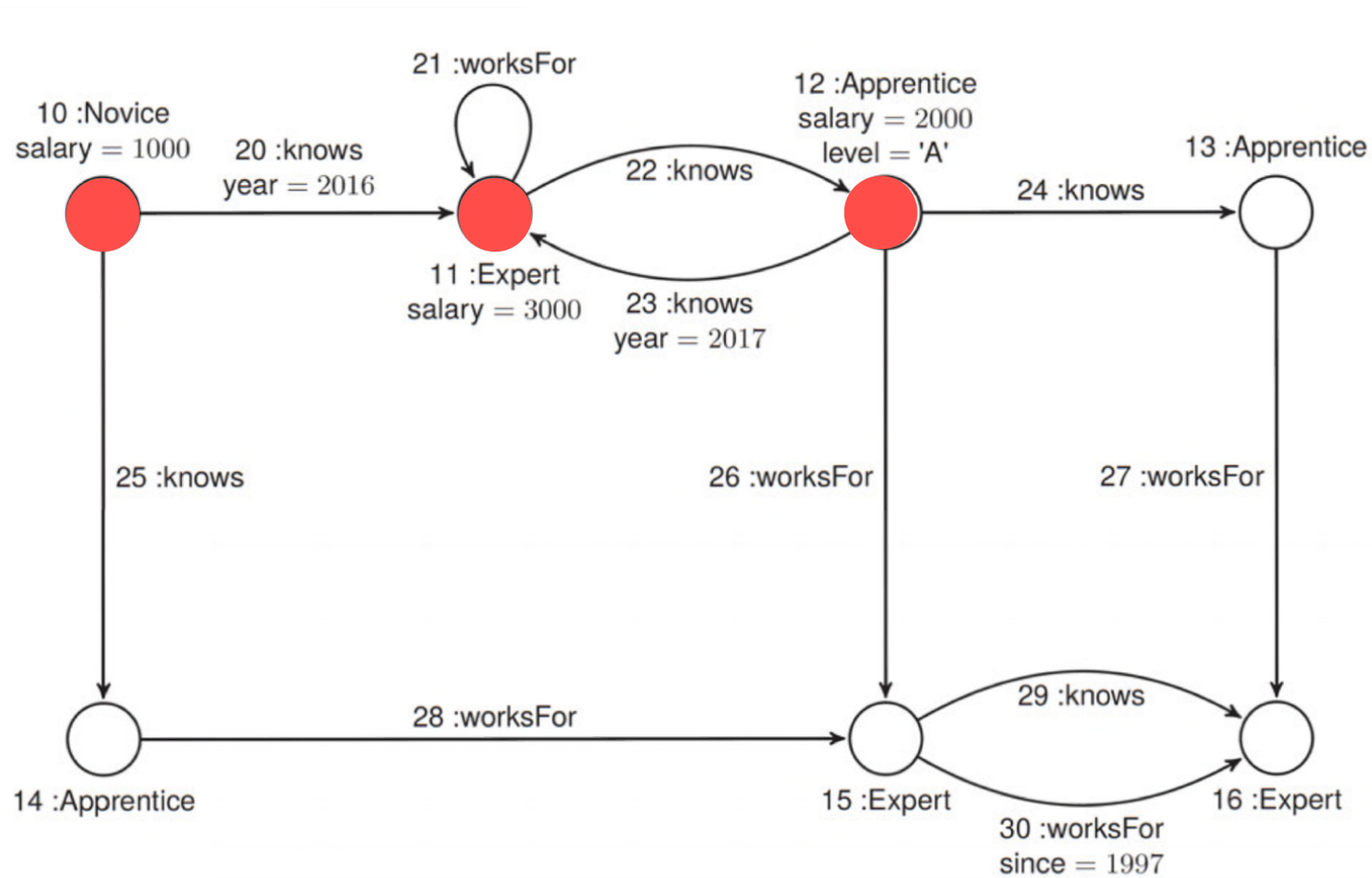

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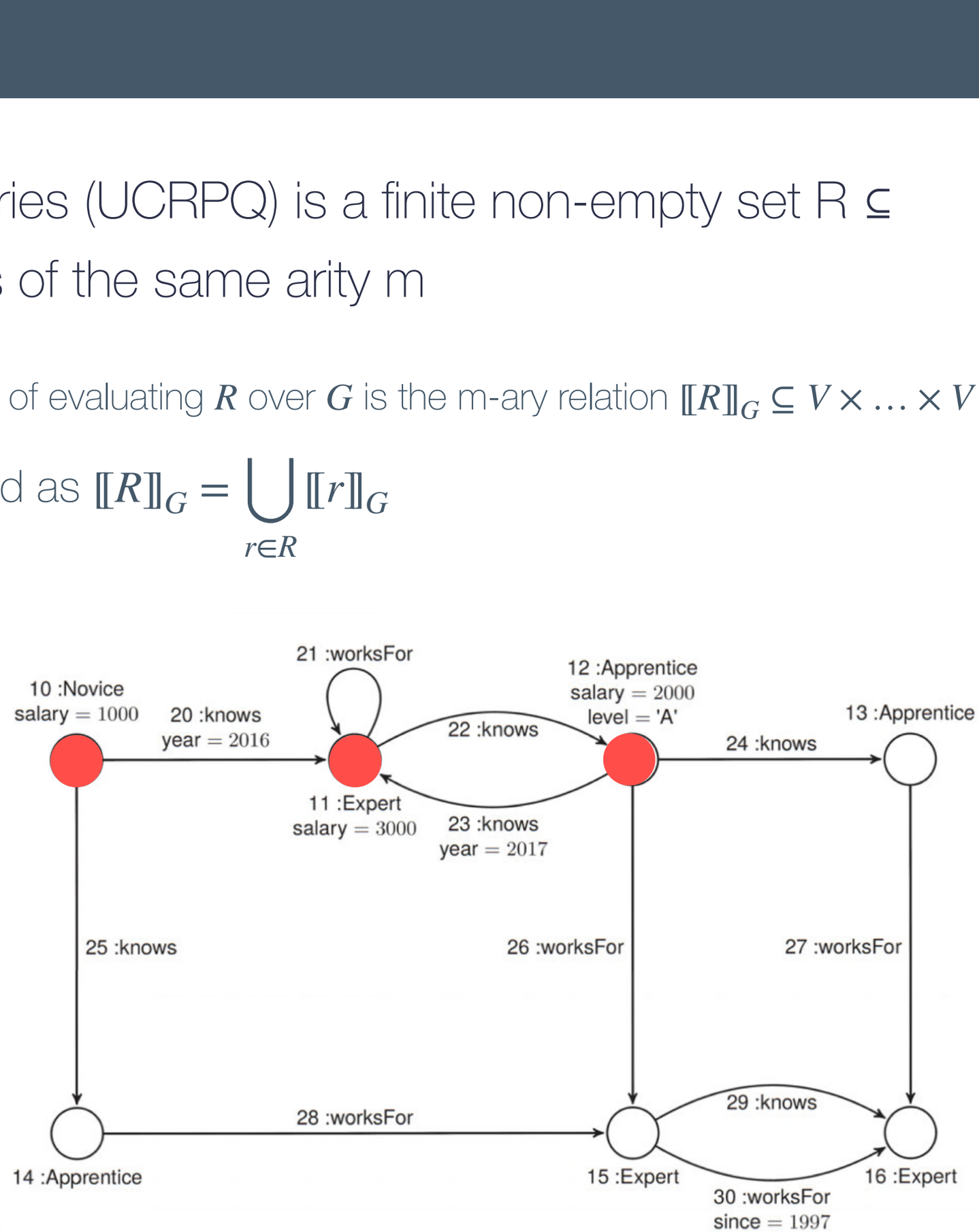
$x = \{10,11,12\}$



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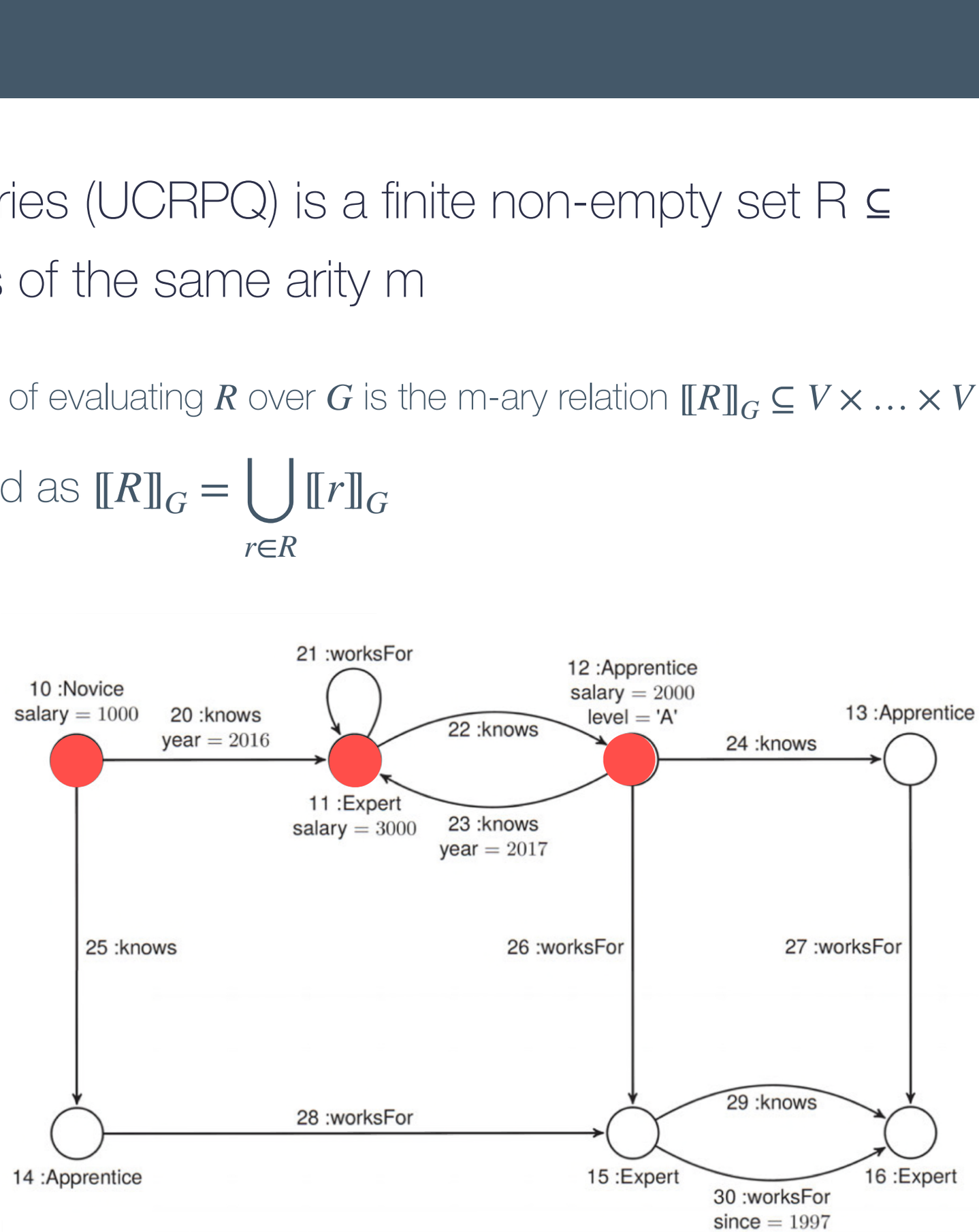

Union of CRP queries

Union of conjunctive regular queries (UCRPQ) is a finite non-empty set R ⊆ CRPQ. each element of which is of the same arity m

Given $R \in$ UCRPQ of arity m, the semantics of evaluating R over G is the m-ary relation $[\![R]\!]_G \subseteq V \times \ldots \times V$

defined as $[\![R]\!]_G = \bigcup_{r \in R} [\![r]\!]_G$

Example

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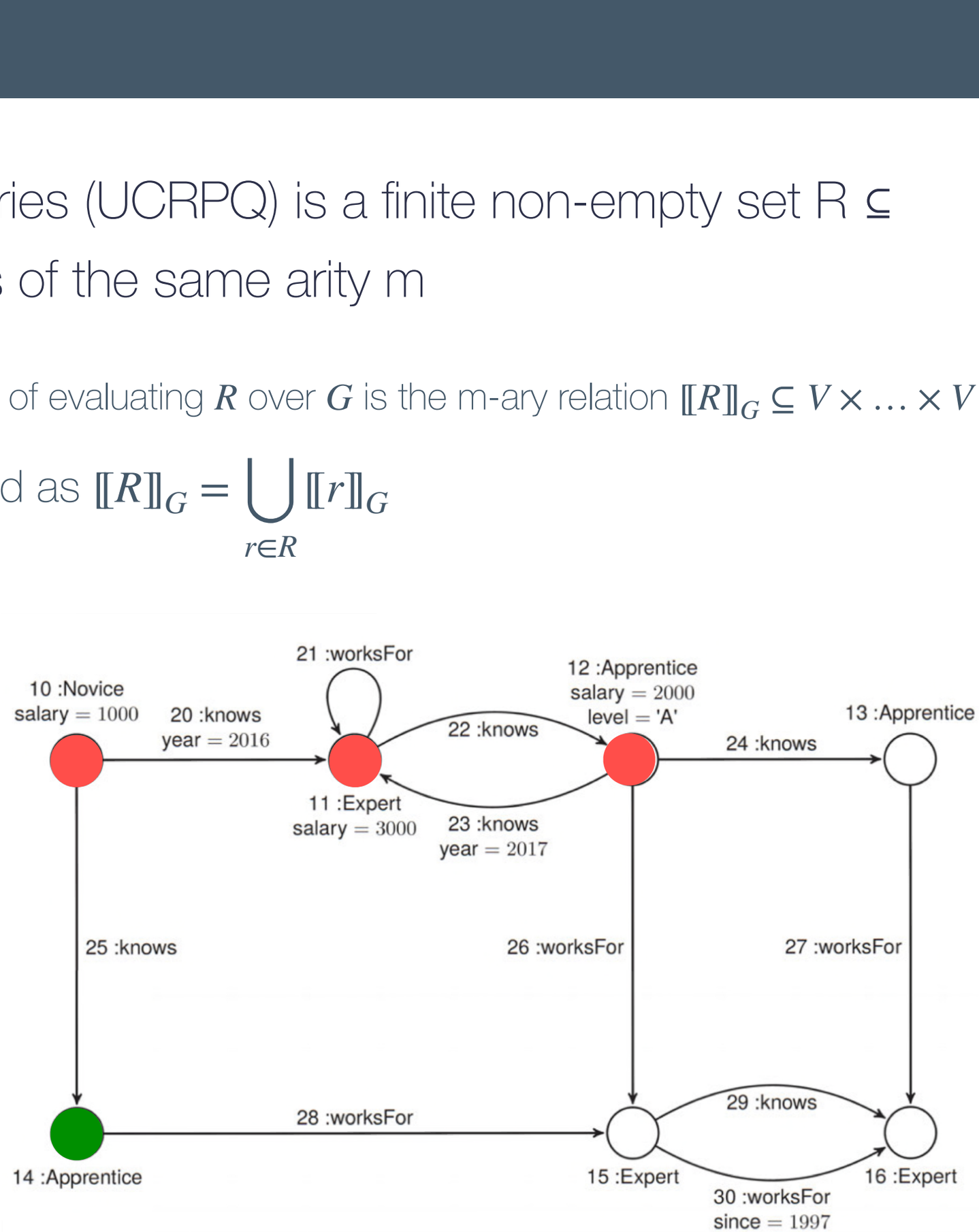

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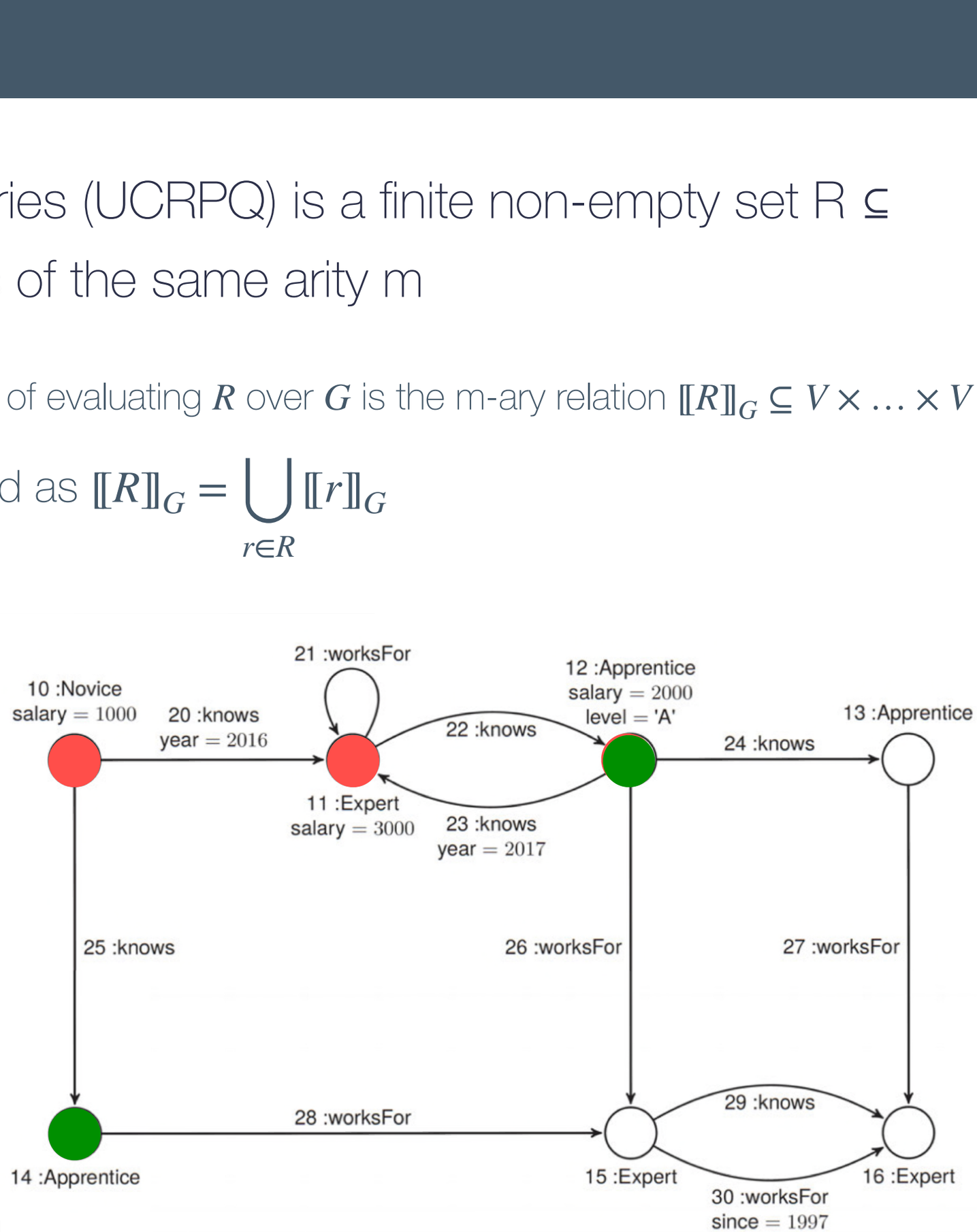

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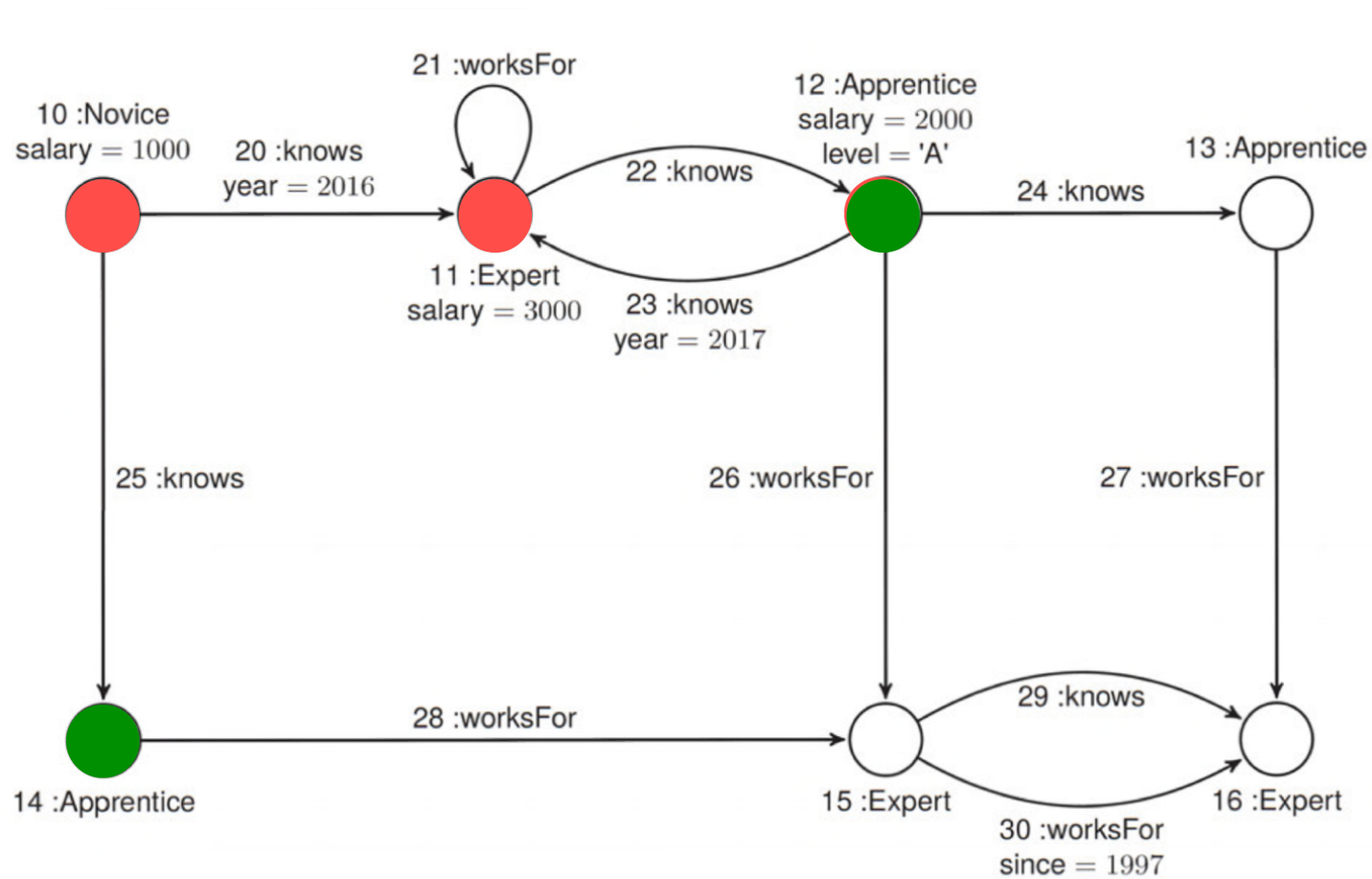

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Solution

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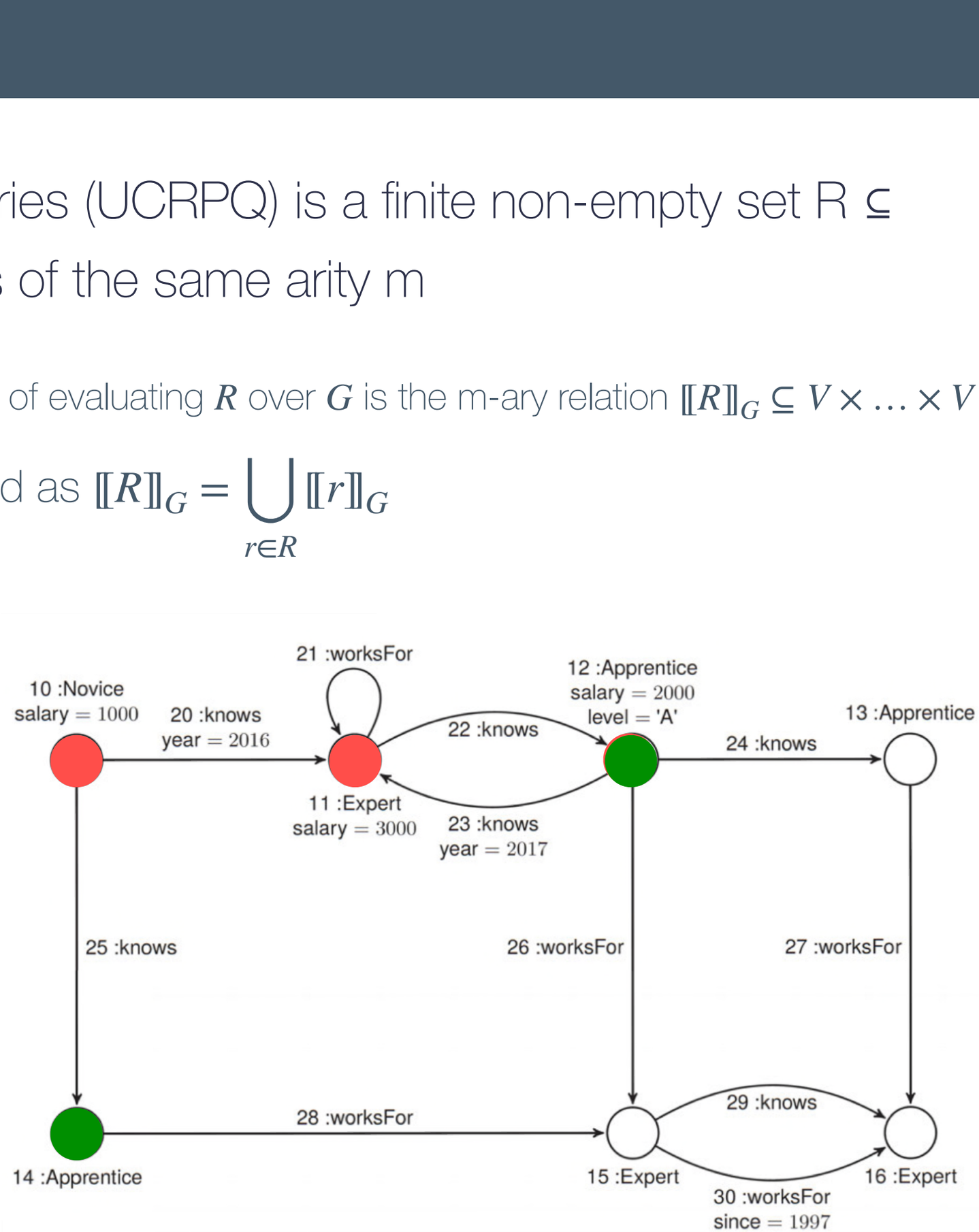
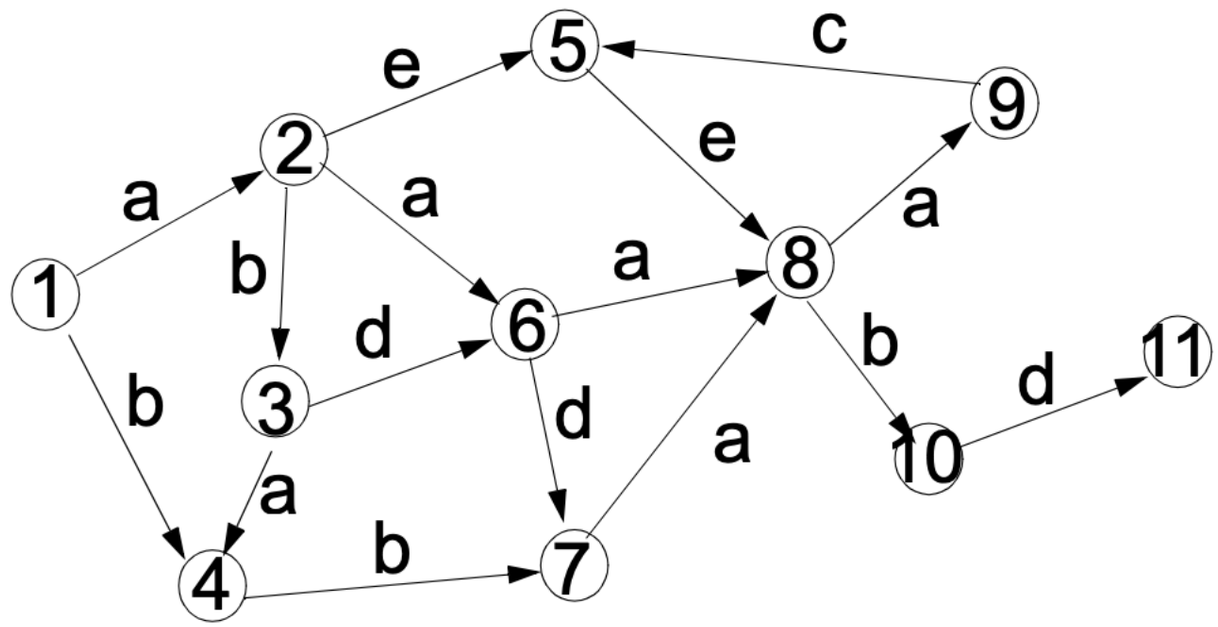


Another example of CRPQ

$$(x, y) \leftarrow a^+(x, y), e^+(x, y)$$



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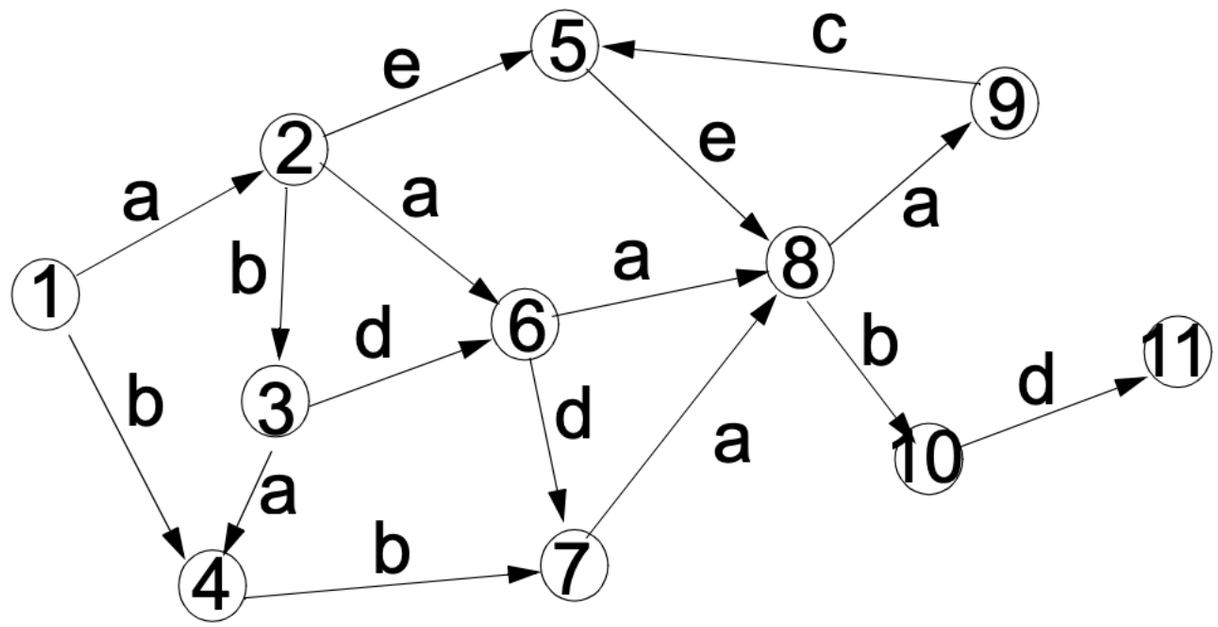

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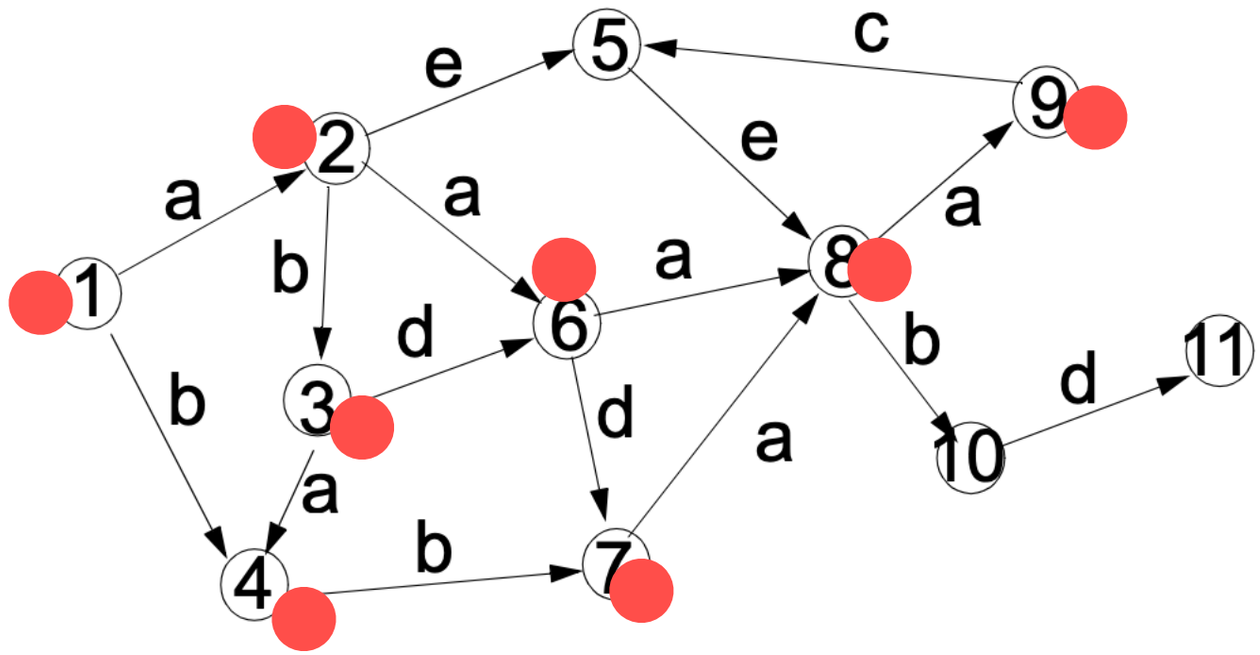

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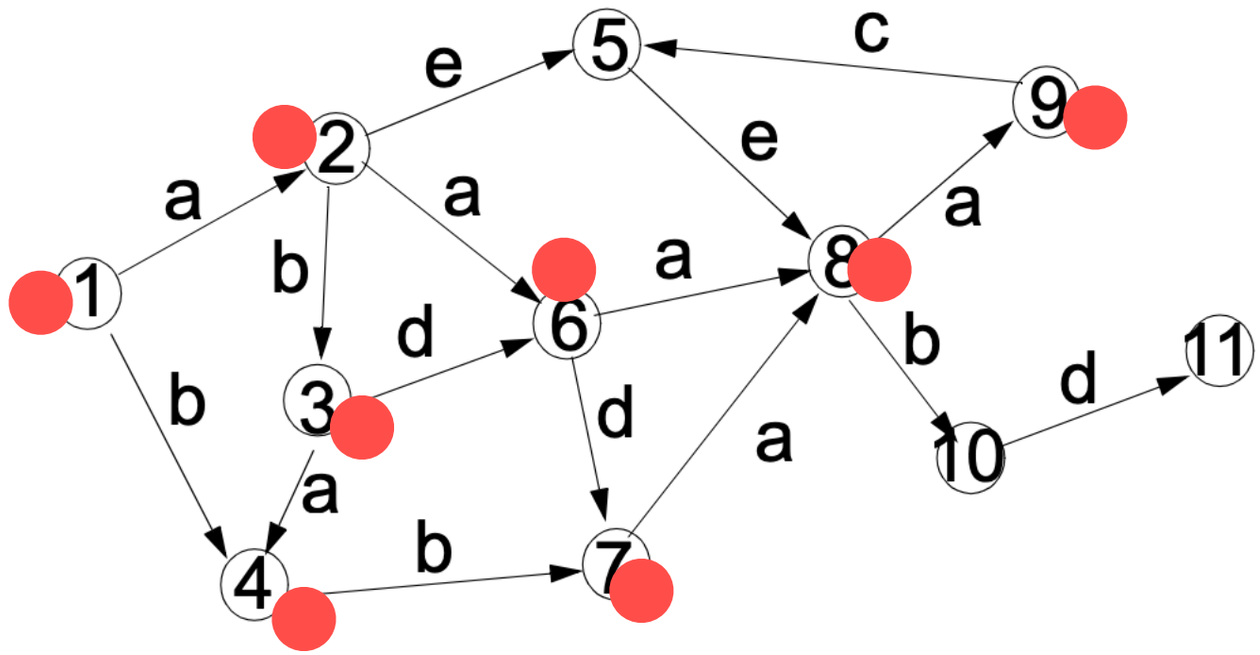

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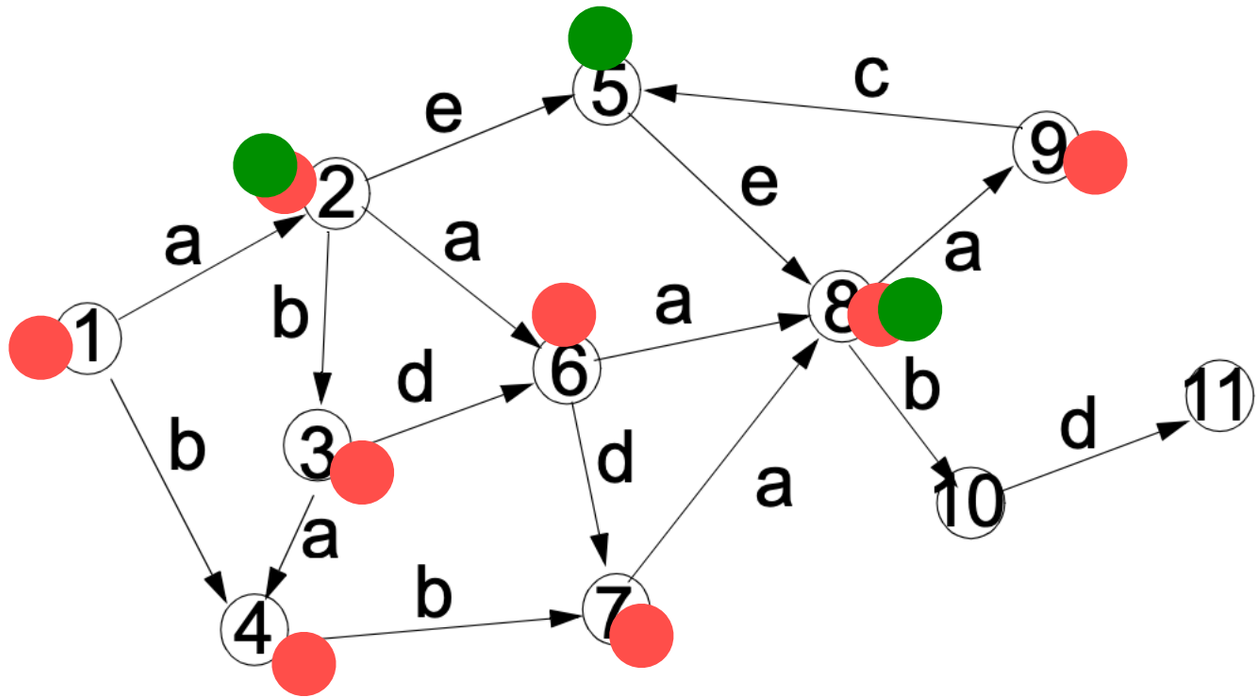

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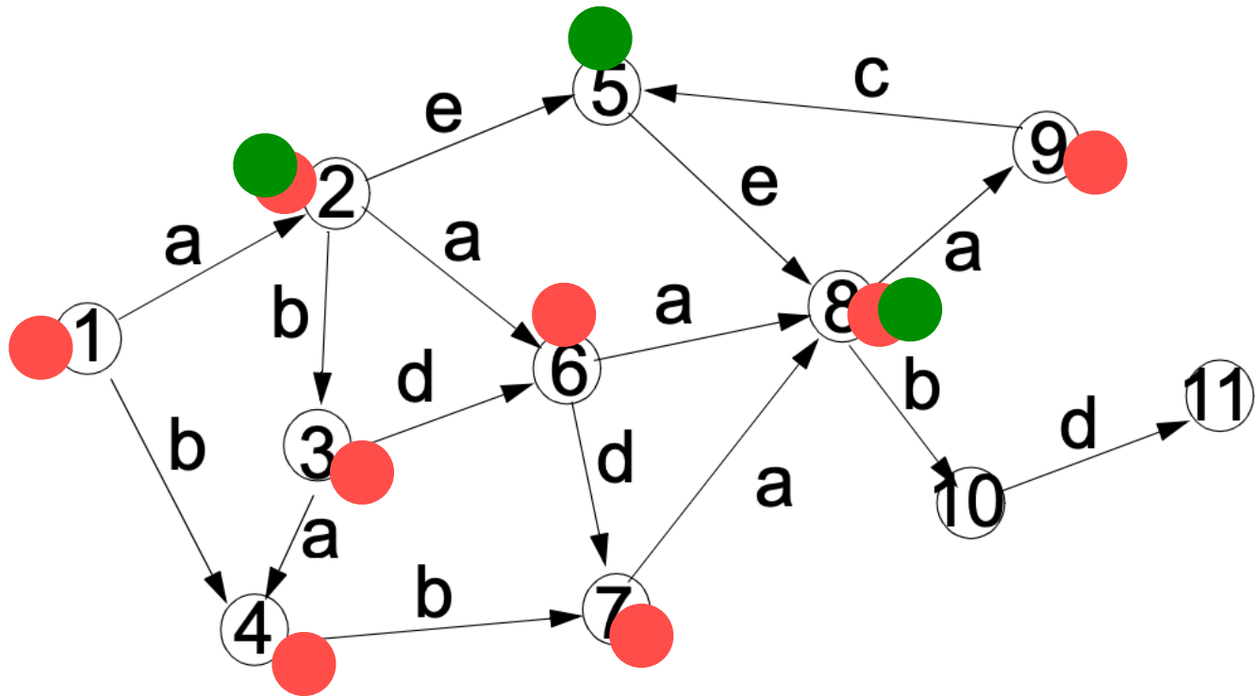

We need to take the intersection between the two sets

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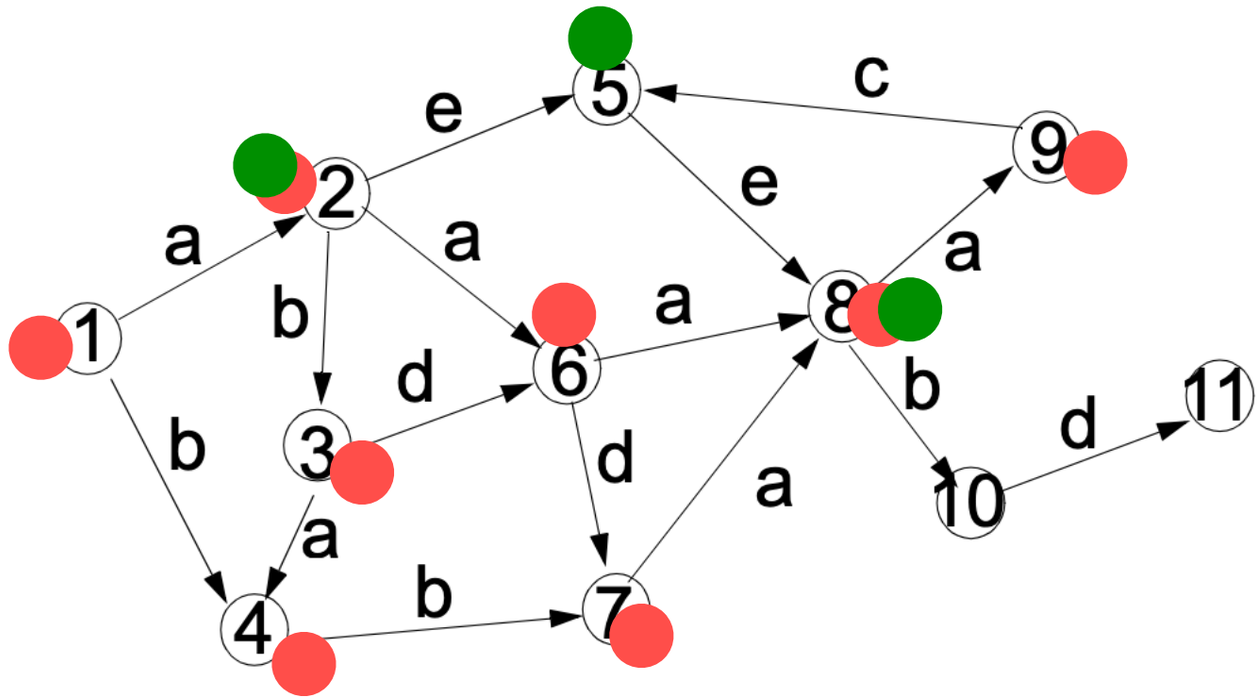
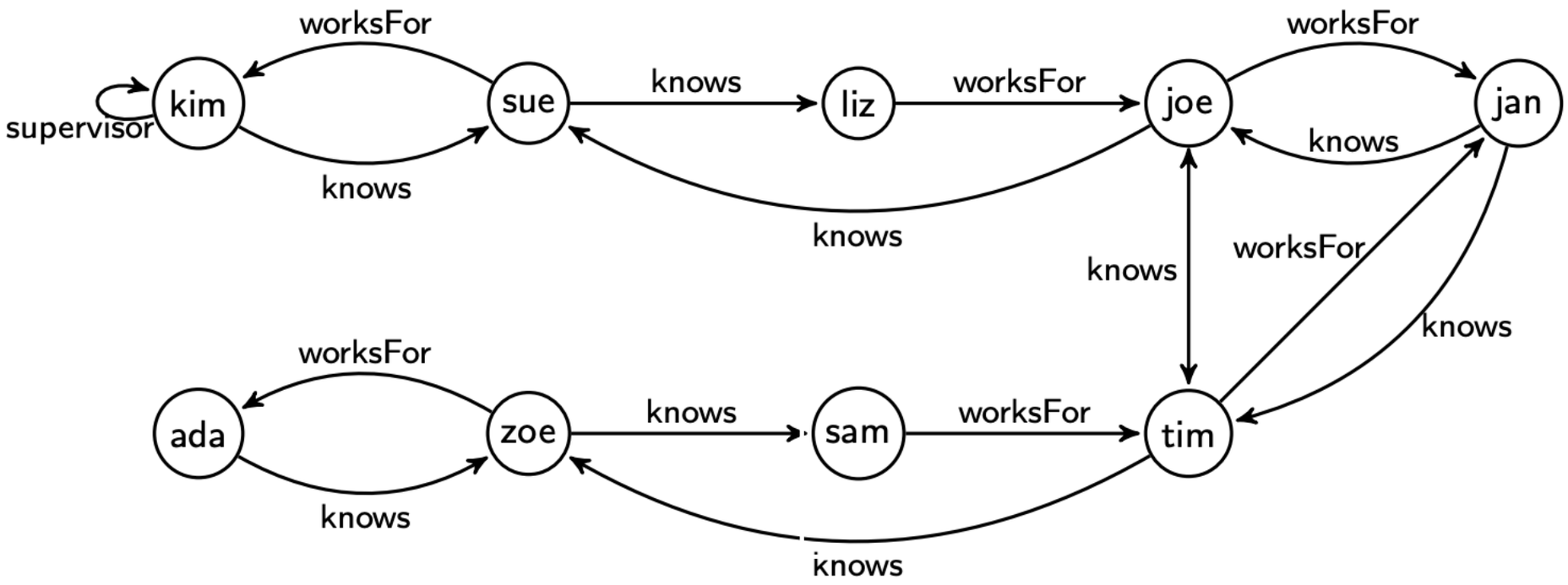


We need to take the intersection between the two sets

Solution: $(x, y) = (2,8)$

Exercise

$$(x, y) \leftarrow \text{supervisor}(x, z)/\text{worksFor}^{-}(x, y)$$



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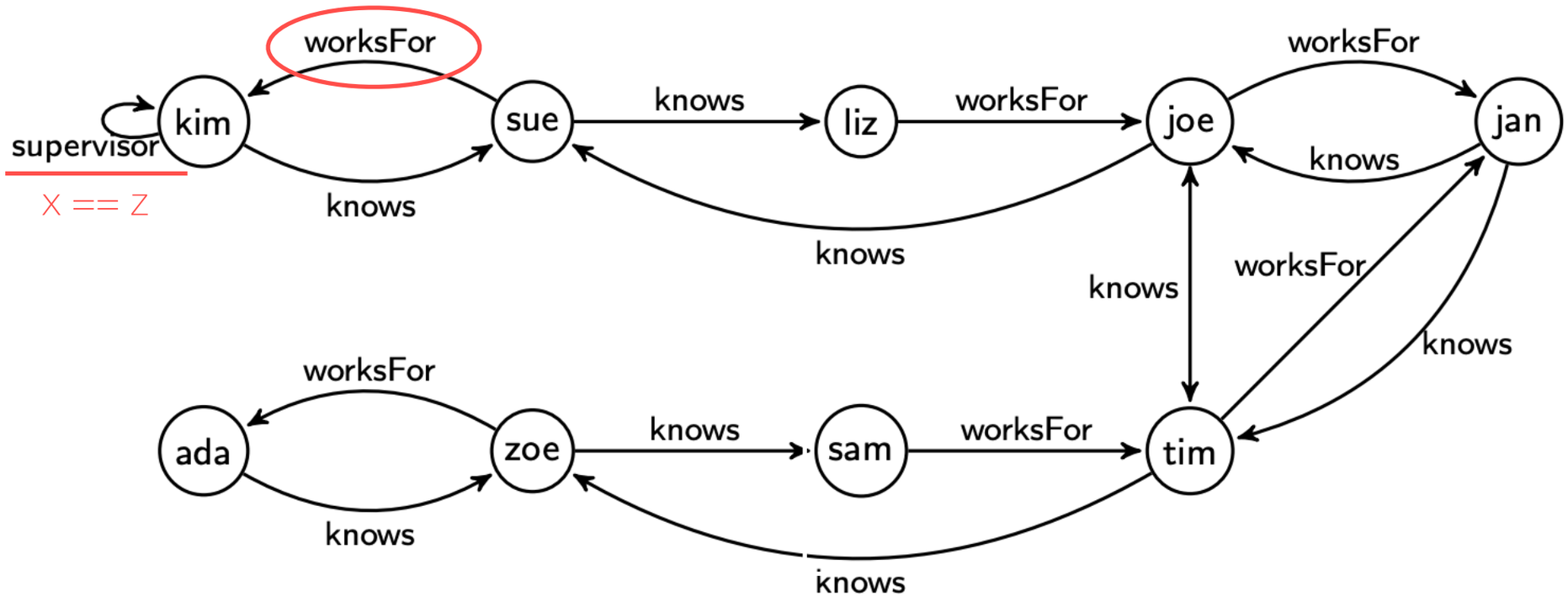

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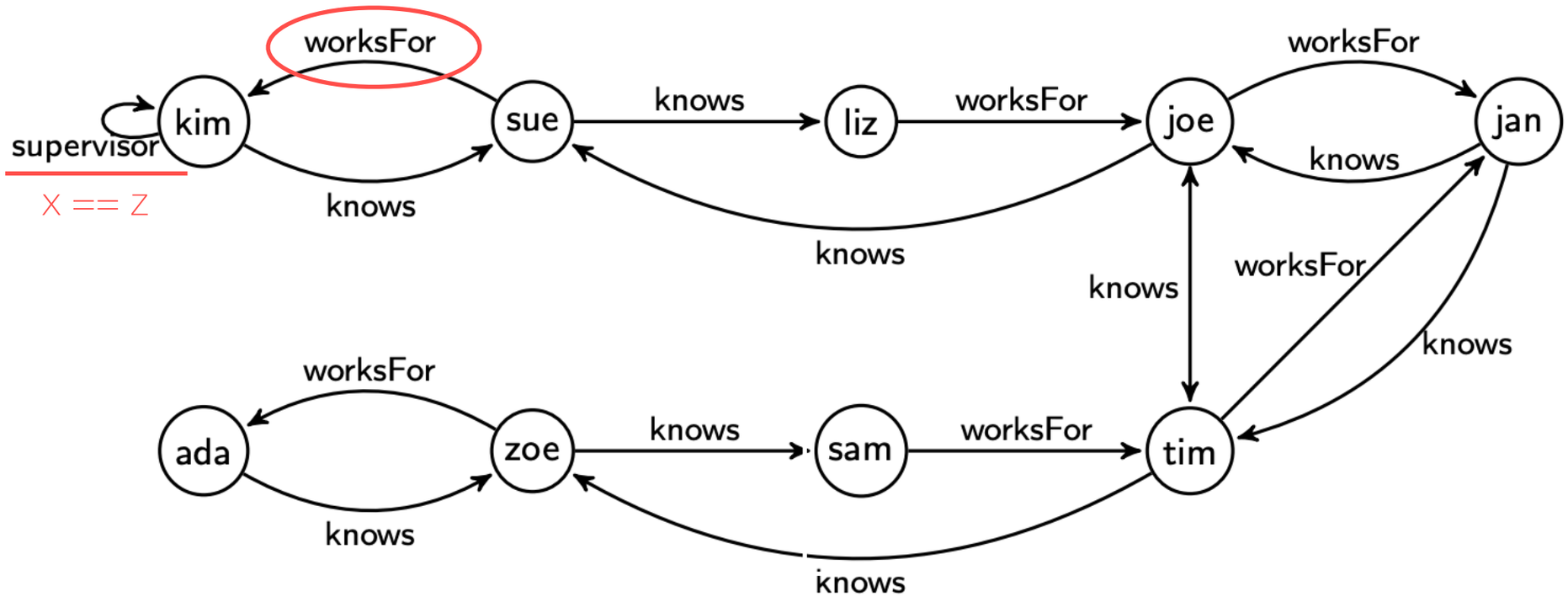


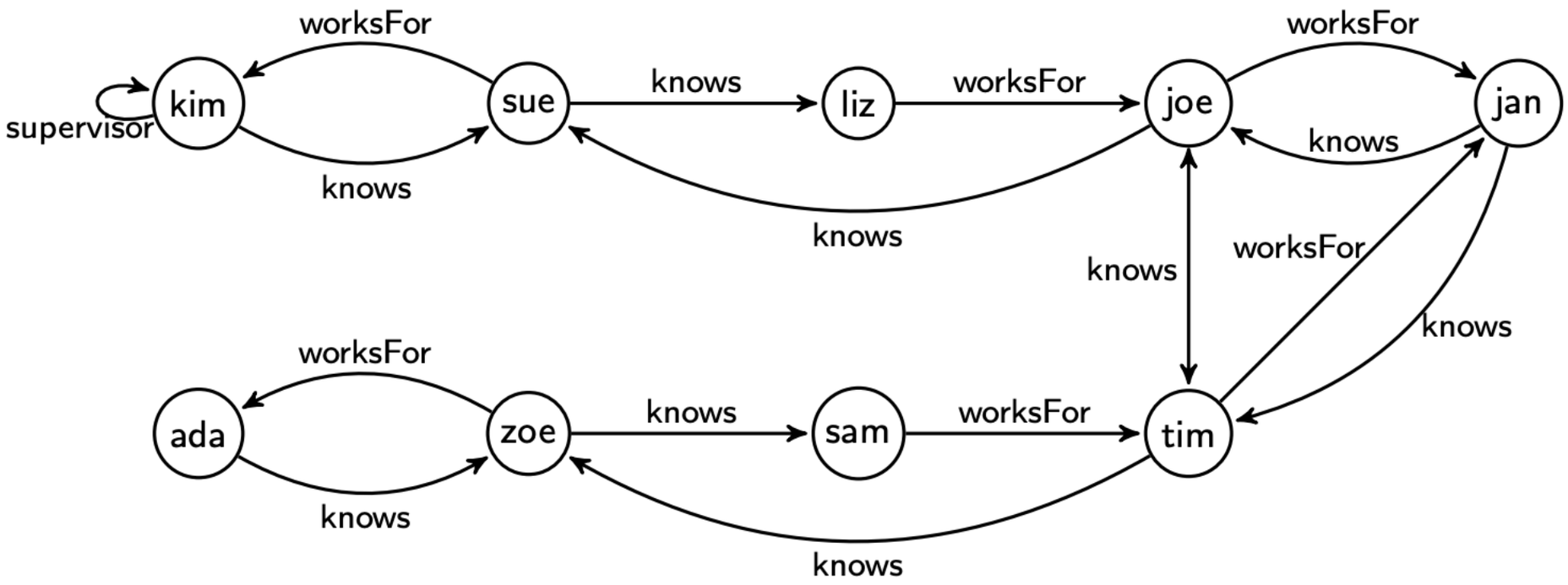
Solution

(x,y) = (kim,sue)

Exercise

Find two people that know the same person

$$(x, y) \leftarrow \text{knows}(x, z)/\text{knows}^{-}(z, y)$$



(x,y) = {(tim, ada), (kim,joe), (jan,tim), (joe,jan)} [note that all the reversed pairs are valid results too]